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Volume Title: Exchange Rates and International Macroeconomics

Volume Author/Editor: Jacob A. Frenkel, ed.

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-26250-2

Volume URL: <http://www.nber.org/books/fren83-1>

Publication Date: 1983

Chapter Title: Real Exchange Rate Overshooting and the Output Cost of Bringing Down Inflation: Some Further Results

Chapter Author: Willem H. Buiter, Marcus Miller

Chapter URL: <http://www.nber.org/chapters/c11384>

Chapter pages in book: (p. 317 - 368)

# Real Exchange Rate Overshooting and the Output Cost of Bringing down Inflation: Some Further Results

Willem H. Buiter and Marcus Miller

## 10.1 Introduction

The proposition that under a floating exchange rate regime restrictive monetary policy can lead to substantial “overshooting” of the nominal and real exchange rate is now accepted fairly widely. The fundamental reason is the presence of nominal stickiness or inertia in domestic factor and product markets combined with a freely flexible nominal exchange rate. Current and anticipated future monetary policy actions are reflected immediately in the nominal exchange rate, set as it is in a forward-looking efficient auction market, while they are reflected only gradually and with a lag in domestic nominal labor costs and/or goods prices. Nominal appreciation of the currency therefore amounts to real appreciation—a loss of competitiveness. Since in most of the simple analytical models used to analyze the overshooting propositions there is no long-run effect of monetary policy on the real exchange rate, any short-run real appreciation implies an overshooting of the long-run equilibrium. The transitory (but potentially quite persistent) loss of competitiveness is associated with a decline in output below its capacity level. This excess capacity is one of the channels through which restrictive monetary policy brings down the rate of domestic cost and price inflation.

One of the virtues claimed for the sharp initial appreciation of the nominal and real exchange rate in response to a previously unanticipated tightening of the stance of monetary policy is its immediate effect on the

Willem H. Buiter is a professor at the London School of Economics and a research associate of the National Bureau of Economic Research. Marcus Miller is a professor in the Department of Economics at the University of Warwick, Coventry.

This paper carries further the analysis contained in Buiter and Miller (1982). The authors have benefited from discussions with Avinash Dixit and Peter Burridge. They would also like to acknowledge financial support from the Leverhulme Trust and the Houblon-Norman Fund, respectively. Opinions expressed are those of the authors and not of the foundations mentioned or of the National Bureau of Economic Research.

domestic price level. The domestic currency prices of those internationally traded goods whose foreign currency prices can be treated as exogenous will decline by the same proportion as the increase in the value of the domestic currency. To a greater or lesser extent the same holds even for those internationally traded goods where home country demand or production is large in relation to the world market. Both through its effect on the prices of internationally traded final goods and through its effect on the prices of imported raw materials and intermediate inputs, a sudden step appreciation of the exchange rate will immediately bring down the domestic price level. In this paper we shall argue that the effect of such exchange rate jumps is merely to redistribute the cost of reducing inflation over time: early gains have to be "handed back" later as the equilibrium level of competitiveness is restored. Crucial to this argument is the assumption of stickiness of some nominal domestic cost component. In our model this is built in by our assumption of a predetermined nominal money wage and through our specification of the behavior of the "core" or underlying rate of inflation,  $\pi$ , the augmentation term in the wage equation. Subject to one quite significant qualification, the core rate of inflation is viewed as predetermined with its behavior over time governed by a first-order partial adjustment mechanism. It can be thought of as an adaptive expectations mechanism for the labor market, although we do not favor that interpretation. In our view the core rate of inflation, which is a distributed lag on current and past rates of inflation, stands for all the factors in the economy that give inertia to built-in trends in wages and prices. Also, while the level of the money wage is always treated as predetermined,  $\pi$ , which is determined by a "backward-looking" process, can make discrete jumps at a point in time. This will happen whenever there is a discrete jump in the general price level. In our model this can occur either if the exchange rate jumps or if there is a change in indirect taxes. Since the exchange rate is a forward-looking price which responds to news about current and future shocks, the underlying rate of inflation indirectly and to a limited extent also responds to such shocks.

We start the paper with a brief review of an earlier paper of ours on the same subject (Buiter and Miller 1981a). This is done in section 10.2 where a simple model of real exchange rate overshooting is discussed. Section 10.3 contains a discussion of the consequences of modifying the wage-price process of the simple model in a number of ways. It is here that we analyze the implications of assuming flexibility of domestic nominal wage costs. While we do not believe that such a "classical" specification is appropriate for the analysis of a mature industrial economy like the United Kingdom, the discussion of this case helps bring out the crucial nature of the assumption of nominal inertia in the behavior of domestic costs. Section 10.4 analyzes in some detail the behavior of the model with sluggish "core" inflation. In section 10.5 we maintain the wage-price

block of section 10.4 but generalize the model in a number of other directions. The long-run real interest rate is substituted for the short-run real interest rate in the IS equation; the expected rate of inflation becomes an argument in the money demand and output demand functions; wealth effects on money demand and output demand are introduced; and wealth adjustment via current account deficits and surpluses is incorporated in the model. Finally, gradual rather than instantaneous adjustment of the level of output is considered. The real exchange rate overshooting proposition survives all these modifications. Section 10.6 contains a discussion of alternative policy combinations for bringing down inflation.

## 10.2 A Simple Model of Real Exchange Rate Overshooting

A slightly simplified version of the model in Buiter and Miller (1981a) is given in equations (1)–(5). All variables except for  $r$ ,  $r_d$ ,  $r^*$ ,  $\pi$ ,  $\theta$ , and  $\tau$  are in logs.

$$(1) \quad m - p - \theta = ky - \lambda(r - r_d), \quad k, \lambda > 0.$$

$$(2) \quad y = -\gamma(r - Dp - D\theta) + \delta(e + p^* - p), \quad \gamma, \delta > 0.$$

$$(3) \quad Dp = \phi y + \pi, \quad \phi > 0.$$

$$(4) \quad \pi = D^+ m.$$

$$(5) \quad De = r - r^* - \tau.$$

The notation is as follows:

- $m$ : nominal money stock (exogenous)
- $p$ : domestic price level "at factor cost," i.e., excluding indirect taxes (predetermined)
- $p^*$ : foreign price level (exogenous)
- $y$ : real output (endogenous)
- $r$ : domestic nominal interest rate on nonmoney assets (endogenous)
- $r_d$ : nominal interest rate paid on domestic money (exogenous)
- $r^*$ : foreign nominal interest rate paid on nonmoney assets (exogenous)
- $\theta$ : rate of indirect tax (exogenous)
- $e$ : exchange rate (domestic currency price of foreign currency) (endogenous)
- $\pi$ : trend or core rate of inflation (endogenous)
- $\tau$ : rate of tax on capital inflows or subsidy on outflows (exogenous)
- $D$ : differential operator, i.e.,  $Dx(t) \equiv (d/dt)x(t)$
- $D^+$ : right-hand side differential operator, i.e.,

$$D^+ x(t) = \lim_{\substack{T \rightarrow t \\ T > t}} \left( \frac{x(T) - x(t)}{T - t} \right)$$

Equation (1) is the LM curve:  $m$  denotes a fairly wide monetary aggregate such as £M3 which consists, to a significant extent (50–60 percent), of interest-bearing deposits. We therefore measure the opportunity cost of holding money by the interest differential between the loan rate,  $r$ , and the own rate on time deposits ( $r_d$ ). Equation (2) is the IS curve. Demand for domestic output depends on the short real interest rate and on the relative price of foreign and domestic goods. The country is small in the world market for its importables so that it takes  $p^*$  as given. It is large in the world market for its exportables. No explicit distinction is made between traded and nontraded goods. Equation (3) is the augmented Phillips curve. By choice of units (the logarithm of) capacity output is set equal to zero. The augmentation term  $\pi$  is identified, in (4), with the right-hand-side time derivative of the money supply. Thus even if  $m$  were to make a discrete jump, the price level would not jump. This is one way of imposing the crucial property of nominal inertia, stickiness, or sluggishness. Equation (5) reflects the assumption of perfect capital mobility and perfect substitutability between domestic and foreign bonds. Risk-neutral speculators equate the uncovered interest differential in favor of the domestic country, net of any tax on capital imports, to the expected rate of depreciation of the domestic currency. The country is small in the world financial markets and  $r^*$  is treated as given. The assumption of rational expectations, employed in Dornbusch (1976), Liviatan (1980), and Buiter and Miller (1981a) is equivalent to perfect foresight in our deterministic model. It is used in equations (2) and (5). For simplicity the foreign price level,  $p^*$ , is assumed to be constant; choice of units sets it equal to zero, so competitiveness is measured by  $e - p$ .

The own rate of interest on money is assumed exogenous. In a competitive banking system with a binding required reserve ratio  $h$ , ( $0 < h < 1$ ) on all bank deposits, the loan rate  $r$  and the deposit rate  $r_d$  are linked by:  $r_d = (1 - h)r(TD + DD)TD^{-1}$ . TD is the volume of interest-bearing time deposits and DD the volume of noninterest-bearing demand deposits. If demand deposits are only a small fraction of the total, then  $r_d \approx (1 - h)r$ . This can be used to eliminate  $r_d$  from the model. The main consequence is to reduce the interest sensitivity of money demand. We prefer treating  $r_d$  as exogenous so that discretionary changes in  $r_d$  can be used to describe policy actions to alter the degree of competitiveness of the banking system. The dynamics of the system are conveniently summarized in terms of the two state variables  $\ell$  and  $c$ :

$$(6a) \quad \ell \equiv m - p.$$

$$(6b) \quad c \equiv e - p.$$

Real liquidity,  $\ell$ , is a backward-looking or predetermined variable. It only makes discrete jumps when the policy instrument  $m$  changes discon-

tinuously. Real competitiveness  $c$  is a forward-looking or jump variable. It jumps whenever  $e$  jumps. The state-space representation of the model of equations (1)–(6) is:<sup>1</sup>

$$\begin{aligned}
 (7) \quad \begin{bmatrix} D & \ell \\ D & c \end{bmatrix} &= \frac{1}{\gamma(\phi\lambda - k) - \lambda} \begin{bmatrix} \phi\gamma & \phi\lambda\delta \\ 1 & \delta(\phi\lambda - k) \end{bmatrix} \begin{bmatrix} \ell \\ c \end{bmatrix} \\
 &+ \frac{1}{\gamma(\phi\lambda - k) - \lambda} \begin{bmatrix} \phi\lambda\gamma & 0 \\ \lambda & -\gamma(\phi\lambda - k) + \lambda \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix} \\
 &\quad \begin{bmatrix} -\phi\gamma & -\phi\gamma\lambda \\ -1 & -\lambda \end{bmatrix}
 \end{aligned}$$

A necessary and sufficient condition for the stationary equilibrium of this model to be a saddlepoint is  $\gamma(\phi\lambda - k) - \lambda < 0$ .<sup>2</sup> This is equivalent to the condition that, at a given real exchange rate, an exogenous increase in aggregate demand raises output.

It is easily verified that the long-run equilibrium has the following properties. Output is equal to its full employment value, 0. The steady-state real interest rate,  $r - Dp$  equals  $r^* - Dp^* + \tau = r^* + \tau$ , since we assume that the foreign rate of inflation is zero. The nominal interest rate  $r$  equals  $r^* + \tau + De = r^* + \tau + Dp - Dp^* = r^* + \tau + Dm - Dp^*$ . Long-run competitiveness is independent of  $Dm$ ,  $\theta$ , and  $r_d$  but improves when  $r^* + \tau$  increases. The steady-state stock of real money balances  $\ell$  decreases when  $Dm$  or  $r^* + \tau$  increases, but increases when  $\theta$  or  $r_d$  increases.

The immediate response of the economy to a variety of policy actions is as follows. Overshooting by the real exchange rate of its long-run equilibrium value occurs in the model in response to (a) previously unantici-

1. For simplicity, the term  $D\theta$  in equation (2) has been ignored. We consider it in section 10.6.

2. The equilibrium is a saddlepoint if the state matrix has one stable and one unstable characteristic root. A necessary and sufficient condition for this is that the determinant of the state matrix be negative.

pated immediate or future announced reductions in either the level or the rate of growth of the money stock, (b) previously unanticipated immediate or future announced increases in indirect taxes ( $\theta$ ), and (c) previously unanticipated current or future increases in the own rate on money,  $r_d$ . All three kinds of shocks can be argued to have jolted the British economy in the two years after mid-1979 (Buiter and Miller 1981a, b).

### 10.3 Real Exchange Rate Overshooting and the Wage-Price Process

A crucial component of all models exhibiting disequilibrium overshooting of the real exchange rate is the wage-price process. The price equation used in this paper so far, as in many others (e.g., Buiter and Miller 1981a; Dornbusch 1976), has a number of weaknesses. It is important to perform a "sensitivity analysis" of the specification of this equation to establish the robustness of the overshooting proposition.

#### 10.3.1 A Direct Effect of the Exchange Rate on the Domestic Price Level

Even if domestic wage costs are sticky in nominal terms, so that the money wage rate,  $w$ , can be treated as predetermined, the domestic price level might in an open economy still be capable of making discrete jumps at a point in time. This will be the case if the domestic currency price of internationally traded goods is a function of the exchange rate. A convenient way of representing this notion is to express the domestic price level,  $p$ , as a weighted average of the sticky domestic money wage and the domestic currency value of an appropriate (trade-weighted) index of world prices,  $p^*$ . Making the small country assumption that  $p^*$  is given and choosing units such that  $p^* = 0$ , we have:

$$(8) \quad p = \alpha w + (1 - \alpha)e, \quad 0 \leq \alpha \leq 1.^3$$

Equation (3) is then replaced by:

$$(9) \quad Dw = \phi y + \pi.$$

For the time being we still assume that

3. A more general approach is the following. Let  $p_H$  be the price of domestically produced goods. It is a weighted average of unit labor costs,  $w$ , and unit imported intermediate input costs:  $e + p^{*I}$ , i.e.,

$$(8') \quad p_H = \beta_1 w + (1 - \beta_1)(e + p^{*I}), \quad 0 \leq \beta_1 \leq 1.$$

The domestic price level or consumer price index is a weighted average of the price of domestically produced goods and the price of imported final goods:  $e + p^{*F}$ , i.e.,

$$(8'') \quad p = \beta_2 p_H + (1 - \beta_2)(e + p^{*F}), \quad 0 \leq \beta_2 \leq 1.$$

For our purposes not much is lost by using the simpler formulation in (8). An alternative interpretation in terms of traded and nontraded goods is also possible.

$$(4) \quad \pi = D^+ m.$$

With  $\alpha < 1$ , the domestic price level is no longer predetermined. The jump appreciation of the nominal (and real) exchange rate in response to (for example) an unanticipated reduction in the rate of monetary growth will have the immediate effect of lowering the price level. However, as long as  $\alpha > 0$ , the earlier analysis is not affected qualitatively. We redefine our state variables as follows:

$$(10a) \quad \ell = m - w.$$

$$(10b) \quad c = e - w.$$

As before,  $\ell$  is predetermined (except when  $m$  jumps), and  $c$  is a jump variable. The state-space representation of the model given in equations (1), (2), (8), (9), (4), and (5) is:

$$(11) \quad \begin{bmatrix} D\ell \\ Dc \end{bmatrix} = \frac{1}{\alpha\gamma(\lambda\phi - k) - \lambda} \begin{bmatrix} \phi\alpha\gamma & \phi\alpha[\lambda\delta - \gamma(1 - \alpha)] \\ 1 & \alpha\delta(\phi\lambda - k) + \alpha - 1 \end{bmatrix} \begin{bmatrix} \ell \\ c \end{bmatrix} \\ + \frac{1}{\alpha\gamma(\lambda\phi - k) - \lambda} \begin{bmatrix} \alpha\gamma\lambda\phi & -\phi\lambda\gamma(1 - \alpha) \\ \lambda & \lambda + \gamma(k - \phi\lambda) \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix} \\ - \begin{bmatrix} -\phi\alpha\gamma & -\phi\alpha\gamma\lambda \\ -1 & -\lambda \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix}.$$

It is easily seen that (7) is the special case of (11) with  $\alpha = 1$ . A necessary and sufficient condition for the existence of a unique saddlepoint equilibrium is

$$(12) \quad \alpha\gamma(\lambda\phi - k) - \lambda < 0.$$



This again has the interpretation that, at a given level of competitiveness, an increase in aggregate demand raises output. The single convergent path is again upward-sloping, and the real exchange rate overshooting results of section 10.2 carry over to the more plausible model under consideration here. The main change from the previous analysis, with  $\alpha = 1$ , is that the exchange rate appreciation consequent upon restrictive monetary policy actions (or increases in  $\theta$  or  $r_d$ ) now has an immediate beneficial effect on the price level, although, as long as  $\alpha > 0$ , a given percentage appreciation of  $e$  will be associated with a smaller percentage reduction in  $p$ .

The special case  $\alpha = 0$  represents the "law of one price" for all goods or instantaneous purchasing power parity (PPP). Although few propositions in economics have been rejected more convincingly by the data than PPP (Kravis and Lipsey 1978; Frenkel 1981; Isard 1977), it is mentioned here briefly for completeness. With the domestic price level moving perfectly in line with the exchange rate, the wage equation (9), which still incorporates stickiness in the level of the money wage, ceases to be relevant to the rest of the model. The relative price of domestic and foreign goods is constant. Real output is a function of the exogenous real interest rate. Unless we impose the requirement that steady-state real wages are constant, output need not be at its full employment level. Alternatively, we could add an equation making output a (decreasing) function of the real wage. As this model has little to recommend it, we shall not pursue it any further here.

### 10.3.2 Money Wage Flexibility and Real Wage Flexibility

We now consider the case where both the money wage and the real wage are perfectly flexible, and output is always at its equilibrium or capacity value, 0. We can view this as the case where the core rate of wage inflation,  $\pi$ , equals the expected (and actual) rate of wage inflation, i.e.,

$$(4') \quad \pi = Dw.$$

The model of equations (1), (2), (8), (9), (4'), and (5) has the following very simple state-space representation:

$$(13) \quad \begin{bmatrix} D\ell \\ Dc \end{bmatrix} = \begin{bmatrix} \lambda^{-1} & \gamma^{-1}\delta - (1-\alpha)\lambda^{-1} \\ 0 & \gamma^{-1}\delta \end{bmatrix} \begin{bmatrix} \ell \\ c \end{bmatrix} \\ + \begin{bmatrix} 1 & -\alpha^{-1}(1-\alpha) \\ 0 & -\alpha^{-1} \end{bmatrix} \\ \quad \quad \quad \begin{bmatrix} -\lambda^{-1} & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix}.$$

With both  $e$  and  $w$  freely flexible, neither of the two state variables,  $\ell$  and  $c$ , is predetermined. A unique convergent solution trajectory exists because there are now two unstable characteristic roots ( $\lambda^{-1}$  and  $\gamma^{-1}\delta$ ). The system is also recursive, with  $Dc$  independent of  $\ell$  and also of the policy instruments  $Dm$ ,  $r_f$ , and  $\theta$ . Only a real shock (such as a change in the foreign real interest rate  $r^* + \tau$ ) will affect the dynamics and steady-state behavior of  $c$ .

The diagrammatic representation of the system is given in figure 10.1. Without loss of generality, we assume that the  $D\ell = 0$  locus is downward sloping. Consider an unexpected, immediately implemented reduction in  $Dm$ . The initial equilibrium is at  $E_1$ , the new equilibrium at  $E_2$ . Note that these equilibria are completely unstable. Since the cut in the monetary growth rate is immediately implemented,  $\ell$  jumps immediately from  $E_1$  to  $E_2$  with no change in  $c$ . Monetary disinflation is costless. If we consider a previously unanticipated future reduction in  $Dm$ ,  $\ell$  will jump to an intermediate position like  $E_{12}$  between  $E_1$  and  $E_2$  at the moment the future policy change is announced. After that it moves gradually in a straight line from  $E_{12}$  to  $E_2$ , where the system arrives at the moment that  $Dm$  is actually reduced. Again, there is no effect on competitiveness in the short run or in the long run.

It is instructive to contrast monetary disturbances with a real shock, such as an increase in  $r^* + \tau$ , analyzed in figure 10.2. The steady-state effect is to alter the long-run equilibrium from  $E_1$  to  $E_2$ , lowering  $\ell$  and raising  $c$ . If the increase in  $r^* + \tau$  occurs immediately, both  $c$  and  $\ell$  jump

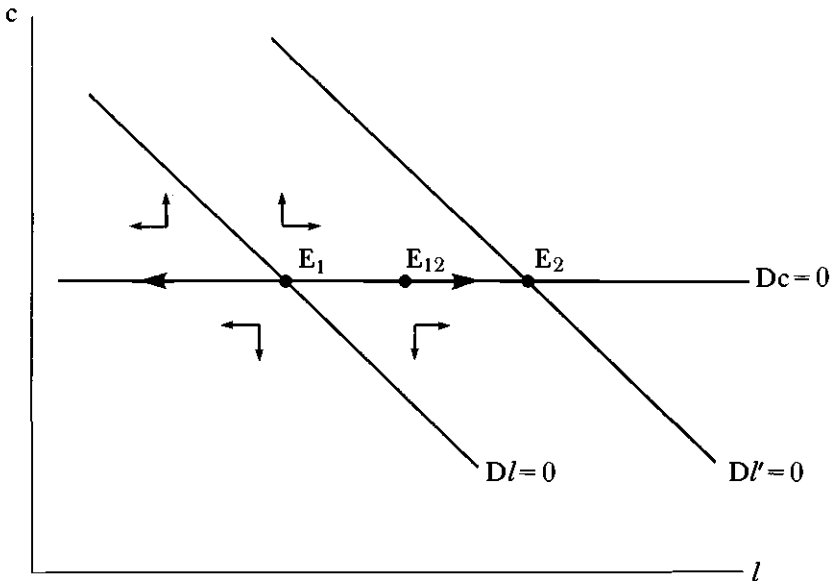


Fig. 10.1 Monetary disturbances (with money and real wage flexibility).

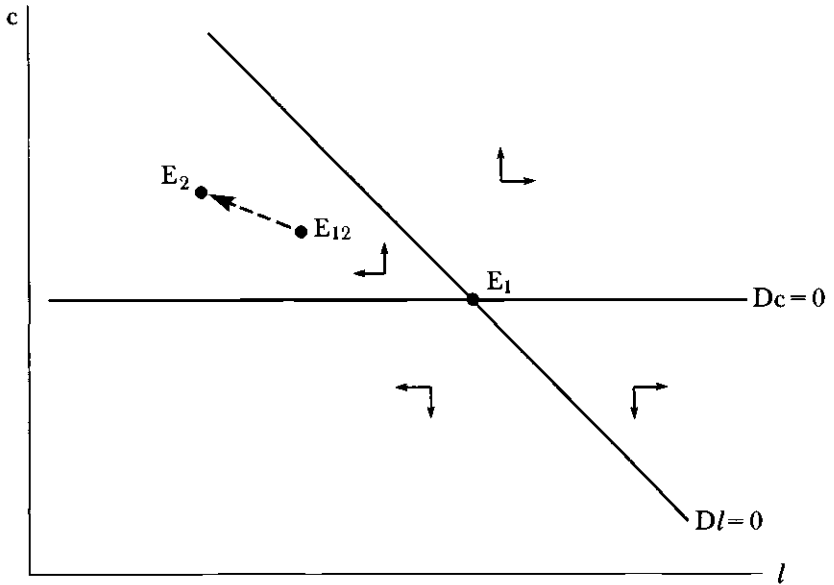


Fig. 10.2 Real disturbances (with money and real wage flexibility).

to  $E_2$  without delay. If we have a future increase in  $r^* + \tau$ , the system jumps to an intermediate position such as  $E_{12}$  after which it proceeds gradually to  $E_2$  where it arrives when  $r^* + \tau$  is actually raised. Note that this adjustment of the real exchange rate is an equilibrium phenomenon, taking place at a constant level of output.

### 10.3.3 Money Wage Flexibility and Real Wage Rigidity

Some recent work on wage and price behavior can be interpreted as combining the assumption of perfectly flexible money wages with the assumption of sluggish adjustment in the real wage. The latter is treated as predetermined because of (generally unspecified) transactions and adjustment costs.

Consider, for example, the following specification for  $\pi$ :

$$(4'') \quad \pi = Dp - \eta(w - p), \quad \eta \geq 0.$$

Equation (4''), in combination with (9), yields

$$(14) \quad Dw = \phi y + Dp - \eta(w - p),$$

or

$$(14') \quad D(w - p) = \phi y - \eta(w - p).$$

Equation (14) can be viewed as a rational expectations version of the kind

of equation proposed by Sargan (1980). It is also very close to an equation found in Minford (1980), although his equation incorporates nominal stickiness. The state-space representation of the model with nominal flexibility and real stickiness is given in equation (15):

$$(15) \quad \begin{bmatrix} D\ell \\ Dc \end{bmatrix} = \begin{bmatrix} \lambda^{-1} & -\frac{(1-\alpha)[\eta\lambda + k\alpha\gamma\eta + 1 - \alpha(1 + \gamma\phi)] + \alpha\delta k\lambda}{\lambda[1 - \alpha(1 + \gamma\phi)]} \\ 0 & -\frac{[\eta(1 - \alpha) + \phi\alpha\delta]}{1 - \alpha(1 + \gamma\phi)} \end{bmatrix} \begin{bmatrix} \ell \\ c \end{bmatrix} \\ + \begin{bmatrix} 1 & \frac{\lambda[1 - \alpha(1 + \gamma\phi)] + \lambda\phi\gamma + k\gamma(1 - \alpha)}{\lambda[1 - \alpha(1 + \gamma\phi)]} & -\lambda^{-1} & -1 \\ 0 & \frac{\phi\gamma}{1 - \alpha(1 + \gamma\phi)} & 0 & 0 \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix}$$

Note that real wage rigidity implies real exchange rate rigidity, as  $w - p = (\alpha - 1)c$ . With a flexible money wage,  $\ell$  now is a jump variable. The roles of  $\ell$  and  $c$  as predetermined and jump variables are the exact reverse of what they were in the model with sticky money wages (and flexible real wages) of section 10.2. The two characteristic roots of equation (15) are  $\lambda^{-1}$  and  $-\{[\eta(1 - \alpha) + \phi\alpha\delta]/[1 - \alpha(1 + \gamma\phi)]\}$ . The sign of the second root—the one governing the behavior of  $c$ —depends on the sign of  $1 - \alpha(1 + \gamma\phi)$ . This has the following interpretation. Add an exogenous demand shock  $f$  to the IS equation (2). This yields  $y = -\gamma(r - Dp) + \delta(e - p) + f$ . It is readily verified that

$$(16a) \quad e - p = \alpha c,$$

$$(16b) \quad r - Dp = r^* + \tau + \alpha Dc,$$

and

$$(16c) \quad Dc = \frac{\phi}{\alpha - 1} y - \eta c.$$

The IS curve can therefore be written as

$$(16d) \quad y = -\frac{\gamma(1-\alpha)}{1-\alpha(1+\gamma\phi)}(r^* + \tau) + \frac{\alpha(1-\alpha)(\gamma\eta + \delta)}{1-\alpha(1+\gamma\phi)}c \\ + \frac{(1-\alpha)}{1-\alpha(1+\gamma\phi)}f.$$

For  $0 \leq \alpha < 1$ ,  $1 - \alpha(1 + \gamma\phi)$  must be positive if an exogenous increase in demand is to raise output at a given level of competitiveness. We shall make this assumption. It implies that the root governing  $c$  is negative. Note that with equation (14') governing the behavior of the real wage, there is no automatic tendency for the level of output to converge to its capacity level 0. In long-run equilibrium we have (setting  $D[w - p] = 0$ ),

$$(17) \quad y = \frac{\eta}{\phi}(w - p) = (\alpha - 1)\frac{\eta}{\phi}c.$$

The system is still dichotomized, and the behavior of  $c$ ,  $w - p$ ,  $y$ , and  $r - Dp$  is independent of monetary shocks, but even if we start at full employment, real shocks will not necessarily be followed by a return to full employment. Only if  $\eta$  (minus the coefficient on the lagged real wage in the wage equation) is zero will the system tend to full employment. This can be shown as follows. In long-run equilibrium the IS equation is

$$(18) \quad y = -\gamma(r^* + \tau) + \delta\alpha c + f.$$

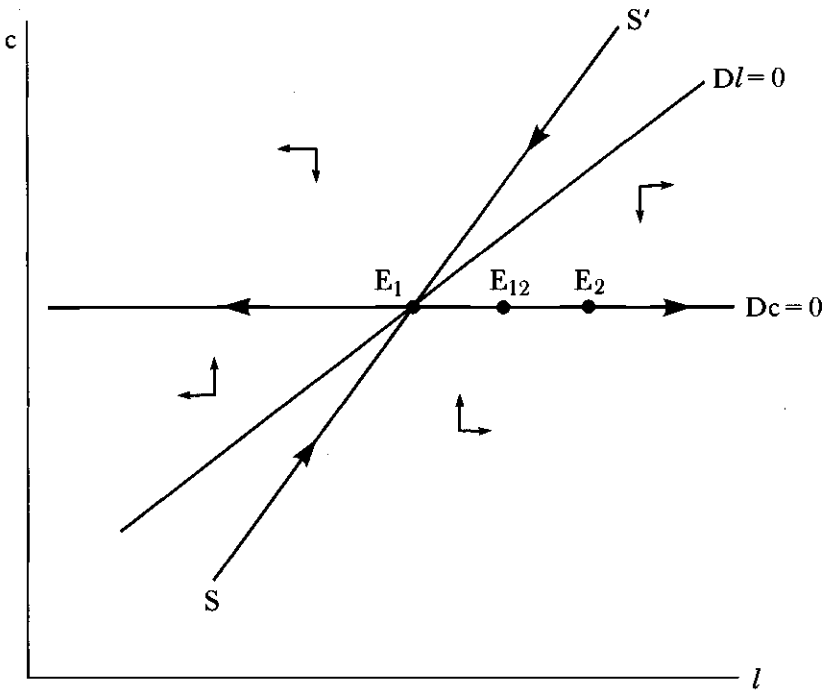
Combining (17) and (18) gives

$$(19) \quad y = \frac{(\alpha - 1)\eta\gamma}{\phi\delta\alpha - (1 - \alpha)\eta}(r^* + \tau) - \frac{(\alpha - 1)\eta\delta\alpha}{\phi\delta\alpha - (1 - \alpha)\eta}f.$$

Apart from the absence of an automatic return to full employment, the behavior of the flexible-money-wage, sticky-real-wage model is qualitatively the same when  $\eta = 0$  and when  $\eta > 0$ . The response to an unanticipated reduction in  $Dm$  is shown in figure 10.3. An unanticipated, immediately implemented reduction in  $Dm$  instantaneously moves the system to the new stationary equilibrium  $E_2$  without any change in  $c$ ,  $y$ , or  $r - Dp$ . An announced future reduction in  $Dm$  instantaneously moves the system to an intermediate position such as  $E_{12}$  between  $E_1$  and  $E_2$ . From there it moves gradually to  $E_2$  where it arrives at the moment that the reduction in  $Dm$  actually occurs. This whole process again takes place without any changes in  $c$ ,  $y$ , or  $r - Dp$ .

Now consider the effect of an increase in  $r^* + \tau$  in this model, which changes the long-run equilibrium in figure 10.4 from  $E_1$  to a point such as  $E_2$ .

With  $c$  predetermined, an immediate, unanticipated increase in  $r^* + \tau$  causes an equal jump increase in  $e$  and  $w$ , lowering  $\ell$  to  $E_{12}$ . From there  $c$  and  $\ell$  converge gradually to the new long-run equilibrium  $E_2$  along the



**Fig. 10.3** Money disturbances (with money wage flexibility and real wage rigidity).

unique convergent trajectory  $S'S'$ . A previously unanticipated future increase in  $r^* + \tau$  leads to an immediate jump in  $\ell$  down to a point intermediate between  $E_1$  and  $E_{12}$ , such as  $E'_{12}$ . From there  $\ell$  declines gradually to  $E_{12}$  where it arrives when  $r^* + \tau$  is actually raised. Then  $c$  and  $\ell$  increase gradually along  $S'S'$  toward  $E_2$ .

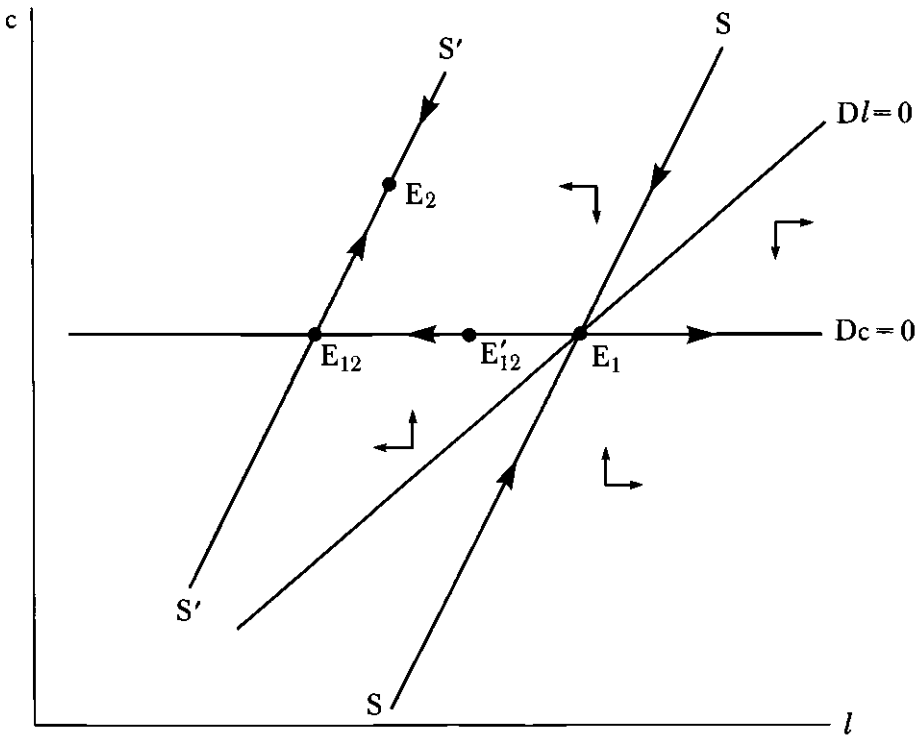
It is interesting to see what happens to the wage equation (14') when the exchange rate has no effect on the price level, that is, when  $\alpha = 1$ . In that case the price equation (8) becomes

$$(20a) \quad p = w,$$

while the wage equation reduces to

$$(20b) \quad \phi y = \eta(w - p).$$

Equations (20a, b) imply that  $y = 0$  at each instant. The model now is in many ways the same as the model with money wage and real wage flexibility discussed in section 10.3.2 and summarized in equation (13). The link between the real wage and the real exchange rate, given by  $w - p = (\alpha - 1)c$  in the general model, disappears. Even though the real wage is still predetermined (and indeed remains constant throughout at



**Fig. 10.4** Real disturbances (with money wage flexibility and real wage rigidity).

0), the real exchange rate again becomes a jump variable. Because  $w$  still is a jump variable,  $\ell$  also stays that way. The state-space representation of this version of the model is given in (21)

$$(21) \begin{bmatrix} D\ell \\ Dc \end{bmatrix} = \begin{bmatrix} \lambda^{-1} & \gamma^{-1}\delta \\ 0 & \gamma^{-1}\delta \end{bmatrix} \begin{bmatrix} \ell \\ c \end{bmatrix} + \begin{bmatrix} 1 & 0 & -\lambda^{-1} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix}.$$

The response of this system to nominal and real shocks is qualitatively similar to that described in section 10.3.2 and figures 10.1 and 10.2.

### 10.3.4 Rational Expectations in the Labor Market with Money Wage Stickiness

Without changing the equation for the core rate of inflation (4'') or the associated wage equation (14) of the previous section, a single change of assumption concerning the behavior of the money wage destroys the

classical policy implications of that model. The crucial change in assumption is to rule out discrete jumps in  $w$ , that is, to require  $w$  to be a continuous function of time. The exchange rate, however, is still free to make discrete jumps at a point in time. This change in assumption does not rule out a rational expectations interpretation of (14). This is particularly obvious if we assume that  $\eta = 0$ . The behavior of this rational expectations model of the labor market is, however, very different from the classical behavior of the models of sections 10.3.2 and 10.3.3. Instead it resembles the behavior of the sticky money wage model of sections 10.2 and 10.3.1. Monetary shocks lead to real exchange rate overshooting and departures of actual output from capacity output. Note that this kind of behavior is ruled out when  $\alpha = 1$ . This "closed economy" representation means that rational expectations automatically rule out departures of actual output from capacity output.<sup>4</sup> With the assumed asymmetry in the behavior of  $c$  and  $w$ , and with a direct effect of  $e$  on  $p$ , monetary shocks will alter the real wage and the real exchange rate and cause departures from full employment.

With the sticky money wage interpretation of equation (14),  $\ell$  and  $c$  again assume the roles of section 10.2, where  $\ell$  is predetermined while  $c$  (via  $e$ ) can jump in response to news.

The response of the system to an unanticipated reduction in  $Dm$  is illustrated in figure 10.5.

If the reduction in  $Dm$  takes place immediately,  $c$  jump-appreciates to  $E_{12}^0$ . After that it moves gradually to  $E_2$  along  $S'S'$ . From equation (16d) we see that this jump-appreciation of  $c$  will be associated with a fall in output. An anticipated future reduction in  $Dm$  will be associated with a smaller immediate jump-appreciation of  $c$  when the news arrives, say to  $E_{12}^1$ . This jump places  $c$  and  $\ell$  on the divergent path, driven by the values of the forcing variables determining  $E_1$ , that will put them on the convergent path through  $E_2$  ( $S'S'$ ) when the cut in  $Dm$  is actually implemented. An equal reduction in  $Dm$  at a more distant future time will again be associated with a smaller initial jump-appreciation of  $c$  (say to  $E_{12}^2$ ), after which  $c$  and  $\ell$  follow the unstable trajectory (drawn with reference to  $E_1$ ) that will put it on  $S'S'$  when  $Dm$  is actually cut. There always will be a finite initial jump in  $c$  when the news of a future reduction in  $Dm$  arrives, except in the limiting case when the announced monetary growth reduction is infinitely far in the future. One implication is that if a monetary deceleration is planned, the loss of output and competitiveness is smaller, the further in advance the proposed policy action is announced.

From equation (16c) with  $\eta = 0$ , we obtain:

$$(22) \quad y = \frac{\alpha - 1}{\phi} Dc.$$

4. These issues are discussed for the fixed exchange rate case in Buiter (1978, 1979).



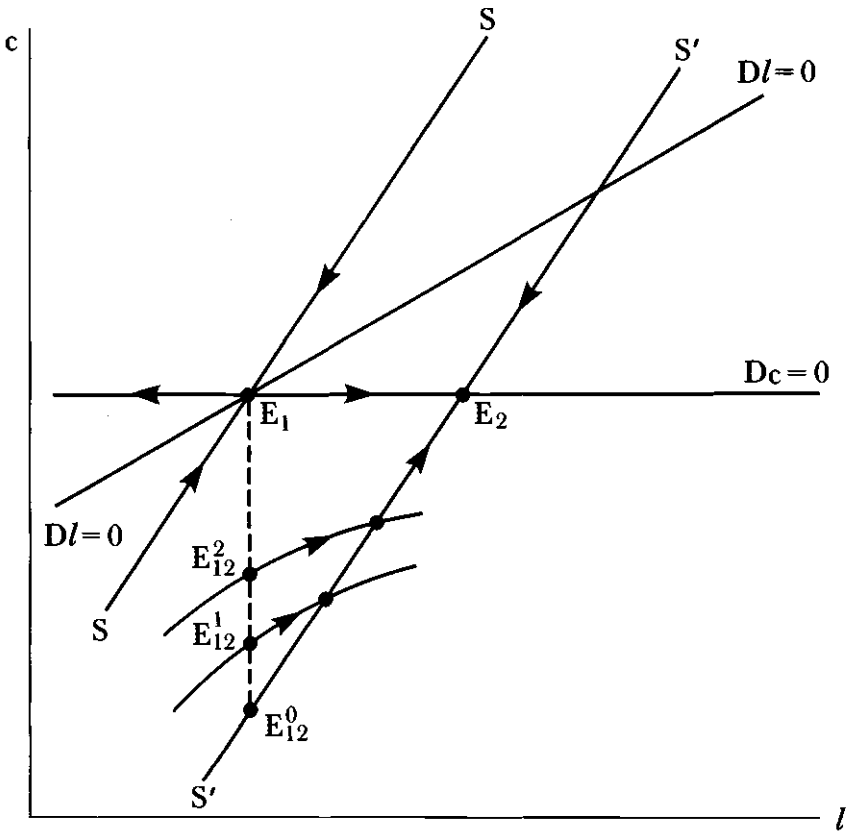


Fig. 10.5 Reduction of monetary growth (with money wage stickiness).

Assume the system starts in long-run equilibrium at  $t = 0$ . The net cumulative loss of output<sup>5</sup> following a monetary deceleration is

$$(23) \quad \int_0^{\infty} y(t) dt = \frac{\alpha - 1}{\phi} [c(\infty) - c(0)].$$

Here  $c(\infty)$  is the steady-state real exchange rate which is the same in the initial and the final long-run equilibrium. Therefore  $c(\infty) - c(0)$  is just the initial jump in the real exchange rate. The cumulative loss of output is minimized by minimizing the initial jump in  $c$ . This is achieved, for a given proposed reduction in  $Dm$ , by announcing the reduction as early as possible.

The assumption made so far, that  $w(t)$  is a continuous function of time, can be derived from two more basic assumptions. The first is that  $w(t)$

5. Since the adjustment path of output is monotonic in this example,  $y$  does not change sign during the transition. The net output loss therefore also equals the gross output loss.

cannot jump instantaneously in response to new information. In principle this would still permit discrete jumps in  $w(t)$  at some  $t > t_0$ , where  $t_0$  is the instant at which new information becomes available. The second assumption is an arbitrage condition for the labor market, which asserts that efficient speculative behavior in the labor market eliminates all profit opportunities associated with anticipated future jumps in  $w$ . This assumption is analogous to the arbitrage condition we have used to rule out anticipated future jumps in  $e$ , although its application to the labor market is rather less convincing than its use in the foreign exchange market.

### 10.3.5 Gradual Adjustment of Core Inflation with Occasional Jumps

The final specification of the equation for the core rate of inflation that we shall consider is given in equation (4'''):

$$(4''') \quad D\pi = \xi(Dp + D\theta - \pi), \quad \xi > 0.$$

This adaptive process for  $\pi$  does not rule out discrete jumps in  $\pi$ , although it is consistent with our assumption that the *level* of the money wage is predetermined. Equation (4''') defines  $\pi$  as a backward-looking weighted average of past rates of inflation with exponentially declining weights,

$$\pi(t) = \xi \int_{-\infty}^t e^{-\xi(t-s)} [Dp(s) + D\theta(s)] ds.$$

Because  $\pi(t)$  is backward looking, it will be associated with a stable eigenvalue. It is, however, not predetermined, since  $\pi(t)$  also depends on current  $Dp(t)$  and  $D\theta(s)$ . If  $p + \theta$  makes a discontinuous jump at  $t = \bar{t}$ ,  $Dp + D\theta$  becomes unbounded and so does  $D\pi$ . Therefore  $\pi$  jumps. This characteristic of  $\pi$  as a "dependent" jump variable—it jumps at  $t = \bar{t}$  if and only if  $p + \theta$  jumps at  $t = \bar{t}$ —will be important when we come to consider the specification of the boundary conditions of this model—our preferred model.

It is easily verified that the following relation holds for  $\pi$ :

$$(24) \quad \pi(t) = \pi(t^-) + \xi[p(t) - p(t^-) + \theta(t) - \theta(t^-)],$$

where

$$\pi(t^-) = \lim_{\substack{\tau \rightarrow t \\ \tau < t}} \pi(\tau) \quad \text{etc.}$$

The jump in  $\pi$  is  $\xi$  times the sum of the jumps in  $p$  and  $\theta$ . Rewriting (24) in terms of the state variables, we get

$$(24') \quad \pi(t) - \pi(t^-) = \xi \{ (1 - \alpha)[c(t) - c(t^-)] + \theta(t) - \theta(t^-) \\ + \ell(t) - \ell(t^-) - [m(t) - m(t^-)] \}.$$

If there are no level jumps in  $m$ , this becomes

$$(24'') \quad \pi(t) - \pi(t^-) = \xi \{ (1 - \alpha)[c(t) - c(t^-)] + \theta(t) - \theta(t^-) \}.$$

For convenience we reproduce the complete model (to be used in section 10.4) below.

- (1)  $m - p - \theta = ky - \lambda(r - r_d).$
- (2)  $y = -\gamma(r - Dp - D\theta) + \delta(e - p).$
- (8)  $p = \alpha w + (1 - \alpha)e.$
- (9)  $Dw = \phi y + \pi.$
- (4''')  $D\pi = \xi(Dp + D\theta - \pi).$
- (5)  $De = r - r^* - \tau.$
- (10a)  $\ell = m - w.$
- (10b)  $c = e - w.$

The model's state-space representation is given in equations (25) and (26).<sup>6</sup>

$$(25) \begin{bmatrix} D\ell \\ D\pi \\ Dc \end{bmatrix} = \Delta^{-1} \begin{bmatrix} \phi\alpha\gamma & \lambda + \alpha\gamma k \\ \xi[1 - \alpha(1 + \gamma\phi)] & \xi\lambda[1 - \alpha(1 + \gamma\phi)] \\ 1 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} \phi\alpha[\lambda\delta - \gamma(1 - \alpha)] \\ \xi\{\alpha\phi[\gamma(1 - \alpha) - \alpha\delta\lambda] \\ - (1 - \alpha)[1 - \alpha(1 - \delta k)]\} \\ \alpha\delta(\phi\lambda - k) - (1 - \alpha) \end{bmatrix} \begin{bmatrix} \ell \\ \pi \\ c \end{bmatrix}$$

$$+ \Delta^{-1} \begin{bmatrix} \Delta & -\phi\lambda\gamma(1 - \alpha) & -\phi\alpha\gamma \\ 0 & \xi(1 - \alpha)(\lambda + \gamma k) & -\xi[1 - \alpha(1 + \gamma\phi)] \\ 0 & \lambda + \gamma(k - \phi\lambda) & -1 \end{bmatrix}$$

$$\begin{bmatrix} -\phi\alpha\gamma\lambda \\ -\xi\lambda[1 - \alpha(1 + \gamma\phi)] \\ -\lambda \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix},$$

where  $\Delta = \alpha\gamma(\phi\lambda - k) - \lambda < 0.$

6. The  $D\theta$  term is again omitted. It will be discussed in section 10.6.

$$(26) \begin{bmatrix} r \\ y \\ Dw \\ Dp \\ De \end{bmatrix} = \Delta^{-1} \begin{bmatrix} 1 - \alpha\gamma\phi & -k\alpha\gamma \\ -\alpha\gamma & -\alpha\lambda\gamma \\ -\alpha\gamma\phi & -(\lambda + \alpha\gamma k) \\ 1 - \alpha(1 + \gamma\phi) & -\alpha(\lambda + \gamma k) \\ 1 - \alpha\gamma\phi & -k\alpha\gamma \end{bmatrix} \begin{bmatrix} \ell \\ \pi \\ c \end{bmatrix} \\
 + \Delta^{-1} \begin{bmatrix} 0 & k\gamma(1 - \alpha) & -(1 - \alpha\gamma\phi) \\ 0 & \lambda\gamma(1 - \alpha) & \alpha\gamma \\ 0 & \phi\lambda\gamma(1 - \alpha) & \phi\alpha\gamma \\ 0 & (1 - \alpha)(\lambda + k\gamma) & -[1 - \alpha(1 + \gamma\phi)] \\ 0 & \lambda + \gamma(k - \alpha\lambda\phi) & -(1 - \alpha\gamma\phi) \end{bmatrix} \begin{bmatrix} Dm \\ r^* + \tau \\ \theta \\ r_d \end{bmatrix} \\
 - \begin{bmatrix} [k\alpha\delta + (1 - \alpha\gamma\phi)(1 - \alpha)] \\ -\alpha[\lambda\delta - \gamma(1 - \alpha)] \\ -\alpha\phi[\lambda\delta - \gamma(1 - \alpha)] \\ \{\alpha\phi[\gamma(1 - \alpha) - \alpha\delta\gamma] \\ -(1 - \alpha)[1 - \alpha(1 - \delta k)] \\ -[k\alpha\delta + (1 - \alpha\gamma\phi)(1 - \alpha)] \end{bmatrix}$$

We now turn to a more detailed study of the behavior of the model of equations (25) and (26) in section 10.4. For notational convenience, we set  $r^* + \tau = \theta = r_d = 0$ .

### 10.4 The Real Exchange Rate and the Output Cost of Monetary Disinflation in a Model with Sluggish “Core” Inflation

In this section we solve the model of equations (25) and (26) for the time paths of selected variables, using a particular set of “plausible” parameter values. The numerical example is designed to focus on the role of exchange rates and interest rates in monetary disinflation. We examine the mechanism through which inflation is reduced and calculate the costs, in terms of lost output, incurred in the process.

In our earlier paper on this subject (Buiter and Miller 1982), the effects of real interest rates on aggregate demand were suppressed, but here real interest rates as well as the real exchange rate are assumed to determine

aggregate demand. As we show numerically, however, variations in the parameters affecting the *inflationary* process systematically change the relative contributions which each of these two "channels" of monetary policy makes to fluctuations in output.

#### 10.4.1 Parameter Values

To illustrate the operation of the model, we construct a "central" case by choosing particular values for the parameters as follows:

$$\lambda = 2, k = 1, \alpha = 3/4, \phi_c = \xi = \delta = \gamma = 1/2.$$

With these values substituted into equation (25), the system becomes

$$(27) \quad \begin{bmatrix} D\ell \\ D\pi \\ Dc \end{bmatrix} = \begin{bmatrix} -0.0938 & -1.1875 & -0.1641 \\ -0.0156 & -0.0313 & -0.0977 \\ -0.5 & -1.0 & 0.125 \end{bmatrix} \begin{bmatrix} \ell \\ \pi \\ c \end{bmatrix} + \begin{bmatrix} Dm \\ 0 \\ 0 \end{bmatrix}.$$

The characteristic roots of the state matrix  $A$  in equation (27) are 0.375 and  $-0.1875 \pm 0.2977i$ , and the row eigenvector associated with the positive root  $\hat{\rho} = 0.375$  is found as follows.

The row eigenvector  $v'$  associated with *any* root must satisfy the condition that  $v'(\rho I - A) = Z$ , where  $Z$  denotes the zero vector. Normalizing the eigenvector appropriately, this implies (for the parameter values shown above) that

$$[v_1 \quad v_2 \quad -1] \begin{bmatrix} \rho + (3/32) & 19/16 & -a_{13} \\ 1/64 & \rho + (1/32) & -a_{23} \\ 1/2 & 1 & \rho - a_{33} \end{bmatrix} = [0 \quad 0 \quad 0],$$

where exact values are shown in the first two columns, but the last column is not given for simplicity. On inserting the value of 0.375 for  $\hat{\rho}$ , we find the elements of the associated eigenvector  $\hat{v}$  to be

$$[\hat{v}_1, \hat{v}_2, -1] = [12/11, -8/11, -1],$$

as may readily be confirmed.

To ensure stability, the path to be followed by the homogenous system must not depend on the positive (unstable) root. Avinash Dixit (1980) has shown that for a previously unanticipated, immediately implemented shock this can be achieved by ensuring that the product of the eigenvector  $\hat{v}$  with the initial values of the variables (measured as deviations from the new long-run equilibrium) equals zero, that is,

$$\hat{v}_1[\ell(0) - \bar{\ell}] + \hat{v}_2[\pi(0) - \bar{\pi}] - [c(0) - \bar{c}] = 0,$$

where  $\bar{\ell}$ ,  $\bar{\pi}$ , and  $\bar{c}$  are the new long-run equilibrium values of  $\ell$ ,  $\pi$ , and  $c$ , and  $\hat{v}_1$  and  $\hat{v}_2$  are elements associated with the unstable root.

Let  $Dm \equiv \mu$ . The terms measuring initial disequilibrium following an unanticipated change in  $\mu$ , denoted  $d\mu$ , can be evaluated as follows:

$$\ell(0) - \bar{\ell} = \lambda d\mu.$$

$$\pi(0) - \bar{\pi} = \pi(0) - \bar{\mu}, \text{ where } \bar{\mu} \text{ denotes the new value of } \mu.$$

$$c(0) - \bar{c} = dc, \text{ the jump in competitiveness.}$$

From equation (24''), we know that

$$d\pi = \xi(1 - \alpha)dc,$$

so the initial disequilibrium in  $\pi$  becomes

$$\pi(0) - \bar{\pi} = \xi(1 - \alpha)dc - d\mu.$$

Hence the product of the eigenvector and the initial values can be rewritten as

$$\hat{v}_1 \lambda d\mu + \hat{v}_2 [\xi(1 - \alpha)dc - d\mu] - dc = 0,$$

so the initial change in competitiveness is found to be

$$dc = \frac{\hat{v}_1 \lambda - \hat{v}_2}{1 - \hat{v}_2 \xi(1 - \alpha)} d\mu.$$

#### 10.4.2 The Impact Effects of an Unanticipated Change in Monetary Growth ( $d\mu$ )

Using these values for  $\hat{\rho}$ ,  $\hat{v}_1$ , and  $\hat{v}_2$ , we find the initial jump in competitiveness is  $dc = 8d\mu/3$ , that is, the initial percentage change in competitiveness will be just under three times the percentage change in monetary growth announced by the monetary authorities. The immediate effect that this has on the "core" rate of inflation is

$$d\pi = \xi(1 - \alpha)dc = \frac{dc}{8} = d\mu/3.$$

Given the simple structure of the model, the change in competitiveness will be associated with an immediate change in output,

$$dy = \delta\alpha dc = \frac{3}{8}dc = d\mu,$$

so real output will change on impact by the change in the trend rate of monetary growth.

The rate of wage settlements will jump on impact as a result of both the shift in  $\pi$  and the recession, as follows:

$$d(Dw) = \phi dy + d\pi = [\phi\delta\alpha + \xi(1 - \alpha)]dc = \frac{5dc}{16} = 5d\mu/6.$$

### 10.4.3 The Long-Run Equilibrium

In a system that is superneutral, one would not expect a change in monetary growth to affect the equilibrium real interest rate or the real exchange rate, though nominal interest rates will reflect the monetary slowdown. By setting the left-hand side of equation (27) at zero and differentiating with respect to  $\mu$ , we can confirm that a change in  $\mu$  has no long-run effect on  $c$ , but changes  $\pi$  one-for-one. As the equilibrium nominal rate of interest will also move in line with  $\mu$ , the impact on real balances in the long run is  $-\lambda d\mu$ .

### 10.4.4 The Dynamic Behavior of the System

The dynamic behavior of the variables in the system is summarized in table 10.1. In the first column of panel (a) are shown the "starting values" for  $\ell$ ,  $\pi$ , and  $c$  discussed above, measured as deviations from their new equilibrium values after a one-point slowdown in monetary growth at  $t = 0$ . (All variables are scaled by 100, so a one-point slowdown in monetary growth will appear as  $d\mu = -1.0$ ). The second column shows  $D\ell$ ,  $D\pi$ , and  $Dc$  at time zero calculated from equation (27) and from the first column.

These starting values are chosen to place the system on the two-dimensional stable manifold, on which the stable path of  $\ell$ ,  $\pi$ , and  $c$  can be described by

**Table 10.1** Dynamic Behavior of the System

Variable	Starting Values <sup>a</sup>		Dynamic Characteristics <sup>b,c</sup>			
	$x(0)$	$Dx(0)$	$B_1$	$B_2$	$B$	$\epsilon$
(a) $\ell$	-2.0	-0.1665	-2.0	-1.8068	2.6953	3.8763
$\pi$	0.6667	-0.2502	0.6667	-0.4178	0.7868	5.7234
$c$	-2.6667	0.0	-2.6667	-1.6683	3.1456	3.7001
(b) $y$	-1.0	-0.1251	-1.0	-1.0430	1.4449	3.9480
$Dp$	0.1667	-0.2294	0.1667	-0.6609	0.6816	4.9595
$Dw$	0.1665	-0.3127	0.1665	-0.9394	-0.9540	4.8878
Alternative Policy <sup>d</sup>						
(c) $y$	-1.0	-0.1251	-1.0	-1.0430	1.4449	3.9480
$Dp$	0.5	-0.3126	0.5	-0.7302	0.8849	5.3129

<sup>a</sup>See text for derivation of  $x(0)$  for  $\ell$ ,  $\pi$ ,  $c$ ;  $Dx(0)$  can be obtained by multiplying the matrix shown in (27) into  $x(0)$ .

<sup>b</sup>Damping factor  $\rho = -0.1875$ ; frequency  $\omega = 0.2997$ .

<sup>c</sup> $B_1 \equiv x'(0)$ ;  $B_2 \equiv [x'(0) - \rho x(0)]/\omega$ ;  $B = \sqrt{B_1^2 + B_2^2}$ ;  $\epsilon \equiv \tan^{-1}(B_2/B_1)$ .

<sup>d</sup>For which  $\pi(0) = 1.0$ , and  $y$  follows same path as above (see text).

$$x(t) = e^{\rho t}(B_1 \cos \omega t + B_2 \sin \omega t) = B e^{\rho t} \cos(\omega t - \epsilon),$$

where the values for  $B_1$ ,  $B_2$ ,  $B$ , and  $\epsilon$  are calculated from the initial conditions in the first two columns, as explained in the notes to the table.

The same parameters, calculated in the same manner, are shown for  $y$ ,  $Dp$ , and  $Dw$  in panel (b) of table 10.1. These variables are also measured as deviations from their new equilibrium values. For output  $y$ , this is zero by construction, but for wage and price inflation ( $Dp$  and  $Dw$ ), the new steady state will correspond to the new rate of monetary growth ( $\bar{\mu}$ ). For convenience in what follows, we will assume that the newly chosen rate of monetary growth is zero, so that there is *no* inflation in the new equilibrium.

A check on the calculations contained in table 10.1 and some indication of how the policy works are obtained by integrating the paths shown there for  $Dw$  and  $Dp$ . The formula<sup>7</sup> which gives the required integral is

$$\int_0^{\infty} x(t) dt = \frac{-\rho B_1 + \omega B_2}{\rho^2 + \omega^2},$$

where  $B_1$  and  $B_2$  are shown in table 10.1 and the values for  $\rho$  and  $\omega$  are given in note b. Applying this we find:

$$\int_0^{\infty} Dw(t) dt = -2.0, \text{ and } \int_0^{\infty} Dp(t) dt = -1.33.$$

The discrepancy is accounted for by the fact that the price level shows an instantaneous discrete fall at time zero, which is not picked up in the integration. The fall will be simply  $(1 - \alpha)dc$ , where  $dc$  measures the initial impact of the monetary policy for competitiveness. The initial loss of competitiveness is  $-2.6667$  (see the first entry in the third row of the table), and  $1 - \alpha$  is 0.25, which provides a figure of  $-0.67$  for the initial fall in the price level. Together with the integral reported above, this gives a total of  $-2.0$  for the long-run effect on the price level. Thus the real wage is unchanged in the long run, as one would expect from a model which is "superneutral." (The 2 percent fall in the price level is required to increase real balances to satisfy the higher demand for liquidity at the lower nominal interest rate prevailing when prices are stable).

The cyclical path toward this long-run value is illustrated in figure 10.6. In the top panel of figure 10.6, by choice of units, both the price level and the money stock can be represented by the same line,  $CB$ , until the time ( $t = 0$ ) of the monetary slowdown. The money stock levels off at point  $B$ , and the price level jumps down (because of the jump-appreciation of  $e$ ) and then rises for a while, as shown by the path labeled  $AA$ .

In the lower panel of figure 10.6, the paths after  $t = 0$  for monetary growth (the time axis) and the rate of price inflation ( $AA$ ) are shown. It is evident from the fact that the initial rate is only a little above zero that this

7. Kindly provided by Peter Burridge and Avinash Dixit.





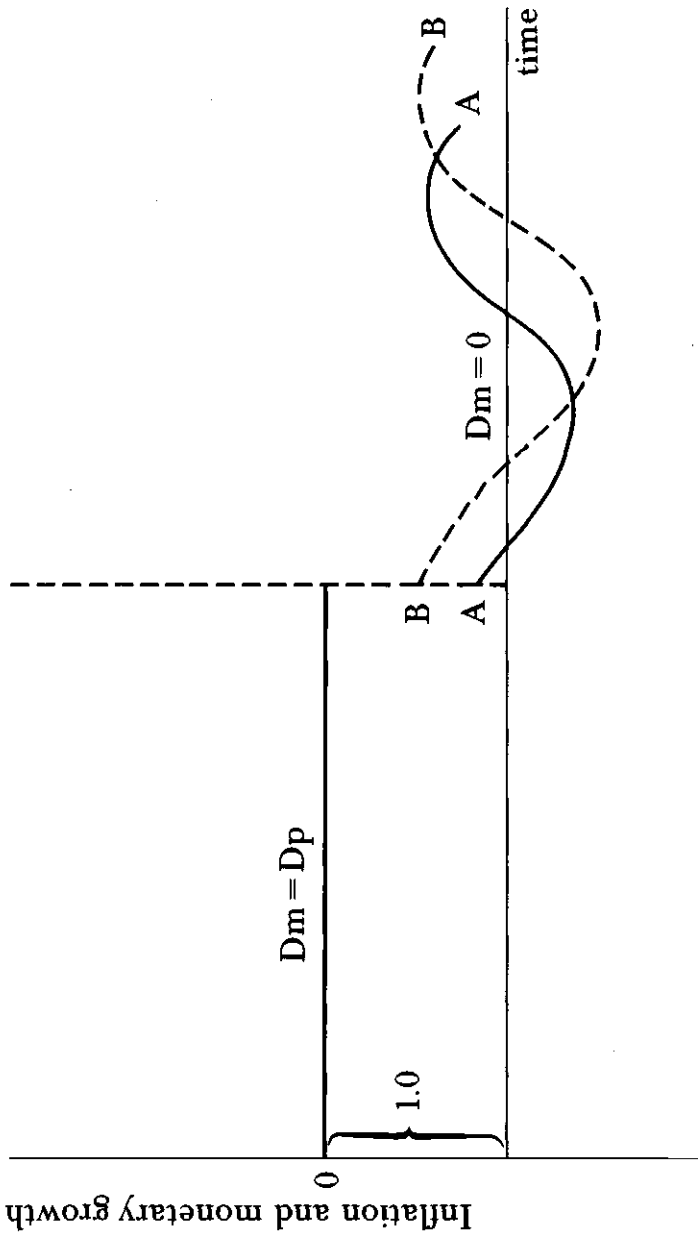


Fig. 10.6 Money and prices.

policy has a prompt effect on inflation. This effect is the result of two factors: the immediate fall in core inflation (by  $\frac{1}{3}$  from 1.0 to 0.6667, see table 10.1, column 1, row 2), and the recession in output (of 1.0, see table 10.1, column 1, row 4). The factors inducing the recession will be examined with the aid of table 10.2 below.

This rapid fall in inflation might lead one to be optimistic about the costs of eliminating inflation as measured by the size and duration of the recession. But if we measure the output costs of checking inflation simply by the unweighted integral of  $y$  (which means that some of the recession is cancelled out by subsequent boom as output cycles toward equilibrium), the following expression for the cumulative net loss of output can be obtained:

$$\int_0^{\infty} y(t) dt = \frac{\bar{\mu} - \underline{\mu}}{\xi\phi}.$$

With  $\bar{\mu} - \underline{\mu} = -1.0$  and  $\xi\phi = 0.25$ , the cumulative net output cost required to bring down the steady-state inflation rate by one percentage point is four "point years" loss of output.

Our model ignores possible benefits from bringing down inflation slowly because of nonlinearities in the Phillips curve which might cause two years with 5 percent excess capacity to have a stronger counterinflationary effect than one year with 10 percent excess capacity. The evidence on this is, however, by no means clear.

As a measure of economic waste,  $\int_0^{\infty} y(t) dt$  may understate the case where  $y(t)$  changes sign on the interval  $[0, \infty)$ , because periods of excess demand will "offset" periods of excess supply. A more appropriate index of economic waste might be the sum of the *absolute* deviations of output from equilibrium. For the present case

$$y(t) = e^{\rho t} (B_1 \cos \omega t + B_2 \sin \omega t) = B e^{\rho t} \cos(\omega t - \epsilon),$$

and  $\rho$ ,  $B_1$ ,  $B_2$ , and  $B$  are all negative (see table 10.1). The formula for the required integral (also provided by Peter Burridge) is

$$\int_0^{\infty} |y(t)| dt = \frac{\rho B_1 - \omega B_2}{\rho^2 + \omega^2} + \frac{2\omega B e^a}{(\rho^2 + \omega^2)(1 - e^b)},$$

where

$$a = (2\epsilon - \pi)\rho/2\omega,$$

and

$$b = \pi\rho/\omega,$$

$$\pi = 3.1415927,$$

which being evaluated with parameters from table 10.1 yields

$$\int_0^{\infty} |y| dt = 4.0 + 1.8205 = 5.8205,$$

an increase of cost of 45½ percent vis-à-vis the other measure ( $\int_0^{\infty} y dt$ ).

## 10.4.5 The Channels of Monetary Policy

From the construction of the model it is apparent that the costs of curing inflation are crucially dependent on the sensitivity of wage inflation to fluctuations in activity (as measured by the parameter  $\phi$ ) and on the speed with which core inflation adapts to the experience of inflation (as measured by the parameter  $\xi$ ). In this section we vary these two parameters and see how the channels of monetary policy are affected by such variations.

Specifically, the parameter  $\phi$  was doubled and doubled again, keeping  $\xi$  constant—it could not be increased much further without violating a stability condition of the model (namely that  $\Delta$  be negative). Resetting  $\phi$  at its original value of  $\frac{1}{2}$ , the speed of adjustment of core inflation was increased from  $\frac{1}{2}$  to 1, 2, and finally 5. The immediate results (of an unanticipated slowdown of monetary growth) on output and inflation, summarized in the top panel of table 10.2, show the recession in output to be fairly uniform (varying from just under 1 percent to just under  $1\frac{2}{3}$

Table 10.2 The Channels of Monetary Policy

A. Impact Effects on Real Output <sup>a</sup> and Inflation <sup>b</sup>					
$\xi$ (1)	$\phi$ (2)	$y(0)$ (3)	$\alpha\delta c(0)$ (4)	$-\gamma[r(0) - Dp(0)]$ (5)	$Dp(0) - 1.0$ (6)
$\frac{1}{2}$	$\frac{1}{2}$	-1.0	-1.0	0	-0.83
$\frac{1}{2}$	1	-1.03	-0.84	-0.19	-0.81
$\frac{1}{2}$	2	-1.61	-0.70	-0.91	-2.85
—	—	—	—	—	—
1	$\frac{1}{2}$	-0.98	-0.87	-0.11	-0.99
2	$\frac{1}{2}$	-1.03	-0.75	-0.28	-1.33
5	$\frac{1}{2}$	-1.36	-0.64	-0.72	-2.34

B. Accumulated Effects on Real Output				
$\xi$ (1)	$\phi$ (2)	$\int_0^{\infty} y(t)dt$ (3)	$\alpha\delta \int_0^{\infty} c(t)dt$ (4)	$-\gamma \int_0^{\infty} [r(t) - Dp(t)]dt$ (5)
$\frac{1}{2}$	$\frac{1}{2}$	-4	-3	-1
$\frac{1}{2}$	1	-2	-1.16	-0.84
$\frac{1}{2}$	2	-1	-0.30	-0.70
—	—	—	—	—
1	$\frac{1}{2}$	-2	-1.13	-0.87
2	$\frac{1}{2}$	-1	-0.25	-0.75
5	$\frac{1}{2}$	-0.4	-0.24	-0.64

<sup>a</sup>As  $y = \delta(e - p) - \gamma(r - Dp)$  and  $e - p = \alpha(e - w) = \alpha c$ , so  $y = \alpha\delta c - \gamma(r - Dp)$ , as shown in columns (3), (4), and (5).

<sup>b</sup>To obtain the "impact" effect on price inflation we have to subtract 1 percent from the figure shown for inflation in table 10.1 (which was measured from the new, lower equilibrium prevailing *after* the monetary slowdown of 1 percent).

percent, see column [3]). The impact of monetary policy on inflation is dramatically improved by increases in these parameters, however, with inflation falling on impact by well over the reduction in the trend of monetary growth for higher values of  $\phi$  and  $\xi$  (column [6]).

Since, according to equation (2), output depends on the level of competitiveness and on the real interest rate, the separate contributions of these two channels of monetary policy to the initial recession are shown in columns (4) and (5) of table 10.2. Because raising either of the parameters cuts inflation more quickly, it also reduces the initial shock to the real exchange rate but accentuates the effect because of the real interest rate.

In the lower panel of table 10.2 are shown the *accumulated* effects on output—a total we have used above as a measure of the costs of fighting inflation. These costs fall sharply as either  $\xi$  or  $\phi$  is increased (column [3]) but the “accumulated contribution” made by the real exchange rate falls even more sharply (column [4]).

The exact equality of “accumulated contribution” of the real interest rate (in the lower panel of the table) and the “impact” effect of the real exchange rate (in the top panel) merits specific comment. It reflects both the connection between initial shocks to the real exchange rate and expected future real interest rate differentials in models of this sort, and the particular elasticities used in the aggregate demand curve in our model. In models where anticipated exchange rate changes are taken to reflect uncovered interest differentials ( $De = r - r^*$ ), we find that current real exchange rate disequilibria reflect *anticipated* real interest rate differentials. Assuming for simplicity that there is no inflation overseas, we find that, in the absence of further shocks,

$$\int_0^{\infty} [r(t) - r^*(t) - Dp(t)] dt = -[e(0) - p(0)] = \alpha c(0),$$

where  $e - p = \alpha(e - w) = \alpha c$ . To obtain the accumulated real interest effect, the left-hand side is simply multiplied by  $-\gamma$ ; while  $\delta$  times the right-hand side shows the impact effect of the real exchange rate. As we have chosen  $\gamma = \delta$ , the two “contributions” are the same. These contributions differ however for  $\gamma \neq \delta$ , despite the relationship of equivalence between real exchange rate disequilibria and future real interest rate differentials in the model.

### 10.5 Real Exchange Rate Overshooting in Some More General Models

As a further test of the robustness of the real exchange rate overshooting proposition, we now generalize the model of equations (1), (2), (8), (9), (4<sup>'''</sup>), (5), (10a), and (10b) in a number of directions. The wage-price

sector (equations [8], [9], and [4<sup>m</sup>]) and the uncovered interest parity condition (5) are maintained throughout. The first modification consists in replacing the short real interest rate,  $r - Dp$ , in the IS curve by the long real interest rate,  $R$ . The (expected) rate of inflation is also entered as an argument in the IS and LM equations. In our second modification we add to the model wealth adjustment through the current account. The last change of specification involves replacing the assumption of instantaneous adjustment of real output by one of sluggish adjustment.

### 10.5.1 A Modified IS-LM Block

We first replace equations (1) and (2) by (28), (29), and (30):

$$(28) \quad m - p - \theta = ky - \lambda_1(r - r_d) - \lambda_2 Dp, \quad k, \lambda_1, \lambda_2 > 0.$$

$$(29) \quad y = -\gamma R + \delta_1(e - p) - \delta_2 Dp, \quad \gamma_1, \delta_1, \delta_2 > 0.$$

$$(30) \quad DR = \nu[R - (r - Dp)], \quad \nu > 0.$$

The inclusion of the rate of inflation in the money demand function allows for the possibility of substitution between money and commodities as well as between money and bonds. The negative effect of inflation on effective demand represents what Tobin has called the dynamic Pigou effect (Tobin 1975). The long real interest rate  $R$  is defined implicitly by (30). This expresses  $R$  as a forward-looking, weighted average of future short real interest rates, with exponentially declining weights:

$$R(t) = \nu \int_t^{\infty} e^{\nu(t-z)} [r(z) - Dp(z)] dz,$$

where  $\nu$  has the interpretation of the steady-state value of  $R$ . Note that  $R$  is a forward-looking or jump variable.

We consider the solution of the model of equations (28), (29), (30), (8), (9), (4<sup>m</sup>), (5), and (10a, b) for the following parameter values:  $k = 1$ ,  $\lambda_1 = 2$ ,  $\lambda_2 = 1$ ,  $\gamma = 1$ ,  $\delta_1 = .5$ ,  $\delta_2 = .25$ ,  $\nu = .05$ ,  $\alpha = .75$ ,  $\phi = .5$ , and  $\xi = .5$ . As before, we consider the effect of a previously unanticipated, immediately implemented one-point reduction in the rate of monetary growth,  $Dm$ .<sup>8</sup> The steady-state effects are easily obtained. The rate of price inflation, core inflation, wage inflation, and exchange rate depreciation decline by one point, as does the nominal interest rate. Real money balances increase by 2.83 percent, more than in the original model, because both  $r$  and  $Dp$  have a negative effect on money demand. Real output and the long-run real interest rate remain unchanged, as does the short-run real interest rate  $r - Dp$ . The real exchange rate appreciates by .67 percent: lower inflation stimulates demand via the dynamic Pigou

8. The solution is obtained by using the algorithm "Saddlepoint" of Austin and Bulter (1982).

Table 10.3 Generalized IS-LM Model

	$\ell$	$\pi$	$c$	$R$	$r$	$y$	$Dp$	$Dw$	$De$
Long-run change in (%)	2.83	-1	-.67	0	-1	0	-1	-1	-1
Initial jump in (%)	0	-.60	-4.80	.14	-.79	-1.62	-1.256	-1.413	-.79
Eigenvalues	-.2402 ± 2795i; .3022; .0457								

effect; to clear the goods market in long-run equilibrium a loss of competitiveness is required.

The impact effects and dynamics are summarized in table 10.3. There are now four state variables,  $\ell$ ,  $\pi$ ,  $c$ , and  $R$ :  $\ell$  is predetermined,  $c$  and  $R$  are pure jump variables, while  $\pi$ , as before, has its initial value determined by equation (24''). There should be two stable and two unstable characteristic roots. This is indeed the case for our choice parameter values, as can be seen from table 10.3.

Speaking loosely, the cyclical behavior imparted to the original model by the wage-price block is carried over to the present model, as evidenced by the pair of stable complex conjugate roots. The unstable root "contributed" by the long real interest rate dynamics can be identified with the small positive root .0457.

The real exchange rate appreciates on impact by 4.8 percent, which represents real exchange rate overshooting of 4.1 percent. Output declines by 1.6 percent.

### 10.5.2 Wealth Adjustment through the Current Account

Current account deficits and surpluses alter the stock of net claims on the rest of the world. If nonhuman wealth is an important argument in the output demand function and/or the money demand function, a satisfactory analysis requires the simultaneous analysis of wealth, price, and exchange rate adjustment. Analytical models such as Dornbusch and Fischer (1980) and Branson and Buiter (1983) are restricted to the analysis of the cases of perfect price flexibility (Dornbusch and Fischer, Branson and Buiter) or of a permanently fixed price level (Branson and Buiter) by the need to limit the number of state variables to two. Our numerical algorithm permits us to analyze dynamic models with many more state variables.

The "current account" model replaces (28) and (29) by (31), (32), and (33):

$$(31) \quad m - p - \theta = ky - \lambda_1(r - r_d) - \lambda_2 Dp + \lambda_3 F, \quad \lambda_3 \geq 0.$$

$$(32) \quad y = -\gamma R + \delta_1(e - p) - \delta_2 Dp + \delta_3(m - p) + \delta_4 F, \quad \delta_3, \delta_4 > 0.$$

$$(33) \quad DF = \epsilon_1(e - p) - \epsilon_2 y + \epsilon_3 F, \quad \epsilon_1, \epsilon_2 > 0; \epsilon_3 \geq 0.$$

**Table 10.4** Current Account Model; No Interest Income from Abroad in the Current Account and No Wealth Effect on Money Demand

	<i>F</i>	$\ell$	$\pi$	<i>c</i>	<i>R</i>	<i>r</i>	<i>y</i>	<i>Dp</i>	<i>Dw</i>	<i>De</i>
Long-run change in (%)	-3.1	3	-1	0	0	-1	0	-1	-1	-1
Initial jump in (%)	0	0	-.56	-4.48	.15	-.73	-1.52	-1.17	-1.31	-.73
Eigenvalues	-.2099 ± 2798i; .3102; *.0485; -.1099									

*F* denotes the level (not the logarithm) of net private sector claims on the rest of the world. Since *F* can be negative, a log-linear specification could be awkward.<sup>9</sup> Dornbusch and Fischer (1980) have a wealth effect on output demand ( $\delta_3, \delta_4 > 0$ ) but not on money demand ( $\lambda_3 = 0$ ). Note that our IS equation now includes both the static Pigou effect ( $\delta_3[m - p]$ ) and the dynamic Pigou effect ( $-\delta_2 Dp$ ). Equation (33) is the current account equation;  $\epsilon_1(e - p) - \epsilon_2 y$  is the trade balance surplus; and  $\epsilon_3 F$  is the net interest, property, and dividend income derived from the ownership of foreign assets. With  $\epsilon_3$  equal to zero, the long-run real exchange rate is invariant under all changes in exogenous variables other than capacity output. In equation (33) the long-run value of *c* is zero. With a positive value for  $\epsilon_3$  (which should be identified with  $r^*$  or *r*), competitiveness and net claims on the rest of the world are always inversely related across steady states; with  $\epsilon_3 > 0$  a larger value of *F* requires a worsening of the trade balance to maintain current account equilibrium. The complete model consists of equations (8), (9), (4'''), (5), (10a, b), (30), (31), (32), and (33).

The following parameter values are used for all current account models:  $k = 1$ ,  $\lambda_1 = 2$ ,  $\lambda_2 = 1$ ,  $\gamma = 1$ ,  $\delta_1 = .5$ ,  $\delta_2 = .25$ ,  $\delta_3 = .015$ ,  $\delta_4 = .06$ ,  $\epsilon_1 = .9$ ,  $\epsilon_2 = .6$ ,  $\alpha = .75$ ,  $\phi = .5$ ,  $\xi = .5$ ,  $\nu = .05$ .

The case of no wealth effect on money demand ( $\lambda_3 = 0$ ) and no net interest income from the rest of the world in the current account ( $\epsilon_3 = 0$ ) is considered first. The long-run and impact effects of a one percent surprise reduction in monetary growth are given in table 10.4 together with the characteristic roots of the dynamic system.

Steady-state claims on the rest of the world decline. There is no long-run change in competitiveness. *F* and  $\ell$  are predetermined. Competitiveness worsens on impact by about 4.5 percent. Core inflation declines by .56 percent. The short nominal interest rate declines, but the long real interest rate rises. Output falls by 1.5 percent. As in the previous model, the recession and the fall in core inflation contribute to reduce

9. *F* is measured in terms of units of the consumption bundle. Capital gains or losses on external assets or liabilities because of exchange rate changes and changes in the general price level are ignored.



inflation initially by more than the reduction in monetary growth. This rather implausible feature disappears when output is treated as predetermined. The current account goes into deficit after the monetary contraction, with the loss of competitiveness dominating the effect of a lower level of output. The adjustment process is cyclical, however, and the current account returns temporarily to surplus during the later phases of the adjustment process.

When we include net interest income from the rest of the world in the current account (with  $\epsilon_3 = .05$ ), the outcome is not very different from that obtained when the current account is equated with the trade balance. Table 10.5 summarizes the results. The percentage long-run reduction in net claims on the rest of the world does not differ noticeably between table 10.5 and table 10.4; the real exchange rate depreciates in the long run in table 10.5. The impact effects are virtually the same.

Including wealth as an argument in the money demand function with  $\lambda_3 = .4$  (keeping  $\epsilon_3 = .05$ ) yields the results summarized in table 10.6. Having wealth as an argument in the LM equation reduces the long-run increase in real money balances associated with a one-point reduction in the monetary growth rate. The reason is that  $F$  declines, thus reducing the demand for real money balances. As a consequence, the magnitude of the initial changes in  $\pi$ ,  $c$ ,  $R$ ,  $r$ ,  $y$ ,  $Dp$ ,  $Dw$ , and  $De$  are all smaller than in the versions of the model that did not have wealth in the money demand function. Qualitatively, however, the behavior of this model is the same as that of the earlier ones.

**Table 10.5** Current Account Model; Interest Income from Abroad in the Current Account but No Wealth Effect on Money Demand

	$F$	$\ell$	$\pi$	$c$	$R$	$r$	$y$	$Dp$	$Dw$	$De$
Long-run change in (%)	-3.1	3.17	-1	.68	0	-1	0	-1	-1	-1
Initial jump in (%)	0	0	-.56	-4.41	.16	-.73	-1.51	-1.16	-1.31	-.73
Eigenvalues	-.2128 ± .2759i; .3120; .05; -.0574									

**Table 10.6** Current Account Model; Interest Income from Abroad in the Current Account and a Wealth Effect on Money Demand

	$F$	$\ell$	$\pi$	$c$	$R$	$r$	$y$	$Dp$	$Dw$	$De$
Long-run change in (%)	-3.1	.06	-1	.57	0	-1	0	-1	-1	-1
Initial jump in (%)	0	0	-.36	-2.85	.10	-.47	-.97	-.75	-.84	-.47
Eigenvalues	-.2485 ± .1968i; .3966; .0501; -.0646									

**Table 10.7** Sluggish Output Model; Interest Income from Abroad in the Current Account but no Wealth Effect on Money Demand

	<i>F</i>	<i>ℓ</i>	<i>π</i>	<i>c</i>	<i>R</i>	<i>r</i>	<i>y</i>	<i>Dp</i>	<i>Dw</i>	<i>De</i>
Long-run change in (%)	-3.1	3.17	-1	.68	0	-1	0	-1	-1	-1
Initial jump in (%)	0	0	-.57	-4.56	.16	-.31	0	-.51	-.57	-.32
Eigenvalues	-.1986 ± .3049i; .3072; .05; -.0572; -2.0807									

### 10.5.3 Sluggish Output Adjustment

In all models considered thus far, the level of output adjusts instantaneously. Monetary contraction, for example, brings about an immediate fall in output. This is clearly an undesirable property of these models: the multiplier process takes time. The simplest way of modeling sluggish output adjustment is to treat the level of output as predetermined and make its rate of change an increasing function of "excess demand," as in equation (34):

$$(34) \quad Dy = \sigma[-\gamma R + \delta_1(e - p) - \delta_2 Dp + \delta_3(m - p) + \delta_4 F - y], \sigma > 0.$$

Substituting (34) for (32) in the models of the previous subsection, we now have a model with six state variables:  $y$ ,  $F$ ,  $\ell$ ,  $\pi$ ,  $c$ , and  $R$ . Variables  $y$ ,  $F$ , and  $\ell$  are predetermined;  $c$  and  $R$  are jump variables; and  $\pi$  is, as before, a "backward-looking" jump variable whose initial value is determined by equation (24").

The model should have four stable and two unstable characteristic roots. Table 10.7 shows that, for the parameter values  $k = 1$ ,  $\lambda_1 = 2$ ,  $\lambda_2 = 1$ ,  $\lambda_3 = 0$ ,  $\gamma = 1$ ,  $\delta_1 = .5$ ,  $\delta_2 = .25$ ,  $\delta_3 = .015$ ,  $\delta_4 = .06$ ,  $\alpha = .75$ ,  $\phi = .5$ ,  $\xi = .5$ ,  $\nu = .05$ ,  $\epsilon_1 = .9$ ,  $\epsilon_2 = .6$ ,  $\epsilon_3 = .05$  and  $\sigma = 2$ , this is indeed the case.<sup>10</sup>

There still is a significant loss of competitiveness in response to a one-point monetary growth deceleration. Because output is predetermined, however, the impact effect on the rate of inflation is smaller than in the case where output can respond instantaneously. Output declines gradually and converges cyclically to its constant capacity level.

Adding wealth as an argument in the money demand function with  $\lambda_3 = .4$  again weakens the long-run effect of monetary deceleration on  $\ell$  and  $c$  and reduces the magnitudes of the impact effects on all non-

10. For reasons of space, only the version with net interest income from the rest of the world in current account is considered. A smaller value of  $\sigma$  reduces the magnitude of the largest stable characteristic root but does not significantly affect the initial changes in the nonpredetermined variables.

predetermined variables without changing the qualitative behavior of the model. This is shown in table 10.8.

This "sensitivity analysis" of the real exchange rate overshooting proposition convinces us that the simple model of equations (1), (2), (8), (9), (4<sup>m</sup>), (5), and (10a, b) is not a bad first approximation. The rest of the analysis is therefore conducted in terms of this model.

### 10.6 Alternative Policies

A sudden loss of competitiveness in response to monetary disinflation was found to be a common characteristic of the various model specifications constructed above. To see how much the initial loss of competitiveness contributes to the speed with which inflation is reduced (and to reducing the output costs of eliminating inflation), we now consider policies where competitiveness is kept *constant*.

For this exercise we return to the central numerical example of section 10.4. The consequences of monetary disinflation shown there (in the upper panel of table 10.1, for example) will be referred to for brevity as the results of pursuing policy A: a policy of permanently slowing the rate of monetary growth without prior warning and without complementary changes in fiscal or exchange rate policy.

If, instead, competitiveness is to be held constant at its equilibrium value, the price level and the wage rate will move together, as can be seen from equation (8). As a consequence, the inflation process in the model reduces to the familiar augmented Phillips curve commonly used to characterize closed economies. We get

$$(35) \quad Dp = \phi y + \pi.$$

$$(4^m) \quad D\pi = \xi(Dp - \pi).$$

The stabilization of the real exchange rate precludes both the discrete adjustment to the core rate of inflation,  $\pi$ , which was a feature of the previous policy, as well as any discrete jumps in the price level. Both  $\pi$  and  $p$  are predetermined. Given the initial level of core inflation  $\pi(0) = \bar{\mu} = 1.0$ , the initial value of the price level  $p(0)$ , and a path for real

**Table 10.8** Sluggish Output Model; Interest Income from Abroad in the Current Account and a Wealth Effect on Money Demand

	<i>F</i>	<i>ℓ</i>	$\pi$	<i>c</i>	<i>R</i>	<i>r</i>	<i>y</i>	<i>Dp</i>	<i>Dw</i>	<i>De</i>
Long-run change in (%)	-3.1	.06	-1	.57	0	-1	0	-1	-1	-1
Initial jump in (%)	0	0	-.35	-2.82	.10	-.20	0	-.31	-.35	-.20
Eigenvalues	-.2288 ± .2337i; .3980; .0501; -.0639; -2.1045									

output, equations (35) and (4''') will by themselves generate the entire path to be followed by the price level. Note that, by construction, money has become a complete "sideshow." One transmission mechanism, the real balance effect, has been ruled out from the start. Now that the real exchange rate and thus the real interest rate is also kept constant, there is no feedback from money on real output. Since the model has the standard homogeneity properties, it will of course still be true that in the long run the rate of inflation equals the rate of growth of the money supply. As will become clear, however, the truism that "inflation is always and everywhere a monetary phenomenon" should in the current example be turned around: money here is an inflation phenomenon.

The first alternative policy to be considered (called B) takes the path of output to be *precisely the same* as that generated by the monetary contraction described in section 10.4 with policy A. This can be achieved, for example, by adding a fiscal instrument  $g$  to the IS curve, so that

$$(36) \quad y = \delta\alpha c - \gamma(r - Dp) + \eta g, \quad \eta > 0.$$

With  $c$  kept constant at its equilibrium value  $\gamma r^*/\alpha\delta$ , we can duplicate policy A's real output path by making  $\eta g_B$  follow the exact path of  $\delta\alpha c_A - \gamma(r_A - Dp_A) - \gamma(1 - \alpha\delta)r^*/\alpha\delta$  under A.<sup>11</sup> The real exchange rate stabilization together with the free movement of capital implies that  $De = Dw = Dp$ . To so manage the exchange rate, the nominal money stock is endogenously determined. When  $De = Dw = Dp$ , the nominal interest rate is given by  $r = r^* + De = r^* + Dp$ . The LM equation (1) then becomes

$$(37) \quad m = p - \lambda(r^* + Dp) + ky = p - \lambda r^* + (k - \lambda\phi)y - \lambda\pi.$$

With  $p$  and  $\pi$  predetermined and  $y$  determined by (36), and with  $c = \gamma r^*/\alpha\delta$  and  $r - Dp = r^*$ , equation (37) determines the nominal money stock. (Other ways of stabilizing the real exchange rate, such as a variable tax on capital inflows [subsidy on outflows], can be thought of.) In the present example the inflation generated by equations (35), (4'''), and (36) is the dog wagging the money supply tail through equation (37). Given equa-

11. The constant level of the real exchange rate to be maintained under policy B is that prevailing in equilibrium under policy A, that is, where  $y = \delta\alpha c - \gamma(r - Dp) = 0$ , and  $Dc = 0$ . These conditions imply  $De = Dw = Dp = r - r^*$ , so the equilibrium rate is found to be  $\bar{c} = \gamma r^*/\alpha\delta$ . If capital movements are not to be constrained under policy B but yet the rate is to be stabilized at  $\bar{c}$ , then one must ensure that over time  $De = Dp = r - r^*$ . To find the path of  $\eta g$  which will achieve this, we equate

$$y_A = \delta\alpha c_A - \gamma(r_A - Dp_A) = \delta\alpha\bar{c} - \gamma r^* + \eta g_B = y_B,$$

which implies

$$\eta g = \delta\alpha c_A - \gamma(r_A - Dp_A) - \gamma(1 - \alpha\delta)r^*/\alpha\delta,$$

the condition shown in the text.

tions (35) and (4<sup>m</sup>), it will of course always be the case that the path of inflation is determined once a path for output is specified.

The results of the alternative policy for inflation are shown in the bottom row of table 10.1. Inflation starts at a significantly higher level than with policy A because the starting value for core inflation  $\pi$  is now  $\bar{\mu} = 1.0$ . The path followed by inflation has the same damping factor  $\rho$  and frequency  $\omega$  as the output path, but the amplitude is smaller and the inflation cycle leads the output cycle. The rate of inflation will, as before, be reduced to zero at a net cost of four point-years of output. The price level toward which the system converges under the alternative policy is, however, higher than the long-run price level of policy A. This can be seen from the coefficients in table 10.1. While  $\int_0^{\infty} Dp_A(t) dt = -2$  for policy A, as we have already discussed, integrating the path for inflation under policy B yields a smaller fall, as follows:

$$\int_0^{\infty} Dp_B(t) dt = \frac{-\rho \cdot 0.5 - 0.7302\omega}{\rho^2 + \omega^2} = -1.$$

These results are illustrated in figure 10.6 where the path followed by the price level and the rate of inflation under policy B are plotted alongside those already described for policy A. In the top panel the price level under the alternative policy proceeds from point B, without any "jump," along a path which cycles around a steady-state level which is one point below the line BF showing the level of the money stock, which is one point above the corresponding steady state shown for policy A.

In the bottom panel the rate of inflation is shown starting at point B and cycling toward zero along the path *BB*. Thus inflation starts at a higher level under the alternative policy than under the floating exchange rate case. The determination of those "starting values" and the subsequent comparison of inflation can be seen from figure 10.7. There, labeled  $SRPC_B$ , is the "short-run Phillips curve" which determines initial inflation under policy B, where  $\pi_B(0) = \bar{\mu}$ . This value of  $\pi$  determines the intercept of  $SRPC_B$ , and the value of  $y(0)$  determines the value of inflation shown as  $Dp_B(0)$ . From this point inflation and output cycle toward the origin, as shown by the path labeled *BB*.

By contrast the relationship determining inflation under policy A, (after the initial jump at time zero),

$$Dp_A(0) = \phi y(0) + \pi_A(0) + (1 - \alpha)Dc(0) = \phi y(0) + \pi_A(0) \text{ as } Dc(0) = 0,$$

yields the Phillips curve shown as  $SRPC_A$  which has an intercept of  $\pi_A(0)$  which is lower than  $\pi_B(0)$  because of the jump induced by the revaluation of the currency at the inception of the monetary slowdown under floating rates. Thereafter inflation falls away following the path shown as *AA*.

What is apparent from the above is that the policy of fighting inflation by cycles in output and in the real exchange rate (with an initial recession

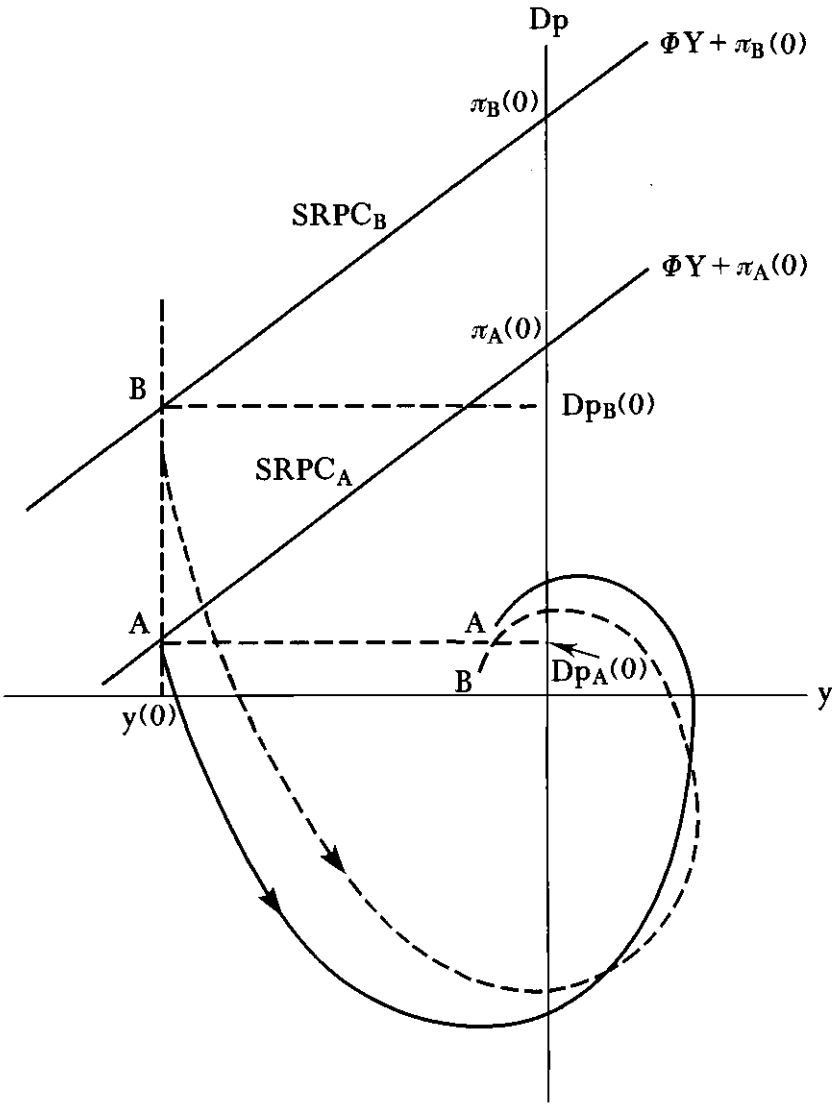


Fig. 10.7 Inflation under the two policies.

associated with an overvalued exchange rate) does not lead to any change in long-run inflation, compared to the same output cycle and a stable real exchange rate. The loss of competitiveness does, however, reduce inflation more quickly early on, as shown in figure 10.7; the early lead established by this policy over the alternative is whittled away later when competitiveness is regained in the boom, but we are left with the conclusions that inflation is brought down more quickly with policy A, as shown

by the dotted line in figure 10.7. (Inflation under the alternative policy is shown by the solid line.)

The fluctuations of the real exchange rate can therefore be seen to have effects on inflation not unlike those attributed to temporary incomes policies by those who argue that the latter hold down inflation in the short run, but have *no* effect on the inflation rate in the long run. If that is true, then a temporary bout of incomes policy would change only the long-run price level without changing the long-run growth of prices, which is what we have found to be characteristic of the policy of permitting the real exchange rate to vary.

### 10.6.1 "Efficient Disinflationary Policies"<sup>12</sup>

Once we permit changes in the indirect tax rate,  $\theta$ , we have a way of costlessly reducing inflation. Consider the money demand function and the wage equation:

$$m - p - \theta = -\lambda r + ky.$$

$$Dw = \phi y + \pi.$$

Given a reduction in the rate of monetary growth by  $d\mu = \bar{\mu} - \mu$ , we can easily calculate the change in  $\theta$  required to jump  $\pi$  to its new long-run equilibrium value,  $\bar{\mu}$ . Holding  $p$  constant, equation (24) shows that  $d\pi(t) = \xi d\theta(t)$ . The required change in  $\theta$  is therefore given by

$$d\theta = \xi^{-1} d\pi = \xi^{-1} d\mu.$$

From the money market equilibrium condition, we obtain that, since the long-run interest rate changes in line with  $\mu$ ,

$$d(m - p - \theta) = -\lambda dr = -\lambda d\mu.$$

Holding  $p$  constant, this yields

$$dm - d\theta = -\lambda d\mu,$$

or

$$dm = (-\lambda + \xi^{-1})d\mu.$$

Therefore a change in the rate of monetary growth by  $d\mu$ , accompanied by a change in indirect taxes by  $\xi^{-1}d\mu$  and a change in the *level* of the nominal money stock of  $(-\lambda + \xi^{-1})d\mu$ , will immediately move the system to the new steady-state equilibrium with the lower rate of inflation. Output and the real exchange rate are unaffected. With our choice of parameter values ( $\lambda = 2$  and  $\xi = .5$ ),  $d\theta = 2d\mu$  and  $dm = 0$ .

The mechanism permitting the required reduction in inflation to be brought about without loss of output is the following. First, the cut in  $\theta$  (in general, together with the change in  $m$ ) permits the long-run change in

12. See Okun (1978).

the stock of real money balances to be brought about immediately without any need for a jump in nominal prices, including the exchange rate. Second, the cut in  $\theta$  achieves the immediate change in the core rate of inflation to its new long-run value. The use of indirect taxes to facilitate the process of bringing down inflation has been specifically advocated by Okun (1978).

Alternatively, incomes policy can be used to jump  $\pi$ . If *incomes policy* can be identified with a once-and-for-all reduction in  $\pi$ , without any other "overwriting" of the behavioral equations of the model, then it will lower the cost of disinflation. If *monetary policy* changes or announcements themselves directly change  $\pi$ , as is the case in the model of section 10.2, then disinflationary monetary policies will also be "efficient," in the sense that we are using that term here.

## 10.7 Conclusion

After a summary of various approaches to the modeling of the inflationary process in an open economy with a floating exchange rate, we have studied the way in which a monetary slowdown might be expected to work in an economy where the money wage is predetermined and core inflation is sluggish and adjusts to actual inflation with a significant lag. The proposition that real exchange rate overshooting occurs in response to nominal shocks is found to survive a wide range of generalizations of the Dornbusch (1976) model. These include the addition of static and dynamic Pigou effects to the IS curve, the inclusion of wealth adjustment through the current account, and the replacement of instantaneous output adjustment by a gradual process.

Despite the sluggishness in the core inflation process, we found that core inflation can be reduced quickly by jumps in the price level induced by jumps in the exchange rate. In the many variants of the model used to focus on this particular aspect of the monetary transmission mechanism, we found that a monetary slowdown will cut inflation promptly via its impact on the nominal and the real exchange rate. Indeed, in our central numerical example of sections 10.4 and 10.6, the inflation rate responds *immediately* by almost the full extent of the monetary slowdown! This reduction of inflation follows, in our example, from the effects of announced monetary policy on the exchange rate and, less significantly, on the current real interest rate. The appreciation of the exchange rate cuts the inflation rate in two ways, first by reducing core inflation, and second by cutting the level of output. Both of these effects involve sharp changes in the real exchange rate.

Since in the central example money is "superneutral," however, such changes in the real exchange rate and in real output must ultimately be reversed. Pursuit of a constant growth rate of money generates a cyclical



convergence for these variables. The net output costs associated with a steady-state reduction in inflation are found to be given by a simple formula,  $d\mu/\xi\phi$ , where  $d\mu$  shows the change in steady-state inflation,  $\phi$  is the slope of the short-run Phillips curve, and  $\xi$  measures the speed of adjustment of core inflation. Thus for  $\xi = \phi = 1/2$ , the net output loss associated with reducing monetary growth and steady-state inflation by 1 percent is 4 point-years of GNP.

For comparison we considered an alternative where the real exchange rate was held constant but output was constrained to follow the same path as before. Such an alternative, whose net output loss is, of course, identical, was found to achieve the same effect on steady-state inflation, but the path taken by inflation was different. The prompt anti-inflation success from the loss of competitiveness being absent, inflation starts from a higher level under this alternative.

In a superneutral model, with core inflation modeled in the way we have, it turns out that *any* path for output which exhibits a cumulative four point-year loss of output will (if  $\xi\phi = 1/4$ ) reduce steady-state inflation by 1 percent, irrespective of the path taken by the real exchange rate (provided it starts and finishes at the same level). Thus the freedom to vary the real exchange rate to reduce inflation does not succeed in reducing the output costs of changing steady-state inflation; it does, however, change the time path of inflation relative to other policies which exhibit the same output path.

While the numerical model makes no claim to being a realistic model of the United Kingdom, we would point out that the sort of output costs associated with reductions in medium-term inflation in Treasury evidence to the Treasury Committee suggested a figure of four point-years of output for each one point off medium-term inflation. A more detailed analysis of simulations on an earlier version of the Treasury model, when the slope of the Phillips curve was flatter and the mean lag of core inflation longer, showed even higher costs (see Miller 1979).

In considering "efficient" disinflationary policies, we noted that a cut in *indirect taxes* could reduce the output costs of curing inflation by securing an immediate jump reduction in the price index at market prices and in the core rate of inflation. In our model a reduction in the rate of growth of money by  $d\mu$  accompanied by a cut in indirect taxes of  $\xi^{-1}d\mu$  will immediately and costlessly achieve a reduction in steady-state inflation of  $d\mu$ . (In general, a change in the *level* of the money stock will also be required.)

In the United Kingdom, a one point cut in value-added tax (VAT) is reckoned to cut the rate of indirect taxation by half a point. A four point reduction in VAT would therefore avoid the four point year of output loss otherwise associated with a point reduction in monetary approach.

The Thatcher administration's decision to raise VAT by eight points early in its first term of office, at the same time that a program of successive reductions in monetary growth was announced, would in our model increase the cost of bringing down inflation. In the short run, however, the adverse consequences of the VAT increase on the price level are countered by the appreciation of the exchange rate.

Those who argue that *incomes policies* can secure a step reduction in core inflation would, of course, advocate their use as a way of cutting the output costs of reducing inflation (see, for example, Tobin 1977). We have not examined this case in detail in this paper. We have, however, considered (in Buiter and Miller 1981a) the possibility that announced *monetary* policy could immediately and directly reduce core inflation in just such a fashion. If announced monetary policy has this sort of direct expectational effect, then it will save output costs, just as a similarly successful incomes policy would. If, however, monetary targets only secure immediate effects on core inflation by a sudden loss of competitiveness, this will not constitute an "efficient" way of reducing inflation. In any case, full immediate adjustment of core inflation to its new equilibrium value is not sufficient in our model for avoiding all output costs of bringing down inflation. The long-run increase in the stock of real balances associated with a reduction in the rate of growth of the money stock requires an immediate increase in the *level* of the nominal money stock path if the costly alternative of lowering the level of the path of the sluggish money wage is to be avoided.

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## Comment Robert P. Flood

The NBER Conference on Exchange Rates and International Macroeconomics has presented me with the welcome opportunity to comment formally, for the second time, on the work of Buiter and Miller (B-M)<sup>1</sup> concerning the output cost of disinflation. In my first comment<sup>2</sup> I discussed some technical aspects of the sticky-price rule adopted by B-M. Presently, however, I will discuss some deeper aspects of the B-M methodology, which are typical in modern macroeconomics. In particular, B-M assume that an elected policy authority may reduce drastically a country's rate of monetary growth without opposition from other important political groups in the country. This assumption manifests itself when B-M solve their model conditional on agents' believing that the current policy regime will last indefinitely.

An alternative methodology is to model agents' beliefs concerning the possibility that the current political regime will not last indefinitely. In particular, if a regime's policies are perceived to be too costly, then the regime may be ousted at the next election or the regime may disavow its former policies in order to be reelected. In either case, a policy which is perceived to be too costly is reversed at the next election.

When a political regime comes to power and attempts to lower the inflation rate by lowering the money growth rate, the effectiveness of their policy will depend on two elements: first, their ability actually to lower the money growth rate, and second, their ability to convince agents that the lowering of money growth is permanent. Since a regime can be ousted, a fine line must be tread: the lower the money growth rate, the higher the probability agents attach to the demise of the current regime at the next election. If the opposing political group favors a higher money growth rate than does the current regime, then raising the probability of the current regime's demise will raise the expected long-term rate of money growth. Thus, a lower current rate of money growth may raise the expected long-term rate of money growth and may not have the desired effect of lowering the inflation rate.

Sargent (1981) has discussed the importance of a country's political climate as a precondition for a successful low-cost transition from high

Robert P. Flood is a professor in the Department of Economics at Northwestern University and a research associate of the National Bureau of Economic Research.

The views expressed here are those of the author and do not necessarily reflect the views of the Governors of the Federal Reserve System or other members of their staff. The author would like to thank the National Science Foundation for support and Robert Hodrick for comments.

1. This comment is intended to apply equally to Buiter and Miller (1982) and Buiter and Miller in this volume.
2. My first comment is Flood (1982).

inflation to low inflation. My intention in this comment is to provide a formal model of the political-economic interaction determining the success or failure of a political regime's attempt to reduce a country's inflation rate. The methods I use are drawn from a series of papers I have written with Peter Garber; especially relevant is Flood and Garber (1982).

### A Model with Political Stochastic Process Switching

The framework for analysis is a stripped-down version of the B-M model. Its primary equations are:

$$(1) \quad m_t - p_t = -\lambda(E_t e_{t+1} - e_t), \quad \lambda > 0,$$

$$(2) \quad p_{t+1} - p_t = \phi y_t + E_t \bar{p}_{t+1} - \bar{p}_t, \quad \phi > 0,$$

$$(3) \quad y_t = \delta(e_t - p_t), \quad \delta > 0,$$

where, in logarithms,  $m_t$  = money stock;  $p_t$  = home goods price level;  $e_t$  = exchange rate;  $y_t$  = demand for home goods;  $\bar{p}_t$  = the value of  $p_t$  such that  $y_t = 0$ ; and  $E_t$  = mathematical expectation operator conditioned by full information at time  $t$ , which includes all variables dated  $t$  or earlier. Equation (1) gives money market equilibrium; the supply of real money balances ( $m_t - p_t$ ) equals the demand for them  $[-\lambda(E_t e_{t+1} - e_t)]$ .<sup>3</sup> Equation (2) is a price adjustment function stating that inflation ( $p_{t+1} - p_t$ ) equals a function of excess demand for goods ( $\phi y_t$ ) plus a "core" inflation rate,  $E_t \bar{p}_{t+1} - \bar{p}_t$ . The variable  $y_t$  is the excess demand for home goods, since the natural rate of output is normalized to zero;  $p_t$  is a predetermined variable, and output is assumed to be equal to the demand for it,  $y_t$ . Equation (3) specifies that output demand depends on the relative price of the home goods. The foreign price of foreign goods is constant and normalized to zero.

Since my version of the model appears to be quite different from that of B-M, I will describe it in more detail. First, I have used a discrete-time model, while B-M use a continuous-time one. This alteration is intended only to facilitate the stochastic analysis which follows. Second, I have ignored the foreign interest rate and a scale variable in the money demand function and the real rate of interest in the demand function for home goods; these are first-pass analytical simplifications. Third, the "core inflation" term in (2),  $E_t \bar{p}_{t+1} - \bar{p}_t$ , reflects my own preferred specification rather than that of B-M. In the present setup,  $\bar{p}_t = e_t$ .<sup>4</sup>

3. I have followed B-M, section 10.2, in deflating nominal money balances by the domestic price of home goods. Apparently, B-M follow Dornbusch (1976) in adopting this simplification. Using this deflator leads to the B-M, section 10.2, definition of inflation,  $p_{t+1} - p_t$ , which I also use. The algebra of the model would be more difficult if a consumer price index deflated money balances and defined inflation. However, such complications do not reverse B-M's results or mine; thus they are avoided.

4. The price adjustment rule I use is based largely on the work of Michael Mussa and is applied to the open economy in his 1982 paper.

In this model,  $e_t$  is a “jumping” variable and  $p_t$  is a predetermined variable. Since  $\bar{p}_t = e_t$ ,  $\bar{p}_t$  also is a “jumping” variable.

The only exogenous variable in the model is the money supply. Thus, an exchange rate solution for the model is an expression giving  $e_t$  as a function of the predetermined variable  $p_t$  and all expected future values of  $m_t$ . Such a solution is

$$(4) \quad e_t = \beta p_t + \sum_{i=0}^{\infty} \gamma_i E_t m_{t+i},$$

where

$$\beta = \frac{-1}{\delta\phi\lambda} < 0,$$

$$\gamma_0 = \frac{1 + (1/\delta\phi\lambda)}{1 + \lambda},$$

$$\gamma_i = \gamma_0 \left( \frac{\lambda}{1 + \lambda} \right)^i; \quad i = 0, 1, 2, \dots$$

Note that  $\sum_{i=0}^{\infty} \gamma_i = 1 + (1/\delta\phi\lambda) > 1$ , as in Dornbusch (1976).

I assume that two money supply processes are relevant to agents’ calculations of the infinite sum in (4). The current regime expands money according to

$$(5) \quad m_{t+i} = \mu_1 + m_{t+i-1} + v_{t+i}, \quad i = 1, 2, 3, \dots,$$

where  $v_{t+i}$  is a zero mean disturbance which is not serially correlated. The opposition regime, if in power, would expand money according to

$$(6) \quad m_{t+i} = \mu_2 + m_{t+i-1} + v_{t+i}; \quad i = 1, 2, 3, \dots, \mu_2 \geq \mu_1.$$

The current regime, though, is definitely in power from  $t$  to  $t + 1$ . However, at  $t + 1$  there will be an election and the current regime will be deposed with probability  $\pi$ , where  $\pi$  is the probability agents at  $t$  attach to the current regime’s being defeated in the  $t + 1$  election. For simplicity, I assume that the  $t + 1$  election is the only future election.<sup>5</sup> The probability  $\pi$  is determined by a simple model of the political process. In particular,

$$(7) \quad \pi = pr\{[y_{t+1}|OR] < \bar{y}\},$$

where  $pr\{x < y\}$  is the probability that  $x$  is less than  $y$  for any  $x$  and  $y$ . Thus,  $\pi$  is the probability that  $y_{t+1}$ , conditional on the continuation of the old regime,  $[y_{t+1}|OR]$ , is less than the minimum level required to maintain the current regime’s majority,  $\bar{y}$ . Intuitively, (7) formalizes a political process which works as follows. The electorate at  $t + 1$  examines the state

5. Allowing a single election makes the present setup a special case of Flood and Garber (1982). My paper with Garber is more general than the present model in that our joint work allows calculation of an equilibrium when the endogenous stochastic variable ( $y_{t+i}$ ) may pass a critical barrier ( $\bar{y}$ ) at any future date.

of the world at  $t + 1$  and calculates the level of  $y_{t+1}$  which would prevail if the old regime were left in power. If this conditional  $y_{t+1}$  is below the minimum ( $y, \bar{y}$ ) required to preserve the current regime's majority, then the electorate ousts the old regime and elects a new regime. By assumption, the new regime will follow a monetary policy no more restrictive than that of the old regime.

To complete the solution of the model, we must calculate  $\pi$  and use  $\pi$  in the infinite sum in (4) to find  $e_t$ . Once  $e_t$  is found as a function of the current state, we may use the expression for  $e_t$  in (2) to find the inflation rate. The first task then is to determine  $\pi$ .

Forwarding equation (3) and using the definition of  $\pi$  from equation (7), obtain

$$(8) \quad \pi = pr\{\delta([e_{t+1}|\text{OR}] - p_{t+1}) < \bar{y}\},$$

where  $[e_{t+1}|\text{OR}]$  is the value of  $e_{t+1}$  conditional on the old regime's continuing in power at  $t + 1$ ;  $p_{t+1}$  is determined at  $t$  and is an unconditional realization. To find  $[e_{t+1}|\text{OR}]$ , use (5) in (4), both forwarded appropriately, to obtain

$$(9) \quad [e_{t+1}|\text{OR}] = \beta p_{t+1} + (1 - \beta)m_{t+1} + (1 - \beta)\mu_1\lambda.$$

Since  $m_{t+1}$  is generated by equation (5), we may use (9) and (5) in (8) to obtain

$$(10) \quad \pi = pr[v_{t+1} < Z + p_{t+1} - m_t - \mu_1(1 + \lambda)],$$

where  $Z \equiv \bar{y}/\delta(1 - \beta)$ .

To make more progress, a specific assumption must be made about the probability density function (p.d.f.) governing  $v_{t+1}$ . In particular, I assume that  $v_{t+i}$  ( $i = 0, 1, 2, \dots$ ) has a uniform p.d.f., which is

$$(11) \quad \begin{aligned} f(v_{t+i}) &= 1/(2\eta); & -\eta \leq v_{t+i} \leq \eta, & i = 1, 2, 3, \dots, \\ f(v_{t+i}) &= 0; & v_{t+i} > \eta, & i = 1, 2, 3, \dots, \\ & & v_{t+i} < -\eta, & i = 1, 2, 3, \dots. \end{aligned}$$

It follows that the probability that  $v_{t+1}$  is less than any number  $x$  is given by

$$(12) \quad pr(v_{t+1} < x) = \int_{-\eta}^x \frac{1}{2\eta} dv_{t+1} = \frac{x + \eta}{2\eta},$$

when  $-\eta \leq x \leq \eta$ . If  $x$  lies outside the  $[-\eta, \eta]$  bounds, then  $pr(v_{t+1} < x)$  becomes either 0 or 1 depending on which bound is violated. In what follows I assume that  $\eta$  is large enough that the formula given in (12) is applicable.

Combine (12) and (10) to obtain

$$(13) \quad \pi = \frac{1}{2\eta} [\eta + Z + p_{t+1} - m_t - \mu_1(1 + \lambda)].$$

The terms  $m_t$ ,  $\eta$ ,  $Z$ , and  $\mu_1(1 + \lambda)$  are either state variables at  $t$  or parameters; however,  $p_{t+1}$  is an endogenous variable at  $t$  which may be expressed in terms of the state variables and parameters.

Using equations (2), (3), and the definition of  $\bar{p}_{t+i}$ , obtain

$$(14) \quad p_{t+1} = (1 - \theta)p_t + (\theta - 1)e_t + E_t e_{t+1},$$

where  $\theta \equiv \phi\delta$  and the stability of the model requires  $\theta < 2$ , which I assume.<sup>6</sup> Equation (4) allows us to express  $e_t$  and  $E_t e_{t+1}$  as functions of  $p_t$ ,  $p_{t+1}$  and expected future values of money. These expressions are

$$(15) \quad \begin{aligned} e_t &= \beta p_t + \gamma_0 m_t + \gamma_1 E_t m_{t+1} \\ &+ (1 - \pi) \sum_{i=2}^{\infty} \gamma_i E_t(m_{t+i} | \text{OR}) \\ &+ \pi \sum_{i=2}^{\infty} \gamma_i E_t(m_{t+i} | \text{NR}), \end{aligned}$$

and

$$(16) \quad \begin{aligned} E_t e_{t+1} &= \beta p_{t+1} + \gamma_0 E_t m_{t+1} \\ &+ (1 - \pi) \frac{1 + \lambda}{\lambda} \sum_{i=2}^{\infty} \gamma_i E_t(m_{t+i} | \text{OR}) \\ &+ \pi \frac{(1 + \lambda)}{\lambda} \sum_{i=2}^{\infty} \gamma_i E_t(m_{t+i} | \text{NR}), \end{aligned}$$

where  $E_t(m_{t+i} | \text{OR})$  and  $E_t(m_{t+i} | \text{NR})$  are expectations of  $m_{t+i}$  conditioned on time  $t$  information and on the old regime (OR) and new regime (NR), respectively. The law of iterated expectations allows us to convert the  $t + 1$  expectations in the expression for  $e_{t+1}$  into  $t$  expectations in (16).

Under condition (OR), equation (5) governs  $m_{t+i}$ , and under (NR) equation (6) governs  $m_{t+i}$ . Use this information in (15) and (16) and combine those results in (14) to obtain

$$(17) \quad \begin{aligned} p_{t+1} &= (1 - \theta)p_t + \theta m_t \\ &+ \theta(1 - \beta)\lambda\mu_1 + \frac{\theta(1 - \beta)\lambda^2\pi(\mu_2 - \mu_1)}{1 + \lambda}. \end{aligned}$$

To determine  $\pi$ , substitute (17) into (13) and rearrange to yield

6. This is not a stability condition in the sense of assuming away "bubbles." It is a condition for the stability of "market fundamentals" in the sense of Flood and Garber (1980). To see this point clearly, examine equation (17) with  $\mu_1 = \mu_2 = 0$ .



$$(18) \quad \pi = \frac{(1 + \lambda)[\eta + Z + (1 - \theta)(p_t - m_t - \lambda\mu_1)]}{2\eta(1 + \lambda) + \lambda^2\theta(1 - \beta)(\mu_1 - \mu_2)}.$$

In interpreting (18), recall that  $\eta$  must be large enough to ensure  $-\eta \leq Z + p_{t+1} - m_t - \mu_1(1 + \lambda) \leq \eta$ . This condition, along with the stability condition  $\theta < 1$ , allows  $\partial\pi/\partial\mu_1 < 0$ , which says that a reduction in the current money growth rate increases the chance that the old regime will be voted out at the next election.

The current inflation rate is  $p_{t+1} - p_t$ . Since  $p_t$  is predetermined at  $t$ , we may use (17) to obtain

$$(19) \quad \frac{\partial(p_{t+1} - p_t)}{\partial\mu_1} = \theta(1 - \beta)\lambda \left[ \left( 1 - \frac{\pi\lambda}{1 + \lambda} \right) + \frac{(\mu_2 - \mu_1)\lambda}{1 + \lambda} \left( \frac{\partial\pi}{\partial\mu_1} \right) \right].$$

From (19), when  $\mu_1 = \mu_2$ , a reduction in current money growth ( $\mu_1$ ) must reduce inflation as  $\pi\lambda/(1 + \lambda) < 1$ . However, when  $\mu_1$  is less than  $\mu_2$ , then a further reduction in  $\mu_1$  may increase inflation as  $\partial\pi/\partial\mu_1 < 0$ . Indeed, by setting (19) equal to zero, we find that value of  $\mu_1$  which minimizes the current inflation rate. This value is obtained by substituting the derivative of (18) into (19) and equating the result to zero to yield

$$(20) \quad \hat{\mu}_1 = \mu_2 - \frac{2\eta[1 + (1 - \hat{\pi})\lambda]}{\lambda(1 + \lambda)},$$

where  $\hat{\mu}_1$  is the inflation minimizing value of  $\mu_1$  and  $\hat{\pi} = \pi(\hat{\mu}_1)$ .

### Final Remarks

This comment is not an appropriate place to draw out the nonstandard implications and interpretations of the type of political-based stochastic process switching model presented here. However, this simple model does serve to point out the importance of such considerations. Few propositions in monetary economics are as widely accepted as the proposition that decreasing the money growth rate will lower the inflation rate. However, it has been shown presently, in a simple example, that when an opposition party is lurking in the background with a policy more expansionary than that of the current regime, reducing the money growth rate need not reduce inflation.

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## Comment      Jürg Niehans

It is a pleasure to comment on the interesting and important paper by Buiter and Miller. The paper is rich both in analytical content and in policy implications, and it contains the seeds for fruitful modifications and elaborations. It can generally be described as an effort to apply the underemployment version of Dornbusch's exchange dynamics model to the stylized facts of British monetary policy. By and large, this effort is successful and I believe with relatively few modifications and extensions it could be made even more successful. The first four of my comments concern the policy implications of the paper, while the remaining comments have to do with the way it stylizes the facts.

1. In comparing policy alternatives, Buiter and Miller use the gain or loss of output, measured in percent years, as their main yardstick. They estimate that a 10 percent reduction of inflation by a retardation of monetary expansion costs 4 percent of output for ten years. This figure seems high to me, but it should be noted that Buiter and Miller do not really use it as an empirical estimate but rather as an illustrative benchmark, based on plausible, but freely chosen parameters.

2. Buiter and Miller compare this policy with an alternative in which cuts in government expenditures are used to move output along the same track while monetary policy is reduced to the passive role of preventing exchange overshooting. They demonstrate that this "fiscal" variant reduces inflation by the same amount as the "monetary" variant but leaves the price level permanently higher. In fact, the aggregate output costs of disinflation are the same, regardless of the behavior of the exchange rate. The message from this strong result seems to be threefold. First, while the

overshooting of the exchange rate strengthens the impact effect of a disinflationary policy, this early benefit is balanced by the apparent recurrence of inflation in later stages. Second, to minimize both early illusions and later disappointments, the success or failure of disinflationary policies should be measured by their effect on "sticky" domestic prices and not on exchange-sensitive international prices. Third, once the variability of output is added to the loss function, overshooting may make things much worse, and it may be dangerous to regard it with "benign neglect." After all, governments may not survive to the later stages.

3. In the model, a reduction in money growth by 1 percent produces an instantaneous appreciation of the real exchange rate by almost 3 percent. The order of magnitude seems plausible, but while Buiter and Miller envisage monetary contraction as gradual, British monetary policy, though committed to gradualism, actually produced an instantaneous reduction by about 10 percent. According to the model, this should have produced an appreciation of sterling by 28 percent—and this is not far from what it actually did.

4. Buiter and Miller suggest that inflation can be eliminated painlessly by combining a reduction in the growth rate of the money stock with a one-time increase in its level and a one-time reduction in indirect taxes. I confess that I find this proposal unconvincing. The monetary part of the proposal reminds me of a general who orders his embattled troops to counterattack from a point ten miles behind their present lines. While it is easy to explain such an order on the map, it clearly invites disaster. The fiscal part of the proposal is based on the notion that a one-time cut in indirect taxes lowers the rate of expected inflation, which I find hard to understand.

5. My remaining comments have to do with the way Buiter and Miller model British monetary policy. They define the money stock as something like sterling M3. Disinflation policy is described as a retardation in the supply of this aggregate, and it is shown how this would produce a real appreciation of sterling and a recession in output. In fact, in 1979–80 the Thatcher government produced an acceleration in the supply of £M3 by about 50 percent. But far from producing a real depreciation of sterling and a stimulation of output, the effect was similar to what Buiter and Miller expect from monetary contraction. I conclude that there must be some flaw in the specification of the money supply. The problem disappears if the money supply is reinterpreted as the monetary base (or perhaps M1). As a matter of fact, the Thatcher government, largely unintentionally, reduced the rate of expansion of base money drastically, cutting it from about 14 percent to less than 4 percent virtually overnight. Overshooting and recession appear as the normal concomitants of such abrupt contraction, just as indicated by the model.

6. There is a related problem with interest rates. The model contains two rates, one for monetary assets like time deposits, the other for nonmonetary assets. The rate on monetary assets is regarded as exogenous and so is the money supply. I do not see how this can be correct. Either the government can set the quantity of money and let the interest rate adjust or vice versa, but it cannot set both. (It should be noted that the zero rate on currency is exogenously given in any case.) In the United Kingdom, the relevant exogenous variable was, in fact, the minimum lending rate, represented in the model by  $r_d$ . (It remains to be seen to what extent recent modifications have really changed this.) The money supply, on the other hand, was endogenous, public and private borrowers being supplied with any amounts demanded at that rate. It would be interesting to know what the model has to say about the implications of using the discount rate as the main policy variable. Of particular interest would be the question of whether a system that is stable for a money supply policy is also stable for an interest policy. Wicksell taught us that usually this is not so, at least if the accumulation of real capital and international reserves are disregarded.

7. If an endogenous variable is treated as exogenous, one would expect a model to be overdetermined. The Buiter and Miller model, however, is not. Overdetermination is avoided, I am inclined to conjecture, by omitting an equation that really should be part of the model, namely the interest-parity condition for monetary assets. Shouldn't the time deposit rate on sterling be related to the Eurodollar rate and the rate of sterling depreciation? I see no valid reason to recognize interest parity for nonmonetary assets but not for monetary assets. If both relationships are included, the differential between the two interest rates is determined by the rest of the world and thus exogenous. (Alternatively, the two interest-parity conditions could perhaps be interpreted as relating to different maturities. The term structure of interest rates would then reflect the term structure of expected exchange rates.)

8. Buiter and Miller let the demand for real balances depend on the difference between the two interest rates but not on their absolute level. Once this difference, as just suggested, is exogenously determined, U.K. interest rates can no longer influence the demand for real cash balances. The consequences for the model would be far-reaching. I believe that the demand for real balances should, in fact, also depend on the absolute level of interest rates. (After all, a substantial part of the money supply still bears no, or virtually no, interest.) As a matter of fact, it was precisely the exogenous increase in the MLR that called forth the contraction in the demand for base money.

9. In the Buiter and Miller model, an acceleration in the money supply results in an increase in interest rates, not only in the long run but at once.

This is consistent with the observation that the expansion of the growth rate of £M3 in 1979–80 was indeed associated with a marked rise in interest rates. However, the reason for the parallel short-run association of monetary expansion and interest rates seems to be different in reality from what it is in the Buiter and Miller model. In the model, an expansion in the money supply raises the demand for real cash balances, and thus interest rates, through an expansion of output. Actually, the British economy experienced recession. I would be inclined to suggest that the exogenous increase in the MLR, while leading to a sharp contraction of output and a marked decline in the demand for base money, also brought about a still larger increase in the demand for interest-bearing deposits and thus an expansion in (endogenous) £M3. As a result, £M3 gave tragically misleading signals about monetary policy, acting like a distorting mirror in which a person sees himself getting fatter while in fact he is losing weight.

10. In the Buiter and Miller model, as in the underlying Dornbusch model, the effects of monetary policy and overshooting for foreign trade and capital flows are not made explicit. I believe this is a major limitation which, in the light of recent contributions in this area, could probably be remedied without excessive difficulty. This extension would have to take into account that any trade deficits or surpluses caused by changing competitiveness must be consistent with the desired accumulation or reduction of foreign assets. To be relevant for policy questions, this extension would also have to take account of the *J*-curve lags in the effects of exchange rates on trade.

To sum up, during the last two years, British monetary policy greatly suffered from the Radcliffian notion that the differences between money and other liquid assets are too small to matter. This notion, it seems to me, also casts a slight shadow over Buiter and Miller's excellent paper. Nevertheless, there is enough light in this paper to make it an important contribution to our understanding of monetary policy under floating exchange rates.