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APPROXIMATE ADAPTIVE CONTROL SOLUTIONS TO U.S. BEEF TRADE POLICY*

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In this paper, the U.S. beef trade policy is specified as an adaptive control problem. Since this problem is not analytically soluble, a number of approximate solution procedures are presented and compared. These include certainty equivalent, stochastic, sequential stochastic, sequential adaptive covariance, and M-measurement feedback controls. After an exposition of the theory associated with each of these approximate control strategies, the empirical components of the beef trade policy problem are briefly described. Of particular interest is the trade off between proxy measures for consumer and producer welfare in the selection of the "optimal" beef import quotas. On the basis of the developed empirical components, the M-measurement feedback controls proved to generate the largest expected gains followed closely by adaptive covariance and sequential stochastic controls. For the certainty equivalent controls serious specification errors were revealed. In the case of stochastic controls, less important specification errors were obtained due to the nature of beef trade policy problem examined.

1. INTRODUCTION

This study examines one of the measures utilized by the U.S. government to partially control domestic consumer meat prices and beef producer profits, viz., the maximum quota level on beef imports. The empirical importance of U.S. beef quota policies to domestic consumers and producers is revealed in U.S. Congress (1969) and U.S. Tariff Commission (1964) reports. As indicated in the Congressional report, there has been some controversy over U.S. beef import quota policies. Consumer groups have argued that recent increases in beef prices are due, in part, to import quota restrictions that have been imposed while beef producers contend that unrestricted beef imports "... could cause irreparable harm to the domestic livestock industry" (U.S. Congress [1969, p. 51]). Very recently, consumer meat prices have increased substantially and the administration has not imposed beef import quotas for the year 1973.

In determining beef import quota levels, the President, his advisors, and other public decision makers are obviously uncertain about the current and future effects of such actions. The uncertain policy possibility set, however, is typically altered as additional information concerning the livestock sector becomes available. In particular, new observations measuring the recent performance and current state of the livestock sector, e.g., of price, quantity, and stock changes, provide a more knowledgeable basis for determining current period decisions and for evaluating the effects of alternative policy actions.

As the above discussion suggests a proper analysis of U.S. beef trade policy (as do most economic policy problems) requires the formulation of a rational, multiperiod decision problem under conditions of imperfect information. In other words, for a quantitative policy formulation to be of some assistance to public decision makers it should be advanced in the context of an adaptive control framework. Such a framework involves the specification of (i) the relevant policy

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maker(s) and the control, or instrument variables which he (they) can manipulate; (ii) a criterion function; (iii) the state transformation functions defining the policy possibility set; and (iv) the process of information generation.

In an operational context, although uncertainty arises for each of specifications (i) through (iv), it occurs principally with respect to the effects of alternative policy actions on various performance variables, i.e., with respect to the state transformation functions (iii). Typically, these functions are based on an econometric model of the system under consideration. A common (simplifying) procedure has been to first estimate the parameters of the econometric model and then derive the "optimal" policy, assuming the estimated parameters are equal to their "true" values; while possibly recognizing uncertainty in the future exogenous and additive random disturbances which enter the specified model. Treating the parameters as known with certainty, however, is obviously unsatisfactory since they are generally only the point estimates of the true but unknown, or perhaps even random, parameters. In general, imperfect knowledge of the relationships comprising the constraints, emanates from the following major sources: many approximations including omitted variables, simplifying mathematical functions, and various forms of aggregation lead to the specification of stochastic rather than deterministic relationships; we may capture only (small) sample estimates of the parameters entering the relationships; structural changes; and the future environment, i.e., only imperfect information on the future values of the noncontrollable exogenous variables is available.

In this setting, the specific purposes of the present analysis include a comparative performance evaluation of various control strategies and a determination of how alternative preference weighting (among consumer and producer groups) affect the selected beef import quota levels. A general objective of our analysis is to investigate the applicability of adaptive control theory as a medium for providing information to public decision makers concerned with U.S. beef trade policy. Since the complete mathematical formulation of the adaptive control problem cannot be solved analytically, a number of approximate solution procedures are presented in Section 2. The properties of these approximate solutions are briefly developed and the economic "value" of additional information is discussed. The empirical components of the U.S. beef trade decision problem are the concern of the following section. Specifically, in Section 3, the set of criteria or policy preference functions, an econometric model of the U.S. livestock sector, and the updating or revision estimators required to derive the approximate control strategies are described. Given these empirical components, the numerical results for the approximate adaptive control solutions advanced in Section 2, are reported and compared in Section 4. Finally, Section 5 contains a summary of the empirical results and suggestions for additional research on the beef trade policy problem.

2. Adaptive Control and Approximate Solution Procedures

2.1. *Adaptive Control*

Adaptive control methods recognize that as a system progresses through the controlling periods more data become available with which to update or revise the decision maker's perception of the policy possibility set. These revisions, in general, should not be regarded as separate from the derivation of an optimal policy. To be sure, it is possible that various decisions may reveal more or less information about the actual system via different sets of the resulting data obtained. The inherent benefits of the additional information depends upon whether or not an "improved" representation of the structure results in superior future control. The incurred costs of such information emanates, in part, from choosing a current policy which is less than optimal from a pure control point of view.

The adaptive control approach to economic policy corresponds to Bellman's (1961, pp. 198–209) as well as Fel'dbaum's (1965, pp. 24–31) third class of control systems. This class is characterized by some unknown quantities about which uncertainty changes as the process evolves. This class includes active learning or accumulation of information, i.e., the accumulation of information does not take place independently of the control process. In effect, optimal adaptive controls require a simultaneous solution to a combined control and sequential design of experiments problem and thus are dual in nature.[1] The design of experiments dimension will prove important if losses associated with selecting a current policy which is nonoptimal from a pure control standpoint can be recovered in subsequent periods by utilizing improved model representations.

The above considerations and implied models have been notably lacking in empirical treatments of economic policy. Among economists, perhaps the best known works, illustrative of these considerations, are Prescott (1971, 1972), MacRae (1972), and Zellner (1971).[2] A number of engineers have also examined the applicability of these concepts to economic problems.[3] Several formulations of adaptive control models have, of course, been employed in mathematical and engineering fields, at least, in a theoretical context.[4] From the viewpoint of economic policy, the adaptive control formulation represents an extension of the pioneering models advanced by Tinbergen (1952) and Ramsey (1928).

For the beef trade policy problem, a special case of the more general formulation presented in Rausser and Freebairn (1972a) will be employed. In particular, the objective function will be specified as

$$J = E\left\{\sum_{t=1}^{T} \beta^{t-1}(2k_t'y_t + 2h_t'u_t - y_t'K_ty_t - u_t'H_tu_t) + 2\beta^T k_{T+1}'y_T - \beta^T y_T'K_{T+1}y_T\right\}. \tag{1}$$

$$= E\left\{\sum_{t=1}^{T} W_t(u_t, y_t) + W_{T+1}(y_T)\right\}.$$

i.e., as a time additive, quadratic function: where W is the criterion function, E is the

[1] As indicated in Rausser and Freebairn (1972a), the dual nature of the adaptive control formulation may be characterized by three major dimensions, viz., direct control, learning, and design of experiments.

[2] A rather complete summarization of Prescott's work may be found in Zellner (1971, pp. 331–357). A recently completed Ph.D. thesis at the University of Illinois [Popović (1972)] is also concerned with adaptive control procedures in the context of economic policy problems. See Marschak (1963), Ying (1967), and MacRae (1972) for further suggestions along these lines.

[3] These include the work of Murfy (1965), Buchanan and Norton (1971), Athans (1972), and Perkins, *et al.* (1972).

[4] See, for example, Aoki (1967), Åström and Wittenmark (1971), Bar-Shalom and Sivan (1969), Bellman and Kalaba (1959), Curry (1970), Early and Early (1972), Gunckel and Franklin (1963), Kogan (1966), Ku and Athans (1972), Lainiotis, *et al.* (1972), Murphy (1968), Tarn (1971), Tse and Athans (1970, 1972), Tse and Bar-Shalom (1972), and Tse, *et al.* (1972).

expectation operator. T is the terminal point of the planning horizon, the parameters contained in k_t, h_t, K_t, H_t, ($t = 1, \ldots, T$), k_{T+1} and K_{T+1} are assumed known. K_t and H_t are both symmetric $n \times n$ and $m \times m$ matrices, respectively with $K_t \geq 0$ and $H_t > 0$ for all t. $\beta = 1/1 + \gamma$ and $\gamma > 0$ is a known preference discount rate. y_t is an $n \times 1$ vector of endogenous or state variables, and u_t is an $m \times 1$ vector of control variables. Note that T is assumed to be finite and the inclusion of the terminal component $W_T(y_{T+1})$ provides for continuity with future periods beyond T of the system under examination. The state transformation functions will be specified as

$$(2) \qquad y_t = A_t y_{t-1} + B_t u_t + C_t x_t + e_t, \; t = 1, \ldots, T$$
$$= D_t z_t + e_t$$

i.e., as linear with parameters which are allowed to differ over time: where x_t is an $(p-n-m) \times 1$ vector of noncontrollable exogenous variables, and e_t is an $n \times 1$ vector of disturbance terms. $D_t = [A_t, B_t, C_t]$, and $z_t' = [y_{t-1}', u_t', x_t']$. The general processes by which information is generated will be denoted as

$$(3) \qquad P^t(D_t^T, e_t^T, x_t^T) = I_t[P^{t-1}(D_{t-1}^T, e_{t-1}^T, x_{t-1}^T), y_t, z_t]$$
$$= P_d^t(D_t^T)P_e^t(e_t^T)P_x^t(x_t^T), \; t = 1, \ldots, T$$

i.e., the joint probability distribution (or set of sufficient statistics) $P^t(\cdot)$, conceived at time t, is a function of $P^{t-1}(\cdot)$ and the most recent observations y_t and z_t: where $D_t^T = (D_t, D_{t+1}, \ldots, D_T)$, and e_t^T and x_t^T are similarly defined. The second statement in (3) assumes that the stochastic elements of the problem, viz., the parameters entering the constraint functions (A_t, B_t, C_t), the disturbance terms e_t, and the noncontrollable exogenous variables x_t, are independently distributed. This specification for the probability distribution or updating functions $P^t(\cdot)$, $t = 1, \ldots, T$ is sufficiently general to allow for the case in which the distribution of the stochastic elements are known as well as the case in which future moments of the stochastic elements are random. The latter case is assumed to hold for the beef trade policy problem, i.e., future means and covariances of the probability distributions are assumed to be stochastic and some *a priori* probability density for these moments is presumed to be available. As usual, we shall assume that the disturbances, e_t, are intertemporally independent, normally distributed random variables with zero expectation and stationary covariance Ω. In addition to (2) and (3), the maximization of (1) will also be constrained by initial conditions on the state variables and the initial prior probability distribution function, i.e.,

$$(4) \qquad y_0 = y(0),$$

and

$$(5) \qquad P^0(D_0^T, e_0^T, x_0^T) = P(0).$$

Note that the formulation (1)–(5) assumes the state vectors, y_t, are measured accurately, i.e., the state of the system is completely accessible in each of the t periods.[5] The mathematical tractability of the quadratic specification (1) is an

[5] See Aoki (1967), Athans (1972), or Popović (1972) for a treatment of the case in which state vector measurements are noisy. Perkins, *et al* (1972) have recently examined economic control systems in which some state variables cannot be measured at all or only with a delay.

obvious advantage. Even though the actual criterion function is not quadratic, such a form might provide a reasonable approximation. In this regard, appeal may be made to a Taylor series expansion in which only the linear and quadratic terms are retained, and to Zellner and Geisel's (1968) results which suggest that quadratic criterion functions provide satisfactory approximations to a number of more general functions when asymmetry is not an important consideration. Similar justifications could, of course, be offered for the set of linear, equality constraints (2). These discrete time, dynamic equations clearly simplify the derivation of the optimal controls as well as the application of econometric techniques. In the present investigation (2) will represent some of the equations entering the reduced form of an econometric model approximating the livestock sector. The assumption of independence of the random elements (3) should cause no particular problems once the distinction between the application and sample period for the econometric model is recognized. Over the application period ($t = 1, \ldots, T$), for the beef trade policy problem, it seems reasonable to presume that forecasts of the exogenous variables would be independent of coefficient estimates of the econometric model as well as the disturbance terms. In the case of the stochastic parameters and disturbance terms, the assumption of serially independent e_t suggests that the sampling distribution estimates of the parameters (based on the sample period) may be regarded as independent of the disturbance terms emanating over the application period.[6]

Unfortunately, for the specification (1)–(5), it has not as yet been possible to express the adaptive or dual control solutions in analytical form.[7] The intractable nature of the problem is due to the interaction between the transformation functions in y_t (2), and the probability updating functions (3). This interaction results in highly nonlinear functions, the expected value of which can only be evaluated numerically. Moreover, numerical solution procedures rapidly encounter the "curse of dimensionality" for even modest size control problems[8] of the sort considered here. Hence, given present knowledge and available computer facilities, we shall turn to approximate solution procedures which involve some alterations of the original structure of the problem. The severity of these alterations will

[6] The specification (1) through (5) admits a number of special formulations which may be found in the literature. The deterministic form, of course, follows from this specification when e_t is a null vector, x_t is fixed, and A_t, B_t, and C_t are known constants for all t. The certainty equivalent formulation advanced by Simon (1956), Theil (1964), Holt (1962), and Chow (1972) follows when A_t, B_t and C_t are known constants for all t and x_t is either fixed or stochastic, but independent of e_t. Two stochastic formulations frequently found in the engineering literature [Aoki (1967)] are also special cases of this specification. The first presumes that $P^t(D, e, x) = P^0(D, e, X) = P(0)$, for all t, and that the first two moments of the various distributions are known while the second again assumes knowledge of the first two moments but allows the probability distributions to change independently over time. That is, this second stochastic form presumes that random variables of the decision problem are distributed independently in the current and future periods. Both stochastic forms are non-adaptive since the probability distributions of these formulations are indepedent of new information sequences. At most (the second form), they allow for only passive (independent of controls) accumulation of information. This subclass of control systems is characterized by Fel-dbaum (1965, pp. 339–341) as neutral. The neutral class also includes the case in which $P^t(\,.\,)$ is independent of y_t and z_t; for this case the experimental gain component and thus the dual nature of the optimal controls disappear.

[7] For a demonstration of this well-known result, see Aoki (1967, pp. 111–113) or Rausser and Freebairn (1972a, pp. 12–14).

[8] The simple pedagogic models investigated by Marshak (1963) and Ying (1967) illustrate the computation burden involved for numerical solutions.

dictate the degree of approximation of the proposed control strategies to the (optimal) adaptive control solutions.

2.2. *Approximate Solution Procedures*

The approximate solution procedures which have been advanced in the literature include[9] *inter alia*, (i) replacing the nonlinear information updating functions (3) by linear or piecewise linear approximating functions: (ii) replacing the optimum of J for periods $t+1$ through T by its Taylor series expansion in which only the linear and quadratic terms in control variables are retained: (iii) compute the certainty equivalent controls: (iv) compute the stochastic controls: (v) compute the sequential stochastic controls: (vi) compute the sequential, adaptive covariance controls, and (vii) compute the M-measurements feedback controls. In the analysis which follows, we shall be concerned only with the approximate solution procedures (iii) through (vi). They will be treated in order of increasing complexity, i.e., the order in which they are presented above. As will become obvious, each of these approximate solutions is a special case of the subsequent approximate solution procedure, and thus (iii) through (v) are each special cases of (vi). The derivation of analytical expressions for the approximate solutions (iii)-(v) to (1)-(5) may be characterized by either the Pontryagin maximum principle (in its discrete form)[10] or by stochastic dynamic programming. We shall utilize the latter and the approximate solution procedures (iii)-(v), upon invoking Bellman's (1957) principle of optimality, will be conceptualized by a number of optimization problems, one for each period $t = 1, \ldots, T$.

2.2.1. *Certainty equivalent controls* (c). The approximate certainty equivalent solution is obtained by treating D_t as though it was a known constant matrix, for all t. The only stochastic elements which will be recognized by this approximate procedure are e_t and x_t. These random vectors are independent by (3), each with known Gaussian distribution having mean and nonnegative definite covariance $(0, \Omega)$, and $(\bar{x}_t, \Gamma_t^x)$, $t = 1, \ldots, T$, respectively.

Under the simplifying approximation on the D_t matrices, the state of the system is described by y_{t-1} in period t, and thus the maximum gain for periods t through T may be represented as

$$\Lambda_{ct}(y_{t-1}) = \max_{u_t,\ldots,u_T} E\left\{\sum_{i=t}^{T} \bar{W}_i + \bar{W}_{T+1}\right\}, \qquad t = 1, \ldots, T. \tag{6}$$

where $\bar{W}_i$ is obtained by substituting (2) into (1) with D_t replaced by $\bar{D}_t$. The solution to (6) results in the maximization of (1) subject to (2)-(5) where the unknown parameters contained in D_t of (2) and (3) are treated as though they were constant at their mean values, $\bar{D}_t$. The derivation of certainty equivalent controls have been reported in a number of places [see, for example, Chow (1972) or Theil (1964)] and thus need not detain us here. They may be represented as

$$u_t^c = -N_{ct}^{-1}(F_{ct}y_{t-1} + f_{ct}) \tag{7}$$

[9] See, for example, Aoki (1967, Chapter VII), Curry (1970, pp. 84-86), Early and Early (1972), Prescott (1971), Popović (1972) and Zellner (1971).

[10] For a discussion of this principle in its discrete form, see Halkin (1966), Athans (1967), or Cannon, *et al.* (1970).

where

(8) $$N_{ct}^{-1} = [\bar{B}_t' S_{ct} \bar{B}_t + H_t]^{-1}$$

(9) $$F_{ct} = \bar{B}_t' S_{ct} \bar{A}_t$$

(10) $$f_{ct} = \bar{B}_t' S_{ct} \bar{C}_t \bar{x}_t - \bar{B}_t' R_{ct} - h_t$$

and R_{ct}' and S_{ct} are defined as

(11) $$R_{ct}' = k_t' + \beta g_{ct+1}'$$
$$t = 1, \ldots, T-1$$
(12) $$S_{ct} = K_t + \beta G_{ct+1}$$

with $R_T = k_T' + \beta k_{T+1}', S_T = K_T + \beta K_{T+1}$, and

(13) $$g_{ct}' = R_{ct}' \bar{A}_t + f_{ct}' N_{ct}^{-1} F_{ct} - \bar{x}_t' \bar{C}_t' S_{ct} \bar{A}_t$$

(14) $$G_{ct} = \bar{A}_t' S_{ct} \bar{A}_t - F_{ct}' N_{ct}^{-1} F_{ct}.$$

The vector g_{ct}' and matrix G_{ct} also appear in the linear and quadratic terms, respectively, of the c control "maximum" expected gain for period t, i.e.,

$$\Lambda_{ct}(y_{t-1}) = 2g_{ct}' y_{t-1} - y_{t-1}' G_{ct} y_{t-1} + Q_{ct},$$

where Q_{ct} does not involve the control or state variables. However, Q_{ct} is the only term of $\Lambda_{ct}(y_{t-1})$ which does involve the covariance matrices (Ω and Γ^x) of the stochastic elements (e_t and x_t) which are recognized by the c controls.

2.2.2. *Stochastic controls* (s). This approximate solution procedure is obtained from the original structure (1)–(5) if (3) is replaced by $P^t(\cdot) = P^0(\cdot)$ for all t. For this altered structure, the dynamic programming method results in a set of recursive equations which begin with the last period of the decision horizon and end with the first decision period. Applying Bellman's principle of optimality, the general form of the t-th period subproblem is given by

(15) $$\Lambda_{st}(y_{t-1}) = \max\,[E\{2k_t' y_t + 2h_t' u_t - y_t' K_t y_t - u_t' H_t u_t + \beta\Lambda_{st+1}(y_t)\} | y_{t-1}].$$

Since the difference equations represented in (2) are assumed linear with a Gaussian noise term, once the expectation operator E is applied, the covariance matrix of D_t (conceived at the beginning of period t) will enter (15). This covariance matrix of the elements appearing in D_t arranged by rows, e.g., $(D') = [a_{11}, a_{12}, \ldots, b_{11}, b_{12}, \ldots c_{11}, c_{12}, \ldots]$, will be denoted as Γ_t; it is of dimension $np \times np$ and in terms of A, B and C it may be represented as

(16) $$\Gamma_t = \begin{bmatrix} \Gamma_t^{AA} & \Gamma_t^{AB} & \Gamma_t^{AC} \\ \Gamma_t^{BA} & \Gamma_t^{BB} & \Gamma_t^{BC} \\ \Gamma_t^{CA} & \Gamma_t^{CB} & \Gamma_t^{CC} \end{bmatrix}.$$

To simplify the exposition, we assume that $D_{t'} = D_t$ and $\Gamma_{t'} = \Gamma_t$ for all $t' \geq t = 1, \ldots, T$. Substituting $y_t = D_t z_t + e_t$ into (15), applying the expectation operator, and expanding in terms of A, B, and C we have

$$(17)\ \Lambda_{st}(y_{t-1}) = \max_{u_t} \{2R'_{st}\bar{A}_t y_{t-1} + 2(R'_{st}B_t + h'_t)u_t + 2R'_{st}\bar{C}_t\bar{x}_t - y'_{t-1}(\bar{A}'_t S_{st}\bar{A}_t + S_{st} \circledast \Gamma_t^{AA})y_{t-1} - u'_t(\bar{B}'_t S_{st}\bar{B}_t + S_{st} \circledast \Gamma_t^{BB} + H_t)u_t - \bar{x}'_t(\bar{C}'_t S_{st}\bar{C}_t + S_{st} \circledast \Gamma_t^{CC})\bar{x}_t - (\bar{C}'_t S_{st}\bar{C}_t + S_{st} \circledast \Gamma_t^{CC}) \circledast \Gamma_t^x - 2u'_t(\bar{B}'_t S_{st}\bar{A}_t + S_{st} \circledast \Gamma_t^{BA})y_{t-1} - 2u'_t(\bar{B}'_t S_{st}\bar{A}_t + S_{st} \circledast \Gamma_t^{BC})\bar{x}_t - 2y'_{t-1}(\bar{A}'_t S_{st}\bar{C}_t + S_{st} \circledast \Gamma_t^{AC})\bar{x}_t - S_{st} \circledast \Omega + \beta Q_{st+1}\}.$$

The set of recurrence relations obtained from maximizing $\{\ \}$ in (17) may be stated as:

$$(18)\qquad u_t^s = -N_{st}^{-1}(F_{st}y_{t-1} + f_{st}).$$

where

$$(19)\qquad N_{st}^{-1} = [\bar{B}'_t S_{st}\bar{B}_t + S_{st} \circledast \Gamma_t^{BB} + H_t]^{-1},$$

$$(20)\qquad F_{st} = [\bar{B}'_t S_{st}\bar{A}_t + S_{st} \circledast \Gamma_t^{BA}]$$

$$(21)\qquad f_{st} = (\bar{B}'_t S_{st}\bar{C}_t + S_{st} \circledast \Gamma_t^{BC})\bar{x}_t - \bar{B}'_t R_{st} - h_t,$$

and R'_{st} and S'_{st} are obtained from

$$(22)\qquad R'_{st} = k'_t + \beta g'_{st+1},$$

$$(23)\qquad S_{st} = K_t + \beta G_{st+1}, \qquad t = 1, \ldots, T-1$$

with R'_{sT} and S_{sT} defined as in (11) and (12). The "maximum" expected gain (in period t) obtained from following the open-loop stochastic control strategy is

$$(24)\qquad \Lambda_{st}(y_{t-1}) = 2g'_{st}y_{t-1} - y'_{t-1}G_{st}y_{t-1} + Q_{st},$$

where

$$(25)\qquad g'_{st} = R'_{st}\bar{A}_t + f'_{st}N_{st}^{-1}F_{st} - x'_t(\bar{C}'_t S_t\bar{A}_t + S_{st} \circledast \Gamma_t^{CA}).$$

$$(26)\qquad G_{st} = \bar{A}'_t S_t\bar{A}_t + S_{st} \circledast \Gamma_t^{AA} - F'_{st}N_{st}^{-1}F_{st}.$$

$$(27)\qquad Q_{st} = f'_{st}N_{st}f_{st} + 2R'_{st}\bar{C}\bar{x}_t - S_{st} \circledast \Omega - \bar{x}'_t(\bar{C}'_t S_{st}\bar{C}_t + S_{st} \circledast \Gamma_t^{CC})\bar{x}_t - (\bar{C}'_t S_{st}\bar{C}_t + S_{st} \circledast \Gamma_t^{CC}) \circledast \Gamma_t^x + \sum_{r=1}^{T-t} \beta^r Q_{st+r}.$$

As statements (18) through (27) indicate the computation of u_t^s and the resulting expected gain Λ_{st} requires the expected value ($\bar{D}'_t$) and covariance matrix (Γ_t). This information could be provided by a number of classical (consistent) as well as Bayesian estimators. Note that for the expected gains emanating from the s controls (24), the covariance matrices Ω and Γ_t^x of the stochastic elements e_t and x_t only appear in Q_{st} while elements of the covariance matrix Γ associated with the stochastic matrix D appear in g_{st}, G_{st}, as well as Q_{st}.[11] Since the *a priori* values $\bar{D}$ and Γ are treated as if they were known exactly, the s controls are "optimal." However, if these assumptions are not satisfied as (3) in general suggests, then this control strategy involves a specification error.

2.2.3. *Sequential stochastic controls* (*ss*). The u_t^s controls can be generalized by constructing a sequence of open-loop subproblems. These subproblems begin in each period of the planning horizon with only the initial policy values actually implemented. As policy decisions are made and time progresses, additional data become available which are utilized to update $\bar{D}_t$ and Γ_t. The revised estimates are then employed as prior information for the next open-loop subproblem. This approach essentially assumes that in each period no additional information will be forthcoming, but this assumption is revised after each period.[12] By neglecting the availability of new information over future periods of the planning horizon (for each open-loop subproblem), this procedure forces an otherwise dual system to be neutral and hence allows only independent or passive accumulation of information.[13] Thus, the sequential stochastic control strategy recognizes the control and learning dimensions of the optimal adaptive control, but it ignores the experimental dimension. In linear feedback form, these controls may be represented as

$$u_t^{ss} = -N_{sst}^{-1}[F_{sst}y_{t-1} + f_{sst}], \tag{28}$$

where N_{sst}, F_{sst}, and f_{sst} are conditional on $P^{t-1}(\cdot)$ of (3) rather than $P^0(\cdot)$ as in the case of the stochastic controls (18). In general, for each initial period (t), $P^{t'-1}(\cdot) = P^{t-1}(\cdot)$, for all $t' \geq t$.

It should also be noted that certainty equivalent controls may be updated in a similar fashion. This involves revising the expected parameter values $\bar{D}_t$ on the basis of new sample information and implementing policies only after actual observations on the state of the system for the previous period become available. These controls in feedback form may be denoted as

$$u_t^{cc} = -N_{cc}^{-1}[F_{cct}y_{t-1} + f_{cct}]. \tag{29}$$

2.2.4. *Adaptive covariance, sequential controls* (*sf*). Up to this point, there has been no need to be specific about the nature of probability updating functions

[11] The matrix operator ⊛, appearing in expressions (17)–(27), is defined by MacRae (1971) as the star product. To illustrate the properties of this operator, let A be an m by n matrix, and let B be an mp by nq matrix. The star product of A and B is a p by q matrix C, i.e., $C = A \circledast B = \sum_{i,j} a_{ij}B_{ij}$ where a_{ij} is the ijth element of A and B_{ij} is the ijth submatrix of B. Clearly, for the case in which A and B are of the same dimension, $A \circledast B = \operatorname{tr} A'B$. Thus the third and fifth terms of Q_{st}, for example, could be represented as traces of the appropriate product matrices. This operator along with Nissen's (1968) stacking operator $\mathscr{S}$ will prove especially useful in setting out the s and subsequent control strategies which involve the expected value of random matrices. For example, if X and Y are the random matrices and A is non-random, then $E\{X'AY\} = A \circledast E\{\mathscr{S}(X')\mathscr{S}'(Y)\} = \bar{X}'A\bar{Y} + A \circledast \Gamma$, where Γ is the covariance matrix for the elements of X and Y, arranged by rows, and $\bar{X} = E(X)$, $\bar{Y} = E(Y)$.

[12] This is one form of open-loop feedback control strategy first introduced by Dreyfus (1964). See Murphy (1968), Prescott (1971), Popovič (1972), Tse, *et al.* (1971), and Zellner (1971), among others, for alternative treatments of this approach.

[13] It is optimal, i.e., it corresponds to the dual control optimal strategies, only if the set of admissible controls is sufficiently restricted or, as previously noted, if all random variables entering the decision problems are independently distributed in the current and future periods of the planning horizon.

$P_d'(\cdot)$. Recognizing that the elements of the matrix D are unknown and neglecting any overidentifying restrictions in the underlying structural model for (2), we may capture uncertainty associated with the D matrix by modelling its elements as normal random variables with given prior means, variances, and covariances. The latter will typically be data based, i.e., the initial prior will be estimated on the basis of sample data which precedes the planning horizon. Since the state transformation functions are linear with a Gaussian disturbance term, the posterior distribution of D, given data up through some period t, will also be multivariate normal. Thus changes in $P_d(\cdot)$ resulting from the availability of additional observations may be summarized by treating movements in $\bar{D}_t$ and Γ_t over the planning horizon. More specifically, the conditional mean and covariance matrix of D can be determined recursively by

(30) $$\Gamma_t^{-1} = \Gamma_{t-1}^{-1} + (I_n \otimes z_t)\Omega^{-1}(I_n \otimes z_t')$$

and

(31) $$\mathscr{S}(\bar{D}_t') = \Gamma_t[\Gamma_{t-1}^{-1}\mathscr{S}(\bar{D}_{t-1}') + (I_n \otimes z_t)\Omega^{-1}y_t],$$

where $\mathscr{S}$ denotes the stacking operator [Nissen (1968)].[14]

The mathematical difficulty arising in the optimization problem represented by (1), (2), (30), (31), and (4) results not only from the random matrix D, but as well from the random conditional means (31) and covariance matrices (30). The randomness of these latter elements is clearly due to the dependence of Γ_t^{-1} and $\mathscr{S}(\bar{D}_t')$ on the random states y and stochastic exogenous variables x (appearing in z). Following MacRae (1972), much of this randomness may be avoided by operating with the modified updating equations

(32) $$\Gamma_t^{-1} = \Gamma_{t-1}^{-1} + E\{(I_n \otimes z_t)\Omega^{-1}(I_n \otimes z_t')|y_0\}$$

and

(33) $$\mathscr{S}(\bar{D}_t') = \Gamma_t[\Gamma_{t-1}\mathscr{S}(\bar{D}_{t-1}') + E\{(I_n \otimes z_t)\Omega^{-1}y_t|y_0\}].$$

Utilizing these two equations in place (30) and (31) eliminates the uncertainty associated with the mean vector and covariance matrix while the uncertainty of the D matrix itself is retained.

As may be easily demonstrated, this approximation results in the elimination of the need for the updating relation (33).[15] Hence, the use of (32) and (33) in place

[14] Note that if identities appear in (2) or some elements of D are known with certainty, the inverse operation in Ω^{-1} and Γ^{-1} of equation (30) applies only to the non-singular portion of each matrix; the remaining rows and columns are, of course, zero.

[15] In other words, since $\mathscr{S}(\bar{D}_t') = \mathscr{S}(\bar{D}_{t-1}')$ for all t, the modified update rule for the means is not a constraint to the altered optimization problem. To obtain this equivalence first evaluate

$$E\{(I_n \otimes z_t)\Omega^{-1}y_t|y_0\} = E\{(I_n \otimes z_t)\Omega^{-1}E(y_t|y_{t-1})|y_0\}$$

then substitute in

$$E(y_t|y_{t-1}) = \bar{D}_{t-1}z_t = (I_n \otimes z_t')\mathscr{S}(\bar{D}_{t-1}')$$

to obtain

$$\begin{aligned}\mathscr{S}(\bar{D}_t') &= \Gamma_t[\Gamma_{t-1}^{-1} + E\{(I_n \otimes z_t)\Omega^{-1}(I_n \otimes z_t')|y_0\}]\mathscr{S}(\bar{D}_{t-1}') \\ &= \Gamma_t\Gamma_t^{-1}\mathscr{S}(\bar{D}_{t-1}') = \mathscr{S}(\bar{D}_{t-1}').\end{aligned}$$

of (30) and (31) allows the original problem to be converted into a sequence of open-loop problems. The modified updating relation (32) is employed for designing a single open-loop path while the actual updating equations (30) and (31) are used to compute the new means and covariances to begin the next open-loop path. Given the actual observations on the state vector of the previous period, this essentially open-loop feedback approach involves the maximization of (1) subject to the stochastic constraints in (2) and the deterministic updating constraints (32). For this modified problem some interaction between direct control and experimentation remains since current control settings influence the future values of both y and Γ. As the adaptive feature of this approximation is based on the covariance updating rule (32) the resulting controls might be characterized as sequential, adaptive covariance controls.

To formalize this approach, we may operate with an augmented criterion function which includes J of (1) and the deterministic constraints (32) along with an associated matrix of Lagrangean multipliers, V_t. That is, since the covariance constraints (32) are deterministic, J of (1) may be replaced by

$$J^* = J - \sum_{t=1}^{T} V_t \circledast [\Gamma_t^{-1} - \Gamma_{t-1}^{-1} - E\{(I_n \otimes z_t)\Omega^{-1}(I_n \otimes z_t')|y_0\}]$$

where J^* is the desired augmented criterion function. V_t is an $np \times np$ matrix of Lagrangian multipliers which may be partitioned in the same fashion as Γ_t in (16), and $\circledast$ is the matrix operator defined and discussed in footnote 11. Proceeding as before, the t-th period subproblem after some simplifications is given as

$$(34) \qquad \Lambda_{sft}(y_{t-1}) = \max_{u_t} [E\{2k_t'y_t + 2h_t'u_t - y_t'K_ty_t - u_t'H_tu_t + V_t \circledast (I_n \otimes z_t)\Omega^{-1}(I_n \otimes z_t') - (V_t - V_{t+1}) \circledast \Gamma_t^{-1} + \beta\Lambda_{sft+1}(y_t)\}|y_{t-1}].$$

Substituting (2) in for y_t and expanding in terms of A, B, and C, the solution to (34) may be represented by

$$(35) \qquad u_t^{sf} = -N_{sft}^{-1}(F_{sft}y_{t-1} + f_{sft}), \qquad t = 1, \dots, T,$$

in addition to the requirements that $\partial E\{\ \}/\partial\Gamma_t^{-1} = 0$, i.e.,

$$(36) \qquad V_t = V_{t+1} + \Gamma_t[E(I_n \otimes z_t)S_{sft}(I_n \otimes z_t')]\Gamma_t, \qquad t = 1, \dots, T,$$

with $V_{T+1} = 0$, and that $\partial E\{\ \}/\partial V_t = 0$, i.e.,

$$(37) \qquad \Gamma_t^{-1} = \Gamma_{t-1}^{-1} + E\{(I_n \otimes z_t)\Omega^{-1}(I_n \otimes z_t')|y_{t-1}\}, \qquad t = 1, \dots, T+1;$$

where

$$(38) \qquad N_{sft}^{-1} = [\bar{B}_t'S_{sft}\bar{B}_t + S_{sft} \circledast \Gamma_t^{BB} - \Omega^{-1} \circledast V_t^{BB} + H_t]^{-1}$$

$$(39) \qquad F_{sft} = [\bar{B}_t'S_{sft}\bar{A}_t + S_{sft} \circledast \Gamma_t^{BA} - \Omega^{-1} \circledast V_t^{BA}]$$

$$(40) \qquad f_{sft} = (\bar{B}_t'S_{sft}\bar{C}_t + S_{sft} \circledast \Gamma_t^{BC} - \Omega^{-1} \circledast V_t^{BC})\bar{x}_t - \bar{B}'R_{sft} - h_t$$

and R_{sft}' and S_{sft} are defined as

$$(41) \qquad R_{sft}' = k_t' + \beta g_{sft+1}'$$

and

$$S_{sft} = K_t + \beta G_{sft+1} \tag{42}$$

with $R'_{sfT} = k'_T + \beta k'_{T+1}$ and $S_{sft} = K_T + \beta K_{T+1}$. As before, the vector g'_{sft+1} and the matrix G_{sft+1} enter the "maximum" expected gain $\Lambda_{sft}(y_{t-1})$ obtained from the u_t^{sf} controls. This expected value, when simplified, may be represented as

$$\Lambda_{sft}(y_{t-1}) = 2g'_{stf}y_{t-1} - y'_{t-1}G_{sft}y_{t-1} + Q_{sft}, \tag{43}$$

where

$$g'_{sft} = R'_{sft}\bar{A}_t + f'_{sft}N_{sft}^{-1}F_{sft} - \bar{x}'_t(\bar{C}'_tS_{sft}\bar{C}_t + S_{sft} \circledast \Gamma_t^{CA} - \Omega^{-1} \circledast V_t^{CA}) \tag{44}$$

$$G_{sft} = \bar{A}'_tS_{sft}\bar{A}_t + S_{sft} \circledast \Gamma_t^{AA} - \Omega^{-1} \circledast V_t^{AA} - F'_{sft}N_{sft}^{-1}F_{sft} \tag{45}$$

$$\begin{aligned} Q_{sft} = {} & f'_{sft}N_{sft}f_{sft} + 2R'_{sft}\bar{C}_t\bar{x}_t - S_{sft} \circledast \Omega - \bar{x}'_t(\bar{C}'_tS_{sft}\bar{C}_t \\ & + S_{sft} \circledast \Gamma_t^{CC} - \Omega^{-1} \circledast V_t^{CC})\bar{x}_t - (\bar{C}'_tS_{sft}\bar{C}_t + S_{sft} \circledast \Gamma_t^{CC} \\ & - \Omega^{-1} \circledast V_t^{CC}) \circledast \Gamma_t^{x} - (V_t - V_{t+1}) \circledast \Gamma_t^{-1} + \sum_{r=1}^{T-t} \beta^r Q_{sf,t+r}. \end{aligned} \tag{46}$$

The system of equations (35)–(42), (44), (45), along with (2) characterize the "optimal" solution in the current period t for the sequential adaptive covariance approximation to the original problem (1)–(5). Although these approximate controls recognize that the parameter matrices are unknown, they assume all parameters entering (2) are invariant with respect to time. The uncertainty regarding the parameter matrices is captured by the use of Bayesian analysis, (30) and (31), to update conditional means and covariances but is subsequently altered via the approximations (32) and (33). These modifications lead to a deterministic treatment of (3) but do allow the controls in each open-loop to affect future parameter variance and covariance values (Γ) as well as future states of the system (y).

2.2.5. *M-Measurement feedback controls* (M). This approximate solution procedure recently suggested by Curry (1970), Early and Early (1972), and Popović (1972), appears promising. In effect, it represents an intermediate approach to the optimal dual controls, by assuming that in each period t new information about the system will become available only at some M future stages of the controlling horizon. This method obviously permits a degree of active information accumulation. Here again, as analytical solutions are not yet possible, numerical techniques are required. Nevertheless, the application of such techniques are substantially simpler than those required for the original dual control problem (1)–(5) particularly when further approximations are imposed upon the M-measurement specifications.[16]

Since there has been no investigation of what constitutes (in each period t) the optimal distribution in the set $\{t, \ldots, T\}$ of the M future measurements we shall assume that the relevant stages are the M successive periods in the immediate future, i.e., $t+1, \ldots, t+M$. The M-measurement feedback controls (u_t^M, $t = 1, \ldots, T$) are then those policy decisions which utilize all past and present information

[16] For suggestions along these lines, see Popović (1972) and Tse, *et al.* (1972).

as well as the knowledge that over the next M periods new information (observations) will become available. More formally, these controls require the specification of an integer (l_t) which is the smaller of M and the remaining periods in the planning horizon, i.e., $l_t = \min\{M, T - t - 1\}$, $t = 1, \ldots, T$. In other words, this integer is equal to M if the *assumed* number of future observations taken into account is less than the *actual* number of future observations which will become available during the controlling process.

Given l_t, it is possible to specify the M-measurement feedback controls as generalization of each control strategy $u_t^c, u_t^s, u_t^{ss}, u_t^{cc}$, and u_t^{sf}. For example, u_t^M reduces to u_t^{ss} for $M = 0$ if u_t^M along with $u_{t+1}^M, \ldots, u_{t+l_t}^M$ are determined by

$$(47) \qquad A_{Mt}(y_{t-1}, P^{t-1}) = \max_{u_t \ldots u_{t+l_t}} \left[E\left\{ \sum_{i=t}^{t+l_t} W_i(u_i, y_i) + \beta^{l_t+1} \Lambda_{ss,t+l_t+1}(y_{t+l_t}, P^{t+l_t} \middle| P^{t+l_t+1}, u_t^{t-l_t}, x_t^{t+l_t}) \right\} \middle| y_{t-1} \right].$$

where $u_t^{t+l_t} = (u_t, \ldots, u_{t+l_t})$, and $x_t^{t+l_t}$ is similarly defined. For the u_t^M controls resulting from (47), two limiting forms are immediate. First, if $M = 0$, the availability of future information is neglected and thus u_t^M is equivalent to u_t^{ss}. Second, if $M = T - t - 1$, the availability of future information over all remaining periods of the planning horizon is taken into account and thus u_t^M is equivalent to the adaptive or dual control solution for period t. In addition, note that if the second term on the right hand side of (47) is appropriately modified, the u_t^M controls for $M = 0$ could reduce to either u_t^c, u_t^s, u_t^{cc}, or u_t^{sf}. Hence, we could characterize the various M-measurement feedback controls as u_t^{Mj}; each type of control strategy j being obtained from $\Lambda_{M_j t}$, where $j = c, s, ss, cc, sf$.

Since each of the j approximate solution procedures has an analytical form, the controls contained in $u_{t+l_t+1}^{r}$ may be determined conditionally on $u_t^{t+l_t}$. However, in order to find $u_t, u_{t+1}^{t+l_t}$ which satisfies $\Lambda_{M_j t}$, an $l_t + 1$-fold, m-dimensional control space must be searched and an analytic solution for the M_j-measurement feedback control in the current period appears to be precluded. For beef trade policy problems the original $l_t + 1$-fold m-dimensional search space will be replaced by a finite search in the n-dimensional state space. The resulting numerical approximation will not be detailed here; it may be found in Popović (1972, pp. 127–132).

2.3. *Comparison of Approximate Control Strategies*

The computation of the initial or current period policies for each approximate control procedure presume that the state of the system, y_{t-1}, is observable without error, as are the parameters of the criterion function. Aside from these common features, as previously noted, an appropriate specification of (47) allows each approximate control (c, s, ss, cc, sf) to be treated as a special case of the M-measurement feedback controls. Furthermore, if the unknown parameter matrices A, B, and C are treated as constant over time, the approximate controls (c, s, ss, cc) are special cases of the sequential, adaptive covariance controls (sf). The latter controls reduce to the cc controls (or c controls, neglecting revisions) if the parameter matrices A, B, and C are considered fixed at their mean values since Γ_t and V_t are null. Similarly the ss controls (or the s controls, neglecting revisions) are obtained from the sf controls when the adaptive covariance equation (32) is neglected and thus V no longer appears in (38)–(42). It should also be noted that, in general, it is not possible to infer whether one control strategy will call for smaller policy responses or be less aggressive than another. This observation can be confirmed in a number of ways: the simplest (although perhaps not the most revealing) is to examine the difference between the expressions for any two control strategies and demonstrate that this difference can be either positive or negative. As shown below, such differences are also important in determining the degree of "sub-optimality" associated with utilizing one approximate control strategy rather than another.

The degree of suboptimality of the various approximate controls relative to the optimum adaptive controls depends in part upon the importance of the experimental dimension. The importance of this dimension is reflected by the extent of uncertainty as well as the extent to which alternative settings of the control variables might increase the precision of the coefficient estimates. Unfortunately, due to the lack of analytical results for the adaptive control strategy, it has not been possible to quantify this expected degree of suboptimality.[17] It is, however, possible to provide a general comparison of the relative performance of the control solutions. This simply involves an examination of the expected loss (or gain) for current period policies of utilizing one approximate control strategy rather than another. More specifically, assuming one control strategy is obtained from the "proper" specification, expected losses associated with the remaining approximate controls may be evaluated.

If the sequential, adaptive covariance controls are treated as the proper specification, this approach first involves substituting (2) for y_t in (35) and applying the expectation operator at period t for policy u_t. The result may be represented as

$$J_{sft}(u_t|y_{t-1}) = 2u_t'\psi_t - u_t'N_tu_t + q_t \tag{48}$$

where $\psi_t = -F_{sft}y_{t-1} - f_{sft}$ and $N_t = N_{sft}$ (for the definitions of N_{sft}, F_{sft}, and f_{sft} see expressions (38)–(40)) and q_t is a function of y_{t-1} and $\bar{x}_t$, i.e., it is a group of terms not involving u_t. This formulation assumes that the "optimal" sequential, adaptive covariance decisions are followed over the interval $t+1, \ldots, T$. The "optimal" control strategy for (48) is $u_t^{sf} = N_t^{-1}\psi_t$, i.e. (35).

If this result is substituted back into (48) we obtain $J_{sft}(u_t^{sf}|y_{t-1})$ which can be employed to evaluate the expected loss of using some other control strategy u_t^j relative to u_t^{sf}. This expected loss may be defined as

$$\begin{aligned} L_{sf}(u_t^{sf}, u_t^j) &= J_{sft}(u_t^{sf}|y_{t-1}) - J_{sft}(u_t^j|y_{t-1}) \\ &= (u_t^{sf} - u_t^j)N_{sft}(u_t^{sf} - u_t^j). \end{aligned} \tag{49}$$

Since N_{sft} is positive definite, the expected loss (49) resulting from a less optimal first period decision than u_t^{sf} is a quadratic function of the control error $u_t^{sf} - u_t^j$.

[17] For a special example (a single constraint equation and a single decision variable, i.e., the scalar case), Prescott (1971) has used numerical procedures to investigate this issue. He found the expected value of the criterion function to be roughly equivalent for the stochastic and (numerical) dual control strategies when the ratio of the mean coefficient estimate to its estimated standard deviation exceeds one in absolute value.

For $j = s$ (or ss), given (18) and (37), the degree of suboptimality is a function of the matrices $\Omega^{-1} \circledast V_t^{BB}$, $\Omega^{-1} \circledast V_t^{BC}$, $\Omega^{-1} \circledast V_t^{BA}$, and the differences between S_{sft} and S_{st} and R_{sft} and R_{st}; while for $j = c$, (49) may be expressed as a function of these same matrices along with the covariance matrices $S_{sft} \circledast \Gamma_t^{BB}$, $S_{sft} \circledast \Gamma_t^{BA}$, and $S_{sft} \circledast \Gamma_t^{BC}$ in addition to the differences between S_{sft} and S_{ct} and R_{sft} and R_{ct}. Proceeding in a similar fashion for the stochastic controls, the expected loss function $L_s(u_t^s, u_t^c)$ may be derived. It should be obvious that the degree of suboptimality of u_t^c relative to u_t^s, obtained from this loss measure, is a function of the covariance matrices $S_{st} \circledast \Gamma_t^{BB}$, $S_{st} \circledast \Gamma_t^{BA}$, $S_{st} \circledast \Gamma_t^{BC}$ in addition to the differences between S_{st} and S_{ct} and R_{st} and R_{ct}.

2.4. *Value of Additional Information*

For each of the analytical approximate control strategies (c, s, ss, cc, sf), the value of additional information regarding the stochastic elements of the decision problem may be characterized. Such characterizations allow an assessment of superior probabilistic information and thus provide a basis for determining whether or not additional information should be purchased.[18] The specifications involved in this determination treat the various covariance matrices Γ_t^{-1}, Γ_t^{x-1}, and Ω^{-1} as stocks of information. Thus additional information may refer either to more efficient estimates of the coefficients entering the state transformation functions[19] (2), of the noncontrollable exogenous variables, or reductions in elements of the error covariance matrix.

The additional information values may be ascertained by deriving the imputed price associated with the above stocks of information. Since $\Lambda_{jt}(y_{t-1})$—the maximum expected value for the j-th control strategy—is a function of these stocks or covariance matrices, the relevant imputed prices may be obtained by evaluating the partial derivatives of $\Lambda_{jt}(y_{t-1})$ with respect to the covariance terms. For example, in the case of the stochastic controls, (24)–(27), we have

$$\frac{\partial \Lambda_{st}}{\partial \Omega^{-1}} = \sum_{r=0}^{T-t} \beta^r \Omega S_{st+r} \Omega \quad \text{and} \quad \frac{\partial \Lambda_{st}}{\partial \Gamma_t^{x-1}} = \Gamma_t^x(\bar{C}_t' S_{st} \bar{C}_t + S_{st} \circledast \Gamma_t^{cc}) \Gamma_t^x.$$

Since each of these terms is positive semidefinite, the price of information is larger, i.e., is more positive definite, for "larger" values of S_{st}, the latter reflecting the cost of imperfectly estimating the stochastic elements. Clearly, a smaller stock of information, i.e., a "larger" value of Ω or Γ^x, leads to higher imputed values. Similar, although more complicated, derivations may be found in Rausser and Freebairn (1973a) for the s and ss control imputed prices associated with the

[18] A possible framework for this determination involves specifying the expected (welfare) gain of the additional information and comparing it to the costs of collecting the additional information. The costly activities would, of course, include the collection of additional and perhaps more accurate data, the funding of further research, etc. To be sure, in the dual control framework, the allocation of data-collecting resources should be incorporated as part of the entire optimization and control process.

[19] Hence, it is implicity assumed, that the coefficient estimates are unbiased. Although the analysis of reductions in the bias of these estimates in an important consideration, it will not be treated here.

elements of Γ_t.[20] For the sequential, adaptive controls the result of differentiating Λ_{sft} with respect to Γ_t^{-1} is represented by (36). The matrix J_t is the sum of terms which may be interpreted as the stream of future rents resulting from an increment in the current stock of information associated with Γ_t^{-1}. For the *sf* controls, it should also be noted that the term $\Omega^{-1} \circledast J_t$ may be interpreted as the value of estimating (2) [MacRae (1972, p. 443)].

3. Empirical Components of Beef Trade Policy

The empirical components needed to implement the approximate control strategies (*c*, *s*, *ss*, *cc*, *sf*, *M*) for the U.S. beef trade policy problem are treated in this section. Since the construction and evaluation of these components are contained in other papers only a brief summarization of this material will be provided here. These components include a set of criteria functions (1), the state transformation or stochastic difference equations (2), and the information or probability updating functions (3).

3.1. *Criterion Function Set*

As argued in Rausser and Freebairn (1972, 1973), it is both unnecessary and unrealistic to attempt to specify a unique or single-valued criterion function for the analysis of public policy. In the environment of public policy making the importance of bargaining and the resulting compromises between different political groups, the range of preferences of these groups, and the lack of an explicitly stated unambiguous value consensus suggests the construction of several criterion functions. These functions should reflect the extreme viewpoints and preferences of various decision makers actively involved in the policy-making process, as well as the preference sets lying between these extremes. A parametric treatment of the resulting set of preferences in the derivation of the decision strategies would then provide decision makers with rational policy outcomes conditional on the representation of policy preferences. The generation of such information might even contribute to the efficiency of the bargaining process in reaching a consensus.

To specify the set of preference functions an analysis of the political process is required, particularly the major leverage points in this process [Bauer and Gergen (1968)]. Operational elements of the process, as well as its formal structure, should be ascertained. The current policy structure and some historical sketches of recent policy decisions may provide useful vehicles for characterizing the underlying processes. In Rausser and Freebairn (1973) an attempt along these lines was made for the beef policy problem and a formal framework was advanced for isolating the desired set. This framework involves a selection of the relevant arguments of W, a specification of the mathematical form of W, and the estimation

[20] In Rausser and Freebairn (1973a) these imputed prices are also determined within the context of a discrete Pontryagin type maximum problem. The costate variables of this problem (associated with the conditional covariance matrices of the stochastic elements) are determined by solving a two point boundary value problem, i.e., the canonical equations. For the stochastic or sequential stochastic controls this involves the derivation of a matrix Ricatti equation and the resulting time paths of the costate variables provide an explicit result for the value of information regarding Γ_t^{-1}.

of a range or set of values for the parameters of W. Information which may be used for this purpose includes interviews of decision makers (direct approach), implicit inferences based on an interpretation of past policy actions (indirect approach), and the investigator's knowledge of existing preferences or his value judgments, i.e., what he believes the "preference weights" ought to be (arbitrary approach).[21] Each of these sources of information were utilized in our attempt to capture an empirical representation of the beef policy preference function.

3.1.1. *Arguments of W.* The performance variables investigated as arguments of W were based on representative measures of consumer welfare, of beef producer welfare, and of preferences for the policy instrument variable (the level of import quota). The welfare effects of beef trade policy on the consuming segment of the U.S. populous were evaluated in terms of the market basket costs of selected meat commodities (y_1). Applying some separable utility function theorems we reduced the scope of the analysis by restricting it to the effects of trade policy on a subset of food items: i.e., the four meat commodities, fed or quality beef (q_1), other beef (q_2), pork (q_3), and poultry (q_4) were treated as separable commodity group. Furthermore consumers were disaggregated into five classes according to income per household[22] and distributional preferences for the various household income categories were employed. With respect to the latter, we assume that the inverse of the marginal personal income taxation rate is a reasonable index of decision makers' distributional preferences among consumers.[23] The resulting measure of consumer meat costs (y_1) is specified as a time varying linear combination of the retail prices for quality beef (p_1^r), other beef (p_2^r), pork (p_3^r), and poultry (p_4^r).

The second set of performance variables entering W provide measures of U.S. beef producer welfare. Empirical evidence presented in Rausser and Freebairn (1972) suggests that beef goes through two production stages and, to a large extent, different individuals are involved in these two stages. These two groups are beef breeding cow-calf producers and cattle feeders. Changes in beef trade policy might be expected to have different effects on the returns to the two activities. Moreover, there appears to be a tendency for public decision makers to place greater weight on the welfare of breeding beef cow-calf producers than on the welfare of cattle feeders. Therefore, the welfare of beef producers are represented by two variables; one measuring the aggregate returns to breeding cow-calf producers (y_2) and the other the aggregate returns to cattle feeders (y_3). The former measure is specified as a time varying linear combination of the stock of breeding beef cows (K_b), the producer price of feeder calves (p_5^f), the producer price of other beef (p_2^f), and a vector composed of calf survival rates, heifer replacement rates, cow death rates, average cow sale weight, average calf sale weight, as well as variable input expenditures for the breeding cow activity. Similarly, the latter measure for the cattle feeding activity is specified as a time varying linear combination of the stock of cattle on feed (I_f), the producer price of quality beef

[21] This approach embraces the imaginary interviewing procedures suggested by van Eijk and Sandee (1959).

[22] These classes are: <2.000, 2.000–3.999, 4.000–5.999, 6.000–7.999, and ≥ 8.000.

[23] To be sure, progressive taxation is but one of many devices used to redistribute wealth. For further details on this and other possible measures, see Rausser and Freebairn [1972].

(p_1^f), the producer price of feeder calves (p_5^f), the producer price of corn (p_c^f), and a vector composed of death rates, average purchase and sale weights, as well as variable input expenditures.

3.1.2. *Estimation of parameter set.* Given the justification for an additive, quadratic specification of W presented in Rausser and Freebairn (1973), procedures are developed there for estimating a set of preference weights. On the basis of implicit inferences from past policy actions, preferences for higher producer returns were given greater weight than the preferences for lower food cost to consumers. Taking producers as a collective group, preference weights for aggregate consumer meat cost relative to aggregate producer returns ranging from 0.25:1.0 to 1.0:1.0 were isolated. With respect to the two types of producers, weights were obtained for cow-calf producer returns relative to cattle feeder returns over the range 2.0:1.0 to 1.0:1.0. For the policy variable u two cases were considered, one in which a zero weight is attached to preferences for this variable and another in which a million pound change in u is equated to a million dollar increase in consumer meat cost.

The explanatory properties of the estimated set of criteria functions were evaluated by implicitly deriving the weights associated with the various performance and control variables over the period 1959–1969 [Rausser and Freebairn (1973)]. On the basis of this evaluation, it was found that trade-off ratios in the vicinity of 1:2:2:0 were consistent with actual beef trade policy decisions over the indicated period. This evaluation, of course, only provides an *ex post* justification for the estimates derived and assumes that for the sample period a reasonable approximation is obtained from treating the estimation and control problems separately. For purposes of the beef trade policy analysis, it does, however, support or at least does not refute, the presumption that values for the parameters of the criterion function set may be based on a relative weight range of 1:1:1:0 to 1:4:4:2.

3.2 *Econometric Model of U.S. Livestock Sector*

Since the performance variables of W are determined as a linear combination of the state variables p_i^r ($i = 1, 2, 3, 4$), p_1^f, p_2^f, p_5^f, K_b, and I_f, it would appear that nine state transformation equations (2) are required. However, if these state variables are embedded in a larger structural system, i.e., they are interdependent with a number of other endogenous variables, more than nine state transformation equations will be involved. In the present investigation, available evidence suggests that the nine endogenous variables mentioned above are either interdependent or seemingly unrelated with a number of other (current) endogenous variables characterizing the U.S. livestock sector. Hence, although our ultimate concern is with the reduced form relations of the state or endogenous variables entering W, a complete structural model of the U.S. livestock sector was formulated and estimated.

In developing this model, an attempt was made to represent the significant components of the aggregate (annual) behavior of economic units involved in the production, consumption and trade of meat products. As usual, it is not maintained that the real world in every detail is actually represented by the constructed model. However, we propose that the model does provide a "reasonable" approximation of the more important causal behavior patterns. Its specific components may be described as (i) consumer meat demand. (ii) margin and producer meat prices. (iii) cattle producers. (iv) beef imports. (v) pork producers. and (vi) poultry producers and marketing. These components are collectively represented in the structural model by 30 equations, of which 20 are stochastic and 10 are identities.

The theoretical foundations underlying the structural model. knowledge of technical relationships influencing consumer and producer decisions related to meat products, sample data, the complete econometric model specification. estimators employed, the estimated relationships, and various model evaluations are completely described in Freebairn and Rausser (1973). One of the principal features of the theoretical model is the recognition that cattle, pork and poultry producers behave under risk and uncertainty. The length of the sample time series data (1956–1969) is severely limited by the specification of two beef quality components (fed and other beef) for both consumers and producers.[24] Evaluations of the estimated model involved an examination of impact and dynamic multipliers. frequency response characteristics. stability properties. forecasting performance. and stochastic (simulated) properties. The model's behavior in each of these respects conformed to *a priori* notions: it was found to be stable with an average cyclical period length of 5.7 years: forecasts for both 1970 and 1971 were relatively close to observed values and mean forecast errors (changes and levels) were deemed acceptable: and as expected interim multipliers for beef import quotas were significant for a relatively large number of future periods. This latter result suggests that future period effects of changes in the current levels of beef import quotas are fairly substantial.

3.2.1. *State transformation functions.* As indicated above, not all of the reduced form relations obtained from the estimated structural model are required for the state transformation functions of the beef trade decision problem. The argument variables of the criteria function set discussed in Section 3.1 suggest that state transformation equations are needed for p_i^r $(i = 1, 2, 3, 4)$, p_1^f, p_2^f, p_5^f, K_b, and I_f. However, in addition to these nine equations, relations are needed for those lagged endogenous variables which appear as explanatory variables. These variables which are not represented as arguments in the criterion function set include the producer price of pork (p_3^f), the producer prices of poultry (p_4^f), the stock of calves available for feedlots (K_f). the stock of farrowing sows (K_h). and births of beef calves less beef calf deaths and meat sales (K_{cs}). Combining the reduced form equations for these five variables with the nine listed above results in a specification for (2) which contains fourteen state transformation equations or endogenous variables. Thus, although 30 reduced form relations have been derived from the estimated structural model. 16 of these are not needed for the decision model application. The reduced form equations required for the application. of course.

[24] Three reasons may be cited for separating beef into two commodities of differing qualities. First. fed beef, representing the higher quality. satisfies different wants and has a higher income elasticity of demand [Langemeir and Thompson (1967)] than does other beef. representing the lower quality ground. stewing and processed beef products; second. while fed beef is the main output from feedlot operations. most of the other beef category is produced by a separate group of firms, viz.. dairy and breeding beef cow-calf firms. Last and perhaps most importantly. almost all the imported beef is of comparable quality to domestic produced other beef.

reflect information which is contained in all of the structural relations of the econometric model and thus behavior patterns for the entire set of 30 endogenous variables. The control vector u in (2) is represented by a single instrument variable, the level of the beef import quota. In the case of the noncontrollable exogenous variables of (2), x is of dimension 16×1. These latter variables are defined and their relative importance is discussed in Freebairn and Rausser (1973).

3.3. *Information Updating Functions*

Turning to the updating functions (3), prior estimates of the probability distribution for the uncertain elements (D, e) at the beginning of the control period, i.e., $P_d^0(\cdot)$ and $P_e^0(\cdot)$ are obtained from the sampling distribution estimates of the coefficients and disturbances entering the reduced form relations. Given the Gaussian specification on the structural disturbances, the initial (joint) probability distribution $P_d^0(\cdot)$ may be stated in terms of the estimated reduced form coefficients $(\bar{A}_0, \bar{B}_0, \bar{C}_0)$ and covariance matrix (Γ_0).[25] As indicated in 3.2.1, these estimates refer only to the 14 state transformation equations which were derived from the estimated livestock sector model. The prior $P_e^0(\cdot)$ was also obtained from this source, while the mean vector and covariance matrix of $P_x^0(\cdot)$ were obtained from estimated linear and quadratic trend equations utilized to forecast the non-controllable exogenous variables.[26] In the case of the control strategies which involve a sequence of open-loops, the priors for Ω, $\bar{x}$, and Γ^x were updated independently of $P_d(\cdot)$.[27]

For the period beyond the endpoint of the sample (1969) over which observations are available, viz., 1970 through 1972 ($t = 1, 2, 3$), equations (30) and (31) were utilized to derive the updated mean vector $\bar{D}_t$ and covariance matrix Γ_t.[28] These updated estimates were then employed as priors in the derivation of the *ss*, *ce*, *sf*, and *M* control strategies for period t, $t > 1$. In case of the stochastic controls, only the prior estimates based on the sample period are required, i.e., $\bar{D}_0$ and Γ_0. Similarly, for the certainty equivalent controls only the prior mean estimate $\bar{D}_0$ is needed.

[25] Given the maintained hypothesis of the livestock econometric model and the estimators which were utilized, these estimates are, of course, valid only in an asymptotic sense.

[26] For further details, see Freebairn and Rausser (1972, Appendix C).

[27] For cases in which the covariance matrix of the disturbance vector e is treated as unknown, the initial prior is the product of a normal distribution on D and a Wishart distribution on Ω. The resulting posteriors obtained after each observation period will all be of the same normal-Wishart form and will lead to updating equations of the type (30) and (31) for $\bar{D}_t$ and Γ_t as well as an updating equation for $\bar{\Omega}$. The details on this derivation may be found in a number of places; see, for example, Rausser and Freebairn (1973a).

[28] In other words, for sake of simplicity we ignored the overidentifying restrictions associated with the original structural model of the U.S. livestock sector. We could have updated $\bar{D}$ and Γ (assuming no changes in the maintained hypothesis) by re-estimating the structural model after each additional observation, i.e., estimate the original structure on the basis of an augmented sample and derive the new $\bar{D}$ and Γ associated with the fourteen reduced form equations of 3.2.1. For a treatment of this and other alternative updating procedures (stated in computationally efficient recursive forms) along with relative computational efficiency in the context of various types of systems, see Rausser and Freebairn (1973a).

4. Approximate Control Solution Results

Some results obtained from the approximate control analysis for the U.S. beef trade policy problem are reported in this section.[29] First period certainty equivalent decisions for an eight-year planning horizon, a time preference factor of 0.9, and ten functions of the criterion set for W are recorded in Table 1. Comparing these results (particularly the first five criteria functions listed) to those reported for the stochastic controls (Table 2) suggests that assuming the coefficient matrices A, B, and C are known, when in fact they are not, involves a serious specification error. This observation is not surprising in view of the uncertainty present in the decision problem under examination.

From the information in Tables 1 and 2, it is also clear that even though $-N_{s1}^{-1}f_{s1}$ exceeds $-N_{c1}^{-1}f_{c1}$, the c controls are far less aggressive than the s controls. Although this outcome is surprising at first glance, it results from the difference between S_{st} and S_{ct}, the relative magnitudes of the elements contained in Γ_t^{BC} and Γ_t^{BA}, and the fact that both $-N_{c1}^{-1}F_{c1}y_{t-1}$ and $-N_{s1}^{-1}F_{s1}y_{t-1}$ are less than zero, with the latter absolutely smaller than the former. Hence, the net effect of recognizing uncertainty in coefficient matrices A, B, and C is to place more weight on the proxy measures of consumer welfare relative to producer welfare.

The 10 criterion functions presented in Table 2 represent the extremes of the relative weight attached to the control variable (u) and the range of relative weights associated with the performance variables. For each of the different welfare weightings indicated, the derived first period s controls are larger, in some cases substantially, than the actual 1970 U.S. beef import quota of approximately 1,800 million pounds (carcass weight equivalent). For the most part, this control along with the c control results are consistent with our prior expectations: in particular, they conform to the belief that the desired import quota levels should increase as more weight is attached to the welfare of consumers relative to that of producers. Note also that c and s (as in the case of other approximate control settings) are positively related, in general, to producer and retail prices for the various meats with p_1^f having the largest relative influence. Each of the stock variables, cattle on feed (I_f), beef cows (K_b), and the combined calf inventory variable (K_f and K_{cs})[30] have a negative influence on the various approximate control decisions. In addition, for all approximate control strategies, the scalar term $-N_{jt}^{-1}f_{jt}$ indicates that the

[29] In a preliminary examination we investigated the sensitivity of the various first period (1970) approximate controls to the length of the planning horizon, T, and the time preference factor, β. Much of these results are treated in some detail elsewhere [Rausser and Freebairn (1972a)]. In general, the various approximate controls are quite sensitive to T, increasing from $T = 3$ to $T = 6$ years and declining thereafter. From a relative standpoint, beyond $T = 6$ the length of the planning horizon has less influence on the first period controls than $T \leq 6$. For the time preference factor, variation over the range $0.75 \leq \beta \leq 0.95$ was investigated. Although the first period controls were influenced by β, the effects of this factor were minimal and less marked than those associated with variation in T or parameters of the criterion function set.

[30] The variables K_f and K_{cs} are related by the identity $K_{ft} = K_{cs,t-1} - I_{dt} + M_{ft}$, where I_d denoted dairy stock replacements and M_f denotes calf imports. Since the values of I_d and M_f are relatively constant and small, it is meaningful to evaluate the combined effects of the closely related variables K_f and K_{cs}. In this regard, the weighted average (where the weights are based on recent observations) effect on K_f and K_{cs} is negative.

TABLE 1

FIRST PERIOD (1970) CERTAINTY EQUIVALENT CONTROLS ASSUMING AN EIGHT YEAR PLANNING HORIZON, A TIME PREFERENCE RATE OF 0.9, AND DIFFERENT PREFERENCE WEIGHTINGS

Welfare Weighting	$-N_{c1}^{-1}F_{c1}$									$-N_{c1}^{-1}f_{c1}$	Import Quota (mill. lb.)
	p_1^f	p_3^f	p_4^f	p_5^f	K_f	I_f	K_h	K_b	K_{cx}		
1:1:1:0	223.05	10.68	28.42	−1.58	−0.2643	−0.9859	−0.1502	−0.0852	0.0215	34.714	3.531
1:2:2:0	207.64	17.23	34.56	−1.66	−0.2672	−0.9277	−0.0252	−0.0833	0.0216	31.065	2.723
1:3:2:0	233.32	20.84	35.28	−1.56	−0.2670	−0.9228	−0.0151	−0.0814	0.0226	28.756	2.018
1:3:3:0	196.73	28.15	38.39	−1.61	−0.2691	−0.8927	0.0544	−0.0807	0.0183	28.610	2.048
1:4:4:0	189.91	35.22	40.91	−1.51	−0.2704	−0.8719	0.1067	−0.0784	0.0146	27.042	1.552
1:1:1:1	88.40	−15.34	8.64	−10.03	−0.1406	−0.2073	−0.1843	−0.0977	0.2568	7.465	3.293
1:2:2:1	95.76	−7.34	13.07	−9.59	−0.1564	−0.2561	−0.1339	−0.0967	0.2478	7.924	2.781
1:3:2:1	108.49	−5.64	13.91	−9.60	−0.1613	−0.2650	−0.1197	−0.1032	0.2411	7.793	2.381
1:3:3:1	101.44	−0.90	16.59	−9.16	−0.1687	−0.2964	−0.0932	−0.0958	0.2377	8.405	2.358
1:4:4:1	106.07	4.46	19.48	−8.74	−0.1784	−0.3304	−0.0593	−0.0949	0.2275	8.885	2.032

TABLE 2

FIRST PERIOD (1970) STOCHASTIC CONTROLS ASSUMING AN EIGHT YEAR PLANNING HORIZON, A TIME PREFERENCE RATE OF 0.9, AND DIFFERENT PREFERENCE WEIGHTINGS

Welfare Weighting	$-N_{s1}^{-1}F_{s1}$									$-N_{s1}^{-1}f_{s1}$	Import Quota (mill. lb.)
	p_1^f	p_3^f	p_4^f	p_5^f	K_f	I_f	K_h	K_b	K_{cx}		
1:1:1:0	232.18	8.14	26.90	−3.77	−0.2599	−0.8618	−0.0685	−0.0791	0.1126	27.743	4.708
1:2:2:0	220.67	16.29	31.68	−3.56	−0.2619	−0.8309	0.0220	−0.0725	0.1039	25.757	4.424
1:3:2:0	237.32	18.33	32.14	−3.47	−0.2619	−0.8292	0.0454	−0.0729	0.1006	24.559	4.014
1:3:3:0	214.48	23.21	34.28	−3.41	−0.2631	−0.8148	0.0718	−0.0687	0.0984	24.664	4.343
1:4:4:0	210.48	27.62	35.91	−3.31	−0.2638	−0.8049	0.1033	−0.0662	0.0948	23.995	4.206
1:1:1:2	63.56	−4.91	6.34	−5.62	−0.0847	−0.1299	−0.0565	−0.0304	0.1292	5.391	3.948
1:2:2:2	73.39	−3.37	8.26	−6.24	−0.0993	−0.1568	−0.0495	−0.0342	0.1458	5.453	3.841
1:3:2:2	79.45	−3.16	8.75	−6.53	−0.1044	−0.1629	−0.0483	−0.0385	0.1510	5.517	3.808
1:3:3:2	81.34	−2.00	9.86	−6.69	−0.1112	−0.1796	−0.0431	−0.0377	0.1586	5.546	3.558
1:4:4:2	87.89	−0.76	1.12	−7.04	−0.1210	−0.1993	−0.0371	−0.0408	0.1687	5.696	3.293

TABLE 3

APPROXIMATE CONTROL SOLUTIONS (1970–1973) ASSUMING A TEN YEAR PLANNING HORIZON, A PREFERENCE WEIGHTING OF 1:3.2:2 AND A TIME PREFERENCE RATE OF 0.9

Decision Period	Approximate Control Policies					
	u_t^c	u_t^s	u_t^{cc}	u_t^{ss}	u_t^{sf}	u_t^M
1970	2.406	3,808	3,406	3.808	4,062	4,153
1971	2.749	4.211	2,698	4.181	4,271	4,365
1972	2.685	3.903	2.933	4,261	4,394	4,437
1973	2.833	4.106	3,593	4,626	4.568	4.522

TABLE 4

RATIO OF EXPECTED APPROXIMATE CONTROL GAIN TO EXPECTED SEQUENTIAL, ADAPTIVE COVARIANCE CONTROL GAIN (1970–1973) ASSUMING A TEN YEAR PLANNING HORIZON, A PREFERENCE WEIGHTING OF 1:2:2:1 AND A TIME PREFERENCE RATE OF 0.9

Decision Period	Relative Expected Gain*					
	$J_{sft}(u_t^c)/\alpha_t$	$J_{sft}(u_t^s)/\alpha_t$	$J_{sft}(u_t^{cc})\alpha_t$	$J_{sft}(u_t^{ss})/\alpha_t$	$J_{sft}(u_t^{sf})/\alpha_t$	$J_{Mt}(u_t^M)/\alpha_t$
1970	0.62	0.88	0.62	0.88	1.00	1.11
1971	0.58	0.96	0.56	0.94	1.00	1.09
1972	0.52	0.85	0.69	0.93	1.00	1.06
1973	0.64	0.82	0.78	0.97	1.00	1.03

* $\alpha_t = J_{sft}(u_t^{sf})$ and $J_{Mt}(u_t^M)$ is computed by numerical approximation.

net effect of the noncontrollable variables is to call for increasing levels of beef import quotas over the planning horizon.

Representative results for the comparison of c and s to sequential controls are reported in Tables 3 and 4. As noted in Section 2, control variable settings, *ceteris paribus*, will be more extreme the larger the value of additional information $(\partial\Lambda_{jt}/\partial\Gamma_t^{-1})$ which in turn reflects, in part, the level of uncertainty (Γ_t). This experimental aspect is tempered by the unknown effect of extreme policy actions on y and the negative weight, if any, on changes in levels of the policy variable u. In the case of beef quotas, two of the approximate control strategies, sf and M (the M-measurement controls were computed for $M = 2$ and as a generalization of the sf controls), partially recognize this dimension. In general, these two approximate approaches led to more (less) extreme settings of control levels in the first (last) few periods of a given planning horizon than the sequential stochastic controls (ss). Furthermore, these two approximations generally resulting in (i) control settings which exceeded, in some cases by substantial amounts, solutions obtained for c, s, and cc approximations (Table 3); and (ii) expected gains which exceeded all other control strategies (Table 4). These results suggest that beef trade public decision makers may find it beneficial to incur learning (mitigating uncertainty) costs by substituting knowledge accumulation in current periods for expected gains at some later date.

For almost all cases examined, c controls and to a smaller extent s controls performed poorly in comparison to ss, sf, and M controls. This was principally due to their failure to record the impact of recent observations on the estimated system (2). In particular, note the change between the 1971 and 1972 decision periods: all other controls increased while c and s controls decreased from 1971 to 1972. These relative movements were caused, in part, by structural changes in consumer meat demand functions over the period 1970–1972 which were reflected (in a significant fashion) only in the updated structure (2) for 1972 and 1973 ($\bar{D}_3$ and $\bar{D}_4$). Since c and s controls are conditioned on an estimated system pertaining to the initial decision period, they obviously do not reflect any structural changes that might occur subsequently. These structural changes along with the general growth in f_t (especially $\bar{x}_t$) and the elements of y_{t-1} also assist in explaining the substantial increases in the cc control setting from 1972 to 1973.

The relative magnitudes of the control settings reported in Table 3 and the ordering among strategies suggested by the expected gains (Table 4) over t(1, 2, 3, 4) remained fairly robust against changes in the planning horizon length beyond six years; reasonable variations in the time preference rate (β): the preference weighting interval of 1:1:1:0 to 1:4:4:2; and changes in the current state of the system (y_{t-1}). All approximate controls generally increased with increments in the length of the planning horizon, the time preference rate, and "consumer welfare"; they decreased with increases in the preference weights attached to the control variable, and "producer welfare." In addition, all approximate control settings generally increased over time due to changes in the probability updating functions (3), the state of the system, and the levels of the exogenous variables. The relative expected gain measures suggest, as anticipated, that sequential controls ss and sf outperform the c and s controls, especially the certainty equivalent approximation as well as its updated form (cc). Moreover, the performance of ss, sf, and M controls is proximal with a fairly consistent advantage given to M-measurement feedback controls. The ordering $ss < sf < M$, without substantial differences, is also supported by preliminary (stochastic) simulation experiments with the beef trade policy model over the period 1973 through 1982.

5. Conclusion

The adaptive control model formulation of economic policy examined in this paper appears to capture some important characteristics of many economic decision making problems. Imperfect knowledge about the effects of alternative decisions is a dominant feature. The process of sequential decision making permits the utilization of forthcoming sample information so as to learn about the uncertain elements as the process evolves. In the general formulation decisions are allowed to influence in an active way the type of information generated in the learning phase and thus the resulting controls are dual in nature. These closed-loop control strategies require the simultaneous optimization of the direct control, learning, and design of experiment dimensions. Operationally, however, seldom can we expect to derive the optimal decision strategies: analytical solutions are not yet available and the computational cost of numerical procedures is burdensome for all but the simplest problems. Thus we turned to approximate control strategies.

The derived properties of the approximate adaptive controls are largely of a qualitative rather than a quantitative nature. The stochastic controls (s) recognize uncertainty and in the case of the sequential stochastic control strategies (*ss*) they allow for the passive accumulation of information. The latter controls ignore the dual experimental dimension of the optimal adaptive controls; the importance of this dimension is reflected by the precision of the estimated means for the random coefficients and its information value is enhanced with extreme settings of the policy variables. This dimension is partially recognized by the approximate sequential, adaptive covariance (*sf*) control formulation as well as the *M*-measurement feedback control specification (*M*). If parameter uncertainty is an important consideration, the certainty equivalent controls (*c*) provide a poor approximation: if, in addition, learning is an important consideration, both the *c* and *s* controls provide poor approximations while if the experimental dimension is also an important consideration the *ss* controls provide an inferior approximation. The degree of approximation obtained, in general, by utilizing the *sf* or *M* controls remains an open question.

For the beef trade policy problem, the level of uncertainty resulted in *c* controls which were crude approximations to the *s*, *ss*, *sf*, *M* or presumably the optimal adaptive control strategies. It should be noted, however, that (i) the actual import quota level for 1970 was consistent with the certainty equivalent decisions derived from objective functions which weighted a decrease in consumer meat costs to an increase in aggregate producer returns of 1:3 or more; (ii) the actual level of the import quota for 1972 was slightly below that suggested by these decisions for the criterion function set considered: (iii) as expected, these decision strategies relate future levels of the import quota positively to the prices of beef products and negatively with beef stock variables; and (iv) these strategies indicate that the optimal control setting for the quota instrument is sensitive to changes in the levels of price and stock variables. In contrast to (i) and (ii), the *s*, *ss*, *sf*, and *M* control strategies suggested that it would be desirable to expand the maximum import quota substantially. The properties of these strategies were similar to those noted above in (iii) and (iv) for the certainty equivalent controls.

From the standpoint of historical U.S. beef import quota decisions, the *ss*, *sf*, and *M* control solutions represent extreme policy actions. These controls as well as the *s* control solutions, for some of the criterion functions examined, exceed by substantial amounts recent import quota levels. Moreover, these controls are likely to be nonbinding in the sense that they exceed the level of beef imports under a free trade situation for at least some years of the planning horizon. Hence it appears reasonable to argue that the derived *ss*, *sf*, and *M* strategies provide an acceptable approximation, in an operational context, to the optimal dual controls for the U.S. beef trade policy problem.

Finally, since desirable import quotas are sensitive to variables measuring the state of the livestock sectors and since estimated coefficients appearing in the model representation for this sector have relatively large variances, it would seem useful to explicitly incorporate sequential procedures for adjusting the quota level in any future legislation influencing the import of beef. In addition, a large potential payoff is indicated for employing new sample information to update coefficient estimates of the model representation. Such updating might also be extended to the original structural representation and to revisions in the existing maintained hypothesis: in particular the specification of a more detailed subsystem for supply response in the principal beef exporting countries. The apparent importance of structural change also suggests that limited memory filters [Jazwinski (1970)] may prove useful in future modeling efforts concerned with the U.S. Livestock Sector. The above aspects along with the noisy state measurement specification found in much of the engineering literature have been employed by the authors in other applications of adaptive control. Preliminary results obtained from these models appear promising and provide further support for the view that the *ss*, *sf*, and *M* control strategies, especially the latter, are worth the effort.

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REFERENCES

[1] Aoki, M. [1967]. *Optimisation of Stochastic Systems*. New York: Academic Press, Inc.
[2] Astrom, K. J. and B. Wittenmark [1971]. "Problems of Identification and Control." *Journal of Mathematic Analysis and Applications*, 34.
[3] Athans, M. [1967]. "The Matrix Minimum Principle." *Information and Control*, 11.
[4] Athans, M. [1972]. "The Discrete Time Linear Quadratic Gaussian Stochastic Control Problem." *Annals of Economic and Social Measurement*, 1.
[5] Bar-Shalom, Y. and R. Sivan [1969]. "On the Optimal Control of Discrete-Time Linear Systems with Random Parameters." *IEEE Trans. on Automatic Control*, AC-14.
[6] Bauer, R. A. and K. J. Gergen [1968]. *The Study of Policy Formation*. New York: Free Press.
[7] Bellman, R. E. [1961]. *Adaptive Control Processes: A Guided Tour*. Princeton, New Jersey: Princeton University Press.
[8] Bellman, R. [1957]. *Dynamic Programming*. Princeton, New Jersey: Princeton University Press.
[9] Bellman, R. and R. Kalaba [1959]. "A Mathematical Theory of Adaptive Control Processes." *Proc. Nat. Acad. Sci. USA*, 45.
[10] Buchanan, L. F. and F. E. Norton [1971]. "Optimal Control Applications in Economic Systems." in C. T. Leondes, ed. *Advances in Control Systems: Theory and Applications*, 8. New York: Academic Press.
[11] Cannon, M. D., C. D. Cullum, and E. Polak [1970]. *Theory of Optimal Control and Mathematical Programming*. New York: McGraw-Hill.
[12] Chow, G. C. [1972]. "Optimal Control of Linear Econometric Systems with Finite Time Horizon." *International Economic Review*, 13.
[13] Curry, R. E. [1970]. *Estimation and Control with Quantized Measurements*. Cambridge, Mass.: The M.I.T. Press.
[14] Dreyfus, S. E. [1964]. "Some Types of Optimal Control of Stochastic Systems." *SIAM Journal of Control*, 2.
[15] Early, R. H. and B. N. Early [1972]. "On the Relative Performance of the Optimal Control System with M Measurements." *IEEE Trans. on Automatic Control*, AC-17.
[16] Fel'dbaum, A. A. [1965]. *Optimal Control Systems*. New York: Academic Press, Inc.
[17] Fisher, W. D. [1962]. "Estimation in the Linear Decision Model." *International Economic Review*, 3.
[18] Freebairn, J. W. and G. C. Rausser [1973]. "An Econometric Model of the U.S. Livestock Sector with Emphasis on Beef Imports." unpublished manuscript, University of California, Davis.
[19] Gunckel, T. L. and G. F. Franklin [1963]. "A General Solution for Linear Sampled-Data Control." *Journal of Basic Engineering (Trans. ASME Ser. D)*, 85.
[20] Halkin, A. [1966]. "A Maximum Principle of the Pontryagin Type for Systems Described by Nonlinear Difference Equations." *SIAM Journal of Control*, 4.
[21] Holt, C. [1962]. "Linear Decision Rules for Economic Stabilisation and Growth." *Quarterly Journal of Economics*, 67.
[22] Jazwinski, A. H. [1970]. "Stochastic Processes and Filtering Theory." New York: Academic Press.
[23] Kogan, Y. A. [1966]. "Comparison of Non-Optimal and Optimal Strategies in Dual Control Problems." *Automation and Remote Control*, 27.
[24] Ku, R. and M. Athans [1972]. "On the Adaptive Control of Linear Systems Using the Open-Loop-Feedback-Optimal Approach." IEEE Decision and Control Conference, New Orleans, La., December.

[25] Langemeir. L. and R. G. Thompson [1967]. "Demand Supply and Price Relationships for the Beef Sector. Post World War II Period." *Journal of Farm Economics*, 49.
[26] Lainiotis, D. G., J. G. Deshpande, and T. N. Upadhyay [1972]. "Optimal Adaptive Control: A Nonlinear Separation Theorem." *International Journal on Control*. 15.
[27] MacRae, E. C. [1972]. "Linear Decision with Experimentation." *Annals of Economic and Social Measurement*. 1.
[28] MacRae. E. C. [1971]. "Matrix Derivatives with an Application to the Analysis of Covariance Structures." Special Studies Paper No. 20. Federal Reserve Board. September.
[29] Marschak. J. [1963]. "On Adaptive Programming." *Management Science*. 9.
[30] Murfy. R. E. [1965]. *Adaptive Processes in Economic Systems*. New York: Academic Press.
[31] Murphy. W. J. [1968]. "Optimal Stochastic Control of Discrete Linear Systems with Unknown Gain." *IEEE Trans. on Automatic Control*. AC-13.
[32] Nissen. D. H. [1968]. "A Note on the Variance of a Matrix." *Econometrica*. 36.
[33] Perkins. W. R., J. B. Cruz. Jr., and N. Sundararajan [1972]. "Feedback Control of a Macroeconomic System Using an Observer." *IEEE Trans. on Systems Management and Cybernetics*. SMC-2.
[34] Popović. B. [1972]. *Analytic Solutions to Some Stochastic Adaptive, and Dual Optimal Control Problems for Economic Decision Making*. unpublished Ph.D. thesis. University of Illinois. Urbana-Champaign.
[35] Prescott. E. C. [1971]. "Adaptive Decision Rules for Macroeconomic Planning." *Western Economic Journal*. 9.
[36] Prescott. E. C. [1972]. "The Multi-Period Control Problem Under Uncertainty." *Econometrica*. 40.
[37] Ramsey. F. P. [1928]. "A Mathematical Theory of Saving." *The Economic Journal*. 38.
[38] Rausser. G. C. and J. W. Freebairn [1972]. "Estimation of Social Preference Functions for the Analysis of Economic Policy Problems." 1972 American Agricultural Economic Association Meeting. Gainesville. Florida. August.
[39] Rausser. G. C. and J. W. Freebairn [1972a]. "Adaptive Control Methods: An Application to U.S. Beef Trade Policy." 1972 Econometric Society Meetings. Toronto. Canada. December.
[40] Rausser. G. C. and J. W. Freebairn [1973]. "Estimation of Policy Preference Functions: An Application to U.S. Beef Import Policy." *Rev. of Econ. Statist.*. forthcoming.
[41] Rausser. G. C. and J. W. Freebairn [1973a]. "Alternative Updating Estimators and Filters for Economic Systems." unpublished manuscript. University of California. Davis.
[42] Simon. H. A. [1956]. "Dynamic Programming Under Certainty with a Quadratic Criterion Function." *Econometrica*. 24.
[43] Tarn. T. J. [1971]. "Extended Separation Theorem and Exact Analytical Solution of Stochastic Control." *Automatica*. 7.
[44] Theil. H. [1964]. *Optimal Decision Rules for Government and Industry*. Amsterdam: North-Holland.
[45] Tinbergen. J. [1952]. *On the Theory of Economic Policy*. Amsterdam: North-Holland.
[46] Tse. E. and M. Athans [1970]. "Optimal Minimal-Order Observer-Estimator for Discrete Linear Time-Varying System." *IEEE Trans. on Automatic Control*. AC-15.
[47] Tse. E. and M. Athans [1972]. "Adaptive Stochastic Control for a Class of Linear Systems." *IEEE Trans. on Automatic Control*. AC-17.
[48] Tse, E., L. Meier. and Y. Bar-Shalom [1972]. "Wide-Sense Adaptive Dual Control for Nonlinear Stochastic Systems." *IEEE Trans. on Automatic Control*. AC-17.
[49] Tse. E. and Y. Bar-Shalom [1972]. "An Actively Adaptive Control for Linear Systems with Random Parameters via the Dual Control Approach." *IEEE Trans. on Automatic Control*. AC-17.
[50] U.S. Congress [1969]. Committee on Government Operations. *Federal Responsibility for Retail Price Increases for Beef*. Washington. D.C.
[51] U.S. Tariff Commission [1964] *Beef and Beef Products*. Tariff Commission Publication 128. Washington. D.C.
[52] Van Eijk. C. J. and J. Sandee [1959]. "Quantitative Determination of an Optimum Economic Policy." *Econometrica*. 27.
[53] Ying. C. C. [1967]. "Learning by Doing - An Adaptive Approach to Multiperiod Decisions." *Operations Research*. 15.
[54] Zellner. A. [1971]. *An Introduction to Bayesian Inference in Economics*. New York: John Wiley and Son.
[55] Zellner. A. and M. S. Geisel [1968]. "Sensitivity of Control to Uncertainty and Form of the Criterion Function." in D. G. Watts (ed.). *The Future of Statistics*. New York: Academic Press.