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**UNEMPLOYMENT AND ENTREPRENEURSHIP: A  
CYCLICAL RELATION?**

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# Unemployment and entrepreneurship: A cyclical relation?

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**Abstract:** This paper presents a cyclical model for unemployment and entrepreneurship. The estimated periodicity of the cycles for the US, the UK, Spain and Ireland is between 5 and 10 years, and the orders of integration are smaller (greater) than 1 if the underlying disturbances are autocorrelated (white noise), corresponding to dampen cycles (limit cycle).

**Keywords:** New firms; Employment creation; Cycles;

**JEL Classification:** J69, L26, M13

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# Unemployment and entrepreneurship: A cyclical relation?

## Introduction

What is the relationship between entrepreneurship and unemployment? There is a large literature dealing with this question [e.g., Oxenfeldt, 1943; Blau, 1987; Evans and Jovanovic, 1989; Evans and Leighton, 1990; Blanchflower and Meyer, 1994; Pfeifer and Reize, 2000a, 2000b; Audretsch et al., 2001]. On the one hand new firm startups hire workers, which may result in a fall of unemployment. On the other hand, high unemployment may lead to an increase in startup activity, since the opportunity cost of starting a new firm is lower for the unemployed. This suggests that both variables impact each other dynamically.

This paper presents a cyclical model between firm creation and unemployment. The model can generate a unique stable limit cycle, or dampen cycles. The estimated periodicity of the cycles for the US, the UK, Spain and Ireland is between 5 and 10 years. The orders of integration are above 1 if the underlying disturbances are white noise, which is consistent with a limit cycle. If autocorrelation is allowed, the orders of integration are below unity, implying mean reverting behavior consistent with dampen cycles.

## The model

The variation of unemployment rate over time,  $\dot{u} \equiv du/dt$ , is associated with entrepreneurship ( $e$ ), through function  $f(e)$ . Assuming that new firm startups increase competition, through the creation of new goods and services, or direct competition in existing industries reducing the monopoly power of the incumbent firms, the

increasing competition is reflected in the increase in production, which can only be achieved, for a given level of technology and managerial skills, through the increase in labor employment, leading to a reduction in the unemployment rate:

$$\dot{u} = -f(e). \quad (1)$$

Entrepreneurship varies over time,  $\dot{e} \equiv de/dt$ , as a function of existing firms and unemployment. We assume a positive relation between the unemployed and business creation, since the opportunity cost to create a brand new business is smaller for the unemployed. Of course, we consider entrepreneurial skills and the probability of being unemployed to be the same across the population. We also assume that the unemployed have unrestricted access to the credit market<sup>1</sup>, and tax structure and social security do not represent barriers for firm creation. The impact of existing firms, captured by the function  $g(e)$ , is negatively related to the creation of new firms because business creation is smaller in environments with greater competition, since the profitability is smaller:

$$\dot{e} = u - g(e). \quad (2)$$

Deriving eq. (2) with respect to time, and using eq. (1), allows us to rewrite the dynamic system formed by equations (1) and (2) as a Liénard type differential equation for  $e$ :

$$\ddot{e} + g_e(e)\dot{e} + f(e) = 0. \quad (3)$$

It is well-known that eq. (3) admits a unique stable limit cycle as a solution. The existence of the limit cycle depends on the following specific properties of functions  $f$  and  $g$ :

- a)  $f$ , and  $g$ , are odd functions of  $e$ ;

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<sup>1</sup> This is a Schumpeterian position (Schumpeter, 1934) as opposed to the view of Knight (1921) that the entrepreneur bears all the risk because capital markets provide too little capital to entrepreneurs.

b)  $g$  is characterized by: i)  $g(0) = g(e^*) = 0$ ;

ii)  $g(e) \rightarrow \infty$  as  $e \rightarrow \infty, \forall e > e^*$ ;

iii)  $g_e(0) < 0$ ; and

c)  $e f(e) > 0$  for all  $|e| > 0$  [e.g., Perko, 2001, p. 254]<sup>2</sup>.

If  $f$  and  $g$  are linear functions of  $e$ , such as  $f(e) = e - a$ ;  $g(e) = e$ , then equation (3) is no longer a Liénard type equation, and becomes a second order linear differential equation with constant coefficients and constant term:  $\ddot{e} + \dot{e} + e = a$ . Notice that by taking the coefficients of  $\dot{e}$  and  $e$  into account this equation has two complex characteristic roots, and the time path for business creation  $e$  is cyclical. Since the coefficient of  $\dot{e}$  is positive, each successive cycle has a smaller amplitude than the preceding one, so the time path is characterized by damped fluctuation, converging to the equilibrium value<sup>3</sup> of  $e$ ,  $a$ . The same dynamical path holds true for  $u$ . As a consequence, this simpler linear model is able to generate cyclical paths for both variables.

If the cycle exists, it has a very intuitive economic explanation. When unemployment is high, more people create new businesses and successful new firm startups create new job posts, reducing unemployment. An increase in the number of firms means greater competition, which leads to a reduction in firm creation. It can also lead to the closing of less competitive existing firms, increasing unemployment. And the cycle repeats itself.

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<sup>2</sup> For economic applications of the Liénard equation see Schinasi (1981) and Tu (1994).

<sup>3</sup> See Chiang and Wainwright (2005, pp. 504-527)

## The statistical model

Let us assume that  $\{y_t, t = 1, 2, \dots, T\}$  is the observed time series data. We consider the following model,

$$(1 - 2 \cos w_r L + L^2)^d y_t = \varepsilon_t, \quad t = 0, 1, \dots, \quad (4)$$

where  $L$  is the lag-operator ( $Ly_t = y_{t-1}$ );  $w_r = 2\pi/r$  and  $r$  is an integer value indicating the number of time periods per cycle;  $d$  may be any real value, and  $\varepsilon_t$  is an  $I(0)$  process, defined as a covariance stationary process with spectral density function that is positive and finite at any frequency. In this context,  $d$  plays a crucial role to describe the persistence of the cycles in the time series. The higher  $d$  is, the higher is the level of association between cycles far away in the past<sup>4</sup>.

Note that the polynomial in the left-hand-side in (4) can be expressed in terms of the Gegenbauer polynomials, such that calling  $\mu = \cos w_r$ , for all  $d \neq 0$ ,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j,$$

where  $C_{j,d}(\mu) = \sum_{k=0}^j \frac{(-1)^k (d)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}$ ;  $(d)_j = \frac{\Gamma(d+j)}{\Gamma(d)}$ , and  $\Gamma(x)$  is the

Gamma function. Alternatively, we can use the recursive formula

$$C_{0,d}(\mu) = 1, \quad C_{1,d}(\mu) = 2\mu d, \quad \text{and}$$

$$C_{j,d}(\mu) = 2\mu \left( \frac{d-1}{j} + 1 \right) C_{j-1,d}(\mu) - \left( 2 \frac{d-1}{j} + 1 \right) C_{j-2,d}(\mu), \quad j = 2, 3, \dots .$$

(See Magnus et al., 1966, or Rainville, 1960, for further details on Gegenbauer polynomials). Gray et al. (1989) showed that  $y_t$  in (4) is stationary if  $d < 0.5$  for  $|\mu| < 1$  and if  $d < 0.25$  for  $|\mu| = 1$ . In the following section, we use Robinson (1994)

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<sup>4</sup> These processes were introduced by Gray, Yhang and Woodward (1989, 1994) and have been employed with integer orders of integration (e.g., Bierens, 2001) or fractional values (Gil-Alana, 2001).

parametric approach<sup>5</sup> along with an estimate of  $d$  based on the Whittle function in the frequency domain.

### **Empirical results**

The data for the US, the UK, Ireland and Spain<sup>6</sup>, consist of two variables: Unemployment rates ( $u_t$ ) and self employment (business ownership per labor force) ( $e_t$ ), the latter is a proxy of entrepreneurship, with annual observations from 1972 to 2004. The data have been obtained from the Comparative Entrepreneurship Data for International Analysis (COMPENDIA) data base.

Table 1 displays the estimates of  $r$  and  $d$  in the model given by equation (4) assuming first (in the left hand side of the table) that  $\varepsilon_t$  is white noise. The periodicity of the cycles is constrained between 5 and 10 years depending on the variable and the country under analysis. We notice that all the orders of integration are above 1 implying a strong degree of dependence in the cyclical structure of the series. Starting with the unemployment series, it is observed that the cycles repeat themselves every six years ( $r = 6$ ) in the cases of Ireland and the United States, eight years in the case of Spain and 10 years for United Kingdom. In the cases of Ireland and USA the unit root cyclical model (i.e.,  $d = 1$ ) cannot be rejected at conventional statistical levels. If we look at the entrepreneurship we see that the periodicity is a little bit shorter in all countries except Ireland, and the orders of integration are all above 1. Only for USA the unit root cannot be rejected though the interval is rather wide in this case.

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<sup>5</sup> See Lobato and Robinson (1998) for a semiparametric method and Dalla and Hidalgo (2005) for another parametric approach.

<sup>6</sup> We have selected a pool of EU countries with different unemployment rates behaviour in order to compare our results. The UK and Ireland can be considered two of the countries with the most dynamic labour markets, whereas Spain is probably the EU-15 country that has suffered the worst episode of unemployment during the last 20 years.



A very different picture emerges if the disturbances follow an AR(1) process. Here, we observe (in the right hand side of Table 1) that for Ireland, Spain and the US,  $d$  is smaller than 1, and only the UK displays an order of integration above 1. This happens for unemployment as well as for the entrepreneurship series.

Finally, we make a linear regression of entrepreneurship on unemployment, and assume that the resulting residuals follow the cyclical model described by equation (4). In other words, we consider now a model of form:

$$e_t = \alpha u_t + x_t; \quad (1 - 2 \cos w_r L + L^2)^d x_t = \varepsilon_t, \quad t = 0, 1, \dots, \quad (5)$$

again assuming white noise and AR(1) disturbances  $\varepsilon_t$ . The results for the four countries are displayed in Table 2. If  $\varepsilon_t$  is white noise,  $r = 5$  in all cases, implying that the cycles have a periodicity of five years. Moreover,  $d$  is found to be above 1 in all cases, ranging from 1.50 (Ireland) to 1.63 (UK). Thus, following this specification the series are nonstationary and non-mean reverting with respect to the cyclical structure. If  $\varepsilon_t$  is autocorrelated,  $r$  is slightly higher for most of the series and  $d$  is positive though smaller than 1, implying then mean reversion. In terms of the theoretical model, the white noise specification is consistent with the limit cycle of the Liénard model, while the AR(1) specification corresponds to the case of dampen cycles. Performing LR-type tests the AR(1) specification seems to be preferred for the four countries.

### **Concluding remarks**

This paper presents a model where unemployment and entrepreneurship impact each other dynamically. The model can generate a unique stable limit cycle, or dampen cycles. The estimated periodicity of the cycles for the US, the UK, Spain and Ireland is between 5 and 10 years. The orders of integration of the series seem to be very sensitive to the specification of the error term. If there is no autocorrelation,  $d$  is above

1, while under autocorrelation  $d$  is smaller than 1, showing mean reversion. The model with autocorrelation (white noise) specification corresponds to dampen cycles (limit cycle).

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**Table 1: Estimates of r and d in the model given by eq. (5)**

| i) Unemployment series (u) |                             |      |                |                       |      |                |
|----------------------------|-----------------------------|------|----------------|-----------------------|------|----------------|
|                            | White noise $\varepsilon_t$ |      |                | AR(1) $\varepsilon_t$ |      |                |
| Country                    | r                           | D    | Conf. interval | r                     | d    | Conf. interval |
| IRELAND                    | 6                           | 1.31 | [0.63, 1.66]   | 9                     | 0.37 | [0.25, 0.48]   |
| SPAIN                      | 8                           | 1.65 | [1.48, 1.80]   | 5                     | 0.70 | [0.12, 1.41]   |
| UK                         | 10                          | 1.14 | [1.04, 1.24]   | 4                     | 1.20 | [0.17, 1.14]   |
| USA                        | 6                           | 1.28 | [0.92, 1.53]   | 8                     | 0.62 | [0.03, 1.13]   |
| ii) Enterpreunership (e)   |                             |      |                |                       |      |                |
|                            | White noise $\varepsilon_t$ |      |                | AR(1) $\varepsilon_t$ |      |                |
| Country                    | r                           | d    | Conf. interval | r                     | d    | Conf. interval |
| IRELAND                    | 7                           | 1.27 | [1.16, 1.37]   | 6                     | 0.83 | [0.41, 1.74]   |
| SPAIN                      | 5                           | 1.54 | [1.10, 1.88]   | 3                     | 0.63 | [0.40, 1.49]   |
| UK                         | 8                           | 1.31 | [1.20, 1.39]   | 6                     | 0.45 | [0.54, 1.28]   |
| USA                        | 5                           | 1.58 | [0.99, 2.01]   | 10                    | 0.57 | [0.29, 1.62]   |

The confidence interval refers to the 95% level.

**Table 2: Estimates of r and d in the model given by eq. (6)**

|         | White noise $\varepsilon_t$ |      |                | AR(1) $\varepsilon_t$ |      |                |
|---------|-----------------------------|------|----------------|-----------------------|------|----------------|
| Country | r                           | d    | Conf. interval | r                     | D    | Conf. interval |
| IRELAND | 5                           | 1.50 | [1.01, 1.86]   | 7                     | 0.83 | [0.03, 1.04]   |
| SPAIN   | 5                           | 1.55 | [1.10, 1.88]   | 8                     | 0.63 | [0.09, 0.98]   |
| UK      | 5                           | 1.63 | [1.00, 2.00]   | 5                     | 0.45 | [0.01, 0.92]   |
| USA     | 5                           | 1.58 | [1.02, 2.01]   | 8                     | 0.57 | [0.09, 0.94]   |

The confidence interval refers to the 95% level.

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