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IS THERE CHRONIC EXCESS SUPPLY OF LABOR?: DESIGNING A STATISTICAL TEST

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ABSTRACT

In this paper we present and implement a statistical test of the hypothesis that the labor market has chronic excess supply. The procedure is to estimate a disequilibrium labor market model, and construct a test statistic based on the unconditional probability that there is excess supply each period. We find that the data reject the hypothesis of chronic excess supply. Hence, one cannot assume that all observations lie on the demand curve.

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Is There Chronic Excess Supply of Labor?:

Designing A Statistical Test

by

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1. Introduction

Since the late 1970's, a number of attempts have been made to estimate disequilibrium models of the aggregate labor market. (See, e.g., Rosen and Quandt [1978, 1984], Romer [1981], Eaton and Quandt [1983], Sarantis [1981], Briguglio [1984], and Smyth [1984].) These models have generally been fairly successful in the sense of being able to replicate key labor market developments over a long period of time.

A problem with disequilibrium models is that they are difficult to deal with computationally. As a consequence of the "min condition" present in these models, difficult nonlinear estimation problems emerge, and standard econometric software packages generally cannot be used. These facts give rise to the following thoughts, which have been expressed to us by a number of people: "Why go to all the trouble? Suppose you are right, and the labor market really can be characterized by a disequilibrium model. Then surely it must be the case that there is chronic excess supply. After all, even in World War II, the unemployment rate was positive. Hence, the min condition implies that all observations are on the demand curve, and that is all you need to estimate."

One response to this assertion is that the official government unemployment statistics upon which we condition our intuitions about the labor market do not correspond to the notions of unemployment used in most theoretical models. This was pointed out a long time ago by Lucas and Rapping [1970]. Still, the thought is troubling to those who estimate disequilibrium models, and is worth careful consideration. In this paper we present and implement a statistical test of the hypothesis that the labor market has chronic excess supply.

We proceed by estimating a disequilibrium labor model, and constructing a test statistic based on the unconditional probability that there is excess supply each period.* The model is briefly summarized in Section 2. The statistical procedure is presented in Section 3. Section 4 contains the results and conclusions.

2. The Model

Here we sketch the disequilibrium model. Since our focus is on the test for chronic excess supply, we refer the reader to Quandt and Rosen [1984] for a detailed discussion of economic, statistical and data issues.

The model consists of six equations. The first three are the notional demand for labor, the notional supply, and a condition which states that the observed quantity is the minimum of the quantity demanded and quantity supplied. The next two equations describe the dynamic adjustment patterns of nominal

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*Necessarily, the validity of the test is conditioned on the validity of the model as a whole. This, of course, is a standard problem in applied econometrics.

wages and prices, respectively. The sixth equation relates excess demand to observed unemployment rates.

1. Demand for labor:

$$\ln D_t = \alpha_0 + \alpha_1 \ln(W_t/P_t) + \alpha_2 \ln Q_t + \alpha_3 t + u_{1t} , \tag{2-1}$$

where t indexes time periods, D_t = notional demand for hours of work, W_t = nominal wage rate, P_t = price level, Q_t = GNP, and u_{1t} is a random error.

2. Supply of labor:

$$\ln S_t = \beta_0 + \beta_1 \ln(W_{nt}/P_t) + \beta_2 \ln H_t + u_{2t} \tag{2-2}$$

where S_t = notional supply, W_{nt} = after tax wage, H_t = potential number of hours of work available.

3. Observed quantity of labor:

$$\ln L_t = \min(\ln S_t, \ln D_t) \tag{2-3}$$

4. Wage adjustment:

$$\ln W_t = \gamma_0 + \gamma_1 \ln W_{t-1} + \gamma_2 U_t + \gamma_3(\ln P_t - P_{t-1})$$
$$+ \gamma_4(\ln P_{t-1} - \ln P_{t-2}) + \gamma_5 \ln W_{t-2} + u_{3t} , \tag{2-4}$$

where U_t = reported unemployment rate.

5. Price adjustment:

$$\ln P_t = \delta_0 + \delta_1 \ln P_{t-1} + \delta_2(\ln W_t - \ln W_{t-1})$$
$$+ \delta_3(\ln W_{t-1} - \ln W_{t-2}) + u_{4t} \tag{2-5}$$

6. Vacancy - unemployment rate relationship:

$$\ln D_t - \ln S_t = \frac{\lambda}{U_t} - U_t + u_{5t} \tag{2-6}$$

Equation (2-6) is based on the approximation $\ln D_t - \ln S_t = VA_t - U_t$, where VA_t is the vacancy rate, and then assuming $VA_t = \lambda/U_t$.

The tests below are based on estimates of the model obtained from annual observations on the U.S. economy for the years 1929 through 1979.

3. Testing the Null Hypothesis of 'All-Excess-Supply'

Various tests can be devised for the 'all-excess-supply' hypothesis. They differ principally in terms of the meaning attributed to the statement that only excess supply occurs. One possibility is that the conditional probability $\Pr\{D_t > S_t | L_t\}$ is small for all t . Another is that the unconditional probability $\Pr\{D_t > S_t\}$ is small for all t . The former concept permits a quasi-likelihood ratio test of H_0 but may be difficult to implement, because it requires estimation of the parameters under H_0 which may not be possible since some of them may not be identified under H_0 (Quandt [1985], Portes, Quandt, Yeo [1985]).

It is relatively straightforward to test H_0 using the second interpretation by employing the Rogers [1984] test.

The Rogers Test. Rogers [1984] has analyzed the following problem. Consider a latent variable problem with exogenous variables x_i and latent endogenous variables y_i^* , with a joint density function that can be written

$$f^*(y^*|x;\ \theta)h(x)\ .$$

As a rule, the form of f^* is stated but that of $h(x)$ is not, and instead of y_i^* , some y_i is observed. The hypotheses investigated by Rogers pertain to the probability that y^* belongs to some subset of the space containing all y^* , i.e., to

$$\Pr\{y^* \in Y_H | x;\ \theta\} = p(x;\ \theta)$$

where θ is the parameter of the f^* density. In the simple disequilibrium model consisting only of demand and supply functions and a min condition, one can easily obtain the density of the transacted quantity Q, $f(Q|x;\theta)$ and the density of excess demand $F = D - S$, denoted by $g(F|x;\theta)$, where x denotes the exogenous variables. Then the probability that excess demand is positive is

$$\Pr\{F > 0|x;\theta\} = \int_0^\infty g(F|x;\theta)dF = p(x;\theta)$$

and

$$\Pr\{F > 0|\theta\} = \int_x p(x;\theta)h(x)dx = Ep(x;\theta)$$

In order to test the hypothesis that $Ep(x;\theta) \geqq c$, Rogers introduces the statistic

$$M_n = n^{-\frac{1}{2}} \sum_i (p(x_i;\hat{\theta}) - c)/\hat{v}$$

where $\hat{\theta}$ is the maximum likelihood estimator of θ,

$$\hat{v}^2 = n^{-1} \sum_i (p(x_i;\hat{\theta}) - c)^2 + \hat{G}'\hat{V}^{-1}\hat{G}$$

$$\hat{G} = n^{-1} \sum_i \frac{\partial}{\partial\theta} p(x_i;\hat{\theta})$$

$$\hat{V} = -n^{-1} \sum_i \frac{\partial^2}{\partial\theta\partial\theta'} \ln f(Q_i|x_i;\hat{\theta})$$

Rogers shows that on the hypothesis that $Ep(x;\theta) = c$, the quantity M_n is asymptotically distributed as $N(0,1)$.

Derivation of $\Pr\{D_t - S_t > 0\}$. We rewrite the model in the following compact form, where now all variables except U_t and t (time) are in logarithmic form:

$$D_t = \alpha_1 W_t - \alpha_1 P_t + z_{1t} + u_{1t} \qquad (3\text{-}1)$$

$$S_t = \beta_1 W_t - \beta_1 P_t + z_{2t} + u_{2t} \qquad (3\text{-}2)$$

$$w_t = \gamma_2 U_t + \gamma_3 P_t + z_{3t} + u_{3t} \qquad (3\text{-}3)$$

$$P_t = \delta_2 W_t + z_{4t} + u_{4t} \qquad (3\text{-}4)$$

$$D_t - S_t = \lambda/U_t + u_{5t} \qquad (3\text{-}5)$$

$$L_t = \min(D_t, S_t) \qquad (3\text{-}6)$$

where z_{1t}, z_{2t}, z_{3t}, z_{4t} are functions of parameters and predetermined variables whose definitions are given implicitly by comparing (3-1) to (3-4) with (2-1), (2-2), (2-4), and (2-5). Defining $F_t = D_t - S_t$, we seek $\Pr\{F_t < 0\}$.

First solve (3-3) and (3-4) for W_t and P_t , yielding

$$W_t = c_{11} U_t + c_{12t} + c_{13} u_{3t} + c_{14} u_{4t}$$

$$P_t = c_{21} U_t + c_{22t} + c_{23} u_{3t} + c_{13} u_{4t}$$

where $c_{11} = \gamma_2/(1-\delta_2\gamma_3)$, $c_{12t} = (z_{3t} - \gamma_3 z_{4t})/(1-\delta_2\gamma_3)$, $c_{13} = 1/(1-\delta_2\gamma_3)$, $c_{14} = \delta_2\gamma_2/(1-\delta_2\gamma_3)$, $c_{22t} = (\delta_2 z_{3t} + z_{4t})/(1-\delta_2\gamma_3)$, $c_{23} = \delta_2/(1-\delta_2\gamma_3)$. Now subtract (3-2) from (3-1) and substitute for W_t, P_t from the last two equations, yielding

$$F_t = A_1 U_t + A_{2t} + A_3 u_{3t} + A_4 u_{4t} \qquad (3\text{-}7)$$

where $A_1 = (\alpha_1-\beta_1)(c_{11}-c_{21})$, $A_{2t} = (\alpha_1-\beta_1)(c_{12t}-c_{22t}) + z_{1t} - z_{2t}$,

$A_3 = (\alpha_1-\beta_1)(c_{13}-c_{23})$, $A_4 = (\alpha_1-\beta_1)(c_{14}-c_{13})$. Equations (3-5) and (3-7) can now be written as

$$F_t - \lambda/U_t + U_t = u_{5t} \equiv v_{1t}$$

$$F_t - A_1U_t - A_{2t} = A_3u_{3t} + A_4u_{4t}+u_{1t}-u_{2t} \equiv v_{2t}$$

v_{1t} and v_{2t} are jointly normally distributed with means zero and variance $\sigma_{v1}^2 = \sigma_5^2$ and $\sigma_{v2}^2 = A_3^2\sigma_3^2 + A_4^2\sigma_4^2 + \sigma_1^2 + \sigma_2^2$. Transforming to F_t, U_t, we obtain the joint pdf $f(F_t,U_t)$ as

$$f(F_t,U_t) = \frac{|A_1+1+\lambda/U_t^2|}{2\pi\sigma_{v1}\sigma_{v2}} . \exp\{ -\tfrac{1}{2} [\frac{(F_t-\lambda/U_t+U_t)^2}{\sigma_{v1}^2} + \frac{(F_t-A_2U_t-A_{2t})^2}{\sigma_{v2}^2}] \} \qquad (3\text{-}8)$$

where $|A_1 + 1 + \lambda/U_t^2|$ is the Jacobian of the transformation. We require

$$\Pr\{F_t < 0\} = \int_0^\infty \int_{-\infty}^0 f(F_t,U_t)dF_t dU_t \qquad (3\text{-}9)$$

Completing the square on F_t allows (3-9) to be written as

$$\Pr\{F_t < 0 \}= \int_0^\infty \frac{|A_1+1+\lambda/U_t^2|}{\sqrt{2\pi}\ (\sigma_{v1}^2+\sigma_{v2}^2)^{\frac{1}{2}}} \exp \{-\tfrac{1}{2} \frac{(A_{2t}+(1+A_1)U_t-\lambda/U_t)^2}{\sigma_{v1}^2 + \sigma_{v2}^2} \} \Phi (\frac{-B_1}{\sigma_{v1}\sigma_{v2}/(\sigma_{v1}^2+\sigma_{v2}^2)^{\frac{1}{2}}})dU_t \qquad (3\text{-}10)$$

where $B_1 = [\sigma_{v1}^2(A_1U_t+A_{2t}) + \sigma_{v2}^2(\lambda/U_t-U_t)]/(\sigma_{v1}^2 + \sigma_{v2}^2)$ and $\Phi(\ )$ is the cumulative standard normal integral. The integral in (3-10) is univariate and may be obtained in straightforward fashion by numerical integration. The model was estimated by maximum likelihood, and also with the additional feature that Equs. (3-3) and (3-4) were assumed to have serially correlated error terms according to first order Markov process. The analogue of (3-10) was then evaluated at the maximum likelihood estimates.

4. Results and Conclusions

Equ. (3-10) was calculated by Gaussian quadrature and the first and second derivatives required for the Rogers test ($\hat{G}$ and $\hat{V}$) were obtained by numerical differentiation. First we display in Table 1 the conditional probabilities $\Pr\{F_t < 0 | L_t\}$ and the unconditional ones computed from (3-10).

Table 1. Probabilities that Excess Demand is Negative

Year	$\Pr\{F_t < 0 \| L_t\}$	$\Pr\{F_t < 0\}$
1932	1.00	1.00
3	1.00	1.00
4	1.00	1.00
5	1.00	1.00
6	1.00	1.00
7	1.00	1.00
8	1.00	1.00
9	1.00	1.00
40	1.00	0.97
1	1.00	1.00
2	1.00	0.23
3	1.00	0.00
4	0.00	0.00
5	0.00	0.00
6	0.00	1.00
7	1.00	1.00
8	1.00	0.44
9	0.92	1.00
50	1.00	1.00
1	1.00	0.59
2	0.00	0.21
3	0.00	0.28
4	0.00	0.81
5	1.00	0.74
6	1.00	0.88
7	1.00	1.00
8	1.00	1.00
9	1.00	1.00

Table 1 (continued)

Year	$Pr\{F_t < 0 \mid L_t\}$	$Pr\{F_t < 0\}$
1960	1.00	1.00
1	1.00	1.00
2	1.00	1.00
3	1.00	1.00
4	1.00	0.97
5	1.00	0.88
6	1.00	0.61
7	0.92	0.54
8	0.92	0.49
9	0.13	0.54
70	0.01	0.75
1	1.00	0.78
2	1.00	0.70
3	1.00	0.57
4	1.00	0.62
5	1.00	0.05
6	1.00	0.29
7	1.00	0.21
8	1.00	0.07
9	1.00	0.02

The two sets of probabilities are qualitatively similar but conditioning on the observed quantity appears to allow much sharper discrimination between regimes.

The Rogers statistic is based on the unconditional probabilities and Table 2 displays its values for alternative c's with and without the correction for autocorrelation of residuals. Each value of c corresponds to a different null hypothesis and asks the question "are the unconditional probabilities of excess supply on the whole larger than c?" If the answer were yes, we would expect the test statistic to be significantly positive. A value of $c = .99$ is quite stringent, whereas a value of $c = .80$ is the opposite.

Table 2. Rogers Statistics

M	c = 0.80	c = 0.90	c = 0.95	c = 0.99
without autocorrelation correction	-1.996	-3.284	-3.807	-4.169
with autocorrelation correction	-0.980	-1.996	-2.460	-2.806

Since a one-tailed test is appropriate, H_0 is rejected at the .05 level for every c, except in the case of c = 0.80 when we estimate autocorrelation coefficients in Equs. (3-3) and (3-4). Hence, the data reject the hypothesis of chronic excess supply in the labor market. One cannot assume that all observations lie on the demand curve , and the estimation technique must take this into account.

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