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ON THE OPTIMALITY OF RESERVE REQUIREMENTS

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ABSTRACT

An implicit rationale for a bank reserve requirement is that a central monetary authority is in a unique position (as "social planner") to impose a "socially superior" outcome to that yielded by a free banking system. We illustrate how this can be true in the context of a simple economy modeled to mimic certain basic characteristics of a monetary economy with banks and agents who trade with one another. Banks exist in our model because by pooling liquidation risks they provide liquidity otherwise unavailable to depositors, which, in turn, provides the incentive for using deposit claims as the medium of exchange.

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I. Introduction

Recent deregulation of certain aspects of the banking industry has rekindled debate on the pros and cons of free versus regulated banking, and renewed interest in questions about the optimality of banking restrictions.[1] This paper focuses on the optimality of but one of these restrictions -- bank reserve requirements. An implicit rationale for a bank reserve requirement is that a central monetary authority is in a unique position (as "social planner") to impose a "socially superior" outcome to that yielded by a free banking system. We illustrate how this can be true in the context of a simple economy modeled to mimic certain basic characteristics of a monetary economy with banks and agents who trade with one another. Banks exist in our model because by pooling liquidation risks they provide liquidity otherwise unavailable to depositors, which, in turn, provides the incentive for using deposit claims as the medium of exchange.

One argument for government intervention in banking stresses the instability due to bank runs caused by depositors' withdrawing deposits because of fear of other depositors' withdrawing. Dybvig and Diamond (1983) argue that government deposit insurance can eliminate this instability. Another argument for intervention, particularly the imposition of reserve requirements, emphasizes the role of asymmetric information which leads banks to pursue different objectives than depositors -- see, for example, King (1983) and Cothren (1987). We abstract from both the problem of bank runs and of asymmetric information whereby one agent is able to exploit an information advantage over another. In our model, there is a Nash equilibrium where each bank behaves in the best interest of its depositors given the behavior of other banks. Our analysis focuses on the way individual bank reserves affect the return and risk associated with bank investments and how this return and risk affects the extent of trading among agents -- that is, the level of economic activity. At this Nash equilibrium a free banking system can yield a suboptimal outcome according to certain welfare criteria.

That free banking may be suboptimal hinges on a tension between behavior that is optimal before the realization of a random shock and that which is optimal afterward. This tension is most easily demonstrated if agents are not expected utility maximizers; therefore we consider such agents in our analysis. However, this tension arises in general equilibrium models where agents are expected utility maximizers and thus our results are of a more general interest. For example, see Lucas (1977), Muench (1977), Polemarchakis and Weiss (1977), and Azariadis (1981).

In section II we describe the economy's agents, production and trading technology, preferences, and the nature of equilibrium. Section III describes the economy's banks and the determination of optimal bank reserves. Section IV describes the search decision involved in trade. In section V we demonstrate that there is a tension in this model between optimizing *ex post*, conditional utility (conditional on the realization of certain random variables) versus optimizing *ex ante*, unconditional utility. Section VI examines the competitive, Nash equilibrium that arises in our economy. Section VII concludes the paper by discussing the implications of our analysis for the optimality of a reserve requirement.

II. The Model

Before filling in the details a brief overview of the model is helpful. There are three agent types in the economy: banks, type A individuals, and type B individuals. The economy evolves through three discrete time periods.

Each type A agent lives for the first two periods, is endowed with one unit of a homogenous good, and wishes to consume one unit of a labor service in period two. Each type B agent lives for three periods, wishes to consume the type A's goods in period three, and is capable of providing an indivisible unit of labor in period two.[2] These facts provide a motive for trade between A and B agents in period two; however, trades are executed only after costly search by type B agents. To simplify the analysis we assume there are N type A and N type B agents.

There is an investment technology whereby the endowment good of a type A agent can be invested in period one to yield like goods in period three. As the type A agents will not be alive in period three, they will either trade or liquidate their investments in period two; but liquidation is subject to a cost. Trading is preferable to liquidation. However some known fraction of type A agents, unable to trade, will be forced to consume their own goods in period two rather than labor services. They must incur the liquidation cost. This fact explains why there is a role for banks. A bank, exploiting the law of large numbers by pooling deposits in period one and through its knowledge of the fraction of type A's forced to liquidate, can eliminate any liquidation costs by holding some deposits in reserve.

In the remainder of this section assumptions concerning type A and B agents are laid out and the nature of equilibrium is characterized when there are no banks. In the next section banks are introduced.

Type A and B individuals: Each type A is endowed with a unit (a bushel, say) of oats at the beginning of period one. Type A's live in periods one and two and consume only in the second. Each type B is endowed with a unit of labor service, lives three periods, and consumes only in period three. There are an equal number N of A's and B's.

Production, Trading, and Information Assumptions

A1: In period one the ith type A individual ($i=1,\ldots,N$) chooses to invest $\theta_i \epsilon [0,1]$ of the endowment unit of oats; $1-\theta_i$ is stored costlessly; the endowment good cannot be invested after period one. Each type B must decide in period one whether to search for a type A trading partner in period two. A fraction y ($0 \leq y \leq 1$) of type B's will decide to search, depending on each B's fixed search cost x (measured in foregone utility).[3] The cost x, $x \epsilon [0,\infty)$, is independently distributed across type B agents according to the cumulative distribution F(x) and each B's x is revealed to him in period one.

A2: The fraction θ_i of a type A's endowment unit invested in period one yields a net return $\theta_i r_3$ in period three, where r_3 is a random variable given by

$$r_3 \equiv m_1 + \epsilon_3 \tag{1}$$

where the random variables m_1 and ϵ_3 are realized in periods one and three, are distributed independently with means $\bar{m}_1$ and 0 and variances $\sigma^2_{m_1}$ and λ^2 respectively. The realization of m_1 is known to type A agents in period one, but it is only made known in period two to those type B agents who have incurred the cost of search. The realization of ϵ_3 is known to type B agents in period three when investments reach fruition.

A3: In period two a type A may liquidate his investment θ_i and obtain $(1-c)\theta_i$ where c is the liquidation cost per unit of liquidated investment $(0<c<1)$. While the fraction $1-\theta_i$ of a type A's endowment not invested in period one can be stored costlessly for consumption in a later period, it cannot be invested after period one.

A4: In period two each searching type B views m_i and each type A's θ_i. Each B selects a type A trading partner given the matches of all others. Labor being indivisible, each type A who is paired with a type B trades his entire portfolio, consisting of the fraction $(1-\theta_i)$ of the unit of A's oats endowment uninvested plus the fraction θ_i invested, for a type B's unit of labor. The type A's who are paired each consume one unit of labor service and no oats in period two, realizing a value of D. The remaining type A's (those not paired) must each liquidate their investment, incurring liquidation cost c per unit. They each consume the portion $(1-c)\theta_i$ of the unit of oats endowment retrieved from investment plus the portion $1-\theta_i$ that was initially stored, realizing a value δ per unit of endowment good consumed, $\delta<D$.

A5: Letting $p(\theta)$ equal the probability that a type A with $\theta_i=\theta$ will execute a trade of his portfolio for one unit of a type B's labor in period two, and assuming type A's are expected value maximizers, a type A will invest $\theta_i=\theta$ in period one only if

$$p(\theta)D + (1-p(\theta))(1-c\theta_i)\delta > \delta \ . \tag{2}$$

A6: A type B agent who decides not to search for a type A trading partner will be autarkic, consuming his own labor service, realizing zero utility. A type B agent who decides to search wishes to consume oats in period three, obtained in trade from a type A in period two. The utility from consuming oats in period three, evaluated at any previous period, will depend on the mean and variance of the period three consumption evaluated as of the previous period. Specifically, if in period j (j=1 or 2) a type B's period thr consumption has mean μ_j and variance σ_j^2, his utility is

$$U(\mu_j,\sigma_j^2) = \mu_j - k\sigma_j^2 - x \tag{3}$$

where k is a constant greater than zero, and x is the cost of search defined in A1.[4]

The essence of the decision process embodied in A1-A6 above can be summarized as follows.

Step 1: in period one, not yet knowing m_1, B's decide whether to search for a type A trading partner in period two.

Step 2: in period one A's observe m_1 and then choose θ.

Step 3: in period two searching B's observe the m_1 realization and select A's on the basis of their knowledge of each A's θ_i.

We will now anticipate and sketch out the results on the existence and characterization of equilibrium which are formally spelled out in the propositions to follow. Type A's and B's have rational expectations; each knows the other's utility function and all functions characterizing the model. The determination of optimal behavior is illustrated by considering steps 1-3 in reverse order. At step 3 the B's will choose those A's having the most favorable values of θ. At step 2, observing the m_1 realization the A's choose the θ value optimal for the B's in period two because A's anticipate the B's subsequent optimal behavior. Thus there is a specific function of m_1, $\theta(m_1)$, that gives the optimal θ from the standpoint of the B's. This function will govern the behavior of the A's at step 2. At step 1, the B's are able to anticipate the optimal behavior of the A's and, of course, their own optimal behavior at step 3 when they will know the m_1 realization and choose type A trading partners. This means that at step 1 B's are able to calculate the $\theta(m_1)$ function and use it to determine whether to search. This precisely determines y, the fraction of B's who decide to search, and allows the A's, anticipating the optimal behavior of the B's, to determine the value of $p(\theta)$ in (2). In equilibrium each A selects the same value of θ so $p(\theta) = y$.

Proposition 1: The fraction of type B's choosing to search is $y = F(\hat{x})$, where from (3)

$$\hat{x} \equiv \mu_1 - k\sigma_1^2 . \tag{4}$$

Proof: From A1 the cost of search x is distributed with function F. Since a type B will search based upon information available to himself as of period one (see A1 and A2) a type B will search if and only if $x \leq u_1 - k\sigma_1^2$, that is, if and only if the cost of search is less than or equal to the utility gain. Since this inequality holds for a fraction $F(\hat{x})$ of type B's the proposition is proven. Q.E.D.

Since it is not necessary to the argument at this point, we defer explicit calculation of μ_1 and σ_1^2 until (10) and (11) below.

The Search Process and Equilibrium

In period two the yN type B's (recall $0 \leq y \leq 1$ from A1) who have decided to search are randomly ordered at the entrance to the trading ground where the N type A's are assembled, each displaying his value of θ_i ($i=1,\ldots,N$) selected in period one (from A1) conditional on the realization of m_1 (from A2). The type B's enter the trading ground in turn. Upon entry a type B observes the realization of m_1, m_1', and all the θ_i's. Each type B, given the matches of all preceding B's, will select from the remaining unmatched A's that type A offering the best available portfolio (conditional on m_1'). This leads to Lemma 1.

Lemma 1: Type A's who choose to invest will select $\theta_i = \theta^*$, that value of θ, given $m_1=m_1'$, maximizing the type B's period two utility. That is, given $m_1 = m_1'$, θ is selected to maximize,

$$U(\mu_2,\sigma_2^2) = \theta_i[m_1' - k\theta_i\lambda^2] - x \tag{5}$$

Proof: That (5) is period 2 utility follows from (3) and A2 with $j=2$. As in A5 above, let $p(\theta)$ be the probability of a type A with $\theta_i=\theta$ making a match. A type A will select $\theta_i=\theta$ only if $p(\theta)>0$ (see equation 2). Suppose $p(\theta)>0$ for some $\theta<\theta^*$. Then clearly $p(\theta^*)>p(\theta)$, since each type B will select the best available match. Thus no type A will select $\theta_i=\theta<\theta^*$. By a similar argument no type A will select $\theta>\theta^*$. Q.E.D.

Let $\hat{N}$, $N \geq \hat{N}$, be the number of type A's who choose $\theta_i = \theta^*$. Let $M^* \equiv yN$, the number of type B's who search, recalling that there are N type A's and N type B's. There will always be some type B's who don't search, that is $N > M^*$ since search cost $x\epsilon[0,\infty)$. The trading equilibrium in period two is described by the following proposition.

Proposition 2: Given $\theta^* = \theta^*(m_1')$, let $\hat{N}$ solve

$$\frac{\hat{N}-M^*}{\hat{N}}(1-c\theta^*)\delta + \frac{M^*}{\hat{N}}D = \delta \tag{6}$$

and let $N^* = \min(N, \hat{N})$. The economy characterized by assumptions A1-A6 has a unique equilibrium such that N^* A's choose $\theta_i = \theta^*$, $N - N^*$ A's choose $\theta_i = 0$ (i.e., they store), and $N \geq N^* > M^*$.

Proof: Since $\delta < D$, $\hat{N}$ solving (6) exists. At $p(\theta^*) = \frac{M^*}{\hat{N}}$, type A's are indifferent between investing or not. Thus if $N > \hat{N}$, not all type A's will choose to invest $\theta_i = \theta^*$, since the probability of a trade, $\frac{M^*}{N}$, when all A's invest is insufficient to offset the probable cost of liquidation of an investment. In this case $N-\hat{N}$ A's will not invest and $\hat{N}$ will. The former have a zero probability of making a trade, the latter a probability $\frac{M^*}{\hat{N}}$. The utility of each type A is δ. If $\hat{N} \geq N$ all type A's invest θ^*, because the probability of any one making a trade is $\frac{M^*}{N}$ and the left side of (6) is greater than the right for N^*. Q.E.D.

<u>Corollary</u>: If $N = N^*$ then $p(\theta)$ in (2) equals y of Proposition 1, while if $N > N^*$ then p equals $\frac{M^*}{N^*} > \frac{M^*}{N} \equiv y$.

Proof: The proof is immediate from (2), (6), and Propositions 1 and 2. Q.E.D.

Henceforth we will assume for simplicity and without loss of generality that $N=N^*$ so that (2) and (6) become

$$yD + (1-y)(1-c\theta^*)\delta \geq \delta. \tag{7}$$

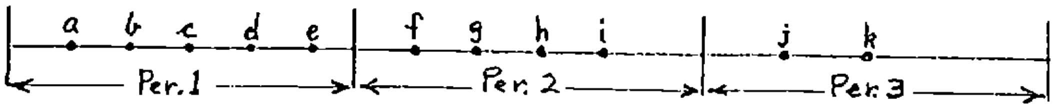
The left side of (7) indicates that a fraction y of the type A's and y of the type B's (all B's who search) will be matched and make a trade at the trading ground in period two. A fraction 1-y of the type A's assembled at the trading ground will not be matched and therefore must consume their endowment, equal the fraction $1-\theta^*$ that was stored plus the fraction θ^* that was invested in period one but now must be liquidated at liquidation cost c per unit.

Table 1 summarizes our discussion of the world without banks by itemizing the sequence of events along the time line. The reader will find it helpful to reexamine the story as summarized in Table 1 before proceeding.

III. Banks

We now introduce banks into this economy. Before proceeding in detail, a brief sketch of the role and reason for banks is helpful. Banks can provide the same storage and investment service to type A agents as an individual type A can provide for himself, but in addition can provide liquidity.[5] This liquidity is available because a bank through its ability to pool deposits can avoid the liquidation cost described in assumption A3. Thus in period one a type A agent has an incentive to deposit his unit endowment of oats in any bank that will invest θ^* and store $1-\theta^*$ of it. The type A can then trade the deposit slip to a searching type B at the trading ground in period two for B's labor service. However if the type A is among the $N-M^*$ A's who are not matched with a searching B in period two, the A can withdraw his entire deposit from the bank without the liquidation cost $c\theta^*$ otherwise incurred if he had not used a bank--hence the incentive for type A's to deposit in a bank.

Table 1



a. Each type A endowed with unit of good. Each type B endowed with unit of labor service.

b. Each type B draws search cost x from F(x) distribution.

c. Each type B calculates utility as of period one, $U(\mu_1, \sigma_1^2)$. Those type B's with $U(\mu_1, \sigma_1^2) - x \geq 0$, decide to search in next period.

d. Realization m_1' of random return variable m_1 occurs and is observed by type A's, but not by B's.

e. Given m_1', each type A makes investment decision by choosing a value of θ.

f. Type B's who search cue up at entrance to trading ground and observe m_1'. Type A's assemble at trading ground, each displaying his chosen θ

g. Type B's enter the trading ground sequentially. Each chooses that (as yet unchosen) type A displaying a θ which gives the type B the highest period two utility $U(\mu_2, \sigma_2^2)$, given m_1'.

h. Type A's and B's matched at g now trade. Type A's not matched liquidate the portion of their endowment good unit invested and consume that, net of liquidation cost, plus the portion stored.

i. Type A's die.

j. Realization ϵ_3' of random return variable ϵ_3 occurs and is observed by type B's each of whom now consumes a unit of the endowment good <u>plus</u> its realized return $r_3' = m_1' + \epsilon_3'$.

k. Type B's die.

Searching type B's each having acquired a deposit claim (slip) through trade in period two, withdraw and consume the deposit in period three--the $1-\theta^*$ of the unit endowment stored plus the fruition of the θ^* portion invested. Competition among banks forces each bank to choose θ^* since this value for θ will maximize a depositor's probability of making a trade with a type B. In equilibrium type A's deposit only in banks that invest θ^* because searching type B's at the trading ground will select only those A's holding deposit slips with a θ^* on them. We now elaborate on this scenario, beginning with a description of banks.

B1: A single bank can costlessly service a fraction z of the economy's N type A depositors. The marginal cost of servicing depositors beyond z is infinite. The banking service is provided in a competitive market with free entry so banks will earn zero profits.

B2: Given its period one deposits from type A's, each bank determines its reserve policy in period one given knowledge of y and of the realization m_1' of m_1, and given the fraction of type B searchers (see Proposition 1) and hence the fraction of type A depositors who are matched with type B's in period two. This period one reserve decision by a bank is analogous to the type A's period one choice of θ_i in the world without banks.

B3: Banks are assumed to be mutually owned ("mutuals") by depositors and are liquidated in period three.[6] A type A's bank deposit in a representative bank can be viewed as an equity claim. These claims or "deposit slips" can be traded to type B's who claim the period three gross investment proceeds of the invested endowment good backing each deposit, or they can be "liquidated" by a non-trading type A in period two. To this extent equity claims are also a medium of exchange similar to a redeemable bank note issued by a privately owned competitive bank.

Type A's will choose to deposit their unit endowment in a bank in period one, rather than invest it on their own, if banks can reduce or eliminate the liquidation cost c (incurred by those type A's who are not matched with a type B trading partner in period two) without lowering a type A's probability of being matched with a type B. From B1 it follows that in a competitive equilibrium 1/z banks can provide services to N type A agents at no cost and at no profit to themselves. Given B2 a bank can exploit the law of large numbers to hold at least a fraction 1-y of its deposits in storage to accommodate withdrawals in period two by the fraction 1-y of type A depositors who are not matched with type B traders. In this way investment liquidation costs are eliminated. Since the banking service will be provided at zero cost and banks will earn zero profits, the savings in liquidation costs can be passed on to the type A depositors. These savings induce type A depositors to utilize the banking service, since with c=0 a type A's prospects given by (7) are increased.[7] The remaining fraction y of deposit claims will be redeemed in period three by type B agents who will have acquired them at the trading ground in period two in exchange for their labor services. In sum, <u>banks exist because they eliminate liquidation costs by pooling the risks that type A's may not make a trade with a type B</u>.

<u>Proposition 3</u>: Given that each bank will hold a fraction 1-y of its deposits in reserve to cover period two withdrawals, let $1-\theta$ of the remaining fraction also be held in reserve and θ invested. Then each deposit claim held until period three by type B agents, conditional on the realization m_1' of m_1, will have a mean net rate of return and variance as of period two of

$$\mu_2 = \theta m_1' \tag{8}$$

$$\sigma_2^2 = \theta^2\lambda^2 \qquad (9)$$

Proof: Since banks provide their services at zero cost and earn zero profits (from B1), the return from investments will accrue to depositors. Since period two withdrawals yield no net return the total return on investment will be distributed to claim holders in period three. It follows from A2 and (1) that, conditional on m_1, the mean and variance of this return are given by (8) and (9). Q.E.D.

The next proposition and proof show how and why banks assure that a type A's probability of being matched with a type B is the same whether the type A uses a bank or invests on his own.

<u>Proposition 4</u>: Given y and conditional on m_1, in period one each bank will select $\theta = \theta^*$ to maximize the period two utility of type B's, $\mu_2 - k\sigma_2^2 - x$, where μ_2 and σ_2^2 are given by equations (8) and (9). (Recall the maximizing value θ^* is conditional on m_1, $\theta^*=\theta^*(m_1)$.

Proof: The crucial fact here is that the banking industry is competitive (B1). In a competitive market, each bank will attract customers in period one by offering type A's the best possible chance of making a trade at the trading ground in period two. As follows from Lemma 1, this means each bank selects $\theta = \theta^* \equiv \theta(m_1)$ to maximize $\mu_2 - k\sigma_2^2 - x$, given m_1, where μ_2^2 and σ_2 are given by (8) and (9) (see (5)). Q.E.D.

IV. <u>The Search Decision</u>

We now specify μ_1 and σ_1^2, the mean and variance of a type B's period three consumption calculated as of period one. These values enter (4) of Proposition 1 to determine the fraction y of type B's who decide in period one to search in period two.

Proposition 5: The value attached to a deposit claim by a type B agent in period one, ex ante his period two reading of m_1, is $\mu_1 - k\sigma_1^2 - x$, where

$$\mu_1 = E_{m_1}(\mu_2) = E_{m_1}[m_1\theta(m_1)] \tag{10}$$

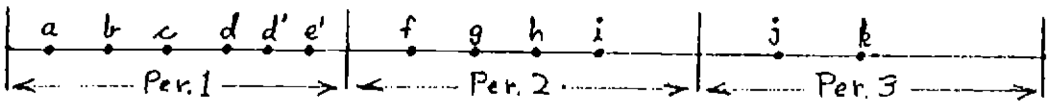
$$\sigma_1^2 = E_{m_1}[m_1\theta(m_1) - \mu_1]^2 + E_{m_1}(\theta(m_1))^2\lambda^2 \tag{11}$$

where $\theta(m_1)$ is defined in Proposition 4, and E_{m_1} is the expectation operator over m_1.

Proof: If acquired, the type B agent will hold a deposit claim until period three. The mean net return is $E(\theta(m_1))(m_1+\epsilon_3)$ (from A1, A2 and Proposition 4). Taking the expected value of this first over ϵ_3 and then m_1 yields μ_1 above. The variance of this return is $V[\theta(m_1)(m_1+\epsilon_3)] = V[m_1\theta(m_1)] + V[\epsilon_3\theta(m_1)] + 2Cov[m_1\theta(m_1), \epsilon_3\theta(m_1)]$, which is σ_1^2 from the fact that μ_1 is as given and that m_1 and ϵ_3 are independently distributed (see A2). Q.E.D.

Table 2 summarizes our discussion of the world with banks by indicating where events along the time line of Table 1 (world without banks) are changed as a consequence of the introduction of banks. The reader should closely compare Tables 1 and 2.

Table 2



a. Same as Table 1.

b. Same as Table 1.

c. Same as Table 1.

d. Same as Table 1 except m_1' also observed by banks.

d'. Type A's deposit endowment good in banks.

e'. Each bank puts fraction $1-\theta y$ of its deposits in reserve and, given m_1', invests θy.

f. Same as Table 1 except each A displays deposit ticket marked with θ chosen by bank.

g. Same as Table 1.

h. Same as Table 1 except type A's not matched withdraw deposit of endowment good (with no liquidation cost) and consume it.

i. Same as Table 1.

j. Same as Table 1 except Type B's withdraw deposit from bank and consume it (endowment good plus realized return).

k. Same as Table 1.

V. Ex ante versus Ex post Optimization

There is an obvious distinction between type B's utility in period one, given by (3) (with j=1), (10) and (11), determined ex ante B's observation of m_1, and type B's period two utility, given by (3) (with j=2), (8) and (9), ex post the search decision, and B's observation of m_1. This raises an interesting question. Is it possible that a bank's setting of $\theta=\theta^*$ according to Proposition 4 to maximize type B's period two utility is not optimal from the vantage point of type B's period one utility? This question is of interest because the number of type B searchers is determined by a type B's period one utility according to Proposition 1, while by Proposition 4 banks are compelled by competition to maximize a type B's period two utility. If maximizing period two utility does not maximize period one utility then there may be a sub-optimal number of type B searchers. The following example demonstrates that this can be the case under certain conditions.

First of all, note that the mean-variance tradeoff (the slope of the opportunity locus) available to a bank in period two by its selection of θ conditional on m_1 (i.e. ex post the realization m_1' of m_1), is, from (8) and (9)

$$\frac{d\mu_2}{d\sigma_2^2} = \frac{d\mu_2}{d\theta}\frac{d\theta}{d\sigma_2^2} = \frac{m_1'}{2\theta\lambda^2} \quad . \tag{12}$$

Examples of such loci are given by 0a and 0b in Figure 1, where 0b corresponds to a higher value of m_1 than 0a, and the endpoints a and b correspond to zero reserves, $\theta = 1$, in excess of 1-y. Second, note that each type B has linear indifference curves with slope k in mean-variance space (given by equation (3)) -- such as U_a and U_b in Figure 1. Now consider the following assumption.

Assumption M: m_1 is distributed over the positive interval $[\alpha,\beta]$, where

$$k < \frac{d\mu_2}{d\sigma_2^2} = \frac{m_1}{2\theta\lambda^2} \tag{13}$$

at $m_1 = \alpha$, θ=1, or

$$k < \frac{\alpha}{2\lambda^2} \tag{14}$$

The condition (14) implies that a type B's period two utility is maximized when banks choose θ=1 regardless of m_1's value -- that is, when banks hold zero reserves in excess of 1-y. This case is illustrated in Figure 1 where the bank opportunity locus associated with $m_1 = \alpha$ is Oa, and the maximum type B period two utility level U_a is attained at the endpoint of the bank opportunity locus, point a, corresponding to a zero level of reserve holdings. Recognizing that when θ=1 condition (14) is true a fortiori for all $m_1 > \alpha$, it follows that a type B's period two utility is maximized for all $m_1 \in [\alpha,\beta]$ when banks chose θ=1. For example, the slope of the opportunity locus Ob corresponding to an $m_1 > \alpha$ is steeper at point b than the slope of Oa at point a.

Now to see that θ=1 may not maximize type B's period one utility. First suppose that θ is a constant, the same for all banks and all m_1's as in the case under consideration. Then assuming type B's form their period one expectations rationally to obtain μ_1 and σ_1^2 according to (10) and (11),

$$\mu_1 = \theta\bar{m}_1 \tag{10'}$$

and

$$\sigma_1^2 = \theta^2\lambda^2 + E[\theta m_1 - \theta\bar{m}_1]^2$$

or

$$\sigma_1^2 = \theta^2[\lambda^2 + \sigma_{m_1}^2] \ , \tag{11'}$$

where $\bar{m}_1$ and $\sigma_{m_1}^2$ are respectively the mean and variance of m_1. The period one mean-variance tradeoff is, from (10') and (11'),

$$\frac{d\mu_1}{d\sigma_1^2} = \frac{\bar{m}_1}{2\theta(\lambda^2+\sigma_{m_1}^2)} \ . \tag{15}$$

The period one opportunity locus is shown as Oc in Figure 2. Evaluating (15) at θ=1 reveals that

$$\frac{\bar{m}_1}{2(\lambda^2+\sigma_{m_1}^2)} \gtrless k \ . \tag{15'}$$

Comparing (14) and (15') it is clear that it is possible for the period one opportunity locus to be less steeply sloped at its endpoint, point c where θ=1, than is the case for the period two opportunity locus. In this event the period one utility level will not be maximized when $\theta=1$, as illustrated in Figure 2, since the level of utility U_c (given by (3), (10) and (11)) associated with the indifference curve passing through point c is lower than the maximum attainable level U_d associated with the indifference curve passing through point d.[8] This discussion culminates in the following proposition and corollary.

Proposition 6: Given Assumption M, if

$$\left.\frac{d\mu_2}{d\sigma_2^2}\right|_{\theta=1} \equiv \frac{\alpha}{2\lambda^2} > k > \frac{\bar{m}_1}{2(\lambda^2+\sigma_{m_1}^2)} \equiv \left.\frac{d\mu_1}{d\sigma_1^2}\right|_{\theta=1} \tag{16}$$

then a type B's period one utility is maximized when $\theta=\theta'$, where $1>\theta'>0$ solves

$$\frac{d\mu_1}{d\sigma_1^2} \equiv \frac{\bar{m}_1}{2\theta'(\lambda^2+\sigma_{m_1}^2)} = k \ ,$$

and period two utility is maximized at $\theta=1$.

Proof: Given inequality (16) and equation (15), such a θ' will exist and is optimal ex ante (i.e. in period one) since at θ', a type B's indifference curve will be just tangent to the mean-variance locus. That ex post (i.e. in period two) utility is optimized at $\theta=1$ follows from the above discussion.

Q.E.D.

Corollary: Given Assumption M, equation (15), and inequality (16), a requirement that $\theta=\theta'$ will also maximize the period one utility of type A agents, given by (2) of A5, as well as the period one utility of type B agents.

Proof: Requiring that $\theta=\theta'$ maximizes the period one utility of type B's and hence will maximize the probability of a type B's searching given by $y=F(\hat{x})$, equations (4), (10′), and (11′). Since the probability that a type A executes a trade is $p \geq y$ by the corollary to Proposition 2, to maximize y is to maximize the utility of type A's given by (2) of A5. Q.E.D.

The requirement that $\theta=\theta'$ is of course a binding constraint because $1>\theta'>0$, while banks would prefer to choose $\theta=1$. This is equivalent to a reserve requirement of $1-y+y(1-\theta')=1-y\theta'$ since banks know that 1-y will be withdrawn in period two by non-trading type A's.

VI. The Nash Equilibrium

Why don't individual banks choose to hold the level of reserves corresponding to the maximization of the type B's period one utility at point d in Figure 2? Certainly if all banks did so it would increase the chances of their type A depositors making trades as demonstrated by the corollary to Proposition 6. However no single bank will hold reserves to maximize a type B's period one utility if all others are doing so.

To see why, suppose that all banks other than bank i are holding reserves in excess of 1-y (i.e., $\theta' > 0$) to maximize a type B's period one utility, corresponding to point d in Figure 2 for period one utility, and d in Figure 1 for period two utility. Then in this case, since bank i's behavior has no impact on the period one search decision of type B agents, bank i will select its portfolio to maximize a type B's period two utility (i.e., choose θ = 0) given by point a or b (depending upon the realization of m_i) in Figure 1. Bank i will do this because it increases to one the probability that each of its depositors will make a trade -- the bank can extract more than a competitive equilibrium normal profit from this situation. This follows since if all other banks offer a lower period two utility, such as point d in Figure 1, then at the trading ground searching type B's will prefer to trade with bank i's type A depositors.[9] The conclusion is that banks' choosing to optimize period one type B utility is not an equilibrium. Clearly the Nash equilibrium is for all banks to maximize period two type B utility. Although all banks might agree to maximize period one type B utility and hence maximize the number of traders, in the absence of some kind of enforcement mechanism this agreement would not be time consistent. Of course, type B agents recognize this fact and always search for type A's who deposit at banks which maximize their period two utility; type B's know such search will be successful.

VII. The Optimality of a Reserve Requirement[10]

The simplistic economy we have modeled above is intended to mimic certain basic characteristics of a monetary economy with banks and agents who trade. The banks provide liquidity otherwise unavailable to depositors. This in turn necessitates the use of deposit claims as the medium of exchange when type A depositors buy labor services from type B's; deposit claims are willingly accepted by type B's precisely because they represent a claim on the period three proceeds of an invested endowment good which a type B wants to consume. Thus in this economy the existence of banks and deposit claims serving as money increases welfare by eliminating the liquidation costs otherwise incurred by type A's who are unable to execute a trade with a type B in period two.

Within this framework we have seen that under certain conditions (given by Proposition 6 and its corollary) the imposition of a reserve requirement unambiguously increases the welfare of type A individuals. The reserve requirement also increases period one utility $U(\mu_1, \sigma_1^2)$ for all type B's at point c in Table 2. Given the drawing of the search cost x at point c in period one, some type B's will still decide not to search (those drawing the highest x's, since $x\epsilon[0,\infty)$), but the number of these will be less than in the absence of a reserve requirement. Some type B's who decide not to search when there is no reserve requirement will decide to search when the requirement is imposed. And finally, some type B's (those drawing the lowest x's) will decide to search under either regime. Thus in period one at point c, Table 2, some type B's are better off and none are worse off with the imposition of the reserve requirement.

Consider the effect of the reserve requirement on agents in period two. More type A's will make trades with type B's than when there is no requirement

-- hence some type A's are better off and none are worse off. Type B's who would trade in a regime without reserve requirements are worse off in a regime with reserve requirements for every realization of m_1: judged by period one utility the reserve requirement makes this group better off, but worse off according to period two utility. What is the "appropriate" welfare criterion? There is no unambiguous answer to this question. As Lucas (1977, p. 352) has noted in a similar context, "The optimality criterion one <u>should</u> adopt is, of course, a controversial issue, one which cannot be settled by unsupported assertions as to what is 'the only appropriate criterion'."[11] Our own preference is to judge according to period one utility, that is, when agents confront the most uncertainty about the world at the very beginning. By this criterion the reserve requirement is unambiguously welfare improving.

We have dwelled on the case where a reserve requirement is welfare improving, given the conditions of Proposition 6 and its corollary. However, it should be emphasized by way of conclusion that if these conditions don't hold then a reserve requirement reduces welfare.

Footnotes

1. See, for example, Diamond and Dybvig (1983), Fama (1980), King (1983), Rolnick and Weber (1983), White (1984), and Cothren (1985, 1987).
2. The assumption of two types of individuals with different lifespans, one living two periods and the other three, is not new. Diamond and Dybvig (1983) make the same assumption. In their three period model individuals do not know which type they are going to be until the second period; here individuals know which type they are in the first period.
3. The cost of search may be simply thought of as the aggravation of search.
4. The type B agent is not an expected utility maximizer. The linearity of U is for convenience. A more general specification is possible without voiding the results to follow.
5. Diamond and Dybvig (1983) and Cothren (1987) also model bank deposits as deposits of goods. Of course, in reality bank deposits are <u>assets</u> that can be converted into consumable goods subject to uncertainty as to the exchange ratio. Modeling deposits as goods themselves then is a simplifying assumption that is not a distortion of reality when the deposit/good exchange ratio is stable as is the case here. This is the same as the story about how goldsmiths accepted gold as deposits and issued deposit claims thereby becoming bankers -- a story often told in elementary textbooks when describing the origin of banks.
6. Diamond and Dybvig (1983, p. 408) also assume their banks are mutuals.

7. It is not necessary to pass the entire cost saving on to type A's. Any reduction in c would induce them to deposit in banks. For simplicity we assume the entire saving is passed on to the type A's.

8. The restriction that $\theta(m)=1$ for all m can be relaxed. Details are available upon request.

9. Recall that by assumption A1 the endowment good can only be invested in period one as it takes three periods for the investment to come to fruition. Therefore banks must make their decision about θ -- how much of the fraction (1-y) to invest -- in period one.

10. It is often argued that a principal reason for a mandatory reserve requirement is that it gives the Fed control over the money supply. However such an argument does not negate the fact that imposition of a reserve requirement affects the risk-return trade-off of a bank's asset portfolio, the focus of this analysis.

11. Although our agents are not expected utility maximizers, the tension between the period one-period two utility criteria need not hinge on this fact. In general equilibrium models, such tension can also arise in the context of expected utility analysis, as has been illustrated, for example, in the exchange between Lucas (1977), Muench (1977), and Polemarchakis and Weiss (1977), and the work of Azariadis (1981).

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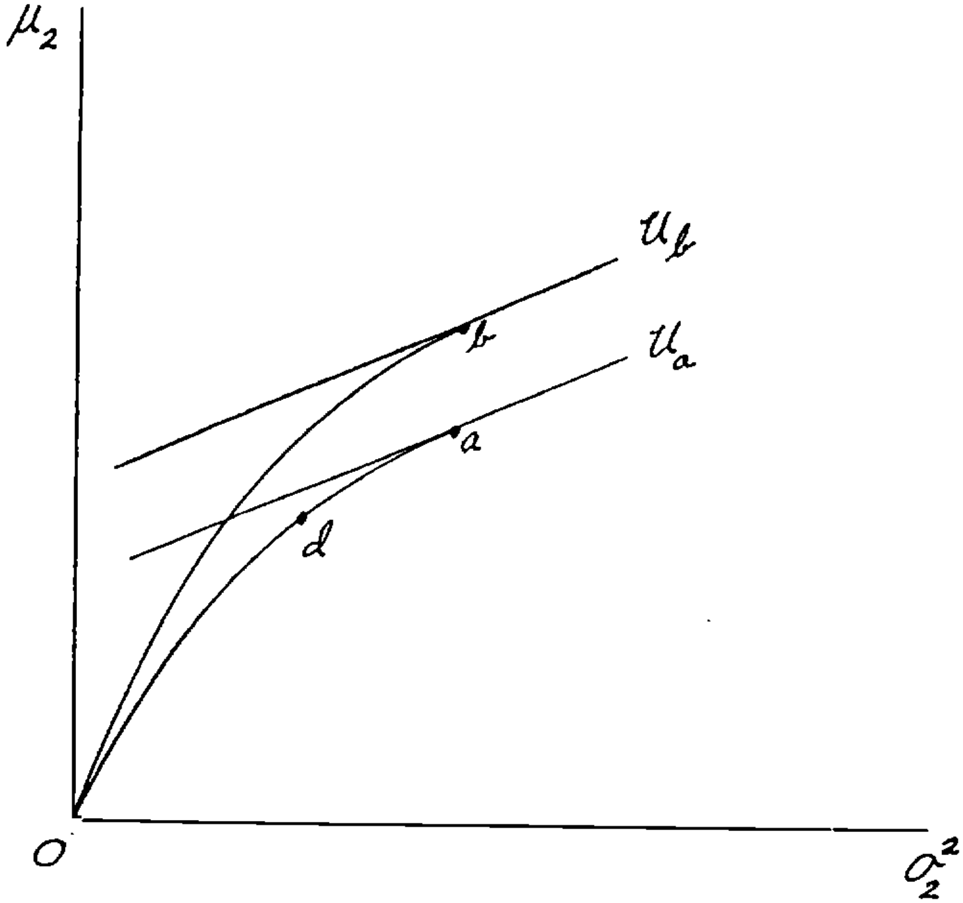

Figure 1

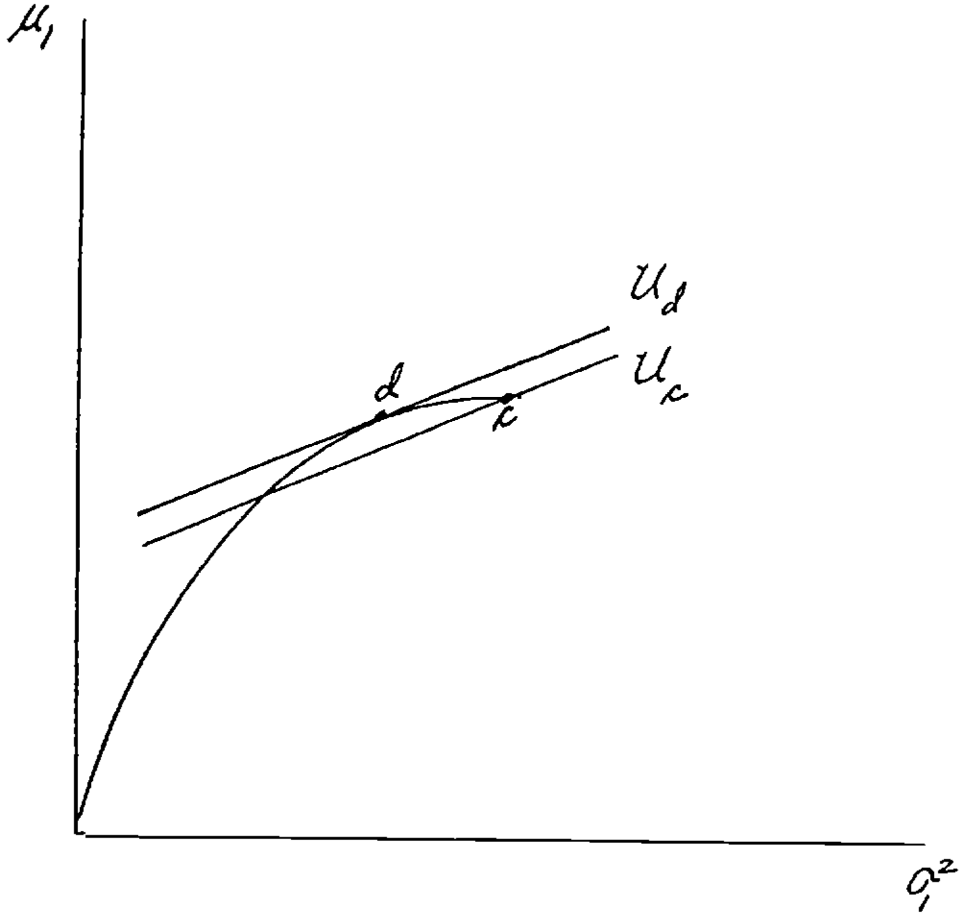

Figure 2