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ABSTRACT

In dynamic stochastic welfare comparisons, a failure clearly to distinguish between risk aversion and intertemporal substitutability can result in misleading assessments of the impact of risk aversion on the welfare costs of consumption-risk changes. The problem arises in any setting in which uncertainty is propagated over time, notably, but not exclusively, in economies with stochastic consumption trends. Regardless of the preference setup adopted, an increase in risk aversion amplifies the per-period costs of risks. The weights consumers use to cumulate the per-period costs of risks with persistent effects should, however, depend on intertemporal substitutability as well as on risk aversion. Under time-separable expected-utility preferences, an increase in the period utility function's curvature therefore alters the welfare effect of risk for reasons that in part are unrelated to risk aversion.

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Welfare evaluations of economic policies and institutions are increasingly conducted in dynamic stochastic models. Recent examples include Lucas's (1987) and İmrohoroğlu's (1989) calculations on the welfare cost of consumption uncertainty; Cole and Obstfeld's (1991) estimates of the gains from international risk sharing; Dixit and Rob's (1991) study of intersectoral labor mobility with nontradable labor-income risk; and İmrohoroğlu and Prescott's (1991) examination of the cost of inflation. In all these models, a key parameter underlying the quantitative results is the degree of risk aversion embedded in consumer preferences.

It is common practice in this research to employ social welfare functions of the expected-utility sort, in which risk aversion and intertemporal substitutability cannot vary independently. This paper shows that a failure clearly to distinguish the two concepts can result in a misleading picture of the influence of risk aversion on the welfare costs of consumption-risk changes. The problem arises in any setting in which uncertainty is propagated over time, notably, but not exclusively, in economies with stochastic consumption trends. Regardless of the preference setup adopted, an increase in risk aversion amplifies the per-period costs of risks. The weights consumers use to cumulate the per-period costs of risks with persistent effects should, however, depend on intertemporal subsitutability as well as on risk aversion. Under time-separable expected-utility preferences, an increase in the period utility function's curvature therefore alters the welfare effect of risk for reasons that in part are unrelated to risk aversion.[1]

To place the discussion in a concrete and familiar setting, I re-examine Lucas's (1987) calculation of the aggregate cost of United States consumption variability. This calculation is controversial. The purpose of studying it is not to endorse or question a particular view of business cycles. Instead, the goal is to clarify, through the simple examples that Lucas's model suggests, an issue of interpretation likely to arise in any welfare comparison of alternative stochastic consumption paths.

The paper is organized as follows. Section I describes how to evaluate changes in consumption risk and trend consumption growth under two benchmark assumptions: log consumption is independently and identically distributed (i.i.d.) around a time trend, and log consumption follows a martingale. In the second of these cases, though not in the first, the welfare effect of a change in risk depends on an intertemporal-substitution as well as a risk-aversion parameter, as section II shows. Section III illustrates these results by recalculating Lucas's (1987) welfare-cost measures under alternative assumptions on preferences and on the stochastic process for log consumption. Section IV summarizes. An appendix explores more realistic consumption processes. It shows that it is consumption persistence, rather than a unit root per se, that implies conceptually distinct roles for risk aversion and intertemporal substitutability in measures of the welfare cost of consumption-risk changes.

I. Deterministic versus Stochastic Consumption Trends under Expected-Utility Preferences

The main point of this paper is made in the context of a specific exercise, Lucas's (1987) calculation of the amount a representative U.S. consumer would gain if the variability of U.S. consumption could be eliminated with no change in expected consumption levels. In this section I show how persistence in consumption shocks changes the analysis of Lucas (1987), whose calculations take the natural logarithm of consumption to be i.i.d. around a deterministic trend. To be concrete as well as clear, I take as the alternative hypothesis that log consumption follows a martingale. The intuition from this exercise motivates the adoption of nonexpected-utility preferences in section II.

Let C_t denote the level of real per capita consumption on date t, c_t its natural logarithm. Lucas (1987) in effect assumes that c_t is generated by the trend-stationary process

$$(1)\qquad c_t = \bar{c} + \mu t - \tfrac{1}{2}\sigma_z^2 + z_t, \qquad\qquad z_t \sim NIID(0,\sigma_z^2),$$

where $\bar{c}$ is a constant.[2]

The rational-expectations permanent-income theory suggests, however, that the c_t process could contain a unit root.[3] The simplest such alternative process is the martingale

(2) $$c_t = c_{t-1} + \mu - \tfrac{1}{2}\sigma_\zeta^2 + \zeta_t, \qquad \zeta_t \sim NIID(0,\sigma_\zeta^2).$$

Merton (1971) shows the optimality of (2) for an infinitely-lived optimizing consumer who has continuous trading opportunities and faces i.i.d. investment uncertainty.

Cases (1) and (2) are meant to highlight, in as simple a manner as possible, the effect of persistent or cumulative risks on dynamic welfare comparisons. For this purpose, the assumption of a unit root in (2) is not essential. As is shown by simulation in the appendix, the evaluation of stationary but persistent consumption processes raises all of the issues discussed below (albeit in a less acute form).

Under (1), c_t can be written as

$$c_t = c_0 + \mu t + z_t - z_0,$$

while under (2),

$$c_t = c_0 + (\mu - \tfrac{1}{2}\sigma_\zeta^2)t + \zeta_t + \zeta_{t-1} + \dots + \zeta_1.$$

Given $t = 0$ information, the conditional variance $V(c_t|c_0)$ is σ_z^2 under (1), but is $V\left(\sum_{i=1}^{t}\zeta_i\right) = t\sigma_\zeta^2$ under (2). The important point is that while the martingale (2) implies permanent shocks that cumulate over time, the i.i.d. process (1) implies that shocks are transitory, one-shot affairs. Except at more or less short horizons, process (2) thus implies greater consumption uncertainty than does (1). Whether this also translates into lower ex ante welfare depends on the discount rate for future utilities, the ratio of σ_z^2 to σ_ζ^2, and--the main focus in this paper--individual tolerances for risk and for intertemporal substitution.

Closed-form solutions are available for the isoelastic class of time-separable expected-utility functions:

$$(3) \qquad U_0 = \mathbf{E}_0\left\{\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma}\left(C_t^{1-\gamma} - 1\right)\right\}, \qquad 0 < \beta < 1,\ \gamma \geq 0.$$

Direct calculation shows that when consumption follows the martingale process (2),

$$(4) \quad U_0 = U(C_0,\mu,\sigma_\zeta^2) = \begin{cases} \dfrac{C_0^{1-\gamma}}{1-\gamma} \times \left[1 - \beta e^{(1-\gamma)(\mu - \gamma\sigma_\zeta^2/2)}\right]^{-1} & (\gamma\neq 1) \\[2ex] \dfrac{1}{1-\beta}\left[\log(C_0) + \dfrac{\beta}{1-\beta}(\mu - \tfrac{1}{2}\sigma_\zeta^2)\right] & (\gamma=1) \end{cases}$$

(apart from an irrelevant additive constant when $\gamma \neq 1$).

Equation (4) makes it easy to evaluate welfare under different consumption processes. A standard way to compare welfare under alternative parameter settings (μ,σ_ζ^2) and $(\mu',\sigma_\zeta^{2\prime})$ is to compute the *compensating variation* in consumption: the uniform percentage increase in consumption, effective on all dates and in all states of nature, that leaves consumers indifferent between the mean-variance pairs (μ,σ_ζ^2) and $(\mu',\sigma_\zeta^{2\prime})$. This measure, denoted κ, is defined by $U[(1+\kappa)C_0,\mu,\sigma_\zeta^2] = U(C_0,\mu',\sigma_\zeta^{2\prime})$. From (4), $\kappa = \kappa[(\mu,\sigma_\zeta^2),(\mu',\sigma_\zeta^{2\prime});\gamma]$. So under (2),

$$(5)\quad \kappa[(\mu,\sigma_\zeta^2),(\mu',\sigma_\zeta^{2\prime});\gamma] = \begin{cases} \left[\dfrac{1-\beta e^{(1-\gamma)(\mu-\gamma\sigma_\zeta^2/2)}}{1-\beta e^{(1-\gamma)(\mu'-\gamma\sigma_\zeta^{2\prime}/2)}}\right]^{\frac{1}{1-\gamma}} - 1 & (\gamma\neq 1) \\ \left[e^{\mu'-\mu-(\sigma_\zeta^{2\prime}-\sigma_\zeta^2)/2}\right]^{\frac{\beta}{1-\beta}} - 1 & (\gamma=1). \end{cases}$$

The welfare comparison changes if a consumption process like (1) is posited instead. Assume that expected utility (3) is evaluated *before* z_0 is revealed. Then lifetime expected utility, denoted by $\tilde{U}(\bar{c},\mu,\sigma_z^2)$ for process (1), is:

$$\tilde{U}(\bar{c},\mu,\sigma_z^2) = \begin{cases} \dfrac{e^{(1-\gamma)[\bar{c}-\gamma\sigma_z^2/2]}}{1-\gamma} \times \dfrac{1}{1-\beta e^{\mu(1-\gamma)}} & (\gamma\neq 1) \\ \dfrac{1}{1-\beta}\left[\bar{c} - \dfrac{1}{2}\sigma_z^2 + \dfrac{\beta}{1-\beta}\mu\right] & (\gamma=1). \end{cases}$$

Under (1), the fraction by which $e^{\bar{c}}$ must be raised to generate the same welfare change as a shift from (μ,σ_z^2) to $(\mu',\sigma_z^{2\prime})$ is:

$$(6)\quad \tilde{\kappa}[(\mu,\sigma_z^2),(\mu',\sigma_z^{2\prime});\gamma] = \begin{cases} e^{-\gamma(\sigma_z^{2\prime}-\sigma_z^2)/2}\left[\dfrac{1-\beta e^{\mu(1-\gamma)}}{1-\beta e^{\mu'(1-\gamma)}}\right]^{\frac{1}{1-\gamma}} - 1 & (\gamma\neq 1) \\ e^{-(\sigma_z^{2\prime}-\sigma_z^2)/2}\left[e^{\mu'-\mu}\right]^{\frac{\beta}{1-\beta}} - 1 & (\gamma=1). \end{cases}$$

Now we want to compare the effects of changes in the consumption process under (2) as against (1). For this purpose, it is illuminating to look at first-order approximations to (5) and (6). Consider small changes $\Delta\sigma_\zeta^2 = \sigma_\zeta^{2\prime} - \sigma_\zeta^2$, $\Delta\sigma_z^2 = \sigma_z^{2\prime} - \sigma_z^2$, and $\Delta\mu = \mu' - \mu$. The following formulas approximate (5) and (6), respectively:

$$(7)\quad \kappa[(\mu,\sigma_z^2),(\mu',\sigma_\zeta^{2\prime});\gamma] \approx \frac{\beta e^{(1-\gamma)(\mu-\gamma\sigma_\zeta^2/2)}}{1-\beta e^{(1-\gamma)(\mu-\gamma\sigma_\zeta^2/2)}}\left(-\gamma\Delta\sigma_\zeta^2/2 + \Delta\mu\right),$$

$$(8)\quad \tilde{\kappa}[(\mu,\sigma_z^2),(\mu',\sigma_z^{2\prime});\gamma] \approx -\gamma\Delta\sigma_z^2/2 + \frac{e^{\beta(1-\gamma)\mu}}{1-\beta e^{(1-\gamma)\mu}}\Delta\mu.$$

Formula (8) includes a standard measure of welfare loss due to higher consumption variability (the first term on the right-hand side).[4] It also includes a measure of the gain from higher trend growth (the second term). According to (1), log consumption shocks are one-time deviations from trend that leave the trend itself unchanged. Thus, the *per-period* percentage consumption loss due to variability in (8) is just the "static" cost, $\gamma\Delta\sigma_z^2/2$.

Expression (7), in contrast, reflects that innovations in growth, like trend growth, have cumulative effects on the level of consumption under a unit-root assumption. Since a consumption shock ζ_t has a proportional effect on consumption that persists for all future periods, the *per-period* cost of variability is a discounted sum of "static" losses. To interpret (7) in this light, notice that the term multiplying $-\gamma\Delta\sigma_\zeta^2 + \Delta\mu$ is the ex-dividend market value of per-capita wealth, divided by initial consumption C_0.[5] (Basically, it is the inverse of the propensity to consume out of wealth.) Thus, the right-hand side of (7) can be viewed as the expected present discounted value of the flow of percentage income changes due to higher variability or growth.

Expressions (7) and (8) imply comparable measures of the welfare effects of changes in trend growth, provided the changes are small and $\gamma\sigma_\zeta^2$ is not too big. In contrast, the welfare effect of a small change in variance under (2) is

$$\frac{\beta e^{(1-\gamma)(\mu-\gamma\sigma_\zeta^2/2)}}{1-\beta e^{(1-\gamma)(\mu-\gamma\sigma_\zeta^2/2)}}$$

times its effect under (1). As the next section shows, it is essentially through this term that intertemporal substitutability affects the welfare cost of changes in risk under (2).

II. Intertemporal Substitutability versus Risk Aversion

I now show why dynamic welfare evaluations of changes in consumption risk inevitably encounter the need to separate consumers' risk aversion from their willingness to substitute over time. The expected-utility criterion (3) confounds these two factors, because the coefficient of relative risk aversion it implies is γ, while the elasticity of intertemporal substitution is $1/\gamma$ (Hall 1988). As the appendix shows, this confusion is problematic not only when consumption contains a stochastic trend, but even when it is stationary and serially correlated.

An easy way to understand the restrictive nature of preference class (3) is to assume temporarily that there is no uncertainty, so as to focus attention on the welfare effects of changes in trend growth. Equation (7) or equation (8) gives the welfare effect under (3) of a small change $\Delta\mu$ in growth:

$$(9) \qquad \frac{\beta e^{(1-\gamma)\mu}}{1-\beta e^{(1-\gamma)\mu}}\Delta\mu.$$

How does an increase in γ affect (9)? If trend growth μ is positive, (9) falls as γ rises. The reason is clear. A higher γ implies that, other things equal, the marginal utility of consumption, is falling more swiftly over time. This effect, however, diminishes the welfare benefit of the change $\Delta\mu$ by reducing the contribution of growth to future flow utility levels.

Alternatively, the rise in γ raises "shadow" real interest rates, reducing the value of future trend increases in consumption.[6]

When trend growth is negative, the marginal utility of consumption rises more swiftly over time the higher is γ. Thus the welfare gain from faster growth is greater the higher is γ.

It is clear that this effect of an increase in γ is entirely due to the implied reduction in the intertemporal substitutability of consumption. Risk aversion, by assumption, does not enter the picture. But we can draw on this analysis to understand the effect of changes in γ on computations of the cost of higher variability.

Under an i.i.d. process of the form (1), the cost of higher variability is roughly proportional to γ, where that coefficient appears as a measure of risk aversion [recall (8)]. Because risk is not cumulative when consumption is i.i.d., its income-equivalent welfare cost can be evaluated without reference to an intertemporal substitution parameter [see (6) or (8)]. There is thus no danger that the experiment of raising γ will confound risk-aversion with intertemporal-substitution effects.[7]

Under the martingale process (2), however, variability is cumulative, and it thus enters *as a downward adjustment* $\gamma\sigma_\zeta^2/2$ *to the trend growth rate* μ [recall (5) and (7)]. When γ rises, therefore, two distinct changes occur. First, the downward risk adjustment applied to the growth rate rises in proportion to the increase in γ. Second, the welfare benefit of growth is itself scaled downward when $\mu - \gamma\sigma_\zeta^2/2 > 0$, upward when $\mu - \gamma\sigma_\zeta^2/2 < 0$.[8]

Evidently, assessing the impact of pure changes in risk aversion on the costs of consumption-risk changes calls for a parametric class of preferences broader than (3). Appropriate preferences have been proposed by Epstein and Zin (1989) and by Weil (1990). Under Weil's formulation, for example, lifetime utility is given by the recursion

$$(10)\quad U_t = \frac{\left\{(1-\beta)C_t^{1-\theta} + \beta[1+(1-\beta)(1-\gamma)\mathrm{E}_t U_{t+1}]^{\frac{1-\theta}{1-\gamma}}\right\}^{\frac{1-\gamma}{1-\theta}} - 1}{(1-\beta)(1-\gamma)},$$

where $0 < \beta < 1$ and $\gamma, \theta \geq 0$. In (10), γ is the coefficient of relative risk aversion while $1/\theta$ is the elasticity of intertemporal substitution for nonrandom consumption paths.

When log consumption is generated by (2), lifetime utility is given by

$$U_0 = U(C_0, \mu, \sigma_\zeta^2) = \begin{cases} \dfrac{C_0^{1-\gamma}}{1-\gamma} \times \dfrac{(1-\beta)^{\frac{\theta-\gamma}{1-\theta}}}{\left[1-\beta e^{(1-\theta)(\mu-\gamma\sigma_\zeta^2/2)}\right]^{\frac{1-\gamma}{1-\theta}}} & (\gamma, \theta \neq 1) \\[2em] \dfrac{1}{1-\beta}\left[\log(C_0) + \dfrac{1}{1-\theta}\log\left(\dfrac{1-\beta}{1-\beta e^{(1-\theta)(\mu-\sigma_\zeta^2/2)}}\right)\right] & (\gamma=1, \theta\neq 1) \end{cases}$$

[apart from an irrelevant additive constant when $\gamma \neq 1$; compare with (4)].[9] As before, the compensating variation measure of the welfare change caused by a parameter shift from (μ,σ_ζ^2) to $(\mu',\sigma_\zeta^{2\prime})$ in (2) is defined by $U[(1+\kappa)C_0,\mu,\sigma_\zeta^2] = U(C_0,\mu',\sigma_\zeta^{2\prime})$. Thus $\kappa = \kappa[(\mu,\sigma_\zeta^2),(\mu',\sigma_\zeta^{2\prime});\gamma,\theta]$, where

$$(11)\quad \kappa[(\mu,\sigma_\zeta^2),(\mu',\sigma_\zeta^{2\prime});\gamma,\theta] = \left[\frac{1-\beta e^{(1-\theta)(\mu-\gamma\sigma_\zeta^2/2)}}{1-\beta e^{(1-\theta)(\mu'-\gamma\sigma_\zeta^{2\prime}/2)}}\right]^{\frac{1}{1-\theta}} - 1 \qquad (\theta\neq 1).$$

An illuminating linear approximation to this expression is:

$$(12)\quad \kappa[(\mu,\sigma_\zeta^2),(\mu',\sigma_\zeta^{2\prime});\gamma,\theta] \approx \frac{\beta e^{(1-\theta)(\mu-\gamma\sigma_\zeta^2/2)}}{1-\beta e^{(1-\theta)(\mu-\gamma\sigma_\zeta^2/2)}}\left(-\gamma\Delta\sigma_\zeta^2/2 + \Delta\mu\right).$$

Comparison of (12) with (7) [or of (11) with (5)] underscores the earlier motivation for moving to a more generously parameterized preference class. Now, the risk aversion parameter, γ, simply governs the amount by which growth is adjusted downward to reflect the negative welfare effect of variability. The welfare effects of changes in *adjusted* growth, $\mu - \gamma\sigma_\zeta^2/2$, depend on the intertemporal substitution parameter, θ, alone. All else the same, higher θ implies that marginal utility is declining more quickly when adjusted growth is positive, so that a given change in growth is of less importance. This effect of higher θ is reversed when adjusted growth is negative.[10]

As (12) shows most clearly, $e^{(1-\theta)(\mu-\gamma\sigma_\zeta^2/2)}$ plays the role of a second discount factor on future consumption: one can think of first-order welfare effects as depending on an overall discount rate equal to $(\theta - 1)(\mu - \gamma\sigma_\zeta^2/2) - \log\beta$. With the risk-adjusted growth rate $\mu - \gamma\sigma_\zeta^2/2$ held constant, the effect of changing γ under expected-utility preferences thus corresponds to that of a simultaneous change in risk aversion *and* in the subjective discount rate.[11]

Equation (12) also implies that, given the risk-adjusted growth rate, the welfare cost of higher variability is roughly linear in the risk aversion coefficient γ. This linearity holds under expected-utility preferences when consumption is trend-stationary [as shown by (8)], but not when consumption is serially correlated [as shown by (7)].

A final implication of equations (11) and (12) is that for $\theta > 1$, variability hurts more when growth is lower. This result follows from the fact that when $\theta > 1$, extra growth has a smaller positive effect on income the higher growth already is. The dependence on θ reflects the opposing effects of a higher initial growth rate on base income levels and on the marginal utility of consumption. These effects offset each other exactly when $\theta = 1$.

III. Application to United States Consumption Data

The theoretical results derived above are illustrated by an application to U.S. data. Following Lucas (1987), I attach the label "the cost of consumption instability" to the amount a representative agent would gain if the randomness in consumption could be eliminated with no change in μ. (A representative agent is defined as one whose consumption is average per capita U.S. consumption.) The compensating variations $\tilde{\kappa}[(\mu,\sigma_z^2),(\mu,0);\gamma]$, $\kappa[(\mu,\sigma_\zeta^2),(\mu,0);\gamma]$, and $\kappa[(\mu,\sigma_\zeta^2),(\mu,0);\gamma,\theta]$ can then be computed from equations (6), (5), and (11), respectively, using estimates of the moments μ, σ_z^2, and σ_ζ^2 from U.S. data. A useful metric for assessing the cost of consumption instability comes from comparing it, as does Lucas (1987), to the cost of a forgone percentage point of trend consumption growth.

National-accounts data on consumption are not entirely appropriate for this purpose, as the accounts treat expenditures on durable goods as current consumption. To control for the problem I report calculations for per capita consumption of nondurables and services (C^{nds}) as well as for total per capita personal consumption (C^{tot}). My calculations assume $\beta = 0.95$.[12]

Consumption data are annual and cover 1950-1990. C^{tot} is (total) personal consumption expenditure in 1982 dollars, from the *Economic Report of the President* (February 1991), Table B-15. C^{nds} is the sum of consumption of nondurable goods and services, also in 1982 dollars, from the same table. The consumption series are converted to per capita terms using data on total population from Table B-31 of the 1991 *Economic Report*. Table 1 reports estimates of the key parameters in equations (1) and (2), the trend annual growth rate and the annual standard deviation.

Table 1

Estimates of U.S. Consumption's Trend Growth and Variability
(Annual Data, 1950-1990)

A. *Specification (1):* $c_t = \bar{c} + \mu t - \frac{1}{2}\sigma_z^2 + z_t$

Consumption measure	$\hat{\mu}$	$\hat{\sigma}_z$
C^{tot}	0.0214	0.0266
C^{nds}	0.0197	0.0228

B. *Specification (2):* $c_t = c_{t-1} + \mu - \frac{1}{2}\sigma_\zeta^2 + \zeta_t$

Consumption measure	$\hat{\mu}$	$\hat{\sigma}_\zeta$
C^{tot}	0.0202	0.0163
C^{nds}	0.0185	0.0112

Note: C^{tot} denotes total U.S. consumption per capita in 1982 dollars, C^{nds} consumption of nondurables and services. Annual data cover 1950-1990 and come from *Economic Report of the President* (February 1991). Point estimates for μ in specification (1) are obtained by ordinary least squares regression of the logarithm of per capita consumption on a constant and a time trend. The estimate $\hat{\sigma}_z$ is the estimated standard error of that regression. Point estimates for μ in specification (2) come from regressing the first-differenced logarithm of per capita consumption on a constant and adding $(1/2)\hat{\sigma}_\zeta^2$, where $\hat{\sigma}_\zeta$ is the estimated standard error of the regression.

Table 2 uses those estimates to compute the cost of consumption instability both under (1) and under (2).[13]

Table 3 provides a benchmark of comparison for the numbers in Table 2 by computing the welfare benefit of raising trend consumption growth by 1 percentage point per year. Reported in brackets below each estimate is its ratio to the corresponding cost of consumption instability from Table 2.

Table 2 illustrates the following points:

1. Since total consumption is more volatile than consumption of nondurables and services--in part spuriously so--the losses calculated using C^{tot} are higher than those based on C^{nds}.

2. Even specification (1) (trend-stationary consumption) implies welfare losses three to four-and-a-half times the size of those Lucas (1987, p. 26) reports. But Lucas's detrending method, based on applying the Hodrick-Prescott filter to quarterly data, implies a σ_z estimate of only 0.013. (Compare Table 1, panel A.)

3. For low γ, the losses under the integrated consumption process (panel B) substantially exceed those based on the i.i.d. specification (panel A). But while in panel A losses rise roughly in proportion to the parameter γ, they rise less than proportionally in panel B. Thus, by the time γ reaches 10, the loss in panel A is clearly overtaking that in panel B. When C^{nds} is used, for example, the maximal reported loss (γ = 20) is slightly over a half a percent of output per year under (1), but

Table 2

Cost of Consumption Instability for U.S. Consumers
(Per Year, as a Percent of Consumption)

A. *Specification (1):* $c_t = \bar{c} + \mu t - \frac{1}{2}\sigma_z^2 + z_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	0.0354	0.0708	0.1771	0.3544	0.7101
C^{nds}	0.0260	0.0520	0.1300	0.2603	0.5212

B. *Specification (2):* $c_t = c_{t-1} + \mu - \frac{1}{2}\sigma_\zeta^2 + \zeta_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	0.2514	0.3572	0.4720	0.5167	0.5198
C^{nds}	0.1190	0.1735	0.2359	0.2612	0.2620

Note: Calculations are based on the estimated parameters reported in Table 1, with $\beta = 0.95$ assumed. Panel A reports values of $\tilde{\kappa}[(\mu,\hat{\sigma}_z^2),(\hat{\mu},0);\gamma] = e^{\gamma\hat{\sigma}_z^2/2} - 1$. Panel B reports the numbers:

$$\kappa[(\hat{\mu},\hat{\sigma}_\zeta^2),(\hat{\mu},0);\gamma] = \begin{cases} \left[\dfrac{1 - \beta e^{(1-\gamma)(\hat{\mu}-\gamma\hat{\sigma}_\zeta^2/2)}}{1 - \beta e^{(1-\gamma)\hat{\mu}}}\right]^{\frac{1}{1-\gamma}} - 1 & (\gamma\neq 1), \\ \left(e^{\hat{\sigma}_\zeta^2/2}\right)^{\frac{\beta}{1-\beta}} - 1 & (\gamma=1). \end{cases}$$

only about half as big under (2). This underscores the point made above, that under expected utility preferences an increase in the risk aversion parameter simultaneously affects the effective discount factor applied to future consumption. For U.S. data, and for the range of γ considered here, an increase in γ sharply raises the effective discounting of future consumption. In principle the cost of instability could actually fall as γ rises.

In addition, Table 3 discloses that:

4. For $\gamma = 1$, the benefit from an extra percentage point of growth is very high--over 20 percent of consumption per year, regardless of the specification or consumption measure. But this benefit falls off as γ rises with an elasticity around 2/3.[14] Thus, when $\gamma = 10$, the extra point of growth is worth only between 3 and 3.5 percent of consumption per year. The falling gain from additional growth as γ rises reflects the same phenomenon just discussed under point 3.

5. For low γ, the *ratio* of the gain from increased growth to the cost of instability is extremely high, but much higher under the i.i.d. consumption specification (1). For example, the C^{nds} consumption measure implies a ratio of 804.81 under (1) when $\gamma = 1$ (panel A), but a ratio of 175.79 under (2) (panel B). All of these ratios, however, become more nearly comparable as γ rises. For example, when $\gamma = 10$ and C^{nds}, is used, the ratios are close to 13 under both specifications.

Table 3

Benefit of an Extra Percentage Point of Trend Consumption Growth (Per Year, as a Percent of Consumption [and as a Ratio to the Cost of Consumption Instability])

A. *Specification (1):* $c_t = \bar{c} + \mu t - \frac{1}{2}\sigma_z^2 + z_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	20.925 [591.10]	13.188 [186.28]	6.093 [34.40]	3.055 [8.62]	1.379 [1.94]
C^{nds}	20.925 [804.81]	13.532 [260.23]	6.420 [49.38]	3.269 [12.56]	1.450 [2.78]

B. *Specification (2):* $c_t = c_{t-1} + \mu - \frac{1}{2}\sigma_\zeta^2 + \zeta_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	20.925 [83.24]	13.452 [37.66]	6.418 [13.60]	3.353 [6.49]	1.654 [3.18]
C^{nds}	20.925 [175.79]	13.787 [79.46]	6.712 [28.45]	3.508 [13.43]	1.691 [6.45]

Note: Calculations are based on the estimated parameters reported in Table 1, with $\beta = 0.95$ assumed. Panel A reports values of $\tilde{\kappa}[(\hat{\mu},\hat{\sigma}_z^2),(\hat{\mu}+0.01,\hat{\sigma}_z^2);\gamma]$, as defined in equation (6) in the text. Panel B reports the numbers $\kappa[(\hat{\mu},\hat{\sigma}_\zeta^2),(\hat{\mu}+0.01,\hat{\sigma}_\zeta^2);\gamma]$, as defined in equation (5) in the text. Numbers in square brackets are these entries divided by the corresponding entries in Table 2.

Consider next the cost of consumption instability when the coefficient of relative risk aversion and the elasticity of intertemporal substitution are not constrained to be reciprocals. Table 4 gives values for $\kappa[(\mu,\sigma_{\zeta}^2),(\mu,0);\gamma,\theta]$, defined by (11), over a grid of (γ,θ) pairs. For a fixed intertemporal substitution parameter θ, the cost of instability rises approximately in proportion to the risk-aversion parameter γ. For fixed γ, however, the cost of instability declines sharply as θ rises, that is, as the intertemporal substitution elasticity falls. These results are in line with analysis of the last section.

Table 4 raises the theoretical possibility that high risk aversion, when coupled with high intertemporal substitutability, leads to a high cost of consumption instability. The empirical results of Hall (1988) and of Campbell and Mankiw (1989) suggest, however, a very small intertemporal substitution elasticity, perhaps on the order of 0.10. This point estimate would imply θ = 10. The likely level of consumer risk aversion is probably more controversial. Conventional estimates are in the range of 2 to 6, but Kandel and Stambaugh (1991) have argued that even values as high as 30 cannot be ruled out on the basis of available data.

Recent work by Epstein and Zin (1991) reports Euler-equation estimates of the parameters γ and θ based on a variety of consumption measures, asset returns, and estimation techniques. Their γ estimates cluster around unity (logarithmic risk aversion), but are not grossly inconsistent with the hypothesis

Table 4

Cost of Consumption Instability for U.S. Consumers
(Per Year, as a Percent of Consumption)

Consumption measure		$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}						
	$\theta = 2$	0.1783	0.3572	0.8981	1.8131	3.6957
	$\theta = 5$	0.0934	0.1873	0.4720	0.9567	1.9665
	$\theta = 10$	0.0502	0.1007	0.2542	0.5167	1.0680
	$\theta = 20$	0.0242	0.0485	0.1226	0.2499	0.5198
C^{nds}		$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
	$\theta = 2$	0.0867	0.1735	0.4349	0.8738	1.7635
	$\theta = 5$	0.0469	0.0940	0.2359	0.4750	0.9627
	$\theta = 10$	0.0258	0.0516	0.1296	0.2612	0.5310
	$\theta = 20$	0.0126	0.0253	0.0636	0.1285	0.2620

Note: Calculations are based on the estimated parameters reported in Table 1 for specification B, with $\beta = 0.95$ assumed. The numbers reported are:

$$\kappa[(\hat{\mu},\hat{\sigma}_{\zeta}^{2}),(\hat{\mu},0);\gamma,\theta] = \left[\frac{1-\beta e^{(1-\theta)(\hat{\mu}-\gamma\hat{\sigma}_{\zeta}^{2}/2)}}{1-\beta e^{(1-\theta)\hat{\mu}}}\right]^{\frac{1}{1-\theta}} - 1.$$

that γ is as high as 2. Their estimates for θ exhibit more dispersion: anything from $\theta = 1$ to $\theta = 20$ appears possible. But the point estimates for θ cluster around two (approximate) values, 4 and 1.4. The equations yielding the higher estimates for θ, however, also estimate *negative* rates of time preference (as they must to fit the low average level of the U.S. risk-free interest rate, given per capita consumption growth). Thus, $\theta \approx 2$ (implying an intertemporal substitution elasticity around 0.5) seems a reasonable inference from the Epstein-Zin (1991) results.

Assuming that $\gamma = 1$ and $\theta = 2$ leaves the estimated cost of aggregate U.S. consumption instability quite small: 0.1783 percent of consumption per year if C^{tot} is used, 0.0867 per cent if C^{nds} is used. These estimates are higher than those of Lucas (1987, p. 26), who finds a cost of consumption instability of 0.042 percent of consumption when $\gamma = \theta = 5$, and a cost a fifth that large when $\gamma = \theta = 1$.

These estimates are not as high as those found by İmrohoroğlu (1989), who posits a general-equilibrium model in which agents face idiosyncratic risk and imperfect capital markets. (Consumption is, however, a stationary process in İmrohoroğlu's model.) For $\gamma = \theta = 1.5$, İmrohoroğlu estimates a cost of consumption instability of 0.30 percent of consumption when the only means of intertemporal smoothing is storage. This estimate exceeds those in the northwest corners of Table 4's two panels.

Table 5 turns to the gain from higher growth under the

Table 5

Benefit of an Extra Percentage Point of Trend Consumption Growth (Per Year, as a Percent of Consumption [and as a Ratio to the Cost of Consumption Instability])

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}					
$\theta = 2$	13.427 [75.31]	13.452 [37.66]	13.530 [15.07]	13.662 [7.54]	13.933 [3.77]
$\theta = 5$	6.318 [67.64]	6.343 [33.87]	6.418 [13.60]	6.548 [6.84]	6.822 [3.47]
$\theta = 10$	3.201 [63.76]	3.218 [31.96]	3.267 [12.85]	3.353 [6.49]	3.538 [3.31]
$\theta = 20$	1.462 [60.41]	1.471 [30.33]	1.499 [12.23]	1.548 [6.19]	1.654 [3.18]
C^{nds}	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\theta = 2$	13.774 [158.87]	13.787 [79.46]	13.826 [31.79]	13.890 [15.90]	14.022 [7.95]
$\theta = 5$	6.660 [142.00]	6.673 [70.99]	6.712 [28.45]	6.778 [14.27]	6.914 [7.18]
$\theta = 10$	3.429 [132.91]	3.437 [66.61]	3.463 [26.72]	3.508 [13.43]	3.601 [6.78]
$\theta = 20$	1.591 [126.27]	1.596 [63.08]	1.611 [25.33]	1.637 [12.74]	1.691 [6.45]

Note: Calculations are based on the estimated parameters reported in Table 1, with $\beta = 0.95$ assumed. Entries are values of $\kappa[(\hat{\mu},\hat{\sigma}_{\zeta}^{2}),(\hat{\mu}+0.01,\hat{\sigma}_{\zeta}^{2});\gamma,\theta]$, as defined in equation (11) in the text. Numbers in square brackets are these entries divided by the corresponding entries in Table 4.

integrated log consumption process (2), measuring it in absolute terms and relative to the cost of instability. As expected, the *absolute* benefits vary little with γ but drop sharply as θ rises.

For estimates close to Epstein and Zin's--say, $\gamma = 2$ and $\theta = 2$, implying expected-utility preferences--the benefit of an extra percentage point of growth *relative* to the cost of instability is about 38 based on total consumption, nearly 80 based on nondurables and services. (These numbers appear in square brackets.) In contrast, Lucas's (1987, pp. 25-26) calculations imply a ratio exceeding 1000 for $\gamma = 2$. Growth clearly remains a much more important issue than variability. It is harder to argue, however, that the cost of variability is negligible, particularly if degrees of consumer risk aversion higher than $\gamma = 2$ are possible.[15]

IV. Conclusion

Unless risk aversion and intertemporal substitutability are carefully separated, attempts to measure the welfare cost of changes in consumption risk can yield misleading conclusions about the role of risk aversion. Under expected-utility preferences, which make no such separation, an increase in risk aversion simultaneously alters the effective discount factor that consumers apply to future consumption. When consumption uncertainty is persistent, simply increasing the coefficient of relative risk aversion can lead to a distorted picture of the extent to which more risk averse consumers are adversely affected by risk.

The paper illustrated these points by applying both expected- and nonexpected-utility preferences to a well-known example, Lucas's (1987) inquiry into the welfare gain from the hypothetical elimination of the unpredictable variability in U.S. aggregate consumption. Generalized preferences make little difference under Lucas's implicit assumption that the deviations of log consumption from a deterministic trend are unpredictable. Nonexpected-utility preferences substantially change the estimated effect of higher risk aversion when log consumption contains a stochastic trend (and even when log consumption is stationary but exhibits substantial serial correlation).

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Appendix

Neither of the data-generating processes (1) and (2) examined in the text is a good approximation to the one that actually generates U.S. consumption. A more realistic comparison would work with two processes, one integrated and the other trend-stationary, that both match the true consumption data-generating process reasonably well. To illustrate the difference this makes to the estimates, I briefly examine in this appendix more general ARIMA representations for log consumption, c_t. While the precise estimates of costs and benefits are somewhat different, the qualitative nature of their dependence on γ is not.

The estimates make the important point referred to above, that it is persistence in consumption, rather than a unit root per se, that gives intertemporal substitution a distinct role in analyses of the welfare cost of consumption risk.

Table A1 reports estimates of two autoregressive data-generating processes for the log of U.S. annual consumption. Panel A reports the result of fitting an ARIMA(2,0,0),

$$\text{(A1)} \quad c_t = \bar{c} + \phi_1 c_{t-1} + \phi_2 c_{t-2} + \mu t - \tfrac{1}{2}\sigma_z^2 + z_t.$$

The equations in panel B, in contrast, are constrained to contain a unit root. Here, an ARIMA(1,1,0) process,

$$\text{(A2)} \quad \Delta c_t = \phi \Delta c_{t-1} + \mu - \tfrac{1}{2}\sigma_\zeta^2 + \zeta_t,$$

Table A1

Alternative Data-Generating Processes for U.S. Consumption
(Annual Data, 1950-1990)

A. *Specification (A1):* $c_t = \bar{c} + \phi_1 c_{t-1} + \phi_2 c_{t-2} + \mu t - \frac{1}{2}\sigma_z^2 + z_t$

Consumption measure	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\mu}$	$\hat{\sigma}_z$
C^{tot}	1.0859	−0.3277	0.0051	0.0152
C^{nds}	1.2357	−0.3751	0.0026	0.0105

B. *Specification (A2):* $\Delta c_t = \phi \Delta c_{t-1} + \mu - \frac{1}{2}\sigma_\zeta^2 + \zeta_t$

Consumption measure	$\hat{\phi}$	$\hat{\mu}$	$\hat{\sigma}_\zeta$
C^{tot}	0.2233	0.0158	0.0161
C^{nds}	0.3108	0.0128	0.0107

Note: C^{tot} denotes total U.S. consumption per capita in 1982 dollars, C^{nds} consumption of nondurables and services. Annual data cover 1950-1990 and come from *Economic Report of the President* (February 1991). Coefficient estimates are obtained by ordinary least squares. Point estimates for μ in panel B are obtained by adding $(1/2)\hat{\sigma}_\zeta^2$ to the estimated regression constant. Both regressions use 1948 and 1949 data for lags.

appears to fit well. Tests for additional lags produced no evidence of their presence.[16]

Panel A of Table A1 shows lower one-period ahead forecast variabilities of c_t, σ_z, than does panel A of Table 1. However, the estimated persistence of c_t is considerable, which confirms that z_t is far from being i.i.d. (as Table 2 assumed).

Panel B of Table A1 reports one-period ahead forecast variabilities, σ_ζ, that are much the same as those reported in panel B of Table 1. However, the autoregressive representation of Δc_t contains a significantly positive lagged term that contributes importantly to its unconditional variability.

Table A2 reports welfare losses under processes (A1) and (A2).[17] Panel A shows that losses under the trend-stationary process (A1) are somewhat higher than those in Table 2, panel A, at low levels of γ, but somewhat lower at high levels. According to panel B of Table A2, losses under (A2) are higher than those under (2) for all the values of γ examined (see Table 2, panel B). Furthermore, comparison of the panels of Table A2 shows that of the two competing log consumption specifications, the integrated process implies uniformly higher losses.

It is still true, as under the simple martingale (2), that the losses in panel B rise less than linearly with γ. The dynamics implied by (A1) yield a similar result in panel A: even under a stationary process the cost of consumption instability is now a strictly concave function of γ. This is a result of the serial

Table A2

Cost of Consumption Instability for U.S. Consumers
(Per Year, as a Percent of Consumption)

A. *Specification (A1):* $c_t = \bar{c} + \phi_1 c_{t-1} + \phi_2 c_{t-2} + \mu t - \frac{1}{2}\sigma_z^2 + z_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	0.0442	0.0785	0.1718	0.3043	0.5187
	(0.0035)	(0.0041)	(0.0048)	(0.0056)	(0.0063)
C^{nds}	0.0340	0.0608	0.1293	0.2149	0.3293
	(0.0039)	(0.0043)	(0.0048)	(0.0051)	(0.0052)

B. *Specification (A2):* $\Delta c_t = \phi \Delta c_{t-1} + \mu - \frac{1}{2}\sigma_\zeta^2 + \zeta_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	0.3211	0.5350	0.7873	0.9394	1.1008
	(0.0238)	(0.0204)	(0.0156)	(0.0127)	(0.0111)
C^{nds}	0.1614	0.2915	0.4471	0.5312	0.6004
	(0.0178)	(0.0154)	(0.0117)	(0.0093)	(0.0076)

Note: Calculations are based on the estimated parameters reported in Table A1, with $\beta = 0.95$ assumed. Welfare costs calculated as follows: Given historical values for c_{1948} and c_{1949}, each consumption process is simulated stochastically over a 100-year horizon. The average of realized lifetime utility levels over 75,000 repetitions, $\hat{U}^S$, gives an estimate of (3). A single deterministic simulation of the same consumption process, with σ_z^2 [in (A1)] or σ_ζ^2 [in (A2)] set to zero, approximates lifetime utility under a deterministic consumption path, U^d. The reported loss measure is $(U^d/\hat{U}^S)^{1/(1-\gamma)} - 1$ $(\gamma \neq 1)$, $e^{(1-\beta)(U^d-\hat{U}^S)/(1-\beta^{100})} - 1$ $(\gamma = 1)$. Standard errors appear in parentheses; if $\kappa(\hat{U}^S)$ is the calculated welfare cost and $\hat{\sigma}^2$ the estimated variance of the utility estimate $\hat{U}^S$, the approximate standard error is $|\kappa'(\hat{U}^S)\hat{\sigma}|$.

correlation in consumption that is assumed absent in panel A of Table 2, but that is accounted for in panel A of Table A2. Notice, however, that the concavity of losses in γ is less marked in the upper panel of Table A2 than in the lower panel's unit-root case.

Table A3 re-evaluates the welfare benefit from an extra percentage point of unconditional trend consumption growth.[18] Under more realistic consumption processes the measured benefit is apparently higher than in Table 3, but the difference is dramatic only when a trend-stationary process is assumed (panel A).[19] When divided by the relevant costs of consumption instability (yielding the numbers in square brackets), the entries in panel A are uniformly higher than those in panel B, and by factors ranging from 3 to more than 10. The ratios in square brackets fall sharply as γ rises, as under the simpler consumption specifications examined in the main text.

I have not replicated Tables A2 and A3 for nonexpected-utility preferences.[20] The qualitative features that the results would have should, however, be apparent.

Table A3

Benefit of an Extra Percentage Point of Trend Consumption Growth (Per Year, as a Percent of Consumption [and as a Ratio to the Cost of Consumption Instability])

A. *Specification (A1):* $c_t = \bar{c} + \phi_1 c_{t-1} + \phi_2 c_{t-2} + \mu t - \frac{1}{2}\sigma_z^2 + z_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	30.814 [697.15]	22.670 [288.79]	13.851 [80.62]	9.254 [30.41]	6.128 [11.81]
C^{nds}	28.149 [827.91]	20.449 [336.33]	11.580 [89.56]	6.993 [32.54]	4.166 [12.65]

B. *Specification (A2):* $\Delta c_t = \phi \Delta c_{t-1} + \mu - \frac{1}{2}\sigma_\zeta^2 + \zeta_t$

Consumption measure	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
C^{tot}	21.071 [65.62]	14.236 [26.61]	7.267 [9.23]	4.226 [4.50]	2.563 [2.33]
C^{nds}	20.881 [129.37]	14.367 [49.29]	7.383 [16.51]	4.222 [7.95]	2.455 [4.09]

Note: Calculations are based on the estimated parameters reported in Table A1, with $\beta = 0.95$ assumed. Welfare costs calculated as follows: Given historical values for c_{1948} and c_{1949}, each consumption process is twice simulated stochastically over a 100-year horizon. The first simulation sets μ equal to the value in Table A1; the second adjusts μ upward so that the *unconditional* trend growth rate of c_t is higher by 0.01. The average of realized lifetime utility levels over 75,000 repetitions, $\hat{U}$ and $\hat{U}^+$, give estimates of (3) under, respectively, the baseline and higher-growth parameter configurations. The reported loss measure is $(\hat{U}^+/\hat{U})^{1/(1-\gamma)} - 1$ $(\gamma \neq 1)$, $e^{(1-\beta)(\hat{U}^+ - \hat{U})/(1-\beta^{100})} - 1$ $(\gamma = 1)$. Numbers in square brackets are these entries divided by the corresponding entries in Table A2.

Endnotes

1. This problem is noted by Cole and Obstfeld (1991, p. 20). Van Wincoop (1991) independently makes the same point.

2. The term $\frac{1}{2}\sigma_z^2$ is subtracted from the log of consumption to ensure that increases in the variance of z_t are mean-preserving spreads on the *level* of consumption.

3. Lucas (1987, pp. 22-23) notes this possibility, but is unpersuaded of its empirical importance.

4. Lucas (1987, p. 26) derives this measure.

5. It is straightforward to verify that under (2),

$$\frac{\beta e^{(1-\gamma)(\mu-\gamma\sigma_\zeta^2/2)}}{1-\beta e^{(1-\gamma)(\mu-\gamma\sigma_\zeta^2/2)}} = \mathrm{E}_0\left\{\sum_{t=1}^{\infty} \beta^t \frac{C_t^{-\gamma}}{C_0^{-\gamma}} C_t\right\}/C_0.$$

The right-hand expectation is the ex-dividend shadow stock-market value of the national consumption process.

6. In general, the market risk-free interest rate in this economy is:

$$r^f = (1/\beta)e^{\gamma\mu - (1+\gamma)\gamma\sigma_\zeta^2/2} - 1.$$

7. If we were *simultaneously* varying σ_z^2 and μ, raising γ would mix risk-aversion with intertemporal-substitution effects on welfare cost even under i.i.d. uncertainty. The concern here, however, is with changes in consumption risk that leave μ unchanged.

8. Under specification (2), there is a simple tradeoff between changes in growth and variability. A change $\Delta\sigma_\zeta^2$ in variability translates into a change $\Delta\mu = \gamma\Delta\sigma_\zeta^2$ in trend growth. This formula captures the essence of why aggregate growth effects are empirically larger than aggregate variability effects. Since growth standard deviations tend to be of the same order of magnitude as growth rates, growth variances will tend to be two orders of magnitude smaller than growth rates. Thus, even if γ = 10, it is likely that the total elimination of consumption variability translates into something substantially less than a percentage point increase in trend growth.

9. The case $\theta = 1$ probably is not empirically relevant, as discussed below. The expressions that apply in that case are readily derived with the help of L'Hospital's rule.

10. The market risk-free rate of interest is now (compare with footnote 6):

$$r^f = (1/\beta)e^{\theta\mu - (1+\theta)\gamma\sigma_\zeta^2/2} - 1.$$

11. Similar reasoning is behind Kocherlakota's (1990) result that β and θ cannot be separately identified in some econometric asset-pricing models.

12. This value of β seems unrealistically low; using it mutes the difference between costs based on trend-stationary and non-stationary processes by reducing the latter. However, I retain β = 0.95 for comparability with Lucas's results.

13. The calculations below are intended to illustrate the analytical points made in the last two sections; they do not aim at empirical accuracy. In fact, neither of the consumption processes examined in this section adequately reflects the autocorrelation in U.S. consumption data. As the purpose of this paper is methodological, I don't explore this question in detail. A more accurate account of the stochastic properties of consumption would, however, lead to somewhat different estimates of the cost of consumption instability and the gains from added growth. The simulations in the appendix indicate the orders of magnitude involved.

14. Lucas (1987, p. 25) analyzes only the case $\gamma = 1$.

15. The results above are specific to the postwar United States, of course. I have repeated the exercise for developing countries, using consumption data from the Penn World Table (Mark 5). Even under the conservative assumption that $\gamma = 1$ and $\theta = 4$, there are many cases in which the cost of consumption instability exceeds 1 percent of consumption per year.

16. Specifications (1), (2), (A1), and (A2) are all nested within the general specification $c_t = \alpha + \delta t + u_t$, where $(1 - \xi L)(1 - \psi L)u_t = \varepsilon_t$, L is the backward-shift operator, and ε_t is a white noise.

17. Expected utilities were calculated by Monte Carlo simulation. See the notes to Table A2 for details.

18. For the specification in panel A, the conditional time trend μ therefore is increased by the amount $0.01 \times (1 - \hat{\phi}_1 - \hat{\phi}_2)$. In panel B, μ is increased by $0.01 \times (1 - \hat{\phi})$.

19. Standard errors for these estimates were several orders of magnitude smaller than the estimates themselves, and are not reported.

20. The simulation methodology underlying Tables A2 and A3 is not feasible in this case. In principle one could approach the utility calculation using the approximation methods advocated by Judd (1991). In practice I have been unable to obtain approximations close enough to isolate the subtle welfare effects under study here.