NBER WORKING PAPER SERIES

EQUILIBRIUM EXCHANGE RATE HEDGING

Fischer Black

Working Paper No. 2947

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 1989

I am grateful for comments on earlier drafts by Michael Adler, Bernard Dumas, Louis Kingsland, Robert Merton, Bhaskar Prasad, Barr Rosenberg, Stephen Ross, Richard Stern Rene Stultz, and Lee Thomas. This paper is part of NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the author not those of the National Bureau of Economic Research.

NBER Working Paper #2947
April 1989

EQUILIBRIUM EXCHANGE RATE HEDGING

ABSTRACT

In a one-period model where each investor consumes a single good, and where borrowing and lending are private and real, there is a universal constant that tells how much each investor hedges his foreign investments. The constant depends only on average risk tolerance across investors. The same constant applies to every real foreign investment held by every investor. Foreign investors are those with different consumption goods, not necessarily those who live in different countries. In equilibrium, the price of the world market portfolio will adjust so that the constant will be related to an average of world market risk premia, an average of world market volatilities, and an average of exchange rate volatilities, where we take the averages over all investors. The constant will not be related to exchange rate means or covariances. In the limiting case when exchange risk approaches zero, the constant will be equal to one minus the ratio of the variance of the world market return to its mean. Jensen's inequality, or "Siegel's paradox," makes investors want significant amounts of exchange rate risk in their portfolios. It also makes investors prefer a world with more exchange rate risk to a similar world with less exchange rate risk.

Fischer Black
Goldman, Sachs & Co.
85 Broad Street, 29th Floor
New York, NY 10004

UNIVERSAL HEDGING FORMULA

$$1 - \lambda = \frac{\mu_m - \sigma_m^2}{\mu_m - \frac{1}{2}\sigma_e^2}$$

λ	average risk tolerance
$1 - \lambda$	fraction of foreign investments hedged
μ_m	average world market portfolio expected excess return
σ_m^2	average world market portfolio return variance
σ_e^2	average exchange rate return variance

INTRODUCTION

Solnik (1974), Grauer, Litzenberger, and Stehle (1976), Sercu (1980), Stulz (1981), Adler and Dumas (1983), and Trevor (1986) have equilibrium models of international investment in equities or real assets. Ross and Walsh (1983) have a similar model for individuals in a single country with their own price indexes. Each of their models assumes that the typical investor in each country consumes a single good or basket of goods. All but Solnik find that investors will hold shares of a single world market portfolio of real assets.

In fact, when all borrowing and lending is real, they find that every investor will hold a mix of a "universal logarithmic portfolio" with domestic lending, where the universal logarithmic portfolio may include foreign borrowing or lending along with the world market portfolio of real assets.

This result leads directly to one result we give below: that there is a universal constant giving the fraction that each investor hedges of his investments in foreign real assets.

In contrast, most people who have looked at exchange rate hedging, like Adler and Dumas (1984), Adler and Simon (1986), Eun and Resnick (1988), Thomas (1988) and Perold and Schulman (1988), have noted that the best hedge depends on mean changes in exchange rates and on covariances of exchange rate changes with one another and with asset returns. Their results are correct, but do not make full use of the equilibrium conditions. Roll and Solnik (1977) have used those conditions to explore the relation between mean changes in exchange rates and covariances in an equilibrium model like Solnik's.

AN EQUILIBRIUM MODEL

Let's imagine a world where all investors in a single country consume the same good. The technology is such that at each moment, any consumption good may be converted to any other consumption good at a fixed exchange rate. Future exchange rates are uncertain.

An asset may pay off in any combination of goods. An investor will use the technology to convert the goods he receives into the good he consumes. Or he may trade the goods he receives for the one he consumes. Assets are not associated with the countries, so we will not distinguish domestic and foreign assets.

Investors create real borrowing and lending in each good. There is no government, so all borrowing and lending is private. Investors create exchange rate contracts by borrowing in one good and lending in another.

This world will last only one infinitesimal period, so we may treat returns on all assets and contracts as following a joint normal distribution. An investor wants to minimize the variance of his portfolio return for a given mean return.

The key to our result is that every investor holds the same portfolio of risky assets, including both equities and foreign borrowing or lending. We also use the fact that borrowing and lending is private, so one person's lending must be another person's borrowing.

Write a_i for the fraction of world wealth held by investors who consume good i and b_i for the fraction of real assets such as common stocks held by investors who consume good i. Even though different investors use different units of account, they all come up with the same values for these fractions. Total world wealth is equal to total world assets.

Write c_i for gross domestic lending by investors who consume good i, again expressed as a fraction of world wealth. When an investor lends, investors in other countries borrow in relation to their holdings of all risky assets. We will even assume that an investor borrows a similar share of his own lending. Thus his net lending in his own good is less than his gross lending.

Since each investor holds the same share of all risky assets, and even the same share of riskless gross domestic borrowing, we can say that borrowing in good j by investors who consume good i is $b_i c_j$. When $j = i$, this refers to gross borrowing, not net borrowing.

An investor in good i will have total gross borrowing equal to $\sum_j b_i c_j$, expressed as a fraction of world wealth or world assets. Since gross lending is c_i, his net lending is $c_i - \sum_j b_i c_j$. But his net lending must equal his wealth minus his real assets, since all borrowing is private.

$$a_i - b_i = c_i - \sum_j b_i c_j \tag{1}$$

We assume the fractions a_i and b_i of world wealth and world assets are both given, though the asset fractions b_i will depend in turn on investor risk preferences. Thus equation (1) gives a separate equation for each good i. These equations help specify the gross lending fractions c_i.

Summing equation (1) over i gives an identity, since the fractions a_i and b_i both sum to 1.0. The equations are not independent, so the solution is not unique. In fact, a general solution to the equations in (1) is:

$$c_i = a_i - \lambda b_i \tag{2}$$

In other words, the investor lends his wealth less a multiple of his holdings of real assets. The multiple λ is the same for all investors. Summing over investors, we see that $1 - \lambda$ is equal to total gross lending for the world, expressed as a fraction of world wealth. It need not be positive: it can also be zero or negative.

We can also interpret $1 - \lambda$ as the amount of hedging each investor does for his foreign investments. A zero value for λ represents 100% exchange rate hedging for foreign real investments. Gross borrowing will then be:

$$\sum_j b_i c_j = b_i(1 - \lambda) \tag{3}$$

In other words, gross borrowing when λ is zero will equal investment in real assets. When λ is zero, we have 100% hedging of foreign investments. When λ

is one, we have no hedging of foreign investments. When λ is greater than one, we have negative hedging: investors add to the exchange risk in their foreign investments. Whatever the level of hedging, it applies equally to investors all over the world, even though they vary in wealth and risk tolerance, and even though the expected exchange rate changes and the covariances between exchange rate changes and asset returns will differ across investors.

A SIMPLE EXAMPLE

To understand why investors will bear some exchange rate risk in equilibrium, let's study a simple symmetric world with two investors and two goods.

Investor 1 consumes good 1, and investor 2 consumes good 2. Investors are endowed with equal amounts of goods 1 and 2 at the horizon.

The exchange technology at the horizon will let investors change from one good to the other at the prevailing exchange rate. The exchange rate will be 2:1 or 1:2 with equal probability.

The investors can trade their endowments one-for-one at the start of the period so that each holds only claims on the good he consumes. Thus each investor can hold a riskless position.

But the payoff from holding the other good is substantial. The expected payoff from holding one unit of the foreign good unhedged is the average of 2.0 and 0.5, or 1.25 units of the domestic good. Everything is symmetric, so each investor gains in expected return (and in risk), from holding some of the foreign good. An investor always gains in expected utility from taking some amount of risk when he faces a positive expected payoff from risk-taking.

Note that the gain in expected return comes entirely from Jensen's inequality or Siegel's (1972, 1975) "paradox." It comes from the difference between the expected value of an inverse and the inverse of expected value. It is substantial, even though it comes from a fact often thought to have mathematical significance but not economic significance. For example, see McCulloch (1975) and Roper (1975). Krugman (1981) and Frankel (1986) show that Siegel's paradox is economically significant in a model more general than this example.

Note also that each investor gains from the existence of exchange rate risk. Both prefer this world to an otherwise identical world where the exchange rate at the horizon will be 1:1 for sure. Similarly, they will have a still stronger preference for a world where the exchange rate will be 3:1 or 1:3 with equal probability.

A GENERAL CASE

We will assume that there are no taxes or other barriers to international investment or disinvestment. When such barriers exist, they will generally cause investors to move away from a world market portfolio and toward a domestic one.

We will assume that borrowing and lending are entirely real. They take the form of contracts for a fixed amount of a single good at the horizon. Exchange rate hedging involves borrowing a foreign good. This is equivalent to taking a forward position in that good.

Actual exchange rate hedging is generally nominal rather than real. This means it is a "noisy" form of hedging. When investors can do only nominal hedging, they may tend to do less of it. On the other hand, if price level changes have real effects, investors may do more nominal hedging than real hedging.

We will assume that each investor consumes a single good, and that there are hedging contracts for that good. Even if an investor consumes goods produced in many parts of the world, the model will hold if he hedges in a contract that reflects his exact consumption basket, so long as the proportions of different goods in the basket are fixed.

In fact, though, even real contracts fail to reflect an investor's actual consumption basket in most cases. This is another source of noise that may affect the amount he hedges.

Actually, we will assume nothing about national boundaries *per se*. We assume that there are hedging contracts for each investor's consumption good, whether or not investors live in different countries.

We will assume that the investor's horizon is an infinitesimal time in the future. The results when investors consume continuously into the future should be similar so long as the inputs to the model fit the horizon. But the difference between nominal hedging and real hedging may be greater with a longer horizon.

To make this model correct for the first instant of a full continuous time model, we will want to assume that future tastes and technology are known at the start. If there are uncertain state variables that affect future tastes or technology, investors will hedge against unfavorable outcomes for those state variables.

We will assume a single real asset representing the world market portfolio. Write "y_{mi}" for the payoff in good i of one unit of the market portfolio, and "f_{mi}" for the forward price of one unit of the market portfolio. The uncertain payoff y_{mi} is worth f_{mi} units of good i for sure.

Write "x_{ij}" for the exchange rate, at the horizon, from good i to good j. We assume that the exchange can go in either direction at this rate. An exchange from i to j and back again returns the investor to his starting point.

As in the simple example, we are assuming a technology that allows conversion of one good into another. Our results do not, however, depend on such a technology. We can also assume uncertain endowments of the goods, where the exchange rates are the equilibrium prices for exchanging the goods. With the right choices for the endowments, this will lead to the same equilibrium.

Write "f_{ij}" for the forward rate from good i to good j. In other words, f_{ij} is the forward price of good i in units of good j. Now we can write the following relations among these variables:

$$y_{mj} = y_{mi}x_{ij} \tag{4}$$

$$f_{mj} = f_{mi}f_{ij} \tag{5}$$

$$x_{ik} = x_{ij}x_{jk} \tag{6}$$

$$x_{ji} = 1/x_{ij} \tag{7}$$

From (4) and (5), we have:

$$y_{mj}/f_{mj} = (y_{mi}/f_{mi})(x_{ij}/f_{ij}) \tag{8}$$

Write "d_{mi}" for the market return over the infinitesimal interval in units of good i. Write "e_{ij}" for the return on good i in units of good j. In other words, d_{mi} is the fractional difference between the actual and forward values of the market portfolio at the horizon, and e_{ij} is the fractional difference between the actual and forward exchange rates. This means:

$$1 + d_{mi} = y_{mi}/f_{mi} \tag{9}$$

$$1 + e_{ij} = x_{ij}/f_{ij} \tag{10}$$

From equations (6), (7), and (8), we have:

$$1 + d_{mj} = (1 + d_{mi})(1 + e_{ij}) \tag{11}$$

$$1 + e_{ik} = (1 + e_{ij})(1 + e_{jk}) \tag{12}$$

$$1 + e_{ji} = 1/(1 + e_{ij}) \tag{13}$$

We will generally use the subscript "m" to refer to the market, and subscripts "i", "j", and "k" to refer to goods. Write "h_{mi}" for the mean of d_{mi}, and "h_{ij}" for the mean of e_{ij}.

Write "g_{mi}" for the variance of d_{mi}, and "g_{ij}" for the variance of e_{ij}. Write "g_{mij}" for the covariance of d_{mj} and e_{ij}, and "g_{ijk}" for the covariance of e_{ik} and e_{jk}.

Returns are stochastic, but variances and covariances are nonstochastic, as explained in Merton (1982). Starting from equation (13), Jensen's inequality or Siegel's paradox comes out like equations (14) and (15).

$$e_{ji} = -e_{ij} + g_{ij} \tag{14}$$

$$e_{ij} + e_{ji} = g_{ij} \tag{15}$$

Since the last term in equation (14) is nonstochastic, we can multiply both sides by e_{ji} or e_{ij} to give:

$$g_{ji} = g_{ij} = g_{iij} = g_{jji} \tag{16}$$

From the definition of g_{ijk}, we have:

$$g_{jik} = g_{ijk} \tag{17}$$

From equation (12), we have:

$$e_{ik} = e_{ij} + e_{jk} - g_{ikj} \tag{18}$$

Again in equation (18), we have an extra "Jensen's inequality" term. From equations (14) and (18) and the definition of g_{ijk}, we have:

$$g_{ijk} + g_{ikj} = g_{jk} \tag{19}$$

From (11), we have:

$$d_{mj} = d_{mi} + e_{ij} - g_{mji} \tag{20}$$

From (20) and (14) and the definition of g_{mij}, we have:

$$g_{mij} = -g_{mji} + g_{ij} \tag{21}$$

$$g_{mij} + g_{mji} = g_{ij} \tag{22}$$

From (15), we have:

$$h_{ij} + h_{ji} = g_{ij} \tag{23}$$

To set up the investor's optimization problem, write "w_{mj}" for the fraction of wealth that investors who consume good j hold in the market. Note that we use "forward values" to figure this and related fractions. No present values are defined in this model.

Write "w_{ij}" for gross borrowing of good j by investors who consume good i. For all goods but the home good, gross borrowing is the same as net borrowing, and is equal to the amount of hedging in good j done by investors who consume good i. Gross borrowing is expressed as a fraction of wealth.

Write "d_{pj}" for the portfolio return for investors who consume j, with mean "h_{pj}" and variance "g_{pj}". The investor wants to minimize g_{pj} for given h_{pj}.

$$d_{pj} = w_{mj}d_{mj} - \sum_k w_{jk}e_{kj} \tag{24}$$

From (24), we can write the investor's problem as:

$$\text{minimize} \quad g_{pj} = w_{mj}^2 g_{mj} - 2w_{mj}\sum_k w_{jk}g_{mkj} + \sum_{ik} w_{ji}w_{jk}g_{ikj} \tag{25}$$

$$\text{subject to} \quad h_{pj} = w_{mj}h_{mj} - \sum_k w_{jk}h_{kj} \tag{26}$$

Taking derivatives of equation (25) subject to (26), and using Lagrange multipliers "λ_j", we have:

$$w_{mj} g_{mj} - \sum_i w_{ji} g_{mij} = \lambda_j h_{mj} \tag{27}$$

$$- w_{mj} g_{mkj} + \sum_i w_{ji} g_{ikj} = -\lambda_j h_{kj} \tag{28}$$

Let's try values for w_{mj} and w_{ji} as solutions to (27) and (28) as follows:

$$w_{mj} = b_j / a_j = \lambda_j / \lambda \tag{29}$$

$$w_{ji} = b_j c_j / a_j = \lambda_j c_i / \lambda \tag{30}$$

Then (27) and (28) become:

$$g_{mj} - \sum_i c_i g_{mij} = \lambda h_{mj} \tag{31}$$

$$- g_{mkj} + \sum_i c_i g_{ikj} = -\lambda h_{kj} \tag{32}$$

Reversing two subscripts in (32), we have:

$$- g_{mjk} + \sum_i c_i g_{ijk} = -\lambda h_{jk} \tag{33}$$

Adding (32) and (33), and using (19), (22), and (23), we have:

$$- g_{jk} + \sum_i c_i g_{jk} = -\lambda g_{jk} \tag{34}$$

All these equations (34) will be satisfied if:

$$\sum_i c_i = 1 - \lambda \tag{35}$$

Multiplying (31) by c_j and summing, we have:

$$\sum_j c_j g_{mj} - \sum_{ij} c_i c_j g_{mij} = \lambda \sum_j c_j h_{mj} \tag{36}$$

We know:

$$\sum_{ij} c_i c_j g_{mij} = \sum_{ji} c_j c_i g_{mji} = \sum_{ij} c_i c_j g_{mji} \tag{37}$$

From (22) and (37), we have

$$\sum_{ij} c_i c_j g_{mij} = \frac{1}{2} \sum_{ij} c_i c_j g_{ij} \tag{38}$$

From (36) and (38), we have:

$$\sum_j c_j g_{mj} - \frac{1}{2} \sum_{ij} c_i c_j g_{ij} = \lambda \sum_j c_j h_{mj} \tag{39}$$

Write "s_j" for gross domestic lending as a fraction of world gross lending.

$$s_j = c_j / \sum_j c_j \tag{40}$$

Using (35), we have:

$$s_j = c_j/(1 - \lambda) \tag{41}$$

Note that:

$$\sum_j s_j = 1 \tag{42}$$

Thus we can take s_j to be the weight on good j in an average across investors. Write "g_m", "g", "h_m", and "h", for the averages of g_{mj}, g_{ij}, h_{mj}, and h_{ij}.

$$g_m = \sum_j s_j g_{mj} \tag{43}$$

$$g = \sum_{ij} s_i s_j g_{ij} \tag{44}$$

$$h_m = \sum_j s_j h_{mj} \tag{45}$$

$$h = \sum_{ij} s_i s_j h_{ij} \tag{46}$$

Using (39), (41), (43), (44), and (45), we have:

$$\lambda = (g_m - \tfrac{1}{2}g)/(h_m - \tfrac{1}{2}g) \tag{47}$$

$$1 - \lambda = (h_m - g_m)/(h_m - \tfrac{1}{2}g) \tag{48}$$

Note that only market means h_{mj} , market variances g_{mj} , and exchange rate variances g_{ij} appear in (58) and (59). There are no exchange rate means or covariances, though these are related indirectly to the market means and variances.

From (29), we have:

$$\lambda = \sum_j a_j \lambda_j \tag{49}$$

Thus λ is a weighted average of investor risk tolerances, where the weight a_j is the fraction of world wealth owned by investors with risk tolerance λ_j . Since the λ_j's are positive, equation (49) also tells us that λ must be positive. Adler and Dumas (1983, p. 952) give a result that leads directly to the fact that optimal hedging depends on this average of investor risk tolerances.

From (23), we have:

$$\sum_{ij} s_i s_j h_{ij} + \sum_{ij} s_i s_j h_{ji} = \sum_{ij} s_i s_j g_{ij} \tag{50}$$

$$h = \tfrac{1}{2} g \tag{51}$$

Thus we can write (47) and (48) as:

$$\lambda = (g_m - h)/(h_m - h) \tag{52}$$

$$1 - \lambda = (h_m - g_m)/(h_m - h) \tag{53}$$

Write "d_m" for the average of d_{mi} across investors, and "g_{mm}" for the variance of d_m .

$$d_m = \sum_i s_i d_{mi} \tag{54}$$

$$g_{mm} = \sum_{ik} s_i s_j d_{mi} d_{mj} \tag{55}$$

Write "g_n" for the average of g_{mij} across all pairs of investors.

$$g_n = \sum_{ij} s_i s_j g_{mij} \tag{56}$$

From (38), we have

$$g_n = \frac{1}{2} g \tag{57}$$

From (20), (55), and (56), we have

$$g_m = g_{mm} + g_n \tag{58}$$

From (57) and (58), we have

$$g_m - \frac{1}{2} g = g_{mm} \tag{59}$$

Since g_{mm} is a variance, this means

$$g_m - \frac{1}{2} g \geq 0 \tag{60}$$

Since λ is positive, equations (52) and (60) give:

$$h_m - h \geq 0 \tag{61}$$

Write "μ_m", "σ_m^2", and "σ_e^2" for h_m, g_m, and g. Then we can write equation (48) as:

$$1 - \lambda = \frac{\mu_m - \sigma_m^2}{\mu_m - \frac{1}{2}\sigma_e^2} \tag{62}$$

It may be easier to remember the definitions of the inputs in this form.

Equations (2), (35), and (39) define λ. As we have noted, λ measures the degree to which foreign investment remains unhedged. We measure foreign investment as in (40) by the wealth of foreign investors, not by the locations of foreign assets. Thus $1 - \lambda$ measures the degree to which investors hedge their foreign investments.

Since Siegel's paradox implies that exchange rate risk adds to investors' expected returns, we expect that investors will generally be less than fully hedged.

It's even possible that investors will want so much exchange rate risk that we will have λ greater than 1.0. Then they will add to the exchange rate risk in their foreign investments. The amount hedged will be negative.

From (25), (26), (29), (30), (31), and (32), we have:

$$g_{pj}/h_{pj} = \lambda_j = w_{mj}\lambda \tag{63}$$

Recall that we defined portfolio return in (24), expressing the weights w_{mi} and w_{ij} as fractions of investor wealth. Write "h_{bj}" and "g_{bj}" as the mean and variance of portfolio returns for investor j where the returns are defined in terms of the investor's holdings of the world market portfolio. We have:

$$g_{bj}/h_{bj} = \lambda \tag{64}$$

In other words, λ is the ratio of variance to mean for the overall portfolio of risks held by each investor, including both market risk and exchange rate risks. It is the same for every investor even though investors use different goods in evaluating the payoffs from their investments.

Thus λ measures average risk tolerance across investors. The greater the risk tolerance of the world's investors, the smaller the market's expected return will be for given endowments, and the more exchange rate risk they will take. Breeden (1979) also makes use of the aggregate risk tolerance in his intertemporal consumption-based asset pricing model.

Note that the amount of exchange rate risk investors will take on does not approach zero as exchange rate volatility approaches zero. Write λ_0 for the exposure to exchange rate risk in the limiting case of zero exchange rate volatility. We have:

$$\lambda_0 = g_m/h_m \tag{65}$$

When exchange rate volatility is exactly zero, investors are indifferent to the amount of hedging they do. The hedge will have no effect.

The market's risk premium is not observable, so it will be hard to estimate λ even when the world is in equilibrium. Right now, investors have far too little international diversification, assuming investment barriers are not a problem. So it's even harder to estimate what λ will be when the world is in equilibrium.

Many people, including Friend and Blume (1975), have tried to estimate average risk tolerance in ways other than this. If other methods give reliable estimates, we can use them instead of formulas like (62). Mehra and Prescott (1985) discuss several other estimates of average risk tolerance. For reasons discussed in Black (1989), I think (62) is the most reliable method for estimating λ .

When making your estimates, recall that μ_m is like "expected excess return" over and above interest. Also, σ_e^2 is an average of exchange rate volatility over all possible pairs of countries, including a country paired with itself. For these self-pairs, the volatility is zero. Finally, recall that both σ_m^2 and σ_e^2 are averages of variances, not standard deviations. The average of variances will generally be higher than the squared average of standard deviations. Frankel (1988) discusses methods for estimating exchange rate volatilities. Edwards (1987, 1988) gives estimates of exchange rate volatilities for developing countries.

In Tables 1-6, we give some historical data that may help you create inputs for the formula. Table 1 just gives the weights that you can apply to different countries in estimating the three averages. Most of the weight will be on Japan, the US, and the UK. In Tables 2-4, we have historical statistics for 1986-88, and in Tables 5-7, we have historical statistics for 1981-85.

When averaging exchange rate volatilities over pairs of countries, we include the volatility of a country's exchange rate with itself. That volatility is always zero, so the average exchange rate volatilities in Tables 4 and 7 are lower than the averages of the positive numbers in Tables 2 and 5.

The excess returns in Tables 3 and 6 are averages across countries of the world market return minus the interest rate in that country. They differ between countries because of differences in exchange rate movements. The excess returns are not national market returns. For example, in 1987 the Japanese market did better than the US market, but the world market portfolio did better relative to interest rates in the US than in Japan.

Exchange rate volatility contributes to average stock market volatility; as we shall see, it even contributes to the average return on the world market. Thus, for consistency, both μ_m and σ_m^2 should be greater than $\frac{1}{2}\sigma_e^2$.

Looking at Tables 4 and 7, here is one way to create inputs for the formula. The average excess return on the world market was 3% in the earlier period and 11% in the later period. A possible estimate for the future is 8%. The world market volatility was higher in the more recent period, but that included the crash, so we may want to use the 15% from the earlier period. The average exchange rate volatility of 10% in the earlier period may also be a better guess for the future than the more recent 8%.

Thus some possible values for the inputs are:

μ_m	8%
σ_m	15%
σ_e	10%

With these inputs, the fraction hedged comes to 77%.

For comparison, let's see what happens when we use the historical averages from either the earlier period or the later period in the formula.

	1981-85	1986-88
μ_m	3	11
σ_m	15	18
σ_e	10	8

With the historical averages from the earlier period as inputs, the fraction hedged comes to 30%, while the historical averages from the later period give a fraction hedged of 73%.

We generally won't use straight historical averages, because they can vary so much. We want estimates of future volatilities for the formula. Taylor (1987) discusses some general methods for forecasting exchange rate volatilities.

DISCUSSION

Note that in this model, everything but the average world market risk premium h_m is exogenous. So we can take an equation like (47) as fixing h_m in terms of inputs λ, g_m, and g. Still, if we feel we can estimate h_m, we can use (47) in figuring an estimate for λ.

Why don't the equations like (47) involve means or covariances of exchange rate changes? Because the impact of the means exactly offsets the impact of the covariances.

Roughly, investors in country A can hedge their foreign investments in B only if investors in B hedge their foreign investments in A. Every trade has two sides.

Suppose that exchange rate risks are such that a hedge reduces portfolio risk for investors in A, but not for investors in B. Then investors in A will be willing to pay investors in B to take on a hedge. The mean exchange rate change will adjust until both sides are happy putting on the hedge.

In equilibrium, the expected exchange rate changes and the correlations between exchange rate changes and stock returns cancel one another.

In the same way, the Black-Scholes option pricing formula includes neither the underlying stock's expected return nor its beta. In equilibrium, they cancel one another.

The capital asset pricing model is similar. The optimal portfolio for any investor could depend on expected returns and volatilities for all available assets. In equilibrium, though, the optimal portfolio for any investor is a mix of the market portfolio with borrowing or lending. The expected returns and volatilities cancel one another (except for the market as a whole), so neither affects the investor's optimal holdings.

In the end, investors in A will hedge because it reduces risk, even though it also reduces expected return; while investors in B will hedge because it increases expected return, even though it also increases risk.

I am surprised by the ease with which we can aggregate in this model. Even though people differ in wealth, risk tolerance, and consumption good, we take simple weighted averages of volatility, expected excess return, and risk tolerance across investors.

TABLE 1

Capitalizations and Capitalization Weights

	Domestic companies listed on the major stock exchange as of December 31, 1987[†]		Companies in the FT-Actuaries World Indices^{TM} [‡] as of December 31, 1987 [§]	
	Capitalization (US $ billions)	Weight (%)	Capitalization (US $ billions)	Weight (%)
Japan	2700	40	2100	41
US	2100	31	1800	34
UK	680	10	560	11
Canada	220	3.2	110	2.1
Germany	220	3.2	160	3.1
France	160	2.3	100	2.0
Australia	140	2.0	64	1.2
Switzerland	130	1.9	58	1.1
Italy	120	1.8	85	1.6
Netherlands	87	1.3	66	1.3
Sweden	70	1.0	17	.32
Hong Kong	54	.79	38	.72
Belgium	42	.61	29	.56
Denmark	20	.30	11	.20
Singapore	18	.26	6.2	.12
New Zealand	16	.23	7.4	.14
Norway	12	.17	2.2	.042
Austria	7.9	.12	3.9	.074
Total	6800	100	5300	100

[†] "Activities and Statistics: 1987 Report" by Federation Internationale des Bourses de Valeurs (page 16).

[‡] The FT-Actuaries World Indices^{TM} are jointly compiled by The Financial Times Limited, Goldman, Sachs & Co., and County NatWest/Wood Mackenzie in conjunction with the Institute of Actuaries and the Faculty of Actuaries.

[§] Here we exclude Finland, Ireland, Malaysia, Mexico, South Africa, and Spain.

TABLE 2
Exchange Rate Volatilities for 1986-88

	Japan	US	UK	Canada	Germany	France	Australia	Switzerland	Italy	Netherlands	Sweden	Hong Kong	Belgium	Denmark	Singapore	New Zealand	Norway	Austria
Japan	0	11	9	12	7	7	14	7	8	7	7	11	9	8	10	17	9	8
US	11	0	11	5	11	11	11	12	10	11	8	4	11	11	6	15	10	11
UK	9	10	0	11	8	8	14	9	8	8	7	11	9	8	10	16	9	9
Canada	12	5	11	0	12	11	12	13	11	11	9	6	12	11	8	15	10	12
Germany	7	11	8	12	0	3	15	4	3	2	5	11	6	4	10	17	8	5
France	7	11	8	11	2	0	14	5	3	3	5	11	6	4	10	17	7	5
Australia	14	11	14	12	14	14	0	15	14	14	12	11	14	14	12	14	14	14
Switzerland	7	12	9	13	4	5	15	0	5	5	7	12	8	6	11	18	9	7
Italy	8	10	8	11	3	3	14	5	0	3	5	11	6	4	10	17	7	5
Netherlands	7	11	8	11	2	3	14	5	3	0	5	11	6	4	10	17	7	5
Sweden	7	8	7	9	5	5	12	7	5	5	0	8	6	4	8	16	6	5
Hong Kong	11	4	11	6	11	11	11	12	10	11	8	0	11	11	5	14	10	11
Belgium	9	11	9	12	6	6	14	8	6	6	6	11	0	6	10	17	8	6
Denmark	8	11	8	11	4	4	14	6	4	4	4	11	6	0	10	17	7	5
Singapore	10	6	10	8	10	10	12	11	10	10	8	5	10	10	0	15	10	10
New Zealand	17	15	16	15	17	17	14	18	17	17	15	14	17	17	15	0	16	17
Norway	9	10	9	10	7	7	13	9	7	7	5	10	8	7	10	16	0	7
Austria	8	11	9	12	5	5	15	7	5	5	5	11	6	5	10	17	8	0

TABLE 3
World Market Excess Returns and Return Volatilities in Different Currencies for 1986-88

	Excess Return			Return Volatility		
Currency	1986	1987	1988	1986	1987	1988
Japan	8	-12	21	14	26	15
US	29	12	14	13	25	11
UK	23	-14	16	14	26	15
Canada	26	4	5	14	24	11
Germany	8	-5	30	15	27	14
France	11	-7	27	14	26	14
Australia	23	-2	-6	19	25	14
Switzerland	8	-8	36	15	27	15
Italy	2	-6	23	15	27	14
Netherlands	8	-7	30	15	27	14
Sweden	16	-6	19	13	25	13
Hong Kong	30	13	17	13	25	11
Belgium	7	-8	28	15	27	14
Denmark	8	-10	26	15	27	14
Singapore	36	6	16	12	25	12
New Zealand	15	-22	13	20	29	14
Norway	19	-11	15	14	26	12
Austria	7	-6	30	15	27	14

TABLE 4
World Average Values for 1986-88

	Excess Return	Return Volatility	Exchange Rate Volatility
1986	17	14	9
1987	-3	26	8
1988	18	13	8
1986-88	11	18	8

TABLE 5
Exchange Rate Volatilities for 1981-85

	Japan	US	UK	Canada	Germany	France	Australia	Switzerland	Italy	Netherlands
Japan	0	12	13	11	10	10	12	11	9	10
US	11	0	12	4	12	13	11	13	10	12
UK	12	13	0	12	10	11	14	12	11	10
Canada	11	4	11	0	11	12	10	12	10	11
Germany	10	12	10	12	0	5	13	7	5	2
France	10	13	11	12	4	0	12	8	5	5
Australia	12	10	13	10	12	12	0	13	11	12
Switzerland	11	14	12	13	7	8	14	0	8	7
Italy	9	10	11	10	5	5	12	8	0	5
Netherlands	10	12	10	11	2	5	12	7	5	0

TABLE 6
World Market Excess Returns
and Return Volatilities
in Different Currencies for 1981-85

Currency	Excess Return	Return Volatility
Japan	3	17
US	-1	13
UK	10	16
Canada	2	13
Germany	8	15
France	7	16
Australia	7	18
Switzerland	9	16
Italy	4	15
Netherlands	8	15

TABLE 7
World Average Values
for 1981-85

Excess Return	Return Volatility	Exchange Rate Volatility
3	15	10

REFERENCES

Adler, Michael, and Bernard Dumas. "International Portfolio Choice and Corporation Finance: A Synthesis." *Journal of Finance* 38 (June, 1983), pp. 925-984.

Adler, Michael, and Bernard Dumas. "Exposure to Currency Risk: Definition and Measurement." *Financial Management* 13 (Summer, 1984), pp. 41-50.

Adler, Michael, and David Simon. "Exchange Risk Surprises in International Portfolios." *Journal of Portfolio Management* 12 (Winter, 1986), pp. 44-53.

Black, Fischer. "Mean Reversion and Consumption Smoothing." Goldman, Sachs & Co., (March, 1989).

Breeden, Douglas T. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." *Journal of Financial Economics* 7 (September, 1979), pp. 265-296.

Edwards, Sebastian. "Real Exchange Rate Variability: An Empirical Analysis of the Developing Countries Case." *International Economic Journal* 1 (Spring, 1987), pp. 91-106.

Edwards, Sebastian. "Real Exchange Rate Behavior in Developing Countries: The Cross Country Evidence." UCLA Working Paper No. 510, (September, 1988).

Eun, Cheol S., and Bruce G. Resnick. "Exchange Rate Uncertainty, Forward Contracts, and International Portfolio Selection." *Journal of Finance* 43 (March, 1988), pp. 197-215.

Frankel, Jeffrey A. "The Implications of Mean-Variance Optimization for Four Questions in International Macroeconomics." *Journal of International Money and Finance* 5 (March, 1986), pp. 53-75.

Frankel, Jeffrey A. "Recent Estimates of Time-Variation in the Conditional Variance and in the Exchange Risk Premium." *Journal of International Money and Finance* 7 (May, 1988), pp. 115-125.

Friend, Irwin, and Marshall Blume. "The Demand for Risky Assets." *American Economic Review* 65 (December, 1975), pp. 900-922.

Grauer, Frederick L.A., Robert H. Litzenberger, and Richard E. Stehle. "Sharing Rules and Equilibrium in an International Capital Market Under Uncertainty." *Journal of Financial Economics* 3 (June, 1976), pp. 233-256.

Krugman, Paul. "Consumption Preferences, Asset Demands, and Distribution Effects in International Finance Markets." National Bureau of Economic Research Working Paper No. 651 (March, 1981).

McCulloch, J. Huston. "Operational Aspects of the Siegel Paradox." *Quarterly Journal of Economics* 89 (February, 1975), pp. 170-172.

Mehra, Rajnish, and Edward C. Prescott. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (March, 1985), pp. 145-161.

Merton, Robert C. "On the Mathematics and Economic Assumptions of Continuous-Time Financial Models," in *Financial Economics: Essays in Honor of Paul Cootner*, W.F. Sharpe and C.M. Cootner (eds.), Prentice Hall, 1982, pp. 19-51.

Perold, André F., and Evan C. Schulman. "The Free Lunch in Currency Hedging: Implications for Investment Policy and Performance Standards." *Financial Analysts Journal* 44 (May, June 1988), pp. 45-50.

Roll, Richard, and Bruno Solnik. "A Pure Foreign Exchange Asset Pricing Model." *Journal of International Economics* 7 (February, 1977), pp. 161-179.

Roper, Don E. "The Role of Expected Value Analysis for Speculative Decisions in the Forward Currency Market." *Quarterly Journal of Economics* 89 (February, 1975), pp. 157-169.

Ross, Stephen A., and Michael M. Walsh, "A Simple Approach to the Pricing of Risky Assets with Uncertain Exchange Rates." in *Research in International Business and Finance*, Vol. 3, R.G. Hawkins, R.M. Levich, and C.G. Wihlborg (eds.), JAI Press Inc., 1983, pp. 39-54.

Sercu, Piet. "A Generalization of the International Asset Pricing Model." *Revue de l'Association Francaise de Finance* 1 (Juin, 1980), pp. 91-135.

Siegel, Jeremy J. "Risk, Interest Rates, and the Forward Exchange." *Quarterly Journal of Economics* (May, 1972), pp. 303-309.

Siegel, Jeremy J. "Reply - Risk, Interest Rates, and the Forward Exchange." *Quarterly Journal of Economics* (February, 1975), pp. 173-175.

Solnik, Bruno H. "An Equilibrium Model of the International Capital Market." *Journal of Economic Theory* 8 (August, 1974), pp. 500-524.

Stulz, René. "A Model of International Asset Pricing." *Journal of Financial Economics* 9 (December, 1981), pp. 383-406.

Taylor, Stephen J. "Forecasting the Volatility of Currency Exchange Rates." *International Journal of Forecasting* 3 (April, 1987), pp. 159-170.

Thomas, Lee R. III. "Currency Risks in International Equity Portfolios." *Financial Analysts Journal* 44 (March-April, 1988), pp. 68-71.

Trevor, Robert G. "The Micro-Foundations of International Asset Demands with Endogenous Exchange Rates." Reserve Bank of Australia Working Paper, (August, 1986).