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DEMAND VARIABILITY, SUPPLY SHOCKS AND THE OUTPUT-INFLATION TRADEOFF

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ABSTRACT

This paper examines the shift in the relation between the inflation rate and the rate of growth of real output which has occurred in the United States over the past three decades, and attempts to assess the relative importance of three possible lines of explanation: a) the new classical view of the output-inflation tradeoff, initially specified by Lucas; b) the effect of supply-side shocks, such as energy prices; c) the effect of inflation variability on the natural rate of real output, as hypothesized by Milton Friedman. The paper concludes that b) and c) seem to have played a significant role in the observed shift from a positive to a negative correlation between the rate of inflation and the rate of real output growth, but that a) did not.

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I. Introduction

Over the past three decades there has been a shift in the relation between the rate of inflation and the unemployment rate, and analogously, between the rate of inflation and the rate of change of real output. As pointed out by Milton Friedman (1977), as this shift evolved, the traditional Phillips curve view that inflation and unemployment are inversely related was replaced by the "natural rate" or "accelerationist" or "expectations adjusted Phillips curve" hypothesis (as it is variously called), which in turn has had to give ground to the more recent empirical phenomenon of an apparent positive relation between inflation and unemployment, or a negative relation between inflation and the rate of change of real output. In the United States, while the rate of inflation and the unemployment rate were negatively correlated for the period 1957 to 1968 (-.86), for the period 1969 to 1980 the correlation between these variables was positive (.44). Similarly, the correlation between the rate of inflation and the rate of change of real output was positive from 1957 to 1968 (.16), and negative over the period 1969 to 1980 (-.65).[1] The purpose of this paper is to sort out and assess empirically the possible contribution of each of the following in explaining the shift in the relation between the rate of inflation and the rate of change of real output:

a) the new classical view of the output-inflation tradeoff, initially specified by Lucas (1972, 1973);

b) the effect of supply-side shocks, particularly energy price shocks;

c) the effect of inflation variability on the natural rate of real output, hypothesized by Friedman (1977).

The new classical view hypothesizes that only unanticipated changes in aggregate demand would affect real output. Furthermore, within the new classical view the response of real output to unanticipated changes in aggregate demand is specified to be inversely related to the variability of inflation and aggregate demand. Hence in the new classical view a deteriorating output-inflation tradeoff can be explained by increased variability of inflation and aggregate demand.

Other things equal, supply-side shocks such as the dramatic increases in the price of energy would be expected to cause an increase in the rate of inflation and a decrease in the rate of change of real output. Hence the energy price increases of the 1970s might account, at least in part, for the observed shift from a positive to a negative relation between the rate of inflation and the rate of change of real output.

Friedman's view (1977, pp. 464-468) essentially is that, due to institutional rigidities, increasing variability of the rate of inflation causes a reduction in the efficiency of the price system in guiding economic activity, hence a possible increase in the unemployment rate and a reduction in the rate of change of real output. Since high inflation rates tend to be associated with greater inflation variability this is reflected in the data as a positive relationship between the inflation rate and unemployment, or a negative relationship between inflation and the rate of change of real output.

The model constructed here incorporates each of the above influences. The model, to be described in Section II, is generically related to the Lucas-type model (1973) as amended by Cukierman and Wachtel (1979). This Lucas-type model is further modified to explicitly incorporate the effects of supply-side factors, in particular energy prices, as well as to allow for a variable natural rate of output as suggested by Milton Friedman's analysis referred to above. Section III of the paper presents estimates of the model for the United States for the years 1959-1980. Section IV examines the implications of these estimates concerning the causes of the change in the output-inflation relationship. Section V contains concluding comments.

II. Model Specification

As noted above, the model used in this study is generically related to the Lucas-type model (1973) as amended by Cukierman and Wachtel (1979). These models incorporate the new classical view (a) in the introduction, but do not explicitly allow for influences (b) and (c) -- effects of supply-side shocks and the effects of inflation variability on the natural rate of output. In this section we consider first how supply shocks can be incorporated into the model.[2] We then consider the modifications of the model that are required to allow for variability in the natural rate as suggested by Milton Friedman's analysis.

II.1 Supply-Side Shocks

Following Lucas we assume the economy consists of a large number, m, of "scattered, competitive markets" (Lucas, 1973, p. 327). We derive output supply schedules for each of these markets. Then we specify the demand side of the model and describe expectations formation. Finally, we derive the aggregate output equation for the model. In this discussion we assume that the supply shock comes from a change in the price of the energy input to the production process. For estimation, as explained in Section III we employ an energy price as well as an import price measure of supply shocks.

II.1.a Market Supply Equations

The supply equations derived here are based on factor demand equations for energy and labor as well as labor supply functions at the individual market level. (For a more detailed derivation, see the appendix, section A.I.) The equations are for the short run -- that is, the capital stock is taken as given.

The factor demand equations take the following form:

$$\begin{bmatrix} Q_t(v) \\ N_t(v) \end{bmatrix} = \begin{bmatrix} a_{10} a_{11} a_{12} a_{13} a_{14} \\ a_{20} a_{21} a_{22} a_{23} a_{24} \end{bmatrix} \begin{bmatrix} 1 \\ p_t(v) \\ W_t(v) \\ q_t(v) \\ K_t(v) \end{bmatrix} , \quad v = 1,\ldots,m \tag{1}$$

where v indexes the market and for each market,

$Q_t(v)$ = quantity of energy; $N_t(v)$ = number of labor hours;

$p_t(v)$ = market specific producer price; $W_t(v)$ = money wage;

$q_t(v)$ = price of energy; $K_t(v)$ = quantity of capital

where all variables are in logs.

Equations (1) express the demands for energy and labor as functions of product and factor prices. These functions are derived on the assumption that firms maximize profit subject to the production function constraint. The log linearity of equations (1) can be viewed as deriving from the assumption that the production function is Cobb-Douglas or, more generally, as an approximation to factor demand equations based on production functions of the generalized CES type.[3]

The supply of labor to firms in a specific market is taken to be a log linear function of the money wage in that market and laborers' expectation of the general price level p_t^* (conditioned on information in market v).

$$N_t(v) = d_0 + d_1 p_t^* + d_2 W_t(v) \,. \tag{2}$$

Equation (2) indicates laborers know the market-specific money wage but must form an expectation of the economy-wide aggregate price level. Laborers' formulation of expected price p_t^* will be modeled below.

The labor supply function (2) is used to substitute $W_t(v)$ out of equations (1). We can then express the quantities of labor and energy as functions of product price, laborers' expectations of the aggregate price level, the price of energy, and the capital stock,

$$\begin{bmatrix} Q_t(v) \\ N_t(v) \end{bmatrix} = \begin{bmatrix} b_{10} b_{11} b_{12} b_{13} b_{14} \\ b_{20} b_{21} b_{22} b_{23} b_{24} \end{bmatrix} \begin{bmatrix} 1 \\ p_t(v) \\ p_t^* \\ q_t(v) \\ K_t(v) \end{bmatrix}, \quad v = 1,\dots,m. \qquad (3)$$

The supply function for market v is derived by substituting equations (3) into the production function. The resulting supply functions are

$$y_t(v) = g_0 + g_1 p_t(v) + g_2 p_t^* + g_3 q_t(v) + g_4 K_t(v) \;, \qquad (4)$$

where g_1, $g_4 > 0$ and g_2, $g_3 < 0$.

II.1.b Demand and Expectations Formation

On the demand side of the model, following Cukierman and Wachtel (1979), market demand can be specified as

$$p_t(v) = x_t + w_t(v) - y_t(v) \qquad (5)$$

here $w_t(v)$ is the market specific demand shock, $y_t(v)$ is market specific real output, and x_t is economy-wide aggregate demand. (Again all variables are in logs.) Cukierman and Wachtel take x_t to be nominal income, assuming the aggregate demand curve to be unit elastic -- as did Lucas (1973).[4]

The expectation of economy-wide aggregate price p_t^* is modeled in a manner consistent with the way actual price is determined in the model. The information conditioning expectations in market v is assumed to be the current market specific product price $p_t(v)$, the distributions of market specific and aggregate demand shocks, and the lagged values of aggregate demand. The market specific demand shock $w_t(v)$ and the aggregate demand shock are assumed to be distributed as follows.

$$w_t(v) \sim N\ (0,\ \sigma_w^2) \qquad (6)$$

$$x_t = x_{t-1} + \Delta x_t,\ \Delta x_t \sim N\ (\delta, \sigma_x^2) \qquad (7)$$

We assume that current market specific product price is used together with aggregate information to form an optimal expectation of the aggregate price p_t^*. This optimal expectation is given by

$$p_t^* = (1 - \theta)\ p_t(v) + \theta\bar{p}_t \qquad (8)$$

where $\bar{p}_t$ is the expectation of aggregate price conditional on information prior to time period t, i.e., conditional on available aggregate information; where θ will be seen to be a function of the variances of market specific and aggregate demand shocks as well as other variances and parameters to be introduced below (see below, equation 15; also section A.IV of the Appendix); and where there is a separate equation (8) for each market (conditioned on the individual $p_t(v)$).

To find $\bar{p}_t$ we equate market supply, equation (4), and demand, equation (5), substituting equation (7) for x_t and equation (8) for p_t^* to obtain equilibrium $p_t(v)$. (For details on this and what follows, see appendix section A.II). Next we aggregate the equilibrium expression for $p_t(v)$ across markets to obtain equilibrium p_t. Taking the expectation of p_t conditional on information through period t - 1 yields the following expression for $\bar{p}_t$,

$$\bar{p}_t = \delta + x_{t-1} - g_0 - g_4K_t - g_3\phi(t) \qquad (9)$$

where K_t is the aggregate capital stock and $\phi(t)$ will be defined presently. To derive (9) we have made the following assumptions concerning the market specific and aggregate energy prices,[5]

$$q_t(v) = q_t + \eta_t(v) \qquad (10)$$

$$q_t = p_t + \phi(t) + \mu_t \qquad (11)$$

where: $q_t(v)$ is the market-specific energy price, q_t is the economy-wide aggregate energy price, and $\eta_t(v)$ is the market-specific energy price disturbance; p_t is the aggregate output price, $\phi(t)$ is a linear time trend in the relative price of energy, and μ_t is the aggregate energy price disturbance; with

$$\eta_t(v) \sim N(0, \sigma_\eta^2) \quad \text{for all } v, \tag{12}$$

$$\mu_t \sim N(0, \sigma_\mu^2) \tag{13}$$

and $\eta_t(v)$ and μ_t are independently distributed and serially uncorrelated.[6,7]

II.1.c Aggregate Output

We can now derive the aggregate output equation. (For details of the derivations in this section see section A.III of the appendix). Equations (5), (8), and (9) are used to eliminate $p_t(v)$ and p_t^* from the market-specific supply equation (4). Aggregating across markets the resulting aggregate output equation is

$$y_t = g_0 - \frac{g_2\theta}{1-g_2\theta}(\Delta x_t - \delta) + \frac{g_3}{1-g_2\theta}\mu_t + g_3\,\phi(t) + g_4K_t \tag{14}$$

where in the derivation of (14) we make use of the facts that $q_t = p_t + \phi(t) + \mu_t$ and that by definition $p_t = x_t - y_t$.

II.2 The Terms of the Output-Inflation Tradeoff in the Extended Model

Equation (14) indicates that the determinants of output y_t are: the difference between the actual change in nominal income Δx_t and the expected change in nominal income δ, the aggregate demand shock; the aggregate energy price disturbance μ_t, the aggregate supply shock in the model; the time trend in the relative price of energy $\phi(t)$; and the aggregate capital stock.

The coefficients in (14) are functions of supply equation parameters (the g's) and the parameter θ which characterizes the information structure of the model. That is, θ can be shown to be a function of the variances of economy-wide and market-specific disturbances[8]

$$\theta = \frac{\frac{\sigma_w^2+g_3^2\sigma_\eta^2}{B}}{\frac{\sigma_x^2+g_3^2\sigma_\mu^2}{A} + \frac{\sigma_w^2+g_3^2\sigma_\eta^2}{B}}, \qquad (15)$$

where $A = (1-g_2\theta)^2$ $\quad B = (1-g_2\theta-g_3)^2$

Although (15) is not an explicit expression for θ it can be shown by use of the implicit function theorem (see section A.IV of the appendix) that θ is an increasing function of the market-specific variances (σ_w^2 and σ_η^2) and a decreasing function of the variances of the aggregate demand and supply disturbances (σ_x^2 and σ_μ^2, respectively). Since θ is a function of these variances the coefficients in (14) which characterize the real output response to aggregate demand and aggregate supply shocks, the coefficients which contain θ, will also depend on these market-specific and aggregate variances.

II.2.a Twisting the Tradeoff Curve

Inspection of the coefficients in (14) indicates that the response of real output to aggregate demand shocks is a declining function of the variability of the aggregate demand shock and an increasing function of the variability of market-specific demand disturbances, a result analogous to that of previous Lucas-type models. When the model is extended to include supply shocks, it can also be seen from inspection of the coefficients of equation (14) that the real output response to an aggregate demand shock is also a declining function of the variability of aggregate supply shocks and an increasing function of the variability of market-specific supply shocks. Hence in our framework the real output response to aggregate demand shocks is a function of the variability of both demand and supply-side shocks.

This is illustrated in Figure 1 where the aggregate demand shock (the change in nominal income) Δx_t is plotted on the vertical axis and real output y_t on the horizontal axis. The slope of the tradeoff curve T equals the inverse of the coefficient on $(\Delta x_t - \delta)$ in equation (14). The natural rate of output in t, $y_{n,t}$, equals $g_0 + g_3\phi(t) + g_4K_t$ in (14), and this determines the location of the vertical axis in Figure 1. The mean δ of Δx is the intercept of the tradeoff curve on the vertical axis, with μ_t assumed equal to zero. Along a given tradeoff curve such as T_0, an aggregate demand shock such as $\Delta x_t'$ would cause real output to increase from $y_{n,t}$ to y_{t0}. Associated with the increase in real output would be an increase in the economy's price level. That the price level will also rise as the result of a positive shock to aggregate demand can be seen in the figure by noting that the tradeoff curve (T_0) is drawn to the left of the 45° line; the coefficient on Δx_t, the change in the log of nominal income, is less than one (the inverse of the coefficient is greater than one). The portion of the increase in nominal income which does not go into increased real output goes into an increase in the price level. Therefore, movements along a given tradeoff curve in response to aggregate demand shocks will give rise to a <u>positive</u> association between price changes and changes in real output.

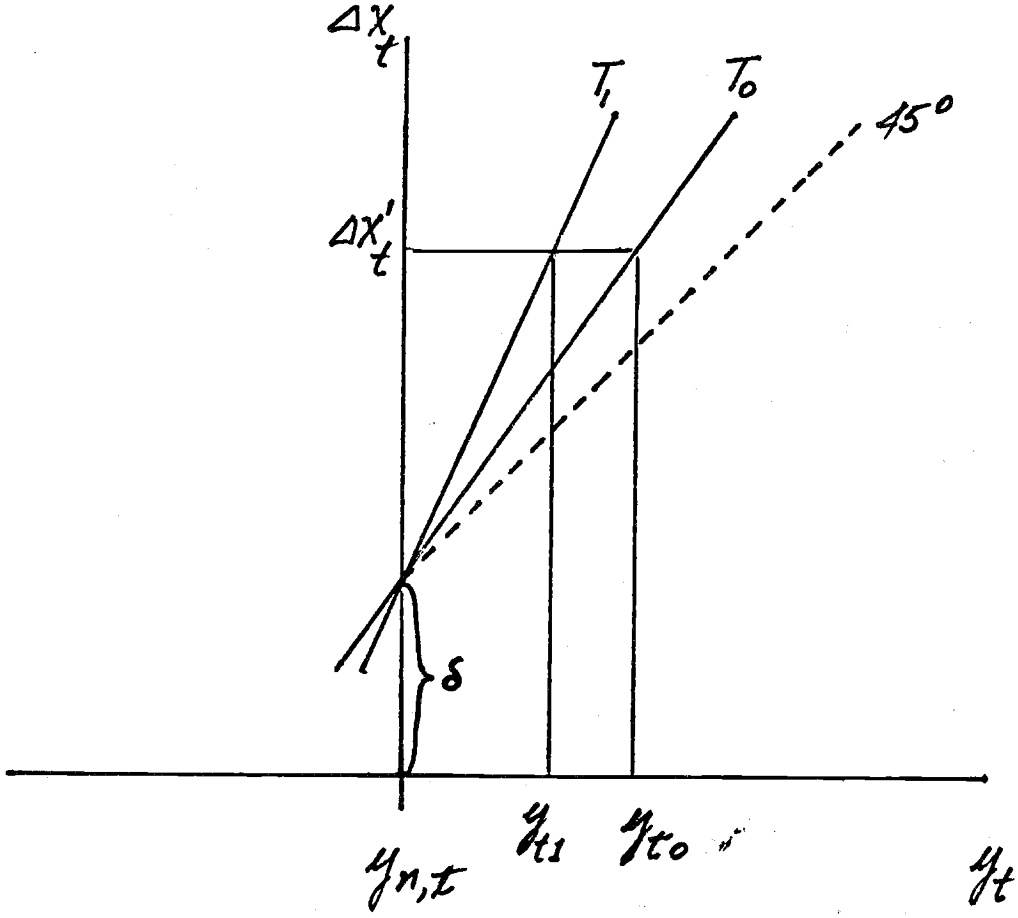

Figure 1

If the variability of aggregate demand and/or aggregate supply shocks increases, the tradeoff curve twists counterclockwise to a position such as T_1. Due to this twist in the tradeoff curve a given value of the aggregate demand shock $\Delta x_t'$ (and therefore a given nominal income $x_{t-1} + \Delta x_t'$) would now correspond to a lower level of real output (y_{t1}) and consequently a higher price level. Clearly, this twisting of the tradeoff curve would result in a deterioration in the terms of the output-inflation tradeoff. To explain the observed emergence of a negative correlation between price changes and output changes within the new classical view it is necessary to argue that this twisting effect (which does produce changes in real output which are in the opposite direction from the associated price changes) dominates movements along a given tradeoff schedule (which produce positively associated movements in price and real output).

II.2.b The Effect of Supply-Side Shocks

While the variability of supply shocks affects the output-inflation tradeoff through θ, supply shocks also have a direct effect on aggregate real output and the price level. For example, a positive shock μ_t in the energy price will cause real output to decline [since $g_3/(1-g_2\theta)$ in (14) is negative]; given aggregate demand the positive energy price shock will cause the price level to rise. Furthermore the size of the output response to μ_t will be an <u>increasing</u> function of the variability of both the aggregate demand and the aggregate supply shocks, and a decreasing function of the variability of market-specific demand and supply shocks. Aggregate and market-specific demand and supply variability affect the output response to μ_t because they affect the size of θ. Increases (decreases) in the variability of aggregate supply or demand shocks will lower (raise) θ and hence raise (lower) the absolute value of the supply response coefficient ($g_3/(1-g_2\theta)$ in (14)),while increases (decreases) in the variability of market-specific demand and supply shocks will raise (lower) θ and lower (raise) the absolute value of the supply response coefficient. The economic interpretation of the relationships between output responses to aggregate supply and demand shocks and the variability of aggregate and market-specific shocks may be clarified by reference to aggregate demand and supply curves in aggregate price and output space. Increases in the variability of aggregate shocks, whether on the supply or demand side, will cause the aggregate supply curve to become more steeply sloped: with the effect that a given aggregate demand shock, represented by a horizontal shift in the aggregate demand curve along the aggregate supply curve, will cause output to change less; and the effect that a given aggregate supply shock, represented by a horizontal shift in the aggregate supply curve along the aggregate demand curve, will cause output to change more

The direct effect of supply shocks on the relationship between real output and price level changes is illustrated in terms of the tradeoff curve in Figure 2. As noted before, aside from Δx_t, the position of the tradeoff curve is determined by the other parameters and variables in (14), in particular the supply shock μ_t, which was earlier assumed to be zero. Since the coefficient on μ_t is negative in (14), an increase in μ_t will cause real output y_t to decline and the economy's price level to rise, given Δx_t. In terms of Figure 2, an

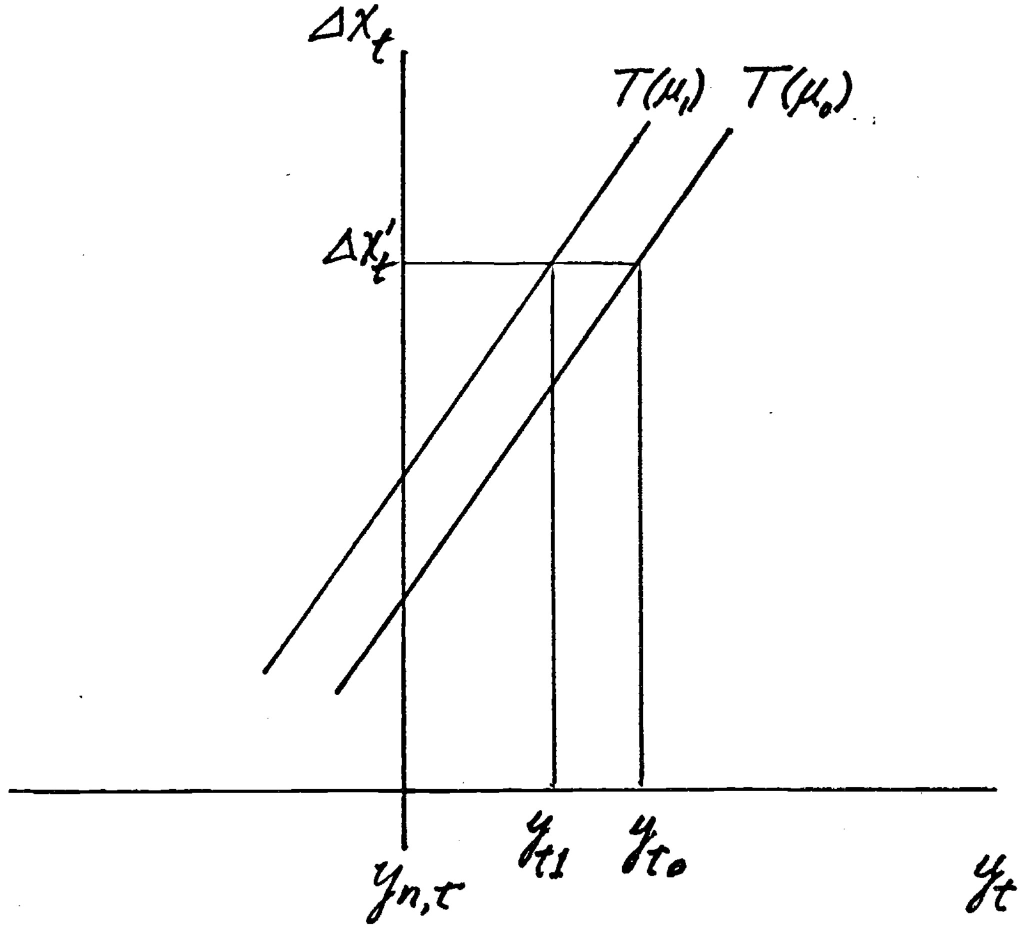

Figure 2

increase in μ_t from μ_0 to μ_1 shifts the tradeoff curve leftward from $T(\mu_0)$ to $T(\mu_1)$ so that for a given level of the aggregate demand shock, $\Delta x_t'$, real output falls from y_{t0} to y_{t1}. Given $\Delta x_t'$ (the change in nominal demand), the economy's price level must rise. Therefore supply shocks give rise to an observed negative correlation between price and real output changes, other things the same. Moreover, the greater the variability of the aggregate demand and/or the aggregate supply shocks, the larger the coefficient on μ_t in (14) and the larger the shift in the tradeoff curve in Figure 2 in response to a given supply shock μ_t.

II.3 Aggregate Demand and Supply Variability and the Natural Rate of Output

Milton Friedman's (1977) analysis suggests that because of rigidities due to political and institutional arrangements, high variability of inflation -- whether caused by variability of aggregate demand or supply -- would lead to a loss of efficiency in the price system and a likely rise in unemployment. Since high inflation rates and greater inflation variability have tended to go together, Friedman would expect this positive relationship between inflation variability and unemployment to be reflected in the data by a positive association between the level of the inflation rate and level of the unemployment rate. It also appears logical that a rise in the unemployment rate will imply a decline in real output and we would therefore observe a negative correlation between the rate of inflation and rate of growth in real output.[9] It should be noted, however, that all Friedman suggests is that the positive (negative) relationship between inflation variability and unemployment (output) "seems plausible." He does not argue that such a relationship follows as a necessary implication of a theory;[10] rather Friedman's view is an empirical proposition.

Consider how we might incorporate Friedman's view within our model. Along lines similar to Lucas (1973) or Cukierman and Wachtel (1979) we can divide the factors which influence output in equation (14) into those affecting the natural rate ($y_{n,t}$) and those causing cyclical fluctuations around the natural rate ($y_{c,t}$)

$$y_t = y_{n,t} + y_{c,t} \tag{16}$$

$$y_{n,t} = g_0 + g_3\,\phi(t) + g_4 K_t \tag{17}$$

$$y_{c,t} = \frac{-g_2\theta}{1-g_2\theta}(\Delta x_t - \delta) + \frac{g_3}{1-g_2\theta}\mu_t \tag{18}$$

Lucas's or Cukierman and Wachtel's specification of the natural rate

$$y_{n,t} = a + bt \tag{16'}$$

would result if we made the further assumption that the relative price of energy as well as K_t (the log of the capital stock) follow a linear time trend.

Friedman's analysis suggests that, in addition, the natural rate of output will depend on the variability of inflation. Within our model the variability of inflation will depend on the variability of aggregate demand and supply.[11] Therefore Friedman's analysis suggests the following specification of the natural rate

$$y_{n,t} = a + bt + \alpha_1\sigma^2_{x,t} + \alpha_2\sigma^2_{\mu,t} \qquad \alpha_1 < 0 \qquad \alpha_2 < 0 \tag{16''}$$

where $\sigma^2_{x,t}$ and $\sigma^2_{\mu,t}$ are the time-dependent variances of the aggregate demand and supply shocks respectively, empirical measures of which will be described below. An increase in the variability of aggregate demand or aggregate supply would lead to an increase in the variability of the inflation rates and hence would result in a decline in the natural rate of output according to Friedman's view.

The way that the variability of the inflation rate affects the relationship between price level changes and real output level changes via the natural rate of output in our model is illustrated in Figure 3. If the natural rate of output

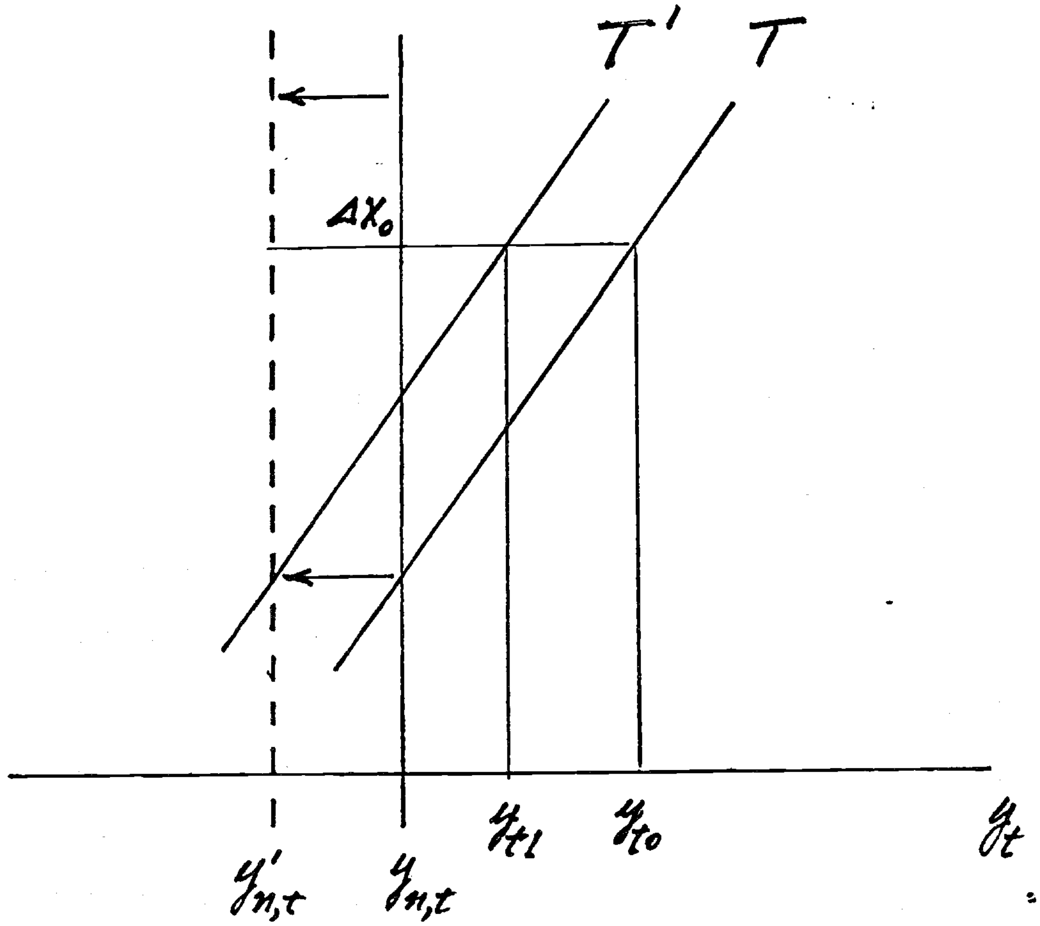

Figure 3

is an inverse function of the variability of the inflation rate, as Friedman suggests, then an increase in that variability will cause the natural rate of output to fall from $y_{n,t}$ to $y'_{n,t}$ say, as indicated by the leftward shift of the vertical axis in Figure 3. Likewise, the tradeoff curve shifts leftward from T to T' along with the vertical axis. Suppose that previous to the decline in the natural rate, an aggregate demand shock Δx_0 would give rise to a real output level y_{t0}. After the decline, the real output level corresponding to Δx_0 would be the lower level y_{t1}. Therefore, after the decline in the natural rate, the price level corresponding to Δx_0 would be higher. Hence <u>increases in inflation variability that cause the natural rate of output to decline will give rise to an observed negative correlation between price and real output level changes, all other things equal</u>.

III. Empirical Specification and Initial Model Estimates

III.1 Empirical Specification

This subsection explains some details of the empirical specification of the independent variables in equations (14) and (16").

III.1.a Measures of the Variability of Aggregate Demand and Supply

If it could be assumed that aggregate demand and supply variability had been constant over time, then the variance terms, σ_x^2, σ_μ^2, would simply form a constant term in equation 14 (or 16"). If instead we assumed that there were discrete shifts in aggregate demand or supply variability between distinguishable subperiods, then the variance terms would still be subsumed in the constant terms of separate subperiod regressions. We will consider two subperiods, 1959-68 and 1969-80, which do appear quite different, at least in terms of supply variability, and we will make comparisons of our model estimates for the two subperiods. Still, it appears reasonable to believe that the variability of aggregate demand and supply have fluctuated within these subperiods as well as perhaps being characterized by a shift between subperiods. To be able to examine the effects of continuous movements in the variability of aggregate supply and demand we must first construct measures of such movements.

The variability of aggregate demand is measured in our model by the variance in the change in the log of nominal income (σ_x^2), the model's measure of the aggregate demand shock. The variance of aggregate supply is measured by the variance of the (detrended) relative price of energy, or alternatively of the (detrended) relative price of imports (σ_μ^2),[12] the model's measures of supply shocks. The difficulty is that we observe only one outcome from the distribution of nominal income growth and the energy (or import) price at each point in time and this alone is not enough to construct estimates of σ_x^2 and σ_μ^2 at each point in time. Hence, as a proxy for a time-varying σ_x^2 we construct a moving variance of actual changes in nominal income, $\hat{\sigma}_{x,t}^2$, and similarly as a proxy for a time-varying σ_μ^2 we construct a moving variance of the actual (detrended relative) energy (or import) price, $\hat{\sigma}_{\mu,t}^2$. The number of periods used to construct these moving variances is unavoidably an arbitrary choice. At each point in time we have computed a variance using observations from the past 8 periods (quarters), exclusive of the current period. We compute the moving variance at time t from data in past periods since we do not want our proxy to contain information unavailable to agents at time t.[13]

III.1.b Removing the Trend from the Growth Rate in Aggregate Demand

An examination of the data over our sample period (1957-80) revealed a statistically significant upward trend in Δx, the change in log of nominal income, rather than the constant mean (δ) specified in equation 7. To allow for this we respecify the aggregate demand shock as the detrended change in the log of nominal income $\Delta\tilde{x}$ (i.e. $\Delta x_t = \delta_0 + \delta_1 t + e_t$, $\Delta\tilde{x}_t$ being the estimate of the residual e_t, which is the unanticipated aggregate demand shock). The trend growth in Δx can be shown to have no effect on output within the structure of our model since such growth will be anticipated by rational economic agents. Therefore the effect of this specification change is simply that $\Delta\tilde{x}$ replaces $(\Delta x - \delta)$ in equation 14.

III.1.c Autocorrelation of the Supply Shock

The specification of the model in section II.1 was based on the assumption that the supply shock μ was serially uncorrelated. An examination of the data for μ, as measured by either the energy price or import price shock, revealed a significant pattern of first order autocorrelation. Consistent with the data, the (aggregate) energy price shock should be specified as

$$\mu_t = \xi\mu_{t-1} + \varepsilon_t \qquad 1 > \xi > 0 \qquad (19)$$

With this modification, the equation for the cyclical component of output would become[14]

$$y_{c,t} = \frac{g_2\theta}{1-g_2\theta}\,\tilde{\Delta}x_t + g_3\,\xi\mu_{t-1} + \frac{g_3}{1-g_2\theta}\,\varepsilon_t \qquad (18')$$

In contrast to the case of demand-side shocks, equation (18') implies that for a supply shock both the anticipated and unanticipated components will affect real output. In fact the effect of the anticipated portion of the supply shock will be larger in magnitude than for the unanticipated component ($g_3 > g_3/1-g_2\theta$ in absolute value in 18'). An anticipated supply shock, such as a rise in energy prices, will in addition to its direct affect on output also increase labor suppliers' expectation of the aggregate price level with consequent upward pressure on the money wage and, therefore, a further effect on output.

Since we would expect differential effects from the anticipated and unanticipated components of supply shocks, it would be preferable to split our measures of supply shocks into these two components ($\xi\mu_{t-1}$ and ε_t). We explain how this is done below, but in general our estimates of the output effects of supply shocks were not significantly affected by whether we made such a decomposition of supply shocks and the estimates presented are those where the whole supply shock is entered as an independent variable.

III.1.d <u>An Adjustment Lag</u>

As in Lucas's (1973) model we assume that there is persistence to movements in output due to adjustment lags. This persistence is represented by the inclusion of the lagged value of the dependent variable in our output equation. Rationales for such persistence are developed in Lucas (1975) and Sargent (1977).

III.2 <u>Initial Model Estimates</u>

With the modification to the specification of the aggregate demand shock and the addition of the lagged dependent variable, our model implies the following specification for detrended real output ($\tilde{y}_t$)

$$\tilde{y}_t = \alpha_1 \hat{\sigma}^2_{x,t} + \alpha_2 \hat{\sigma}^2_{\mu,t} - \frac{g_2\theta}{1-g_2\theta} \Delta\tilde{x}_t + \frac{g_3}{1-g_2\theta} \mu_t + \lambda \tilde{y}_{t-1} \qquad (20)$$

where we have included the composite supply shock in the equation.[15] [Results where the energy shock is decomposed into the expected part ($\xi\mu_{t-1}$) and unanticipated part (ε_t) are discussed below]. The dependent variable is detrended real output, but such deviations of output from trend have a different interpretation here than in Lucas (1973) or Cukierman and Wachtel (1979). In those studies the deviation of output from trend are the cyclical component of output. Here output may deviate from trend due to cyclical factors ($\Delta\tilde{x}$ and μ_t) or due to the effects of supply and demand variability on the natural rate of output.[16]

The data for this study cover the period 1957:I to 1980:IV. Since one quarter is lost in first differencing nominal income (Δx) and eight quarters will be used to create the proxies for the moving variances of aggregate demand and supply, for estimation our sample period is 1959:II - 1980:IV. In addition to this sample period we estimate equation 20 for two subperiods 1959:II - 1968:IV and 1969:I - 1980:IV. The breakpoint separates our sample period into an earlier subperiod of relatively low inflation variability and a later subperiod of higher inflation variability.

This increase in inflation variability can be seen from the last column of section A of Table 1 which gives the calculated variance of the inflation rate.[17] The ratio of the variance of the inflation rate in the second subperiod to that of the first subperiod ($\hat{\sigma}^2_{\Delta p2}/\hat{\sigma}^2_{\Delta p1}$) equals 2.42. The variances of aggregate demand ($\hat{\sigma}^2_x$) as measured by the variance of the change in the log of nominal income, and the variance of aggregate supply ($\hat{\sigma}^2_\mu$), as measured by the variance of the energy price shock in section A of Table 1 or the import price shock in section B of Table 1, indicate that it was an increase in the variability of the supply shock which was responsible for the rise in inflation variability. The ratio of the calculated variance of the aggregate demand shock in the second subperiod to that in the first subperiod ($\hat{\sigma}_{x2}/\hat{\sigma}^2_{x1}$) is .98. The ratio of the variance of the supply shock in the second subperiod to the same variance in the first subperiod ($\hat{\sigma}^2_{\mu2}/\hat{\sigma}^2_{\mu1}$) is 3.18 for the energy price measure and 2.25 for the import price measure.

The estimates of equation 20, as shown in Table 1, assume that the distribution of both aggregate demand and supply shocks have been constant over time; or where the estimates are for subperiods we presume that these distributions were constant over the subperiod but perhaps shifted between those subperiods. The moving variance terms in equation 20 are in this case subsumed in the constant term. The estimation technique is the modified "three pass least squares" procedure suggested by Wallis (1967) for equations which include the lagged value of the dependent variable in the presence of an autocorrelated error term. This procedure was used because preliminary ordinary least squares estimates showed evidence of significant first-order autocorrelation.

The model estimates in Table 1 indicate that both demand side influences, as measured by changes in nominal income, and supply-side factors, whether measured by the energy price variable (section A of Table 1) or the import price variable (section B of Table 1), were significant determinants of deviations of output from its time trend over this period.[18]

The estimates in Table 2 include the proxy measures for the changing variances of aggregate demand ($\hat{\sigma}^2_{x,t}$) and aggregate supply ($\hat{\sigma}^2_{\mu,t}$). According to Friedman's (1977) analysis we would expect there to be a negative relationship between these variances and the level of detrended real output. With the energy price measure of the supply shock (section A of Table 2), the estimate for the whole period, given in the first line of the table, shows a significant negative effect on output for the proxy for aggregate supply variability. The proxy for the variability of aggregate demand has the expected negative sign but is not significant ($t = -1.657$). The inclusion of the measure of supply variability in the equation also results in a decline in the size of the coefficient on the supply shock variable (μ_t) from 0.01248 (Table 1, section A) to 0.00541 and this coefficient is no longer significant at the 5% level ($t = -1.775$).

The estimates given in the second and third lines of section A of Table 2 reveal significant differences in the estimates for the two subperiods. The negative output effect for the variability of aggregate supply ($\hat{\sigma}^2_{\mu,t}$) is significant only in the second subperiod. In the first subperiod there is a significant negative effect for the variability of aggregate demand ($\hat{\sigma}^2_{x,t}$). Notice that the energy price supply shock μ_t is not statistically significant in the first subperiod, though it is in the second subperiod. The coefficient estimates which are based on the import price measure of the supply shock (Table 2, section B) reveal essentially the same pattern as those for the energy price measure, with the exception that for the whole period estimates using the import price measure the coefficient on the level of the supply shock (μ_t) remains significant when the proxies for aggregate supply and demand variability are included.

IV. Further Model Estimates and the Implications of the Results

In light of the results presented in Tables 1 and 2, we evaluate each of the influences a), b), c), as listed in the introduction, as separate determinants of the change in the nature of the output-inflation tradeoff.

a. The New Classical View

According to the new classical view we would expect the increase in inflation variability between the two subperiods to have caused the output response to an aggregate demand shock to decline; the coefficient on $\Delta\tilde{x}$ should be lower for the second subperiod than for the first subperiod. In terms of Figure 1 in Section II.2.a, the increase in inflation variability should have rotated the tradeoff curve in a counter-clockwise direction. Additionally as explained in Section II.2.b, we would expect the rise in aggregate price variability to have caused an increase in the size of the output response to a supply shock; the (negative) coefficient on the energy or import price variables (μ_t) in Tables 1 and 2 should be larger in absolute value for the second subperiod than for the first. In terms of Figure 2 in Section II.2.b a given energy price shock would cause a larger leftward shift in the tradeoff curve.

As can be seen from Tables 1 and 2 there is no evidence of a decline in the coefficient on the aggregate demand term ($\Delta\tilde{x}$) between the two subperiods. The estimates in Table 1 also fail to show the expected increase in the (absolute value of) the coefficient on the supply shock measures. But in Table 2 where we allow for the effects on the natural rate of output from changes in aggregate demand and supply variability, we do observe an increase in the estimated output response to a given supply shock.

Allowing for a one time shift in the responses to aggregate demand and supply shocks in the output equation is a crude representation of the new classical view. If the variability of inflation is changing in a continuous fashion within sub-periods as suggested in Section III.1.a, it would be preferable to specify these responses directly as functions of a measure of inflation variability. Specifically, we let the coefficients on $\Delta\tilde{x}$ and μ in equation 20, denoted $\beta_{\Delta x}$ and β_{μ}, be linear functions of a time-varying measure of inflation variability

$$\begin{aligned} \beta_{\Delta x,t} &= \alpha_{10} + \alpha_{11}\ \hat{\sigma}^2_{\Delta p,t} \\ \beta_{\mu,t} &= \alpha_{20} + \alpha_{21}\ \hat{\sigma}^2_{\Delta p,t} \end{aligned} \qquad (21)$$

where $\hat{\sigma}^2_{\Delta p,t}$ is a proxy for the time moving variance of the distribution of the inflation rate $(\sigma^2_{\Delta p})$[19]. We construct this proxy for $\sigma^2_{\Delta p,t}$, in exactly the same way that the proxies for $\sigma^2_{x,t}$ and $\sigma^2_{\mu,t}$ were constructed (see Section III.1.a).

When equation 20 is respecified making the substitution shown in (21) for the coefficients on $\Delta\tilde{x}_t$ and μ_t, the resulting equation contains two interaction terms $\hat{\sigma}^2_{\Delta p,t}\Delta\tilde{x}_t$ and $\hat{\sigma}^2_{\Delta p,t}\mu_t$.[20] Significant estimates for the coefficients on these terms α_{11}, and α_{21} would be evidence that the output response to a demand or supply shock depends upon aggregate price variability.[21] When equation 20 was estimated with this modification, using either the energy price or import price measures of the supply shock, these interaction coefficients α_{11} and α_{21} were not significant at the 5% level. This was true for estimates of the whole period (1959:II - 1980:IV) and for the two subperiods considered (1959:II - 1968:IV and 1969:I - 1980:IV).[22,23]

Overall then our estimates do not show evidence of the "twisting" of the output-inflation trade-off curve, as depicted in Figure 1. The results in Table 2 did indicate that a given supply shock would have generated a larger output response in the second of the subperiods we considered. Within the new classical view this would be consistent with the greater variability of aggregate supply and, therefore, of inflation during that time period.

b. The Effect of Supply-Side Shocks

Our estimates in Tables 1 and 2 indicate that supply-side shocks had a significant negative impact on real output, at least during the second subperiod. Also, in the estimates where we allow for a Friedman-type effect on the natural rate of output (Table 2) we find that the estimated output response to a given supply shock was somewhat larger in the second subperiod than in the first. Therefore, our estimates are consistent with the view that the large supply shocks of the 1970's in the energy sector as well as in the world market for other basic commodities were an important factor in the change in the nature of the output inflation relationship.

c. The Effect of Inflation Variability on the Natural Rate of Output

The estimated coefficients in Table 2 indicate that, for a given value of the aggregate supply shock variable (μ_t) and the aggregate demand measure ($\Delta\tilde{x}_t$), the level of aggregate demand variability had a significant negative effect on real output in the first subperiod while the level of aggregate supply variability had a significant negative effect in the second subperiod. The estimate for the whole period shows a significant negative effect for the measures of aggregate supply variability. Our estimates are therefore not inconsistent with Milton Friedman's view that an increase in the variability of inflation will have a negative effect on the natural rate of output. As illustrated in Figure 3 in Section II.3, an increase in the variability of aggregate supply or demand would have caused the vertical axis, the position of which measures the natural rate of output, to shift leftward. There would then have to be a leftward shift in the tradeoff curve of equal magnitude. The absence of a significant effect for supply variability in the estimate for the earlier subperiod (1959:II - 1968:IV) is perhaps not surprising due to the relative stability of supply factors during that period. There may simply be too little movement in our measure of supply variability to pick up the effects of this variable in the data. The absence of a significant effect of demand variability in the second subperiod is more puzzling.

According to Friedman's analysis, an increase in the variability of inflation will cause the natural rate of output to decline. In our model the variability of inflation depends upon the variability of aggregate demand and supply -- the underlying aggregate disturbances in the model. This is the motivation for including proxies for the aggregate disturbances ($\hat{\sigma}^2_{\Delta x,t}$, $\hat{\sigma}^2_{\mu,t}$) in our output equation. As in the previous subsection, however, we can test more directly for the effects of changes in inflation variability, in this case effects on the natural rate of output, by including a proxy for inflation variability in our equation.

Table 3 gives the estimates of our output equation when the same proxy variable, $\hat{\sigma}^2_{\Delta p,t}$, as was used in the previous subsection is included in the equation in place of the proxies for aggregate demand and supply variability. In the estimated equation for the whole sample period the coefficient on the proxy for the variability of inflation has the correct negative sign but is not statistically significant. In the first subperiod where our previous estimates (Table 2) indicated that aggregate demand variability had a significant negative effect on the natural rate of output, we also find a significant affect for our direct proxy for the variability of inflation. In the second subperiod where earlier estimates indicated that it was aggregate supply variability which had a negative effect on the natural rate of output, the direct proxy for inflation variability is insignificant and has the wrong sign. The estimate in the table is for the energy price measure of the supply shock but the same pattern is evidenced in the estimates using the import price measure.

For the first subperiod our estimates indicate that changes in the variability of inflation, primarily as a result of the variability of aggregate demand, had a negative impact on real output. For the second subperiod the variability of aggregate supply seems to have had a negative impact on real output but one which is not picked up when a more direct measure of inflation variability ($\hat{\sigma}^2_{\Delta p,t}$) is entered in our income equation.

One explanation of this latter result, one which stems from Friedman's own analysis, would be that the increased variability of aggregate supply shocks did lower the natural rate of output but that due to government intervention in the price process this increased variability in supply is not closely mirrored in the variability of a published price index such as the GNP deflator.[24] As Friedman puts it, "In practice, the distorting effects of uncertainty, rigidity of voluntary long-term contracts, and the contamination of price signals will almost certainly be reinforced by legal restrictions on price change." In addition to rigidity of the prices of government provided services and prices which are regulated by the government, Friedman cites attempts by the government to repress inflation via mandatory or "voluntary" wage-price controls. Friedman concludes that it is not only increased price volatility per se but also increased government intervention with the price system which has negative effects on the natural rate of output.

An alternative, though not essentially contradictory, explanation for the significant coefficient on supply variability but insignificant coefficient on the direct measure of inflation variability is that the negative effect of supply variability on the level of the natural rate of output is not an effect which comes through an increase in inflation variability whether open or supressed. A supply shock, for example the oil embargo of the Arab states in 1974, may have direct effects on output by creating shortages, diversion of factor services to exploring alternative energy sources, and temporarily inefficient combinations of factor inputs. These effects may exist even in the absence of government intervention. Further such effects might be expected to be associated with sharp movements in energy prices and hence with the variance, not the level of our supply shock measure.

V. Conclusion

The purpose of this paper has been to assess empirically the contributions of the factors listed as a), b), and c) above as explanations of the apparent change in the relation between output and inflation in the United States. Our results can be summarized as follows:

a) The new classical view. We did not find that the terms of the U.S. output-inflation tradeoff, measured as the output response to a given aggregate demand shock, have deteriorated as rational economic agents adjusted to the increased variability of inflation. There was no evidence of a *twisting* of the output-inflation tradeoff curve such as that depicted in Figure 1.

b) The effect of supply-side shocks. Our estimates suggest that supply shocks played a significant role in the observed shift in the relation between the rate of inflation and the rate of change of real output. Both the increase in the price of energy (or imports), and the increase in the size of the output response to a given supply shock (from the first to the second sub-period) appear to have contributed to the shift.

c) The effect of inflation variability on the natural rate of real output. Our evidence concerning the effect of increased inflation variability on the natural rate of output was somewhat ambiguous. The significant output effects of the variability of aggregate supply which we found may stem from their effect on inflation variability and particularly their effect upon the degree of government intervention in the price process as hypothesized by Milton Friedman. On the other hand, the variability of aggregate supply may have more direct effects on the natural rate of output, as discussed above. In either case, our estimates suggest that the increased variability of aggregate supply also played a role in the observed shift from a positive to a negative correlation between the rate of inflation and the rate of real output growth.

FOOTNOTES

[1] These correlation coefficients were computed from annual data for the unemployment rate, change in the GNP deflator, and rate of growth in real GNP.

[2] Froyen and Waud (1980) suggest that adding a supply shock to the Lucas model is necessary for a consistent explanation of the recent behavior of movements in aggregate demand variance, inflation variance and the terms of the output inflation tradeoff within several industrial countries. Blinder (1981) contains an extension of Lucas' original model to include an energy price variable.

[3] See Sato (1972).

[4] This assumption considerably simplifies the analysis since with it no detailed specification of the elements of aggregate demand is required. Nelson's (1979), (1981) estimates provide support for such a recursive structure between nominal income and real output. Alberro (1981) tested the Lucas specification of aggregate demand and found little evidence to refute that specification. (See also Froyen and Waud [1980, p. 420]).

[5] Equation 9 also reflects the assumption that a proportional increase in product price and the prices of each of the two variable factors of production leaves desired output supply unchanged. It can be shown that this assumption implies $g_1 = -(g_2 + g_3)$ in equation 4.

[6] The case where the aggregate energy price shock is serially correlated is, in fact, relevant and will be analyzed below.

[7] Equation (11) assumes that oil prices are fully indexed to the aggregate price level (the coefficient on p_t is one). We have also examined the case where energy prices are only partially indexed (the coefficient on p_t in (11) is between zero and one). In this latter case aggregate demand management policy can, by changing p_t, affect the real price of energy. The potential role of monetary policy in this case is analyzed by Blinder (1981).

[8] See the Appendix, section A.IV, for an explanation of the derivation of θ.

[9] Evans (1978) points out that Keynes (1924) also posited a negative relationship between instability of the aggregate price level and the level of output. Okun (1981) recently argued, along somewhat different lines, that increased variability of aggregate demand would both steepen the Phillips curve and cause the curve to shift upwards, increasing the "inflation rate associated with the cycle average unemployment rate."

[10] Evans (1978) has shown that within a model where both the supply and demand for labor depend on the degree of uncertainty about the aggregate price level, employment may either increase or decrease in response to an increase in uncertainty. The ambiguity stems from the uncertain response of labor supply to an increase in aggregate price uncertainty. A recent paper by Azariadis (1981) demonstrates the ambiguity of the relationship between price level uncertainty and the natural rate of output within a general equilibrium model. Evans (1978), Levi and Makin (1980), and Mullineaux (1980) provide empirical evidence supporting the view that increased aggregate price uncertainty depresses the natural rate of employment or output. As Levi and Makin (1980, p. 1023) note, the relationship between employment and inflation <u>uncertainty</u> is somewhat different from the relationship between inflation <u>variability</u> and output or employment suggested by Friedman. Friedman's notion would seem broader than those investigated by Evans, Makin and Levi, or Mullineaux in that increased uncertainty is only one channel by which increased inflation variability might affect output or employment.

[11] For the derivation of the variance of the aggregate price level see the Appendix, section A.IV. Within our model, since the lagged value of the price level is given, the variance of the inflation rate can be shown to equal the variance of the aggregate price level. Within the model, the variance of the aggregate price level will (through θ) also depend upon the variances of the market specific demand and supply shocks. These variances are unobservable and are modeled here as part of the additive error term in our final estimating equation.

[12] The relative price of imports is measured as the index of U.S. import prices divided by the GNP deflator. The price of energy is measured by the producer price index for fuels, related products, and electricity. The relative price of energy is computed as this index divided by the GNP deflator. The supply shock in each case is the detrended relative price. Data for energy prices are from <u>Producer Prices and Price Indices</u>, (BLS, 1956-80). Other data are from the IMF, <u>International Financial Statistics</u> computer tape.

[13] In Friedman's analysis changes in the variability of inflation, and therefore of aggregate demand or supply, would affect output whether these changes were perceived or not. Therefore, whether our proxies for the moving variances contain information not available to market participants would not appear to matter. The role of changes in aggregate demand and supply variability within the Lucas model does, however, depend on whether the changes in variability are perceived by market participants. Since we plan to use these same proxies to measure the effects of continuous changes in demand and supply variability on the terms of the output-inflation tradeoff within the modified Lucas model of cyclical fluctuations in income, we construct the proxies using only information available to market participants.

[14] With the energy price shock given by (19), the lagged value of the energy price now conveys information about the current energy price, and, therefore, about the current aggregate price level. Equation (9), for p_t, must be recomputed taking account of this fact. Equation (18') is not derived simply by substituting (19) into (18).

[15] Notice that the trend term in the supply shock variable, $\phi(t)$, will affect only the trend in real output and therefore does not appear in equation(20) for detrended real output.

[16] Nelson (1979) (1981) provides evidence that the residuals from a deterministic trend representation of real output are nonstationary and therefore do not measure the cyclical proportion of real output. Nelson suggests that, rather than a fixed trend for natural output, there is instead a negative relationship between the natural rate of output and the level of inflation, along lines suggested by Friedman. Our specification of detrended real output would appear to be consistent with this view.

[17] The calculated variances of the inflation rate and the aggregate supply and demand shocks in the table are for the whole period for which we have data, 1957:II - 1980:IV. The first subperiod calculated variances refer to the period 1957:II - 1968:IV.

[18] As explained in Section III.1.c, since the anticipated and unanticipated components of the supply shock ($\xi\mu_{t-1}$ and ε_t, respectively in equation 19) will in theory effect output differently we have also decomposed our supply shock measures into these two components and re-estimated equation 20 with these two separate parts of the supply shock as independent variables. To break the supply shock into these two parts we ran regressions of the relative energy or import price against time using an iterative Cochrane-Orcutt technique to estimate the first-order autocorrelation coefficient (ξ). We then broke this equation's residual into $\hat{\varepsilon}_t$ and $\xi\hat{\mu}_{t-1}$. Model estimates with the two separate supply shocks generally showed significant effects for the anticipated portion of the supply shock (import or energy price measure) which conformed to the estimated effects of μ_t in Table 1. The coefficient on the $\hat{\varepsilon}_t$ was generally insignificant.

[19] While Friedman's (1977) analysis and the new classical view, as expressed in Section II.2.a, focus on the relationship between inflation variability and the nature of the output-inflation tradeoff, other research concentrates on the relationship between inflation uncertainty and this tradeoff [Levi and Makin (1980), Evans (1978), Mullineaux (1980)]. To test for the latter relationship we have constructed an alternative time-varying proxy for inflation uncertainty. To construct this measure we first detrend the inflation rate and then proceed in the same manner as described in Section III.1.a for the construction of $\hat{\sigma}^2_{x,t}$ and $\hat{\sigma}^2_{\mu,t}$. The result using this alternative proxy did not differ from the results using the measure of inflation variability ($\hat{\sigma}^2_{\Delta p,t}$) in any significant respect.

[20] The proxy for inflation variance $\hat{\sigma}^2_{\Delta p,t}$ also appears separately in the equation. The coefficient on this variable is discussed in the next section where we consider the effects of inflation variability on the natural rate of output.

[21] In assessing these estimates, a caveat needs to be kept in mind. The actual relationship between the coefficients on $\Delta\tilde{x}$ and μ_t and $\hat{\sigma}^2_{\Delta p}$ which comes via θ is nonlinear (see Appendix, Section A.IV). Our expressions for these coefficients given by equation (21) are therefore only approximations.

[22] In addition to specifying the coefficients on μ_t and $\Delta\tilde{x}_t$ as functions of inflation variability, we also estimate our output equation (20) specifying the coefficients as linear functions of $\hat{\sigma}^2_{x,t}$ and $\hat{\sigma}^2_{\mu,t}$ -- proxies for the structural aggregate variances that determine θ in the model -- in a manner analogous to the specification in equation 21. These estimates (for the whole period as well as for the two subperiods) also provided little evidence that the output response to aggregate demand or supply shocks was significantly affected by changes in the variability of such shocks. For the energy price measure of the supply shock only one of the coefficients on the four interaction terms between μ_t, $\hat{\Delta x}_t$ and $\hat{\sigma}^2_{x,t}$, $\hat{\sigma}^2_{\mu,t}$ was significant at the 5% level. This was the coefficient for the interaction between the variance of aggregate supply and the aggregate demand measure ($\hat{\sigma}^2_{\mu,t}\Delta\tilde{x}$) for the whole period estimate. This coefficient should be negative but the estimated coefficient was positive. For the import price measure, for the time period as a whole, none of the four interaction terms was significant. For the first subperiod, only the coefficient on the interaction term between the variance of the supply shock and the output response to the supply shock ($\hat{\sigma}^2_{\mu,t}\mu_t$) was significant. This coefficient was positive, but should be negative. For the second subperiod again only one of the coefficients on the interaction terms was significant, in this case the coefficient for the interaction term between the variability of aggregate demand and the level of the supply shock variable. This coefficient is negative, consistent with the sign implied by the new classical theory.

[23] Previous research [see Abrams, Froyen and Waud (1983)] suggests that the degree of inflation variability and aggregate demand variability in the United States over a period similar to that considered here may have been too low to produce evidence of a variable output inflation tradeoff, in the sense of the type 1 effect here. This previous work did not, however, allow for explicit type 2 and 3 effects.

[24] In this light, it is of interest to note that the simple correlation coefficient between the proxy for the variability of the aggregate supply shock, $\hat{\sigma}^2_{\mu,t}$ (energy price measure) and the proxy for inflation variability ($\hat{\sigma}^2_{\Delta p,t}$) is only .11 for the second subperiod.

TABLE 1[a]

ESTIMATES OF THE BASIC MODEL

A. Energy Price Measure of the Supply Shock (μ_t)

Sample Period	Constant	$\Delta\tilde{x}_t$	μ_t	$\tilde{y}_{t-1}$	$\bar{R}^2$	ρ	$\hat{\sigma}^2_x$	$\hat{\sigma}^{2\,b}_{\mu}$	$\hat{\sigma}^2_{\Delta p}$
1959:II - 1980:IV	-0.00070	0.88149*	-0.01248*	0.90588*	0.989	0.56	0.000098	0.045389	0.000049
	(-0.903)	(24.18)	(-2.740)	(29.66)					
1959:II - 1968:IV	0.00089*	0.84590*	-0.01818*	0.92000*	0.995	-0.06	0.000102	0.021545	0.000012
	(2.572)	(18.82)	(-2.710)	(36.70)					
1969:I - 1980:IV	-0.00197	0.89873*	-0.01249*	0.89296*	0.985	0.55	0.000097	0.068520	0.000029
	(-1.776)	(18.07)	(-2.393)	(20.64)					

TABLE 1 (Continued)

B. Import Price Measure of the Supply Shock (μ_t)

Sample Period	Constant	$\Delta\tilde{x}$	μ_t	$\tilde{y}_{t-1}$	R^2	ρ	$\hat{\sigma}^{2\,c}_{\mu}$
1959:II - 1980:IV	-0.00079	0.88484*	-0.01889*	0.90850*	0.989	0.47	0.02518
	(-1.237)	(24.47)	(-3.613)	(35.30)			
1959:II - 1968:IV	0.00058	0.85443*	-0.01795*	0.93705*	0.995	-0.33	0.015536
	(1.479)	(19.08)	(-2.50)	(44.92)			
1969:I - 1980:IV	-0.00187*	0.90168*	-0.01832*	0.89504*	0.986	0.47	0.035021
	(-2.057)	(18.15)	(-3.023)	(24.18)			

a. The dependent variable here and in the table that follows is detrended real income. The numbers in parentheses beneath the coefficients are t statistics. Coefficients marked with an asterisk are significant at the five percent level using a two-tailed t test. The second to the last column in the table (ρ) is the estimated value of the first order autoregressive coefficient.

b. Energy price measure of the supply shock.

c. Import price measure of the supply shock.

TABLE 2

MODEL ESTIMATES: THE EFFECTS OF DEMAND AND SUPPLY VARIABILITY

A. <u>Energy Price Measure of the Supply Shock</u>

Sample Period	Constant	$\widetilde{\Delta x}_t$	μ_t	$\tilde{y}_{t-1}$	$\hat{\sigma}^2_{x,t}$	$\hat{\sigma}^2_{\mu,t}$	R^2	ρ
1959:II - 1980:IV	-0.00190*	0.88397*	-0.00541	0.89841*	-14.067	-0.40020*	0.991	0.35
	(-2.116)	(26.38)	(-1.775)	(42.51)	(-1.657)	(-5.557)		
1959:II - 1968:IV	0.00325*	0.86729*	0.00449	0.97260*	-21.21*	-0.36094	0.996	-0.23
	(4.138)	(22.23)	(0.514)	(38.74)	(-3.200)	(-0.551)		
1969:I - 1980:IV	-0.00077	0.88280*	-0.00774*	0.86150*	15.10	-4.2947*	0.990	0.19
	(-0.497)	(20.14)	(-2.934)	(31.82)	(0.761)	(-6.222)		

TABLE 2 (Continued)

B. Import Price Measure of the Supply Shock

Sample Period	Constant	$\Delta\tilde{x}$	μ_t	$\tilde{y}_{t-1}$	$\hat{\sigma}^2_{x,t}$	$\hat{\sigma}^2_{\mu,t}$	R^2	ρ
1959:II - 1980:IV	0.00128	0.88744*	-0.00977*	0.91020*	-9.986	-0.50511*	0.992	0.27
	(1.565)	(26.70)	(-2.540)	(49.69)	(-1.276)	(-5.330)		
1959:II - 1968:IV	0.00317*	0.87338*	0.00456	0.97037*	-17.88*	-0.67886	0.996	-0.24
	(4.174)	(23.08)	(0.544)	(52.22)	(-2.361)	(-0.900)		
1969:I - 1980:IV	-0.00075	0.89346*	-0.01197*	0.88546*	7.664	-0.47821*	0.990	0.19
	(-0.467)	(20.00)	(-3.170)	(34.05)	(0.376)	(-5.013)		

TABLE 3

INFLATION VARIABILITY AND THE NATURAL RATE OF OUTPUT: A DIRECT TEST

Sample Period	Constant	$\Delta\tilde{x}_t$	μ_t	$\tilde{y}_{t-1}$	$\hat{\sigma}^2_{\Delta p}$	R^2	ρ
1959:II - 1980:IV	-0.00061	0.88052*	-0.01250*	0.90504*	-8.796	0.989	0.57
	(-0.584)	(23.94)	(-2.727)	(29.35)	(-0.126)		
1959:II - 1968:IV	0.00402*	0.83621*	-0.01971*	0.92650*	-55.35*	0.996	-0.26
	(3.279)	(20.62)	(-3.576)	(45.59)	(-2.610)		
1969:I - 1980:IV	-0.00241	0.90280*	-0.01252*	0.89642*	3.330	0.985	0.55
	(-1.564)	(17.89)	(-2.419)	(20.55)	(0.408)		

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Appendix

This appendix provides a more detailed derivation of the equations reported in the text.

A.I Market Supply Equations

The output supply equations for each market v (v=1,...,m) are based on the derived factor demand equations for energy and labor.

$$\begin{bmatrix} Q_t(v) \\ N_t(v) \end{bmatrix} = \begin{bmatrix} a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} 1 \\ p_t(v) \\ W_t(v) \\ q_t(v) \\ K_t(v) \end{bmatrix}, \quad v=1, \ldots, m \quad (1)$$

where v indexes the market and for each market,

$Q_t(v)$ = quantity of energy

$N_t(v)$ = number of labor hours

$p_t(v)$ = market specific producer price

$W_t(v)$ = money wage

$q_t(v)$ = price of energy

$K_t(v)$ = quantity of capital

where all variables are in logs.

The factor demand equations (1) may be derived in the usual way by assuming that firms maximize profit subject to the production function constraint. The log linearity of (1) would follow either from the assumption that the production function is Cobb-Douglas or, more generally, as an approximation to factor demand equations based on production functions of the generalized CES type (see K. Sato [1972]).

It is assumed that laborers know the market-specific money wage but must form an expectation of the economy-wide aggregate price level p_t^* (conditioned on information in market v), so that the supply of labor to firms in a specific market is taken to be the log-linear function

$$N_t(v) = d_0 + d_1 p_t^* + d_2 W_t(v). \tag{2}$$

When (2) is used to substitute $W_t(v)$ out of (1) we get

$$\begin{bmatrix} Q_t(v) \\ N_t(v) \end{bmatrix} = \begin{bmatrix} b_{10} & b_{11} & b_{12} & b_{13} & b_{14} \\ b_{20} & b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix} \begin{bmatrix} 1 \\ p_t(v) \\ p_t^* \\ q_t(v) \\ K_t(v) \end{bmatrix}, \quad v=1,\dots,m. \tag{3}$$

where

$$b_{10} = a_{10} + \frac{a_{12}}{d_2}(b_{20} - d_0),$$

$$b_{11} = a_{11} + \frac{a_{12}b_{21}}{d_2},$$

$$b_{12} = \frac{a_{12}}{d_2}(b_{22} - d_1),$$

$$b_{13} = a_{13} + \frac{a_{12}b_{23}}{d_2},$$

$$b_{14} = a_{14} + \frac{a_{12}b_{24}}{d_2},$$

$$b_{20} = \frac{a_{20}d_2 - a_{22}d_0}{(d_2 - a_{22})},$$

$$b_{21} = \frac{a_{21}d_2}{(d_2 - a_{22})},$$

$$b_{22} = \frac{-a_{22}d_1}{(d_2 - a_{22})},$$

$$b_{23} = \frac{a_{23}d_2}{(d_2 - a_{22})},$$

$$b_{24} = \frac{a_{24}d_2}{(d_2 - a_{22})},$$

The production function, in accordance with our earlier remarks, is assumed to be log-linear of the form

$$y_t(v) = g_0' + g_1'K_t(v) + g_2'N_t(v) + g_3'Q_t(v).$$

The supply function for market v is derived by substituting equations (3) into the production function to give

$$y_t(v) = g_0 + g_1 p_t(v) + g_2 p_t^* + g_3 q_t(v) + g_4 K_t(v), \quad (4)$$

where

$$g_0 = (g_0' + g_2'b_{20} + g_3'b_{10})$$

$$g_1 = (g_2'b_{21} + g_3'b_{11})$$

$$g_2 = (g_2'b_{22} + g_3'b_{12})$$

$$g_3 = (g_2'b_{23} + g_3'b_{13})$$

$$g_4 = (g_1' + g_2'b_{24} + g_3'b_{14}).$$

A.II Demand and Expectations Formation

Market demand is specified (the same as in Cukierman and Wachtel (1979), all variables in logs)

$$p_t(v) = x_t + w_t(v) - y_t(v) \tag{5}$$

where $w_t(v)$ is the market specific demand shock, $y_t(v)$ is market specific real output, and x_t is economy-wide aggregate demand taken to be nominal income. $w_t(v)$ and x_t are assumed to be distributed as follows:

$$w_t(v) \sim N(0, \sigma_w^2) \tag{6}$$

$$x_t = x_{t-1} + \Delta x_t, \ \Delta x_t \sim N(\delta, \sigma_x^2). \tag{7}$$

The information conditioning expectations in market v is the current market specific product price $p_t(v)$, the distributions of market specific and aggregate demand shocks, $w_t(v)$ and Δx_t respectively, and the lagged values of aggregate demand. The expectation of the economy-wide aggregate price p_t^* is modeled consistent with the way actual aggregate price is determined in the model. This expectation is given by

$$p_t^* = (1-\Theta)\ p_t(v) + \Theta \overline{p}_t \tag{8}$$

where $\overline{p}_t$ is the expectation of aggregate price conditioned on information prior to time period t, i.e., conditioned on available aggregate information, and Θ is a function (to be explained below) of the variances of market specific and aggregate demand shocks as well as other variances and parameters to be introduced below.

To find $\overline{p}_t$ we first equate market supply, equation (4), and demand, equation (5) to get

$$p_t(v) - x_t - w_t(v) = g_0 + g_1 p_t(v) + g_2 p_t^* + g_3 q_t(v) + g_4 K_t(v) \tag{9}$$

If it is assumed that a proportional increase in product price and the prices of each of the two variable factors of production leaves desired output supply unchanged, then $g_1 = -(g_2 + g_3)$ and the above equation can be rewritten

$$g_0 + g_2(p_t^* - p_t(v)) + g_3(q_t(v) - p_t(v)) + g_4 K_t(v) = x_t + w_t(v) - p_t(v).$$

Substituting from equation (8) for p_t^* and rearranging gives the equilibrium expression for $p_t(v)$,

$$p_t(v) = \frac{1}{1-g_2\Theta-g_3}\,[x_t + w_t(v) - g_0 - g_2\Theta\overline{p}_t - g_3 q_t(v) - g_4 K_t(v)]. \tag{9'}$$

Next we make the following assumptions about energy prices

$$q_t(v) = q_t + \eta_t(v) \tag{10}$$

$$q_t = p_t + \phi(t) + \mu_t \tag{11}$$

where: $q_t(v)$ is the market-specific energy price, q_t is the economy-wide aggregate energy price, and $\eta_t(v)$ is the market-specific energy price disturbance; p_t is the aggregate output price, $\phi(t)$ is a function of time, and μ_t is the aggregate energy price disturbance; and where

$$\eta_t(v) \sim N(0,\sigma_\eta^2) \text{ for all } v, \tag{12}$$

$$\mu_t \sim N(0,\sigma_\mu^2) \tag{13}$$

and $\eta_t(v)$ and μ_t are independently distributed and serially uncorrelated. Using the assumptions (6), (7), (10), (11), (12), and (13), aggregating (9') across markets gives

$$p_t = \frac{1}{1-g_2\Theta-g_3}\,[\Delta x_t + x_{t-1} - g_0 - g_2\Theta\overline{p}_t - g_3 q_t - g_4 K_t] \tag{9''}$$

(For the theoretical underpinnings of such an aggregation procedure see appendix A, p. 607, of Cukierman and Wachtel [1979]). Taking the expectation of p_t conditional on information through period t-1 gives the expression for $\bar{p}_t$,

$$\bar{p}_t = \delta + x_{t-1} - g_0 - g_3\phi(t) - g_4K_t \ . \tag{9}$$

where K_t is the aggregate capital stock.

A.III Aggregate Output

To derive the aggregate output equation we proceed as follows. Using the assumption that $g_1 = -(g_2 + g_3)$, the market-specific supply equation (4) can be rewritten as

$$y_t(v) = g_0 + g_2(p_t^*-p_t(v)) + g_3(q_t(v)-p_t(v)) + g_4K_t(v)$$

or, substituting from (8),

$$y_t(v) = g_0 - g_2\Theta(p_t(v)-\bar{p}_t) + g_3(q_t(v)-p_t(v)) + g_4K_t(v)$$

which, upon substituting from (9), becomes

$$y_t(v) = g_0-g_2\Theta(p_t(v)-\delta-x_{t-1}+g_0+g_3\phi(t)+g_4K_t)+g_3(q_t(v)-p_t(v))+g_4K_t(v). \tag{14'}$$

Now note that (5) may be written as

$$p_t(v) = \Delta x_t + x_{t-1} + w_t(v) - y_t(v)$$

and, given that $p_t = x_t - y_t$, from (10) and (11) that

$$q_t(v) = x_t - y_t + \phi(t) + \mu_t + \eta_t(v).$$

Substitute these expressions for $p_t(v)$ and $q_t(v)$ into (14') and remember the fact that $x_t = \Delta x_t + x_{t-1}$ to get

$$y_t(v) = g_o - g_2\theta(\Delta x_t + w_t(v) - y_t(v) - \delta + g_0 + g_3\phi(t) + g_4K_t) + g_3[\mu_t - y_t + \phi(t) + \eta_t(v) - w_t(v) + y_t(v)] + g_4K_t(v).$$

Aggregating this equation across markets (again see appendix A of Cukierman and Watchel [1979]) gives aggregate output y_t as

$$y_t = g_0 - \frac{g_2\theta}{1-g_2\theta}(\Delta x_t - \delta) + \frac{g_3}{1-g_2\theta}\mu_t + g_3\phi(t) + g_4K_t \quad (14)$$

A.IV Expectations Formation

We will now derive the optimal expectation of the aggregate price p_t^*, given by (8), and show how θ is a function of the market-specific demand and supply variances (σ_w^2 and σ_η^2 respectively), the aggregate demand and supply variances (σ_x^2 and σ_μ^2 respectively), and the parameters g_2 and g_3. The information conditioning the expectation p_t^* in market v is assumed to be the current market product price $p_t(v)$ and the distributions given by (6), (7), (12) and (13). The optimal expectation of the aggregate price p_t^* conditioned on this information is then given by (see for example Hogg and Craig, pp. 211-13, Introduction to Mathematical Statistics, Macmillan, New York, 1959).

$$p_t^* = \rho_{p_t p_t(v)} \frac{\sigma_{p_t}}{\sigma_{p_t(v)}} [p_t(v) - \bar{p}_t] + \bar{p}_t \quad (i)$$

where $\sigma_{p_t}^2$ and $\sigma_{p_t(v)}^2$ are the variances of the aggregate price and market-specific prices respectively, and $\rho_{p_t p_t(v)}$ is the correlation coefficient between p_t and $p_t(v)$.

To obtain $\sigma_{p_t}^2$ use (9") and (11) to express p_t as

$$p_t = -g_0 - \frac{g_2\theta\delta}{1-g_2\theta} - g_3\phi(t) + x_{t-1} + \frac{1}{1-g_2\theta}\Delta x_t - \frac{g_3}{1-g_2\theta}\mu_t - g_4K_t.$$

Substituting this expression for p_t together with (9) for $\overline{p}_t$ into

$$\sigma^2_{p_t} = E(p_t - \overline{p}_t)^2$$

we get

$$\sigma^2_{p_t} = E\left[\frac{\Delta x_t - \delta - g_3 \mu_t}{1 - g_2 \Theta}\right]^2$$

or

$$\sigma^2_{p_t} = \frac{1}{(1-g_2\Theta)^2}\,(\sigma^2_x + g_3^2\sigma^2_\mu) \qquad \text{(ii)}$$

assuming Δx_t and μ_t are distributed independently. Note from (ii) that the variance of the aggregate price depends upon the variance of the aggregate demand shock and the variance of the aggregate supply shock, as well as the market specific variances (via θ).

The variance of the market-specific price $\sigma^2_{p_t(v)}$ is equal to the sum of the variance of the aggregate price σ^2_p and the variance of market-specific price about the aggregate price level σ^2_τ, or

$$\sigma^2_{p_t(v)} = \sigma^2_{p_t} + \sigma^2_\tau \qquad \text{(iii)}$$

where

$$\sigma^2_\tau = E\left[p_t(v) - p_t\right]^2.$$

Substituting (9') and (9") for $p_t(v)$ and p_t respectively and using (10) gives

$$p_t(v) - p_t = \frac{1}{1 - g_2\Theta - g_3}\left(w_t(v) - g_3\eta_t(v)\right).$$

From (6) and (12) it follows that

$$\sigma_\tau^2 = \frac{1}{(1-g_2\theta-g_3)^2}(\sigma_w^2 + g_3^2\sigma_\eta^2). \qquad \text{(iv)}$$

assuming that w and η are independently distributed. Note from (iv) that the variance of market-specific price about the aggregate price depends upon the variance of the market-specific demand disturbance and the variance of the market-specific energy price disturbance, as well as the aggregate supply and demand variances (via θ). Substituting (ii) and (iv) into (iii) gives

$$\sigma_{p_t(v)}^2 = \frac{1}{A}(\sigma_x^2 + g_3^2\sigma_\mu^2) + \frac{1}{B}(\sigma_w^2 + g_3^2\sigma_\eta^2) \qquad \text{(v)}$$

where

$$A = (1-g_2\theta)^2$$

$$B = (1-g_2\theta-g_3)^2 .$$

Now note that

$$\mathrm{Cov}(p_t,p_t(v)) = E(p_t(v)-\bar{p}_t)(p_t-\bar{p}_t) = E(p_t(v)p_t) - \bar{p}^2.$$

Since

$$p_t(v) = p_t + \frac{1}{1-g_2\theta-g_3}(w_t(v)-g_3\eta_t(v))$$

it follows that

$$E(p_t(v)p_t) = E(p_t^2) + \frac{1}{1-g_2\theta-g_3}E[p_t(w_t(v)-g_3\eta_t(v))]$$

and therefore, since $E[p_t(w_t(v)-g_3\eta_t(v))] = 0$,

$$\mathrm{Cov}(p_t,p_t(v)) = \sigma_{p_t}^2 .$$

Hence

$$\rho_{p_t p_t(v)} = \frac{\mathrm{Cov}(p_t,p_t(v))}{\sigma_{p_t}\cdot\sigma_{p_t(v)}} = \frac{\sigma_{p_t}}{\sigma_{p_t(v)}}$$

and (i) may be written

$$p_t^* = \frac{\sigma^2_{p_t}}{\sigma^2_{p_t(v)}} [p_t(v) - \bar{p}_t] + \bar{p}_t. \qquad \text{(i')}$$

Now from (8), (i'), (ii), and (v) it can readily be seen that

$$\theta = \frac{\dfrac{\sigma_w^2 + g_3^2\sigma_\eta^2}{B}}{\dfrac{\sigma_x^2 + {}_3^2\sigma_\mu^2}{A} + \dfrac{\sigma_w^2 + g_3^2\sigma_\eta^2}{B}} \qquad \text{(vi)}$$

To show that θ is inversely related to σ_x^2 denote the right-hand side of (vi) as X and rewrite (vi) as the implicit function

$$\phi = \theta - X = 0.$$

Then

$$\frac{d\phi}{d\sigma_x^2} = -\frac{\phi_{\sigma_x^2}}{\phi_\theta} = \frac{X_{\sigma_x^2}}{1 - X_\theta} \qquad \text{(vii)}$$

Note that X, the right-hand side of (vi), can be rewritten

$$X = \left[\frac{B(\sigma_x^2 + g_3^2\sigma_\mu^2)}{A(\sigma_w^2 + g_3^2\sigma_\eta^2)} + 1\right]^{-1}$$

and that

$$X_{\sigma_x^2} = -\left[\frac{B(\sigma_x^2 + g_3^2\,\sigma_\mu^2)}{A(\sigma_w^2 + g_3^2\sigma_\eta^2)} + 1\right]^{-2} \frac{B}{A(\sigma_w^2 + g_3^2\sigma_\eta^2)} < 0. \qquad \text{(viii)}$$

Letting $a = \sigma_x^2 + g_3^2\sigma_\mu^2$ and $b = \sigma_w^2 + g_3^2\sigma_\eta^2$ also note that

$$X_\theta = - \left[\frac{Ba}{AB} + 1\right]^{-2} \frac{a(B_\theta A - A_\theta B)}{A^2 b} > 0 \qquad \text{(ix)}$$

since

$$B_\theta A - A_\theta B = -2g_2(1-g_2\theta-g_3)(1-g_2\theta)g_3 < 0,$$

where

$$A_\theta = -2g_2(1-g_2\theta)$$

and

$$B_\theta = -2g_2(1-g_2\theta-g_3).$$

It reasonably can be argued that

$$X_\theta < 1$$

since then

$$-ab(B_\theta A - A_\theta B) < (Ba+Ab)^2$$

or, substituting for A, B, a, b, A_θ and B_θ,

$$2\, g_2 g_3(\sigma_x^2+g_3^2\sigma_\mu^2)(\sigma_w^2+g_3^2\sigma_\eta^2)(1-g_2\theta)(1-g_2\theta-g_3)$$
$$< (1-g_2\theta-g_3)^4(\sigma_x^2+g_3^2\sigma_\mu^2)^2$$
$$+ 2(\sigma_x^2+g_3^2\sigma_\mu^2)(\sigma_w^2+g_3^2\sigma_\eta^2)(1-g_2\theta)^2(1-g_2\theta-g_3)^2$$
$$+ (1-g_2\theta)^4(\sigma_w^2+g_3^2\sigma_\eta^2)^2$$

which can be seen by inspection to be true for economically reasonable values of g_2 and g_3. Hence from (vii), (viii), and (ix) it follows that

$$\frac{d\theta}{d\sigma_x^2} < 0. \qquad \text{(x)}$$

A symmetric argument will show that

$$\frac{d\theta}{d\sigma_{\mu}^{2}} < 0. \qquad \text{(xi)}$$

Also note, from (x), (xi) and inspection of (ii), it can be seen that

$$\frac{d\sigma_{p_t}^{2}}{d\sigma_{x}^{2}} > 0$$

and

$$\frac{d\sigma_{p_t}^{2}}{d\sigma_{\mu}^{2}} > 0.$$