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SPECIFIC VERSUS GENERAL ENFORCEMENT OF LAW

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ABSTRACT

The problem of optimal public enforcement of law is examined in a model in which two types of enforcement effort are distinguished: specific enforcement effort, activity devoted to apprehending and penalizing individuals who have committed a single type of harmful act; and general enforcement effort, activity affecting the likelihood of apprehension of individuals who have committed any of a range of harmful acts. (A policeman on the beat, for instance, is able to apprehend many types of violators of law.) If all enforcement effort is specific, then under wide assumptions it is optimal for sanctions to be extreme for all acts. However, if all enforcement effort is general, optimal sanctions are low for acts of small harmfulness, increase with the degree of harmfulness, and reach the extreme only for the most harmful acts (the main result of the paper). Also considered is the assumption that enforcement effort may be both general and specific.

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I. Introduction

The problem of optimal public enforcement of law -- the problem of selecting probabilities and magnitudes of sanctions that best deter violations -- is examined here in a model in which two types of enforcement effort, specific and general, are distinguished. By specific enforcement effort, I mean activity devoted to apprehending and penalizing individuals who have committed a single type of harmful act. The activity of an employee of a traffic department whose sole duty is to ticket people for overtime parking exemplifies specific enforcement effort; so does, typically, investigative or prosecutorial effort made after the commission of a harmful act, for such effort by its nature concerns a single act. In contrast, general enforcement effort is activity affecting the likelihood of apprehension of individuals who have committed any of a range of harmful acts. A policeman on the beat, for instance, is able to apprehend many types of violators of law, from those who shoplift, to those who engage in assault, to those who commit murder. Whenever an enforcement agent's activity naturally allows him to detect different types of violators, the enforcement activity is what is called here general.[1]

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[1] The term "specific enforcement" should not be confused with "specific deterrence" or with "particular deterrence," which often are taken in the literature on deterrence to refer to the tendency of punishment of a particular individual to induce him not to commit bad acts in the future. Nor should "general enforcement" be confused with "general deterrence," which refers to the tendency of the threat of punishment to dissuade people generally from committing bad acts. On the terms "particular deterrence" and "general deterrence," see, for example, La Fave and Scott (1972), pp.

To understand the importance of the distinction between the two types of enforcement effort, consider initially the assumption that all enforcement effort is specific. This means that the enforcement of law concerning one type of act is independent of the enforcement concerning any other; society may devote one level of specific enforcement effort toward apprehension of individuals who commit one act (and set one sanction for it) and may devote a very different level of specific enforcement effort toward apprehension of those who commit another act. This implies, under wide assumptions, that it is optimal for sanctions to be extreme, as high as possible,[3] for all acts.

The reasoning is well known and is due essentially to Becker (1968). To review, if the sanction for an act is not extreme, society should enjoy an opportunity to conserve enforcement resources without sacrificing deterrence: Society should be able to reduce enforcement effort and to augment the (less than extreme) sanction by an amount calculated to leave the expected sanction -- and thus deterrence of the act -- unchanged. At the optimum, it must be impossible for society to use this beneficial strategem involving an increase in the sanction; that is, it is optimal for the sanction to be extreme.[4] Because this argument applies independently to each act, it appears that the optimal sanction for each act is extreme.

22-23, and the articles cited therein.

[3]It is assumed that sanctions have some bound; see note 9.

[4]While Becker (1968) was the first to notice that society may have a beneficial opportunity to reduce enforcement effort and increase sanctions, he did not stress that this point leads to the conclusion that extreme sanctions are optimal, and he proceeded for the most part as if less than extreme sanctions are optimal. That his argument implies that extreme sanctions are optimal has, however, been noted by others; see, for example, Carr-Hill and Stern (1979) and Polinsky and Shavell (1979).

In the analysis below this conclusion is considered formally and is verified to be correct when sanctions are solely monetary or are solely non-monetary.[5] When sanctions are of combined form, the conclusion is modified somewhat; only the monetary component need be extreme.

Of course, the conclusion that sanctions should toward the extreme is at odds with what is observed in fact. Extreme sanctions are not the norm but the exception.

A conclusion about optimal sanctions more in accord with what is observed is reached when one takes into account general enforcement effort. Assume for simplicity, as is done in part of the analysis, that all enforcement effort is general and that enforcement effort results in the same probability of apprehension for all acts. Now to deter reasonably well the totality of harmful acts, a certain probability of apprehension will be required. Because this probability of apprehension will apply in particular to those who commit less harmful acts, the probability will be more than sufficient to deter these acts appropriately if extreme sanctions are employed, so that extreme sanctions will not be needed.[6]

This point may be restated less abstractly. Society wants a certain number of police on the streets to deter the whole range of crimes,

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[5]The precise result shown is that optimal sanctions are extreme if they are positive; however, optimal sanctions are zero for all acts resulting in harm below a certain threshold.

[6]Another theoretical justification for less than extreme sanctions involves risk aversion on the part of sanctioned parties; see Polinsky and Shavell (1979) and Kaplow (1989). In the present paper, individuals are assumed to be risk neutral.

An additional justification for less than extreme sanctions concerns "marginal deterrence," inducing the undeterred to commit less harmful rather than more harmful acts by setting a lower sanction for less harmful acts than for more harmful acts; see Stigler (1970). This justification, however, actually depends implicitly on an assumption of general enforcement effort; see Shavell (1989).

including, especially, serious ones. But given that these police are on the streets, they will be present to apprehend those who commit lesser crimes. Society therefore does not need to threaten those who would commit lesser crimes with the very high sanctions it employs for serious crimes.

More precisely, what will be shown in the analysis where enforcement effort is general is that optimal sanctions are low for acts of small harmfulness, increase with the degree of harmfulness, and reach the extreme only for the most harmful acts.[7] This is true whether sanctions are solely monetary, are solely non-monetary, or may be of combined form. In the last case, optimal sanctions are at first purely monetary and rise with the degree of harm to the highest level, an individual's wealth; then these extreme monetary sanctions are accompanied by non-monetary sanctions that increase with the level of harm. This result, it may be remarked, is in rough accord with reality in that criminal sanctions are reserved for seriously harmful acts and increase with the harmfulness of acts.

Also considered in the analysis is the assumption that enforcement effort may be both general and specific. Under this assumption, the conclusions resemble those applying when all enforcement effort is general. The conclusions are similar to those just discussed in that it is optimal for less than extreme sanctions to be employed for all but the most harmful

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[7]A numerical example is suggestive of the conclusion. Suppose that there are two types of acts, those causing harm of $1 and those causing harm of $100. To deter properly the more harmful acts, the expected sanction should equal $100. Suppose that, following Becker, this is done as cheaply as possible, by using the extreme sanction of a person's entire wealth -- say it is $10,000 -- and along with it a low probability of apprehension -- here a probability of only 1% (for 1%x$10,000 = $100). Because the probability of 1% is general and applies also to those who commit the act causing only the $1 harm, a sanction of just $100 will be optimal for the act (as 1%x$100 = $1); in other words, a sanction far less than a person's entire wealth will be optimal.

acts; the reason is again that since there is general enforcement effort, the probability of apprehending those who commit less harmful acts is more than enough to deter adequately if extreme sanctions are used. However, society is able to undertake specific enforcement effort as well; and it will be worth society's while for this to be done for acts that are sufficiently harmful.[8]

The paper closes with several comments on possible extensions of the analysis.

II. The Model

Risk neutral individuals decide whether to commit harmful acts. Individuals differ; a particular type of individual is identified by the benefit he would obtain from his act and by its harmfulness. Define

b = benefit from committing an act; $b \geq 0$;

$f(b)$ = probability density of b; f is continuous, bounded, and positive on $[0,\infty)$;

h = harm due to an act; $h \geq 0$;

$g(h)$ = probability density of h; g is continuous, bounded, and positive on $[0,\infty)$.

The distribution of benefits is assumed for simplicity to be the same for different h.

If an individual commits a harmful act, he will suffer a sanction with a probability. The sanction may be solely monetary, solely non-monetary, or of combined form. Let

$s(h)$ = monetary sanction for committing an act causing harm h;

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[8] Mookherjee and Png (1989) obtain a similar result in a model considering the optimal joint use of monitoring effort and of investigation of reported violations (their paper and the present one were written independently of each other). See note 16 below.

z(h) = non-monetary sanction for committing an act causing harm h;

w = wealth of individuals;

p = probability of apprehension; p may or may not depend on h, as specified.

It is assumed that the social authority imposing sanctions can observe h; thus the sanctions can be made a function of h. A monetary sanction cannot be higher than an individual's wealth, which is assumed to be equal for all individuals (but see the comment on this assumption in the concluding section). Hence,

(1) $0 \leq s(h) \leq w$.

It is assumed also that non-monetary sanctions are bounded by some maximal sanction $\bar{z}$. This is justified by the usual axioms of expected utility theory; they imply that utility, or disutility, is bounded.[9] Hence,

(2) $0 \leq z(h) \leq \bar{z}$.

If an individual bears a non-monetary sanction, it is assumed that society bears a cost; let

σz = social cost if a non-monetary sanction z is imposed; $\sigma > 0$.

That imposition of non-monetary sanctions (notably, imprisonment) is assumed socially costly is motivated by two considerations. First, the disutility suffered by a sanctioned individual may be considered a social cost.[10]

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[9] See, for example, Arrow (1971). Block and Lind (1975) emphasize the boundedness of utility in an early discussion of the use of sanctions.

[10] Note by contrast that the imposition of a monetary sanction is not natural to consider as a social cost, for what the penalized party pays someone else receives; imposition of monetary sanctions involves only a transfer of command over resources. Imposition of non-monetary sanctions creates a disutility that is not balanced in any automatic way by an increase in the utility of another.

Second, imposition of non-monetary sanctions may involve resource costs (the expenses of operating the prison system).

Because individuals are risk neutral, an individual will commit an act if and only if his benefit is at least as large as the expected sanction,[11]

(3) $b \geq p(s(h) + z(h))$.

The probability of apprehension p is determined by enforcement effort of which, as explained in the Introduction, there are two types, specific and general. Specific enforcement effort raises the probability of apprehension for those who commit a specific type of harmful act, identified by h. General enforcement effort raises the probability of apprehension of all individuals who commit harmful acts, whatever is h. Let

$x(h)$ = enforcement effort specific to apprehending those who commit acts causing harm h;

y = general enforcement effort.

As stated in the Introduction, three cases will be studied. In the first, all enforcement effort is specific; here it is assumed that

(4) $p = p(x(h))$,

where $p(0) = 0$, $0 \leq p(x) < 1$; $p'(x) > 0$; $p''(x) < 0$; that is, the probability of apprehending any given type of individual is zero if no effort is made and increases with enforcement effort but at a decreasing rate. (In the concluding section the assumption implicitly made here that the probability is the same function of x for all h is briefly discussed.) Total specific enforcement effort is

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[11] I assume for concreteness that if there is equality, in (3) the individual will commit the act even though he is indifferent between doing so and not.

$$(5)\ \int_0^{\infty} x(h)dh.$$

In the second case all enforcement effort is general. In this case

$$(6)\ p = p(y),$$

where p has the same properties as before and where total enforcement effort is y. In the third case enforcement effort is both specific and general, and

$$(7)\ p = p(x(h),y),$$

where p is increasing and concave in x and y, and where total enforcement effort is given by exp. (5) plus y.

Social welfare is defined to be the benefits individuals obtain from committing acts, less the harm done, less the social costs of imposing any non-monetary sanctions, less total enforcement effort.

I will now consider the problem of choosing sanctions and enforcement effort -- and thus the probability of apprehension -- so as to maximize social welfare in the three cases. The three cases will be examined first where sanctions are solely monetary, then where they are solely non-monetary, and finally where they are of combined form. This will allow us to build a fairly complete understanding of the solution to the enforcement problem.

A. Sanctions Are Solely Monetary

If enforcement effort is specific, the social problem is a set of entirely independent problems; for each h, enforcement effort and a sanction must be optimally selected. Social welfare is given by

$$(8)\ \int_0^{\infty} \int_{p(x(h))s(h)}^{\infty} (b - h)f(b)dbg(h)dh - \int_0^{\infty} x(h)dh;$$

the social problem is to maximize (8) over functions x(h) and s(h). Equivalently, the social problem is to choose for each h, enforcement effort x and a sanction s to maximize

$$(9) \quad \int_{p(x)s}^{\infty} (b - h)f(b)db - x.$$

The solutions to this problem will be denoted x* (or x*(h)) and s* (or s*(h)) and * will generally denote optimal values below. The following result will be shown.

Proposition 1a. Suppose that enforcement effort is specific and that monetary sanctions alone are employed. Then for all harms h below a threshold, optimal enforcement effort is zero. Above this threshold of harm, optimal enforcement effort -- together with the probability of apprehension -- is positive and increases with h, and the optimal sanction is maximal, equal to wealth. In addition, the expected sanction is less than harm; there is always underdeterrence.■

Remarks. The Proposition is illustrated in Figure 1. That it is not worthwhile expending enforcement effort for small harms is readily explained: the marginal cost of effort is one, but the social benefit due to deterrence of harms tends to zero as the harms tend to zero. That enforcement effort should increase with harm once optimal effort becomes positive makes obvious sense; higher harms are more worthwhile deterring. That the sanction should always equal wealth when enforcement effort is positive is due to Becker's argument: if the sanction were less than wealth, it could be raised and enforcement effort lowered so as to save resources but maintain deterrence. That there is always underdeterrence follows from two points: if one begins with a situation of perfect deterrence and allows the expected sanction to decline, the first-order social loss from failing

Specific Enforcement Effort with Monetary Sanctions

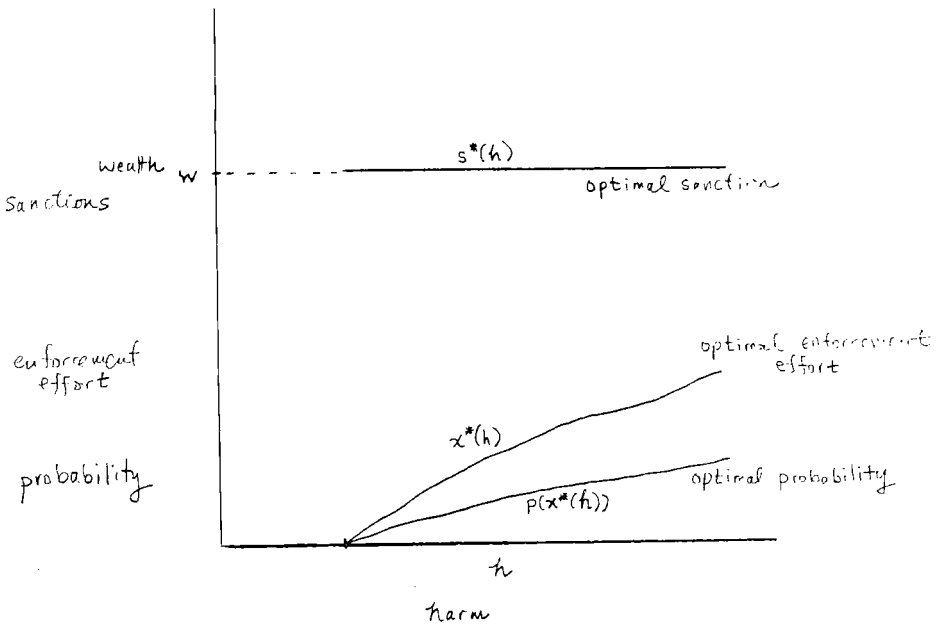

FIGURE 1

to deter is zero; yet the first-order social gain from reducing enforcement effort is strictly positive, namely one.[12]

Proof. The argument consists of several steps.

(i) If $x^*(h) > 0$, then $s^*(h) = w$: Assume otherwise, that $x^* > 0$ and $s^* < w$. Raise s to w and lower x to x' such that $p(x')w = p(x^*)s^*$. (This is obviously possible.) Then the integral in (9) remains the same -- the same individuals commit the act -- but enforcement effort is lower. Thus, (9) is higher, which contradicts the assumption that x^* and s^* were optimal.

(ii) $x^*(h) = 0$ for all h sufficiently low. If $x^*(h) > 0$, it increases with h: From (i), we know that if x^* is positive, $s^* = w$, so that x^* in fact maximizes

$$(10) \qquad \int_{p(x)w}^{\infty} (b - h)f(b)db - x$$

over x. Differentiating (10) with respect to x, we obtain

$$(11) \qquad -p'(x)w(pw - h)f(pw) - 1.$$

The first term in (11) is the marginal gain due to increased deterrence (h - pw is the net social loss avoided when the marginal individual is deterred) and 1 is the marginal cost of raising enforcement effort. If (11) is negative for all x, then $x^* = 0$; and (11) is negative for all x if h sufficiently small.[13] However, if h is high enough so that x^* is positive, then x^* rises with h. This is evident from the first-order condition determining a positive x^*,

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[12] That some degree of underdeterrence is optimal when sanctions are monetary (and enforcement effort is specific) is noted in Polinsky and Shavell (1984) at p. 93.

[13] The first term of (11) equals $-p'(x)pw^2f(pw) + hp'(x)wf(pw)$. Now $hp'(x)wf(pw)$, which is positive, is bounded over all x by $hp'(0)wf_b$, where f_b is a bound for the density f. Hence, for all h sufficiently small, $hp'(x)wf(pw)$ is dominated by -1, so (11) is indeed negative for all x for such h.

(12) $-p'(x)w(pw - h)f(pw) - 1 = 0;$

for the sign of $x^{*\prime}(h)$ equals the sign of the partial derivative of the left-hand side of (12) with respect to h.[14] The latter is $p'(x)wf(pw) > 0$, so $x^{*\prime}(h) > 0$.

(iii) If $x^*(h) > 0$, the expected sanction $p(x^*(h))w$ is less than h: If $x^* > 0$, (12) holds, from which it is clear that $p(x^*)w < h$.∎

When enforcement effort is general, the social problem is no longer a set of independent problems, one for each h. Instead, the enforcement problems for different h are interconnected because a single probability of apprehension applies for all h. The social problem is to choose general enforcement effort y and sanctions s(h) to maximize social welfare,

$$(13)\quad \int_0^{\infty} \int_{p(y)s(h)}^{\infty} (b - h)f(b)db\,g(h)dh - y.$$

I assume for simplicity that optimal general enforcement effort y^* is positive; the probability $p(y^*)$ will be denoted p^*. The following result will be shown.

Proposition 1b. Suppose that enforcement effort is general and that monetary sanctions alone are employed. Then for all harms h below the threshold p^*w, the optimal sanction is given by the formula h/p^*; the expected sanction thus equals the harm h and rises with the level of harm. For harms above the threshold, the optimal sanction is maximal, equal to wealth, w, and there is underdeterrence. The optimal probability p^* is determined by (17) below.∎

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[14] The condition (12) is of the form $W(x,h) = 0$. Implicitly differentiating with respect to h, one obtains $W_x x^{*\prime} + W_h = 0$, so that $x^{*\prime} = -W_h/W_x$. But $W_x < 0$ -- this is the second-order condition for x^* to be a maximum. Hence, the sign of $x^{*\prime}$ is the sign of W_h.

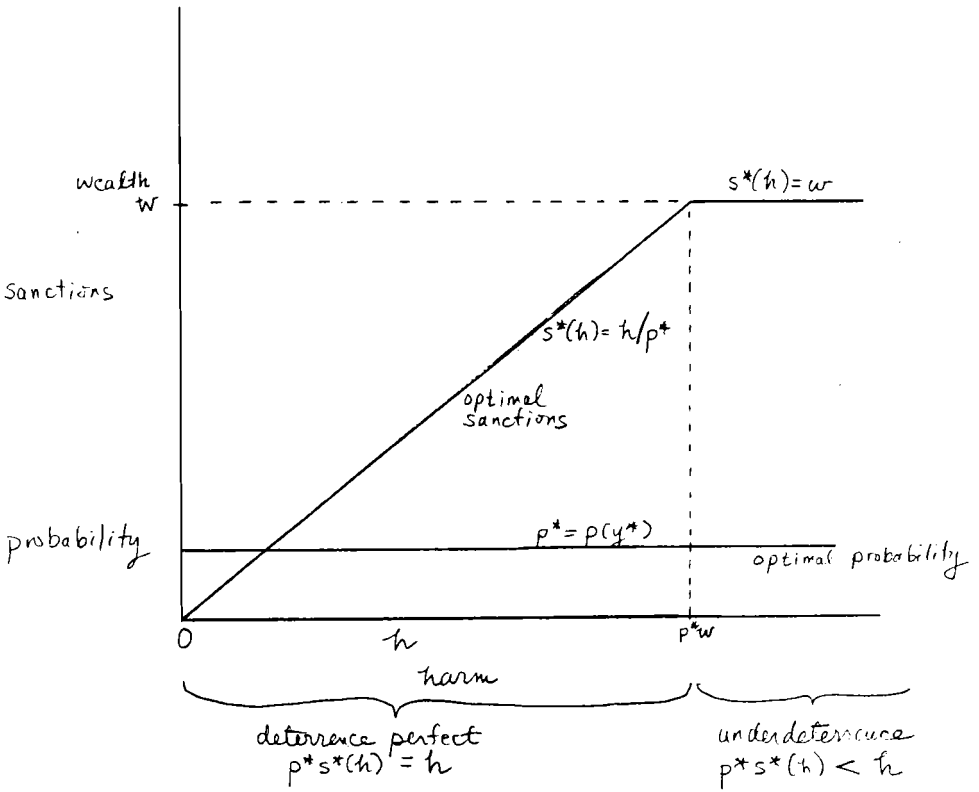
wealth
w
s*(h)= w
Sanctions
s*(h)= h/p*
optimal
sanctions
probability
p* = p(y*)
optimal probability
0
h
p*w
harm
deterrence perfect
p*s*(h) = h
underdeterrence
p*s*(h) < h

FIGURE 2

Remarks. The Proposition is illustrated in Figure 2. Because enforcement effort is general and one probability of apprehension applies for all h, the probability will be high enough to allow achievement of perfect deterrence for h below a threshold. This threshold is at the point where the maximum expected sanction p*w equals the harm.

The condition (17) determining p* equates the marginal cost of raising p to the marginal benefit. The marginal benefit inheres in the fact that raising p increases deterrence in the region beyond the threshold, p*w, for in that region there is underdeterrence and the sanction is at the maximum, so that increasing p is the only way to increase deterrence.∎

Proof. The proof consists of two steps.

(i) $s^*(h) = h/p^*$ -- so that $p^*s^*(h) = h$ -- for $h \leq p^*w$; $s^*(h) = w$ for larger h: Given y and p, the social problem for any h is to maximize over s

$$(14)\quad \int_{ps}^{\infty} (b - h)f(b)db,$$

the derivative of which with respect to s is

$$(15)\quad -p(ps - h)f(ps).$$

This is positive when $ps < h$ or when $s < h/p$; it is zero at $s = h/p$; and it is negative for larger s. It follows that (14) is maximized at $s = h/p$ if this s is feasible, that is, $s^*(h) = h/p^*$ if $h/p^* \leq w$, or if $h \leq p^*w$. Otherwise, $s^*(h) = w$ since (15) is positive when $p^*s < h$.

(ii) It follows from (i) that social welfare (13) may be written as

$$(16)\quad \int_0^{p(y)w} \int_h^{\infty} (b - h)f(b)dbg(h)dh + \int_{p(y)w}^{\infty} \int_{p(y)w}^{\infty} (b - h)f(b)dbg(h)dh - y.$$

The first term is associated with the region of h over which deterrence is perfect, since ps(h) = ph/p = h; the second term is associated with the region of h over which s(h) = w and there is underdeterrence. Differentiating (16) with respect to y, we obtain the first-order condition

$$(17) \quad p'(y)wf(p(y)w)\int_{p(y)w}^{\infty}(h - p(y)w)g(h)dh = 1,$$

determining y* and p*.[15] The left-hand side is the marginal benefit from increasing y and p, which (as was remarked) inheres in reducing social losses by h - p(y)w for persons just deterred in the region of h above p(y)w.■

When both general enforcement effort and specific enforcement effort may be employed, the social problem is to choose specific enforcement effort x(h), general enforcement effort y, and sanctions s(h) to maximize social welfare,

$$(18) \quad \int_0^{\infty}\int_{p(x(h),y)s(h)}^{\infty}(b - h)f(b)dbg(h)dh - \int_0^{\infty}x(h)dh - y.$$

I assume that optimal general enforcement effort y* is positive and that x*(h) is positive for some h; otherwise the social problem devolves into one of the two problems that has already been considered. I will show

Proposition 1c. Suppose that general enforcement effort may be augmented by specific effort and that monetary sanctions alone are employed. Then for all harms below the threshold p*w, the optimal sanction equals h/p* -- hence the expected sanction equals the harm h and rises with the level of harm -- and optimal specific enforcement effort is zero. Beyond the

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[15] Although I have assumed that y* > 0, it is of interest to observe that this must be true if E(h), the mean of h, is sufficiently large. For y* > 0 if the derivative of (16) evaluated at 0 is positive. This derivative at 0 is, from (17), equal to p'(0)wf(0)E(h) - 1, which is positive if E(h) is large enough.

threshold p*w, the optimal sanction equals the maximal level, wealth, and positive specific effort becomes optimal at a level of harm strictly greater than the threshold; when that occurs, optimal specific enforcement effort rises with the level of harm.■

Remarks. The Proposition is illustrated in Figure 3. The explanation for the results is that in the first region of Figure 3, perfect deterrence is possible without supplementing general enforcement effort with specific enforcement effort. After the sanction becomes maximal, there is a problem of underdeterrence; when this problem becomes important enough, specific enforcement effort is worthwhile.[16]■

Proof. (i) If $x^*(h) > 0$, then $s^*(h) = w$: The social problem for any h is to choose s and x to maximize

$$(19)\quad \int_{p(x,y)s}^{\infty} (b - h)f(b)db - x.$$

If $x^* > 0$ but $s^* < w$, raise s to w and lower x to x' such that $p(x',y)w = p(x^*,y)s^*$. Then the integral in (19) remains the same, but since $x' < x^*$, (19) is higher, a contradiction.

(ii) If $h < p^*w$, then $s^*(h) = h/p^*$ and $x^*(h) = 0$, where $p^* = p(0,y^*)$: Assume first that $s^* < w$. Then by (i) $x^* = 0$. This means that (19) reduces to (14), which we know is maximized at $s^* = h/p^*$ (for $h/p^* < w$ since $h < p^*w$), where $p^*s^* = h$. Now assume that $s^* = w$. In this case, however, social welfare is lower: Because $p^*s^* = p^*w > h$, too few individuals commit the act, and if $x^* > 0$, there are additional enforcement expenses incurred. Hence, it must be that $s^* = h/p^*$, and the claim follows.

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[16]The explanation for a similar result of Mookherjee and Png (1989) (see their Proposition 3) is related, although their model is different from the present one (notably, in their model each individual chooses from among a continuum of possible acts).

General and Specific Enforcement Effort with Monetary Sanctions

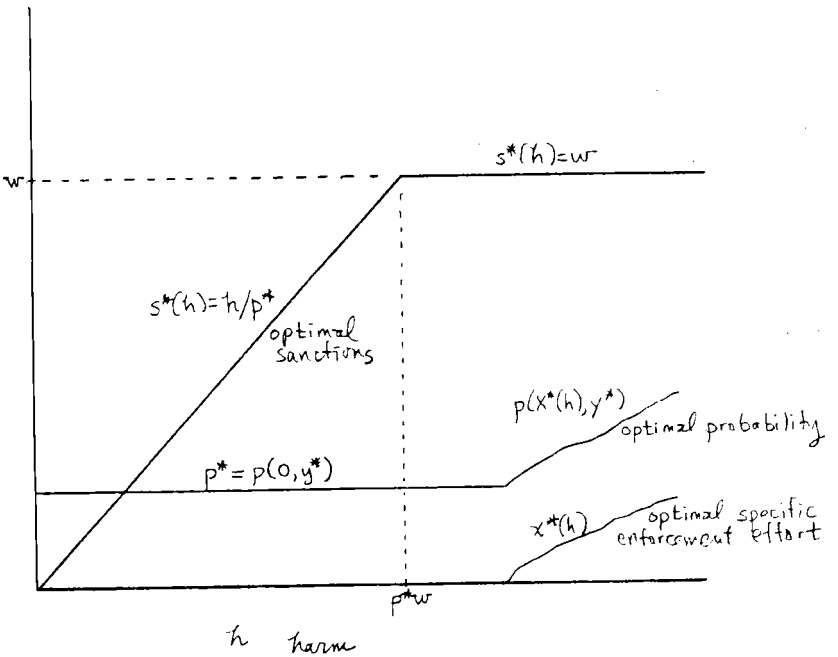

FIGURE 3

(iii) If $h \geq p^*w$, then $s^*(h) = w$; also, for such h, $x^*(h)$ is at first zero; when it becomes positive, it rises with h: If $s^* < w$, then by (i), $x^* = 0$. But since $p^*s^* < p^*w \leq h$, it is socially beneficial to raise s, a contradiction. Thus $s^* = w$. Therefore, (19) equals

$$(20) \quad \int_{p(x,y)w}^{\infty} (b - h)f(b)db - x.$$

The derivative of (20) with respect to x is

$$(21) \quad -p_x w(pw - h)f(pw) - 1.$$

This is negative in a neighborhood of h above $h = p^*w$, so that $x^* = 0$ in the neighborhood. We assumed, recall, that $x^*(h)$ is positive for some h, and when this is so, $x^*(h)$ is determined by the condition that exp. (21) equals zero. Because the sign of the partial derivative of (21) with respect to h is $p_x wf(pw) > 0$, $x^*(h)$ must rise with h.■

B. Sanctions Are Solely Non-Monetary

In this case, the results and proofs are in most respects similar to those where sanctions were monetary.

If enforcement effort is specific, social welfare is

$$(22) \quad \int_0^{\infty} \int_{p(x(h))z(h)}^{\infty} (b - h - \sigma p(x(h))z(h))f(b)dbg(h)dh - \int_0^{\infty} x(h)dh;$$

the social problem is to maximize (22) over functions x(h) and z(h). We have

Proposition 2a. Suppose that enforcement effort is specific and that non-monetary sanctions alone are employed. Then for all harms below a threshold, optimal enforcement effort is zero. Above this threshold, optimal enforcement effort (together with the probability of apprehension) is positive and increases with h, and the optimal sanction is always maximal, equal to $\bar{z}$.■

Remarks. The explanation for this result is like that for Proposition 1a. It should be noted that Becker's argument that the optimal sanction is maximal still applies. In particular, if the sanction is not maximal and is raised and enforcement effort is reduced so that the expected sanction is not altered, then the expected social cost of imposing sanctions also is left unchanged. It should be observed as well that the expected sanction may be above or below h: The first-order condition associated with (25) below implies that pz is less than h + σpz. This allows pz either to be less than or greater than h.■

Proof. The social problem for each h is to choose x and z to maximize

$$(23)\quad \int_{p(x)z}^{\infty} (b - h - \sigma p(x)z) f(b)db - x.$$

Let us now demonstrate two claims.

(i) If $x^*(h) > 0$, then $z^*(h) = \bar{z}$: Assume otherwise, that $x^* > 0$ and $z^* < \bar{z}$. Raise z to $\bar{z}$ and lower x to x' such that $p(x')\bar{z} = p(x^*)z^*$. Then the integral in (23) is unchanged -- the same individuals commit the act and the expected social cost of punishment $\sigma p(x)z$ is unaltered -- but enforcement effort is lower. Thus, (23) is higher, a contradiction.[17]

(ii) $x^*(h) = 0$ for all h sufficiently low. $x^*(h) > 0$ for some h. If $x^*(h) > 0$, it increases with h: From (i), we know that if x^* is positive, x^* maximizes

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[17]It is straightforward to verify that the conclusion that optimal non-monetary sanctions are maximal continues to hold if the marginal disutility of sanctions increases with their magnitude (imprisonment becomes harder as time passes) or if the marginal social cost of imposing sanctions decreases. On the other hand, the conclusion does not necessarily hold if the marginal disutility of sanctions decreases or if the marginal social cost of imposing sanctions increases.

$$(24)\quad \int_{p(x)\bar{z}}^{\infty}(b - h - \sigma p(x)\bar{z})f(b)db - x$$

over x. Using (24), it can be shown that $x^* = 0$ if h is sufficiently small.[18] The derivative of (24) with respect to x is

$$(25)\quad -p'(x)\bar{z}(p\bar{z} - h - \sigma p\bar{z})f(p\bar{z}) - \sigma p'(x)\bar{z}(1 - F(p\bar{z})) - 1.$$

The first term of (25) is the marginal gain due to increased deterrence, and the second term is the social cost due to imposing sanctions with greater likelihood (F is the cumulative distribution function of f). At $x = 0$, (25) equals $p'(0)\bar{z}hf(0) - 1$, which is positive for h sufficiently large. Hence, $x^*(h) > 0$ for h sufficiently large. If $x^*(h) > 0$, it is determined by the first-order condition that exp.(25) equals zero. Moreover, since the partial derivative of (25) with respect to h is positive, we know that x^* rises with h if x^* is positive.∎

If enforcement effort is general, the social problem is to choose y and z(h) to maximize

$$(26)\quad \int_0^{\infty}\int_{p(y)z(h)}^{\infty}(b - h - \sigma p(y)z(h))f(b)dbg(h)dh - y,$$

and we have, assuming as before that $y^* > 0$,

Proposition 2b. Suppose that enforcement effort is general and that non-monetary sanctions alone are employed. Then for all harms below a threshold, the optimal sanction is zero; above this threshold optimal

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[18] An indirect argument demonstrates this. Consider the problem of maximizing social welfare (24) assuming that $\sigma = 0$. It is clear that, for any $x > 0$, (24) is higher if $\sigma = 0$ than if σ is positive. Hence, if $x = 0$ maximizes (24) when $\sigma = 0$ for all h sufficiently low, $x = 0$ must maximize (24) for such h when σ is positive as well. But when $\sigma = 0$, the problem of maximizing (24) is identical in form to maximizing (10) ($\bar{z}$ plays the role of w). And for this problem we know that $x = 0$ is optimal for all h sufficiently small.

sanctions are positive and rise with the level of harm, attaining the maximal level, $\bar{z}$, for all harms beyond some point.■

Remarks. It is best for no sanctions to be imposed for small h because of the social cost of imposing sanctions; this is a difference, note, from the situation in Proposition 1b. Optimal sanctions are not always maximal when they are positive because the same probability p* applies to all h; at p*, less than the maximal sanction is called for for low harms.■

Proof. (i) $z^*(h) = 0$ for all h sufficiently small. $z^*(h) > 0$ for some h. If $0 < z^*(h) < \bar{z}$, then $z^*(h)$ increases with h: Given p, the social problem for any h is to maximize over z

$$(27)\quad \int_{pz}^{\infty} (b - h - \sigma pz) f(b)db.$$

The derivative of this with respect to z is

$$(28)\quad -p(pz - h - \sigma pz)f(pz) - \sigma p(1 - F(pz)).$$

When $z = 0$, (28) equals $phf(0) - \sigma p$. Hence, for h sufficiently small (28) is negative, and $z = 0$ is a local maximum; $z = 0$ can also be shown to be a global maximum.[19] Also, because when $z = 0$ (28) is positive for h

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[19]Since (28) is negative when $h = 0$ and $z = 0$ and is continuous in h and z, (28) must be negative for all h and z in some square $[0,\delta]x[0,\delta]$, where $\delta > 0$. Hence, if there is a global maximum at a positive z at any h in $[0,\delta]$, z must be above δ. If $z > \delta$, then for any $h < p\delta$, individuals with b in $[h,p\delta]$ are discouraged from committing acts (the expected sanction is $pz > p\delta$) even though social welfare would be increased if they did commit harmful acts. Hence, there is a loss relative to first-best behavior of at least

$$(a)\quad \int_{h}^{p\delta} (b - h)f(b)db.$$

(There is also a loss due to the social cost of imposing sanctions.) But the only loss relative to first-best behavior if $z = 0$ is

$$(b)\quad \int_{0}^{h} (b - h)f(b)db.$$

For all h sufficiently small, (b) is dominated by (a), so that $z > \delta$ cannot be optimal for such h. Hence, $z = 0$ must be the global optimum for all such small h, as claimed.

sufficiently large, $z^*(h) > 0$ for such h. If z is an interior optimum, it is determined by the first-order condition

(29) $-p(pz - h - \sigma pz)f(pz) = \sigma p(1 - F(pz))$.

Since the partial derivative of this with respect to h is $pf(pz) > 0$, $z^{*\prime}(h) > 0$.

(ii) If $z^*(h) = \bar{z}$ for some h, it equals $\bar{z}$ for all higher h: If not, then (assuming that $z^*(h)$ is continuous), $z^*(h)$ must fall with h over some region, but this contradicts (i).

(iii) $z^*(h) = \bar{z}$ for all h sufficiently high: If not, then (ii) implies that $z^*(h) < \bar{z}$ for all h. Since, by (i), $z^*(h)$ is positive for all h sufficiently large, we have that $0 < z^*(h) < \bar{z}$ for all h sufficiently large. This leads to a contradiction. On one hand, (29) must hold for h sufficiently large since $z^*(h)$ is an interior solution. On the other hand, $z^*(h)$ must approach a limit, say $\hat{z}$ as $h \to \infty$, since $z^*(h)$ is, by (i), increasing in h. But as $h \to \infty$, the left-hand side of (29) tends toward $-p(p\hat{z} - h - \sigma p\hat{z})$, which grows unboundedly, whereas the right-hand side of (29) tends toward $\sigma p(1 - F(p\hat{z}))$. Thus, (29) cannot hold as $h \to \infty$, a contradiction.■

When enforcement effort is both general and specific, social welfare equals

$$(30)\quad \int_0^{\infty}\int_{p(x(h),y)z(h)}^{\infty}(b - h - \sigma pz)f(b)dbg(h)dh - \int_0^{\infty}x(h)dh - y,$$

and, assuming, as before, that $y^* > 0$ and that $x^*(h) > 0$ for some h, we have

Proposition 2c. Suppose that general enforcement effort may be augmented by specific enforcement effort and that non-monetary sanctions alone are employed. Then the optimal sanction is at first zero and

subsequently rises with harm until it equals the maximal amount $\bar{z}$. Optimal specific enforcement effort is zero until sanctions become maximal, after which optimal specific enforcement effort becomes positive.■

I will only sketch the arguments; given what has been said, it will be apparent that the claims can be established and it would be tedious to supply all the details. Observe first that $x^*(h) > 0$ implies that $z^*(h) = \bar{z}$. For the social problem for any h is to choose x and z to maximize

$$(31)\quad \int_{p(x,y)z}^{\infty}(b - h - \sigma pz)f(b)db - x.$$

Assume that $x^* > 0$ but that $z^* < \bar{z}$. Raise z to $\bar{z}$ and lower x so that pz is constant. Then the integral in (31) is constant, but since x is lower, (31) is higher, a contradiction. Since $x^*(h)$ is 0 until $z^*(h)$ is at its maximum, $z^*(h)$ is determined essentially as described in Proposition 2b until $z^*(h)$ equals $\bar{z}$. (That $z^*(h)$ must equal $\bar{z}$ at some point is implied by the assumption that $x^*(h) > 0$ at some point; for if $z^*(h) < 0$ for all h, then we have just shown that $x^*(h)$ must be zero for all h.) If $z^*(h)$ equals $\bar{z}$ and $x^*(h) > 0$, $x^*(h)$ must increase with h, by the essentially the reasoning in (ii) of the proof of Proposition 2a.

C. Sanctions Are of Combined Form

If sanctions may be monetary as well as non-monetary and enforcement is specific, the social problem is to choose x(h), s(h), and z(h) to maximize

$$(32)\quad \int_0^{\infty}\int_{p(x(h))(s(h)+z(h))}^{\infty}(b - h - \sigma p(x(h))z(h))f(b)dbg(h)dh - \int_0^{\infty}x(h)dh,$$

and we have

Proposition 3a. Suppose that enforcement effort is specific and that both monetary and non-monetary sanctions may be employed. Then for all harms h below a threshold, optimal enforcement effort is zero. Above this

threshold, optimal enforcement effort is positive; it may or may not increase with h if the non-monetary sanction is not maximal, but does increase with h if the non-monetary sanction is maximal. The optimal monetary sanction is always maximal, equal to wealth, but the non-monetary sanction may not be maximal.∎

Remarks. It is worth discussing why the non-monetary sanction may not be maximal (and could be zero) when enforcement effort is positive. By now familiar logic, the optimal monetary sanction equals wealth w. This, however, means that the argument of Becker does not necessarily apply to non-monetary sanctions. Specifically, suppose that the non-monetary sanction z is less than maximal, and raise z slightly and lower enforcement effort so that the expected sanction is held constant. But when enforcement effort and the likelihood of apprehension p are lowered, the likelihood of imposing the monetary sanction w is lowered. This means that to maintain the level of the expected sanction, p cannot be reduced in proportion to the increase in z; p must be reduced less than in proportion. This implies that the social cost of imposing non-monetary sanctions rises, so that it is not clear that social welfare rises. (This argument is what is supplied in the proof below at (34).)∎

Proof. Given h, the problem is to choose x, s, and z to maximize

$$(33)\quad \int_{p(x)(s+z)}^{\infty} (b - h - \sigma p(x)z) f(b)db - x.$$

I establish several claims about the solution.

(i) If $x^*(h) > 0$, then $s^*(h) = w$: Assume otherwise, that $x^* > 0$ and $s^* < w$. Raise s to w and lower x to x′ such that $p(x')(w + z) = p(x^*)(s^* + z)$. Then the integral in (33) can only rise: the same individuals commit the act

and the social cost of imposing non-monetary sanctions, $\sigma p(x')z$, falls if $z > 0$. Since enforcement effort is lower, (33) is higher, a contradiction.

(ii) If $x^*(h) > 0$, then $z^*(h) < \bar{z}$ is possible: Assume that $x^* > 0$ and that $z^* = \bar{z}$. Lower z slightly to z' and raise x to x' so as to keep the expected sanction constant. Thus, the set of individuals who commit the harmful act is unchanged given z' and x'. However, the expected non-monetary sanction <u>falls</u>: Since, by (i), $s^* = w$, we have

(34) $p(x')(w + z') = p(x^*)(w + \bar{z})$

or, equivalently,

(34') $p(x')z' = p(x^*)\bar{z} - (p(x') - p(x^*))w;$

the term $(p(x') - p(x^*))w$ is positive since $x' > x^*$. Hence, $p(x')z' < p(x^*)\bar{z}$, as asserted. Therefore, the integral in (33) rises by

(35) $\sigma(p(x') - p(x^*))w(1 - F(p(x^*)(w + \bar{z}))$.

If σ is sufficiently high, (35) will exceed the increase in enforcement effort, $x' - x^*$,[20] and (33) will rise, a contradiction. Hence, $z^* < \bar{z}$ will hold. (It should be noted that the argument just supplied does not establish that $z^* < \bar{z}$ must hold, only that it can hold.)

If $0 < z^* < \bar{z}$, then z^* is determined by the first-order condition

(36) $-p(x)(p(w + z) - h - \sigma pz)f(p(w + z))$

$$- \sigma p(1 - F(p(w + z)) = 0.$$

(iii) $x^*(h) = 0$ for all h sufficiently low. If $x^*(h) > 0$ and $z^*(h) < \bar{z}$, $x^*(h)$ may increase or decrease with h, as may $z^*(h)$; if $x^*(h) > 0$ and $z^*(h) = \bar{z}$, then x^* increases with h: That $x^*(h) = 0$ if h is sufficiently small follows from an argument similar to that given above.

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[20] The derivative of (35) with respect to x', evaluated at x^*, is $\sigma p'(x^*)w(1 - F)$; the derivative of the increase in enforcement effort is 1; hence, if σ is such that $\sigma p'(x^*)w(1 - F) > 1$, it is clear that the statement in the text is valid.

If $x^* > 0$, it is determined by the first-order condition

$$(37)\quad -p'(x)(w + z)(p(w + z) - h - \sigma pz)f(p(w + z)) - \sigma p'(x)z(1 - F(p(w + z)) - 1 = 0.$$

If z^* is determined by (37), the signs of $x^{*\prime}(h)$ and of $z^{*\prime}(h)$ can be positive or negative: if one differentiates (36) and (37) implicitly with respect to h and solves for $x^{*\prime}(h)$ and $z^{*\prime}(h)$, one finds that the sign of each may be positive or negative.[21]

If $z^* = \bar{z}$ and the constraint (2) is binding, the sign of $x^{*\prime}(h)$ equals the partial derivative of the left-hand side of (37) with respect to h, which is positive.∎

If enforcement effort is general, the social problem is to choose y, s(h), and z(h) to maximize

$$(38)\quad \int_0^\infty \int_{p(y)(s(h)+z(h))}^\infty (b - h - \sigma p(y)z(h))f(b)dbg(h)dh - y,$$

and assuming that y^* is positive, we will show

Proposition 3b. Suppose that enforcement effort is general and that both monetary and non-monetary sanctions may be employed. Then for all harms below the threshold p^*w, the optimal sanction is purely monetary and equals h/p^*; the expected sanction thus equals the harm h and rises with harm. Above the threshold, the optimal monetary sanction is maximal, equal to wealth w, and the optimal non-monetary sanction is at first zero and then becomes positive, in which case it increases with harm.∎

Remarks. The Proposition is illustrated in Figure 4. The reason that purely monetary sanctions are initially employed is that it is wasteful to

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[21]The explanation is that x and z can be substitutes or complements; when x and the probability of apprehension rise, it may be worthwhile reducing the socially costly sanction z since the expected sanction can be preserved; but it also may be worthwhile increasing the sanction z to take advantage of the higher probability of apprehension.

General Enforcement Effort

with Monetary and Non-Monetary Sanctions

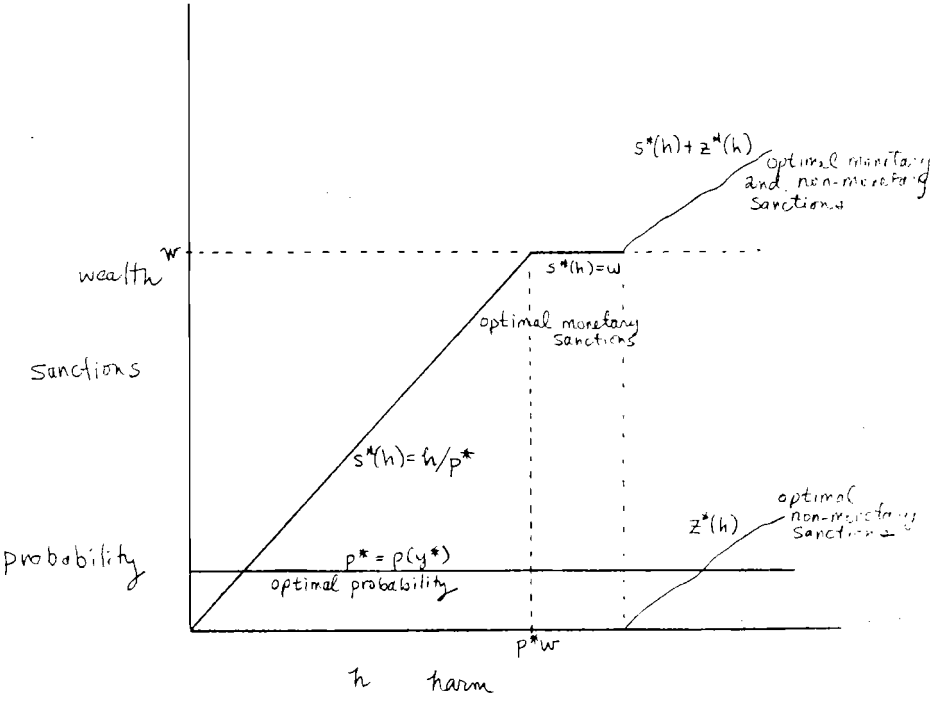

FIGURE 4

impose socially costly non-monetary sanctions when socially costless monetary sanctions can be used in their place. However, beyond the threshold p^*w, the wealth constraint on monetary sanctions implies that there is underdeterrence, and it thus becomes desirable to employ non-monetary sanctions as well.∎

Proof. (i) $s^*(h) < w$ implies $z^*(h) = 0$; that is, non-monetary sanctions are not employed unless maximal monetary sanctions are: Given p, the problem for any h is to choose s and z to maximize

$$(39) \quad \int_{p(s+z)}^{\infty} (b - h - \sigma pz) f(b) db.$$

Assume that $s^* < w$ but $z^* > 0$. Then increase s slightly and decrease z by the same amount, so that their sum is constant. This means that the same individuals commit the act, but since z is lower, the integrand is higher, so (39) is higher, a contradiction.

(ii) If $h < p^*w$, then $s^*(h) = h/p^*$ and $z^*(h) = 0$: From (i), we know there exist two possibilities for the solution to (39): that $s < w$ and $z = 0$, or that $s = w$ and $z \geq 0$. If $s < w$ and $z = 0$, the problem (39) is the same as the problem with monetary sanctions (14). But for this problem, we know that the optimal s is h/p^* (which is less than w). On the other hand, if $s = w$, then (39) is clearly less than if $s = h/p^*$ and $z = 0$. (If $s = w$, fewer individuals for whom $b > h$ commit the act since $p^*w > h$; and if they do commit the act, the integrand will be lower if $z > 0$.) Hence, $s^* = h/p^*$ and $z^* = 0$, as claimed.

(iii) If $h \geq p^*w$, then $s^*(h) = w$: If $s^* < w$, then by (i) $z^* = 0$. Hence, (39) becomes (14). But since $p^*s^* < p^*w \leq h$, increasing s increases (14). This contradicts the supposition that s^* was optimal.

(iv) $z^*(h) = 0$ in an interval $[p^*w, h']$, where $h' > p^*w$: From (iii), we know that for any $h \geq p^*w$, (39) is

$$(40) \quad \int_{p^*(w+z)}^{\infty} (b - h - \sigma p^* z) f(b) db.$$

The derivative of (40) with respect to z is

$$(41) \quad -p^*(p^*(w + z) - h - \sigma p^* z) f(p^*(w + z)) - \sigma p^*(1 - F(p^*(w+z))).$$

Evaluating (41) at $h = p^*w$, we obtain $-p^*(p^*z - \sigma p^* z) f(p^*(w + z)) - \sigma p^*(1 - F(p^*(w+z))$. At $z = 0$, this equals $-\sigma p^*(1 - F(p^*w))$, so that $z = 0$ is a local maximum. By continuity, (41) is negative at $z = 0$ for h in a neighborhood above $h = p^*w$, so that $z = 0$ is a local maximum in such a neighborhood. By an argument analogous to that in note 19, $z = 0$ can also be shown to be a global maximum in a neighborhood above $h = p^*w$.

On the other hand, (41) evaluated at $z = 0$ is $-p^*(p^*w - h) f(p^*w) - \sigma p^*(1 - F(p^*w))$. Since this is positive for h sufficiently large, $z^*(h) > 0$ for such h.

(v) If $0 < z^*(h) < \bar{z}$, then $z^*(h)$ increases with h: This follows since the partial derivative of (41) with respect to h is positive.∎

If enforcement effort is both general and specific, the social problem is to choose x(h), y, s(h), and z(h) to maximize social welfare,

$$(42) \quad \int_0^{\infty} \int_{p(x(h),y)(s(h)+z(h))}^{\infty} (b - h - \sigma p z(h)) f(b) db g(h) dh - \int_0^{\infty} x(h) dh - y,$$

and, assuming as before that y^* is positive and that $x^*(h)$ is positive for some h, we have

Proposition 3c. Suppose that general enforcement effort may be augmented by specific effort and that sanctions may be both monetary and non-monetary. Then below the threshold p^*w, the optimal sanction is purely monetary and equals h/p^* -- so that the expected sanction equals the harm h

and rises with the level of harm; also, optimal specific enforcement effort is zero. Beyond h/p*, the optimal monetary sanction is maximal, wealth, and optimal non-monetary sanctions and specific enforcement effort eventually become positive.■

I will only outline the argument. First, $s^*(h) < w$ implies $z^*(h) = 0$ and $x^*(h) = 0$; that is, neither non-monetary sanctions nor specific enforcement effort is employed unless maximal monetary sanctions are imposed. This is true because the social problem for any h is to choose x, s, and z to maximize

$$(43) \quad \int_{p(x,y)(s+z)}^{\infty} (b - h - \sigma pz) f(b)db - x.$$

If $s^* < w$ but $z^* > 0$, then by increasing s slightly and reducing z, so that s + z is constant, (43) can be raised. Hence z^* equals 0. Second, $s^*(h) < w$ implies $x^*(h) = 0$, for if $s^* < w$ and $x^* > 0$, then by increasing s slightly and reducing x, ps can be held constant. Thus the integral in (43) will be unchanged (for z^* must equal 0, as just shown), and (43) will therefore rise since x is lower. Hence, x^* equals 0.

Also, using the arguments in the proof of Proposition 3b, it may be shown that $s^*(h)$ equals $h/p(0,y^*)$ -- so that the expected sanction is h -- over the interval $[0,p(0,y^*)w]$. Beyond this interval, monetary sanctions are maximal, and non-monetary sanctions and specific enforcement effort become positive.

III. Conclusion

It is worthwhile indicating how relaxation of several of the assumptions of the model would alter the conclusions. One assumption was that all individuals have the same wealth. Were wealth allowed to vary among individuals, then, presumably, it would become optimal to impose non-

monetary sanctions on those who, because of their inadequate wealth, could not pay an otherwise optimal solely monetary sanction. In other words, for certain violations, relatively wealthy individuals would bear only monetary sanctions, while other individuals would suffer non-monetary sanctions as well (and the lower their wealth, the higher the non-monetary sanctions).

Another assumption was that general enforcement effort resulted in the same probability of apprehension for those committing different harmful acts. This assumption could be altered to allow for the effect of general enforcement effort to vary according to the act. We know in fact that when a policeman walks a beat, the likelihood of his apprehending different types of violators is different; the chance of his catching a burglar may be lower than the chance of his catching a person who commits an assault. In formal terms, the probability of apprehension p could be a function not only of general enforcement effort y but also of the type of act h, that is, $p = p(y,h)$. Were this the assumption, the formula for (less than extreme) optimal sanctions would be $h/p(y^*,h)$ rather than $h/p(y^*)$. Hence, it might not be the case that sanctions rise with harm; for if p happens to rise with h over some range, optimal sanctions might fall.

A similar assumption was that the probability of apprehension was the same function of specific enforcement effort for each type of harmful act. Were this assumption altered, the conclusions would change in obvious ways; for instance, optimal specific enforcement effort would tend to be higher than we found where such effort would be very productive in raising the probability.

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