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TESTING PORTFOLIO EFFICIENCY WITH CONDITIONING INFORMATION

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The Capital Asset Pricing Model (CAPM, Sharpe, 1964) implies that a market portfolio should be mean variance efficient. Multiple-beta asset pricing models such as Merton (1973) imply that a combination of the factor portfolios is minimum variance efficient (Chamberlain, 1983; Grinblatt and Titman, 1987). The consumption CAPM implies that a maximum correlation portfolio for consumption is efficient (Breeden, 1979). More generally, any stochastic discount factor model implies that a maximum correlation portfolio for the stochastic discount factor is minimum variance efficient (e.g., Hansen and Richard, 1987).

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ABSTRACT

We develop asset pricing models' implications for portfolio efficiency when there is conditioning information in the form of a set of lagged instruments. A model of expected returns identifies a portfolio that should be minimum variance efficient with respect to the conditioning information. Our tests refine previous tests of portfolio efficiency, using the conditioning information optimally. We reject the efficiency of all static or time-varying combinations of the three Fama-French (1996) factors with respect to the conditioning information and also the conditional efficiency of time-varying combinations of the factors, given standard lagged instruments.

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Introduction

Testing the efficiency of a given portfolio has long been an important topic in empirical asset pricing. The Capital Asset Pricing Model (CAPM, Sharpe, 1964) implies that a market portfolio should be mean variance efficient. Multiple-beta asset pricing models such as Merton (1973) imply that a combination of the factor portfolios is minimum variance efficient (e.g., Chamberlain, 1983; Grinblatt and Titman, 1987). The consumption CAPM implies that a maximum correlation portfolio for consumption is efficient (Breeden, 1979). More generally, stochastic discount factor models imply that a maximum correlation portfolio for the stochastic discount factor is minimum variance efficient (e.g., Hansen and Richard, 1987).

Classical efficiency tests, as studied by Gibbons (1982), Jobson and Korkie (1982), Stambaugh (1982), MacKinlay (1987), Gibbons, Ross and Shanken (1989) and others, ask if a tested portfolio lies "significantly" inside a sample mean variance boundary. These studies form the boundary from fixed-weight combinations of the tested asset returns. However, many studies in asset pricing now condition on predetermined variables to model conditional expected returns, correlations and volatility, and portfolio weights may be functions of the predetermined variables. This paper considers tests of portfolio efficiency in the presence of such conditioning information.

Recent studies using conditioning information expand the set of returns by including a specific collection of *ad-hoc* "dynamic strategies" based on the information. For example, the "factors" or assets' returns may be multiplied by lagged instruments, as in Shanken (1990), Hansen and Jagannathan (1991), Cochrane (1996), Jagannathan and Wang (1996) or Ferson and Schadt (1996). This "multiplicative" approach corresponds to dynamic strategies whose portfolio weights are linear functions of the lagged instruments. In this paper we develop tests of efficiency where the dynamic strategies include *all possible* portfolios formed from a given set of returns, with weights that may be any well-behaved function of the given conditioning information. This expands the set of portfolio returns to the maximum possible extent, thereby using the conditioning information efficiently.

Our paper contributes more to the literature than the specific efficiency tests. We develop a new framework for testing asset pricing theories in the presence of conditioning information. Our framework uses the concept of "unconditional" efficiency as defined by Hansen and Richard (1987). We refer to this concept, using more descriptive language, as efficiency *with respect to the information*, Z. We develop the framework by analogy to well-known results for testing portfolio efficiency when conditioning information is ignored. Along the way, we present generalizations for a number of classical results.

The primary empirical motivation for our refinement of the way conditioning variables are employed is to use the information efficiently. This is important in view of recent evidence calling into question the usefulness of standard lagged instruments, once bias and sampling errors are accounted for (e.g. Ghysels (1997), Carlson and Chapman (2000), Goyal and Welch (2003, 2004), Simin (2003), Ferson, Sarkissian and Simin, 2003). Another motivation is tractability. In a multiplicative approach, with N asset returns and L lagged instruments, a $NL \times NL$ covariance matrix must be inverted. In our approach the matrices are $N \times N$. The third motivation is robustness. As discussed below, our approach should be robust to certain misspecifications. We find that the multiplicative approach, using standard instruments and adjusting for sampling errors, typically has no more ability to reject models than tests that ignore the conditioning information altogether. Our tests that use the same information efficiently perform better. We find that the new tests can reject efficiency in settings where traditional tests do not.

The rest of the paper is organized as follows. Section 1 further motivates tests of minimum variance efficiency with respect to conditioning information and presents the main ideas. Section 2 develops the tests. The data are described in Section 3 and section 4 presents the empirical results. The robustness of the results is addressed in Section 5. Section 6 concludes the paper.

1. Asset Pricing, Portfolio Efficiency and Conditioning Information

Empirical work in asset pricing is often motivated by the fundamental valuation equation:

$$E\{m_{t+1}R_{t+1}|Z_t\} = \underline{1}, \tag{1}$$

where R_{t+1} is an N-vector of test asset gross returns, Z_t is the *conditioning information*, a vector of observable instrumental variables in the public information set at time t, m_{t+1} is a *stochastic discount factor*, and $\underline{1}$ is an N-vector of ones. Most asset pricing models imply a specification for the stochastic discount factor.

A common approach to testing an asset pricing model is to examine necessary conditions of (1) using a method like the Generalized Method of Moments (GMM, see Hansen, 1982). For example, multiplying both sides of (1) by the elements of Z_t and then taking the unconditional expectations leads to a *multiplicative approach*:

$$E\{m_{t+1}(R_{t+1} \otimes Z_t)\} = E\{\underline{1} \otimes Z_t\}. \tag{2}$$

Equation (2) asks the stochastic discount factor to "price" the dynamic strategy payoffs, $R_{t+1} \otimes Z_t$, on average, where $E\{\underline{1} \otimes Z_t\}$ are the average prices. However, the multiplicative approach in Equation (2) captures only a portion of the information in Equation (1).

Equation (1) is equivalent to the following holding for *all* bounded integrable functions f(.):

$$E\{m_{t+1}[R_{t+1}f(Z_t)]\} = E\{\underline{1}f(Z_t)\}. \tag{3}$$

Clearly, Equation (2) is a special case of (3), which may be seen by stacking (3) while taking $f\left(Z_t\right)$ to be each of the instruments in turn. Thus, Equation (2) asks the stochastic discount factor to price only a subset of the strategies allowed by Equation (3).

In this paper we develop tests of asset pricing models based on the following version of Equation (3):

$$E\{m_{t+1}x'(Z_t)R_{t+1}\}=\underline{1} \quad \forall x(Z_t): x'(Z_t)\underline{1}=1\,. \tag{4}$$

Equation (4) uses all portfolio weight functions $x(Z)$ in place of the general functions in Equation (3), subject only to the restrictions that the weights are bounded integral functions that sum to 1.0. Equation (4) follows by multiplying (1) by the elements of the portfolio weight vector $x(Z)$ and summing, using the fact that the weights sum to 1.0, then taking the unconditional expectation.[1]

While studies of conditional asset pricing typically use Equation (2), our objective is to move to Equation (4). There are several strong motivations. The first is to use the information in Z_t efficiently. The intuition is that if we ask the model to price a larger set of dynamic strategies, a smaller set of m_{t+1}'s can do the job, so the tests will be able to reject more models. Equation (4) requires the asset pricing expression to hold for *all* portfolio strategies using Z_t, whereas Equation (2) is restricted to the particular ad hoc strategy in which Z_t is used multiplicatively.

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[1] Because of the portfolio weight restriction, Equation (4) is an implication of, not equivalent to (3). However, in practice (4) is unlikely to leave out much, compared with (3). In Equation (4), the portfolio weights almost always to sum to 1.0 at each realization of *Z*. In equation (3), since both sides of the equation may be arbitrarily scaled by a constant, the unconditional expectation of the portfolio weights sums to 1.0 without loss of generality (see Abhyankar, Basu and Stremme, 2002). Restricting to weights that almost always sum to 1.0 in Equation (4) allows us to work with portfolio returns and portfolio efficiency concepts, as opposed to asset prices and payoffs. Working with prices and payoffs, it would be necessary in any event, to normalize the prices to achieve stationarity for empirical work.

The second motivation for using Equation (4) is tractability. While it may seem difficult to work in the infinite-dimensional space of all possible *x*(Z), closed-form solutions in Ferson and Siegel (2001) provide tractable expressions from which we construct the tests. Implementing the solutions with *N* test assets requires *NxN* covariance matrices, whereas the multiplicative approach requires us to invert matrices with the dimension of ($R \otimes Z$).

The third motivation for our approach is potential robustness. Ferson and Siegel (2001) show that the expressions we use in our tests are likely to be robust to extreme observations. Ferson and Siegel (2003) apply these expressions to the Hansen-Jagannathan (1991) bounds and find evidence of robustness in that setting. Bekaert and Liu (2004) argue that equation (4) is inherently robust to misspecifying the conditional moments of returns. The intuition is that with misspecified moments, the "optimal" *x*(Z) derived by Ferson and Siegel (2001) and used in our tests, is suboptimal. However, it remains a valid, if now ad-hoc, dynamic strategy. Thus the tests may sacrifice power, but remain valid. The key to obtaining these advantages is the relation of Equation (4) to the concept of minimum variance efficient portfolios.

A. Stochastic Discount Factors and Portfolio Efficiency

Minimum variance efficient portfolios are those which have minimum variance among portfolios with the same mean return. Stochastic discount factor models are related to portfolio efficiency because a specification for the stochastic discount factor indicates a portfolio that should be minimum-variance efficient. Consider first the special case where there is no conditioning information, and the asset pricing equation is $E(mR) = \underline{1}$. The following results are well known. Given portfolio return R_m, there exists a stochastic discount factor of the form $m = a + bR_m$, if and only if R_m is minimum variance efficient. An example is the classical CAPM of Sharpe (1964), as discussed by Dybvig and Ingersoll (1982). There exists a stochastic discount factor that is linear in a k-vector of benchmark returns or "factors," $R_B : m = A + B'R_B$, if and only if some combination of the factor returns is minimum variance efficient. This is the case of an exact k-factor beta pricing model, as discussed by Grinblatt and Titman (1987), Shanken (1987), and Ferson and Jagannathan (1996). Finally, if the stochastic discount factor is a fixed function of observable data and parameters: $m = m(X, \theta)$, a portfolio that maximizes the squared correlation with $m(X, \theta)$ must be minimum variance efficient. Examples include the consumption-based model of Lucas (1978) and Breeden (1979), and its more recent generalizations. See Ferson (1995) for a review of these results.

We extend these examples to the context of Equation (4). We show that a specification of the stochastic discount factor implies that particular portfolios are minimum variance efficient with respect to the information Z, as defined below. Using Equation (4), we then develop tests of the hypothesis that a portfolio is efficient in this sense.

B. Portfolio Efficiency with Respect to Conditioning Information

We first define efficiency with respect to the information, Z_t. Consider a portfolio of the N test assets in R_{t+1}, where the weights that determine the portfolio at time t are functions of the information, Z_t. The gross return on such a portfolio with weight $x(Z_t)$, is $x'(Z_t)R_{t+1}$. The restrictions on the portfolio weight function are that the weights must sum to 1.0 (almost surely in Z_t), and that the expected value and second moments of the portfolio return are well defined. Consider now all possible portfolio returns that may be formed, for a given set of test asset returns R_{t+1} and given conditioning information, Z_t. This set determines a mean-standard deviation frontier, as shown by Hansen and Richard (1987). This frontier depicts the *unconditional* means versus the *unconditional* standard deviations of the portfolio returns. A portfolio is defined to be efficient with respect to the information Z_t, if and only if it is on this mean standard deviation frontier.

Proposition 1: Given N test asset gross returns, R_{t+1}, a given portfolio with gross return $R_{p,t+1}$ is **minimum-variance efficient with respect to the information Z_t** if and only if Equation (5) is satisfied (equivalently, Equation (6) is satisfied) for all $x(Z_t) : x'(Z_t)\underline{1} = 1$ almost surely, where the relevant unconditional expectations exist and are finite:

$$Var\left(R_{p,t+1}\right) \le Var\left[x'(Z_t)R_{t+1}\right] \quad \text{if} \quad E\left(R_{p,t+1}\right) = E\left[x'(Z_t)R_{t+1}\right] \tag{5}$$

$$E\left[x'(Z_t)R_{t+1}\right] = \gamma_0 + \gamma_1 Cov\left[x'(Z_t)R_{t+1}; R_{p,t+1}\right]. \tag{6}$$

Equation (5) states that $R_{p,t+1}$ is on the minimum variance boundary formed by all possible portfolios that use the test assets and the conditioning information. Equation (6) states that the familiar expected return - covariance relation from Fama (1973) and Roll (1977) must hold with respect to the efficient portfolio. In Equation (6), the coefficients γ_0 and γ_1 are fixed scalars that do not depend on the functions $x(.)$ or the realizations of Z_t.

C. Efficiency with Respect to Information and Stochastic Discount Factors

Most asset pricing models specify some function for the stochastic discount factor. As a special case, linear factor models say that m is linear in one or more factors. Proposition 2 shows that when there is conditioning information, Z, testing linear stochastic discount factor models in Equation (4) amounts to testing for the efficiency of a portfolio of the factors with respect to Z.

Proposition 2: Given $\{R_{t+1}, Z_t\}$ and a stochastic discount factor m_{t+1} such that Equation (4) holds, then if $m_{t+1} = A + B' R_{B,t+1}$, where $R_{B,t+1}$ is a k-vector of benchmark factor returns, and A and B are a constant and a fixed k-vector, there exists a portfolio, $R_{p,t+1} = w'R_{B,t+1}$, $w'\underline{1} = 1$, where w is a constant N-vector, and $R_{p,t+1}$ is efficient with respect to the information Z_t.

Proof: See the Appendix for all proofs.

We now consider the case of a general $m = m(X, \theta)$, and allow for time-varying weights in the efficient portfolio of factors. This requires the definition of portfolios that are *maximum correlation with respect to Z.*

Definition: A portfolio R_P is **maximum correlation for a random variable, *m*, with respect to conditioning information *Z***, if:

$$\rho^2(R_p, m) \geq \rho^2[x'(Z)R, m] \quad \forall x(Z) : x'(Z)\underline{1} = 1, \tag{7}$$

where $\rho^2(.,.)$ is the squared unconditional correlation coefficient.

Proposition 3 If a given m satisfies Equation (4), then a portfolio R_P that is maximum correlation for m with respect to Z must be minimum variance efficient with respect to Z.

Proposition 2 is clearly a special case of Proposition 3, because if m_{t+1} is linear in $R_{B,t+1}$, a linear regression maximizes the squared correlation. More generally, given a stochastic discount factor, m, we can test the model by constructing a portfolio that is maximum correlation for m with respect to Z, and testing the hypothesis that the portfolio is efficient with respect to Z. Methods for constructing a maximum correlation portfolio with respect to Z are described below.

With the preceding results we can consider a case where the stochastic discount factor is linear in k factor-portfolios, allowing for time-varying weights.

Corrollary Given $\{R_{t+1}, Z_t\}$ and a stochastic discount factor m_{t+1} such that Equation (4) holds, then if a maximum correlation portfolio for m_{t+1} with respect to Z_t has nonzero weights only on the k-vector of benchmark factor returns $R_{B,t+1}$, an efficient-with-respect-to-*Z* portfolio of the factor returns $R_{B,t+1}$ is efficient with respect to *Z* in the full set of test asset returns.

With conditioning information, efficient portfolios generally have time-varying weights. The situation described in the Corrollary is a "dynamic" version of mean variance intersection, as developed by Huberman, Kandel and Stambaugh (1987). For example, one hypothesis that we consider below is that some combination (that depends on *Z*) of the three Fama and French (1996) factors is a mimicking portfolio for a stochastic discount factor. The test is to find the efficient, time-varying combination of the Fama-French factors and see if it is efficient with respect to *Z* in the sample of test assets.

D. Discussion

The presence of conditioning information impacts asset pricing models based on stochastic discount factors in three general ways. First, conditioning information relates to the set of payoffs we ask the model to price. Second, conditioning information relates to the specification of the functional form of the SDF. Third, the asset pricing statement, Equation (1), would ideally apply to conditional moments given a public information set Ω, but an empiricist can only measure *Z*, a proper subset of Ω.

The first issue with respect to conditioning information is the set of payoffs that we ask the model to price. By using the given conditioning information *Z* in different ways we generate different payoffs from the test assets, *R*. As explained above, our approach asks the model to price all portfolios $x(Z)'R$, where $x(Z)'1 = 1$. We thereby expand the set of payoffs, relative to approaches that ignore *Z* or use portfolio functions that are linear in *Z* or ad-hoc functions of *Z*. Expanding the set of payoffs, we restrict the set of m's that can price those payoffs. Our tests should therefore reject models that previous approaches would not reject.

The second related issue is the *functional* form of the SDF. Different asset pricing models imply different functional forms. Our maximum correlation approach can handle general functions of measurable data, $m(X,\theta)$. If we reject the efficiency with respect to *Z*, of a portfolio *that* has maximum correlation with $m(X,\theta)$ with respect to *Z*, we reject the hypothesis that $E(m(X,\theta)R\,|\,Z) = \underline{1}$. By iterated expectations, we therefore reject the model that says $E(m(X,\theta)R|\Omega) = \underline{1}$.

The third issue arises because the asset pricing theory says $E(mR|\Omega)=1$, but the full information set Ω cannot be measured. There are two cases. In the first case, the SDF is a known function of measurable data and parameters and we can test $E(mR|Z) = 1$, a necessary condition which follows from the law of iterated expectations. The inability to measure all of Ω results only in a potential loss of power in this case.

A more difficult case arises when the SDF, $m(\Omega)$, is a function of unobservable parts of Ω. In this case it is not known how to test a model that says $E(m(\Omega)R|\Omega) = 1$. While it remains true that $E(m(\Omega)R|Z)=1$, that is no help if $m(\Omega)$ can not be measured.[2] Hansen and Richard (1987) describe a version of this problem in terms of portfolio efficiency. Consider a conditional version of the CAPM in which $m(\Omega) = a(\Omega) + b(\Omega)R_m$ and the market portfolio R_m is *conditionally efficient* given Ω (meaning minimum conditional variance given Ω subject to the conditional mean return given Ω). Hansen and Richard show that the conditional efficiency of R_m given Ω does not imply conditional efficiency given *Z*. If we can only observe *Z* we can test the efficiency of R_m using *Z*, but such a test does not allow us to reject the conditional CAPM.

[2] Hansen and Jagannathan (1991) develop an SDF given by $m^* = E(m|R)$ and they show how to form the projection m^*. However, m^* can not be used to test the original model because it prices the returns by construction.

Cochrane (2001) calls this the "Hansen-Richard critique." By analogy with the Roll (1977) critique that the CAPM can't be tested because we can't measure the market portfolio, the Hansen-Richard critique implies that the *conditional* CAPM can't be tested (even if we could measure the market portfolio) because we can't measure all the information, Ω. This problem is by no means unique to our paper. In the spirit of virtually all empirical studies, we therefore focus on cases where the SDF is assumed to depend on measurable data only.

E. Testing Conditional Efficiency

Our approach is to test (unconditional) efficiency with respect to Z. An alternative approach is to test the *conditional* efficiency given Z, of a portfolio R_P. While such tests do not imply inferences about the efficiency given Ω, tests of conditional efficiency given observable instruments Z have nevertheless been of historical interest in the asset pricing literature. Hansen and Hodrick (1983) and Gibbons and Ferson (1985) test conditional efficiency given Z, restricting the functional forms of conditional means and betas. Campbell (1987) and Harvey (1989) restrict the form of a market price of risk. Shanken (1990) tests conditional efficiency restricting the form of the conditional betas. Tests of conditional efficiency given Z may be handled in our framework, as a specification of the functional form for $m(X,\theta)$.

The conditional efficiency of a portfolio R_P given Z is equivalent to the existence of an SDF, $m=a(Z) + b(Z)RP$, where $a(Z)$ and $b(Z)$ are particular functions of the conditional first and second moments of R_P and a zero-beta portfolio for R_P. We can also test for the conditional efficiency given Z of a combination of K factor-returns, R_B. In this case $m = A(Z) + B(Z)'R_B$, again with particular coefficients.[3] With our approach we test conditional efficiency given Z by constructing the maximum correlation portfolio to this particular m with respect to Z. The maximum correlation portfolio, call it R_p^*, should be efficient with respect to Z. Note that R_p^* will be different from R_P when the coefficients $a(Z)$ or $b(Z)$ are time varying as functions of Z. Rejecting the efficiency of R_P^* with respect to Z rejects the conditional efficiency of R_P given Z. This is an example of how conditional efficiency (given Z) does not imply unconditional efficiency (with respect to Z) of the same portfolio. However, conditional efficiency does identify a portfolio that should be efficient with respect to Z, and this implication can be tested.

2. Testing Efficiency

A. When There is no Conditioning Information

Classical tests for the efficiency of a given portfolio involve restrictions on the intercepts of a system of time-series regressions. If r_t is the vector of N *excess* returns at time t, measured in excess of a risk-free or zero-beta return, and $r_{p,t}$ is the excess return on the tested portfolio, the regression system is

$$r_t = \alpha + \beta r_{p,t} + u_t; \quad t = 1, \cdots, T; \tag{8}$$

where T is the number of time-series observations, β is the N-vector of betas and α is the N-vector of alphas. The portfolio r_p is minimum-variance efficient only if $\alpha=0$.

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[3] The coefficients are: $A(Z) = [1 + \Sigma_j \lambda_j E(R_{Bj}|Z)/var(R_{Bj}|Z)]/E(R_o|Z)$ and $B_j(Z) = -\lambda_j /[E(R_o|Z)var(R_{Bj}|Z)]$, where $\lambda_j = E(R_{Bj} - R_0|Z)$ and R_0 is the conditional zero-beta return for R_B (that is, $Cov (R_oR_p|Z) = 0$). When $R_{Bj} = R_p$ we have the single-factor coefficients. (See Ferson and Jagannathan, 1996).

It is well known that the classical test statistics for the hypothesis that α=0 in Equation (8) can be written in terms of the squared Sharpe ratios of portfolios (e.g., Jobson and Korkie, 1982). Consider the Wald Statistic:

$$W = T\hat{\alpha}\left[Cov\left(\hat{\alpha}\right)\right]^{-1}\hat{\alpha} = T\left(\frac{\hat{S}^2(R) - \hat{S}^2\left(R_p\right)}{1+\hat{S}^2\left(R_p\right)}\right) \stackrel{a}{\sim} \chi^2(N) \tag{9}$$

where $\hat{\alpha}$ is the OLS or ML estimator of α and $Cov(\hat{\alpha})$ is its asymptotic covariance matrix. The term $\hat{S}^2\left(R_p\right)$ is the sample value of the squared Sharpe ratio of R_p: $S^2\left(R_p\right) = \left[E\left(r_p\right)/\sigma\left(r_p\right)\right]^2$. The term $\hat{S}^2\left(R\right)$ is the sample value of the maximum squared Sharpe ratio that can be obtained by portfolios of the assets in R (including R_P):

$$S^2(R) = \max_x \left\{\frac{[E(x'r)]^2}{Var(x'r)}\right\}. \tag{10}$$

The Wald statistic has an asymptotic chi-squared distribution with N degrees of freedom.

Since the Sharpe ratio is the slope of a line in the mean-standard deviation space, Equation (12) suggests a graphical representation for the Wald statistic in the sample mean standard deviation space. It measures the distance between the sample frontier and the location of the tested portfolio, inside the frontier. Kandel (1984), Roll (1985), Gibbons, Ross and Shanken (1989) and Kandel and Stambaugh (1987,1989) develop this interpretation.

B. Tests with Conditioning Information

To illustrate using conditioning information efficiently, we employ statistics similar to the classical statistic, as in Equation (9). When conditioning information is used, the asymptotic distribution of the statistic in (9) is not known to be chi-squared, and there are many alternative statistics that we could use. Some of these may have better sampling properties. Thus, by moving to Equation (4) and conditioning information we raise some new statistical questions for future research. Our examples focus on the classical-looking statistic as a natural extension of the literature.

Classical tests that ignore conditioning information restrict the maximization in Equation (10) to *fixed-weight* portfolios, where x is a constant vector. In contrast, efficient portfolios with respect to the information Z maximize the squared Sharpe ratio over *all portfolio weight functions*, $x(Z)$. Maximizing over a larger set of weights we get a larger maximum Sharpe ratio. The Appendix describes the closed-form solutions from Ferson and Siegel (2001), for the portfolio weight functions that maximize the squared Sharpe ratio.

Jobson and Korkie (1982) show that the test statistic in Equation (9) may be interpreted as the relative performance of the portfolio of the test assets that is the "most-mispriced" by R_P. This portfolio is also called the "active" portfolio by Gibbons, Ross and Shanken (1989) and the "optimal orthogonal portfolio" by MacKinlay (1995). We use a version of this portfolio in our empirical examples. The portfolio has weights proportional to $\left[Cov(\hat{\alpha})\right]^{-1}\hat{\alpha}$ in the classical case with no conditioning information. With conditioning information the portfolio's weight function is time-varying. We derive the most mispriced portfolio for a general case with an arbitrary fixed "zero-beta" rate, γ_0.

Consider any portfolio formed from the test assets with weights x_P as $R_{P,t+1}=x_P'R_{t+1}$, where x_P may depend on Z_t. The portfolio has unconditional expected return $E(x_P'R_{t+1})=\mu_P$ and variance $Var(x_P'R_{t+1}) = \sigma_p^2$. The ***most mispriced portfolio***, R_C, with respect to R_P maximizes α_c^2/σ_c^2 where σ_c^2 is the variance of R_C, $\mu_c = E(R_c)$ and $\alpha_c + \mu_c - [\gamma_o + (\mu_p - \gamma_o)\sigma_{cp}/\sigma_p^2]$ is the alpha of R_C with respect to R_P, where $\sigma_{cp} = Cov(R_c, R_p)$. Let R_S be the portfolio return that maximizes the squared Sharpe ratio in

(10) over all portfolio weight functions $x(Z)$, when the excess returns $r \equiv R - \gamma_0$. The portfolio R_S has unconditional mean return μ_s and variance, σ_s^2.

Proposition 4: The most mispriced portfolio R_C with respect to a given portfolio R_P, may be found as a fixed linear combination of R_P and the efficient-with-respect to Z portfolio, R_S, that maximizes the squared Sharpe ratio for a given zero beta rate, γ_0, as:

$$R_C = \frac{\left(\frac{\mu_S - \gamma_0}{\sigma_S^2}\right) R_S - \left(\frac{\mu_P - \gamma_0}{\sigma_P^2}\right) R_P}{\left(\frac{\mu_S - \gamma_0}{\sigma_S^2}\right) - \left(\frac{\mu_p - \gamma_o}{\sigma_P^2}\right)}, \tag{11}$$

or

$$R_C = \left(R_S - \frac{\sigma_{SP}}{\sigma_P^2} R_P \right) \Big/ \left(1 - \frac{\sigma_{SP}}{\sigma_P^2} \right). \tag{12}$$

Proposition 4 extends the concept of the "active" or "optimal orthogonal" portfolio to the setting of efficiency with respect to given conditioning information. The most mispriced portfolio R_C has weights that depend on Z; these are presented with the proof in the Appendix. Note that the portfolio R_C is uncorrelated with R_P, according to Equation (12). The most mispriced portfolio is the projection of R_S, orthogonal to R_P, normalized so that the weights sum to 1.0. The portfolio R_P may be found by starting with R_S and then removing its component that is correlated with R_P. A combination of R_P and its most-mispriced R_C is an efficient portfolio with respect to Z.

C. Empirical Strategy

Our empirical examples compare the classical approach using no conditioning information, the multiplicative approach to conditioning information, and the efficient use of the conditioning information. The specifics depend on the example.

When we test the efficiency of a given portfolio, R_P, then $\hat{S}^2\left(R_p\right)$ is formed using the sample mean excess return and sample variance of R_P. $\hat{S}^2\left(R\right)$ differs according to the way conditioning information is used. When there is no conditioning information we use the fixed-weight solution to (10). When the information is used multiplicatively, we define an expanded set of returns as $\hat{R}_t = R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$, where R_{ft} is the one-month Treasury bill return for month t. We then proceed as in the previous case, using the returns $\hat{R}_t$ in place of R_t. When the information is used efficiently, $\hat{S}^2(R)$ is formed using the sample mean and variance of $\hat{x}'(Z)R$ where $\hat{x}(Z)$ is the sample version of the optimal solution from Ferson and Siegel (2001) described in the Appendix.

We evaluate the tests using simulations. To generate data consistent with the null hypothesis that a given portfolio R_P is efficient, we replace its return with a portfolio that is efficient, based on the specification of the asset-return moments in the simulation. With this substitution, we then construct the test statistic using the artificial data in the same way that we get the sample value of the statistic in the actual data. The details are discussed in the Appendix.

3. The Data

To model the conditioning information, we use a number of lagged variables that have long been prominent in the conditional asset pricing literature. These include: (1) the lagged value of a one-month Treasury bill yield (see Fama and Schwert (1977), Ferson (1989), Breen et al. (1989) or Shanken, 1990); (2) the dividend yield of the market index (see Fama and French, 1988); (3) the spread between Moody's Baa and Aaa corporate bond yields (see Keim and Stambaugh, (1986) or Fama, 1990); (4) the spread between ten-year and one-year constant maturity Treasury bond yields (see Fama and French, 1989) and (5); the difference between the one-month lagged returns of a three-month and a one-month Treasury bill (see Campbell, 1987).

We provide results using two alternative methods of grouping common stocks into portfolios. The first sample comprises twenty five industry portfolios (from Harvey and Kirby, 1996) measured for the period February, 1963 to December, 1994.[4] The portfolios are created by grouping common stocks according to their SIC codes and forming value-weighted averages (based on beginning-of-month values) of the total returns within each group of firms. Table 1 shows the industry classifications for the 25 portfolios, and summary statistics of the returns.

The second grouping follows Fama and French (1996). Individual common stocks are placed into five groups according to their prior equity market capitalization, and independently into five groups on the basis of their ratios of book value to market value of equity per share. This 5 by 5 classification scheme results in a sample of 25 portfolio returns. These are the same portfolios used by Ferson and Harvey (1999), who provide details and summary statistics.

This project has matured over a length of time, providing the opportunity to investigate the results over a “hold-out” sample. The hold-out sample period is January, 1995 through December, 2002. We use 25 size x book-to-market and Industry portfolios from Kenneth French

[4] We are grateful to Campbell Harvey for providing these data.

and update the other series with fresh data.[5] The hold-out sample results are interesting in view of recent evidence, cited above, that some of the lagged instruments may have lost their predictive power for stock returns in recent data. Table 1 illustrates this, reporting the adjusted R-squares from regressing the industry returns on the lagged instruments over the 1963-1994 period and the 1995-2002 sample. The R-squares are substantially lower in the more recent period.

4. Empirical Results

A. Inefficiency of the SP500 Relative to Industry Portfolios

Table 2 summarizes results for the 25 industry portfolios for the 1963-94 period, three ten-year subperiods and the holdout sample, 1995-2002. The tested portfolio, R_p, is the SP500. We use the average of the one-month Treasury bill to determine the zero-beta rate. In Panel A there is no conditioning information. Substituting the sample values of $\hat{S}^2(R_p)$ and $\hat{S}^2(R)$ into (9) gives the sample value of the test statistic. Referring to the asymptotic distribution, which is chi-squared with 25 degrees of freedom, the right-tail *p*-value is 0.48 for the full sample and $0.14-0.39$ in the subperiods. The test produces little evidence to reject the hypothesis that the SP500 is efficient in the industry portfolio returns over 1963-1994. During the holdout sample period the sample Sharpe ratios are substantially higher, and so is the value of the test statistic. The asymptotic *p*-value of the test is 0.001 for this period.

[5] We use a subset of the 48 value-weighted industry portfolios provided by French to match the definitions in Table 1. We confirm that the matched industry returns produce similar summary statistics and regression *R*-sequences on the lagged instruments as our original data, over the 1963-1994 period.

Panel A of Table 2 also reports 5% critical values and empirical *p*-values for the tests based on Monte Carlo simulation assuming normality, and based on a resampling approach that does not assume normality. Consistent with Gibbons, Ross and Shanken (1989) the Wald Test rejects a correct null hypothesis too often when the asymptotic distribution is used. The empirical *p*-values are larger than the asymptotic *p*-values in each subperiod, and the full sample period. The smallest empirical *p*-value in the panel is 0.43. Thus, when we correct for finite sample bias there is no evidence against the efficiency of the market index in the industry portfolios, given that no conditioning information is used in the tests.

Panel B of Table 2 summarizes tests using the "multiplicative" returns, $\hat{R}_t = R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$. With 25 industry portfolios, the market return and five instruments plus a constant, there are 156 "returns." One disadvantage of the multiplicative approach is that the size of the system quickly becomes unwieldy. It is not possible to construct the Wald Test for the ten year subperiods, as the sample covariance matrix is singular.

Over the full sample period the value of the Wald Test statistic using the multiplicative returns is 348.6. The asymptotic *p*-value is close to zero. However, we expect a finite-sample bias and the simulations confirm the bias. Based on the empirical *p*-values the tests reject the efficiency of the SP500 at either the 2% (Monte Carlo) or 40% (resampling) levels. Thus, the finite sample results are highly sensitive to the data generating process. This makes sense, because even if R_t is approximately normal, the products of returns and the elements of Z_{t-1} are not normal, and the Monte Carlo simulation assumes normality. We therefore place more trust in the resampling results. Correcting for finite sample bias with the resampling scheme, we find no evidence to reject the efficiency of the market index in the set of dynamic strategy returns that use the conditioning information multiplicatively.

Using the conditioning information Z efficiently, Panel C expands the tests to include *all* portfolios that may be functions of the information. With the efficient portfolio solutions the size of the covariance matrices to be inverted does not increase with the use of conditioning information, so results for the subperiods can be obtained. This illustrates the tractability of our approach, compared to the multiplicative approach. The value of the statistic given by Equation (9) is 161.84 in the full sample, $164.98 - 203.29$ in the ten-year subperiods and 148.2 in the holdout sample. The empirical p-values are 0.5% or less in the full sample and each ten-year subperiod, and 4.4% in the holdout sample. The results also are fairly robust to the method of simulation (Monte Carlo or resampling). Thus, we can reject the hypothesis that the market index is mean variance efficient when the conditioning information is used efficiently. The tests that use the conditioning information efficiently can reject the model when the multiplicative approach cannot. We even find marginal rejections during the holdout sample period, where Table 1 illustrated that the predictive power of the lagged instruments is relatively low.

Figure 1 illustrates the test, showing the sample frontier of fixed-weight portfolios that ignore the conditioning information and the efficient frontier with respect to Z. The test statistics are related to the differences between the squared slopes of the lines drawn through the SP500 versus the lines tangent to the frontiers. The figure shows how the efficient use of conditioning information produces a larger test statistic.

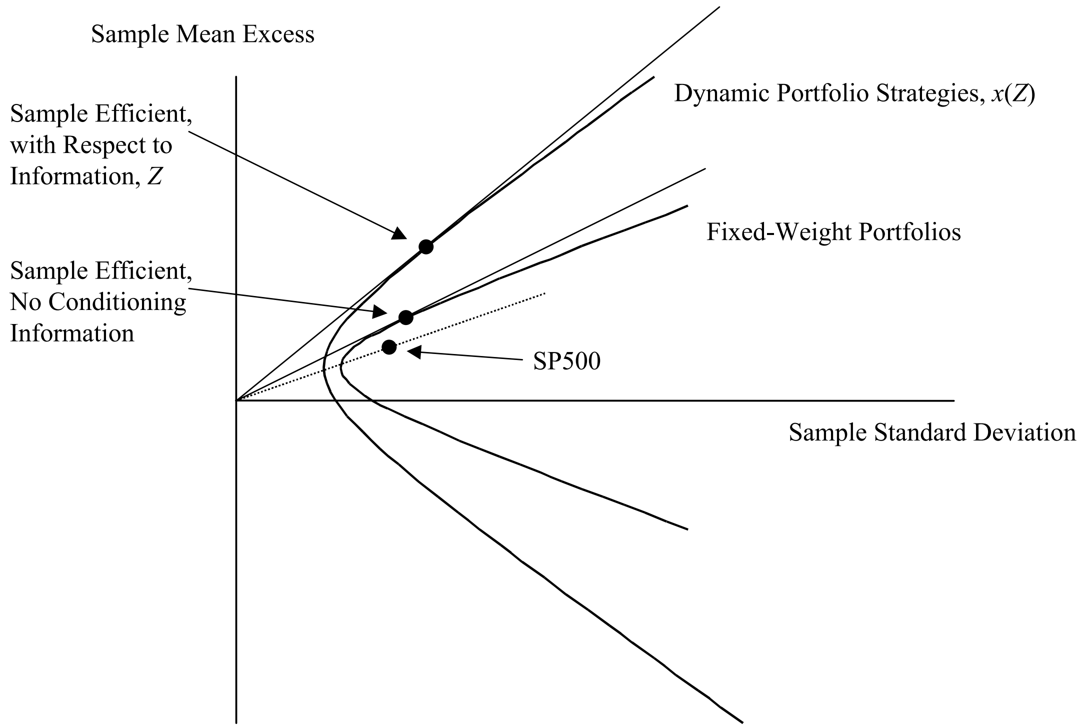

Figure 1. The test statistic for the efficiency of the SP500 compares the squared slope of the line through the tested portfolio with the line through the sample efficient portfolio. As the slopes diverge, the test statistic is larger. Testing for efficiency with respect to the information, Z, the test statistic is larger than when the information is ignored.

B. Alternative Test Assets

Recent studies use portfolios grouped on firm size and book-to-market ratios, and find that a market index is not efficient in these returns (e.g. Fama and French, 1992). Table 3 presents results where the portfolios are grouped on size and book-to-market. The full sample and holdout results for industries from Table 2 are repeated in the right hand column for comparison purposes.

In panel A of Table 3 there is no conditioning information. Consistent with previous studies, the efficiency of the SP500 is rejected in the size x book-to-market portfolio design for the 1963-1994 period. However, in the 1995-2002 period, the efficiency of the market index is not rejected when the finite sample bias in the statistics is corrected. This is consistent with a weakening of the size and book-to-market effects after 1994.

In panel B of Table 3, the test assets are the multiplicative returns. The asymptotic *p*-values suggest rejections of the efficiency hypothesis, but the resampling results indicate a strong, finite-sample bias. The empirical *p*-value based on resampling is 4.4% with the 25 size × book-to-market portfolios over the 1963-1994 sample.

In panel C the test assets are all portfolios of the form $x'(Z_{t-1})R_t$. The resampling *p*-values are 0.3% or less in the size x book-to-market design, including the 1995-2002 subsample. Thus, once again we find that efficiency can be rejected with our approach, in settings where the classical approach does not reject efficiency. The results show that expanding the set of dynamic strategies using our results makes a substantial difference, even in the size *x* book-to-market portfolio design.

C. Expanding the Mean Variance Boundary

The evidence so far shows that the market index return lies "significantly" inside the mean-variance boundaries when the conditioning information is used efficiently. However, these results only indirectly address the question of inferences about the mean variance boundaries themselves. These inferences relate to questions like mean variance intersection and spanning. If the Sharpe ratio of a given portfolio is estimated with greater precision than the maximum Sharpe ratio in a set of returns, as seems likely, then we may be able to draw inferences about efficiency for a given portfolio and yet be unable to draw reliable inferences about the efficient frontiers themselves.

In this section we ask if the use of conditioning information expands the mean variance boundary. Table 4 presents the tests. Here we replace the market index with a portfolio of the test assets whose weights are proportional to $\Sigma^{-1}\mu$, where Σ is the unconditional covariance matrix and μ the mean vector, that determines the excess returns of the test assets in the simulations. This is a portfolio on the "population" mean-variance boundary with no conditioning information. We then test the efficiency of this portfolio instead of the SP500, as in the previous tables. Of course, tests using no conditioning information find the portfolio to be efficient. In panel A the mean variance boundary is constructed using the multiplicative approach. The resampled *p*-values are 0.464 and 0.686, thus providing no evidence that the multiplicative approach expands the boundary. These results are consistent with studies such as Carlson and Chapman (2000) that question the usefulness of the standard lagged instruments in the multiplicative design.

In Panel B of Table 4 the test assets are all portfolios of the form $x'(Z_{t-1})R_t$. In the 1963-94 period the resampled *p*-values are 0.1% and 2.5% for the two portfolio grouping methods, showing that when the conditioning information is used efficiently the mean variance boundary is expanded. However, in the holdout sample we do not reject the null hypothesis. This is consistent with the low explanatory power of the lagged variables during the holdout sample, as indicated in Table 1. While the efficiency of the market index can be rejected during this period, the maximum Sharpe ratio on the fixed-weight frontier is closer to the efficient-with-respect-to-*Z* boundary than is the market index.

The tests of Table 4 have an interesting interpretation when they are applied to the size x book-to-market portfolios and the market index. Fama and French (1996) construct three factors designed to capture the average returns of portfolios grouped by size and book-to-market, the Fama-French "3 factor model." If these factors describe the cross-section of expected returns, a combination of the factors is efficient. A fixed combination of these factors cannot produce a higher Sharpe ratio than the fixed-weight maximum in a sample that includes the three factor portfolios. Logically speaking then, the tests in Table 4 reject a fortiori a static (fixed-weight) version of the Fama-French 3-factor model over 1963-94, but not for 1995-2002. However, given that the statistical noise involved in estimating the maximum Sharpe ratio for 26 test assets will differ from that involving three factors, it is interesting to examine the multifactor models explicitly.

D. Testing Static Combinations of the Fama-French Factors

This section presents tests of the efficiency of a fixed-weight combination of the three Fama and French factors. The hypothesis may be started as $m = a + b_1 R_m + b_2 R_{HML} + b_3 R_{SMB}$, where the coefficients are fixed over time. R_m is the gross return of the market index. R_{HML} is the one-month Treasury bill gross return plus the excess return of high book-to-market over low book-to-market stocks, and R_{SMB} is similarly constructed using small and large market-capitalization stocks. In testing this model we replace the first and 25th portfolios in the industry or size *x* book-to-market design with the returns R_{HML} and R_{SMB}, to insure that the factor portfolios are a subset of the tested portfolio returns.

Table 5 presents the tests. In Panel A there is no conditioning information. Based on the asymptotic *p*-values we would reject the efficiency of the Fama-French factors at the 5% level, except in the size *x* book-to-market portfolio design over 1963-1994. However, adjusting for finite sample bias the only rejection occurs for the industry portfolios. Fama and French (1997) also find that their factors don't explain industry portfolio returns very well.

In Panel B the multiplicative approach to conditioning information is used. The resampled *p*-values strongly reject the model for 1963-94. This is consistent with studies such as Ferson and Harvey (1999) who find that the Fama-French factors do not explain ad-hoc dynamic strategy returns over a similar sample period. Once again, we cannot examine the multiplicative approach over the holdout sample because the covariance matrices are too large to invert.

Panel C of Table 5 presents the tests relative to the efficient-with-respect-to-*Z* frontier. The tests confirm the value of using the conditioning information efficiently. We observe strong rejections of the static version of the Fama-French model, both over 1963-1994 and in the 1995-2002 sample, and for both portfolio designs. The test results are consistent with the intuition that Sharpe ratios can be estimated with greater precision on a smaller number of assets (the Fama French factors in Table 5) than they can on a larger number of assets (the 26 portfolios in Table 4). Thus, the tests using the conditioning information efficiently can reject the Fama French factors even when they could not reject the hypothesis that the mean variance boundary fails to expand, as during the 1995-2002 sample.

E. Testing Dynamic Multifactor Models

The empirical results so far show that the efficient use of conditioning information expands the mean variance boundary of monthly portfolio returns for the sample before 1995, even when a multiplicative approach does not, and that the stock market index and fixed combinations of the Fama-French factors lie inside the expanded boundary, even during the 1995-2002 holdout sample. This section illustrates tests of multifactor and conditional benchmarks with time-varying weights.

The theory indicates two versions of multifactor benchmarks in the presence of conditioning information. Let R_B denote the vector of benchmark factor returns (eg., a market index and the Fama-French factors). The first example specifies $m(Z) = a + b\, w'(Z)R_B$, where a and b are constants and $w'(Z)\underline{1} = 1$. In the language of Huberman, Kandel and Stambaugh (1987), this says there is mean-variance "intersection" of the efficient-with-respect-to-*Z* boundary formed from R_B and the boundary of all the test assets, including R_B. Equivalently, the dynamic portfolio $w'(Z)R_B$ is efficient and there is a single-beta pricing model for the unconditional mean returns of all portfolios of the form $x'(Z)R,$ based on the portfolio $w'(Z)R_B$. We refer to this as the hypothesis of "dynamic intersection."

The second examples implies a multifactor benchmark $m(Z) = A(Z) + B(Z)'R_B$, where $A(Z)$ and $B(Z)$ are the previously-specified scalar and vector-valued functions of the conditional moments of returns and the zero-beta rate (see footnote 3). This says that a time-varying combination of the factor portfolios is conditionally minimum variance efficient given Z. Equivalently, there is a k-beta pricing relation for the conditional mean returns based on the k-vector of factor portfolios, R_B. According to this model, a maximum correlation portfolio with respect to Z for $A(Z) + B(Z)'R_B$, will be efficient with respect to Z. A special case is a conditional CAPM, when $k=1$ and R_B is the market return.

Note that, for a given choice of benchmark factors, the hypotheses of conditional efficiency and dynamic intersection are related. Both hypotheses specify that a particular time-varying combination of the benchmark assets should be efficient with respect to Z. Conditional efficiency specifies that the combination involves all of the test assets through the maximum correlation portfolio. Dynamic intersection restricts the coefficients of the combination to be zero, except for the factor portfolios, but allows the nonzero weights to vary over time to maximize the Sharpe ratio.[6]

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[6] Dynamic intersection in general is stronger than conditional efficiency. Dynamic intersection says that the efficient-with-respect-to-Z boundaries share a common point. Efficient-with-respect-to-Z portfolios must also be conditionally efficient, as shown by Hansen and Richard (1987). Conditional efficiency says the *conditional* boundaries have a common point for each realization of Z. The tangency to the common point is a particular zero-beta rate that may vary with Z over time. Thus, dynamic intersection at a given zero-beta or risk-free rate does not imply conditional efficiency given the same risk-free rate. It follows that rejections of conditional efficiency with a given risk-free rate do not imply a rejection of dynamic intersection.

Table 6 summarizes the tests for dynamic intersection. The tests ask if the efficient-with-respect-to-*Z* frontier formed from all portfolios of the three Fama-French factor returns touches the efficient-with-respect-to-*Z* frontier of the test assets at a point tangent to the risk-free rate. The sample values of the test statistics are smaller, in very case, than the values in Table 5. This is because a time-varying combination of the Fama-French factors has a larger maximum Sharpe ratio in the sample than a fixed-weight combination. The simulations reveal that that the 5% critical values of the test statistics are fairly close to those in Table 5; slightly larger in the 1963-94 samples and slightly smaller in the 1995-2002 samples. The p-values of the test statistics in Table 6 are not as small as the values in Table 5. Still, the hypothesis of dynamic intersection is strongly rejected for the 1963-94 sample. with *p*-values of 0.1% or less.

During 1995-2002 the tests in Table 6 marginally reject intersection, with *p*-values of 3.9% in the industry portfolio design and 5.5% in the size and book/market design. In Panel A of Table 5, when no conditioning information was used, the *p*-values for the tests of intersection were 7.7% and 15.7%. Thus, the evidence against the hypothesis that a combination of the three Fama-French factors can touch the boundary of the test assets is stronger, even during the 1995-2002 period, when the conditioning information is used efficiently.

Table 7 presents tests of the conditional efficiency of the market index and of a combination of the three Fama-French factors. We reject both models over 1963-1994 in both portfolio designs. The bootstrapped *p*-values are 1.2% or less. We also reject conditional efficiency in the 1995-2002 sample period, with *p*-values of 1.6% or less. Thus, when the conditioning information is used efficiently our tests can reject conditional versions of both the CAPM an the Fama-French three-factor model.

5. Robustness

This section discusses the robustness of the results. The tests of portfolio efficiency were illustrated under the assumptions that the conditional mean returns are linear functions of the instruments and the conditional covariance matrix is fixed. While this is a common set of assumptions, there are many ways to model conditional moments and future research should use or framework under alternative specifications. It is important to understand that the rejections of efficiency in our examples are in some sense robust to misspecified conditional moments. We test the efficiency of a given portfolio in sets of returns constructed from the test assets using the conditioning information. If we use the correct specification the solutions for $x(Z)$ are optimal and therefore include all portfolio functions. If we incorrectly specify the conditional moments the portfolio weights $x(Z)$ are not optimal, but they still generate valid dynamic strategy returns. With the wrong conditional moments we essentially test efficiency in a smaller set of constructed returns, but if we reject efficiency on the subset, it implies rejection on the larger set of returns. Therefore, if we reject efficiency of a given portfolio with incorrectly specified conditional moments, it implies a rejection when the conditional moments are correct.

The robustness to misspecified moments does not apply when the tests use a maximum correlation portfolio as the tested portfolio. In these cases, if we get the moments wrong the portfolio is not maximum correlation, and there is no reason that it should be minimum variance efficient. Thus, our rejections of the conditional models in Table 7 could reflect a misspecified data generating process. However, Ferson, Siegel and Xu (2005) shows that the weights of the portfolios that maximize correlation with respect to *Z* share a “robustness” to extreme observations, similar to that of the efficient-with-respect-to-*Z* solutions. Thus it should be interesting for future research to explore the properties of these tests under alternative data generating processes.

While the rejections for a given portfolio are theoretically robust to incorrectly parametrizing the conditional moments, the results of our simulations may be sensitive to the parameters. We conduct experiments to assess this sensitivity. We use a sample of simulated data to estimate alternative parameter values, and then recalibrate the simulations with these parameters. Repeating this experiment 100 times we obtain information on how sensitive our empirical *p*-values are to variation in the parameters. We consider the test for the efficiency of the SP500. The initial simulated sample produces Monte Carlo *p*-values of 0.59, 0.02 and 0.00 for the fixed-weight, multiplicative and efficient-with-respect-to-*Z* frontiers, respectively. After 100 experiments the mean *p*-values are 0.579, 0.027 and 0.000, with standard deviations equal to 0.015, 0.005 and 0.000, respectively. Thus, the results do not appear highly sensitive to variation in the simulation parameters.

Finally, in our empirical examples we use the average Treasury bill return as the fixed risk-free rate. We provide the analytical results for a general zero-beta rate. The empirical results may be sensitive to the choice of the zero beta rate. Therefore, it should be interesting in future research to apply our framework in a setting where the zero beta rate is a parameter to be estimated, perhaps by extending results in Kandel (1984).

6. Summary and Conclusions

We develop a new framework for testing asset pricing models in the presence of lagged conditioning information. The approach requires a model's stochastic discount factor (SDF) to correctly price all the dynamic portfolio returns that may be constructed from a set of test assets, where the portfolio weights may be functions of the conditioning information. By requiring the SDF to price a large set of payoffs, the tests can reject models that previous approaches would not reject.

Our tests examine the (unconditional) mean variance efficiency of a portfolio with respect to the conditioning information, a version of efficiency studied previously by Hansen and Richard (1987) and Ferson and Siegel (2001). We show how different specifications for a model's SDF identify portfolios that should be efficient with respect to the conditioning information. If we reject the efficiency of the portfolio, we reject the asset pricing model. We illustrate the approach with versions of the Capital Asset Pricing model, the Fama-French (1996) factors, and a dynamic version of mean-variance intersection (Huberman, Kandel and Stambaugh, 1987).

Using a standard set of lagged instruments and test portfolios, the efficiency of the SP500 index and all combinations of the Fama-French factor returns are rejected. In the same setting, the commonly-used "multiplicative" approach to conditioning information does not significantly expand the mean variance boundary, when compared with ignoring the conditioning information altogether. A holdout sample illustrates that the predictive power of the lagged variables declines after 1995, but even during this period the efficient use of the conditioning information enhances the results.

Appendix

Efficient Portfolio Solutions

The portfolio weights for efficient portfolios in the presence of conditioning information are derived by Ferson and Siegel (2001). First consider the case with N risky assets with returns R and a riskless asset returning R_f. In $N \times 1$ column-vector notation, we have

$$R = \mu(Z) + \varepsilon \tag{13}$$

The noise term ε is assumed to have conditional mean zero given Z and nonsingular conditional covariance matrix $\Sigma_\varepsilon(Z)$. The conditional expected return vector, $\mu(Z)$, is permitted to have a singular or nonsingular (unconditional) covariance matrix, so there can be any number of conditioning variables.

Define portfolio P by letting the $1 \times N$ row vector $x'(Z) = (x_1(Z), \ldots, x_N(Z))$ denote the portfolio share invested in each of the N risky assets, investing (or borrowing) at the riskless rate the amount $1 - x'(Z)\underline{1}$, where $\underline{1} = (1,\ldots,1)'$ denotes the column vector of ones. The observed return on this portfolio will be $R_f + x'(Z)(R - R_f\underline{1})$, with unconditional expectation and variance (after computing conditional expectations given Z to eliminate the random noise terms) as follows:

$$\mu_p = R_f + E[x'(Z)(\mu(Z) - R_f\underline{1})] \tag{14}$$

$$\sigma_P^2 = E\left\{x'(Z)\left[\left(\mu(Z) - R_f\underline{1}\right)\left(\mu(Z) - R_f\underline{1}\right)' + \Sigma_\varepsilon(Z)\right]x(Z)\right\} - \left(\mu_p - R_f\right)^2 \tag{15}$$

$$= E\left[x'(Z)Q^{-1}x(Z)\right] - \left(\mu_p - R_f\right)^2$$

where we have defined the $N \times N$ matrix

$$Q = Q(Z) = \left\{E\left[(R - R_f\underline{1})(R - R_f\underline{1})' \middle| Z\right]\right\}^{-1} = \left[(\mu(Z) - R_f\underline{1})(\mu(Z) - R_f\underline{1})' + \Sigma_\varepsilon(Z)\right]^{-1} \tag{16}$$

Define the constant ζ as follows:

$$\zeta = E[(\mu(Z)-R_f\underline{1})'Q(\mu(Z)-R_f\underline{1})] \tag{17}$$

Theorem 1. (Ferson and Siegel, 2001) Given the unconditional expected return μ_P, N risky assets, and a riskless asset, the unique portfolio having minimum unconditional variance is determined by the weights:

$$x'(Z) = \frac{\mu_P - R_f}{\zeta}\left[(\mu(Z) - R_f \underline{1})\right]' Q. \tag{18}$$

The portfolio variance is

$$\sigma_P^2 = (\mu_P - R_f)^2\left(\frac{1}{\zeta} - 1\right). \tag{19}$$

Proof. See Ferson and Siegel (2001).

When there is no riskless asset, we define portfolio P by letting $x' = x'(Z) = (x_1(Z),...,x_N(Z))$ denote the shares invested in each of the N risky assets, with the constraint that $x'\underline{1} = 1$. The return on this portfolio, $R_P = x'(Z)R$, has expectation and variance as follows:

$$\mu_P = E\left[x'(Z)\mu(Z)\right],$$

$$\sigma_P^2 = E\left\{x'(Z)\left[\mu(Z)\mu'(Z) + \Sigma_\varepsilon(Z)\right]x(Z)\right\} - \mu_P^2. \tag{20}$$

Define the matrix $\Lambda = \Lambda(Z) = \{E(RR'|Z)\}^{-1} = \{\mu(Z)\mu(Z)' + \Sigma_\varepsilon(Z)\}^{-1}$ and define the following portfolio constants:

$$\delta_1 = E\left(\frac{1}{\underline{1}'\Lambda\underline{1}}\right), \tag{21}$$

$$\delta_2 = E\left(\frac{\underline{1}'\Lambda\mu(Z)}{\underline{1}'\Lambda\underline{1}}\right), \tag{22}$$

$$\delta_3 = E\left[\mu'(Z)\left(\Lambda - \frac{\Lambda \underline{1}\underline{1}'\Lambda}{\underline{1}'\Lambda\underline{1}}\right)\mu(Z)\right]. \tag{23}$$

Theorem 2. (Ferson and Siegel, 2001) Given *N* risky assets and no riskless asset, the unique portfolio having minimum unconditional variance and unconditional expected return μ_P, is determined by the weights:

$$x'(Z) = \frac{\underline{1}'\Lambda}{\underline{1}'\Lambda\underline{1}} + \frac{\mu_P - \delta_2}{\delta_3}\mu'(Z)\left(\Lambda - \frac{\Lambda\underline{1}\underline{1}'\Lambda}{\underline{1}'\Lambda\underline{1}}\right) \tag{24}$$

The efficient-with-respect to *Z* frontier determined by these weights is a parabola relating the variance of return, σ^2, and the mean return, μ, as $\sigma^2 = a\mu^2 + b\mu + c$, where the constants are:

$$a = (1-\delta_3)/\delta_3, \quad b = -2\delta_2/\delta_3 \text{ and } c = \delta_1 + \delta_2^2/\delta_3.$$

Proof of Proposition 2

By the definition of covariance, $E\left[m_{t+1}x'(Z_t)R_{t+1}\right] = 1$ implies

$$E\left[x'(Z_t)R_{t+1}\right] = \left\{1 - Cov\left[m_{t+1}, x'(Z_t)R_{t+1}\right]\right\}/E(m_{t+1}). \tag{25}$$

Now, using $m_{t+1} = A + B'R_{B,t+1}$, we find that Equation (6) is satisfied, with $R_{p,t+1} = w'R_{B,t+1}$, $w \equiv B/(\underline{1}'B)$, $\gamma_0 = [A + B'E(R_{B,t+1})]^{-1}$, and $\gamma_1 = -\gamma_0(\underline{1}'B)$.

Proof of Proposition 3

Regress *m* on R_P using a simple regression: $m = a + bR_P + u$, where without loss of generality *a* and *b* are constants and $E(u) = E(uR_P) = 0$. If R_P is maximum correlation, then the error also satisfies: $E[ux'(Z)R] = 0 \forall x(Z): x'(Z)\underline{1} = 1$. (If this were not true for some $x(Z)$, then $x'(Z)R$ would enter the regression with a nonzero coefficient, contradicting the assumption that

R_P is maximum correlation.) Now, substitute the regression into (4) to obtain $E\{(a+bR_P+u)x'(Z)R\} = 1 = E\{(a+bR_P)x'(Z)R\}\ \forall x(Z): x'(Z)\underline{1}=1$. Proposition 2 now establishes that R_P is efficient with respect to Z.

Theorem 3: (Ferson, Siegel and Xu, 2005). The solution, $x_m(Z)$ to the maximization:

$$\underset{x(Z)}{Max}\,\rho^2\left[x'(Z)R,F\right] \text{ s.t. } x'(Z)\underline{1}=1, \tag{26}$$

where F is any random variable, is given by:

$$x_m(Z)=\frac{\Lambda\underline{1}}{\underline{1}'\Lambda\underline{1}}+\left(\Lambda-\frac{\Lambda\underline{1}\underline{1}'\Lambda}{\underline{1}'\Lambda\underline{1}}\right)\{-\lambda_1\mu(Z)-\lambda_2E(RF|Z)\}, \text{ where} \tag{27}$$

$\gamma_1(Z) = 1/(\underline{1}'\Lambda\underline{1})$, $\gamma_\mu(Z) = \underline{1}'\Lambda\mu(Z)/(\underline{1}'\Lambda\underline{1})$, $\gamma_F(Z) = \underline{1}'\Lambda E(RF'|Z)/(\underline{1}'\Lambda\underline{1})$,

$\Omega(Z) = [\Lambda - \Lambda\underline{1}\,\underline{1}'\Lambda/(\underline{1}'\Lambda\underline{1})]$, $\gamma_{\mu\mu}(Z) = \mu(Z)'\Omega(Z)\mu(Z)$,

and $\gamma_{\mu F}(Z) = \mu(Z)'\Omega(Z)E(RF'|Z)$, where:

$$\lambda_1 = \frac{-\gamma_1[E(F)-\gamma_{\mu F}]+\gamma_\mu\gamma_F}{\gamma_\mu[E(F)-\gamma_{\mu F}]+\gamma_F[\gamma_{\mu\mu}-1]},$$

$$\lambda_2 = \frac{-\gamma_1[\gamma_{\mu\mu}-1]-\gamma_\mu^{\,2}}{\gamma_\mu[E(F)-\gamma_{\mu F}]+\gamma_F[\gamma_{\mu\mu}-1]},$$

and:

$$\gamma_1 = E(\gamma_1(Z)),\quad \gamma_\mu = E(\gamma_\mu(Z)),\quad \gamma_F = E(\gamma_F(Z))$$

$$\gamma_{\mu\mu} = E(\gamma_{\mu\mu}(Z)), \text{ and } \gamma_{\mu F} = E(\gamma_{\mu F}(Z)),$$

Proof of Proposition 4

Observe that $\alpha_C \equiv \mu_C - [\gamma_0 + (\mu_P - \gamma_0)\sigma_{CP} / \sigma_P^2]$ depends on the portfolio R_C only through its mean and covariance with R_P. It follows that R_C must have minimal variance among all portfolios with its mean and covariance with R_P. From Ferson, Siegel and Xu (2005, Eq. 6) the optimal weights $x_C(Z)$ corresponding to R_C are given by:

$$x_C(Z) = \frac{\Lambda \underline{1}}{\underline{1}' \Lambda \underline{1}} + \left(\Lambda - \frac{\Lambda \underline{1}\,\underline{1}' \Lambda}{\underline{1}' \Lambda \underline{1}} \right) [a\mu(Z) + bE(RR_P|Z)], \tag{28}$$

where a and b are constants. We evaluate the final term using $R_P = R'x_P(Z)$:

$$b\left(\Lambda - \frac{\Lambda 1 1' \Lambda}{1' \Lambda 1} \right) E(RR'|Z) x_P(Z) = b\left(\Lambda - \frac{\Lambda 1 1' \Lambda}{1' \Lambda 1} \right) \Lambda^{-1} x_P(Z)$$
$$= b\left(x_P(Z) - \frac{\Lambda 1 1' x_P(Z)}{1' \Lambda 1} \right) = b x_P(Z) - b\frac{\Lambda 1}{1' \Lambda 1}$$

which we may substitute into the expression for x_C to obtain:

$$x_c = (1-b)\frac{\Lambda 1}{1' \Lambda 1} + a\left(\Lambda - \frac{\Lambda 1 1' \Lambda}{1' \Lambda 1} \right) \mu(Z) + b x_P(Z). \tag{29}$$

Comparing equations (24) and (29), we conclude that the most mispriced portfolio must be formed by combining an efficient-with-respect-to-*Z* portfolio with R_P.

Without loss of generality we represent the efficient-with-respect-to-*Z* frontier using the following two portfolios. Let R_0 denote the efficient portfolio with (unconditional) mean $\mu_0 = \gamma_0$ and unconditional variance σ_0^2. Note that the covariance between R_0 and R_S is $\sigma_{0S} = 0$ as R_0 is a zero-beta asset for R_S.

Consider the system of three assets (R_0, R_S, R_P), which has mean vector (γ_0, μ_S, μ_P) and alpha vector $\alpha' = (\alpha_0, \alpha_S, \alpha_P) = (-(\mu_P - \gamma_0)\sigma_{0P} / \sigma_P^2, \mu_S - \gamma_0 - (\mu_P - \gamma_0)\sigma_{SP} / \sigma_P^2, 0)$ with respect to R_P. The covariance matrix for these assets is

$$V = \begin{bmatrix} \sigma_0^2 & 0 & \sigma_{0P} \\ 0 & \sigma_S^2 & \sigma_{SP} \\ \sigma_{0P} & \sigma_{SP} & \sigma_P^2 \end{bmatrix} \tag{30}$$

Note that the efficient-with-respect to Z frontier and portfolio R_P are all accessible as fixed-weight portfolios within this system.

We maximize the mispricing by maximizing the squared Sharpe ratio within an isomorphic system of assets defined as $(R_0^+, R_S^+, R_P^+) = (R_0 - \gamma_0 + \alpha_0, R_S - \mu_S + \alpha_S, R_P - \mu_P)$, constructed so that the alphas of the original system are equal to the means in the isomorphic system: $E(R_0^+, R_S^+, R_P^+) = (\alpha_0, \alpha_S, 0)$ and we define the zero beta rate in the isomorphic system to be zero. Note that the variances for any fixed portfolio weight function will be the same in the original and the isomorphic system, and that the portfolio alpha in the original system is equal to the portfolio mean in the isomorphic system (because of linearity for both means and alphas). Thus the mispricing α_C^2 / σ_C^2 of a portfolio in the original system is equal to its squared Sharpe ratio μ_C^2 / σ_C^2 in the isomorphic system. It follows that the most mispriced portfolio weights x_C are proportional to $V^{-1}\alpha$.

The portfolio R_0 has zero weight in the most mispriced portfolio. When we multiply the first row of V^{-1} by α, the result is proportional to

$$\begin{aligned} &\left(\begin{vmatrix} \sigma_S^2 & \sigma_{SP} \\ \sigma_{SP} & \sigma_P^2 \end{vmatrix}, -\begin{vmatrix} 0 & \sigma_{SP} \\ \sigma_{0P} & \sigma_P^2 \end{vmatrix}, \begin{vmatrix} 0 & \sigma_S^2 \\ \sigma_{0P} & \sigma_{SP} \end{vmatrix} \right) (\alpha_0, \alpha_S, 0) = \alpha_0 (\sigma_S^2 \sigma_P^2 - \sigma_{SP}^2) + \alpha_S \sigma_{0P} \sigma_{SP} \\ &= \left[-(\mu_P - \gamma_0)\sigma_{0P} / \sigma_P^2 \right] (\sigma_S^2 \sigma_P^2 - \sigma_{SP}^2) + \left[\mu_S - \gamma_0 - (\mu_P - \gamma_0)\sigma_{SP} / \sigma_P^2 \right] \sigma_{0P} \sigma_{SP} \\ &= \left[-(\mu_P - \gamma_0)\sigma_{0P}\sigma_S^2 \right] + (\mu_S - \gamma_0)\sigma_{0P}\sigma_{SP} = -\sigma_{0P}\sigma_S^2 \left[(\mu_P - \gamma_0) - (\mu_S - \gamma_0)\sigma_{SP} / \sigma_S^2 \right] = 0 \end{aligned}$$

where the last equality follows from the fact that R_S is efficient with a zero-beta rate of γ_0 and thus must price $R_P : \mu_P = \gamma_0 + (\mu_S - \gamma_0)\sigma_{SP} / \sigma_S^2$.

Since R_0 does not appear in the most mispriced portfolio, we may maximize α_C^2 / σ_C^2 over the restricted isomorphic system (R_S^+, R_P^+). The optimal weights $(w_S, w_P)'$ will be proportional to

$$\begin{pmatrix} \sigma_S^2 & \sigma_{SP} \\ \sigma_{SP} & \sigma_P^2 \end{pmatrix}^{-1} \begin{pmatrix} \alpha_S \\ \alpha_P \end{pmatrix} = \begin{vmatrix} \sigma_S^2 & \sigma_{SP} \\ \sigma_{SP} & \sigma_P^2 \end{vmatrix}^{-1} \begin{pmatrix} \sigma_P^2 & \sigma_{SP} \\ \sigma_{SP} & \sigma_S^2 \end{pmatrix} \begin{pmatrix} \mu_S - \gamma_0 - (\mu_P - \gamma_0)\sigma_{SP} / \sigma_P^2 \\ 0 \end{pmatrix}$$
$$= \begin{vmatrix} \sigma_S^2 & \sigma_{SP} \\ \sigma_{SP} & \sigma_P^2 \end{vmatrix}^{-1} \left[\mu_S - \gamma_0 - (\mu_P - \gamma_0)\sigma_{SP} / \sigma_P^2\right] \sigma_P^2 \begin{pmatrix} 1 \\ -\sigma_{SP} / \sigma_P^2 \end{pmatrix}$$

hence (w_S, w_P) is proportional to $(1, -\sigma_{SP} / \sigma_P^2)$, which established Equation (12) after normalization. To establish Equation (11), substitute for σ_{SP} using $\mu_P = \gamma_0 + (\mu_S - \gamma_0)\sigma_{SP} / \sigma_S^2$.

Evaluating the Tests by Simulation

We conduct simulation experiments to evaluate the test statistics. Consider first a case with no conditioning information. In Monte Carlo experiments we draw random samples from a normal distribution with mean vector and covariance matrix set equal to the maximum likelihood (ML) estimates from our data. Under the null hypothesis the fixed-weight portfolio R_p should be minimum variance efficient. We therefore replace R_p in the simulations by a fixed-weight portfolio whose weights maximize the Sharpe ratio at the ML estimates. Thus, each artificial sample is drawn from a population in which the tested portfolio R_p is efficient.

The Monte Carlo results may be sensitive to the assumption of normally distributed data. We therefore resample from the original data instead of a normal distribution, using a parametric bootstrap approach. For example, a regression of returns on the lagged conditioning information defines the conditional mean function and the matrix of sample residuals. We choose randomly selected rows, with replacement, from the matrix of the sample residuals; the number of draws matches the length of the time series in the relevant subperiod. We use the conditional mean functions, evaluated at the simulated Z, and add the independently resampled residuals (unexpected returns) to obtain the simulated returns.

When conditioning information is involved the distribution of Z is taken from the empirical distribution of the 5 lagged instruments. In order to capture the strong serial dependence of these instruments we model Z_t as a vector AR(1) process. The sample AR (1) coefficient matrix is a parameter of the simulation. We resample from the matrix of residuals from the AR(1) model and build the time series of the Z_t's recursively in each simulation trial.

Under the null hypothesis the artificial samples are drawn from a population in which the tested portfolio R_p is efficient with respect to Z. The precise manner in which this is accomplished depends on the situation. When the null hypothesis places a given portfolio on the efficient-with-respect-to-Z frontier, we simply replace the tested portfolio return with the time-varying combination of test assets that is ex ante efficient given the data generating process (Tables 2 through 4). When the null hypothesis specifies that a fixed weight combination of factors is efficient, we replace the first factor with the ex ante efficient portfolio (Table 5).

The Corrollary to Proposition 3 describes the case of dynamic intersection. In this case we exploit the most mispriced portfolio of Proposition 4 in order to generate data that satisfy the null hypothesis. We first form a portfolio that is efficient with respect to Z within the set of $k-1$ of the factor portfolios for the given data generating process. This portfolio, call it R_{k-1}, will be inefficient in the full set of assets. We then use Proposition 4 to compute the most mispriced portfolio by R_{k-1}. A combination of R_{k-1} and the most-mispriced portfolio is efficient in the full sample of test assets given the data generating process. We replace the k-th factor with the most mispriced portfolio. With this replacement, the k factor-portfolios satisfy the null hypothesis that they are efficient in the full set of test assets (Table 6). When the null hypothesis specifies the conditional efficiency of a combination of the benchmark returns, R_B, we satisfy the null hypothesis by replacing the conditional mean functions of the test assets with the expressions implied by the equivalent conditional beta pricing restriction: $\mu(Z) = \gamma_o + \Sigma_{j=1}^{k} \beta_j(Z)E[R_{Bj} - \gamma_0 | Z]$, where $B_j(Z)$ is the vector of conditional betas on the j-th benchmark return (Table 7).

Each simulation experiment produces 1,000 artificial samples, and we estimate the relevant test statistic on each sample. The empirical 5% critical value is the value above which 5% of the 1,000 statistics lie. The empirical p-value is the fraction of the 1,000 statistics that are larger than the value obtained in the original sample. The logic is that if this p-value is small, it is unlikely that the sample statistic comes from a population in which the null hypothesis is true.

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Table 1. Monthly Industry Returns

Monthly returns on 25 portfolios of common stocks are from Harvey and Kirby (1996). The portfolios are value-weighted within each industry group. The industries and their SIC codes are in the following table. *Mean* is the sample mean of the gross (one plus rate of) return, σ is the sample standard deviation and ρ_1 is the first order autocorrelation of the monthly return. R^2 is the adjusted coefficient of determination in percent from the regression of the returns on the lagged instruments. The sample period is February of 1963 through December of 1994 (383 observations). $R^2_{HOLDOUT}$ is for the 1995-2002 holdout sample (96 observations). Negative adjusted R-squares are reported as 0.0.

	Industry	SIC codes	Mean	σ	ρ_1	R^2	$R^2_{HOLDOUT}$
1	Aerospace	372, 376	1.0107	0.0644	0.13	9.9	1.1
2	Transportation	40, 45	1.0094	0.0648	0.08	9.1	0.0
3	Banking	60	1.0086	0.0631	0.10	4.3	2.4
4	Building materials	24, 32	1.0097	0.0608	0.09	10.4	0.0
5	Chemicals/Plastics	281, 282, 286-289, 308	1.0094	0.0525	-0.01	8.0	2.5
6	Construction	15-17	1.0109	0.0760	0.16	10.2	0.0
7	Entertainment	365, 483, 484, 78	1.0135	0.0662	0.14	5.7	0.0
8	Food/Beverages	20	1.0117	0.0449	0.05	6.6	0.2
9	Healthcare	283, 384, 385, 80	1.0113	0.0524	0.01	2.4	0.0
10	Industrial Mach.	351-356	1.0089	0.0587	0.05	8.2	0.0
11	Insurance/Real Estate	63-65	1.0095	0.0581	0.15	6.4	2.3
12	Investments	62, 67	1.0097	0.0453	0.05	8.7	4.1
13	Metals	33	1.0075	0.0610	-0.02	4.5	0.2
14	Mining	10, 12, 14	1.0108	0.0535	0.01	7.2	0.3
15	Motor Vehicles	371, 551, 552	1.0095	0.0584	0.11	10.6	0.0
16	Paper	26	1.0095	0.0536	-0.02	6.9	2.4
17	Petroleum	13, 29	1.0102	0.0518	-0.02	4.4	0.0
18	Printing/Publishing	27	1.0114	0.0586	0.21	11.3	0.0
19	Professional Services	73, 87	1.0111	0.0693	0.13	8.4	2.8
20	Retailing	53, 56, 57, 59	1.0106	0.0597	0.15	8.7	3.7
21	Semiconductors	357, 367	1.0080	0.0559	0.08	9.0	0.0
22	Telecommunications	366, 381, 481, 482, 489	1.0085	0.0412	-0.05	5.4	8.8
23	Textiles/Apparel	22, 23	1.0100	0.0613	0.21	11.0	0.0
24	Utilities	49	1.0078	0.0392	0.02	6.8	4.3
25	Wholesaling	50, 51	1.0109	0.0614	0.13	10.7	0.0

Table 2

Tests of the mean variance efficiency of the Standard and Poors 500 stock index excess return in a sample of industry portfolio returns. The monthly returns on 25 industry-sorted portfolios of common stocks are measured, for the sample period February 1963 through December 1994 (T=383 observations), and ten-year subperiods. A holdout sample from January, 1995 through December, 2002 (96 observations) is also shown. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. NA denotes not applicable, when the number of assets is larger than the number of time series observations and the covariance matrix of the returns is singular. Asymptotic *p*-values are from the chi-squared distribution.

Subperiod	63-72	73-82	83-92	63-94	95-02
Panel A: Test assets $R_{t,}$ no conditioning information:					
Wald Statistic	32.80	26.26	29.76	24.77	51.26
asymptotic *p*-value	0.14	0.39	0.23	0.48	0.001
Monte Carlo 5% Critical Value	52.82	52.34	50.82	51.93	211.40
empirical *p*-value	0.43	0.71	0.58	0.59	0.70
Resampling 5% Critical Value	60.05	63.86	62.32	40.00	231.41
empirical *p*-value	0.52	0.81	0.65	0.58	0.72
Panel B: Test assets are $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:					
Wald Statistic	NA	NA	NA	348.63	NA
asymptotic *p*-value				0.00	
Monte Carlo 5% Critical Value				327.99	
empirical *p*-value				0.02	
Resampling 5% Critical Value				475.96	
empirical *p*-value				0.44	
Panel C: Test assets are all Portfolios $x'(Z_{t-1})R_t$:					
Test Statistic	203.29	188.56	164.98	161.84	148.2
Monte Carlo 5% Critical Value	125.69	121.58	121.61	133.27	139.93
empirical *p*-value	0.000	0.000	0.001	0.002	0.029
Resampling 5% Critical Value	117.25	130.55	121.62	118.76	144.89
empirical *p*-value	0.003	0.005	0.003	0.001	0.044

Table 3

Tests of the mean variance efficiency of the Standard and Poors 500 stock index excess return. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks, for the sample period February 1963 through December 1994 (T=383 observations). The size/BM returns are 25 portfolios of stocks sorted on market capitalization and book-to-market ratio, for the sample period July 1963 through December 1994 (T=378 observations). A holdout sample covers January 1995 through December, 2002 (96 observations). The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. Asymptotic p-values are from the chi-squared distribution. NA indicates that the sample size does not allow the statistic to be calculated.

Sample	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets $R_{t,}$ no conditioning information:				
Sample Statistic	83.02	74.12	24.77	51.26
asymptotic p-value	0.000	0.000	0.475	0.001
Monte Carlo 5% Critical Value	40.97	114.58	51.93	211.40
empirical p-value	0.000	0.192	0. 59	0.70
Resampling 5% Critical Value	45.13	131.52	39.99	231.41
empirical p-value	0.000	0.277	0.579	0.72
Panel B: Test assets are $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:				
Sample Statistic	517.12	NA	348.63	NA
asymptotic p-value	0.000		0.000	
Monte Carlo 5% Critical Value	342.03		327.99	
empirical p-value	0.000		0.019	
Resampling 5% Critical Value	508.78		475.96	
empirical p-value	0.040		0.440	
Panel C: Test assets are all Portfolios $x'(Z_{t-1})R_t$:				
Sample Statistic	272.66	210.4	161.84	148.2
Monte Carlo 5% Critical Value	120.79	131.92	133.27	139.93
empirical p-value	0.000	0.000	0.002	0.029
Resampling 5% Critical Value	107.59	135.10	118.76	144.89
empirical p-value	0.000	0.003	0.001	0.044

Table 4

Tests of the null hypothesis that conditioning information does not expand the mean variance boundary. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index return. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. Asymptotic *p*-values are from the chi-squared distribution. NA indicates that the sample size does not allow the test statistic to be calculated.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets are $R_{ft}+(R_t-R_{ft})\otimes Z_{t-1}$:				
Sample Statistic	356.29	NA	304.31	NA
asymptotic *p*-value	0.000		0.000	
Monte Carlo 5% Critical Value	338.18		326.70	
empirical *p*-value	0.033		0.141	
Resampling 5% Critical Value	520.38		485.96	
empirical *p*-value	0.464		0.686	
Panel B: Test assets are all Portfolios $x'(Z_{t-1})R_t$:				
Sample Statistic	155.75	77.70	128.83	63.78
Monte Carlo 5% Critical Value	122.29	127.69	133.42	133.96
empirical *p*-value	0.000	0.458	0.067	0.806
Resampling 5% Critical Value	108.69	138.26	118.83	148.08
empirical *p*-value	0.001	0.539	0.025	0.779

Table 5

Tests of the null hypothesis that a fixed-weight combination of the three Fama-French factors is efficient. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a value-weighted index. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratio and a value-weighted return. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors, respectively. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. Asymptotic *p*-values are from the chi-squared distribution. NA indicates that the sample size does not allow the test statistic to be calculated.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets are R_t:				
Sample Statistic	34.96	49.51	43.03	55.53
asymptotic *p*-value	0.089	0.002	0.014	0.004
Resampling 5% Critical Value	41.58	63.97	39.24	61.22
empirical *p*-value	0.117	0.157	0.021	0.077
Panel B: Test assets are all Portfolios $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:				
Sample Statistic	521.87	NA	415.08	NA
asymptotic *p*-value	0.000		0.000	
Resampling 5% Critical Value	319.34	NA	313.73	NA
empirical *p*-value	0.000		0.000	
Panel C: Test assets are $x'(Z_{t-1})R_t$:				
Sample Statistic	340.61	181.55	180.09	174.64
Resampling 5% Critical Value	70.52	128.03	75.55	118.38
empirical *p*-value	0.000	0.003	0.000	0.001

Table 6

Tests of dynamic intersection. The null hypothesis is that an efficient-with-respect-to-*Z* combination of the three Fama French factors touches the efficient-with-respect-to-*Z* frontier of the test assets at a tangency from the risk-free rate. Industry refers to monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. The first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Sample Statistic	268.09	124.3	125.47	118.9
Resampling 5% Critical Value	73.30	127.14	79.59	114.05
empirical *p*-value	0.000	0.055	0.001	0.039

Table 7

Tests of Conditional Efficiency. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Conditional Efficiency of the Market Index				
Sample Statistic	339.16	131.70	189.70	143.0
Resampling 5% Critical Value	101.12	88.19	91.42	83.22
empirical *p*-value	0.000	0.008	0.002	0.006
Panel B: Conditional Efficiency of the Fama-French Factors				
Sample Statistic	362.95	132.50	144.37	137.8
Resampling 5% Critical Value	98.13	97.65	117.75	94.19
empirical *p*-value	0.000	0.016	0.012	0.015