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WEALTH INEQUALITY: DATA AND MODELS

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ABSTRACT

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hold one third of the total wealth in the economy. Understanding the determinants of wealth
inequality is a challenge for many economic models. We summarize some key facts about the
wealth distribution and what economic models have been able to explain so far.

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1 Introduction

In the United States wealth is highly concentrated and very unequally distributed: the richest 1% of the households own one third of the total wealth in the economy. Understanding the determinants of wealth inequality is a challenge for many economic models. In this paper, we summarize what is known about the wealth distribution and what economic models have been able to explain so far.

The development of various data sets in the past 30 years (in particular the Survey of Consumer Finances) has allowed economists to quantify more precisely the degree of wealth concentration in the United States. The picture that emerged from the different waves of these surveys confirmed the fact that a large fraction of the total wealth in the economy is concentrated in the hand of the richest percentiles: the top 1% hold one third, and the richest 5% hold more than half of total wealth. At the other extreme, a significant fraction of the population holds little or no wealth at all.

Income is also unequally distributed, and a large body of work has studied earnings and wage inequality. Income inequality leads to wealth inequality as well, but income is much less concentrated than wealth, and economic models have had difficulties in quantitatively generating the observed degree of wealth concentration from the observed income inequality. The question is what mechanisms are necessary to generate saving behavior that leads to a distribution of asset holdings consistent with the actual data.

In this work, we describe the main framework for studying wealth inequality, that of general equilibrium models with heterogeneous agents, in which some elements of a life-cycle structure and of intergenerational links are present. Some models consider a dynasty as a single, infinitely-lived agent,

1

while others consider more explicitly the life-cycle aspect of the saving decision. Baseline versions of these models are unable to replicate the observed wealth concentration. More recently, however, some works have shown that certain ingredients are necessary, and sometimes enable the model to replicate the data. Bequests are a key determinants of inequality, and careful modelling of bequests is vital to understand wealth concentration. In addition, entrepreneurs constitute a large fraction of the very rich, and models that explicitly consider the entrepreneurial saving decision succeed in dramatically increasing wealth dispersion. The type of earnings risk faced by the richest is also a potential explanation worth investigating.

Considerable work must still be done to better understand the quantitative importance of each factor in determining wealth inequality and to understand which models are most useful and computationally convenient to study it. The recent advances in modelling have however already helped in providing a more precise picture. The challenge now is improve these models even further and to apply them to the study of several problems for which inequality is a key determinant. For instance, the effects of several tax policies (in particular the estate tax) might depend crucially on how wealth is concentrated in the hands of the richest percentiles of the distribution. In the last section of this paper, we highlight some of the areas in which models of inequality could and should be profitably employed and extended.

2   Data

We first summarize the main facts about the wealth distribution in the United States, facts provided mainly by the Survey of Consumer Finances. We also mention some facts about the historical trends, although in this paper we do

not focus on understanding them (an area on which little work has been done so far).

2.1 Data sources

The main source of microeconomic data on wealth for the United States is the Survey of Consumer Finances (SCF)[1] which, starting from 1983, every three years collects detailed information about wealth for a cross-section of households. It also includes a limited panel (between 1983 and 1989), as well as a link to two previous smaller surveys (1962 Survey of Financial Characteristics of Consumers and the 1963 Survey of Changes in Family Finances).

The SCF was explicitly designed to measure the balance sheet of households and the distribution of wealth. It has a large number of detailed questions about different assets and liabilities, which allows highly disaggregated data analysis on each component of the total net worth of the household. More importantly, the SCF oversamples rich households by including, in addition to a national area probability sample (representing the entire population), a list sample drawn from tax records (to extract a list of high income households). Oversampling is especially important given the high degree of wealth concentration observed in the data (see Davies and Shorrocks [31]). For this reason, the SCF is able to provide a more accurate measure of wealth inequality and of total wealth holdings: Curtin et al. [29] and Antoniewicz [6] document that the total net worth implied by the SCF matches quite well the total wealth implied by the (aggregate) Flow of Funds Accounts (although not perfectly, especially when disaggregating the various components).

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[1]The survey is publicly available from the Federal Reserve Board website at http://www.federalreserve.gov/pubs/oss/oss2/scfindex.html.

Unfortunately, the SCF does not follow households over time, unlike the Panel Study of Income Dynamics (PSID). The PSID[2] is a longitudinal study, which begun in 1968, and follows families and individuals over time. It focuses on income and demographic variables, but since 1984 it has also included (every 5 years) a supplement with questions on wealth. The PSID includes a national sample of low-income families, but it does not oversample the rich. As a result, this data set is unable to describe appropriately the right tail of the wealth distribution: Curtin et al. [29] show that the PSID tracks the distribution of total household net worth implied by the SCF only up to the top 2%-3% of richest household, but misses much of the wealth holdings of the top richest. Given that the richest 5% hold more than half of the total net worth in the United States, this is an important shortcoming.

Another important data source is the Health and Retirement Study (HRS), which recently absorbed the Study of Assets and Health Dynamics Among the Oldest Old (AHEAD). This survey focuses on the older households (from before retirement and on), and provides a large amount of information regarding their economic and health condition. However, as the PSID, this survey misses the richest households.

Other data sets also contain some information on wealth and asset holdings (for instance, the U.S. Bureau of Census's Survey of Income and Program Participation, or, for the very richest, the data on the richest 400 people identified by the *Forbes* magazine and, indirectly, the Internal Revenue Service data on estate returns). However, because of its careful sample choice, the SCF remains the main source of information about the distribution of wealth in the United States. Due to their demographic and health data, the PSID and the

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[2]See http://psidonline.isr.umich.edu/.

Percentile			Year		
group	1989	1992	1995	1998	2001
0-49.9	2.7	3.3	3.6	3.0	2.8
50-89.9	29.9	29.7	28.6	28.4	27.4
90-94.9	13.0	12.6	11.9	11.4	12.1
95-98.9	24.1	24.4	21.3	23.3	25.0
99-100	30.3	30.2	34.6	33.9	32.7

Table 1: Percent of net worth held by various groups defined in terms of percentiles of the wealth distribution (taken from Kennickell [58], p. 9).

HRS provide additional information for studying the wealth holdings of most households (except the richest).

2.2 Wealth concentration in the United States

The most striking aspect of the wealth distribution in the United States is its degree of concentration. Table 1 shows that the households in the top 1% of the wealth distribution hold around one third of the total wealth in the economy, and those in the top 5% hold more than half. At the other extreme, many households (more than 10%) have little or no assets at all.

The data in Table 1 and 2 refer to total net worth. There are many possible measures of wealth, the most appropriate one depending on the problem object of study. Net worth includes all assets held by the households (real estate, financial wealth, vehicles) net of all liabilities (mortgages and other debts); it is thus a comprehensive measure of most marketable wealth. This measure thus includes the value of most defined contribution plans (such as IRAs), but excludes the implied values of defined benefit plans and social security. Defined contribution plans can of course be important sources of income after retire-

Net			Year		
worth	1989	1992	1995	1998	2001
< $0	7.3	7.2	7.1	8.0	6.9
$0-$1,000	8.0	6.3	5.2	5.8	5.4
$1,000-$5,000	12.7	14.4	15.0	13.1	12.8
$25,000-$100,000	23.2	25.4	26.4	22.9	22.0
$100,000-$250,000	20.2	21.6	22.1	22.6	19.2
$250,000-$500,000	11.0	9.3	9.3	12.0	13.0
$500,000-$1,000,000	5.4	4.6	5.1	6.0	7.8
≥ $1,000,000	4.7	3.8	3.6	4.9	7.0

Table 2: Percent distribution of household net worth over wealth groups, 2001 dollars(taken from Kennickell [58], p. 9).

ment; but their measure is problematic because their value has to be imputed. To study other questions it may be useful to look at more restricted measures of wealth, that for example exclude less liquid assets (such as housing), and focus on financial wealth instead. Throughout this paper, we focus on net worth.[3]

The key facts about the distribution of wealth have been highlighted in a large number of studies, among others in Wolff [96], [94], and Kennickell [58]. Wealth is extremely concentrated, and much more so than earnings and income, as shown by Díaz-Giménez et al. [34] and Budria et al. [84]. For instance, in 1992 the Gini index for labor earnings, income (inclusive of transfers) and wealth were respectively .63, .57, and .78 (Díaz-Giménez et al. [34]), while in 1995 they were .61, .55 and .80 (Budria et al. [84]). These two studies also

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[3]It must be noted that the exact definition of net worth varies across studies. Therefore, the numbers we cite below when referring to other works are not directly comparable, as they may include different sets of assets. However, the general picture of a highly skewed distribution and the main trends are unchanged and do not depend on the exact measure of wealth.

Top %	1	5	10	20
Whole population				
percentage of total net worth held	30	54	67	81
Entrepreneurs				
percentage of households in a given percentile	63	49	39	28
percentage of net worth held in a given percentile	68	58	53	47

Table 3: Entrepreneurs and the distribution of wealth. SCF 1989.

report that the correlation between these three variables is positive, but far from perfect.

There is also significant wealth inequality within various age and demographic groups. For instance, Venti and Wise [91] and Bernheim at al. [12] show that wealth is highly dispersed at retirement even for people with similar lifetime incomes, and argue that this differences cannot be explained only by events such as family status, health and inheritances, nor by portfolio choice.

Several studies have also highlighted the differences in wealth holdings across different groups. There are large gaps in wealth holdings by race (see for example Altonji and Doraszelski [3] and Smith [88]). Wolff [96] documents that in the 1980s and 1990s the ratio of average net worth of blacks to the one of whites was around 18%. Unfortunately little work has been done to quantitatively understand the sources of this persistent difference across race groups. (See White [92] for a study of how much of current black-white income and wealth inequality can be explained by initial conditions at Emancipation.)

There is also a large difference in wealth holdings between entrepreneurs and non-entrepreneurs, as shown in Table 3 (taken from Cagetti and De Nardi [20]). Entrepreneurs[4] are a small fraction of the population (about

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[4]We classify as *entrepreneurs* the households who declare owning a privately held business

10%), but hold a large share of total wealth (about 40%). Table 3 shows that entrepreneurs constitute a large fraction of the richest households: more than 60% of the households in the top 1%, and almost one-half of those in the top 5%, and they hold, respectively, 68% and 58% of the wealth held by households in those percentiles. As shown also by Gentry and Hubbard [41], Quadrini [81], and Buera [17], entrepreneurship is a key element to understand the wealth concentration among the richest households.

Regarding household mobility, Hurst et al. [56] use PSID data to analyze the wealth dynamics between 1984 and 1994, for different socio-economic groups and for different types of asset holdings, pointing out that most of the mobility occurs in the midrange deciles, while the top and bottom ones show high persistence. Unfortunately, the PSID does not allow to study what happens at the top percentile. Using the same dataset, Quadrini [81] studies the wealth mobility for entrepreneurs and non-entrepreneurs, showing that entrepreneurs are more upwardly mobile. Because of the purely cross-sectional nature of the SCF, it is difficult to characterize the mobility of households across the wealth distribution.

The key feature that we stress here is that the observed degree of wealth concentration is much higher than the one of labor earnings and that, as we will see in the sections about the models, generating saving behavior that is consistent with these facts is not a trivial task.

(or a share of one), who have an active management role in it, and who have invested a positive amount of wealth in such business. Alternative classifications give very similar results.

2.3 Savings, bequests and wealth accumulation

In addition to income differences, wealth inequality may be driven by differences in the saving behavior, or in the intergenerational transfers received. Analyzing the empirical evidence to tell apart different potential sources of wealth inequality is challenging.

Individual saving cannot be measured directly but must be computed from other data, either as the first difference in wealth, or as income minus consumption. For this reason there are fewer studies that document the heterogeneity in saving rates; their findings suggest significant differences in saving behavior across various groups. (See Browning and Lusardi [16] for a review of the literature.) Dynan et al. [35] show that higher-lifetime income households save a larger fraction of their income than lower-income households. Quadrini [81] documents that entrepreneurs, who tend to be among the richest households, also exhibit higher saving rates.

Bequests also play an important role in shaping wealth inequality. Kotlikoff and Summers [60] were the first to argue that life-cycle savings for retirement account for a small fraction of total capital accumulation, while intergenerational transmission of wealth accounts for the vast majority of capital formation (with a baseline estimate of around 80% of the total). Further studies have confirmed the importance of intergenerational transfers; for instance, Gale and Scholz [40] find that bequests account for about 30% of total wealth accumulation, and intended inter-vivos transfers account for an additional 20%.

It is more difficult to measure the size of intended bequests relative to that of purely accidental ones, due to uncertainty about the life-span. Hurd [54] estimates a very low marginal utility from leaving bequests. Altonji and Villanueva [4] also find relatively small values for the elasticity of bequests to

permanent income, although they do show that this number increases with life-time resources. Most of the bequests, however, are concentrated among the top wealth percentiles, a group that these papers ignore. Looking at a sample of TIAA-CREF retirees (whose average wealth is higher than in the other groups), Laitner and Juster [65] find that about half of the households in their sample plan to leave estate and that the amount of wealth attributable to estate building is significant, accounting for half or more of the total for those who plan to leave bequests. While more empirical research is needed in the area, it appears that intergenerational altruism and intended bequests are a crucial element to understand the distribution of wealth, above all for the very rich.

2.4   Wealth inequality outside the United States

While we focus on the United States, it is interesting to compare its wealth distribution to that of other countries. The evidence on a few other, mostly OECD countries is summarized in Wolff [95] and in Davies and Shorrocks [31], to which we refer for the data and a discussion (including the caveats about data quality for some countries). In all countries for which studies exist, wealth is very unequally distributed, with Gini indexes ranging from .50 to .80, and a wealth share for the top 5% of households ranging from around 25% to over 50%. Among these countries, the United States exhibits the highest degree of wealth concentration, with the largest shares of total wealth in the hand of the richest percentiles of the wealth distribution. The lowest values are found in, among others, Australia, Italy, Japan and Sweden, and intermediate values in Canada, France and the United Kingdom.

These data indicate that, while the main forces generating wealth inequal-

10

ity seem to be common across developed countries, certain factors may differ across countries and reduce, or increase, inequality. Among these factors, public policies such as taxation (estate taxation and progressive income taxation in particular) tend to reduce wealth accumulation among the richest households, and hence inequality. More research needs to be done to explain cross-country differences (see De Nardi [32] as an application that tries to understand the various determinants of inequality both in the United States and Sweden).

2.5   Trends in wealth inequality

It is difficult to measure wealth inequality before the second half of the twentieth century. For the United States some data exist (Census surveys in the nineteenth century and other records of estates), but their interpretation is still debated. Some argue that inequality has always been high and has changed little from the end of the eighteenth century to the first decades of the twentieth century (for example, Soltow [89]), while others argue for a sharp increase in inequality over the period (among others, Lindert [68]). It is however interesting to note that wealth inequality has always been substantial, and, even according to Lindert [68], by 1860 the richest 1% held approximately 30% of total wealth, an amount that remained more or less stable until the 1920's.

There is evidence that U.S. wealth inequality decreased significantly between the 1920 and the 1970s (Davies and Shorrocks [31], Wolff and Marley [97], and Kopczuk and Saez [59]). Wolff [94], for instance, documents that the share of total wealth held by the top 1% of individuals fell from 38% in 1922 to 19% in 1976. As explained by Kopczuk and Saez [59], the decrease took place between the onset of the Great Depression and the end of World War II, and was most likely generated by the Depression and the New Deal

11

policies that increased the tax burden for the wealthy. Given the continuing high estate and income taxes, the top shares did not recover in the decades after World War II. Inequality increased again in the 1980s. Wolff [93] argues that while wealth inequality fell during the 1970s, it rose sharply after 1979, with a dramatic increase over the 1980s, to the level off in the 1990s. The trend in the 1990s is much less clear. The decade saw a stockmarket boom and the rise of some large internet fortunes, as well as increased income concentration (Piketty and Saez [78]). While Wolff [96] suggests a small increase in wealth inequality over the decade, Kennickell [58], Kopczuk and Saez [59], and Scholz [86] find that the share of total wealth in the hands of the richest remained stable in the 1990s.

Some of these trends have also been observed in other countries (see Davies and Shorrocks [31], Piketty et al. [77], and Saez [85]). As in the United States, wealth inequality decreased in the U.K., Sweden and France during the first half of the twentieth century. This decrease was especially strong in the U.K., where wealth inequality in previous centuries was at least as high, if not higher, than in the United States. However in these countries, and unlike in the United States, inequality kept decreasing significantly after World War II and until the 1980s. In contrast, the United States experienced an increase in inequality after the 1970s. Different levels of income and estate taxation, which in the last 50 years have been higher in these countries, might be responsible for the difference in these dynamics.

3   Models

In the following sections we describe the models used so far to study wealth concentration. Most of these models are general-equilibrium, quantitative models

with heterogeneous agents. For expositional purposes we classify these works into three sub-categories: models with infinitely-lived dynasties, models with overlapping-generations (OLG), and models that mix both of these features[5].

The first type of models ignore the life-cycle structure, but consider each dynasty as a single agent who lives forever. The second type explicitly introduces an age and life-cycle structure, with various degrees of intergenerational transmission of wealth and abilities. The third type relaxes the infinitely-lived dynasty assumption of the first type of models, but greatly simplifies the life-cycle structure.

Almost all the current general equilibrium, quantitative models of wealth inequality are versions of Bewley models[6]. These are incomplete-markets models in which households are ex-ante identical[7], in the sense that they face the same stochastic labor earnings and ability processes, but are ex-post heterogeneous because they receive different realizations of such shocks. These models are typically solved for stationary equilibria in which, over time, there is a constant distribution of people over the relevant state variables for the economy, but people move around in the distribution, and thus face considerable uncertainty. This type of framework endogenously generate differences in asset holdings, and hence wealth concentration, as a result of the household's desire to save and the realization of the shocks. An exogenous earnings process is typically the source of these shocks, and its properties are usually estimated

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[5]As we further explain later on, this categorization does not always reflect the chronological order of the various contributions. Historically, the starting point of this literature was Modigliani and Brumberg's [74] life-cycle model with certainty, which was then enriched in various ways by several contributors.

[6]See Ljungqvist and Sargent [70] for an exposition of the properties of these models and of their numerical solution.

[7]See Quadrini and Ríos-Rull [83] for a discussion about why we need incomplete market models to study wealth inequality.

using micro-level data sets.

4 Dynasty models

4.1 A general framework

Let us consider the simplest version of a Bewley model with infinitely-lived agents. There is a continuum of agents. All agents have identical preferences, and have the following utility function when they first enter the model economy:

$$E\left\{\sum_{t=1}^{\infty} \beta^t u(c_t)\right\},$$

where $u(c_t)$ is the constant relative-risk aversion flow of utility from consumption. The labor endowment of each household is given by an idiosyncratic labor productivity shock z that assumes a finite number of possible values and follows a first order Markov process with transition matrix $(\Gamma(z))$. There is only one asset, a, that people can use to self-insure against earnings risk.

A constant returns to scale production technology converts aggregate capital (K) and aggregate labor (L) into aggregate output (Y).

During each period each household chooses how much to consume (c) and save for next period by holding risk free assets (a'). The household's state variables are denoted by $x = (a, z)$, where a is asset holdings carried into the period and z is the labor shock endowment.

The household's recursive problem can thus be written as

$$V(x) = \max_{(c,a')}\left\{u(c) + \beta E\left[V(a', z')|x\right]\right\}$$

14

subject to

$$c + a' = (1 + r)a + zw$$

$$c \geq 0, \quad a' \geq \underline{a}$$

where r is the interest rate net of taxes and depreciation, w is the wage, and $\underline{a}$ is a net borrowing limit[8]. For simplicity, we have not explicitly introduce taxes and government policies, but of course the setup can easily accomodate various types of taxes and transfers.

At every point in time this model economy can be described by a probability distribution of people over assets a and earnings shocks z.

A stationary equilibrium for this economy is a set of consumption and saving rules, prices, aggregate capital and labor, and invariant distribution of households over the state variables of the system such that:

1. Given prices, the decision rules solve the household's recursive problem described above.

2. Aggregate capital is equal to total savings of all of the households of the economy, while aggregate labor is equal to total labor supplied by all of the households of the economy.

3. Prices, that is the interest rate and the wage rate, gross of taxes, equal the marginal product of capital, net of depreciation, and the marginal product of labor.

4. The constant distribution of people is the one induced by the law of motion of the system, which is determined by the exogenous earnings

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[8]See Bewley [14], Aiyagari [1], Huggett [52] and Ljungqvist and Sargent [70] for more exhausting descriptions of this framework and its equilibrium.

shocks and by the endogenous policy functions of the households.

Quadrini and Ríos-Rull [83] nicely summarize the results obtained from this type of models until 1997 with the first three lines of Table 4.

		% wealth in top	
Gini	1%	5%	20%
U.S. data			
.78	29	53	80
Baseline Aiyagari			
.38	3.2	12.2	41.0
High variability Aiyagari			
.41	4.0	15.6	44.6
Quadrini: entrepreneurs			
.74	24.9	45.8	73.2

Table 4: Dynasty models of wealth inequality.

Most of the models in Table 4 display significantly less wealth concentration than in the data. The reason why households save in this type of models is to create a buffer stock of assets to self-insure against earnings fluctuations (as discussed by Carroll [24]). Once such buffer stock is reached, the agents don't save any more, and the model is thus not capable to explain the creation of large fortunes.

Differences in wealth holdings are generated by random earnings shocks. Households hit by a series of negative shocks use part of this buffer stock of wealth, and thus become poorer, relative to those who have had a series of positive shocks. Larger earnings shocks may thus generate more dispersion. Lines two and three of the table compare two identical economies, except for the fact that the second one displays much higher earnings variability than the first one (and thus higher cross-sectional earnings inequality) and show

16

that second economy does generate a slightly more concentrated distribution of wealth.

The table shows however that earnings shocks alone cannot generate a level of wealth inequality comparable to that found in the data[9]. The reason is that there is no mechanism that induces the richer people to keep saving at high rates. Indeed, the model implies that richer households, having reached high levels of wealth, should save at lower rates (or even dissave) than poorer households, who instead need to rebuild their buffer stock. This implication is in contrast with the empirical studies on saving rates (see for example Dynan et al. [35] and Quadrini [81]).

Allowing for heterogeneity in households' earnings processes may generate different levels of buffer stocks of wealth across people. This effect is quantitatively not strong enough to generate a significant increase in wealth inequality when the earnings volatility used is consistent with those estimated in micro studies (see Carroll [24] and Quadrini and Ríos-Rull [83]). Regarding heterogeneity in earnings across educational groups, various estimates (see for example Cunha et al. [28]) show that earnings variability decreases with education. This implies, everything else equal, that people with lower education have higher saving rates. If one were to use these estimated differences in earnings shocks in a model with different educational groups, one would thus find (everything else equal) less wealth dispersion than in the case in which the same volatility for all groups is assumed.

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[9]This observation assumes that ones uses micro-level data set to estimate the level of the individual's earnings volatility. As discussed in section 6, much higher variability may give different results.

4.2 Extensions of the basic model

The failure of the basic model to explain wealth inequality suggests that one needs to look at other mechanisms. Two such mechanisms are entrepreneurship and preference heterogeneity.

The setup presented so far assumes implicitly that the agents are employed workers, who receive some labor income. Entrepreneurs, however, face a different decision problem, as their income is related to their business. A more recent contribution by Quadrini [82] introduces entrepreneurial choice in a dynastic framework: during each period the households decide whether to be entrepreneurs or not. Quadrini finds that a calibrated version of his model can generate a much larger amount of wealth concentration in the hands of the richest. In his model, three elements are crucial to generate this result. First, the existence of capital market imperfections induces workers that have entrepreneurial ideas to accumulate more wealth to reach minimal capital requirements. Second, in the presence of costly financial intermediation, the interest rate on borrowing is higher than the return from saving, therefore an entrepreneur whose net worth is negative faces a higher marginal return from saving and reducing his debt. Third, there is additional risk associated with being an entrepreneur, hence risk averse individuals will save more. Quadrini chooses some of the parameters of his model to match moments of the distribution of wealth and he comes much closer to fitting the upper tail of the wealth distribution than the previous models, although his model still does not generate enough asset holdings in the hands of the very richest compared to the data.

Another mechanism that has been used to generate wealth inequality is heterogeneity in preferences. The decision to save depends crucially on the

specific parameter values of the utility function. In particular, a higher degree of patience (summarized by a higher discount factor β) leads people to save more. In the presence of precautionary savings, a higher coefficient of risk aversion may also induce higher savings.

Krusell and Smith [61] generalize the basic framework by adding a stochastic process for the dynasty's preferences (both discount factor and risk aversion). The discount factor (or the risk aversion) changes on average every generation and is meant to recover the fact that parents and children in the same dynasty may have different preferences. Krusell and Smith find that it is possible to find a stochastic process for the dynasties' discount factor to match the variance of the cross-sectional distribution of wealth, while uncertainty about risk aversion does not affect the results much (although, as shown by Cagetti [18], the results are very sensitive to the values for the utility parameters chosen). While capturing the variance, their model fails to match the extreme degree of concentration of wealth in the hands of the richest 1%.

There is empirical evidence of heterogeneity in preferences, in particular in the discount factor (as show for instance by Lawrance [66] and Cagetti [19]), and this may play an important role in shaping wealth inequality. Given Krusell and Smith's results, however, preference heterogeneity alone does not seem sufficient to replicate the observed wealth concentration.

One possibility is to enrich the functional form for the utility function. Díaz, Pijoan-Mas, and Ríos-Rull [33] study the effect of habit formation in preferences and find that introducing habit formation decreases the concentration of wealth generated by this type of models and is hence not helpful in reconciling the models with the key features of wealth concentration.

Yet another possibility is to assume directly that wealth per se enters the utility function. Carroll [25] concentrates on the fact that in the data house-

holds with higher levels of lifetime income have higher lifetime saving rates (see Dynan, Skinner and Zeldes [35] and Lillard and Karoly [67]). He shows that neither standard life-cycle, nor dynastic models can recover the saving behavior of rich and poor families at the same time. To solve this puzzle he suggests a "capitalist spirit" model, in which finitely lived consumers have wealth in the utility function. This can be calibrated to make wealth a luxury good, thus rendering nonhomothetic preferences.

5 Overlapping-generations models

5.1 Life-cycle models

The models described in the previous section ignore the life-cycle dimensions of the saving decision and only roughly approximate intergenerational linkages.

The life-cycle theory of consumption was first developed by Modigliani and Brumberg [74]. In their framework a household chooses consumption by maximizing the discounted utility from consumption over a lifespan of T years, subject to a lifetime resource constraint. In its simplest form, the utility function for a household is given by $U(c_1, \ldots, c_T)$, and the resources available to the household are given by

$$\sum_{s=1}^{N} \frac{y_s}{(1+r)^s} = \sum_{s=1}^{T} \frac{c_s}{(1+r)^s},$$

where N denotes the number of working years. This theory has implications for wealth inequality: households save while working, reach a maximum wealth level at retirement age, and then decumulate their savings after retirement.

The life-cycle component of wealth inequality, however, is too small to ex-

plain the observed wealth concentration in the data. Atkinson [7] first showed that even allowing for earnings growth and uncertain life-spans the model cannot generate the wealth concentration measured in the richest decile. Wolfson [98] argued that the inclusion of further factors such as differences in earnings, rates of return and family formation still cannot replicate the empirical concentration.

A crucial element not incorporated in these models is the intergenerational transmission of wealth. Becker and Tomes [10] were the first to model explicitly the parental decision problem, by assuming that parents are altruistic and thus value transfers to their offspring. They characterize the structure of such transfers across generations, in the form of both human capital and bequests, and show that in the presence of constraints, parental transfers are first in the form of human capital, and only after the optimal amount of human capital has been reached they do take the form of monetary transfers such as bequests. Bequests are thus a luxury good in this framework.

The impact of bequests was soon shown to be potentially relevant. Among the earlier, partial equilibrium exercises, Davies [30] studies the effects of various factors, including bequests, on economic inequality in a one-period model without uncertainty. In his setup one generation of parents care about their children's future consumption, and there is regression to the mean between parents and children's earnings. As a consequence, the income elasticity of bequests is high and inherited wealth is a major cause of wealth inequality.

The stage was thus set to build and study quantitative models with a life-cycle structure and overlapping generation, which was accomplished by Laitner [63], who developed a model with two sided altruism among generations, constraints on net worth being non negative, and random lifetime earnings (abstracting thus from life-span uncertainty and earnings shocks). In this

21

setup intergenerational transfers are a luxury good and liquidity constraints are less binding for generations receiving larger transfers. This economy can also generate realistic capital to output ratios, but Laitner did not explore the implications of his model for wealth inequality.

We now turn to the models that build on this earlier literature and we discuss their quantitative findings.

5.2 A benchmark OLG framework

We use Huggett's [52] formulation as a benchmark OLG. Each period a continuum of agents are born. They live at most N periods, and face an age-dependent survival probability s_t of surviving up to age t, conditional on surviving up to age $t-1$. The demographic patterns are stable, so age t agents make up a constant fraction μ_t of the population at every point in time.

All agents have identical preferences, and have the following utility function when they first enter the model economy:

$$E\left\{\sum_{t=1}^{N} \beta^t \left(\Pi_{j=1}^t s_t\right) u(c_t)\right\},$$

where $u(c_t)$ is the constant relative-risk aversion flow of utility from consumption, and the expected value is computed with respect to the household's earnings shocks.

The labor endowment of each household is given by a function $e(z,t)$, which depends on the agent's age t, and on an idiosyncratic labor productivity shock z, that assumes a finite number of possible values and that follows a first order Markov chain with transition matrix $\Gamma(z)$.

There are no annuity markets[10]. People save to insure against earnings risk, for retirement, and in case they live a long life. People that die prematurely leave accidental bequests.

There is a constant returns to scale production technology that converts aggregate capital (K) and labor (L) into output (Y).

During each period each household choose how much to consume (c) and save for next period by holding risk free assets (a'). The household's state variables are denoted by $x = (a, z)$, where a is asset holdings carried into the period and z is the labor shock endowment.

The household's recursive problem can be written as:

$$V(x,t) = \max_{(c,a')} \left\{ u(c) + \beta s_{t+1} E\left[v(a', z', t+1) | x \right] \right\}$$

subject to

$$c + a' = (1 + r)a + e(z, t)w + T + b_t$$

$$c \geq 0, \quad a' \geq \underline{a} \quad and \quad a' \geq 0 \quad if \quad t = N,$$

where r is the interest rate net of taxes and depreciation, w is the wage net of taxes, T are accidental bequests that left by all of the deceased in a period, which are assumed to be redistributed by the government to all people alive, and b_t are social security payments to the retirees. Modelling explicitly social security is important because social security redistributes a significant fraction of income from the young to the old and thus reduces the saving rate and changes the aggregate capital-output ratio.

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[10]This is a very common assumption given how small the annuity market is in practice. Eichenbaum and Peled [37] show that in the presence of moral hazard people will choose to self-insure rather than use annuity markets even if the rate of return on annuities is high.

At every point in time this model economy can be described by a probability distribution of people over age t, assets a, and earnings shocks z.

A stationary equilibrium for this economy can be defined analogously to the one described for the infinitely-lived model, with the additional requirements that during each period total lump-sum transfers received by the households alive equal accidental bequests left by the deceased, and the government budget constraint balances every period.

Huggett [52] calibrates this model economy to key features of the U.S. data and uses different versions of it to quantify how much wealth inequality can be generated using a pure life-cycle model with labor earnings shocks and uncertain life span. The paper succeeds in matching the U.S. Gini coefficient for wealth, but the concentration is obtained by having too many people holding little wealth and by not concentrating enough wealth in the upper tail of the wealth distribution. The key reason of this failure is that in the data the rich (people with high permanent income) have a very high saving rate, while in the model households that have accumulated a sufficiently high buffer stock of assets and retirement saving don't keep saving until they reach huge levels of wealth. Huggett finds that relaxing the household's borrowing constraint increases the fraction of people bunched at zero or negative wealth, but does not increase much the asset holdings of the rich, and hence does not help in generating a distribution of wealth closer to the observed one.

Huggett also studies the amount of wealth inequality generated by his model at different ages and finds that, starting from age 40, the model underpredicts the amount of wealth inequality by age. This point is further studied by a recent work by Hendricks [49], who focuses on the performance of various models on cross-sectional wealth inequality at retirement age. Hendricks shows that, at retirement age, his (simplified) version of an OLG model

24

overstates wealth differences between earnings-rich and earnings-poor, while it understates the amount of wealth inequality conditional on similar lifetime earnings. Yang [99] uses a framework with a more realistic life cycle structure, borrowing constraints, and voluntary and accidental bequests. She finds that the model's implications are quantitatively consistent with the ones observed in the data. The model that she uses is a version of De Nardi's [32], which we describe next.

5.3 Adding bequest motives

De Nardi [32] introduces two types of intergenerational links in the OLG model used by Huggett: voluntary bequests and transmission of human capital. She models the utility from bequests as providing "warm glow" (as in Andreoni [5]). In this framework parents and their children are linked by voluntary and accidental bequests and by the transmission of earnings ability. The households thus save to self-insure against labor earnings shocks and life-span risk, for retirement, and possibly to leave bequests to their children.

Compared to Huggett, there is thus an extra term in the value function of a retired person that faces a positive probability of death:

$$V(a,t) = \max_{c,a'}\Big\{u(c) + s_t\beta E_t V(a',t+1) + (1-s_t)\phi(b(a'))\Big\} \qquad (1)$$

where

$$\phi(b) = \phi_1\Big(1 + \frac{b}{\phi_2}\Big)^{1-\sigma} \qquad (2)$$

The utility from leaving bequests thus depends on two parameters: ϕ_1, which represents the strength of the bequest motive, and ϕ_2, which measures the extent to which bequests are a luxury good. These two parameters are respec-

25

tively calibrated to match Kotlikoff and Summers's [60] data on the fraction of capital due to intergenerational transfers, and to match one moment of the observed distribution of bequests.

| Transfer wealth ratio | Wealth Gini | Percentage wealth in the top | | | | | Percentage with negative or zero wealth |
		1%	5%	20%	40%	60%	
U.S. data							
.60	.78	29	53	80	93	98	5.8–15.0
No intergenerational links, equal bequests to all							
.67	.67	7	27	69	90	98	17
No intergenerational links, unequal bequests to children							
.38	.68	7	27	69	91	99	17
One link: parent's bequest motive							
.55	.74	14	37	76	95	100	19
Both links: parent's bequest motive and productivity inheritance							
.60	.76	18	42	79	95	100	19

Table 5: OLG models of wealth inequality, from De Nardi [32]

Table 5 summarizes her results. The first line of the table refers to the U.S. data. The second one to a version of Huggett's model economy in which there are only accidental bequests, which are redistributed equally to all people alive every year. The third line also refers to an economy in which there are only accidental bequests, but these are received by the children of the deceased upon their parent's death, and are thus unequally distributed. This experiment shows that accidental bequests, even if unequally distributed, do not generate a more unequal distribution. This is because receipt of a bequest per se does not alter the saving behavior of the richest. This experiment also highlights the fact that the Auerbach and Kotlikoff's measure on intergenerational transfers is sensitive to the timing of transfers: if children inherit only

once, when their parent dies (rather than every year as in line three), this measure generates a fraction of wealth due to intergenerational transfers that is much lower than the one computed by Huggett. The fourth line allows for a voluntary bequest motive, and shows that voluntary bequests can explain the emergence of large estates, which are often accumulated in more than one generation, and characterize the upper tail of the wealth distribution in the data. The fifth line allows for both voluntary bequests and transmission of ability and shows that a human-capital link, through which children partially inherit the productivity of their parents, generates an even more concentrated wealth distribution. More productive parents accumulate larger estates and leave larger bequests to their children, who, in turn, are more productive than average in the workplace.

The presence of a bequest motive also generates lifetime saving profiles more consistent with the data: saving for precautionary purposes (emphasized in particular by Carroll [24]) and saving for retirement are the primary factors for wealth accumulation at the lower tail of the distribution, while saving to leave bequests significantly affects the shape of the upper tail. Also, with this parameterization of the voluntary bequest motive, and consistently with the data, the rich elderly do not decumulate their assets as fast as predicted by a standard a OLG model.

De Nardi finds that ϕ_2 is a large number, so bequests are a luxury good, and that the extent to which they are a luxury good is key in generating more concentration in the hands of the richest and producing a more realistic lifetime savings profiles (many papers that do not find evidence in favor of a bequest motive, such as Hurd [54] and Hendricks [49], assume that $\phi_2 = 0$.) With this parameterization, and consistently with the data, the bequest motive to save is much stronger for the richest households, who, even when very old, keep some

assets to leave to their children. The rich leave more wealth to their offspring, who, in turn, tend to do the same. This behavior generates some large estates that are transmitted across generations because of the voluntary bequests, while being quantitatively consistent with the elasticity of the savings of the elderly to permanent income that has been estimated from microeconomic data (Altonji and Villanueva [4]).

It is clear from this table that, although modeling explicitly both of these mechanisms does help to better explain the the savings of the richest, De Nardi's model is not capable of matching the wealth concentration of the richest 1% of the people.

5.4   Other extensions

Heer [48] adopts a model in which richer and poorer people have different tastes for leaving bequests. His characterization of the labor income process (people can be employed or unemployed) does not generate enough income inequality compared with the data and his model does not produce large wealth concentration.

Hendricks [50] studies the effects of allowing for preference heterogeneity in a life-cycle framework with only accidental bequests. Consistently with Krusell and Smith [61], he finds that heterogeneity in risk aversion has only minimal effects on saving and wealth inequality. Moreover, he shows that time preference heterogeneity only makes a modest contribution in accounting for high wealth observations if the heterogeneity in discount factor is chosen to generate realistic patterns of consumption and wealth inequality as cohorts age.

Hubbard Skinner and Zeldes [51] focus on the effects of social insurance

programs on wealth holdings of poorer people because micro data find a significant group in the population with little wealth. They show that in presence of precautionary savings the asset-based means testing of welfare programs can imply that a significant fraction of people with lower lifetime earnings do not accumulate wealth.

Gokhale et al. [42] aim at evaluating how much wealth inequality at retirement age arises from inheritance inequality. To do so, they construct an overlapping-generations model that allows for random death, random fertility, assortative mating, heterogeneous human capital, progressive income taxation and social security. All of these elements are exogenous and calibrated to the data. The families are assumed not to care about their offspring, hence all bequests are involuntary. To solve the model, they impose that individuals are infinitely risk averse and that the rate of time preference equals the interest rate. In their framework inheritances in the presence of social security play an important role in generating intra-generational wealth inequality at retirement. The intuition is that social security annuitizes completely the savings of poor and middle-income people but is a very small fraction of the wealth of richer people, who thus keep assets to insure against life-span risk.

6   Mixtures of life-cycle and dynastic behavior

The third class of models mixes features of both life-cycle models and infinitely-lived dynasties, simplifying some aspects of either model to make them more computationally tractable.

Among these works, Laitner [64] assumes that all households save for life-cycle purposes, but only some of them care about their own descendants. There are perfect annuity markets, therefore all bequests are voluntary, and

no earning risk over the life cycle, hence no precautionary savings. Laitner's model is simple to compute and provides a number of interesting insights. The concentration in the upper tail of the wealth distribution is matched by choosing the fraction of households that behave as a dynasty and also depends on the assumptions on the distribution of wealth within the dynasty.

Nishiyama [75] adopts an OLG model with bequests and intervivos transfers in which households in the same family line behave strategically. As De Nardi, he concludes that the model with intergenerational transfers better explains, although not fully, the observed wealth distribution.

Castañeda, Díaz-Giménez and Ríos–Rull [27] consider a model economy populated by dynastic households that have some life-cycle flavor: workers have a constant probability of retiring at each period and once they are retired they face a constant probability of dying. Each household is perfectly altruistic toward its household. The paper employs a number of parameters to match some features of the U.S. data, including measures of wealth inequality.

The key feature of the model that generates huge amount of wealth holdings in the hands of the richest is the productivity shocks process. This process is calibrated so that the highest productivity level is more than 100 times higher than the second highest. There thus is an enormous discrepancy between the highest productivity level and all of the others. Moreover, if one is at the highest productivity level, the chance of being 100 times less productive during the next period is more than 20%. High-ability households thus face much higher earnings risk, save at very high rates to self-insure against earnings risk, and thus build huge buffer stocks of assets.

Building on Quadrini [81], Cagetti and De Nardi [20] take seriously the observation that entrepreneurs, that is, households that own and manage privately-held businesses, make up for the largest fraction of rich people in the

30

data. Cagetti and De Nardi [20] assume that households have two types of ability: entrepreneurial ability (θ), and worker's ability (y). The entrepreneurial ability is linked to the capacity to produce income out of capital according to the following production function:

$$y_e = \theta k^{\nu},$$

where y_e is income from being an entrepreneur during a period and investing working capital k, and ν is the degree of decreasing returns to scale, or "span-of-control," as in Lucas [71]. The worker's ability is linked to the ability to earn income when working for someone else, where the worker's income in a period is given by:

$$y_w = w\,y,$$

where w is the market wage.

During each period, the individual observes his abilities (which evolve stochastically over time), and makes an occupational choice for that period. Contracts are imperfectly enforceable: people repay the amount they own only if it is in their own interest to do so, as in Albuquerque and Hopenhayn [2]. Because of this friction, the amount that an entrepreneur can borrow is a function of his own wealth, which thus acts as collateral. The firm size can thus be suboptimal, and richer households are able to borrow more and grow faster. The model adopts a demographic structure similar to Castañeda, Díaz-Giménez and Ríos–Rull [27], and can thus incorporate the transmission of wealth, and of businesses, across generations.

Compared to Quadrini, Cagetti and De Nardi obtain a much better fit of the upper tail of the wealth distribution by endogenizing the firm size distribution.

31

They do not choose any of the parameter of their model to generate this result, which should hence be interpreted as a check of the goodness of the model. The key reason why their model succeeds in generating this large amount of wealth concentration is linked to the fact that, while entrepreneurs could invest capital at a higher rate of return, the presence of borrowing constraints and collateral requirements makes the entrepreneur to save to exploit the high rate of return even when the entrepreneur becomes "rich". This key intuition does not depend on the demographic structure assumed, and would also hold in a dynastic model. Cagetti and De Nardi chose to formulate it in an economy with more realistic life-cycle features to study the effects of government policies such as estate taxation. The life-cycle structure makes it possible to study the effects of government policies such as estate taxation on wealth inequality and capital accumulation (see Cagetti and De Nardi [21]).

7 Future directions

In the previous sections, we have discussed if and to what extent the current economic models have been able to explain the determinants of wealth inequality in the United States. While the baseline versions of the standard economic models badly fail to replicate the degree of wealth concentration observed in the data, some extensions have had a much greater success.

As we learn more about the determinants of wealth concentration, we can start applying new frameworks to study many economic problems for which inequality is a key element. In what follows, we briefly discuss some of these areas. This discussion, of course, is by no means complete.

7.1 Human capital

All quantitative models of wealth inequality that we are aware of take human capital as exogenous. As documented by Huggett et al. [53], modelling human capital investment in presence of heterogenous learning abilities and exogenous shocks is important to reproduce the data on earnings inequality over the life cycle. Modeling human capital explicitly would also allow a better measurement of the relative importance of human capital formation relative to bequests in generating wealth inequality, in the spirit of Becker and Tomes [10], especially when human capital acquisition is limited by imperfect financial markets (as for instance in the analysis of Heckman et al. [47]). For these reasons it would be worthwhile to study saving decisions and wealth inequality in a framework that also considers human capital accumulation and disentagles the permanent and transitory sources of inequality as in Huggett et al. [53].

7.2 Portfolio choice

The models we have discussed typically assume only one riskless asset, or at most two when entrepreneurial investment is included. An important issue, however, is portfolio choice. Gollier [43] provides a survey about the theory of household portfolios, Haliassos and Michaelides [45] discuss techniques for calibrating and solving household portfolio models, while Miniaci and Weber [36] focus on the econometric issues in the estimation of household portfolio models.

Households' portfolio are very heterogeneous by age, income or occupation (see for instance Bertaut and Starr-McCluer [13], Poterba and Samwick [80], Banks et al. [9], Heaton and Lucas [46], Carroll [26], Hurd [55], and Yang [101]).

Some papers have started to study portfolio choice with heterogeneous

agents in a life-cycle setting. For example, Campbell et al. [22] and Campbell and Viceira [23] have shown that the fraction of risky assets in the portfolio should decrease with age as people move closer to retirement, and Benzoni et al. [11] have further qualified the relation between age, income and asset positions.

For entrepreneurial households business wealth constitutes a relevant fraction of their total net worth. Heaton and Lucas [46] study the effect of business assets on portfolio choice and asset pricing.

A large fraction of total wealth for most households is in the form of housing, a relatively illiquid and indivisible type of investment, with unique risk and tax characteristics. While standard finance models focus on other types of risky assets, recent works have explicitly modeled housing. For instance, Yao and Zhang [102] show that inclusion of housing dramatically changes the fraction of risky and riskless assets held in a portfolio, and Flavin and Yamashita [39] examine the life-cycle pattern of portfolio composition induced by lumpy housing investment. Yang [100] develops a quantitative, dynamic general equilibrium model which, consistently with the consumption data over the life cycle, generates both hump-shaped consumption profiles for non-housing goods and lack of decrease of housing consumption later in life.

Poterba [79] studies the effects of taxation on portfolio choice. In the data, households with different wealth levels hold very different portfolios (Carroll [26]). In order to understand the aggregate impact of portfolio decisions and taxation on aggregate investment and equilibrium prices it is vital to consider how wealth is distributed in the population, and in particular, to understand the saving and portfolio of all households, including the richest ones, who hold a disproportionate share of total wealth. The models of wealth inequality presented in this paper may help shed light on these issues.

34

7.3 Public policy: adequacy of savings

Wealth inequality is also vital to understand policy and redistributional issues. While wealth inequality is often seen per se as a negative aspect that must be addressed by redistributional policies, a different question is whether the observed levels of wealth for most households outside of the richest percentiles are in fact in some way suboptimal and inadequate.

There has been some debate on the adequacy of savings. Some economists believe that the wealth holding of many (or most) households are too low. Most of these works are based on some form of myopia or inconsistency in preferences, as for instance in Lusardi [72] or in models with hyperbolic discounting as in Laibson et al. [62]. Given this lack of foresight or commitment, households tend to save less than it is optimal, and thus government intervention may improve welfare.

In contrast, other works have shown that the currently observed levels of wealth are consistent with a rational, optimizing life-cycle model of wealth accumulation. For instance, Engen et al. [38] show that the amount of savings of most households (except those at the bottom quartile of the wealth distribution) is similar if not larger than that implied by a standard life-cycle model of wealth accumulation with social security and retirement benefits, while Scholz et al. [87] argue that, even for most households in the bottom of the distribution, the wealth deficit, relative to the optimal target, is generally small.

The key element of these works is the current level of social security benefits and other transfers after retirement. While the individual decisions may be optimal given those policies, an entirely different questions is whether the current amount of social security is optimal, and whether aggregate welfare may

be improved by different schemes and different saving program incentives such as IRAs. There is a vast literature on this topic, and the question remains, to a large extent, unresolved. We will not try to summarize the various positions, but we point out that careful quantitative models of wealth inequality can help shed light on the issue.

7.4   Public policy: tax reforms

An area of public policy that crucially depend on wealth inequality is taxation, in particular for those taxes that mainly hit the richest households, such as estate and progressive income taxes.

Given the current exemption levels, a very small fraction of people pays any estate tax (approximately 2% of estates are taxed), and the aggregate revenue for the tax is a relatively small .3% of GDP. However, the households that pay the tax are also those who do most of the saving and hold a large fraction of total wealth. Therefore, their behavior may have a large effect on the aggregates in the economy.

Reforms of these taxes are now being actively debated. To understand the impact of such reforms, it is important to understand how many of these rich households are affected, and and how strongly they are affected. Quantitative models that carefully analyze the determinants of wealth inequality are thus key to study the problem. Using such models, Meh [73] studies changes in the degree of tax progressivity in Quadrini's [82] model, and Cagetti and De Nardi [21] study estate and income taxation in their setup.

7.5 Macroeconomics and the representative agent

When analyzing the effects of aggregate shocks, macroeconomics typically assumes a representative agent. This allows a considerable simplification, at the cost of ignoring the effect of heterogeneity in the population. While heterogeneity may be irrelevant for studying some macroeconomic problems, Browning et al. [15] have argued that in certain cases the behavior of an economy with many agents is significantly different from that of a representative agent one. Few works so far have been able to address the issue. The difficulty lies in the fact that with heterogeneity and aggregate shocks, the distribution of people over state variables may change over time, and, at least in theory, one needs to keep track of the distribution as an additional state variable. The Bewley-type of models studied so far consider steady states without aggregate shocks, in which therefore the distribution is constant. But in the presence of aggregate shocks this is not true anymore.

Krusell and Smith [61] were the first to solve a model with aggregate productivity shocks and heterogeneous agents. They find that in their model heterogeneity does not matter for aggregate movements, a result partly confirmed also by Storesletten et al. [90], who extend their setup to a life-cycle economy. In this economy, heterogeneity may have consequences for mobility and individual welfare, but does not affect the aggregate movements due to the business cycle. It is exactly because of this irrelevance that these authors are able to solve their models numerically. When the distribution has little effect on the aggregates, it ceases to be a significant state variable, and one need only keep track of it its mean, or at most its variance.

While they generate wealth inequality, these models fail to replicate the extreme degree of wealth concentration and the fraction of wealth held by the

37

richest percentiles. As argued before, the behavior of this group may be quite different from that of the median households. Entrepreneurship, for instance, may imply different responses of investment and savings to aggregate shocks. Incorporating the insights of the models that study wealth concentration into a setup with aggregate shocks is very interesting, but poses a considerable computational challenge. It is necessary to solve the decision problem for a large number of agents (as in a standard Bewley model), while at the same time keeping track of the distribution of wealth (a function) as a state variable. As computational power increases and new algorithms are developed, more and more of these problems may start to be tackled.

7.6 Entrepreneurship, wealth and growth

In the paper, we have highlighted the role of entrepreneurs in determining capital accumulation and wealth inequality in the United States. Unfortunately, little empirical evidence exists for the role of entrepreneurs in shaping wealth inequality in other countries. However, several studies suggests that entrepreneurs are important also in other countries.

Among these, Banerjee and Newman [8] develop a framework to study how initial wealth inequality shapes entrepreneurial decisions and in turn development. In the presence of capital market imperfections, only richer households can become entrepreneurs and create large firms. Thus, business formation and growth depend on the wealth distribution, which is in turn dynamically determined by entrepreneurial decisions.

The evidence from Thailand studied in Paulson and Townsend [76] shows that financial constraint are key to determine business start-up, and wealthier households are more likely to start a business and face less stringent con-

38

straints. Furthermore, entrepreneurship and financial deepening can account for a significant fraction of the growth in total factor productivity in Thailand (as found by Jeong and Townsend [57]), and their effect depends crucially on the wealth distribution.

More data and empirical research on entrepreneurship may thus be key to understand wealth accumulation and the implied wealth distribution not only for developed, but also for developing countries.

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