NBER WORKING PAPER SERIES

WAGE INDEXATION AND TIME-CONSISTENT MONETARY POLICY

Laurence Ball

Stephen G. Cecchetti

Working Paper No. 2948

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 1989

This paper is part of NBER's research programs in Economic Fluctuations and Financial Markets and Monetary Economics. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

NBER Working Paper #2948
April 1989

WAGE INDEXATION AND TIME-CONSISTENT MONETARY POLICY

ABSTRACT

This paper investigates the effects of wage indexation on the time-consistent level of inflation. Departing from previous work on time-consistent policy, we study a structural model of the economy. Indexation reduces the cost of inflation, which is inflationary, and steepens the Phillips curve, which is anti-inflationary. In most cases, the net effect is to raise inflation but also to raise welfare: the loss from higher inflation is outweighed by the gain from greater protection against inflation.

Laurence Ball
Department of Economics
Princeton University
Princeton, NJ 08544-1017

Stephen G. Cecchetti
Department of Economics
Ohio State University
Columbus, OH 43210-1172

1 Introduction

Since the early 1970's, both academic researchers and policymakers have devoted considerable attention to the macroeconomic effects of wage indexation. But these two groups have focused on different effects. Starting with Gray (1976) and Fischer (1977a), formal models emphasize the role of indexation in stabilizing or destabilizing output. In contrast, informal policy discussions usually focus on the allegation that indexation is inflationary (for example, Okun, 1971; Simonsen, 1983; Cardoso and Dornbusch, 1987). To reduce this gap, this paper presents a model of the effects of indexation on inflation and asks whether, when these effects are taken into account, indexation raises economic welfare.

Until recently it was difficult to formalize the argument that indexation is inflationary, because economists lacked models of the sources of inflation. This paper applies the insight of Barro and Gordon (1983a) that inflation arises because, given the employment gains from unexpected inflation, noninflationary monetary policy is not time-consistent. As stressed by Fischer and Summers (1988), policies that reduce the costs of inflation, such as indexation, cause Barro-Gordon policymakers to choose higher inflation. Indeed, inflation can rise so much that welfare falls despite greater protection against inflation. This effect is appealing because it captures the common argument that indexation weakens policymakers' will to fight inflation.[1]

This is not the end of the story, however, because wage indexation has a second effect: it reduces the employment effects of surprise inflation. That is, unlike most policies to reduce the costs of inflation, such as indexation of interest rates, wage indexation steepens the Phillips curve. In the Barro-Gordon model, a steeper Phillips curve reduces the temptation to inflate, and hence reduces equilibrium inflation. Since indexation has one inflationary effect — lower costs of inflation — and one anti-inflationary effect — a steeper Phillips curve — the net effect appears ambiguous. And even if the net effect on inflation is determined, the welfare effect may remain unclear: if inflation rises, the welfare loss might or might not be outweighed by the lower cost of a given amount of inflation.[2]

[1] For example, Okun (1971) argues that indexation 'may be read as evidence that the government has raised its tolerance level of inflation.... It is quite rational to expect that the more effectively the government can minimize the social costs of inflation, the more inflation it will accept.'

[2] Fischer and Summers mention the effect of wage indexation on the Phillips curve, but they do not include it in their model because they focus on 'inflation mitigation' policies that do not affect the Phillips curve, such

This paper attempts to resolve these ambiguities. We do so by deriving time-consistent monetary policy in the Gray-Fischer model of an indexed economy. In studying a structural model, we depart from the Barro-Gordon tradition of specifying ad hoc costs of inflation and unemployment and an ad hoc Phillips curve. Our approach allows us to derive the effects of indexation rather than simply assuming that indexation changes ad hoc parameters in certain directions. As a result, it is possible to determine the sum of opposite effects.

Following Barro-Gordon, we modify the Gray-Fischer model by introducing a microeconomic distortion (for concreteness, taxation) that causes equilibrium employment to fall short of the social optimum. In addition, we assume that the money stock is chosen by an optimizing policymaker rather than fluctuating randomly. The other essential elements of models of time-consistent policy arise naturally in the Gray-Fischer model. There is a short run Phillips curve because wages do not adjust immediately to nominal shocks. And inflation is costly because, with staggered wage setting, it causes inefficient variation in relative wages.

In our model, perfect indexation completely eliminates both the costs of inflation and the Phillips curve; as a result, the time-consistent inflation rate is indeterminate. But perfect indexation is unrealistic: in actual economies, less than 100% of wages are indexed, and wage adjustments occur at discrete intervals rather than continuously. We incorporate these imperfections and determine both the effects of extending coverage and the effects of more frequent adjustment. The details of the results are complicated, but simple conclusions emerge. It appears likely that indexation raises inflation. On the other hand, the welfare effects of indexation are more likely to be positive than negative.

Since our results arise from specific assumptions about the structure of the economy, we investigate robustness. We focus on the costs of inflation. The cost in our model, relative wage variation, is realistic, and it is the main inflation cost that wage indexation reduces. But there are other costs that wage indexation does not reduce, such as non-neutral changes in taxes and increased costs of consumer search. Adding these costs to the model reduces the effect of indexation on inflation and can even make it negative. In addition, the conclusion that indexation raises welfare is strengthened.

The remainder of the paper consists of five sections. Section 2 presents a simple discrete time

as indexation of bonds and of the tax system.

model of the economy and derives the time-consistent inflation rate. Section 3 adds indexation. Assuming that indexed wages adjust immediately to the price level, we show that extending coverage of indexation unambiguously raises inflation but also unambiguously raises welfare. Section 4 studies a continuous time model that includes adjustment lags as well as incomplete coverage. Here the effects of greater indexation are more complicated. Section 5 introduces additional costs of inflation. Finally, Section 6 offers conclusions.

2 The Simple Model And Time-Consistent Policy

This section derives time-consistent monetary policy in a simple Gray-Fischer model of staggered wage setting. For the moment we ignore indexation. Departing from Gray and Fischer, but following Barro-Gordon, we assume that equilibrium employment is less than the social optimum, and that there are no shocks to the economy. These assumptions imply that employment is constant at the equilibrium level and that inflation is constant and positive even though zero inflation is optimal.

A. Assumptions

The economy contains a continuum of firms indexed by i and distributed uniformly between $i = 0$ and $i = 1$. Each firm has a pool of immobile workers. For simplicity, the firms produce the same product, so the goods market is perfectly competitive. Omitting constants and time indices, firm i's production function is

$$y_i = \alpha l_i \ , \qquad 0 < \alpha < 1 \ , \tag{1}$$

where y is the log of output and l is the log of employment. The production function and profit-maximization yield firm i's labor demand:

$$l_i^D = -\frac{1}{1-\alpha}(w_i - p) \ , \tag{2}$$

where w_i is the log of the firm's nominal wage and p is the log of the price level (the price for the single good).

The labor supply of firm i's workers is

$$l_i^S = \eta(\hat{w}_i - p) \ , \qquad 0 < \eta \ , \tag{3}$$

where $\hat{w}$ is the log of the after-tax wage. In levels, the after-tax wage is a proportion $\phi < 1$ of the pre-tax wage. Because of taxation, the equilibrium pre-tax wage defined by $l_i^S = l_i^D$ is greater than the efficient level; equivalently, equilibrium employment is less than the efficient level. As a normalization (corresponding to choices of omitted constants), let the equilibrium $w_i - p$ be zero. This implies that the efficient real wage — the equilibrium in the absence of taxes — is $-K$, $K = \frac{\alpha\eta-\eta}{1+\eta-\alpha\eta} ln(\phi) > 0$.

Taxes are the simplest modification of the Fischer-Gray model that causes equilibrium and optimal employment to differ. Nothing depends on this source of inefficiency. An alternative is to introduce imperfect competition in the goods market, as in Ball (1987, 1988). In this case K depends on the slope of firm i's demand curve.

Labor is sold through two period contracts. Each period, half of all firms sign contracts. A contract specifies a single wage for the two periods — that is, wages are 'fixed,' as in Taylor (1980) and Blanchard (1986), not merely 'predetermined,' as in Fischer and Gray. (This assumption is essential for the result below that steady inflation causes relative wage movements.) Wages are chosen after the money stock for the first period is observed. After wages are set, firms choose employment, so employment equals labor demand.

We measure the welfare loss from a deviation of firm i's real wage from the efficient level of $-K$ by

$$L_i = (w_i - p + K)^2 \, , \tag{4}$$

which is a second order Taylor approximation. (Since labor demand is log-linear, (4) is proportional to the squared deviation of log employment from the efficient level.)[3] The monetary authority attempts to minimize (4) averaged across firms and time periods (we ignore discounting). Contract signers do *not* minimize (4). Instead, they minimize the private losses (in profits and workers' consumer surplus) from deviations of the real wage from the equilibrium level of zero. We approximate this private loss by $(w_i - p)^2$. Minimizing the private loss over the two

[3]Fischer and Summers show that their results change when they alter their quadratic functional form for the social loss. In our model a quadratic loss is appropriate because it is a Taylor approximation to a true loss function. The alternative functional forms that Fischer and Summers discuss are inappropriate because they contain linear terms and thus imply first order losses from deviations from the optimal real wage. By the envelope theorem, the losses must be second order.

periods that a wage is in effect implies a simple wage-setting rule:

$$x_t = \frac{1}{2}(p_t + E_t p_{t+1}) \ , \tag{5}$$

where x_t is the wage chosen at t for t and $t+1$.

Finally, money enters the model through a quantity equation for money demand:

$$m_t - p_t = y_t \ , \qquad y = \int_{i=0}^{1} y_i di \ , \tag{6}$$

where m is the log money stock. The money stock is chosen each period by a policymaker. As in Barro-Gordon (1983a), the policymaker takes expectations of current and future money as given. Thus we focus on 'discretionary' rather than 'reputational' equilibria (Barro-Gordon, 1983b). The model is closed by assuming that actual and expected money are equal.[4]

B. The Behavior of Wages and the Price Level

Our goal is to solve for the time-consistent inflation rate. The solution depends on how trend money growth influences the welfare effects of monetary surprises. A preliminary step is to determine the behavior of wages and the price level as functions of a given trend money growth, μ, and sequence of surprises, $\{\delta_t\} = \{\Delta m_t - \mu\}$. The derivation is similar to ones in previous work on staggered price setting (for example, Blanchard and Fischer, 1989).

Combining the production function, (1), labor demand, (2), and money demand, (6), yields the price level in terms of money and the aggregate wage:

$$p_t = \alpha w_t + (1-\alpha) m_t \ , \qquad w = \int_{i=0}^{1} w_i di \ . \tag{7}$$

The aggregate wage is the average of wages set in the current and previous periods:

$$w_t = \frac{1}{2}(x_t + x_{t-1}) \ . \tag{8}$$

Substituting (7) and (8) into the wage-setting rule, (5), leads to

$$x_t = \frac{1-\alpha}{2-\alpha}\mu + \frac{2(1-\alpha)}{2-\alpha} m_t + \frac{\alpha}{2(2-\alpha)} x_{t-1} + \frac{\alpha}{2(2-\alpha)} E_t x_{t+1} \ , \tag{9}$$

[4]Our model departs from Barro-Gordon in several realistic ways: the policymaker chooses money growth rather than setting inflation directly; a policy surprise affects inflation in the future as well as in the present; and the costs of inflation depend on whether it is anticipated. These features are shared by other recent models of time-consistent policy.

where we use the fact that $E_t m_{t+1} = m_t + \mu$. The method of undetermined coefficients yields

$$x_t = \lambda x_{t-1} + (1-\lambda)m_t + \frac{1}{2}(\mu + \mu\lambda), \quad \lambda = \frac{1-\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}}. \tag{10}$$

Finally, solving this difference equation and using the fact that $m_t = \mu t + \sum_{s=0}^{\infty} \delta_{t-s}$, we obtain

$$\begin{aligned} x_t &= \mu(t+\frac{1}{2}) + (1-\lambda)\sum_{s=0}^{\infty}\lambda^s \sum_{r=0}^{\infty}\delta_{t-s-r} \\ &= \mu(t+\frac{1}{2}) + \sum_{j=0}^{\infty}(1-\lambda^{j+1})\delta_{t-j}. \end{aligned} \tag{11}$$

According to (11), the wage set at t equals the trend level of money (averaged between t and $t+1$) plus a weighted sum of past surprises. Substituting (11) into (7) and (8) yields similar solutions for the aggregate wage and the price level.

C. Equilibrium Inflation and Welfare

We now solve for equilibrium inflation and welfare. Equilibrium inflation equals trend money growth, which in turn is determined by two conditions. First, since the policymaker minimizes the average of the loss function (4), the derivative of this average with respect to the surprise δ_t must be zero. Second, since actual and expected money growth are equal, all surprises must be zero. Combining these conditions, the equilibrium μ is defined by the first order condition for δ_t evaluated at $\delta_s = 0$ for all s. Intuitively, the condition states that, given expected money growth of μ, the policymaker finds it optimal not to create a surprise.

A monetary surprise affects the policymaker's loss in the current and all future periods. Consider the effect of δ_t on the loss at $t+r$, $r \geq 0$. Averaging across firms in the first and second periods of contracts, whose wages are x_{t+r} and x_{t+r-1}, the loss is

$$L(t+r) = \frac{1}{2}(x_{t+r} - p_{t+r} + K)^2 + \frac{1}{2}(x_{t+r-1} - p_{t+r} + K)^2. \tag{12}$$

The effect of δ_t is

$$\begin{aligned} \frac{dL(t+r)}{d\delta_t} &= (x_{t+r} - p_{t+r} + K)\left(\frac{dx_{t+r}}{d\delta_t} - \frac{dp_{t+r}}{d\delta_t}\right) \\ &\quad +(x_{t+r-1} - p_{t+r} + K)\left(\frac{dx_{t+r-1}}{d\delta_t} - \frac{dp_{t+r}}{d\delta_t}\right) \end{aligned} \tag{13}$$

$$= (x_{t+\tau} - p_{t+\tau} + K)(\frac{\alpha}{2}\lambda^\tau + \frac{\alpha-2}{2}\lambda^{\tau+1})$$
$$+(x_{t+\tau-1} - p_{t+\tau} + K)(\frac{\alpha-2}{2}\lambda^\tau + \frac{\alpha}{2}\lambda^{\tau+1}) ,$$

where the second line uses the solution for x, (11), the equations defining the price level, (7) and (8), and the fact that $\frac{dm_{t+\tau}}{d\delta_t} = 1$.

When all surprises are zero, the solutions for x and p imply $x_{t+\tau} - p_{t+\tau} = \frac{\mu}{2}$ and $x_{t+\tau-1} - p_{t+\tau} = -\frac{\mu}{2}$: with steady inflation, the real wage averages to zero over a contract. Substituting these results into (13) leads to

$$\frac{dL(t+\tau)}{d\delta_t}|_{\delta_s=0\ \forall_s} = [\mu - 2K(1-\alpha) - \mu\lambda - 2K(1-\alpha)\lambda]\frac{\lambda^\tau}{2} . \quad (14)$$

The equilibrium μ is defined by setting to zero the sum of $\frac{dL}{d\delta_t}$ over all periods:

$$\sum_{\tau=0}^{\infty} \frac{dL(t+\tau)}{d\delta_t}|_{\delta_s=0\ \forall_s} = [\mu - 2K(1-\alpha) - \mu\lambda - 2K(1-\alpha)\lambda]\sum_{\tau=0}^{\infty}\frac{\lambda^\tau}{2} = 0 . \quad (15)$$

The solution is

$$\mu = \frac{2K(1-\alpha)(1+\lambda)}{1-\lambda} \quad (16)$$
$$= 2K\sqrt{(1-\alpha)} ,$$

where the second line uses the definition of λ. Finally, substituting the expressions for real wages under steady inflation into (12) yields the equilibrium social loss per period:

$$L = \frac{1}{2}\left(\frac{\mu}{2}+K\right)^2 + \frac{1}{2}\left(-\frac{\mu}{2}+K\right)^2 \quad (17)$$
$$= \frac{\mu^2}{4} + K^2$$
$$= K^2(2-\alpha) ,$$

where the last line uses (16).

D. Discussion

We can gain intuition about the determinants of μ through a different derivation. Using the fact that $w_{t+\tau}$ is the average of $x_{t+\tau}$ and $x_{t+\tau-1}$, the loss function, (12), can be rewritten as

$$L(t+\tau) = (w_{t+\tau} - p_{t+\tau} + K)^2 + \frac{1}{4}(x_{t+\tau} - x_{t+\tau-1})^2 . \quad (18)$$

The first term in the loss is the deviation of the aggregate real wage from the optimum. The second term is the dispersion of wages (and hence employment) across contracts signed in different periods. Dispersion is inefficient because the loss, (12), is convex. In equilibrium, $(x_{t+r} - x_{t+r-1})$ equals trend inflation μ. Thus wage dispersion is the cost of steady inflation in this model.

Differentiating (18) yields

$$\begin{aligned} \frac{dL(t+r)}{d\delta_t} &= 2(w_{t+r} - p_{t+r} + K)\frac{d(w_{t+r} - p_{t+r})}{d\delta_t} \\ &\quad + \frac{1}{2}(x_{t+r} - x_{t+r-1}\frac{d(x_{t+r} - x_{t+r-1})}{d\delta_t}, \end{aligned} \tag{19}$$

$$\frac{dL(t+r)}{d\delta_t}|_{\delta_s=0\ \forall s} = 2K\frac{d(w_{t+r} - p_{t+r})}{d\delta_t} + \left(\frac{\mu}{2}\right)\frac{d(x_{t+r} - x_{t+r-1})}{d\delta_t};$$

where the derivative of the real wage is negative and the derivative of dispersion is positive. Equation (19) shows that a monetary surprise has two effects. First, since wages do not adjust fully, it reduces the aggregate real wage, which pushes it closer to the optimum and thus reduces the loss. Second, the surprise increases dispersion, and thus raises the loss, since wages set in different periods adjust to different degrees. For example, in the period when the shock occurs ($r = 0$), new wages adjust partially, but wages set the previous period do not adjust at all.[5] The welfare effects of the lower aggregate wage and greater dispersion depend on K, the initial deviation of the aggregate from the optimum, and on μ, the initial dispersion.

Equation (14) implies that the μ at which $\frac{dL(t+r)}{d\delta_t}|_{\delta_s=0}$ equals zero is the same for every r. Thus the equilibrium μ can be derived by setting this derivative to zero for a single r (rather than summing over r as in (15)). It follows that

$$\mu = -4K\left[\frac{d(w_{t+r} - p_{t+r})/d\delta_t}{d(x_{t+r} - x_{t+r-1})/d\delta_t}\right] \tag{20}$$

for any r. Equation (20) gives the inflation rate at which the cost of a monetary surprise just balances the benefit. This solution is similar to the one in Barro-Gordon. Inflation is proportional to the underlying microeconomic distortion, K. It is proportional to the effect of a surprise on the aggregate real wage and employment — that is, the slope of the Phillips curve. And it is inversely proportional to the effect of a surprise on the cost of inflation.

[5] More precisely, dispersion rises because the wages that adjust most to a shock — the ones set most recently — are also the wages that are initially highest.

In contrast to Barro-Gordon, our model includes a specific cost of inflation, wage dispersion. This cost is realistic: empirical studies show that inflation increases dispersion in both wages and prices, and economists frequently cite this effect as an important cost of inflation.[6] Of course it is not the only cost, but it is appropriate to emphasize it here because it is the main cost that wage indexation reduces. Wage indexation has no effect on the inflation tax on money, the non-neutralities in the tax system, and so on. In any case, Section 5 adds other costs of inflation to the model.

3 Indexation in the Basic Model

A. Assumptions

We now ask how wage indexation affects inflation and welfare in the model of the last section. An indexed contract is defined as one in which the second period wage is contingent on the price level. An indexed firm is able to set $w = p$ every period, reducing its private loss to zero. (This is accomplished by setting the base wage equal to the first period price level and specifying full adjustment for inflation in the second period.) Thus indexation is equivalent to replacing two period contracts with one period contracts, which also assures $w = p$. As this suggests, the crucial feature of indexation is the increased frequency of wage adjustments. This feature is emphasized in many discussions of indexation policy (for example, Cardoso and Dornbusch, 1987, on Brazil).

Indexation by all firms, by keeping all real wages at zero, would completely eliminate both the real effects of monetary surprises and the wage dispersion that makes inflation costly. In this case, $\frac{dL(t+r)}{d\delta_t}$ would be zero for all μ, so any inflation rate would be an equilibrium. Since inflation would be costless, the equilibrium social loss would be K^2, the loss for $\mu = 0$ in the previous section.

Of course perfect indexation is unrealistic, and so we introduce an imperfection: a proportion $\pi < 1$ of firms have indexed contracts, but the rest do not. We determine the effects of

[6] If we modify the model by introducing differentiated products, then variation in relative wages leads to variation in relative goods prices (Ball, 1987). For evidence on inflation and relative price variability, see for example Vining and Elwertowski (1976). For relative wage variability, see Cecchetti (1987).

increasing π — that is, of extending the coverage of indexation. Incomplete coverage is the easiest imperfection to add to this model; the continuous time model of the next section includes adjustment lags that make indexation imperfect even for indexed firms.

B. Equilibrium Inflation and Welfare

It is easy to add indexation to the basic model. With indexation, wages equal the price level for a proportion π of firms. Thus the aggregate wage is given by

$$w = \pi p + (1 - \pi) w^N \,, \tag{21}$$

where w^N is the average wage for non-indexed firms. Combining (21) with (7) yields

$$p = \hat{\alpha} w^N + (1 - \hat{\alpha}) m \,, \qquad \hat{\alpha} = \frac{\alpha - \alpha\pi}{1 - \alpha\pi} \,. \tag{22}$$

Equation (22) is the same as equation (7) except that $\hat{\alpha}$ replaces α and w^N replaces w. Note that $\hat{\alpha}$ is decreasing in π: greater indexation reduces the effect of non-indexed wages on the price level and increases the direct effect of money. The wage-setting rule (5) still holds for non-indexed firms, and the aggregate non-indexed wage is still given by (8) (with w^N and x^N replacing w and x). As shown in the last section, (5), (7), and (8) determine the behavior of wages and the price level. Thus the solutions for w and p carry over here, except that $\hat{\alpha}$ replaces α (for example, in the definition of λ) and the results for wages apply only to non-indexed firms.

The policymaker chooses the monetary surprise to minimize his loss function averaged over all firms. But the surprise has no effect on indexed firms, since their real wages are fixed at zero. Thus the policymaker's problem reduces to minimizing the average loss for non-indexed firms. Since these firms' wages and the price level are given by the expressions in the last section, with $\hat{\alpha}$ replacing α, so are the policymaker's first order condition and the solution for inflation. Thus equilibrium inflation is

$$\mu = 2K\sqrt{(1 - \hat{\alpha})} \,, \tag{23}$$

which is the analogue of (16). Since $\hat{\alpha}$ is decreasing in π, μ is increasing in π: indexation is inflationary.

Turning to welfare, the equilibrium social loss for an indexed firm is K^2 (the value of (12) when the real wage is zero). As in the last section, the loss for a non-indexed firm is $(\frac{\mu^2}{4} + K^2)$.

These results imply an average loss of

$$L = K^2 + \frac{(1-\pi)\mu^2}{4}\,. \tag{24}$$

Not surprisingly, for a given inflation rate, greater indexation reduces the average loss. Finally, substituting (23) into (24) yields the equilibrium value of the average loss:

$$\begin{aligned} L &= K^2[1 + (1-\pi)(1-\hat{\alpha})] \qquad (25) \\ &= \frac{K^2(2-\pi-\alpha)}{1-\alpha\pi}\,, \end{aligned}$$

where the second line uses the definition of $\hat{\alpha}$. The loss is decreasing in π: the gain from greater protection against inflation outweighs the loss from higher inflation.

C. Discussion

To understand these results, recall that equilibrium inflation is proportional to the ratio of the effects of a monetary surprise on the aggregate real wage and on wage dispersion (equation (20)). With indexation, which sets the real wage to zero for a proportion π of firms, the aggregate real wage becomes $(1-\pi)(w^N_{t+r} - p_{t+r})$ and wage dispersion becomes $(1-\pi)(x^N_{t+r} - x^N_{t+r-1})$, where the superscript again denotes non-indexed firms. Equilibrium inflation can be written as

$$\mu = -4K\left[\frac{(1-\pi)d(w^N_{t+r} - p_{t+r})/d\delta_t}{(1-\pi)d(x^N_{t+r} - x^N_{t+r-1})/d\delta_t}\right]\,. \tag{26}$$

Greater indexation — an increase in π - reduces $(1-\pi)$ and thus reduces both the numerator and denominator in (26). That is, by protecting some firms' real wages from inflation, indexation both steepens the Phillips curve and reduces the effect of a surprise on wage dispersion. But the net effect on inflation is zero, since the $(1-\pi)$'s in the numerator and denominator cancel; as explained above, indexed firms simply become irrelevant to policymakers. Indexation affects inflation only through indirect effects on the wages of *non-indexed* firms — that is, effects on $d(w^N_{t+r} - p_{t+r})/d\delta_t$ and $d(x^N_{t+r} - x^N_{t+r-1})/d\delta_t$.

While indexation eliminates the effects of surprises on indexed firms, in the short run (for small r) it *increases* the effects on non-indexed firms. The reason is that greater indexation makes the price level more responsive to surprises (a greater π reduces $\hat{\alpha}$ and thus raises

$dp_{t+\tau}/d\delta_t$). Since the price level is more responsive to surprises, so is the average real wage of non-indexed firms; thus the derivative in the numerator of (26) rises. In addition, the more rapid price level response implies greater differences between wages adjusted at different times; thus the derivative in the denominator rises. It turns out that the effect on the numerator is larger (although this is not intuitively obvious), and so inflation rises.[7]

Now consider welfare. Fischer and Summers find that policies to reduce the costs of inflation can be undesirable: inflation rises by so much that welfare falls despite greater protection against inflation. In contrast, we find that wage indexation, which reduces the costs of inflation, raises welfare. The reason is that wage indexation (unlike many of the policies considered by Fischer and Summers) has an anti-inflationary effect on the Phillips curve. Overall, indexation raises inflation, but the Phillips curve effect makes the net increase small. The losses from this small rise in inflation are outweighed by the gains from greater inflation protection.

4 The Continuous Time Model

A. Overview

This section presents a continuous time version of our model in which indexation is imperfect for indexed firms as well as incomplete in coverage. We assume that the length of a labor contract is one unit of time, a normalization. Non-indexed firms fix a wage for this period, while indexed firms adjust for inflation during the contract. As in the last section, one can show that indexation is equivalent to shorter contracts, so we simply assume that indexed firms adjust wages every τ periods, $\tau < 1$.[8] The price level changes continuously, and so indexed firms cannot keep their real wages fixed at zero (except in the limiting case of $\tau = 0$). That is, as in actual economies indexation is imperfect because adjustments occur only at discrete intervals. We determine the effects of reducing these intervals as well as increasing the proportion π of indexed firms. Thus we investigate the common claim (e.g., Cardoso and Dornbusch) that more

[7] This explanation assumes that r is small. For a large r, greater indexation *decreases* the derivatives in both the numerator and the denominator of (26). But the net effect of indexation on (26) is the same for all τ.

[8] For example, a one period contract with an indexed adjustment in the middle is equivalent to a half period contract. It is natural to think of τ as a fraction $\frac{1}{N}$, where N is the number of adjustments per contract. For simplicity, we treat τ as a continuous variable.

frequent adjustment is inflationary.

The results are complicated. For the special case of $\tau = 0$ — indexation is perfect for indexed firms — an increase in the coverage of indexation raises both inflation and welfare, as in the last section. On the other hand, for $\pi = 1$ — universal coverage — improving indexation by reducing τ raises inflation but leaves welfare unchanged. (In this case, the cost from greater inflation just equals the benefit from greater protection against inflation.) Finally, when indexation is imperfect along both dimensions ($\pi < 1$ and $\tau > 0$), changes in indexation parameters have ambiguous effects. Numerical calculations suggest, however, that the most likely effects of more perfect indexation are higher inflation and higher welfare.

B. Equilibrium Inflation and Welfare

Assume that the labor demand, labor supply, and money demand equations of Section 2 hold at every instant, so that equation (7), $p = \alpha w + (1-\alpha)m$, holds at every instant. The aggregate wage equals

$$w(t) = \pi w_I(t) + (1-\pi)w_N(t) \ , \tag{27}$$

where w_I and w_N are the average wages for indexed and non-indexed firms. Assume that wage setting is uniformly staggered, so that $w_I(t)$ is the average of indexed wages set between $t-\tau$ and t, and $w_N(t)$ is the average of non-indexed wages set between $t-1$ and t. With these assumptions, (27) becomes

$$w(t) = \frac{\pi}{\tau}\int_{s=0}^{\tau} x_I(t-s)ds + (1-\pi)\int_{s=0}^{1} x_N(t-s)ds \ , \tag{28}$$

where $x_I(t-s)$ and $x_N(t-s)$ are the wages set by indexed and non-indexed firms at $t-s$.

A wage setter minimizes the private loss $(x-p)^2$ averaged over the period that a wage is in effect. This implies a wage equal to the average of the expected price level. For an indexed firm,

$$x_I(t) = \frac{1}{\tau}\int_{s=0}^{\tau} E_t p(t+s)ds \ . \tag{29}$$

For a non-indexed firm, the wage is given by (29) with τ replaced by one.

As in the discrete time model, we must determine the behavior of wages and the price level for a given trend money growth and history of surprises. Here, a monetary surprise is an instantaneous innovation, $dZ(t)$, and the current money stock is[9]

$$m(t) = \mu t + \int_{s=0}^{\infty} dZ(t-s) \ . \tag{30}$$

Starting from equations (7) and (28)–(30), the method of undetermined coefficients and extensive computations yield an expression for the price level (see the Appendix for details):

$$p(t) = \mu t + \int_{s=0}^{\infty} \gamma(s) dZ(t-s) \ . \tag{31}$$

According to (31), trend inflation equals trend money growth. $\gamma(s)$ gives the effect of a monetary surprise on the price level after s periods. There is no closed-form solution for $\gamma(\cdot)$, but the Appendix defines it implicitly and we approximate it numerically for various parameter values. One can show analytically that $\gamma(s)$ lies between zero and one and approaches one as s approaches infinity: the price level adjusts partially to a surprise in the short run and fully in the long run. Numerical results show that $\gamma(s)$ is increasing in π and decreasing in τ: better indexation makes the price level more responsive to surprises. Finally, the behavior of wages can be derived by substituting (31) into (29) and the corresponding expression for non-indexed firms.

We can now determine equilibrium inflation. The monetary authority chooses the surprise $dZ(t)$ to minimize its loss function averaged over time. The loss at time $t+r$ is the average of $(x - p + K)^2$ across all cohorts of indexed and non-indexed firms:

$$\begin{aligned} L(t+r) &= \frac{\pi}{\tau} \int_{s=0}^{\tau} [x_I(t+r-s) - p(t+r) + K]^2 ds \\ &\quad + (1-\pi) \int_{s=0}^{1} [x_N(t+r-s) - p(t+r) + K]^2 ds \ . \end{aligned} \tag{32}$$

In the spirit of the discrete time model, we differentiate (32) with respect to $dZ(t)$ [using (29) and (31) to find $\frac{dx(t+r-s)}{d[dZ(t)]}$ and $\frac{dp(t+r)}{d[dZ(t)]}$], evaluate the derivative at $dZ(\cdot) = 0$, and sum over r. Setting the result equal to zero defines the equilibrium μ (see the Appendix for the complicated details):

$$\mu = \frac{12K}{\pi\tau^2 + (1-\pi)} \left[\pi \int_{j=0}^{\tau} \left(1 - \frac{j}{\tau}\right) \gamma(j) dj + (1-\pi) \int_{j=0}^{1} (1-j)\gamma(j) dj \right] \ . \tag{33}$$

[9] Note that the innovations in money are choice variables of a policymaker rather than random variables. Our specification is the limiting case of a model in which the policymaker chooses money growth for discrete periods, where the length of a period approaches zero.

For given π, τ, K, and α, one can derive the equilibrium μ from (33) and the numerical solutions for $\gamma(\cdot)$.

Finally, given μ it is easy to compute equilibrium welfare. In equilibrium the real wage of an indexed firm starts at $\frac{\mu\tau}{2}$ when the nominal wage is adjusted and falls steadily to $-\frac{\mu\tau}{2}$ at the next adjustment. Real wage behavior for a non-indexed firm is the same except that τ is replaced by one. Thus at each instant the social loss averaged across wage setters is

$$\begin{aligned} L &= \frac{\pi}{\tau}\int_{s=0}^{\tau}(-\mu s + \frac{\mu\tau}{2} + K)^2 ds \\ &\quad +(1-\pi)\int_{s=0}^{1}(-\mu s + \frac{\mu}{2} + K)^2 ds \\ &= K^2 + \frac{\mu^2}{12}(\pi\tau^2 + 1 - \pi)\ . \end{aligned} \tag{34}$$

The loss increases with inflation and, for given inflation, decreases as indexation improves along either dimension.

C. The Effects of Indexation: Special Cases

We now determine the effects of indexation, beginning with special cases in which indexation is imperfect along only one dimension.

<u>The Effects of Varying π When $\tau = 0$.</u> This case is similar to the discrete time model: we vary the proportion of indexed firms given that indexation is perfect for these firms. Not surprisingly, we again find that greater indexation raises both inflation and welfare. The derivations yield no new intuition, and so are left for the Appendix.

<u>The Effects of Varying τ when $\pi = 1$.</u> Here we assume that indexation is universal and vary the interval between adjustments. We show that a shorter interval raises inflation and leaves welfare unchanged.

For $\pi = 1$, the solution for μ, (33), reduces to

$$\mu = \frac{12K}{\tau^2}\int_{j=0}^{\tau}\left(1 - \frac{j}{\tau}\right)\gamma(j)dj\ . \tag{35}$$

This equation can be rewritten as

$$\mu\tau = \frac{12K}{\tau}\int_{j=0}^{\tau}\left(1 - \frac{j}{\tau}\right)\gamma(j)dj\ . \tag{36}$$

Recall that $\gamma(\cdot)$ depends on τ and π as well as j. For $\pi = 1$, one can show (see the Appendix) that j and τ affect γ only through their ratio. That is, the effect of a surprise after j periods depends on the relation of j to the interval between wage adjustments. In (36), write $\gamma(\cdot)$ as $\gamma(\frac{j}{\tau})$. Then change variables by replacing j/τ with z and dj with τdz, which yields

$$\mu\tau = 12K \int_{z=0}^{1} (1-z)\gamma(z)dz \; . \tag{37}$$

In (37), τ has been eliminated from the right hand side. Thus $\mu\tau$ is independent of τ. This implies that μ is decreasing in τ: better indexation is inflationary. And using (34), when $\pi = 1$ and $\mu\tau$ is independent of τ, welfare is independent of τ.

These results make sense. When π equals one, all firms have the same interval between adjustments. This interval is the basic time unit of the model, and all rates of change are constant relative to it.[10] In particular, $\mu\tau$ is constant because it is total inflation per adjustment. To look at it differently, a smaller τ simply means that the real wage moves more quickly through the constant range $(\frac{\mu\tau}{2}, -\frac{\mu\tau}{2})$. Since the real wage is distributed over a constant range, the average social loss is constant. As an example, if all firms adjust annually and inflation is 5% per year, then a switch to semiannual adjustment implies inflation of 10%. In both regimes, the real wage varies from 2.5% above its average to 2.5% below, and there is no difference in welfare.

In the terms of the Barro-Gordon model, a decrease in τ, like an increase in π, reduces both the slope of the Phillips curve and the cost of inflation. But here the second effect is so much larger that inflation not only rises, but rises enough to fully offset the gain from protection against inflation.

D. The Effects of Indexation: The General Case

We now consider the effects of varying π and τ when indexation is imperfect along both dimensions. The absence of a closed-form solution for $\gamma(\cdot)$ precludes analytical results. Therefore, for a variety of parameter values we approximate $\gamma(\cdot)$ numerically and use the results along with (33) and (34) to calculate inflation and the social loss (see the Appendix). Table 1 presents a

[10] Indeed, when $\pi = 1$, the choice of τ can be viewed as a normalization. Reducing τ could mean increasing the length of a unit of time rather than increasing the frequency of adjustment.

sample of the results. Inflation is always proportional to K and the loss to K^2; for simplicity, we set $K = 1$ throughout the table. The table presents results for α equal to 1/4, 1/2, and 3/4. For each α, we consider a wide range of combinations of the indexation parameters π and τ. For each combination, the top entry is equilibrium inflation and the bottom entry in parentheses is the equilibrium loss.

The results in Table 1 generally confirm the conclusion that indexation is inflationary. Increases in π (greater coverage of indexation) and decreases in τ (more frequent adjustment) almost always raise inflation. The only exception is that for small τ, further decreases in τ lower inflation slightly.

The welfare results are less clearcut. Improvements in indexation along either dimension usually lower the loss, but two exceptions are worth noting. First, while more perfect indexation usually lowers the loss for $\alpha = 1/4$ or $1/2$, it often raises the loss for $\alpha = 3/4$. The increases in the loss are small, however: for $\alpha = 3/4$, the loss function is close to constant, while for smaller α greater indexation reduces the loss significantly.[11]

The second exception is that, even for $\alpha = 1/4$ or $1/2$, the loss rises if π starts high and is raised further (specifically, if π is raised from 0.8 to 1). The increase in the loss is large if τ is small. Intuitively, when π is large and τ is small, indexation is almost perfect along both dimensions. As full perfection is approached — so that the costs of inflation approach zero — inflation rises very rapidly, and so welfare can fall.

In sum, indexation is more likely to lower the loss than to raise it, and even when the loss rises the effects are relatively small. The main qualification is that once an economy is almost perfectly indexed, pushing it even closer to perfection may be undesirable.

5 Additional Costs of Inflation

In contrast to most work on time-consistent policy, which simply assumes that inflation is costly, our model includes a specific cost: relative wage variation. This cost is realistic, and our specificity is crucial for deriving the net effects of indexation. On the other hand, relative wage

[11] It is not clear which values of α are realistic. α is the elasticity of output with respect to labor, which suggests $\alpha = 3/4$. On the other hand, $\frac{1}{1-\alpha}$ is the short run elasticity of labor demand, which suggests much smaller α's. To make meaningful choices of parameter values, one would need a more realistic model with additional parameters.

Table 1: Equilibrium Inflation and Social Loss

$\alpha = 0.25$

π \ τ	0.10	0.30	0.50	0.70	0.90
0.20	5.21	5.29	5.29	5.20	5.05
	(2.81)	(2.91)	(2.98)	(3.03)	(3.05)
0.40	5.54	5.78	5.74	5.50	5.15
	(2.54)	(2.77)	(2.92)	(3.01)	(3.04)
0.60	6.06	6.57	6.41	5.88	5.26
	(2.24)	(2.63)	(2.88)	(3.00)	(3.04)
0.80	7.28	8.29	7.53	6.39	5.38
	(1.92)	(2.56)	(2.89)	(3.01)	(3.04)
1.00	49.49	16.51	16.52	7.08	5.51
	(3.05)	(3.05)	(3.05)	(3.05)	(3.05)

$\alpha = 0.50$

π \ τ	0.10	0.30	0.50	0.70	0.90
0.20	4.05	4.07	4.03	3.94	3.81
	(2.10)	(2.13)	(2.15)	(2.16)	(2.16)
0.40	4.47	4.54	4.43	4.19	3.89
	(2.01)	(2.09)	(2.14)	(2.17)	(2.17)
0.60	5.08	5.25	4.98	4.50	3.98
	(1.87)	(2.04)	(2.14)	(2.17)	(2.17)
0.80	6.26	6.65	5.85	4.89	4.08
	(1.68)	(2.00)	(2.14)	(2.18)	(2.17)
1.00	37.96	12.67	7.60	5.42	4.18
	(2.20)	(2.20)	(2.20)	(2.20)	(2.20)

$\alpha = 0.75$

π \ τ	0.10	0.30	0.50	0.70	0.90
0.20	2.54	2.52	2.47	2.39	2.29
	(1.43)	(1.43)	(1.43)	(1.43)	(1.42)
0.40	2.95	2.91	2.77	2.57	2.35
	(1.44)	(1.45)	(1.45)	(1.44)	(1.42)
0.60	3.57	3.48	3.17	2.79	2.41
	(1.43)	(1.46)	(1.46)	(1.45)	(1.43)
0.80	4.71	4.51	3.77	3.06	2.47
	(1.38)	(1.46)	(1.48)	(1.46)	(1.43)
1.00	24.40	8.13	4.88	3.49	2.71
	(1.50)	(1.50)	(1.50)	(1.50)	(1.50)

Source: For each combination of τ and π, the top number is equilibrium inflation [computed from (33)] and the bottom number in parentheses is the equilibrium social loss [computed from (34)].

variation is clearly not the *only* cost of inflation. This section investigates the implications of introducing additional costs.

For simplicity, we work with the discrete time model of Sections 2 and 3. Following Barro-Gordon, we add a cost that depends on the square of inflation. The social loss at $t+r$ becomes

$$\tilde{L}(t+r) = L(t+r) + c(p_{t+r} - p_{t+r-1})^2 , \tag{38}$$

where $L(t+r)$ is the loss in our basic model (as usual, averaged over all firms) and $c(p_{t+r} - p_{t+r-1})^2$ is the new cost. This cost is meant to capture the inflation tax on money holdings, the non-neutralities in income taxes, the costs of consumer search, and so on. Crucially, the parameter c — and hence the added cost of a given level of inflation — is not affected by wage indexation. This assumption is realistic: while wage indexation reduces relative wage variation (as the model captures), it does not reduce tax distortions or make consumer search easier.

This modification of the model has clearcut effects. One can show that the additional costs make indexation *less* inflationary: $d\mu/d\pi$ is decreasing in c. For c large enough, $d\mu/d\pi$ becomes negative. Finally, since indexation is less inflationary, its effect on welfare is more positive: $d\tilde{L}/d\pi$ is decreasing in c.

The Appendix proves these results; here, we demonstrate them heuristically. As in Sections 2–3, equilibrium inflation depends on the ratio of the effects of a monetary surprise on the aggregate real wage and on the costs of inflation. This ratio is[12]

$$\mu = -4K\left(\frac{(1-\pi)[d(w^N_{t+r} - p_{t+r})/d\delta_t]}{(1-\pi)[d(x^N_{t+r} - x^N_{t+r-1})/d\delta_t] + 8c[d(p_{t+r} - p_{t+r-1})/d\delta_t]}\right) . \tag{39}$$

The difference between this expression and (26), its analogue in the basic model, is that the denominator includes a second term, the effect of the surprise on the new cost of inflation. Unlike the other terms in the ratio, the new term is *not* proportional to $(1-\pi)$, because indexation does not protect any sector of the economy from the new cost. As in the basic model, an increase in π reduces both the numerator and denominator of (39) through its effect on $(1-\pi)$. But the presence of a term unaffected by $(1-\pi)$ weakens the proportional effect on the denominator, and therefore weakens the net increase in the ratio. For sufficiently large

[12]Our explanation is only heuristic because (39) is not the same for all r. This means that the equilibrium μ cannot be derived by evaluating (39) for a given r; instead, one must sum the derivative of the loss function over r, as in (15).

c, indexation has a greater effect on the numerator than on the denominator, and so it reduces inflation. Intuitively, a large c means that relative wage variation, which indexation reduces, is a relatively small cost of inflation, and thus that indexation has a small effect on the total cost. And the effect of indexation on the slope of the Phillips curve is unchanged. Thus the inflationary effect of indexation is reduced relative to the anti-inflationary effect.[13]

Finally, the new cost of inflation strengthens the welfare case for indexation. That is, $d\tilde{L}/d\pi$, which is always negative in the discrete time model, is decreasing in c. Intuitively, the drawback of indexation — higher inflation — is reduced or even reversed, while indexation still provides protection against (part of) the costs of inflation.

6 Conclusion

This paper combines the Gray-Fischer model of wage indexation with the Barro-Gordon model of time-consistent monetary policy. We determine the effects of improving indexation both by extending coverage and by increasing the frequency of wage adjustment. Better indexation reduces the cost of inflation, which is inflationary, and steepens the short run Phillips curve, which is anti-inflationary. We show that the first effect is likely to be larger. Thus our model captures the argument that indexation leads to more inflationary monetary policy.

At the same time, we find that indexation is likely to raise welfare. The cost of the net rise in inflation is usually smaller than the gain from greater inflation protection. This result suggests that indexation will increase social welfare even though it produces somewhat higher inflation.

Our basic model assumes that the only cost of inflation is relative wage variation. But we show that introducing additional costs strengthens the conclusion that indexation raises welfare. It appears that other extensions of the model would further strengthen the case for indexation. For example, if shocks to money demand cause output to vary, then indexation has the additional benefit of stabilizing output. And one can show that there is no additional cost. (In particular, there is no change in the effect of indexation on trend inflation.)[14]

[13]This explanation ignores the indirect effects of π on $d(w^N_{t+r} - p_{t+r})/d\delta_t$ and $d(x^N_{t+r} - x^N_{t+r-1})/d\delta_t$, which are smaller than the direct effect on $(1-\pi)$.

[14]The effects of *productivity* shocks are exacerbated by indexation. But a version of our results holds even when

We depart from previous work on time-consistent policy by studying a structural model of the economy. This approach is crucial for deriving the sum of offsetting effects. Variants of our model could prove useful for addressing other issues in the time-consistency literature, such as the effects of reputation or of changes in policy regimes. As in this paper, a structural model might yield more precise results than the usual ad hoc specification.

---

these shocks exist. If firms choose Gray's optimal degree of indexation (which is less than one with productivity shocks), then increases in the coverage of indexation or the frequency of adjustment usually raise welfare.

Appendix

A. The Aggregate Price Level

Here we derive the aggregate price level in the continuous time model, equation (31). Substituting the two wage-setting rules, (29) and the analogue for non-indexed firms, into the equation for the aggregate wage, (28), yields

$$w(t) = \frac{\pi}{\tau^2}\int_{s=0}^{\tau}\int_{r=0}^{\tau} E_{t-s}p(t+r-s)drds + (1-\pi)\int_{s=0}^{1}\int_{r=0}^{1} E_{t-s}p(t+r-s)drds \ . \qquad \text{(A1)}$$

Substituting (A1) and the definition of m(t), (30), into the price level equation (7) yields

$$\begin{aligned} p(t) &= (1-\alpha)\left[\mu t + \int_{l=0}^{\infty} dZ(t-l)\right] + \frac{\alpha\pi}{\tau^2}\int_{s=0}^{\tau}\int_{r=0}^{\tau} E_{t-s}p(t+r-s)drds \qquad \text{(A2)} \\ &\quad +(1-\pi)\alpha\int_{s=0}^{1}\int_{r=0}^{1} E_{t-s}p(t+r-s)drds \ . \end{aligned}$$

Guess the general form of the solution in (31) and note that

$$\begin{aligned} E_{t-s}p(t+r-s) &= \mu(t+r-s) + E_{t-s}\int_{l=0}^{\infty}\gamma(l)dZ(t+r-s-l) \qquad \text{(A3)} \\ &= \mu(t+r-s) + \int_{l=r}^{\infty}\gamma(l)dZ(t+r-s-l) \ . \end{aligned}$$

Substituting (A3) into (A2) yields

$$\begin{aligned} p(t) &= (1-\alpha)\left[\mu t + \int_{l=0}^{\infty} dZ(t-l)\right] \qquad \text{(A4)} \\ &\quad +\frac{\alpha\pi}{\tau^2}\int_{s=0}^{\tau}\int_{r=0}^{\tau}\left[\mu(t+r-s) + \int_{l=r}^{\infty}\gamma(l)dZ(t+r-s-l)\right]drds \\ &\quad +\alpha(1-\pi)\int_{s=0}^{1}\int_{r=0}^{1}\left[\mu(t+r-s) + \int_{l=r}^{\infty}\gamma(l)dZ(t+r-s-l)\right]drds \end{aligned}$$

$$\begin{aligned} &= \mu t + (1-\alpha)\int_{l=0}^{\infty} dZ(t-l) \qquad \text{(A5)} \\ &\quad +\frac{\alpha\pi}{\tau^2}\int_{s=0}^{\tau}\int_{r=0}^{\tau}\int_{l=r}^{\infty}\gamma(l)dZ(t+r-s-l)drds \\ &\quad +\alpha(1-\pi)\int_{s=0}^{1}\int_{r=0}^{1}\int_{l=r}^{\infty}\gamma(l)dZ(t+r-s-l)drds \ . \end{aligned}$$

The triple integrals in (A5) can be simplified through a change of variables. For the first triple integral, the result is

$$\int_{s=0}^{\tau}\int_{r=0}^{\tau}\int_{l=r}^{\infty}\gamma(l)dZ(t+r-s-l)drds = \tag{A6}$$
$$\int_{k=0}^{\tau}\left[\int_{j=-k}^{0}(k+j)\gamma(k+j)dj + \int_{j=0}^{\tau-k}k\gamma(k+j)dj + \int_{j=\tau-k}^{\tau}(\tau-j)\gamma(k+j)dj\right]dZ(t-k)$$
$$+\int_{k=\tau}^{\infty}\left[\int_{j=-\tau}^{0}(\tau+j)\gamma(k+j)dj + \int_{j=0}^{\tau}(\tau-j)\gamma(k+j)dj\right]dZ(t-k)$$

The second triple integral in (A5) is given by (A6) with τ replaced by one.

We can now find an implicit solution for the weighting function $\gamma(\cdot)$ in (31). Substitute (A6) and the analogous expression for the other triple integral into (A5). Then set the coefficient on $dZ(t-k)$ equal to $\gamma(k)$ [that is, match coefficients with (31)]. This yields

For $0 \le k < \tau$,

$$\begin{aligned}\gamma(k) &= (1-\alpha) \\ &+ \frac{\alpha\pi}{\tau^2}\left[\int_{j=-k}^{0}(k+j)\gamma(k+j)dj + \int_{j=0}^{\tau-k}k\gamma(k+j)dj + \int_{j=\tau-k}^{\tau}(\tau-j)\gamma(k+j)dj\right] \\ &+ \alpha(1-\pi)\left[\int_{j=-k}^{0}(k+j)\gamma(k+j)dj + \int_{j=0}^{1-k}k\gamma(k+j)dj + \int_{j=1-k}^{1}(1-j)\gamma(k+j)dj\right];\end{aligned} \tag{A7}$$

For $\tau \le k < 1$,

$$\begin{aligned}\gamma(k) &= (1-\alpha) \\ &+ \frac{\alpha\pi}{\tau^2}\left[\int_{j=-\tau}^{0}(\tau+j)\gamma(k+j)dj + \int_{j=0}^{\tau}(\tau-j)\gamma(k+j)dj\right] \\ &+ \alpha(1-\pi)\left[\int_{j=-k}^{0}(k+j)\gamma(k+j)dj + \int_{j=0}^{1-k}k\gamma(k+j)dj + \int_{j=1-k}^{1}(1-j)\gamma(k+j)dj\right];\end{aligned}$$

For $k \geq 1$,

$$\begin{aligned}\gamma(k) &= (1-\alpha) \\ &+ \frac{\alpha\pi}{\tau^2}\left[\int_{j=-\tau}^{0}(\tau+j)\gamma(k+j)dj + \int_{j=0}^{\tau}(\tau-j)\gamma(k+j)dj\right] \\ &+ \alpha(1-\pi)\left[\int_{j=-1}^{0}(1+j)\gamma(k+j)dj + \int_{j=0}^{1}(1-j)\gamma(k+j)dj\right] .\end{aligned}$$

For given values of α, τ and π, one can derive a numerical approximation to $\gamma(\cdot)$ from (A7). We follow Ball, Mankiw and Romer (1988, pg. 62). Since $\gamma(s)$ converges to one as s approaches infinity, we set $\gamma(s) = 1$ for $s \geq T$, $T = 3$ (the results are not sensitive to the choice of T). We then consider $\gamma(s)$ for $s = 0, h, 2h, ..., T-2h, T-h$ where $h = 0.0002$. We begin by setting all these γ's to one. New values are obtained by substituting the initial values into (A7) and numerically integrating using rectangular approximations. We iterate until the sum of the absolute changes in the γ's is less that 0.2.

B. Equilibrium Inflation

This section derives equilibrium inflation in the continuous time model, equation (33). As described in the text, we first differentiate $L(t+r)$ (equation (32)) with respect to $dZ(t)$:

$$\frac{dL(t+r)}{d[dZ(t)]} = \frac{\pi}{\tau}\int_{s=0}^{\tau} 2[x_I(t+r-s)-p(t+r)+K]\frac{d[x_I(t+r-s)-p(t+r)]}{d[dZ(t)]}ds \qquad \text{(B1)}$$

$$+(1-\pi)\int_{s=0}^{1} 2[x_N(t+r-s)-p(t+r)+K]\frac{d[x_N(t+r-s)-p(t+r)]}{d[dZ(t)]}ds .$$

Combining the wage-setting rule, (29), with the solution for the price level, (31), leads to

$$x_I(t+r-s)-p(t+r) = \qquad \text{(B2)}$$

$$\mu(\frac{\tau}{2}-s)+\frac{1}{\tau}\int_{l=0}^{\tau}\int_{k=l}^{\infty}\gamma(k)dZ(t+r-s+l-k)dl - \int_{l=0}^{\infty}\gamma(l)dZ(t+r-l) \ ;$$

$$\frac{d[x_I(t+r-s)-p(t+r)]}{d[dZ(t)]} = \begin{cases} -\gamma(r) & r < s \\ \frac{1}{\tau}\int_{l=0}^{\tau}\gamma(r-s+l)dl - \gamma(r) & r \geq s \end{cases} , \qquad \text{(B3)}$$

with corresponding expressions for nonindexed firms.

Substituting (B2) and (B3) into (B1), and evaluating the expression at $dZ(s) = 0\ \forall s$ yields

$$\frac{dL(t+\tau)}{d[dZ(t)]}\Big|_{dZ(s)=0\ \forall s} = \tag{B4}$$

$$\frac{\pi}{\tau}\int_{s=0}^{\tau} 2[\mu(\frac{\tau}{2}-s)+K]A(s,\tau)ds + (1-\pi)\int_{s=0}^{1} 2[\mu(\frac{1}{2}-s)+K]B(s,\tau)ds\ ,$$

where

$$A(s,\tau) = \frac{d[x_I(t+\tau-s)-p(t+\tau)]}{d[dZ(t)]}$$

from (B3), and $B(s,\tau)$ equals $A(s,\tau)$ with τ set to 1.

The policymaker chooses $dZ(t)$ to minimize $L(t+\tau)$ summed from $\tau = 0$ to $\tau = \infty$ (we ignore discounting).[15] Thus equilibrium inflation is defined by

$$\int_{\tau=0}^{\infty} \frac{dL(t+\tau)}{d[dZ(t)]}\Big|_{dZ(s)=0\ \forall s}\, d\tau = 0\ . \tag{B5}$$

Substituting (B4) into (B5) and rearranging terms yields

$$\mu = K\frac{\int_{\tau=0}^{\infty}\left\{\frac{\pi}{\tau}\int_{s=0}^{\tau}A(s,\tau)ds + (1-\pi)\int_{s=0}^{1}B(s,\tau)ds\right\}d\tau}{\int_{\tau=0}^{\infty}\left\{\frac{\pi}{\tau}\int_{s=0}^{\tau}(s-\frac{\tau}{2})A(s,\tau)ds + (1-\pi)\int_{s=0}^{1}(s-\frac{1}{2})B(s,\tau)ds\right\}d\tau}\ . \tag{B6}$$

Extensive calculations show that

$$\int_{\tau=0}^{\infty}\int_{s=0}^{\tau}(s-\frac{\tau}{2})A(s,\tau)dsd\tau = -\frac{\tau^3}{12}\ ; \tag{B7}$$

$$\int_{\tau=0}^{\infty}\int_{s=0}^{\tau}A(s,\tau)dsd\tau = \int_{k=0}^{\tau}(k-\tau)\gamma(k)dk\ . \tag{B8}$$

Substituting (B7) and (B8) into (B6) yields equation (33) in the text.

C. Special Cases

Here we consider the two special cases discussed in the text.

The Effects of Varying π When $\tau = 0$. When $\tau = 0$, the solution for μ, (33), reduces to

$$\mu = 12K\int_{k=0}^{1}(1-k)\gamma(k)dk\ . \tag{C1}$$

[15] More precisely, we assume that the policymaker minimizes the discounted sum of losses and consider the limiting case as the discount rate approaches zero. (With no discounting, the sum of losses is infinite.)

The effect of indexation is given by

$$\frac{d\mu}{d\pi} = 12K \int_{k=0}^{1} (1-k) \frac{d\gamma(k)}{d\pi} dk \; . \tag{C2}$$

For $\tau = 0$, numerical calculations using (A7) show that $\frac{d\gamma(s)}{d\pi} > 0$ for all s: greater indexation makes the price level adjust more quickly. This implies that $\frac{d\mu}{d\pi}$ is positive. And for $\tau = 0$ our numerical calculations show that the social loss is decreasing in π.

The Effects of Varying τ when $\pi = 1$. Here we show that, for $\pi = 1$, $\gamma(j)$ depends only on $\frac{j}{\tau}$. That is, $\gamma(j,\tau)$ is homogeneous of degree zero. Given this result, the text derives the effects of indexation.

For $\pi = 1$, (A7) can be written as

$$\begin{aligned} \gamma(j,\tau) \;\; &= \;\; (1-\alpha) + \alpha \left[\int_{k=-min(j,\tau)}^{0} [min(\frac{j}{\tau},1) + \frac{k}{\tau}] \gamma(j+k,\tau) \frac{dk}{\tau} \right. \\ & \left. + \int_{j=0}^{max(0,\tau-j)} \frac{k}{\tau} \gamma(j+k,\tau) \frac{dk}{\tau} + \int_{k=max(0,\tau-j)}^{\tau} (1-\frac{k}{\tau}) \gamma(j+k,\tau) \frac{dk}{\tau} \right] \; . \end{aligned} \tag{C3}$$

Let $z = \frac{k}{\tau}$. (C3) becomes

$$\begin{aligned} \gamma(j,\tau) \;\; &= \;\; (1-\alpha) + \alpha \left[\int_{z=-min(\frac{j}{\tau},1)}^{0} [min(\frac{j}{\tau},1) + z] \gamma(j+\tau z,\tau) dz \right. \\ & \left. + \int_{z=0}^{max(0,1-\frac{j}{\tau})} z \gamma(j+\tau z,\tau) dz + \int_{z=max(0,1-\frac{j}{\tau})}^{1} (1-z) \gamma(j+\tau z,\tau) dz \right] \; . \end{aligned} \tag{C4}$$

Now consider

$$\begin{aligned} \gamma(\nu j,\nu\tau) \;\; &= \;\; (1-\alpha) + \alpha \left[\int_{z=-min(\frac{j}{\tau},1)}^{0} [min(\frac{j}{\tau},1) + z] \gamma(\nu j+\nu\tau z,\nu\tau) dz \right. \\ & \left. + \int_{z=0}^{max(0,1-\frac{j}{\tau})} z \gamma(\nu j+\nu\tau z,\nu\tau) dz + \int_{z=max(0,1-\frac{j}{\tau})}^{1} (1-z) \gamma(\nu j+\nu\tau z,\nu\tau) dz \right] \; . \end{aligned} \tag{C5}$$

where ν is a constant. It is straightforward to show that (C4) and (C5) have unique solutions. If $\gamma(j,\tau)$ is the solution to (C4), then clearly $\gamma(\nu j,\nu\tau) = \gamma(j,\tau)$ solves (C5). This establishes the homogeneity of $\gamma(j,\tau)$.

D. Additional Costs of Inflation

This section derives the implications of introducing additional costs of inflation (Section 5). Starting from the modified loss function, (38), one can derive equilibrium inflation and the equilibrium loss through calculations analogous to (13)–(17):

$$\mu = \frac{2K(1-\pi)\sqrt{1-\hat{\alpha}}}{(1-\pi)+4c[1+\sqrt{1-\hat{\alpha}}]}\,; \tag{D1}$$

$$\bar{L} = K^2 + \left[\frac{(1-\pi)}{4}+c\right]\mu^2\,. \tag{D2}$$

The effects of indexation are given by

$$\frac{d\mu}{d\pi} = K\frac{[4c\mu-(1-\pi)](1-\hat{\alpha})^{-\frac{1}{2}}\frac{d\hat{\alpha}}{d\pi}+2(\mu-\sqrt{1-\hat{\alpha}})}{(1-\pi)+4c(1+\sqrt{1-\hat{\alpha}})}; \tag{D3}$$

$$\frac{d\bar{L}}{d\pi} = 2\mu\left[\frac{(1-\pi)}{4}+c\right]\frac{d\mu}{d\pi}-\frac{\mu^2}{4}\,. \tag{D4}$$

Numerical calculations confirm that, as claimed in the text, that $\frac{d\mu}{d\pi}$ is decreasing in c and becomes negative for c sufficiently large. And $\frac{d\bar{L}}{d\pi}$, which is always negative, is also decreasing in c; that is, the welfare gain from indexation is increasing in c.

REFERENCES

Ball, Laurence. 'Externalities from Contract Length.' *American Economic Review* 77 (September 1987): 615–629.

______. 'Is Equilibrium Indexation Efficient?' *Quarterly Journal of Economics* 103 (May 1988): 299–311.

Barro, Robert, and Gordon, David. 'A Positive Theory of Monetary Policy in a Natural Rate Model.' *Journal of Political Economy* 91 (April 1983): 589–610. (**a**).

______ and ______. 'Rules, Discretion, and Reputation in a Model of Monetary Policy.' *Journal of Monetary Economics* 12 (1983): 101–122. (**b**).

Blanchard, Olivier. 'The Wage-Price Spiral.' *Quarterly Journal of Economics* 101 (August 1986): 543–565.

______ and Fischer, Stanley. **Lectures on Macroeconomics**. Cambridge, Mass.: M.I.T. Press, 1989 forthcoming.

Cardoso, Eliana, and Dornbusch, Rudiger. 'Brazil's Tropical Plan.' *American Economic Review* 77 (May 1987): 288-292.

Cecchetti, Stephen. 'Indexation and Incomes Policy: A Study of Wage Adjustment in Unionized Manufacturing.' *Journal of Labor Economics* 5 (July 1987): 391–412.

Fischer, Stanley. 'Wage Indexation and Macroeconomic Stability.' *Journal of Monetary Economics* 5 (Supplement, 1977): 107–147. (**a**).

______. 'Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule.' *Journal of Political Economy* 85 (April 1977): 191–205 (**b**).

______ and Summers, Lawrence. 'Should Nations Learn to Live with Inflation?' mimeo, revised May 1988.

Gray, Jo Anna. 'Wage Indexation: A Macroeconomic Approach.' *Journal of Monetary Economics* 3 (1976): 221–235.

Okun, Arthur. 'The Mirage of Steady Inflation.' *Brookings Papers on Economic Activity* 2 (1971): 485–498.

Simonsen, Mario H. 'Indexation: Current Theory and the Brazilian Experience.' In **Inflation, Debt and Indexation,** edited by Rudiger Dornbusch and Mario H. Simonsen, Cambridge, Mass.: M.I.T. Press, 1983.

Taylor, John. 'Aggregate Dynamics and Staggered Contracts.' *Journal of Political Economy* 88 (February 1980): 1–23.

Vining, Daniel and Elwertowski, Thomas. 'The Relationship Between Relative Prices and the General Price Level.' *American Economic Review* 66 (September 1976): 699–708.