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ABSTRACT

Standard international economic models with life cycle/permanent income consumption behavior predict that international portfolio diversification leads to high bilateral consumption correlations. Thus international consumption correlations have been empirically estimated as a test of international portfolio diversification and risk sharing. In this paper we investigate the international consumption correlations generated by a more general model which incorporates habit formation in consumption. We show that, in the presence of common interest rate movements, habit formation itself can generate positive international consumption correlations even in the absence of any international risk sharing. Empirical evidence presented in this paper suggests habit formation characterizes consumption behavior among most of the G-7 countries. Thus, the extent of international portfolio diversification may be even lower than that suggested by previous research which studied international consumption correlations.

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Stark economic models often have stark implications. For example, a stark model of international risk sharing includes a multi-country model with complete markets in which the representative consumer in one country is differentiated from her counterpart in another country only by the realization of her stochastic endowment in any period. The stark implication of this model is that proportional changes in consumption across countries are equated.[1] But evidence is strongly at odds with this prediction. The actual bilateral correlations of consumption across industrial countries, those which are most likely to engage in international risk sharing, are far from unity. Moreover, the data do not generally even support the prediction that bilateral consumption correlations are higher than corresponding bilateral income correlations, a prediction consistent with less stringent versions of an international risk sharing model.[2]

A natural response to the challenge raised by these empirical observations is to enrich the model in a way which mitigates the dissonance between theory and evidence. For example, Devereux, Gregory and Smith (1992) show that a model which assumes perfect risk pooling but, unlike the standard approach,

[1] Obstfeld (1994) discusses the particular assumptions required for national per capita consumption levels to exhibit equal proportional movements *ex post*.

[2] See, for example, the results presented in Tesar (1995) and Obstfeld (1994).

allows for preferences which are not separable between consumption and labor supply, generates consumption correlations across countries which are lower than is the case with separable utility. Stockman and Tesar (1995) incorporate nontraded goods in a model in which households have access to complete contingent claims markets. Their model generates consumption correlations and income correlations closer to those found in the data than is the case with models without nontraded goods. But both of these models fail to replicate the observation that consumption correlations are consistently lower than income correlations.

An alternative response is to investigate the role of capital market imperfections in generating consumption correlations. Lewis (1996) studies whether apparently low levels of consumption risk sharing for a panel of 72 countries over 43 years can be attributed to capital market restrictions, as well as the non-separability of tradables and non-tradables in the utility function. She finds that when she allows for both capital market restrictions and non-separabilities, risk sharing among unrestricted countries cannot be rejected. This evidence is consistent with Obstfeld's (1994) study of the G-7 countries in the period from 1973 to 1988 when capital controls were being dismantled. He cannot reject common consumption paths, controlling for a number of other factors, for France, Germany, Japan and the

United Kingdom during this period. This result is not present when using data from the period of 1951-1972 when capital controls were more pervasive in these countries.

In this paper we study the implications for international consumption correlations of another alternative model, one not before used in this context. In the next section of this paper we show that habit formation has important and striking implications for interpreting standard empirical results concerning international consumption correlations as well as regressions of national consumption on world consumption and other variables.[3] In particular, habit formation itself, in the absence of international risk sharing and common income movements, can generate positive consumption correlations across countries and significant coefficients on world consumption in regressions of national consumption. Thus, if habit formation is a good characterization of behavior, the absence of international risk sharing is even lower than standard tests would suggest. This result, at odds with the many papers which attempt to rescue the relevance of the concept of international risk sharing from its empirical performance, is in spirit close to Baxter and Jermann (1997) who show that the international diversification puzzle is, in their words, "worse than you think."

[3]Mansoorian (1998) considers current account dynamics in a model with durable goods and habit formation in consumption.

To show the relevance of our theoretical results, we present evidence on the presence of habit formation for the G-7 countries. Previous work (Fuhrer 1998) has shown the importance of habit formation in characterizing aggregate consumption in the United States. Here we show this result extends to other countries as well. Given this, our theoretical results suggest that existing research may badly overstate the extent to which common consumption movements across countries reflect international risk sharing.

1 A Model of Habit Formation and Risk Sharing

In this section we present a model in which a small country derives some of its endowment income through risk sharing with the rest of the world. The distinction between this model and the standard one investigated in the literature is that consumption is characterized by habit formation in both the small country and the rest of the world. Accordingly, we begin this section with a discussion of the utility function of the representative consumers which incorporates habit formation. We then present the full consumer's problem and its solution. We use this solution to compute analytical model-based counterparts to the empirical estimates of correlations and regression coefficients found in the literature. As we will show, habit formation can lead to

consumption correlations across countries even in the absence of risk sharing or correlated income. Commensurate consumption correlations do not arise in a model which is identical but for the substitution of time-separable utility for habit formation.

1.1 Habit Formation

To isolate the effects of habit formation on international consumption correlations, we depart from a standard two-country analysis with endowment income only in our formulation of consumers' utility. We consider a model in which utility depends upon consumption in the current period relative to consumption in other periods. In particular, we employ a utility function developed by Carroll, Overland, and Weil (1995) which has been widely used. A consumer's t-period utility is[4]

$$U_t = \frac{1}{(1-\sigma)} \left(\frac{C_t}{Z_t^{\gamma}} \right)^{(1-\sigma)} \tag{1.1}$$

where σ is the standard parameter in the class of isoelastic utility functions, Z_t is the reference consumption level, and γ is a parameter which indexes the importance of habit formation in the utility function $(1 \geq \gamma \geq 0)$.[5] The reference consumption

[4] As discussed below, this particular utility function has the advantage of flexibility in that we can separately vary the extent of habit formation and the reference level of consumption.

[5] In the standard time-separable version of this utility function, $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution. In this model of habit formation, however, the intertemporal

level is defined as

$$Z_t = \rho_z Z_{t-1} + (1 - \rho_z) C_{t-1} \tag{1.2}$$

where $0 \leq \rho_z \leq 1$. This definition of Z_t shows that utility is generally not time-separable because the consumption choice today influences all future reference levels of consumption.

This is a very flexible formulation. If $\gamma = 0$, then the utility function becomes a standard isoelastic utility function with no habit formation. The importance of past consumption in determining the utility derived from current consumption rises with γ. Thus the importance of "habit formation" is increasing in γ. The parameter ρ_z indexes the persistence or "memory" in the habit formation reference level. If $\rho_z = 0$, then only last period's consumption is important. The "mean lag" of the habit reference level rises with ρ_z. Intuitively, more distant past consumption becomes a larger determinant of the reference level of consumption with an increase in ρ_z.

The effects of habit formation may be most easily seen through a slightly simplified and rewritten version of this utility function. Setting $\rho_z = 0$, such that the reference level is the previous period's consumption, and rewriting the utility function, we have

$$U_t = \frac{1}{(1-\sigma)} \left(\left[\frac{C_t}{C_{t-1}} \right] C_{t-1}^{1-\gamma} \right)^{(1-\sigma)}. \tag{1.3}$$

elasticity of substitution cannot be identified with $\frac{1}{\sigma}$ alone. Also, we cannot have $\gamma > 1$ since this would imply that steady-state utility is falling in consumption.

This demonstrates that habit formation mixes utility from the level of consumption with utility from the change in consumption. Consequently, habit formation implies smoothing with respect to both the level of consumption and its change.

The smoothing induced by habit formation is illustrated in Figure 1 which demonstrates the impulse response of consumption in the face of an income shock under different assumptions about habit formation.[6] In the standard permanent income/life-cycle case with no habit formation ($\gamma = 0$), the response of consumption is sharp and, after the initial period, monotonically decreasing. In contrast, with habit formation, consumption follows a hump-shaped time path and the response of consumption in the initial period is muted as compared to the permanent income/life-cycle case. Comparing the time paths of consumption for $\gamma = 0.4$ and $\gamma = 0.8$ shows that the initial response of consumption becomes more muted and its hump-shaped response becomes more pronounced as the extent of habit formation increases.

[6]The consumption paths depicted in this figure are derived from the full model described below. In this model, income has an autoregressive component so a one-time shock to income leads to it diverge from its steady state level for many periods. The differences in the responses of consumption to a real interest rate shock across different values of γ follow the same pattern as those for an income shock shown in Figure 1.

1.2 The Representative Consumer's Problem

The single-time period utility function described above is one component of the representative consumer's problem. The full depiction of the problem requires that the representative consumer in the domestic economy solves the multiperiod problem of

$$\max_{C_t} \sum_{i=0}^{\infty} \beta^i U_{t+i} \tag{1.4}$$

where β is the time rate of preference, subject to the budget constraint

$$Y_t^{*\tau} Y_t^{(1-\tau)} + (1 + R_t) B_{t-1}^* = C_t + B_t^* \tag{1.5}$$

where Y_t is stochastic domestic endowment in period t, Y_t^* is stochastic foreign (i.e. rest of world) endowment in period t, B_t^* is a one-period foreign bond purchased in period t, R_t is the stochastic world real interest rate and τ is a parameter representing the extent to which consumers in the domestic economy engage in risk-sharing.

This budget constraint incorporates two distinct notions related to the integration of international capital markets. This model assumes perfect financial capital mobility in that domestic consumers have access to the bond offered by the foreign country and that the real return on this bond is the same across countries. The model also incorporates risk sharing through the

parameter τ which is assumed to be invariant over time.[7] We assume that domestic consumers have purchased claims on foreign income through selling claims to their own income. As a result, the overall endowment available to the domestic consumer in any period, $Y_t^{*\tau} Y_t^{(1-\tau)}$, is a geometrically-weighted average of domestic and foreign endowment in that period.[8] Thus the parameter τ represents the extent to which domestic consumers pool their income and share risk with the rest of the world.[9] A situation of autarky with respect to risk pooling arises when $\tau = 0$. Note that even in this case there will be full capital mobility in financial markets since the domestic consumer can purchase a foreign bond and the return on the bond is the same for the domestic and foreign consumers.

The solution to the consumption problem facing the representative person in the small economy is given by the (nonlinear)

[7]We will, however, vary τ across various cases to assess its effect on consumption correlations and regression coefficients.

[8]The use of a geometric average rather than an arithmetic average simplifies the algebra without changing the nature of any of the results.

[9]One interpretation of full risk sharing through the complete pooling of risk in this model where there is no correlation between domestic and foreign income, no initial wealth and no trend to income in either country is

$$\tau = \frac{E_t Y_t^*}{E_t \left(Y_t + Y_t^* \right)}$$

where $E_t Y_t^*$ is the expected value of foreign income and $E_t \left(Y_t + Y_t^* \right)$ is the expected value of world (i.e., foreign plus domestic) income.

first-order condition[10]

$$\frac{1}{C_t}\left(\frac{C_t}{Z_t^{\gamma}}\right)^{(1-\sigma)} - \gamma(1-\sigma)(1-\rho_z)P_t \tag{1.6}$$
$$= E_t\left[\frac{\beta\left(1+R_{t+1}\right)}{C_{t+1}}\left(\frac{C_{t+1}}{Z_{t+1}^{\gamma}}\right)^{(1-\sigma)} - \beta\gamma\left(1-\sigma\right)\left(1-\rho_z\right)\left(1+R_{t+1}\right)P_{t+1}\right]$$

where P_t is defined as

$$P_t = \beta\rho_z E_t P_{t+1} + \frac{U_t}{Z_t} \tag{1.7}$$

and, in its linearized form, is defined as

$$P_t \equiv \beta\rho E_t P_{t+1} + b_1 c_t - b_2 Z_t \tag{1.8}$$

where ρ represents the discount rate on future income in the budget constraint. In the analysis presented below we use a linearized version of this first-order condition.[11] This linearization gives us an approximate log-linear consumption function

$$c_t - \left[\tau y_t^* + \left(1-\tau\right)y_t\right] \tag{1.9}$$
$$= \sum_{j=1}^{\infty}\rho^j\left(\begin{array}{c}\left[\tau\Delta y_{t+j}^* + \left(1-\tau\right)\Delta y_{t+j}\right] + a_1\left(E_t\left(P_{t+j+1} - P_{t+j}\right)\right) + \\ a_2\left(E_t\left(Z_{t+j+1} - Z_{t+j}\right)\right) - \delta E_t R_{t+j+1}\end{array}\right)$$

where $c_t = \ln\left(C_t\right)$, $y_t = \ln\left(Y_t\right)$, and the parameters a_1, a_2, b_1, b_2 and δ are nonlinear functions of the underlying parameters γ, σ, ρ_z,

[10]This solution is presented in Fuhrer (1998).

[11]See Fuhrer (1998) for the derivation of this linear approximation.

and β, with

$$\begin{aligned} a_1 &= ((\gamma(1-\sigma)(1-\rho))/\sigma) \\ a_2 &= ((1-\sigma)\gamma)/\sigma \\ \delta &= \beta[(-\gamma(1-\sigma)(1-\rho_z)-1)/\sigma] \\ b_1 &= (\rho-\sigma)/(1-\sigma) \\ b_2 &= (\gamma(1-\sigma)-1)(1-\sigma). \end{aligned} \tag{1.10}$$

Note that with no habit formation ($\gamma = 0$), $a_1 = a_2 = 0$ and the consumption equation is

$$c_t - [\tau y_t^* + (1-\tau)\, y_t] = \sum_{j=1}^{\infty} \rho^j \left[\left[\tau \Delta y_{t+j}^* + (1-\tau)\, \Delta y_{t+j} \right] - \frac{\beta}{\sigma} E_t R_{t+j+1} \right] \tag{1.11}$$

which corresponds to the standard life-cycle/permanent income result when endowment income is derived from both domestic and foreign sources. The two consumption equations (1.9) and (1.11) differ in that the latter does not include the P and Z terms which introduce lagged and expected future levels of consumption into the determination of current consumption. This distinction between the standard consumption model and the model incorporating habit formation will be used in our empirical tests in Section 3.

A similar consumption function can be derived for the rest of the world. We assume that the domestic economy is so small that there is no opportunity for the rest of the world to signif-

icantly pool its risk through the purchase of rights to domestic income and, therefore, consumption in the rest of the world is chosen optimally when

$$c_t^* - y_t^* = \sum_{j=1}^{\infty} \rho^j \Delta y_{t+j}^* + a_1 \left(E_t \left(P_{t+j+1}^* - P_{t+j}^* \right) \right) \quad (1.12)$$
$$+ a_2 \left(\left(E_t \left(Z_{t+j+1}^* - Z_{t+j}^* \right) \right) - \delta E_t R_{t+j+1} \right]$$

where a "*" represents the "rest-of-world" variable.

Note that the domestic economy as well as the rest of the world face the same real interest rate, as pointed out above. Thus any shock to the real interest rate would serve as a common shock to both countries. A stochastic income shock in the large country will only directly affect the smaller country to the extent to which the smaller country holds shares in the income stream of the foreign economy (that is, the extent to which τ deviates from zero). Note also that we assume that the numerical value of behavioral parameters are identical in the small country and the rest of the world.

1.3 Stochastic Structure

We close this model by specifying the stochastic process by which incomes and the real interest rate evolve. We assume that income in each country is autoregressive and also depends upon the lagged value of the real interest rate. We also assume

the real interest rate depends upon current values of income in each country, with the relative importance of a particular country's income a function of its relative size. In addition, each variable is perturbed by an i.i.d. shock. These characteristics are captured by

$$\begin{aligned} y_t &= \rho_y y_{t-1} - \eta R_{t-1} + \varepsilon_t^y \\ y_t^* &= \rho_{y^*} y_{t-1}^* - \eta^* R_{t-1} + \varepsilon_t^{y^*} \\ R_t &= \omega\left(\alpha y_t + (1-\alpha) y_t^*\right) + \varepsilon_t^R \end{aligned} \tag{1.13}$$

where ε_t^y, $\varepsilon_t^{y^*}$, and ε_t^R are each i.i.d. normal with mean zero and

$$E_t\left(\varepsilon_t^y, \varepsilon_t^{y^*}\right) = E_t\left(\varepsilon_t^y, \varepsilon_t^R\right) = E_t\left(\varepsilon_t^R, \varepsilon_t^{y^*}\right) = 0.$$

We need to assign values to the parameters of the stochastic equations as well as decide upon the relative size of the variances of the shocks. We choose the size of the conditional variance of income relative to the conditional variance of the real interest rate based upon the ratio of the residual variances from a vector autoregression of United States data and set[12]

$$var\left(\varepsilon^y\right) = var\left(\varepsilon^{y^*}\right) = 5 \cdot var\left(\varepsilon^R\right).$$

[12]The vector autoregression is estimated on quarterly data from 1961 to 1995, and comprises a long-term real rate, the federal funds rate, the log change in the CPI excluding food and energy, and an output gap, defined as detrended log per capita chain-weighted GDP, with a break in the trend in 1974. The long real rate is defined as the difference between the 10-year Treasury constant maturity yield and a 12-quarter trailing moving average of core CPI inflation. Four lags of each variable are included.

We set $\rho_y = \rho_{y^*} = 0.9$ to reflect the approximate autoregressive property of detrended national income in industrial economies. We also set $\eta = \eta^* = 0.5$, implying a 0.5 percent decline in one country's income from a 100 basis point increase in that country's interest rate. In the stochastic equation for the real interest rate, we set $\omega = 0.05$, which implies that a 1 percent increase in world income is associated with a 5 basis-point increase in the real interest rate. The value of α simply represents the relative size of incomes in the two countries.

We solve the model using a rational expectations algorithm developed by Anderson and Moore (1985). This method enables us to analytically compute the unconditional variance-covariance matrix for all the endogenous variables of the model. From the variance-covariance matrix we generate model-based counterparts to empirical estimates of both consumption correlations and regression coefficients which have appeared in the literature on risk sharing behavior.

2 Consumption Correlations and Habit Formation

In this section, we use the model described above to study the extent to which the presence of habit formation could increase the correlation of consumption across countries. These results demonstrate that empirical findings of positive consumption cor-

relations can reflect habit formation rather than risk sharing or common income comovements. More generally, our findings suggest that drawing inferences about the extent of risk sharing from simple consumption correlations or from regressions of domestic consumption on world income or world consumption may be misleading.

2.1 Standard Consumption-Based Tests of Risk Sharing

International portfolio diversification and risk sharing can be empirically tested in a number of ways. One set of tests draws on the implications of risk sharing for common consumption movements across countries. The simplest of these tests presents the correlation of the first difference of the logarithm of domestic consumption with either the first difference of consumption in another country or the first difference of consumption in some other set of countries which is taken to represent the "rest of the world" (Tesar 1995, Pakko 1998). Strong consumption correlations are often viewed as consistent with a high degree of international risk sharing. In particular, international risk sharing implies that consumption correlations should exceed income correlations. Alternatively, some authors (e.g. Obstfeld 1994) run the simple bivariate regression

$$\Delta \ln C_{it} = \delta + \alpha_{iW} \Delta \ln C_{Wt} + \varepsilon_{it} \tag{2.1}$$

where C_{it} is domestic consumption and C_{Wt} is consumption of a group of other countries ("World" consumption). The test for risk sharing is $\alpha_{iW} = 1$. Studying the G-7, Obstfeld cannot reject $\alpha_{iW} = 1$ but can reject $\alpha_{iW} = 0$ for France, Germany, Japan and UK while for the US he cannot reject $\alpha_{iW} > 1$.

Other regression-based tests attempt to control for country-specific factors including domestic income and the presence of capital controls. Obstfeld (1995) runs the regression

$$\Delta \ln C_{it} = \delta + \alpha_{iW} \Delta \ln C_{Wt} + \gamma_i \Delta \ln Y_{it} + \varepsilon_{it} \qquad (2.2)$$

where Y_{it} is a measure of domestic per capita income in country i. He finds α_{iW} is not significantly different from 1 and γ_i is not significantly different from zero for Germany and Japan over the period from 1973-1988. Lewis (1996) also uses regression-based analysis to study whether apparently low levels of consumption risk sharing can be attributed to the non-separability of tradables and non-tradables in the utility function and the effects of capital market restrictions by some countries. She finds that when she allows for both non-separabilities and capital market restrictions, risk sharing among unrestricted countries cannot be rejected.

We compute model-based counterparts to the correlations and regression results presented in previous work. It is important to note that consumption and income in our model should

be thought of as deviations about their long-run trends. Therefore we compute correlations and regression coefficients directly from the log-levels of consumption and income in the model rather than from log-differences. This approach is equivalent to calculating correlations on deterministically-detrended data. Our regressions include the real interest rate as a regressor, in addition to domestic income, because the real rate represents an important source of common covariance across countries. Thus, our regression equation is

$$c_t = \beta_0 + \beta_W c_t^* + \beta_Y y_t + \beta_R R_t. \tag{2.3}$$

Excluding the real interest rate would, according to a standard left-out-variable error bias analysis, bias upward the coefficient on world consumption growth in the equation above.[13]

The standard interpretation of our model-based estimates is that any positive value of β_W in the regression equation (2.3), or any positive international consumption correlation, reflects a corresponding degree of risk sharing (that is a corresponding value of τ) since, by construction, incomes are uncorrelated.[14] We will show, however, that this standard interpreta-

[13]Because the expected sign of the coefficient of the real interest rate in the domestic consumption growth equation is negative, and the expected correlation between the real rate and world income is negative, the sign of the bias should be positive.

[14]This contrasts with standard empirical tests which are concerned only with the presence of complete risk sharing, that is testing the hypothesis that $\beta_W = 1$.

tion is flawed. The presence of habit formation induces positive values of β_W in the test regression and positive international consumption correlations even when there is neither true risk sharing (that is, $\tau = 0$) nor any correlation of incomes across countries. Thus empirical findings of a positive regression coefficients like β_W or positive consumption correlations may in fact be evidence of habit formation rather than evidence of risk sharing.

The ability of habit formation to induce international consumption correlations is demonstrated by our first set of results which are derived from a simplified version of the model discussed above. In order to isolate the mechanism by which habit formation influences these key correlations, we have no feedback from income to real rates ($\omega = 0$) nor from real rates to income ($\eta = \eta^* = 0$). We also have equal-sized countries ($\alpha = 0.5$). We assume that $\sigma = 6.1$, $\beta = 0.98$ (on an annual basis), and $\rho = 0.99$. We set $\rho_Z = 0$ which causes the "reference level" of consumption in period t to be c_{t-1}. We allow for a variety of risk-sharing values, from $\tau = 0$, representing no risk-sharing, to $\tau = 0.5$ where the resident of the domestic economy derives fifty percent of her income from the endowment in the foreign economy.

The top panel of Figure 2 presents the consumption corre-

lations $corr\left(c_t^i, c_t^W\right)$ for this version of the model. This figure shows that consumption correlations increase with the extent of habit formation for any level of risk sharing. For example, with no risk sharing ($\tau = 0$), the correlation of consumption is 0.11 when $\gamma = 0$ and rises to 0.88 when $\gamma = 0.9$. Note that this consumption correlation corresponds to a level of risk sharing in excess of $\tau = 0.5$ with no habit formation (i.e., the case where $\gamma = 0$). Thus, in this model, consumption correlations rise with either an increase in γ or an increase in τ. Not recognizing this, and assuming no habit formation ($\gamma = 0$), one might view these correlations as evidence of risk sharing when, in fact, there is none.

The plots of the coefficients β_W from the regression (2.3) presented in the bottom panel of Figure 2 tell a similar story. Like the correlations, these regression coefficients rise with the habit formation parameter γ. In the case of no risk-sharing ($\tau = 0$), the regression coefficient $\beta_W = 0.0$ with no habit formation. This coefficient rises to $\beta_W = 0.67$ when $\gamma = 0.9$. If one assumes no habit formation, β_W simply reflects the extent of risk sharing and therefore one associates $\beta_W = 0.67$ with a situation where two-thirds of the representative domestic resident's income is derived from foreign sources.

We can provide some intuition for these results by recognizing

that there are two sources of increasing values of $corr\left(c_t^i, c_t^W\right)$ and β_W with increasing values of γ in this version of the model. One source is that δ increases, in absolute value, with γ as shown above.[15] The increasing value of δ makes the common real interest rate shock a larger relative source of the variation in consumption across countries. The increased importance of the common shock serves to increase the correlation of consumption across countries.[16]

There is a second source of the increasing values of $corr\left(c_t^i, c_t^W\right)$ and β_W with increasing values of γ which is linked to the effect of habit formation on the time path of consumption. Recall from Figure 1 and the attendant discussion that an increase in the extent of habit formation causes the initial response of consumption to a shock to be smaller and its overall response to have a hump-shaped pattern. In this version of the model, with no linkages between income and the real interest rate, real interest rate fluctuations have no persistence. With no habit formation, therefore, consumption adjusts immediately and for only

[15] Recall from above that in the full model

$$\delta = \beta\left[\frac{(-\gamma(1-\sigma)(1-\rho_z)-1)}{\sigma}\right]$$

and therefore δ becomes more negative as γ increases.

[16] The dependence of the real rate coefficient on γ seems to be related to the dependence of the intertemporal elasticity of substitution (IES) on γ. The explicit form of the dependence is difficult to derive in the nonlinear model. This dependence has not been recognized in the literature to our knowledge, and will be the subject of future research.

one period to any real rate shock. This generates little persistence in consumption and thus little correlation of consumption across countries in response to the common real rate shock. At higher levels of habit formation, the smoothing of consumption in response to a real interest rate shock, a shock which is common to each country, generates higher consumption correlations than is the case with no habit formation. This is demonstrated by the results in Figure 3 which plots the relationship between $corr\left(c_t^i, c_t^W\right)$ and γ for $\tau = 0$ for a version of the model identical to that used to generate the earlier correlations but holding δ constant at a level corresponding to $\gamma = 0.5$.[17] The increasing correlation of consumption for values of γ greater than 0.2 with a fixed δ (the dashed line in the top panel) arises from the way in which habit formation smooths consumption in response to real interest rate shocks. The decrease in the consumption correlation for values of γ less than 0.2 reflects the dominant effect of the smaller initial response of consumption to a common shock at these low levels of habit formation. The lower panel of Figure 3 compares the relationship between β_W and γ in the complete baseline model discussed above (the solid line) to the relationship in a model where δ is constant (the dashed line). This shows that β_W increases with γ even when the direct re-

[17] As with previous graphs, increasing the value of τ would simply shift this relationship.

sponse of consumption to the real interest rate is held constant across different values of γ.

The effect of habit formation on both the value of δ and the smoothing of consumption serves to raise consumption correlations only because countries face the same interest rate shock in every period. Thus it is of interest to examine the sensitivity of the results to changes in the relative size of the real interest rate shock.[18] The two panels in Figure 4 present $corr\left(c_t^i, c_t^W\right)$ and β_W, respectively, for the baseline case and a case which doubles the relative size of the real rate shock. The upper panel of this figure shows markedly increased consumption correlations for all values of γ including $\gamma = 0$. The lower panel shows that the regression coefficient β_W does not vary with the relative size of the real interest rate shock when $\gamma = 0$ although it does rise as γ increases. With no habit formation, β_W does not depend upon the relative size of the real interest rate shock because the regression controls for the real interest rate and because, with $\gamma = 0$, the immediate effect of this common shock is the sole source of the correlation between domestic and foreign consumption. As habit formation becomes more important, the increasing persistence of the response of consumption in both countries to the common real interest rate shock raises the correlation of con-

[18]Recall that the VAR results discussed above are consistent with a real rate shock of this relative magnitude.

sumption even after controlling for the real interest rate.

The common effect of the real rate across countries is strengthened when the model is altered by allowing the real rate to affect income ($\eta = \eta^* \neq 0$). In this case, the real interest rate has a common effect on income in both countries and the increase in the international correlation of income raises the correlation of consumption. Figure 5 displays the correlations and regressions coefficients for the baseline case (solid lines) and for the case where we augment the baseline model to allow for the effect of (lagged) real rates on income (dashed lines). The influence on the correlations and regression coefficients is generally positive but small for the values of η and η^* we chose.

Studying a version of the model in which income affects the real interest rate (i.e., when $\omega \neq 0$) reveals an asymmetry in the real interest rate channel. There is a tendency for the real interest rate to move in the same direction as income when income shocks across countries are of the same sign, as shown in the third equation in (1.13). This mutes the effect of income on consumption in both countries and thus diminishes the correlation of consumption across countries. When income shocks are of opposite signs, however, the net effect of domestic and foreign income on the real rate is smaller. As a result, the real rate does not mute negatively correlated income shocks as much as it

mutes positively correlated shocks. This asymmetry lends a net negative bias to the correlation between domestic and foreign consumption when $\omega \neq 0$. This negative bias is most evident for low values of γ in the top panel of Figure 6 which plots consumption correlations derived from a model in which $\omega = .05$. As γ increases, the role of the real interest rate as a common shock becomes increasingly important, offsetting the negative bias and closing the gap between this case and the baseline version of the model where $\omega = 0$. The regression coefficient β_W does not reflect the difference between the case where $\omega = 0$ and the case where $\omega \neq 0$ since the regression already controls for both income and the real interest rate.[19]

In the baseline model the countries are of equal size. We can investigate the effect of relative country size by altering the parameter α. This allows for a differential effect across countries on the real interest rate through the third equation in (1.13). The effect of this alteration on consumption correlations and regression coefficients is small, a result consistent with our finding that the pattern of correlations is not particularly sensitive to ω.

[19] An additional consequence of making the real interest rate a function of income is that the persistence in income translates into persistence in the real interest rate. This persistence in the real interest rate strengthens its contribution to the determination of consumption. This, in turn, increases the negative bias in the correlation for low values of γ.

Cross-country consumption correlations are almost always positive in all the versions of the model discussed above. This implies that consumption correlations are almost always larger than income correlations which, by assumption, are always zero.[20] A common feature of all models of risk sharing is that the correlation of consumption across countries is greater than the correlation of income across countries (see, for example, Devereux, Gregory and Smith 1992, Stockman and Tesar 1995, Baxter and Jermann 1997). Empirical estimates, however, typically find income correlations higher than consumption correlations (Obstfeld 1994, Tesar 1995, Pakko 1998). Reconciling this empirical regularity with a model has represented an unmet challenge.

Strikingly, our habit-formation model of consumption can generate consumption correlations which are lower than income correlations even when there is some risk sharing. Figure 7 plots consumption correlations for our model when real interest rate shocks are small relative to income shocks (the scale factor is 0.05), income affects the contemporaneous interest rates ($\omega = 0.05$), interest rates affect income ($\eta = \eta^* = 0.5$) and the smaller country is one-third the size of the larger country ($\alpha = .75$). This figure demonstrates that the consumption cor-

[20] A positive correlation in income across countries generates a positive correlation in consumption in a straightforward way. Introducing positive income correlation shifts up the consumption correlation loci in any of the figures presented above.

relation is less than the income correlation when $\gamma < 0.8$.[21] Even if the actual extent of risk sharing were forty percent ($\tau = 0.4$), the consumption correlation would be less than the income correlation if $\gamma < 0.6$. The lower panel of this figure demonstrates that this configuration of the model also results in a negative regression coefficient β_W for values of $\gamma < 0.67$. This result arises through the asymmetric effect of income shocks on the real rate, as discussed above, in combination with the relatively small relative size of real interest rate shocks. The asymmetric effect of income shocks on the real rate reduces the correlation of consumption. The relatively small size of the real rate shock diminishes the relative size of the common shock in this model.

3 Empirical Evidence of Habit Formation

The empirical relevance of our theoretical results depends upon whether consumption is actually characterized by the presence of habit formation. In this section, we offer a test for the presence of habit formation and employ this test using data from the G-7 countries. The key to this test is that the linearized version of the habit formation model, equation (1.9), nests the

[21] The income correlation is not quite zero since, even though the shocks to income are uncorrelated (i.e. $E_t\left(\varepsilon_t^y, \varepsilon_t^{y^*}\right) = 0$), the real interest rate channel causes some correlation of income across countries.

standard permanent income / life cycle model, equation (1.11). A likelihood ratio test of a slightly simplified version of (1.9) can be used to compare a model motivated by the habit formation specification to a constrained model corresponding to the permanent income / life cycle specification.

We assume $\rho_Z = 0$ to simplify the linearized habit formation model (1.9) for our empirical tests.[22] In this version of the model, the reference level of consumption is $Z_t = C_{t-1}$ and the variables

$$E_t \left(Z_{t+j+1} - Z_{t+j} \right)$$

in (1.9) can be replaced with

$$E_t \left(C_{t+j} - C_{t+j-1} \right).$$

Also, when we assume $\rho_Z = 0$, we have

$$P_t = \frac{1}{1-\sigma} \left(\frac{C_t}{C_{t-1}^{\gamma}} \right)^{(1-\sigma)}$$

and in its linearized form, P_t is a function of C_{t-1} and C_t. Thus, under the assumption that $\rho_Z = 0$, we have the simple linear approximation

$$\Gamma(E_t \left(C_{t+j} - C_{t+j-1} \right)) \approx a_1 \left(E_t \left(P_{t+j+1} - P_{t+j} \right) \right) + a_2 \left(E_t \left(Z_{t+j+1} - Z_{t+j} \right) \right)$$

[22] Fuhrer (1998) cannot reject the hypothesis that ρ_Z does not significantly differ from zero for the United States since his estimate of ρ_Z is 0.0015 with a standard error of 0.0039.

where Γ is a function of parameters of the model. In this case, the habit formation consumption model becomes

$$c_t - [\tau y_t^* + (1-\tau)\, y_t] = \sum_{j=1}^{\infty} \rho^j \left([\tau \Delta y_{t+j}^* + (1-\tau)\, \Delta y_{t+j}] + \Gamma(E_t\,(C_{t+j} - C_{t+j-1})) - \delta E_t R_{t+j+1} \right) + \epsilon_t. \tag{3.1}$$

This nests the permanent income / life cycle model (1.11) since that equation is obtained from (3.1) by setting $\Gamma = 0$. Our tests for the presence of habit formation are, therefore, tests of unconstrained versions of (3.1) against corresponding constrained versions in which $\Gamma = 0$. The unconstrained versions are consistent with a habit-formation model while the constrained versions are consistent with the permanent income / life cycle specification.

The estimating equation differs from (1.9) in two other ways as well. First, we allow for "rule-of-thumb" consumers, as in Campbell and Mankiw (1990). We let λ represent the proportion of income accruing to people who consume solely out of current income. Second, since the foreign income measures represent income across all seven countries in the sample, we include the weights τ_W and τ_D for world and domestic income rather than τ and $(1-\tau)$ as was done in Section 1. With these two adjustments, we have the estimating equation

$$
\begin{aligned}
& c_t - [\tau_W y_t^* + \tau_D y_t] \qquad (3.2) \\
= \; & (1-\lambda)\left[\sum_{j=1}^{\infty} \rho^j \left([\tau_W \Delta y_{t+j}^* + \tau_D \Delta y_{t+j}] + \Gamma(E_t C_{t+j} - C_{t+j-1})\right.\right. \\
& \left.\left. - \delta E_t R_{t+j+1}\right)\right] + \epsilon_t.
\end{aligned}
$$

We also consider a version of the model in which we set $\tau_W = 0$ and $\tau_D = 1$,

$$
c_t - y_t = (1-\lambda)\left[\sum_{j=1}^{\infty} \rho^j \left(\Delta y_{t+j} + \Gamma(E_t C_{t+j} - C_{t+j-1}) - \delta E_t R_{t+j+1}\right)\right] + \epsilon_t. \qquad (3.3)
$$

The likelihood ratio tests compare unconstrained versions of (3.2) or (3.3) to respective constrained versions in which $\Gamma = 0$.

We estimate the parameters of the consumption function with a maximum-likelihood estimation procedure that is documented in Fuhrer and Moore (1995). The procedure uses the Anderson and Moore (1985) solution procedure to solve for the expectations of future consumption, income and interest rates as a restricted linear function of current and lagged values of all of the variables in the system. In order to form expectations of income, interest rates and inflation, we separately estimate reduced-form vector-autoregressive equations for these variables. We combine these with the structural equation for consumption to form a dynamic rational expectations system that allows estimation of the consumption function parameters. The reduced-form equation

parameters are held fixed during estimation of the consumption parameters.

We apply these tests to quarterly time series data for the G-7 countries; Canada, Italy, Japan, France, Germany, the United Kingdom and the United States. The sample covers 1973:1 to 1993:1 for all countries but for Italy (1975:2 - 1993:1) and Germany (1973:1 - 1990:4). The consumption time series represents real per capita consumption of non-durables for all countries but for Germany, for which we use total private consumption. Details on the data used for each country are included in the data appendix.

Table 1 presents the values of the log-likelihood functions for the constrained and unconstrained versions of (3.2) along with their respective $\chi^2(1)$ statistics and p-values. This table shows that the unconstrained version of the model, corresponding to a habit formation framework, significantly outperforms the constrained version of the model, corresponding to the permanent income / life cycle hypothesis, at the 99 percent confidence level in five of the seven cases. The unconstrained version performs significantly better at the 93 percent confidence level in the case of France. Only in the case of Japan is the performance of the permanent income / life cycle hypothesis version of the model not different from the habit formation framework at conven-

tional levels of significance.

Table 1: Likelihood Ratio Tests for Eq'n. 3.2

Country	$L_{Unconstr.}$	$L_{Const.}$	$\chi^2(1)$	p-value
Canada	1894.2	1890.9	6.56	0.01
France	1834.4	1832.8	3.20	0.07
Germany	1600.2	1586.3	27.78	~ 0
Italy	1561.0	1537.6	46.66	~ 0
Japan	1680.4	1680.4	0.06	0.81
U.K.	1659.0	1645.4	27.32	~ 0
U.S.A.	2030.7	2027.4	7.70	0.01

Table 2 presents similar results for the estimation of equation (3.3). Again, we can reject the constrained model in favor of the unconstrained model in a majority of cases. As with (3.2), the habit formation framework does significantly better than the permanent income / life cycle model at the 95 percent confidence level for the estimates of Canada, Germany, Italy and the United States. The case for the habit formation framework is a somewhat less strong for France and the United Kingdom than was the case with (3.2), although the U.K. estimates are still significant at the 10 percent level. These results, like those presented above, suggest the data for Japan are least supportive

of the habit formation framework.

Table 2: Likelihood Ratio Tests for Eq'n. 3.3

Country	$L_{Unconstr.}$	$L_{Const.}$	$\chi^2(1)$	p-value
Canada	1506.5	1504.3	4.44	0.01
France	1561.7	1560.5	2.30	0.13
Germany	1381.4	1366.4	30.00	~ 0
Italy	1347.1	1331.8	30.70	~ 0
Japan	1410.5	1409.9	1.34	0.53
U.K.	1366.7	1365.4	2.66	0.10
U.S.A.	1530.4	1521.3	18.24	~ 0

4 Conclusion

This paper has shown that the simple approach of studying consumption correlations relative to income correlations as a method for gauging the extent of international risk sharing is, in fact, too simple. Consumption correlations in excess of income correlations can arise when consumption behavior is marked by habit formation even though there is no risk sharing. The particular model used here to demonstrate this result nests the permanent income / life cycle model and therefore we have been able

to demonstrate that these consumption correlations do not generally arise due to the stochastic structure of the model economy but only when consumption is characterized by habit formation behavior.

In this paper we have also developed and implemented a new test for whether consumption is better characterized by habit formation or permanent income / life cycle behavior. The results of this test suggest that habit formation is an important characteristic of consumption behavior for most of the G-7 countries. These countries, which enjoy the world's most highly developed capital markets, are often thought of as the most likely candidates for risk sharing. The theoretical and empirical results in this paper, however, suggest that one should interpret any evidence in favor of risk sharing based upon consumption correlations for these countries with a good deal of caution.

DATA APPENDIX

The following are the definitions of variables used in the empirical analysis, by country.

- United States

Consumption: Nondurables and services consumption, 1987 U.S. dollars, per capita, detrended.

Income: Disposable personal income,1987 U.S. dollars, per capita, detrended.

Interest Rate: Federal funds rate, average of daily observations.

Inflation: Annualized log change in the CPI, All Items

- Canada

Consumption: Total consumption, 1986 Canadian dollars, less durable and semi-durable goods, per capita, detrended.

Income: GDP, 1986 Canadian dollars, per capita, detrended.

Interest Rate: 3-month finance paper rate.

Inflation: Annualized log change in the CPI, All Items

- United Kingdom

Consumption: Total consumer expenditures less durable goods, 1990 British pounds, per capita, detrended.

Income: Real disposable personal income, 1990 British pounds, per capita, detrended.

Interest Rate: 91-day government bond yield.

Inflation: Annualized log change in the CPI, All Items

- Germany

Consumption: Total private consumption expenditures, 1991 DM, per capita, detrended.

Income: Real GDP (source: IMF's International Financial Statistics), per capita, detrended.

Interest Rate: 3-month interbank rate.

Inflation: Annualized log change in the CPI, All Items

- France

Consumption: Total consumption less durable and semi-durable goods, 1980 francs, per capita, detrended.

Income: Real GDP, 1980 francs, per capita, detrended.

Interest Rate: 3-month interbank Pibor rate.

Inflation: Annualized log change in the CPI, All Items

- Japan

Consumption: Total consumption less durable and semi-durable goods, 1985 yen, per capita, detrended.

Income: Real GDP, 1985 yen, per capita, detrended.

Interest Rate: Short-term interest rate.

Inflation: Annualized log change in the CPI, All Items

- Italy

Consumption: Total consumption expenditures less durable and semi-durable goods, 1985 lire, per capita, detrended.

Income: Real GDP, 1985 lire, per capita, detrended.

Interest Rate: Short-term interest rate.

Inflation: Annualized log change in the CPI, All Items

Consumption and Income from OECD database for all countries with exception that German GDP is from International Monetary Fund's International Financial Statistics. German income series is extended using nominal GDP series from IFS, deflated with CPI deflator.

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Figure 1

Consumption response to a one-period income shock

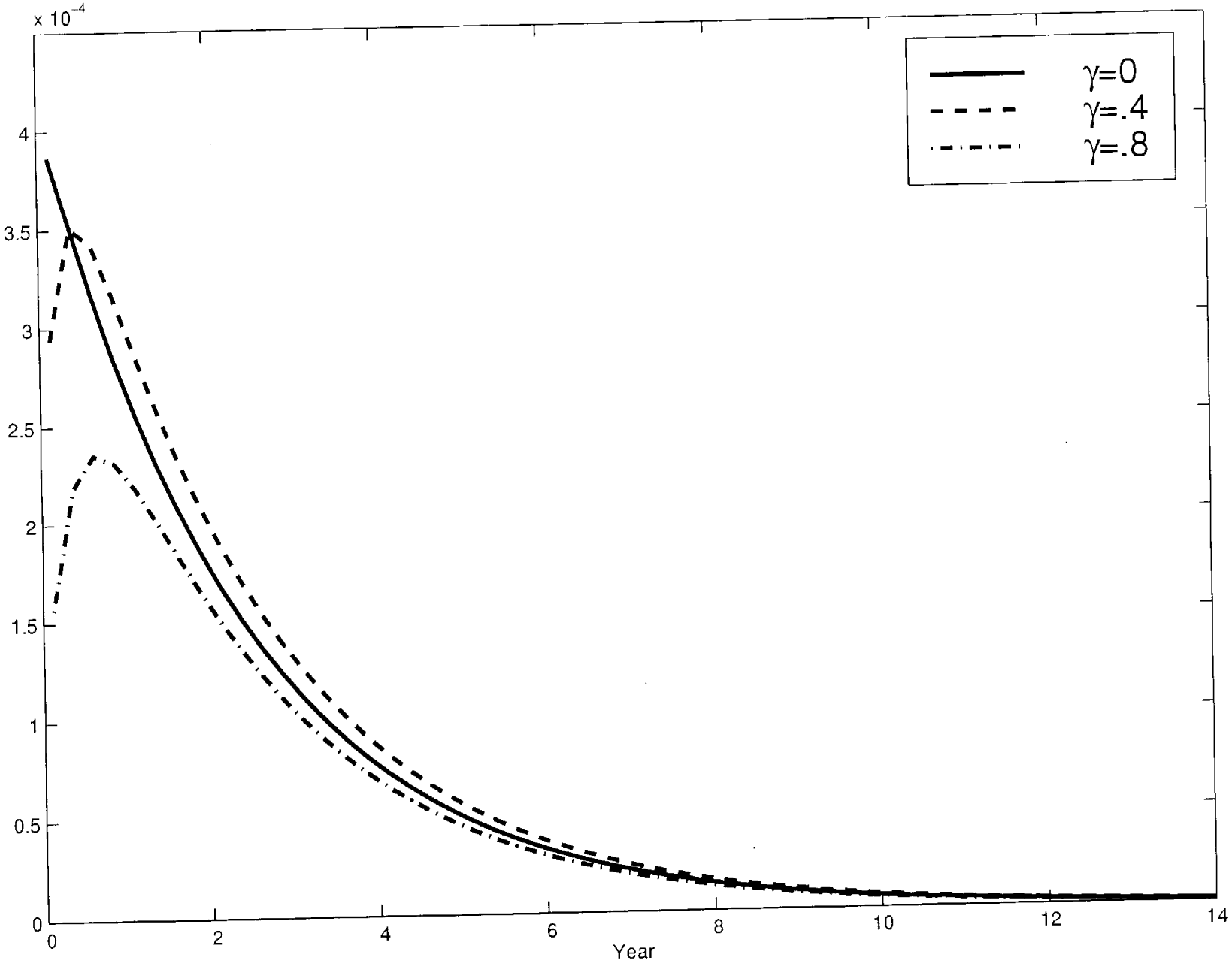
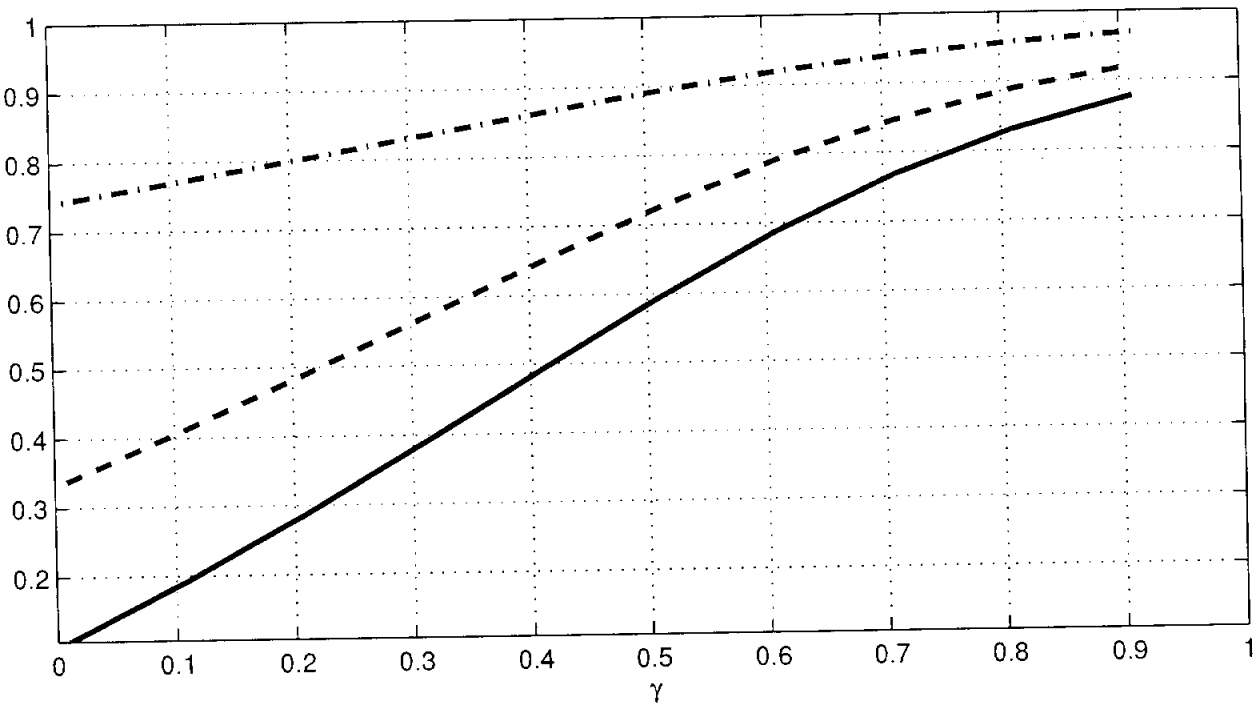


Figure 2

Baseline Case

$\text{Corr}(C_1, C_W)$



β_W from regression: $C_1 = \beta_Y Y_1 + \beta_R R + \beta_W C_W$

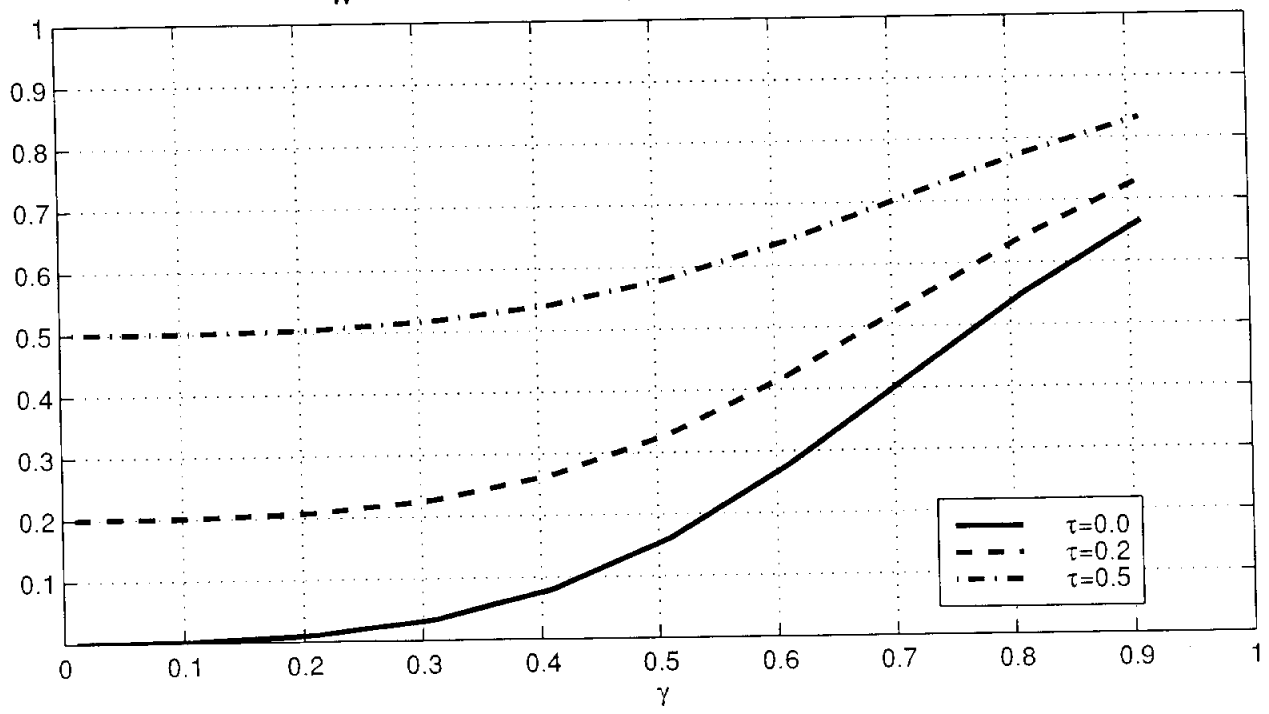
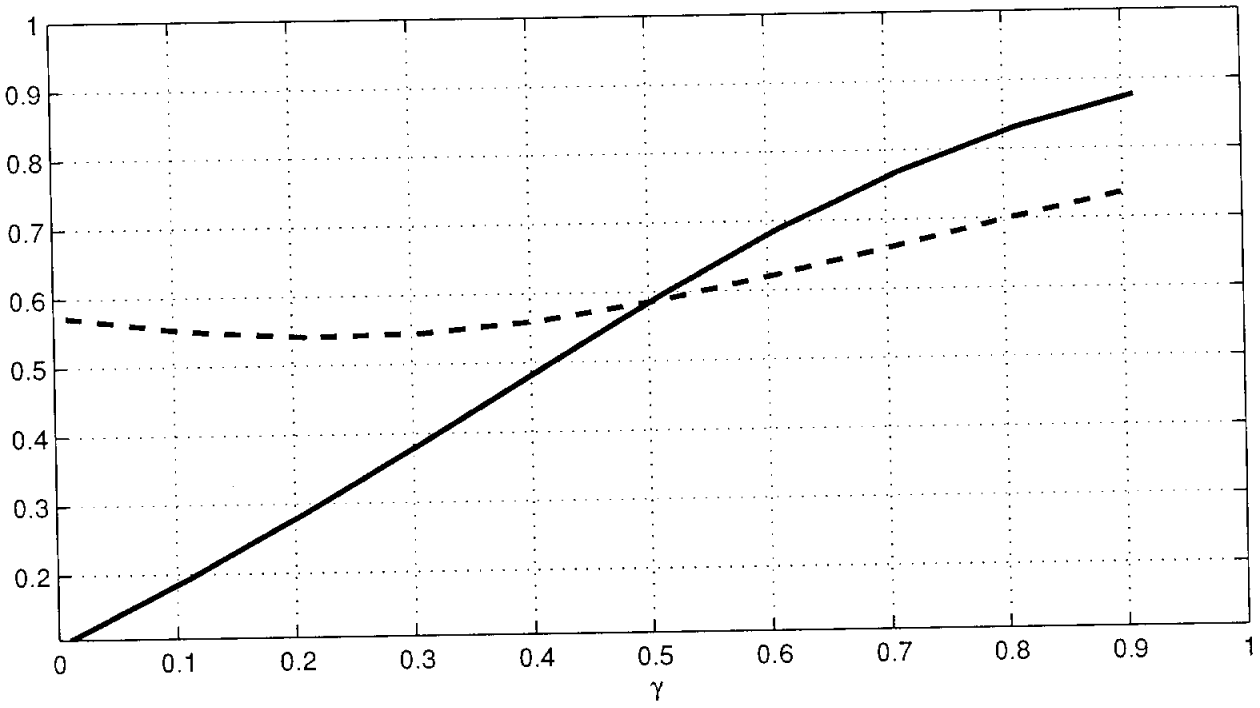


Figure 3

δ constant

Corr(C_1,C_W)



β_W from regression: $C_1 = \beta_Y Y_1 + \beta_R R + \beta_W C_W$

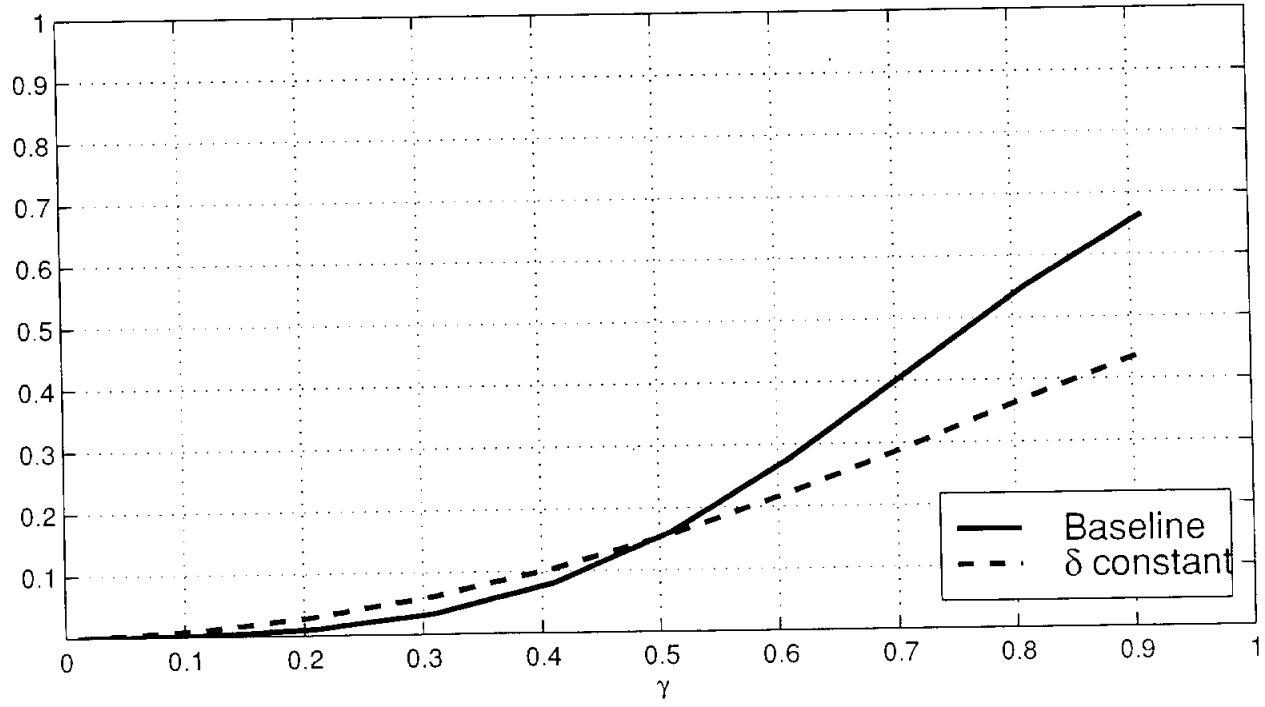
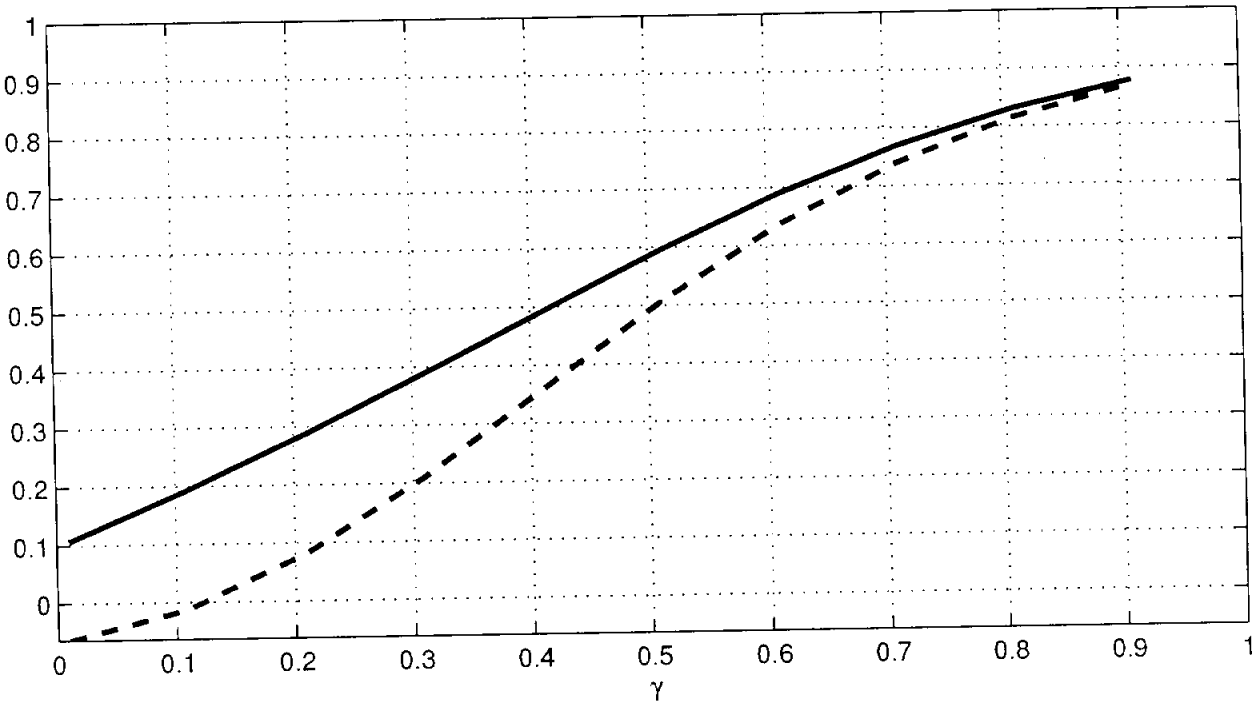


Figure 4

Income Affects Real Rate

$\text{Corr}(C_1, C_W)$



β_W from regression: $C_1 = \beta_Y Y_1 + \beta_R R + \beta_W C_W$

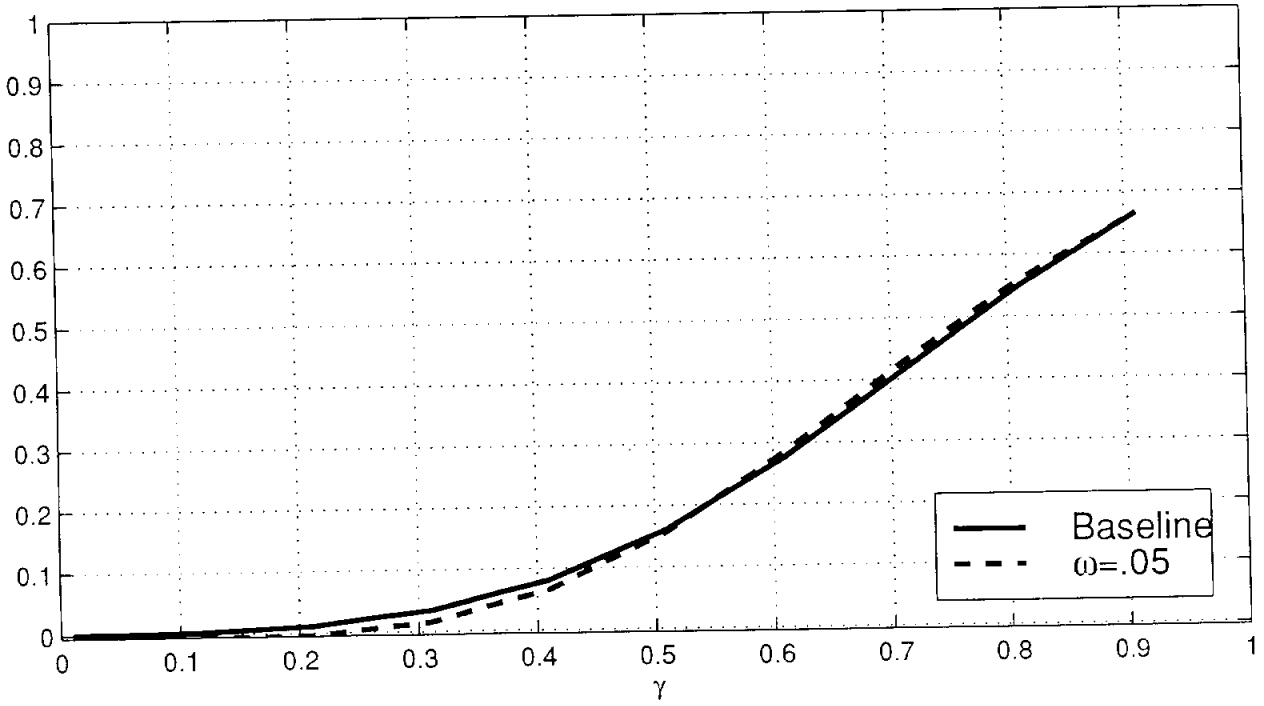
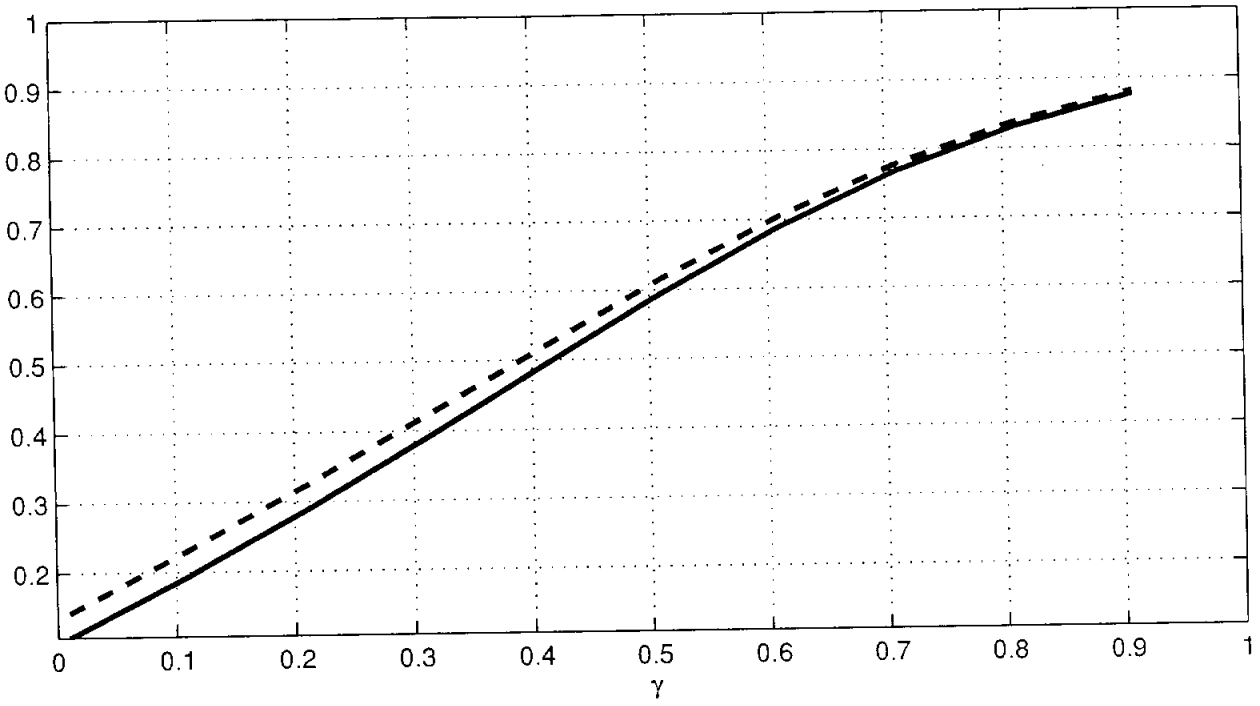


Figure 5

Real Rate Affects Income

$\text{Corr}(C_1, C_W)$



β_W from regression: $C_1 = \beta_Y Y_1 + \beta_R R + \beta_W C_W$

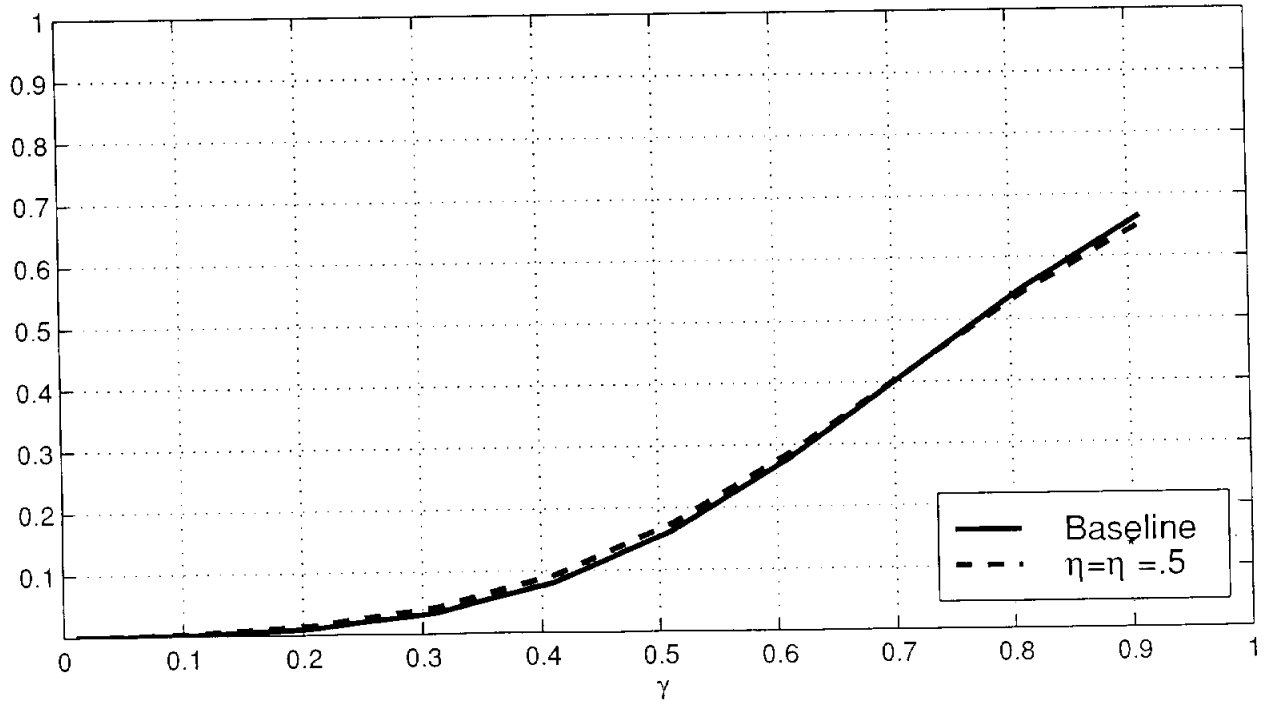
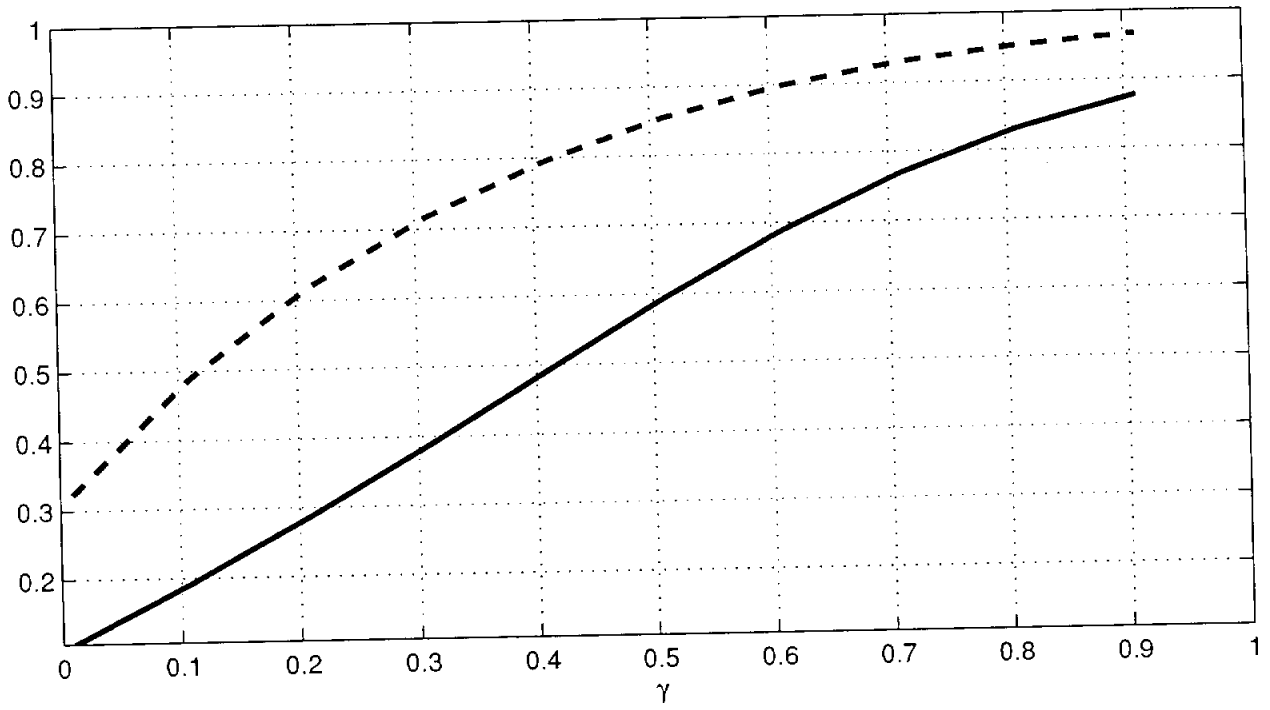


Figure 6

Large Real Rate Shock

Corr(C_1,C_W)



β_W from regression: $C_1 = \beta_Y Y_1 + \beta_R R + \beta_W C_W$