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ABSTRACT

The profitability of contrarian investment strategies need not be the result of stock market overreaction. Even if returns on individual securities are temporally independent, portfolio strategies that attempt to exploit return reversals may still earn positive expected profits. This is due to the effects of cross-autocovariances from which contrarian strategies inadvertently benefit. We provide an informal taxonomy of return-generating processes that yield positive [and negative] expected profits under a particular contrarian portfolio strategy, and use this taxonomy to reconcile the empirical findings of weak negative autocorrelation for returns on individual stocks with the strong positive autocorrelation of portfolio returns. We present empirical evidence against overreaction as the primary source of contrarian profits, and show the presence of important lead–lag relations across securities.

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1. Introduction.

Since the publication of Louis Bachelier's thesis *Theory of Speculation* in 1900, the theoretical and empirical implications of the random walk hypothesis as a model for speculative prices have been subjects of intense interest to financial economists. Although first developed by Bachelier from rudimentary economic considerations of "fair games," the random walk has received broader support from the many early empirical studies confirming the unpredictability of stock price changes.[1] Of course, it is by now well-known that the unforecastability of asset returns is neither a necessary nor a sufficient condition of economic equilibrium.[2] And, in view of recent empirical evidence, it is also apparent that historical stock market prices do not follow random walks.[3]

This fact surprises many economists because the defining property of the random walk is the uncorrelatedness of its increments, and deviations from this hypothesis necessarily imply forecastable price changes.[4] Several recent studies have attributed this forecastability to what has come to be known as the "stock market overreaction" hypothesis, the notion that investors are subject to waves of optimism and pessimism and therefore create a kind of "momentum" which causes prices to temporarily swing away from their fundamental values.[5] Although such a hypothesis may be intuitively appealing, and does yield predictability since what goes down must come up and vice-versa, a well-articulated equilibrium theory of overreaction with sharp empirical implications has yet to be developed. But common to virtually all existing "theories" of overreaction is one very specific empirical implication: price changes must be *negatively* autocorrelated for some holding period.[6] Therefore, the extent to which the data are consistent with stock market overreaction, broadly defined, may be distilled into an

[1] See, for example, the papers in Cootner (1964), and Fama (1965, 1970).

[2] In particular, see Leroy (1973) and Lucas (1978).

[3] See, for example, Lo and MacKinlay (1988). Our usage of the term "random walk" differs slightly from the classical definition of a process with independently and identically distributed increments. We are interested primarily in the uncorrelatedness of increments, and not in either independence or identically distributed innovations. Therefore, a process with uncorrelated but heteroscedastic first-differences would fall into our definition of a random walk; see Lo and MacKinlay (1988) for the exact statement of the random walk hypothesis that we implicitly use here.

[4] However, our surprise must be tempered by the observation that forecasts of stock returns are still subject to random fluctuations, so that profit opportunities are not immediate consequences of forecastability. Nevertheless, recent studies maintain the possibility of significant profits, even after controlling for risk in one way or another.

[5] For example, see DeBondt and Thaler (1985, 1987), De Long et. al. (1989), Lehmann (1988), Poterba and Summers (1988), and Shefrin and Statman (1985).

[6] For example, DeBondt and Thaler (1985) write: "If stock prices systematically overshoot, then their reversal should be predictable from past return data alone . . ." Other studies that consider overreaction also assume this either explicitly or implicitly.

empirically decidable question: are return reversals responsible for the predictability in stock returns?

A more specific consequence of overreaction is the profitability of a contrarian portfolio strategy, a strategy that exploits negative serial dependence in asset returns in particular. The defining characteristic of a contrarian strategy is the purchase of securities that have performed poorly in the past and the sale of securities that have performed well.[7] Selling the "winners" and buying the "losers" will earn positive expected profits because current losers are likely to become future winners and current winners are likely to become future losers when stock returns are negatively autocorrelated. Therefore, it may be said that an implication of stock market overreaction is positive expected profits from a contrarian investment rule. It is the apparent profitability of several contrarian strategies that has led many to conclude that stock markets do indeed overreact.

In this paper we question the reverse implication that the profitability of contrarian investment strategies is evidence of stock market overreaction. Whereas return reversals may be sufficient to yield positive expected profits from a contrarian strategy, they are not necessary. Indeed, as an illustrative example we construct a simple return-generating process in which each security's return is temporally independent, and yet will still yield positive expected profits for a portfolio strategy that buys losers and sells winners. This seemingly counterintuitive result is a consequence of positive *cross-autocovariances* across securities, from which contrarian portfolio strategies inadvertently benefit. For a single security in isolation, negative serial correlation is indeed necessary and sufficient for the contrarian investor to earn positive expected profits. However, when there are many securities to choose from the complex cross-effects among the distinct assets break this link. Therefore, the fact that some contrarian strategies have positive expected profits need not imply that stock markets overreact. In fact, for the particular contrarian strategy we examine, over half of the expected profits is due to cross-effects and not to negative autocorrelation in individual security returns.

However, the most striking aspect of our empirical findings is that these cross-effects are generally positive in sign and have a pronounced lead–lag structure: the

[7] How performance is defined and for what length of time generates as many different kinds of contrarian strategies as there are theories of overreaction.

returns of large capitalization stocks almost always lead those of smaller stocks. This result, coupled with the observation that individual security returns are generally weakly negatively autocorrelated, indicates that the recently documented positive autocorrelation in weekly returns indexes is *completely* attributable to cross-effects. By exploiting our contrarian strategy framework, we show that these cross-autocorrelations are inconsistent with a return-generating process that is the sum of a positively autocorrelated common factor [which generates positive index autocorrelation] plus an idiosyncratic bid-ask spread process [which yields weak negative serial dependence in individual returns]. Although this is a negative result, it does provides important guidance for theoretical models of equilibrium asset prices attempting to explain positive index autocorrelation via time-varying conditional expected returns. Such theories must be capable of generating lead–lag patterns, since it is the cross-autocorrelations that is the source of positive dependence in stock returns.

Since we focus only on the expected profits of the contrarian investment rule and not on its risk, our results have implications for stock market efficiency only insofar as they provide restrictions on economic models that might be consistent [or inconsistent] with the empirical results. We do not assert or deny the existence of "excessive" contrarian profits. Such an issue cannot be addressed without specifying an economic paradigm within which asset prices are rationally determined in equilibrium.[8] However, we have found the contrarian investment strategy to be a convenient tool in exploring the autocorrelation properties of stock returns. Moreover, our analysis of the nature of expected profits does point to more specific sources of risk for contrarian strategies that must be weighed in assessing market efficiency. We leave this more ambitious task to future research.

In Section 2 we provide a summary of the autocorrelation properties of daily, weekly and monthly returns, documenting the positive dependence in portfolio returns and the negative autocorrelations of individual returns. Section 3 presents a formal analysis of the expected profits from a specific contrarian investment strategy under several different return-generating mechanisms, and shows how positive expected profits need not be related to overreaction. In Section 4 we attempt to empirically quantify the proportion of contrarian profits that may be attributed to overreaction and find

[8] Some have accounted for risk in one way or another with mixed results. For example, Chan (1988) claims that DeBondt and Thaler's (1985) excess profits are minimal after properly adjusting for risk, whereas Lehmann (1988) uses a continuous-time argument to conclude that his weekly trading strategy is excessively profitable.

that a substantial portion cannot be. We show that a systematic lead–lag relation among returns of size-sorted portfolios is the primary source of contrarian profits and positive index autocorrelation. In Section 5 we provide some discussion of our use of weekly returns in contrast to the much longer-horizon returns used in previous studies of stock market overreaction, and we conclude in Section 6.

2. A Summary of Current Findings.

In Table 1a we report the first four autocorrelations of weekly equal-weighted and value-weighted returns indexes for the sample period from 6 July 1962 to 31 December 1987, where the indexes are constructed from the CRSP daily returns files.[9] For this sample period the equal-weighted index has a first-order autocorrelation $\hat{\rho}_1$ of approximately 30 percent. Since its heteroscedasticity-consistent standard error is 0.046, this autocorrelation is statistically different from zero at all conventional significance levels. The sub-period autocorrelations indicate that this significance is not an artifact of any particularly influential sub-sample; equal-weighted returns are strongly positively autocorrelated throughout the sample. Higher order autocorrelations are also positive although generally smaller in magnitude, and the decay rate is somewhat slower than the geometric rate of an AR(1) [for example, $\hat{\rho}_1^2$ is 8.8 percent whereas $\hat{\rho}_2$ is 11.6 percent].

To develop a sense of the economic importance of the autocorrelations, recall that the R^2 of a regression of returns on a constant and its first lag is the square of the slope coefficient which is simply the first-order autocorrelation. Therefore, an autocorrelation of 30 percent implies that 9 percent of weekly return variation is predictable by using only the preceding week's returns. In fact, the autocorrelation coefficients implicit in Lo and MacKinlay's (1988) variance ratios are as high as 49 percent for a sub-sample of the portfolio of stocks in the smallest size quintile, implying an R^2 of about 25 percent. This degree of predictability suggests that a profitable trading strategy might be to switch from stocks to bonds when this week's predicted index return falls below the risk-free rate, and vice-versa when it is above. With no transactions costs, the profitability of such a trading rule may be readily verified.

[9] Unless stated otherwise, we take returns to be simple returns and not continuously-compounded. Our construction of weekly returns is described in Lo and MacKinlay (1988).

It may therefore come as some surprise that individual returns are generally weakly negatively autocorrelated. Table 2a reports the cross-sectional average of autocorrelation coefficients across all stocks that have at least 52 non-missing weekly returns during the sample period. For the entire cross-section of 4786 such stocks, the average first-order autocorrelation coefficient, denoted by $\overline{\hat{\rho}_1}$, is –3.4 percent with a cross-sectional standard deviation of 8.4 percent. Therefore, most of the individual first-order autocorrelations fall between –20 percent and 13 percent. This implies that most R^2's of regressions of individual security returns on their return last week fall between 0 and 4 percent, considerably less than the predictability of equal-weighted index returns. Average higher-order autocorrelations are also negative, though smaller in magnitude. The negativity of autocorrelations may be an indication of stock market overreaction for individual stocks, but it is also consistent with the existence of a bid-ask spread. We discuss this further in Section 3.

Table 2a also reports average autocorrelations within size-sorted quintiles.[10] The negative autocorrelations are stronger in the smallest quintile but even the largest quintile has average autocorrelations less than zero. Compared to the 30 percent autocorrelation of the equal-weighted index, the magnitudes of the individual autocorrelations indicated by the means [and standard deviations] in Table 2a are generally much smaller.

For completeness, we also report autocorrelations for returns on daily and monthly indexes in Tables 1b and 1c; cross-sectional averages of autocorrelations for daily and monthly returns on individual stocks are given in Tables 2b and 2c. Similar patterns are observed: autocorrelations are strongly positive for index returns [35.5 and 14.8 percent $\hat{\rho}_1$'s for the equal-weighted daily and monthly indexes respectively], and weakly negative for individual securities [–1.4 and –2.9 percent $\overline{\hat{\rho}_1}$'s for daily and monthly returns respectively].

The general tendency for individual security returns to be negatively serially dependent and for portfolio returns such as those of the equal- and value-weighted market to be positively autocorrelated raises an intriguing issue. We mentioned earlier that because the equal-weighted index exhibits strong positive autocorrelation, a profitable investment strategy would be to allocate assets into equity when equity returns are high

[10] All size-sorted portfolios are constructed by sorting only once [using market values of equity at the middle of the sample period], hence their composition does not change over time.

and into bonds when equity returns are low. This, however, is at odds with virtually any contrarian strategy since it involves buying winners and selling losers. And yet several contrarian strategies have also been shown to yield positive expected profits,[11] even though movements in the aggregate stock market do not support overreaction and the returns of individual stocks are generally only marginally predictable. Is it possible that contrarian profits are due to something other than overreacting investors? We answer these questions in the next two sections.

3. Analysis of Contrarian Profitability.

To reconcile the profitability of contrarian investment strategies with the positive autocorrelation in stock returns indexes. we examine the expected profits of one such strategy under various assumptions on the return-generating process. Consider a collection of N securities and denote by R_t the Nx1–vector of their period t returns $[R_{1t} \cdots R_{Nt}]'$. For convenience, we maintain the following assumption throughout this section:

($A1$) R_t is a jointly covariance-stationary stochastic process with expectation $E[R_t] = \mu \equiv [\mu_1 \; \mu_2 \; \cdots \; \mu_N]'$ and autocovariance matrices $E[(R_{t-k} - \mu)(R_t - \mu)'] = \Gamma_k$ where, with no loss of generality, we take $k \geq 0$ since $\Gamma_k = \Gamma'_{-k}$.[12]

In the spirit of virtually all contrarian investment strategies, consider buying stocks at time t that were "losers" at time $t-k$ and selling stocks at time t that were "winners" at time $t-k$, where winning and losing is with respect to the equal-weighted return on the market. More formally, if $\omega_{it}(k)$ denotes the fraction of the portfolio devoted to security i at time t, let:

[11] For example, DeBondt and Thaler (1985, 1987) and Lehmann (1988).

[12] Assumption ($A1$) is made for notational simplicity, since joint covariance-stationarity allows us to eliminate time-indexes from population moments such as μ and Γ_k; the qualitative features of our results will not change under the weaker assumptions of weakly dependent heterogeneously distributed vectors R_t. This would merely require replacing expectations with corresponding probability limits of suitably defined time-averages. For the results in this section, the added generality does not outweigh the expositional complexity that a weaker set of assumptions requires. However, the empirical results of Section 4 are based on these weaker assumptions; interested readers may refer to conditions ($A2$)–($A4$) in Appendix 2.

$$\omega_{it}(k) = -\frac{1}{N}(R_{it-k} - R_{mt-k}) \qquad i = 1, \ldots, N \tag{3.1}$$

where $R_{mt-k} \equiv \sum_{i=1}^{N} R_{it-k}/N$ is the equally-weighted market index.[13] If, for example, $k = 1$ then the portfolio strategy in period t is to short the winners and buys the losers of the previous period, $t-1$. By construction, $\omega_t(k) \equiv [\omega_{1t}(k)\ \omega_{2t}(k)\ \cdots\ \omega_{Nt}(k)]'$ is an arbitrage portfolio since the weights sum to zero. Therefore, the total investment long [or short] at time t is given by $\mathrm{I}_t(k)$ where:

$$\mathrm{I}_t(k) \equiv \frac{1}{2}\sum_{i=1}^{N} |\omega_{it}(k)| \, . \tag{3.2}$$

Since the portfolio weights are proportional to the differences between the market index and the returns, securities that deviate more positively from the market at time $t-k$ will have greater negative weight in the time t portfolio, and vice-versa. Such a strategy is designed to take advantage of stock market overreactions as characterized, for example, by DeBondt and Thaler (1985): "(1) Extreme movements in stock prices will be followed by extreme movements in the opposite direction. (2) The more extreme the initial price movement, the greater will be the subsequent adjustment." The profit $\pi_t(k)$ from such a strategy is simply:

$$\pi_t(k) = \sum_{i=1}^{N} \omega_{it}(k) R_{it} \, . \tag{3.3}$$

Re-arranging (3.3) and taking expectations yields the following:[14]

[13] This is perhaps the simplest portfolio strategy that captures the essence of the contrarian principle. Lehmann (1988) also considers this strategy, although he employs a more complicated strategy in his empirical analysis in which the portfolio weights (3.1) are re-normalized each period by a random factor of proportionality so that the investment is always one dollar long and short. This portfolio strategy is also similar to that of DeBondt and Thaler (1985, 1987), although in contrast to our use of weekly returns they consider holding periods of three years. See Section 5 for further discussion.

[14] The relatively straightforward derivation of this equation is included in Appendix 1 for completeness. This is the population counterpart of Lehmann's (1988) sample moment equation (5) divided by N.

$$E[\pi_t(k)] = \frac{\iota'\Gamma_k\iota}{N^2} - \frac{1}{N}\operatorname{tr}(\Gamma_k) - \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2 \tag{3.4}$$

where $\mu_m \equiv E[R_{mt}] = \mu'\iota/N$ and $\operatorname{tr}(\cdot)$ denotes the trace operator. The first term of (3.4) is simply the k-th order autocovariance of the equally-weighted market index. The second term is the cross-sectional average of the k-th order autocovariances of the individual securities, and the third term is the cross-sectional variance of the mean returns. Since this last term is independent of the autocovariances Γ_k and does not vary with k, we define the *profitability index* $L_k = L(\Gamma_k)$ and the constant $\sigma^2(\mu)$ as:

$$L_k \equiv \frac{\iota'\Gamma_k\iota}{N^2} - \frac{1}{N}\operatorname{tr}(\Gamma_k) \quad , \quad \sigma^2(\mu) \equiv \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2 . \tag{3.5}$$

thus,

$$E[\pi_t(k)] = L_k - \sigma^2(\mu) . \tag{3.6}$$

For purposes that will become evident below we re-write L_k as the following sum:

$$L_k = \frac{1}{N^2}\left[\iota'\Gamma_k\iota - \operatorname{tr}(\Gamma_k)\right] - \left(\frac{N-1}{N^2}\right)\cdot\operatorname{tr}(\Gamma_k) \equiv C_k + O_k \tag{3.7}$$

where:

$$C_k \equiv \frac{1}{N^2}\left[\iota'\Gamma_k\iota - \operatorname{tr}(\Gamma_k)\right] \quad , \quad O_k \equiv -\left(\frac{N-1}{N^2}\right)\cdot\operatorname{tr}(\Gamma_k) \tag{3.8}$$

hence:

$$E[\pi_t(k)] = C_k + O_k - \sigma^2(\mu) . \tag{3.9}$$

Written this way, it is apparent that expected profits may be decomposed into three terms, one $[C_k]$ depending on only the off-diagonals of the autocovariance matrix Γ_k, the second $[O_k]$ depending on only the diagonals, and a third $[\sigma^2(\mu)]$ which is independent of the autocovariances. This allows us to separate the fraction of expected profits due to the cross-autocovariances C_k, versus the own-autocovariances O_k of returns.

From (3.9), it is clear that the profitability of the contrarian strategy (3.1) may be perfectly consistent with a positively autocorrelated market index and negatively autocorrelated individual security returns. Positive cross-autocovariances imply that the term C_k is positive, and negative autocovariance for individual securities implies that O_k is also positive. Conversely, the empirical finding that equal-weighted indexes are strongly positively autocorrelated and that individual security returns are weakly negatively serially dependent implies, through (3.7), that there must be significant positive cross-autocorrelations across securities. To see this, observe that the first-order autocorrelation of the equally-weighted index R_{mt} is simply:

$$\frac{\mathrm{Cov}[R_{mt-1}, R_{mt}]}{\mathrm{Var}[R_{mt}]} = \frac{\iota'\Gamma_1\iota}{\iota'\Gamma_0\iota} = \frac{\iota'\Gamma_1\iota - \mathrm{tr}(\Gamma_1)}{\iota'\Gamma_0\iota} + \frac{\mathrm{tr}(\Gamma_1)}{\iota'\Gamma_0\iota} . \tag{3.10}$$

The numerator of the second term of the right-hand side of (3.10) is simply the sum of the first-order autocovariances of individual securities which, if negative, implies that the first term must be positive in order for the sum to be positive. Therefore, the positive autocorrelation in weekly returns may be attributed solely to the positive cross-autocorrelations across securities.

The expression for L_k also suggests that stock market overreaction need not be the reason that contrarian investment strategies are profitable. To anticipate the examples below, if returns are positively cross-autocorrelated then a return-reversal strategy will yield positive profits on average, even if individual security returns are *temporally independent*! That is, if a high return for security A today implies that security B's return will probably be high tomorrow, then a contrarian investment strategy will be profitable even if each security's returns are unforecastable using past returns of that security only. The intuition for such a result is straightforward. Suppose there are only the two stocks, A and B; if A's return is higher than the market today, we sell it and buy B. But if A and B are positively cross-autocorrelated, a higher return for A today implies a higher return for B tomorrow [on average], thus we will have profited from our long position in B [on average]. Nowhere do we require that the stock market overreacts, i.e., that individual returns are negatively autocorrelated. Of course, the presence of stock market overreactions enhances the profitability of the return-reversal strategy, but it is not necessary.

To organize our understanding of the sources and nature of contrarian profits, we provide four illustrative examples below. They are highly stylized special cases, nevertheless they yield a useful informal taxonomy of conditions necessary for the average profitability of the investment strategy (3.1).

3.1. The I.I.D. Benchmark.

Let returns R_t be both cross-sectionally and temporally independent. In this case $\Gamma_k = 0$ for all non-zero k hence:

$$L_k = C_k = O_k = 0 \tag{3.11}$$

$$E[\pi_t(k)] = -\sigma^2(\mu) \leq 0 . \tag{3.12}$$

Although returns are both temporally and cross-sectionally unforecastable, the expected profits are negative as long as there is some cross-sectional variation in expected returns. This is a result of the fact that our strategy is shorting the higher and buying the lower mean return securities respectively, a losing proposition even when stock market prices do follow random walks.[15] Since $\sigma^2(\mu)$ is generally of small magnitude and does not depend on the autocovariance structure of R_t, we will focus on L_k and ignore $\sigma^2(\mu)$ for the remainder of Section 3.

3.2. Stock Market Overreaction and Fads.

Almost any operational definition of stock market overreaction implies that individual security returns are negatively autocorrelated over some holding period, so that

[15] This provides a simple counterexample to the somewhat surprising implication that one cannot systematically *lose* money in the stock market if prices follow random walks [and there are no transactions costs]. Multiple securities with distinct means imply the existence of portfolio strategies with positive and negative expected returns.

"what goes up must come down" and vice-versa. If we denote by $\gamma_{ij}(k)$ the i,j-th element of the autocovariance matrix Γ_k, the overreaction hypothesis implies that the diagonal elements of Γ_k are negative, i.e., $\gamma_{ii}(k) < 0$, at least for $k = 1$ when the span of one period corresponds to a complete cycle of overreaction.[16] Since the overreaction hypothesis generally does not restrict the cross-autocovariances, for simplicity we set them to zero, i.e., $\gamma_{ij}(k) = 0, i \neq j$. Hence, we have:

$$\Gamma_k \;=\; \begin{pmatrix} \gamma_{11}(k) & 0 & \cdots & 0 \\ 0 & \gamma_{22}(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_{NN}(k) \end{pmatrix} . \tag{3.13}$$

The profitability index under these assumptions for R_t is then:

$$L_k \;=\; O_k \;=\; -\left(\frac{N-1}{N^2}\right)\cdot \mathrm{tr}(\Gamma_k) \;=\; -\left(\frac{N-1}{N^2}\right)\cdot \sum_{i=1}^{N} \gamma_{ii}(k) \;>\; 0 \tag{3.14}$$

where the cross-autocovariance term C_k is zero; the positivity of L_k follows from the negativity of the own-autocovariances, assuming $N > 1$. Not surprisingly, if stock markets do overreact the contrarian investment strategy is profitable on average.

Another price process for which the return-reversal strategy will yield positive expected profits is the sum of a random walk and an AR(1), which has been recently proposed as a model of "fads" and "animal spirits."[17] Specifically, let the dynamics for the log-price X_{it} of each security i be given by:

$$X_{it} \;=\; Y_{it} \;+\; Z_{it} \tag{3.15}$$

where

$$Y_{it} \;=\; \mu_i \;+\; Y_{it-1} \;+\; \epsilon_{it} \tag{3.16}$$

$$Z_{it} \;=\; \rho_i Z_{it-1} \;+ \nu_{it} \qquad 0 < \rho < 1 \tag{3.17}$$

[16] We discuss this further in Section 5.
[17] See, for example, Summers (1986).

and the disturbances $\{\epsilon_{it}\}$ and $\{\nu_{it}\}$ are temporally, mutually, and cross-sectionally independent at all *positive* leads and lags.[18] The k-th order autocovariance for the return vector R_t is then given by the following diagonal matrix:

$$\Gamma_k \;=\; \text{diag}\,[\; -\rho_1^{k-1}\frac{1-\rho_1}{1+\rho_1}\sigma_{\nu_1}^2 \,,\; \ldots \,,\; -\rho_N^{k-1}\frac{1-\rho_N}{1+\rho_N}\sigma_{\nu_N}^2 \;] \,. \tag{3.18}$$

The profitability index follows immediately:

$$L_k \;=\; O_k \;=\; -\left(\frac{N-1}{N^2}\right)\cdot \text{tr}(\Gamma_k) \;=\; \frac{N-1}{N^2}\cdot\sum_{i=1}^{N}\rho_i^{k-1}\frac{1-\rho_i}{1+\rho_i}\sigma_{\nu_i}^2 \;>\; 0 \tag{3.19}$$

Since the own-autocovariances in (3.18) are all negative this is a special case of (3.13) and may therefore be interpreted as an example of stock market overreaction. However, the fact that returns are negatively autocorrelated at all lags is an artifact of the first-order autoregressive process and need not be true for the sum of a random walk and a general stationary process, a model that has been proposed for both stock market fads and time-varying expected returns.[19] For example, let the "temporary" component of (3.15) be given by the following stationary AR(2) process:

$$Z_{it} \;=\; \frac{9}{7}Z_{it-1} \;-\; \frac{5}{7}Z_{it-2} \;+\; \nu_{it} \,. \tag{3.20}$$

It is easily verified that the first-difference of Z_{it} is positively autocorrelated at lag 1 implying that $L_1 < 0$. Therefore, stock market overreaction necessarily implies the profitability of the portfolio strategy (3.1) [in the absence of cross-autocorrelation], but stock market fads do not.

[18] This last assumption requires only that ϵ_{it} is independent of ϵ_{jt+k} for $k \neq 0$, hence the disturbances may be contemporaneously cross-sectionally dependent without loss of generality.

[19] For example, see Fama and French (1988) and Summers (1986).

3.3. Trading on White Noise and Lead–Lag Relations.

Let the return-generating process for R_t be given by:

$$R_{it} \;=\; \mu_i \;+\; \beta_i \Lambda_{t-i} \;+\; \epsilon_{it} \quad , \quad \beta_i > 0 \quad , \quad i = 1, \ldots, N \qquad (3.21)$$

where Λ_t is a temporally independent common factor with zero mean and variance σ_λ^2, and the ϵ_{it}'s are assumed to be both cross-sectionally and temporally independent. These assumptions imply that for each security i, its returns are white noise [with drift] so that future returns to i are not forecastable from its past returns. This temporal independence is certainly not consistent with either the spirit or form of the stock market overreaction hypothesis. And yet it *is* possible to predict i's returns using past returns of security j, where $j < i$. This is obviously an artifact of (3.21) in which the return on the i-th security depends positively on a lagged common factor, where the lag is determined by the security's index. This implies that the return of security 1 leads that of securities 2, 3, etc.; the return of security 2 leads that of securities 3, 4, etc.; and so on. Alternatively, observing the return on security 1 today will help forecast the return on security 2 tomorrow, but today's return on security 2 provides no information for how security 1 will fare tomorrow. This lead–lag relation will induce positive expected profits for the contrarian strategy (3.1). To see this, observe that:

$$\Gamma_1 \;=\; \begin{pmatrix} 0 & \beta_1\beta_2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \beta_2\beta_3 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \beta_{N-1}\beta_N \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \cdot \sigma_\lambda^2 \qquad (3.22)$$

$$\Gamma_2 \;=\; \begin{pmatrix} 0 & 0 & \beta_1\beta_3 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \beta_2\beta_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \beta_{N-3}\beta_{N-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \cdot \sigma_\lambda^2 \qquad (3.23)$$

and, more generally, when $k < N$ then Γ_k has zeros in all entries except along the k-th super-diagonal, for which $\gamma_{ii+k} = \sigma_\lambda^2 \beta_i \beta_{i+k}$. For future reference, observe that a characteristic of the lead–lag model is the *asymmetry* of the autocovariance matrix Γ_k. The profitability index in this case is:

$$L_k \;=\; C_k \;=\; \frac{\sigma_\lambda^2}{N^2} \sum_{i=1}^{N-k} \beta_i \beta_{i+k} \;>\; 0\;. \tag{3.24}$$

From this example the importance of the cross-effects is evident; although each security is individually unpredictable, a contrarian strategy may still profit if securities are positively cross-correlated at various leads and lags. Moreover, it should be apparent that less contrived return-generating processes will also yield posi*ive expected profits to contrarian strategies as long as the cross-autocovariances are sufficiently large. One source of such cross-effects may be non-synchronous trading, as in the models of Scholes and Williams (1977) and Cohen et. al. (1986). For example, the non-trading process proposed by Cohen et. al. induces the following time series properties in observed returns when true returns are generated by the market model:[20]

1. Individual returns are negatively autocorrelated.
2. A market index composed of observed returns to securities with positive betas will exhibit positive serial dependence.
3. Cross autocorrelations of returns for securities i and j will be non-zero, and of the same sign as $\beta_i \beta_j$.

If securities' betas are generally of the same sign, then non-trading induces positive cross-effects; coupled with the negative individual autocorrelations, this yields positive expected profits for a contrarian investment strategy. However, Lo and MacKinlay (1989) show that the magnitudes of cross-effects documented in Section 4 cannot be completely attributed to non-synchronous trading biases.

[20] See Chapter 6 of Cohen et. al. (1986).

3.4. A Positively Dependent Common Factor and the Bid-Ask Spread.

One plausible return-generating mechanism that is consistent with positive index autocorrelation and negative serial dependence in individual returns is to let each R_{it} be the sum of three components: a positively autocorrelated common factor, idiosyncratic white noise, and a bid-ask spread process.[21] More formally, let:

$$R_{it} = \mu_i + \beta_i \Lambda_t + \eta_{it} + \epsilon_{it} \tag{3.25}$$

where:

$$E[\Lambda_t] = 0 \qquad E[\Lambda_t \Lambda_{t+k}] \equiv \gamma_\lambda(k) > 0 \tag{3.26}$$

$$E[\epsilon_{it}] = E[\eta_{it}] = 0 \qquad \forall\ i, t \tag{3.27}$$

$$E[\epsilon_{it}\epsilon_{jt+k}] = \begin{cases} \sigma_i^2 & \text{if } k = 0 \text{ and } i = j. \\ 0 & \text{otherwise.} \end{cases} \tag{3.28}$$

$$E[\eta_{it}\eta_{jt+k}] = \begin{cases} -\frac{s_i^2}{4} & \text{if } k = 1 \text{ and } i = j. \\ 0 & \text{otherwise.} \end{cases} \tag{3.29}$$

We have implicitly assumed in (3.29) that Roll's (1984) model of the bid-ask spread obtains so that the first-order autocorrelation of η_{it} is the negative of one-fourth the square of the percentage bid-ask spread s_i, and all higher-order autocorrelations and all cross-correlations are zero. Such a return-generating process will yield a positively autocorrelated market index since averaging the white-noise and bid-ask components will trivialize them, leaving the common factor Λ_t. Yet if the bid-ask spread is large enough, it may dominate the common factor for each security, yielding negatively autocorrelated individual security returns.

[21] This is suggested in Lo and MacKinlay (1988). Conrad, Kaul, and Nimalendran (1988) investigate a similar specification.

The autocovariance matrices for (3.25) are given by:

$$\Gamma_1 = \gamma_\lambda(1)\beta\beta' - \frac{1}{4}\operatorname{diag}[s_1^2,\ s_2^2,\ \ldots,\ s_N^2] \tag{3.30}$$

$$\Gamma_k = \gamma_\lambda(k)\beta\beta' \qquad k > 1 \tag{3.31}$$

where $\beta \equiv [\beta_1\ \beta_2\ \cdots\ \beta_N]'$. In contrast to the lead–lag model of Section 3.3, the autocovariance matrices for this return-generating process are all symmetric. This yields an important empirical implication that distinguishes the common factor model from the lead–lag process and will be exploited in our empirical appraisal of overreaction.

Denote by β_m the cross-sectional average $\sum_{i=1}^{N} \beta_i / N$. Then the profitability index is given by:

$$L_1 = -\frac{\gamma_\lambda(1)}{N}\sum_{i=1}^{N}(\beta_i - \beta_m)^2 + \frac{N-1}{N^2}\sum_{i=1}^{N}\frac{s_i^2}{4} \tag{3.32}$$

$$L_k = -\frac{\gamma_\lambda(k)}{N}\sum_{i=1}^{N}(\beta_i - \beta_m)^2 \qquad k > 1\ . \tag{3.33}$$

From (3.32), it is evident that if the bid-ask spreads are large enough and the cross-sectional variation of the β_k's is small enough, the contrarian strategy (3.1) may yield positive expected profits when using only one lag $[k = 1]$ in computing portfolio weights. However, the positivity of the profitability index is due solely to the negative autocorrelations of individual security returns induced by the bid-ask spread. Once this effect is removed, which is the case when portfolio weights are computed using lags 2 or higher, relation (3.33) shows that the profitability index is of the opposite sign of the index autocorrelation coefficient $\gamma_\lambda(k)$; since $\gamma_\lambda(k) > 0$ by assumption, expected profits are negative for lags higher than 1. In view of our empirical analysis of Section 4 which shows that L_k is still positive for $k > 1$, it seems unlikely that the return-generating process (3.25) can account for the weekly autocorrelation patterns of Lo and MacKinlay (1988).

4. An Empirical Appraisal of Overreaction.

To examine the extent to which contrarian profits are due to stock market overreaction, we estimate the expected profits from the return-reversal strategy of Section 3 for several samples of CRSP NYSE–AMEX securities. Recall that $E[\pi_t(k)] = C_k + O_k - \sigma^2(\mu)$ where C_k depends only on the cross-autocovariances of returns and O_k depends only on the own-autocovariances. Table 3a presents estimates of $E[\pi_t(k)]$, C_k, O_k, and $\sigma^2(\mu)$ for the 551 stocks that have no missing weekly returns during the entire sample period from 6 July 1962 to 31 December 1987. Estimates are computed for the sample of all stocks and for three size-sorted quintiles. We develop the appropriate sampling theory in Appendix 2, in which the covariance-stationarity assumption $(A2)$ is relaxed and replaced with assumptions $(A2)$–$(A4)$ that allow for weakly dependent heterogeneously distributed returns.

Consider the last three columns of Table 3a which report the magnitudes of the three terms $\hat{C}_k$, $\hat{O}_k$, and $\sigma^2(\hat{\mu})$ as percentages of expected profits. At lag 1 half the expected profits from the contrarian strategy is due to positive cross-autocovariances. In the central quintile about 67 percent of the expected profits is attributable to these cross-effects. The results at lag 2 are similar; positive cross-autocovariances account for about 50 percent of the expected profits, 66 percent for the smallest quintile.

The positive expected profits at lags 2 and higher provide direct evidence against the common component/bid–ask spread model of Section 3.4. If returns contained a positively autocorrelated common factor and exhibited negative autocorrelation due to "bid-ask bounce," expected profits can be positive only at lag 1; higher lags must exhibit negative expected profits as (3.33) shows. Table 3a shows that estimated expected profits are significantly positive for lags 2 through 4 in all portfolios except one.

The z-statistics for $\hat{C}_k$, $\hat{O}_k$, and $\hat{E}[\pi_t(k)]$ are asymptotically standard normal under the null hypothesis that the population values corresponding to the three estimators are zero. At lag 1 they are almost all significantly different from zero at the 1 percent level. At higher lags, the own- and cross-autocovariance terms are generally insignificant. However, estimated expected profits retains its significance even at lag 4, largely due to the behavior of small stocks. The curious fact that $\hat{E}[\pi_t(k)]$ is statistically different from zero whereas $\hat{C}_k$ and $\hat{O}_k$ are not suggests that there is important negative correlation between the two estimators $\hat{C}_k$ and $\hat{O}_k$.[22] That is, although they are both noisy estimates, the variance of their sum is less than each of their variances because they co-vary negatively. Since $\hat{C}_k$ and $\hat{O}_k$ are both functions of second moments and co-moments, significant correlation of the two estimators implies the importance of fourth co-moments, perhaps as a result of co-skewness or kurtosis. This is beyond the scope of our paper, but bears further investigation.

Table 3a also reports the average long [and hence short] positions generated by the return-reversal strategy over the 1330-week sample period. For all stocks the average weekly long/short position is \$152, corresponding to profits of \$1.69 per week on average. In contrast, applying the same strategy to a portfolio of small stocks yields an expected profit of \$4.53 per week, but requires only \$209 long and short each week on average. The ratio of expected profits to average long investment is 1.1 percent for all stocks, and 2.2 percent for stocks in the smallest quintile. Of course, in the absence of market frictions such comparisons are irrelevant since an arbitrage portfolio strategy may be scaled arbitrarily. However, if the size of one's long/short position is constrained, as is sometimes the case in practice, then the average investment figures reported in Table 3a suggest that applying the contrarian strategy to small firms would be more profitable on average. Alternatively, this may imply that the behavior of small stocks is the more anomalous from the perspective of the efficient markets hypothesis.[23]

Using stocks with continuous listing for over twenty years obviously induces a survivorship bias that is difficult to evaluate. To reduce this bias we perform similar analyses for two sub-samples: stocks with continuous listing for the first and second halves of the 1330-week sample respectively. These results are reported in Tables 3b and 3c. The patterns are virtually identical. In both sub-periods positive cross-effects account for at least 50 percent of expected profits at lag 1, and generally more at higher lags.

To develop further intuition for the pattern of these cross-effects we report in Table 4 cross-autocorrelation matrices $\hat{\Upsilon}_k$ for the vector of returns on the five size-sorted quintiles and the equal-weighted index using the first sample of 551 stocks. Specifically, let Z_t denote the vector $[R_{1t}\ R_{2t}\ R_{3t}\ R_{4t}\ R_{5t}\ R_{mt}]'$ where R_{it} is the return

[22] We have investigated the unlikely possibility that $\sigma^2(\hat{\mu})$ is responsible for this anomaly; it is not.

[23] Of course, we cannot infer from this that the market for small capitalization equity is less efficient than the market for larger stocks since smaller stocks may be more risky. Moreover, no attempt has been made to control for market depth. These two factors might explain the differential in expected profits per dollar long/short between size-sorted portfolios.

on the equal-weighted portfolio of stocks in the i-th quintile and R_{mt} is the return on the equal-weighted portfolio of all stocks. Then we let $\Upsilon_k \equiv D^{-1/2}E[(Z_{t-k}-\mu)(Z_t-\mu)']D^{-1/2}$ where $D \equiv \mathrm{diag}[\sigma_1^2,\ldots,\sigma_5^2,\sigma_m^2]$ and $\mu \equiv E[Z_t]$. By this convention, the i,j-th element of Υ_k is the correlation of R_{it-k} with R_{jt}. The estimator $\hat{\Upsilon}_k$ is the usual sample autocorrelation matrix. Note that it is the upper left 5x5 partition of Υ_k that corresponds to our definition of Γ_k, since the full matrix Υ_k also contains autocovariances between portfolio returns and the equal-weighted market index R_{mt}.[24]

An interesting pattern emerges from Table 4: the entries below the diagonals of $\hat{\Upsilon}_k$ are almost always larger than those above the diagonals [excluding the last row and column, which are the autocovariances between portfolio returns and the market]. This implies that current returns of smaller stocks are correlated with past returns of larger stocks, but not vice-versa. This is strong evidence in favor of a distinct lead–lag relation: the returns of large stocks tend to lead those of small stocks. For example, the first-order autocorrelation between last week's return on large stocks [R_{5t-1}] with this week's return on small stocks [R_{1t}] is 27.6 percent, whereas the first-order autocorrelation between last week's return on small stocks [R_{1t-1}] with this week's return on large stocks [R_{5t}] is only 2.0 percent! Similar patterns may be seen in the higher-order autocorrelation matrices, although the magnitudes are smaller since the higher-order cross-autocorrelations decay. The asymmetry of the $\hat{\Upsilon}_k$ matrices implies that the autocovariance matrix estimators $\hat{\Gamma}_k$ are also asymmetric. This provides further evidence against the return-generating process (3.25) of Section 3.4, since that model implies symmetric autocovariance matrices.

The results in Tables 3 and 4 point to the complex patterns of cross-effects among securities as significant sources of positive index autocorrelation as well as expected profits for contrarian investment rules. Moreover, the presence of these cross-effects has important implications irrespective of the nature of contrarian profits. For example, if such profits are genuine, the fact that at least half may be attributed to cross-autocovariances suggests further investigation of mechanisms by which aggregate shocks to the economy are transmitted from large capitalization companies to small ones.

[24] We include the market return in our autocovariance matrices so that those who are interested may compute portfolio betas and market volatilities from our tables.

5. Long Horizons Versus Short Horizons.

Since many recent studies have employed longer-horizon returns in examining contrarian strategies and predictability of stock returns, we should provide some discussion of our choice to focus exclusively on weekly returns. Because our analysis of the contrarian investment strategy (3.1) uses only short-horizon returns, we have little to say about the behavior of long-horizon returns. Distinguishing between short and long return horizons is important, as it is now well-known that weekly fluctuations in stock returns differ in many ways from movements in three- to five-year returns. Therefore, inferences concerning the performance of the long-horizon strategies cannot be drawn directly from short-horizon results such as ours. Nevertheless, some suggestive comparisons are possible.

Statistically, the predictability of short-horizon returns, especially in weekly and monthly returns, is stronger and more consistent through time. For example, Blume and Friend (1978) have estimated a time series of cross-sectional correlation coefficients of returns in adjacent months using monthly New York Stock Exchange data from 1926 to 1975, and found that for 422 of the 598 months the sample correlation was negative.[25] This proportion of negative correlations is considerably higher than expected if returns are unforecastable. Moreover, in their framework a negative correlation coefficient implies positive expected profits in our equation (3.4) with $k = 1$. Jegadeesh (1988) provides further analysis of monthly data and reaches similar conclusions.

The results are even more striking for weekly stock returns. For example, Lo and MacKinlay (1988) show evidence of strong predictability for portfolio returns using New York and American Stock Exchange data from 1962 to 1985. Using the same data, Lehmann (1988) shows that the profits of a contrarian strategy similar to (3.1) is virtually always profitable. Not surprisingly, such profits are sensitive to the size of the transactions costs; for some cases a one-way transactions cost of 0.40 percent is sufficient to render them positive half the time and negative the other half. The importance of Lehmann's findings obviously hinge on the relevant costs of turning over

[25] Specifically, for every pair of adjacent months $t-1$ and t, Blume and Friend (1978) compute the following statistic ρ_t:

$$\rho_t \equiv \frac{\sum_i (R_{it-1} - R_{mt-1})(R_{it} - R_{mt})}{\sqrt{\sum_i (R_{it-1} - R_{mt-1})^2} \cdot \sqrt{\sum_i (R_{it} - R_{mt})^2}}$$

where $R_{mt} = \sum_i R_{it}/N_t$ and N_t is the number of securities with non-missing returns in months $t-1$ and t. Note that ρ_t is proportional to the profits (3.3) of the contrarian strategy (3.1) [where the factor of proportionality is always negative].

securities frequently, an issue that is not considered in this paper. However, the fact that our Table 3a shows the smallest firms to be the most profitable on average [as measured by the ratio of expected profits to the dollar amount long] may indicate that 0.80 percent roundtrip transactions costs are low. In addition to the bid-ask spread, which is generally $0.125 or larger and will be a larger percentage of the price for smaller stocks,[26] the price impact of trades on these relatively thinly traded securities may become important.

Evidence regarding the predictability of long horizon returns is somewhat mixed. Perhaps the most well-known studies of a contrarian strategy using long horizon returns are those of DeBondt and Thaler (1985, 1987) in which winners are sold and losers are purchased, but where the holding period over which "winning" and "losing" is determined is three years. Based on data from 1926 through 1981 they conclude that the market overreacts since the losers outperform the winners. However, Chan (1988) challenges their conclusion and finds that the performance differences can be largely explained by differences in risk. Moreover, the behavior of DeBondt and Thaler's (1985) cumulative average residual plots, and the results of Lehmann (1989), suggest that short-horizon return reversals may be responsible for the long-horizon effect.

Fama and French (1988) and Poterba and Summers (1988) have also examined the predictability of long horizon returns in a portfolio context, and conclude that there is negative serial correlation in long horizon returns – a result which is consistent with those of DeBondt and Thaler. However, this negative serial dependence is quite sensitive to the sample period employed and may be largely due to the first ten to twenty years of the 1926 to 1989 sample.[27] Furthermore, the statistical procedure on which the long-horizon predictability is based has been questioned by Richardson (1988). Richardson has shown that properly adjusting for the fact that multiple time horizons [and test statistics] are considered simultaneously yields serial correlation estimates that are statistically indistinguishable from zero.

These considerations point convincingly to short-horizon returns as the more immediate source from which evidence of predictability and stock market overreaction might be culled. Of course, this is not to say that nothing may be gleaned from a careful investigation of returns over longer time spans. Indeed, it may be only at these

[26] Smaller stocks tend to have lower prices.
[27] See Kim, Nelson, and Startz (1988).

lower frequencies that the impact of economic factors such as the business cycle is detectable. Moreover, to the extent that transaction costs are greater for strategies exploiting short-horizon predictability, allowing the predictability to persist without representing any unexploited profit opportunities, long-horizon predictability may be the more significant issue.

6. Conclusion.

Traditional tests of the random walk hypothesis for stock market prices have generally focused on either the returns to individual securities or to portfolios of securities. In this paper we show that the cross-sectional interaction of security returns over time is an important aspect of stock price dynamics. As an example, we document the fact that stock returns are often positively cross-autocorrelated, which reconciles the negative serial dependence in individual security returns with the positive autocorrelation in market indexes. This also shows that stock market overreaction need not be the sole explanation for the profitability in contrarian portfolio strategies. Indeed, the empirical evidence suggests that less than 50 percent of the expected profits from a contrarian investment rule may be attributed to overreaction; the majority of such profits is due to the cross-effects among the securities. We have also shown that these cross-effects have a very specific pattern for size-sorted portfolios: they display a lead–lag relation, with the returns of larger stocks generally leading those of smaller ones.

The tantalizing question remains: What are the economic sources of positive cross-autocorrelations across securities? One possibility that is consistent with lead–lag behavior is that different sectors of the economy have different sensitivities to macroeconomic shocks, sensitivities that may be determined by factors such as the degree of vertical and horizontal integration, concentration, market share, etc. Why this should manifest itself in size-sorted portfolios is still a mystery and remains to be investigated.

Appendix 1 – Derivation of (3.4)

$$\pi_t(k) \;=\; \sum_{i=1}^{N} \omega_{it}(k) R_{it} \;=\; -\frac{1}{N}\sum_{i=1}^{N}(R_{it-k} - R_{mt-k})R_{it} \qquad (A1.1)$$

$$= \; -\frac{1}{N}\sum_{i=1}^{N} R_{it-k}R_{it} \; + \; \frac{1}{N}\sum_{i=1}^{N} R_{mt-k}R_{it} \qquad (A1.2)$$

$$= \; -\frac{1}{N}\sum_{i=1}^{N} R_{it-k}R_{it} \; + \; R_{mt-k}R_{mt} \qquad (A1.3)$$

$$E[\pi_t(k)] \;=\; -\frac{1}{N}\sum_{i=1}^{N} E[R_{it-k}R_{it}] \; + \; E[R_{mt-k}R_{mt}] \qquad (A1.4)$$

$$= \; -\frac{1}{N}\sum_{i=1}^{N}\left\{\mathrm{Cov}[R_{it-k}, R_{it}] + \mu_i^2\right\} \; + \; \left\{\mathrm{Cov}[R_{mt-k}, R_{mt}] + \mu_m^2\right\} \qquad (A1.5)$$

$$= \; -\frac{1}{N}\,\mathrm{tr}(\Gamma_k) \; - \; \frac{1}{N}\sum_{i=1}^{N}\mu_i^2 \; + \; \frac{\iota'\Gamma_k\iota}{N^2} \; + \; \mu_m^2 \qquad (A1.6)$$

$$E[\pi_t(k)] \;=\; \frac{\iota'\Gamma_k\iota}{N^2} \; - \; \frac{1}{N}\,\mathrm{tr}(\Gamma_k) \; - \; \frac{1}{N}\sum_{i=1}^{N}(\mu_i^2 - \mu_m)^2 \, . \qquad (A1.7)$$

Appendix 2 – Sampling Theory for $\hat{C}_k$, $\hat{O}_k$, and $\hat{E}[\pi_t(k)]$

To derive the sampling theory for the estimators $\hat{C}_k$, $\hat{O}_k$, and $\hat{E}[\pi_t(k)]$, we re-express them as averages of artificial time series and then apply standard asymptotic theory to those averages. We require the following assumptions:

($A2$) For all t, i, j, and k the following condition is satisfied for finite constants $K > 0$, $\delta > 0$, and $r \geq 0$:

$$E[|R_{it-k}R_{jt}|^{4(r+\delta)}] \;<\; K \;<\; \infty\,. \tag{A2.1}$$

($A3$) The vector of returns R_t is either α-mixing with coefficients of size $2r/(r-1)$ or ϕ-mixing with coefficients of size $2r/(2r-1)$.

These assumptions specify the trade-off between dependence and heterogeneity in R_t that is admissible while still permitting some form of the central limit theorem to obtain. The weaker is the moment condition ($A2$), the quicker the dependence in R_t must decay, and vice-versa.[28] Observe that the covariance-stationarity of R_t is *not* required. Denote by C_{kt} and O_{kt} the following two time series:

$$C_{kt} \;\equiv\; R_{mt-k}R_{mt} \;-\; \hat{\mu}_m^2 \;-\; \frac{1}{N^2}\sum_{i=1}^{N}\left(R_{it-k}R_{it} - \hat{\mu}_i^2\right) \tag{A2.2}$$

$$O_{kt} \;\equiv\; -\,\frac{N-1}{N^2}\sum_{i=1}^{N}\left(R_{it-k}R_{it} - \hat{\mu}_i^2\right) \tag{A2.3}$$

where $\hat{\mu}_i$ and $\hat{\mu}_m$ are the usual sample means of the returns to security i and the equally-weighted market index respectively. Then the estimators $\hat{C}_k$, $\hat{O}_k$, and $\sigma^2(\hat{\mu})$ are given by:

[28] See Phillips (1987) and White (1984) for further discussion of this trade-off.

$$\hat{C}_k = \frac{1}{T-k}\sum_{t=k+1}^{T} C_{kt} \tag{A2.4}$$

$$\hat{O}_k = \frac{1}{T-k}\sum_{t=k+1}^{T} O_{kt} \tag{A2.5}$$

$$\sigma^2(\hat{\mu}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\mu}_i - \hat{\mu}_m)^2 \ . \tag{A2.6}$$

Because we have not assumed covariance-stationarity, the population quantities C_k and O_k obviously need not be interpretable according to (3.8) since the autocovariance matrix of R_t may now be time-dependent. However, we do wish to interpret C_k and O_k as some fixed quantities which are time-independent, thus we require the following:

($A4$) The following limits exist and are finite:

$$\lim_{T\to\infty}\frac{1}{T-k}\sum_{t=k+1}^{T} E[C_{kt}] = C_k \tag{A2.7}$$

$$\lim_{T\to\infty}\frac{1}{T-k}\sum_{t=k+1}^{T} E[O_{kt}] = O_k \ . \tag{A2.8}$$

Although the expectations $E[C_{kt}]$ and $E[O_{kt}]$ may be time-dependent, assumption ($A4$) asserts that their averages converge to well-defined limits, hence the quantities C_k and O_k may be viewed as "average" cross- and own-autocovariance contributions to expected profits. Consistent estimators of the asymptotic variance of the estimators $\hat{C}_k$ and $\hat{O}_k$ may then be obtained along the lines of Newey and West (1987), and are given by $\hat{\sigma}_c^2$ and $\hat{\sigma}_o^2$ respectively, where:

$$\hat{\sigma}_c^2 = \frac{1}{T-k}\left\{ \hat{\gamma}_{c_k}(0) + 2\sum_{j=1}^{q}\alpha_j(q)\hat{\gamma}_{c_k}(j) \right\} \tag{A2.9}$$

$$\hat{\sigma}_o^2 = \frac{1}{T-k}\left\{ \hat{\gamma}_{o_k}(0) + 2\sum_{j=1}^{q} \alpha_j(q)\hat{\gamma}_{o_k}(j) \right\} \tag{A2.10}$$

$$\alpha_j(q) \equiv 1 - \frac{j}{q+1}, \quad q < T$$

and $\hat{\gamma}_{c_k}(j)$ and $\hat{\gamma}_{o_k}(j)$ are the sample j-th order autocovariances of the time series C_{kt} and O_{kt} respectively, i.e.:

$$\hat{\gamma}_{c_k}(j) = \frac{1}{T-k}\sum_{t=k+j+1}^{T} (C_{kt-j} - \hat{C}_k)(C_{kt} - \hat{C}_k) \tag{A2.11}$$

$$\hat{\gamma}_{o_k}(j) = \frac{1}{T-k}\sum_{t=k+j+1}^{T} (O_{kt-j} - \hat{O}_k)(O_{kt} - \hat{O}_k) \, . \tag{A2.12}$$

Assuming that $q \sim o(T^{1/4})$, Newey and West (1987) show the consistency of $\hat{\sigma}_c^2$ and $\hat{\sigma}_o^2$ under our assumptions $(A2)$–$(A4)$.[29] Observe that these asymptotic variance estimators are robust to general forms of heteroscedasticity and autocorrelation in the C_{kt} and O_{kt} time series. Since the derivation of heteroscedasticity- and autocorrelation-consistent standard errors for the estimated expected profits $\hat{E}[\pi_t(k)]$ is virtually identical, we leave this to the reader.

[29] In our empirical work we choose $q = 8$.

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Autocorrelation coefficients for the weekly equal-weighted (Panel A) and value-weighted (Panel B) CRSP NYSE-AMEX stock returns indexes, for the sample period 6 July 1962 to 31 December 1987 and sub-periods. Means and standard deviations are also reported (in percentages), and heteroscedasticity-consistent standard errors for autocorrelation coefficients are given in parentheses.

Time Period	Sample Size	Mean x 100	Std. Dev. x 100	$\hat{\rho}_1$ (SE)	$\hat{\rho}_2$ (SE)	$\hat{\rho}_3$ (SE)	$\hat{\rho}_4$ (SE)
Panel A:							
620706 – 871231	1330	0.359	2.277	0.296 (0.046)	0.116 (0.037)	0.081 (0.034)	0.045 (0.035)
620706 – 750403	665	0.264	2.326	0.338 (0.053)	0.157 (0.048)	0.082 (0.052)	0.044 (0.053)
750404 – 871231	665	0.455	2.225	0.248 (0.076)	0.071 (0.058)	0.078 (0.042)	0.040 (0.045)
Panel B:							
620706 – 871231	1330	0.210	2.058	0.074 (0.040)	0.007 (0.037)	0.021 (0.036)	-0.005 (0.037)
620706 – 750403	665	0.135	1.972	0.055 (0.058)	0.020 (0.055)	0.058 (0.060)	-0.021 (0.058)
750404 – 871231	665	0.285	2.139	0.091 (0.055)	-0.003 (0.049)	-0.014 (0.042)	0.007 (0.046)

Table 1b

Autocorrelation coefficients for the daily equal-weighted (Panel A) and value-weighted (Panel B) CRSP NYSE–AMEX stock returns indexes, for the sample period 3 July 1962 to 31 December 1987 and sub-periods. Means and standard deviations are also reported (in percentages), and heteroscedasticity-consistent standard errors for autocorrelation coefficients are given in parentheses.

Time Period	Sample Size	Mean x 100	Std. Dev. x 100	$\hat{\rho}_1$ (SE)	$\hat{\rho}_2$ (SE)	$\hat{\rho}_3$ (SE)	$\hat{\rho}_4$ (SE)
Panel A:							
620703 – 871231	6408	0.073	0.811	0.355 (0.038)	0.088 (0.041)	0.088 (0.033)	0.104 (0.027)
620703 – 750428	3204	0.054	0.803	0.425 (0.033)	0.127 (0.032)	0.144 (0.029)	0.137 (0.029)
750429 – 871231	3204	0.091	0.818	0.287 (0.067)	0.050 (0.073)	0.034 (0.059)	0.073 (0.044)
Panel B:							
620703 – 871231	6408	0.043	0.842	0.200 (0.028)	-0.009 (0.040)	0.006 (0.031)	-0.001 (0.019)
620703 – 750428	3204	0.029	0.753	0.282 (0.028)	0.016 (0.029)	0.041 (0.027)	0.028 (0.027)
750429 – 871231	3204	0.057	0.922	0.146 (0.042)	-0.026 (0.064)	-0.017 (0.049)	0.021 (0.026)

Autocorrelation coefficients for the monthly equal-weighted (Panel A) and value-weighted (Panel B) CRSP NYSE–AMEX stock returns indexes, for the sample period 31 August 1962 to 31 December 1987 and sub-periods. Means and standard deviations are also reported (in percentages), and heteroscedasticity-consistent standard errors for autocorrelation coefficients are given in parentheses.

Time Period	Sample Size	Mean x 100	Std. Dev. x 100	$\hat{\rho}_1$ (SE)	$\hat{\rho}_2$ (SE)	$\hat{\rho}_3$ (SE)	$\hat{\rho}_4$ (SE)
Panel A:							
620831 – 871231	305	1.293	6.193	0.148 (0.060)	–0.034 (0.061)	–0.011 (0.054)	0.003 (0.060)
620831 – 750430	153	0.889	6.560	0.145 (0.084)	–0.009 (0.093)	0.054 (0.080)	0.004 (0.089)
750530 – 871231	152	1.700	5.794	0.141 (0.079)	–0.087 (0.069)	–0.121 (0.066)	–0.046 (0.064)
Panel B:							
620831 – 871231	305	0.964	4.552	0.042 (0.065)	–0.051 (0.063)	0.020 (0.065)	0.014 (0.058)
620831 – 750430	153	0.678	4.363	0.055 (0.103)	–0.024 (0.097)	0.099 (0.116)	0.063 (0.091)
750530 – 871231	152	1.252	4.732	0.019 (0.082)	–0.089 (0.078)	–0.060 (0.068)	–0.058 (0.070)

Table 2a

Averages of autocorrelation coefficients for weekly returns on individual securities, for the period 6 July 1962 to 31 December 1987. The statistic $\overline{\hat{\rho}_j}$ is the average of j-th order autocorrelation coefficients of returns on individual stocks that have at least 52 non-missing returns. The population standard deviation (SD) is given in parentheses. Since the autocorrelation coefficients are not cross-sectionally independent, the reported standard deviations cannot be used to draw the usual inferences; they are presented merely as a measure of cross-sectional variation in the autocorrelation coefficients.

Sample	Number of Securities	$\overline{\hat{\rho}_1}$ (SD)	$\overline{\hat{\rho}_2}$ (SD)	$\overline{\hat{\rho}_3}$ (SD)	$\overline{\hat{\rho}_4}$ (SD)
All Stocks	4786	-0.034 (0.084)	-0.015 (0.065)	-0.003 (0.062)	-0.003 (0.061)
Quintile 1	957	-0.079 (0.095)	-0.017 (0.077)	-0.007 (0.068)	-0.004 (0.071)
Quintile 3	958	-0.027 (0.082)	-0.015 (0.068)	-0.003 (0.067)	-0.000 (0.065)
Quintile 5	957	-0.013 (0.054)	-0.014 (0.050)	-0.002 (0.050)	-0.005 (0.047)

Table 2b

Averages of autocorrelation coefficients for daily returns on individual securities, for the period 3 July 1962 to 31 December 1987. The statistic $\overline{\hat{\rho}_j}$ is the average of j-th order autocorrelation coefficients of returns on individual stocks that have at least 52 non-missing weekly returns. The population standard deviation (SD) is given in parentheses. Since the autocorrelation coefficients are not cross-sectionally independent, the reported standard deviations cannot be used to draw the usual inferences; they are presented merely as a measure of cross-sectional variation in the autocorrelation coefficients.

Sample	Number of Securities	$\overline{\hat{\rho}_1}$ (SD)	$\overline{\hat{\rho}_2}$ (SD)	$\overline{\hat{\rho}_3}$ (SD)	$\overline{\hat{\rho}_4}$ (SD)
All Stocks	4786	-0.014 (0.101)	-0.016 (0.041)	-0.015 (0.036)	-0.006 (0.034)
Quintile 1	957	-0.093 (0.106)	-0.020 (0.043)	-0.017 (0.040)	-0.008 (0.041)
Quintile 3	958	-0.008 (0.094)	-0.015 (0.042)	-0.017 (0.038)	-0.005 (0.035)
Quintile 5	957	0.048 (0.065)	-0.015 (0.033)	-0.017 (0.029)	-0.008 (0.028)

Table 2c

Averages of autocorrelation coefficients for monthly returns on individual securities, for the period 31 August 1962 to 31 December 1987. The statistic $\overline{\hat{\rho}_j}$ is the average of j-th order autocorrelation coefficients of returns on individual stocks that have at least 24 non-missing monthly returns. The population standard deviation (SD) is given in parentheses. Since the autocorrelation coefficients are not cross-sectionally independent, the reported standard deviations cannot be used to draw the usual inferences; they are presented merely as a measure of cross-sectional variation in the autocorrelation coefficients.

Sample	Number of Securities	$\overline{\hat{\rho}_1}$ (SD)	$\overline{\hat{\rho}_2}$ (SD)	$\overline{\hat{\rho}_3}$ (SD)	$\overline{\hat{\rho}_4}$ (SD)
All Stocks	4472	-0.029 (0.111)	-0.017 (0.100)	-0.002 (0.098)	-0.001 (0.094)
Quintile 1	894	-0.055 (0.131)	-0.011 (0.115)	-0.007 (0.113)	0.005 (0.107)
Quintile 3	894	-0.019 (0.112)	-0.011 (0.101)	0.003 (0.097)	0.002 (0.087)
Quintile 5	894	-0.016 (0.084)	-0.028 (0.079)	-0.005 (0.079)	-0.004 (0.077)

Table 3a

Analysis of the profitability of the return-reversal strategy applied to weekly returns, for the sample of 551 CRSP NYSE-AMEX stocks with non-missing weekly returns from 6 July 1962 to 31 December 1987 (1330 weeks). Expected profits is given by $E[\pi_t(k)] = C_k + O_k - \sigma^2(\mu)$, where C_k depends only on cross-autocovariances and O_k depends only on own-autocovariances. All z-statistics are asymptotically N(0,1) under the null hypothesis that the relevant population value is zero, and are robust to heteroscedasticity and autocorrelation. The average long position $\bar{I}_t(k)$ is also reported, with its sample standard deviation in parentheses underneath. The analysis is conducted for all stocks as well as for the five size-sorted quintiles; to conserve space, results for the second and fourth quintiles have been omitted.

Portfolio	Lag k	$\hat{C}_k$[a] (z-stat)	$\hat{O}_k$[a] (z-stat)	$\sigma^2(\hat{\mu})$[a]	$\hat{E}[\pi_t(k)]$[a] (z-stat)	$\bar{I}_t(k)$[a] (SD[a])	%-$\hat{C}_k$	%-$\hat{O}_k$	%-$\sigma^2(\hat{\mu})$
All Stocks	1	0.841	0.862	0.009	1.694	151.9	49.6	50.9	-0.5
		(4.95)	(4.54)		(20.81)	(31.0)			
Quintile 1	1	2.048	2.493	0.009	4.532	208.8	45.2	55.0	-0.2
		(6.36)	(7.12)		(18.81)	(47.3)			
Quintile 3	1	0.703	0.366	0.011	1.058	138.4	66.5	34.6	-1.0
		(4.67)	(2.03)		(13.84)	(32.2)			
Quintile 5	1	0.188	0.433	0.005	0.617	117.0	30.5	70.3	-0.8
		(1.18)	(2.61)		(11.22)	(28.1)			
All Stocks	2	0.253	0.298	0.009	0.542	151.8	46.7	54.9	-1.6
		(1.64)	(1.67)		(10.63)	(31.0)			
Quintile 1	2	0.803	0.421	0.009	1.216	208.8	66.1	34.7	-0.7
		(3.29)	(1.49)		(8.86)	(47.3)			
Quintile 3	2	0.184	0.308	0.011	0.481	138.3	38.3	64.0	-2.3
		(1.20)	(1.64)		(7.70)	(32.2)			
Quintile 5	2	−0.053	0.366	0.005	0.308	116.9	-17.3	118.9	-1.6
		(−0.39)	(2.28)		(5.89)	(28.1)			
All Stocks	3	0.223	−0.066	0.009	0.149	151.7	149.9	-44.0	-5.9
		(1.60)	(−0.39)		(3.01)	(30.9)			
Quintile 1	3	0.552	0.038	0.009	0.582	208.7	94.9	6.6	-1.5
		(2.73)	(0.14)		(3.96)	(47.3)			
Quintile 3	3	0.237	−0.192	0.011	0.035	138.2	677.6	-546.7	-30.9
		(1.66)	(−1.07)		(0.50)	(32.1)			
Quintile 5	3	0.064	−0.003	0.005	0.056	116.9	114.0	-5.3	-8.8
		(0.39)	(−0.02)		(1.23)	(28.1)			
All Stocks	4	0.056	0.083	0.009	0.130	151.7	43.3	63.5	-6.7
		(0.43)	(0.51)		(2.40)	(30.9)			
Quintile 1	4	0.305	0.159	0.009	0.455	208.7	67.0	34.9	-1.9
		(1.53)	(0.59)		(3.27)	(47.3)			
Quintile 3	4	0.023	−0.045	0.011	−0.033	138.2	–[b]	–[b]	–[b]
		(0.18)	(−0.26)		(−0.44)	(32.0)			
Quintile 5	4	−0.097	0.128	0.005	0.026	116.8	-374.6	493.4	-18.8
		(−0.65)	(0.77)		(0.52)	(28.0)			

[a] Multiplied by 10,000.
[b] Not computed when expected profits are negative.

Table 3b

Analysis of the profitability of the return-reversal strategy applied to weekly returns, for the sample of 949 CRSP NYSE-AMEX stocks with non-missing weekly returns during the period 6 July 1962 to 3 April 1975 (665 weeks). Expected profits is given by $E[\pi_t(k)] = C_k + O_k - \sigma^2(\mu)$, where C_k depends only on cross-autocovariances and O_k depends only on own-autocovariances. All z-statistics are asymptotically N(0,1) under the null hypothesis that the relevant population value is zero, and are robust to heteroscedasticity and autocorrelation. The average long position $\bar{I}_t(k)$ is also reported, with its sample standard deviation in parentheses underneath. The analysis is conducted for all stocks as well as for the five size-sorted quintiles; to conserve space, results for the second and fourth quintiles have been omitted.

Portfolio	Lag k	$\hat{C}_k$[a] (z-stat)	$\hat{O}_k$[a] (z-stat)	$\sigma^2(\hat{\mu})$[a]	$\hat{E}[\pi_t(k)]$[a] (z-stat)	$\bar{I}_t(k)$[a] (SD[a])	%-$\hat{C}_k$	%-$\hat{O}_k$	%-$\sigma^2(\hat{\mu})$
All Stocks	1	1.194 (5.34)	1.191 (4.61)	0.019	2.366 (15.36)	164.0 (35.3)	50.5	50.3	-0.8
Quintile 1	1	2.409 (6.73)	3.533 (8.84)	0.020	5.923 (16.63)	221.8 (49.0)	40.7	59.7	-0.3
Quintile 3	1	1.196 (5.10)	0.445 (1.50)	0.020	1.621 (10.85)	154.4 (35.4)	73.8	27.5	-1.3
Quintile 5	1	0.302 (1.53)	0.380 (1.64)	0.015	0.668 (9.82)	119.8 (28.5)	45.2	57.0	-2.2
All Stocks	2	0.566 (2.80)	0.192 (0.86)	0.019	0.739 (9.16)	164.0 (35.3)	76.6	26.0	-2.6
Quintile 1	2	1.128 (3.83)	0.305 (0.86)	0.020	1.413 (7.71)	221.8 (49.1)	79.8	21.6	-1.4
Quintile 3	2	0.539 (2.59)	0.164 (0.73)	0.020	0.682 (6.46)	154.3 (35.4)	78.9	24.0	-3.0
Quintile 5	2	0.149 (1.09)	0.248 (1.47)	0.015	0.382 (5.52)	119.8 (28.6)	38.9	65.0	-3.9
All Stocks	3	0.314 (1.42)	–0.062 (–0.24)	0.019	0.232 (3.02)	163.9 (35.3)	135.2	-26.9	-8.3
Quintile 1	3	0.583 (2.06)	0.156 (0.45)	0.020	0.719 (4.06)	221.7 (49.1)	81.1	21.7	-2.7
Quintile 3	3	0.385 (1.61)	–0.174 (–0.61)	0.020	0.190 (1.83)	154.2 (35.4)	202.1	-91.4	-10.7
Quintile 5	3	0.227 (1.05)	–0.044 (–0.17)	0.015	0.168 (2.73)	119.7 (28.6)	134.7	-26.0	-8.8
All Stocks	4	0.149 (0.78)	0.030 (0.13)	0.019	0.159 (2.18)	163.8 (35.2)	93.2	19.0	-12.1
Quintile 1	4	0.347 (1.17)	0.103 (0.27)	0.020	0.430 (2.64)	221.6 (49.1)	80.8	23.8	-4.6
Quintile 3	4	0.169 (0.84)	–0.152 (–0.62)	0.020	–0.004 (–0.04)	154.0 (35.1)	–[b]	–[b]	–[b]
Quintile 5	4	–0.025 (–0.14)	0.075 (0.38)	0.015	0.035 (0.69)	119.6 (28.6)	-71.4	213.2	-41.8

[a] Multiplied by 10,000.
[b] Not computed when expected profits are negative.

Table 3c

Analysis of the profitability of the return-reversal strategy applied to weekly returns, for the sample of 1172 CRSP NYSE-AMEX stocks with non-missing weekly returns during the period 4 April 1975 to 31 December 1987 (665 weeks). Expected profits is given by $E[\pi_t(k)] = C_k + O_k - \sigma^2(\mu)$, where C_k depends only on cross-autocovariances and O_k depends only on own-autocovariances. All z-statistics are asymptotically N(0,1) under the null hypothesis that the relevant population value is zero, and are robust to heteroscedasticity and autocorrelation. The average long position $\bar{I}_t(k)$ is also reported, with its sample standard deviation in parentheses underneath. The analysis is conducted for all stocks as well as for the five size-sorted quintiles; to conserve space, results for the second and fourth quintiles have been omitted.

Portfolio	Lag k	$\hat{C}_k$[a] (z-stat)	$\hat{O}_k$[a] (z-stat)	$\sigma^2(\hat{\mu})$[a]	$\hat{E}[\pi_t(k)]$[a] (z-stat)	$\bar{I}_t(k)$[a] (SD[a])	%-$\hat{C}_k$	%-$\hat{O}_k$	%-$\sigma^2(\hat{\mu})$
All Stocks	1	1.022 (3.16)	1.105 (3.10)	0.026	2.101 (17.05)	167.7 (31.5)	48.7	52.6	-1.2
Quintile 1	1	1.897 (3.83)	2.873 (5.18)	0.031	4.739 (15.25)	217.8 (47.2)	40.0	60.6	-0.6
Quintile 3	1	1.038 (3.10)	0.689 (1.88)	0.031	1.696 (12.43)	161.4 (31.7)	61.2	40.6	-1.8
Quintile 5	1	0.465 (1.77)	0.368 (1.47)	0.014	0.819 (9.58)	128.9 (28.8)	56.7	44.9	-1.7
All Stocks	2	0.211 (0.75)	0.438 (1.36)	0.026	0.622 (8.38)	167.5 (31.4)	33.8	70.4	-4.2
Quintile 1	2	0.788 (1.92)	0.665 (1.57)	0.031	1.422 (7.72)	217.7 (47.2)	55.4	46.7	-2.2
Quintile 3	2	0.220 (0.79)	0.237 (0.68)	0.031	0.426 (3.65)	161.2 (31.5)	51.7	55.6	-7.4
Quintile 5	2	-0.083 (-0.33)	0.317 (1.09)	0.014	0.220 (3.04)	128.9 (28.8)	-37.8	144.0	-6.2
All Stocks	3	0.277 (1.30)	-0.025 (-0.10)	0.026	0.225 (3.22)	167.4 (31.2)	122.9	-11.3	-11.6
Quintile 1	3	0.816 (3.05)	-0.178 (-0.54)	0.031	0.608 (3.69)	217.5 (47.1)	134.3	-29.3	-5.1
Quintile 3	3	0.277 (1.19)	0.109 (0.39)	0.031	0.354 (3.83)	161.0 (31.2)	78.2	30.7	-8.8
Quintile 5	3	-0.028 (-0.13)	0.042 (0.18)	0.014	0.000 (0.01)	128.8 (28.7)	-7949.7	11873.6	-3823.9
All Stocks	4	0.098 (0.45)	0.088 (0.34)	0.026	0.160 (2.09)	167.3 (31.1)	61.2	55.2	-16.4
Quintile 1	4	0.407 (1.50)	0.063 (0.19)	0.031	0.439 (2.57)	217.5 (47.2)	92.8	14.2	-7.0
Quintile 3	4	0.124 (0.55)	-0.054 (-0.20)	0.031	0.038 (0.38)	160.9 (31.0)	324.3	-142.2	-82.0
Quintile 5	4	-0.034 (-0.15)	0.104 (0.42)	0.014	0.056 (0.84)	128.7 (28.7)	-61.1	185.3	-24.1

[a] Multiplied by 10,000.

Table 4

Autocorrelation matrices of the vector $Z_t \equiv [R_{1t}\ R_{2t}\ R_{3t}\ R_{4t}\ R_{5t}\ R_{mt}]'$ where R_{it} is the return on the portfolio of stocks in the i-th quintile, $i = 1, \ldots, 5$ and R_{mt} is the return on the equal-weighted index, for the sample of 551 stocks with non-missing weekly returns from 6 July 1962 to 31 December 1987 (1330 observations). Note $\Upsilon_k \equiv D^{-1/2}E[(Z_{t-k} - \mu)(Z_t - \mu)']D^{-1/2}$, where $D \equiv \text{diag}[\sigma_1^2, \ldots, \sigma_5^2, \sigma_m^2]$. Asymptotic standard errors for the autocorrelations under an i.i.d. null hypothesis are given by $\frac{1}{\sqrt{T}} = 0.027$.

$\hat{\Upsilon}_0$ =

	R_1	R_2	R_3	R_4	R_5	R_m
R_1	1.000	0.919	0.857	0.830	0.747	0.918
R_2	0.919	1.000	0.943	0.929	0.865	0.976
R_3	0.857	0.943	1.000	0.964	0.925	0.979
R_4	0.830	0.929	0.964	1.000	0.946	0.974
R_5	0.747	0.865	0.925	0.946	1.000	0.933
R_m	0.918	0.976	0.979	0.974	0.933	1.000

$\hat{\Upsilon}_1$ =

	R_1	R_2	R_3	R_4	R_5	R_m
R_1	0.333	0.244	0.143	0.101	0.020	0.184
R_2	0.334	0.252	0.157	0.122	0.033	0.195
R_3	0.325	0.265	0.175	0.140	0.051	0.207
R_4	0.316	0.262	0.177	0.139	0.050	0.204
R_5	0.276	0.230	0.154	0.122	0.044	0.178
R_m	0.333	0.262	0.168	0.130	0.041	0.202

$\hat{\Upsilon}_2$ =

	R_1	R_2	R_3	R_4	R_5	R_m
R_1	0.130	0.087	0.044	0.022	0.005	0.064
R_2	0.133	0.101	0.058	0.039	0.017	0.076
R_3	0.114	0.088	0.046	0.027	0.002	0.061
R_4	0.101	0.085	0.048	0.029	0.008	0.059
R_5	0.067	0.055	0.020	0.008	−0.012	0.031
R_m	0.115	0.087	0.045	0.026	0.004	0.061

$\hat{\Upsilon}_3$ =

	R_1	R_2	R_3	R_4	R_5	R_m
R_1	0.089	0.047	0.015	0.013	−0.005	0.036
R_2	0.094	0.066	0.038	0.041	0.018	0.056
R_3	0.096	0.079	0.059	0.061	0.041	0.072
R_4	0.084	0.067	0.047	0.049	0.031	0.059
R_5	0.053	0.044	0.031	0.034	0.015	0.038
R_m	0.087	0.063	0.038	0.040	0.020	0.054

$\hat{\Upsilon}_4$ =

	R_1	R_2	R_3	R_4	R_5	R_m
R_1	0.050	0.001	−0.014	−0.029	−0.030	−0.002
R_2	0.064	0.023	−0.002	−0.012	−0.020	0.014
R_3	0.065	0.029	0.006	−0.002	−0.017	0.019
R_4	0.072	0.042	0.017	0.005	−0.008	0.029
R_5	0.048	0.023	0.002	−0.007	−0.022	0.011
R_m	0.062	0.024	0.001	−0.010	−0.021	0.014