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ABSTRACT

The paper generalizes the Taylor principle---the proposition that central banks can stabilize the macroeconomy by raising their interest rate instrument more than one-for-one in response to higher inflation---to an environment in which reaction coefficients in the monetary policy rule evolve according to a Markov process. We derive a long-run Taylor principle that delivers unique bounded equilibria in two standard models. Policy can satisfy the Taylor principle in the long run, even while deviating from it substantially for brief periods or modestly for prolonged periods. Macroeconomic volatility can be higher in periods when the Taylor principle is not satisfied, not because of indeterminacy, but because monetary policy amplifies the impacts of fundamental shocks. Regime change alters the qualitative and quantitative predictions of a conventional new Keynesian model, yielding fresh interpretations of existing empirical work.

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1. INTRODUCTION

Monetary policy making is complex. Central bankers examine a vast array of data, hear from a variety of advisors, use suites of models to interpret the data, and apply judgment to adjust the predictions of models. This process produces a monetary policy rule that is a complicated, probably non-linear, function of a large set of information about the state of the economy.

For both descriptive and prescriptive reasons, macroeconomists seek simple characterizations of policy. Perhaps the most successful simplification is due to Taylor (1993). He finds that a very simple rule does a good job of describing Federal Reserve interest-rate decisions, particularly since 1982. Taylor's rule is

$$i_t = \bar{i} + \alpha(\pi_t - \pi^*) + \gamma x_t + \varepsilon_t, \tag{1}$$

where i_t is the central bank's policy interest rate, $\bar{i}$ is the long-run policy rate, π_t is inflation, π^* is the central bank's inflation target, x is output, and ε is a random variable. With settings of $\alpha = 1.5$ and $\gamma = .5$ or 1, Taylor (1999a) uses this equation to interpret Federal Reserve behavior over several eras since 1960.

The Taylor principle—the proposition that central banks can stabilize the macroeconomy by adjusting their interest rate instrument more than one-for-one with inflation (setting $\alpha > 1$)—and the Taylor rule that embodies it have proven to be powerful devices to simplify the modeling of policy behavior. In many monetary models, the Taylor principle is necessary and sufficient for the existence of a determinate rational

expectations equilibrium. Failure of monetary policy to satisfy the principle can produce undesirable outcomes in two ways. First, the effects of fundamental shocks are amplified and can cause fluctuations in output and inflation that are arbitrarily large. Second, there exist a multiplicity of bounded equilibria in which output and inflation respond to non-fundamental—sunspot—disturbances. If the objective of a central bank is to stabilize output and inflation, these outcomes are clearly undesirable. Taylor (1999a) and Clarida, Gali, and Gertler (2000), among others, have argued that failure of Federal Reserve policy to satisfy the Taylor principle may have been the source of greater macroeconomic instability in the United States in the 1960s and 1970s.

Taylor-inspired rules have been found to perform well in a class of models that is now in heavy use in policy research [Bryant, Hooper, and Mann (1993), Rotemberg and Woodford (1997), Taylor (1999b), Faust, Orphanides, and Reifschneider (2005), Schmitt-Grohe and Uribe (2006)]. Some policy institutions publish the policy interest rate paths produced by simple rules, treating the implied policy prescriptions as useful benchmarks for policy evaluation [Bank for International Settlements (1998), Sveriges Riksbank (2001, 2002), Norges Bank (2005), Federal Reserve Bank of St. Louis (2005)]. In large part because it is a gross simplification of reality, the Taylor rule has been extraordinarily useful.

Gross simplification is both a strength and a weakness of a constant-parameter rule like (1). Because the rule compresses and reduces information about actual policy behavior, it can mask important aspects of that behavior. There are clearly states of the economy in which policy settings of the nominal interest rate deviate from the rule in substantial and serially correlated ways. This confronts researchers with a substantive modeling choice: it matters whether these deviations are shuffled into the ε's or modeled as time-varying feedback coefficients, α_t and γ_t. Positing that policy rules mapping endogenous variables into policy choices evolve according to some probability distribution can fundamentally change dynamics, including conditions that ensure a unique equilibrium, and substantially expand the set of determinate rational expectations equilibria supported by conventional monetary models.

This paper generalizes Taylor's rule and principle by allowing the parameters of that rule to vary stochastically over time.[1] It examines how such time variation affects

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[1] In contrast to our approach, some papers consider changes in processes governing exogenous policy variables [Dotsey (1990), Kaminsky (1993), Ruge-Murcia (1995), Andolfatto and Gomme (2003), Davig (2003, 2004), and Leeper and Zha (2003)]. Each of these considers changes in exogenous processes for policy instruments like a tax rate, money growth rate, or government expenditures. Other papers model policy switching as changes in endogenous policy functions [Davig, Leeper, and Chung (2006) and Davig and Leeper (2006b)].

the nature of equilibrium in popular models of monetary policy. As a first step, in this paper we model parameters as evolving exogenously according to a Markov chain.[2]

Two parts comprise the paper. We use a simple dynamic Fisherian model in the first part to derive interpretable analytical restrictions on monetary policy behavior that are required for the existence of a determinate equilibrium; that model yields intuitive solutions that reveal how regime change alters the nature of equilibrium. In the second, more substantive part, we use a conventional new Keynesian model to examine the practical consequences of regime change for monetary policy.

The Fisherian model of inflation illustrates the following theoretical points:

- A unique bounded equilibrium does not require the Taylor principle to hold in every period. Determinacy does require that monetary policy obey a *long-run Taylor principle*, which permits departures from the Taylor principle that are substantial (but brief) or modest (and prolonged).
- If there are two possible policy rules—one that aggressively reacts to inflation ("more active") and one that reacts less aggressively ("less active" or "passive")—expectations that future policy might be less active can strongly affect the equilibrium under the more-active rule, and vice versa.

These theoretical themes extend to a conventional model of inflation and output determination which has become a workhorse for empirical and theoretical work on monetary policy. The long-run Taylor principle for the new Keynesian model dramatically expands the region of determinacy relative to the constant-parameter setup. On-going regime change creates expectations formation effects that arise from the possibility that future regimes may differ from the prevailing regime. Those effects can change the responses of inflation and output to exogenous disturbances in quantitatively important ways. Regimes that fail to satisfy the Taylor principle can amplify the effects of fundamental exogenous shocks, which increases volatility without resorting to indeterminacy and non-fundamental sources of disturbances.

Having established these theoretical results, we use the new Keynesian model to show that regime change can be important in practice. The illustrations are of interest because the model forms the core of the large class of models being fit to data by academic and central-bank researchers.[3] Illustrations focus on the following questions:

- A number of authors have argued that the U.S. inflation of the 1970s was due to the Federal Reserve's failure to obey the Taylor principle and the resulting indeterminacy of equilibrium. Does this inference hold up when

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[2]Davig and Leeper (2006a) examine the consequences of making regime change endogenous.

[3]For example, Rotemberg and Woodford (1997), Lubik and Schorfheide (2004), Smets and Wouters (2003, 2005), Del Negro and Schorfheide (2004, 2006), Adolfson, Laseen, Linde, and Villani (2005), Harrison, Nikolov, Quinn, Ramsey, Scott, and Thomas (2005).

agents' expectations embed the possibility of regime change? If a central bank is an aggressive inflation fighter today, can the perception that it might revert to 1970s-style accommodative policies make it difficult to stabilize the economy now?

- Over the past 20-plus years, when the Fed arguably has aggressively sought to reduce and stabilize inflation, there are apparent systematic departures from the Taylor principle due to worries about financial instability or concerns about weak real economic activity. What are the consequences of these departures?
- Researchers typically divide data into regime-specific periods to interpret time series as emerging from distinct fixed-regime models. What are the consequences of this practice?

The paper offers some answers, along with some novel interpretations of existing empirical findings. A possible switch from an active to an accommodating monetary policy regime should concern a central bank for two reasons. First, if the accommodating regime is sufficiently passive or sufficiently persistent, the equilibrium can be indeterminate. Second, even in a determinate equilibrium, expectations of a move to a dovish regime can raise aggregate volatility, even if current policy is aggressively hawkish. A realized switch to passive policy dramatically increases inflation volatility even when self-fulfilling expectations are ruled out. Brief departures from the Taylor principle, such as occur during financial crises or business-cycle downturns, are less likely to induce indeterminacy, but can nonetheless create expectations formation effects with quantitatively important impacts on economic performance. Efforts to use theoretical models with fixed policy rules to interpret time series data generated by recurring regime switching are fraught with pitfalls, easily yielding inaccurate inferences.

1.1. **The Relevance of Recurring Regime Change.** Recurring regime change is not the norm in theoretical models of monetary policy, yet a major branch of applied work finds evidence of time variation in monetary policy in the United States.[4] The theoretical norm, which follows Lucas (1976) in treating policy shifts as once-and-for-all rather than as an on-going process, is logically inconsistent, as Cooley, LeRoy, and Raymon (1982, 1984) point out. Once-and-for-all shifts, by definition, are unanticipated, yet once the shift occurs, agents are assumed to believe the new regime is permanent and alternative regimes are impossible. But if regime *has* changed, then regime *can* change; knowing this, private agents will ascribe a probability distribution to regimes. Expectations formation and, therefore, the resulting equilibria will reflect

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[4]For example, Judd and Trehan (1995), Taylor (1999a), Clarida, Gali, and Gertler (2000), Kim and Nelson (2004), Lubik and Schorfheide (2004), Rabanal (2004), Favero and Monacelli (2005), Boivin and Giannoni (2003), Boivin (2005), Cogley and Sargent (2005), Davig and Leeper (2006b), and Sims and Zha (2006).

agents' beliefs that regime change is possible. This paper is a step toward bringing theory in line with evidence.

The paper presumes that policy regimes recur and treats, as a special case, regimes that are permanent. In the United States, monetary policy regimes over the post-World War II period have been shaped largely by particular Federal Reserve chairmen, rather than by institutional or legislative changes that altered the Fed's mandate. Despite this fact, some observers believe that since Alan Greenspan's appointment as Fed chairman in 1987—and possibly even before—U.S. monetary policy has been in an absorbing state. At least this is the implicit assumption in most studies of monetary policy behavior.

We are not persuaded. Appointments of central bank governors are determined by the confluence of economic and political conditions, which fluctuate over time, rather than by any legislated rules. As long as the personalities and preferences of those appointees dictate the policies that central banks follow, fluctuating regimes is a more natural assumption than is permanent regime. Certainly, regime change is a viable working hypothesis.

2. A Fisherian Model of Inflation Determination

An especially simple model of inflation determination emerges from combining a Fisher relation with a monetary policy rule that makes the nominal interest rate respond to inflation. The setup is rich enough to highlight general features that arise in a rational expectations environment with regime change in monetary policy, but simple enough to admit analytical solutions that make transparent the mechanisms at work.

Throughout the paper we define *determinacy* of equilibrium to be the existence of a unique bounded equilibrium. We also place fiscal policy in the background, assuming that lump-sum taxes and transfers adjust passively to ensure fiscal solvency.

This section describes a two-step procedure applicable to purely forward-looking rational expectations models with regime-switching. First, we derive interpretable analytical conditions on the model parameters that ensure a determinate equilibrium. Next, we derive the equilibrium using the method of undetermined coefficients to obtain solutions as functions of the minimum set of state variables.

2.1. **The Setup.** Consider a nominal bond that costs \$1 at date t and pays off \$$(1 + i_t)$ at date $t + 1$. The asset-pricing equation for this bond can be written in log-linearized form as

$$i_t = E_t \pi_{t+1} + r_t, \tag{2}$$

where r_t is the equilibrium (ex-ante) real interest rate at t. For simplicity, the real interest rate is exogenous and evolves according to

$$r_t = \rho r_{t-1} + \upsilon_t, \tag{3}$$

with $|\rho| < 1$ and υ a zero-mean, *i.i.d.* random variable with bounded support $[\underline{\upsilon}, \overline{\upsilon}]$, so that fluctuations in r_t are bounded.

Monetary policy follows a simplified Taylor rule, adjusting the nominal interest rate in response to inflation, where the reaction to inflation evolves stochastically between regimes

$$i_t = \alpha(s_t)\pi_t, \tag{4}$$

where s_t is the realized policy regime, which takes realized values of 1 or 2. Two regimes are sufficient for our purposes, though the methods employed immediately generalize to many regimes. Regime follows a Markov chain with transition probabilities $p_{ij} = P[s_t = j \,|\, s_{t-1} = i]$, where $i, j = 1, 2$. We assume

$$\alpha(s_t) = \begin{cases} \alpha_1 \text{ for } s_t = 1 \\ \alpha_2 \text{ for } s_t = 2 \end{cases} \tag{5}$$

and that the random variables s and υ are independent.

A *monetary policy regime* is a distinct realization of the random variable s_t and a *monetary policy process* consists of all possible α_i's and the transition probabilities of the Markov chain, $(\alpha_1, \alpha_2, p_{11}, p_{22})$. In this model, monetary policy is *active* in regime i if $\alpha_i > 1$ and *passive* if $\alpha_i < 1$, following the terminology of Leeper (1991). If $\alpha_1 > \alpha_2$, then the monetary policy process becomes *more active* if α_1, α_2, or p_{11} increase or p_{22} decreases.

Substituting (4) into (2), the system reduces to the single state-dependent equation

$$\alpha(s_t)\pi_t = E_t\pi_{t+1} + r_t. \tag{6}$$

If only a single, fixed regime were possible, then $\alpha_i = \alpha$ and the expected path of policy depends on the constant α. A unique bounded equilibrium requires active policy behavior ($\alpha > 1$) and the solution to (6) would be

$$\pi_t = \frac{1}{\alpha - \rho} r_t. \tag{7}$$

Stronger responses of policy to inflation (larger values of α) reduce the variability of inflation. The Taylor principle says that $\alpha > 1$ is necessary and sufficient for a unique bounded equilibrium.

When $\alpha < 1$ and regime is fixed, the equilibrium is not unique and a large multiplicity of solutions exist, including stationary sunspot equilibria, in which π_t is a function of (π_{t-1}, r_t) and possibly a sunspot shock.

When regime can change, (6) is a system whose number of equations matches the number of possible regimes. To make this explicit, the conditional expectation in (6) is $E_t\pi_{t+1} = E[\pi_{t+1}|\Omega_t]$, with $\Omega_t = \{s_t, s_{t-1}, \ldots, r_t, r_{t-1}, \ldots\}$.[5] s_t follows a finite two-state Markov chain, so is bounded by construction, and r_t follows a bounded process. It is convenient to define a smaller information set which excludes the current regime, $\Omega_t^{-s} = \{s_{t-1}, \ldots, r_t, r_{t-1}, \ldots\}$, so $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$. Integrating over possible future regimes, we can write

$$E_t\pi_{t+1} = E[\pi_{t+1}|s_t = i, \Omega_t^{-s}] = p_{i1}E[\pi_{1t+1}|\Omega_t^{-s}] + p_{i2}E[\pi_{2t+1}|\Omega_t^{-s}], \qquad (8)$$

where we have introduced the state-contingent notation, $\pi_{it} = \pi_t(s_t = i, r_t)$, for $i = 1, 2$, so π_{it} is the solution to (6) when $s_t = i$. Define $z_t = (s_t, r_t)$ to be the minimum state vector at date t. We shall prove that the minimum state vector solution, $\pi_t(z_t) = (\pi_{1t}, \pi_{2t})'$, is the unique bounded solution to (6).

Shifting notation somewhat by letting $E_t\pi_{it+1}$ denote $E[\pi_{it+1}|\Omega_t^{-s}]$, we now can express (6) as

$$\begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} E_t\pi_{1t+1} \\ E_t\pi_{2t+1} \end{bmatrix} + \begin{bmatrix} r_t \\ r_t \end{bmatrix}. \qquad (9)$$

Define the matrix

$$M = \begin{bmatrix} \alpha_1^{-1} & 0 \\ 0 & \alpha_2^{-1} \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \qquad (10)$$

and write (6) as

$$\pi_t = ME_t\pi_{t+1} + \alpha^{-1}r_t, \qquad (11)$$

where $\pi_t = (\pi_{1t}, \pi_{2t})'$ is now a vector and α^{-1} denotes the matrix that premultiplies the transition probabilities in (10).

2.2. **The Long-Run Taylor Principle.** This section derives necessary and sufficient conditions for the existence of a unique bounded solution to (11), assuming bounded fluctuations in the exogenous disturbances. Our definition of determinacy is consistent with the standard one used in the Taylor-rule literature in the absence of regime switching.

We use the standard definition for two reasons. First, it corresponds to existence of a locally unique solution. Local uniqueness allows us to analyze how small perturbations to the model impact the equilibrium, as Woodford (2003, Appendix A.3) shows. Second, this paper follows most of the literature in studying log-linear approximations

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[5]This assumes that current regime enters the agent's information set, which contrasts with the usual econometric treatment of regime as an unobserved state variable [Hamilton (1989) or Kim and Nelson (1999)]. Some theoretical work treats agents as having to infer the current regime [Andolfatto and Gomme (2003), Leeper and Zha (2003), and Davig (2004)]. Concentrating all uncertainty about policy on future regimes makes clearer how expectations formation, as opposed to inference problems, affects the regime-switching equilibrium.

to underlying nonlinear dynamic stochastic general equilibrium models.[6] Bounded solutions to the linear systems are approximate local solutions to the full nonlinear models when the exogenous shocks are small enough.

Determinacy hinges on the eigenvalues of M. Taking $\alpha_i > 0$ for $i = 1, 2$, those eigenvalues are

$$\lambda_1 = \frac{1}{2\alpha_1\alpha_2}\left(\alpha_2 p_{11} + \alpha_1 p_{22} + \sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1\alpha_2 p_{12} p_{21}}\right), \quad (12)$$

$$\lambda_2 = \frac{1}{2\alpha_1\alpha_2}\left(\alpha_2 p_{11} + \alpha_1 p_{22} - \sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1\alpha_2 p_{12} p_{21}}\right). \quad (13)$$

This leads to one of the main propositions of the paper.

Proposition 1. *When $\alpha_i > 0$, for $i = 1, 2$, a necessary and sufficient condition for determinacy of equilibrium, defined as the existence of a unique bounded solution for $\{\pi_t\}$ in (11), is that all the eigenvalues of M lie inside the unit circle.*

The proof, which appears in appendix A, shows that when all the eigenvalues of M lie inside the unit circle, then all bounded solutions must coincide with the minimum state variable solution, which is a function only of (s_t, r_t). In the case where one of the eigenvalues does not lie inside the unit circle, the proof displays a continuum of bounded solutions, including stationary sunspot equilibria. Hence, even within the standard definition of determinacy of equilibrium, the monetary policy process can generate a large multiplicity of solutions that are a function of an expanded state vector.[7]

Although one could use proposition 1 and work directly with the eigenvalues to characterize the class of policy processes consistent with a determinate equilibrium, it is more convenient and economically intuitive to analyze an equivalent set of conditions. It turns out that requiring both eigenvalues to lie inside the unit circle is equivalent to requiring that policy be active in at least one regime—$\alpha_i > 1$ for some i—and that the policy process satisfies a *long-run Taylor principle*. This is the second proposition of the paper, which appendix A also proves.

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[6]Appendices in Davig, Leeper, and Chung (2004) display a model with regime switching in monetary and fiscal policy rules for which conventional linearization methods will fail to uncover even locally accurate stability conditions for the underlying full nonlinear model. The extent to which solutions to linear systems are approximate local solutions to the nonlinear switching models remains an area for future research.

[7]Farmer, Waggoner, and Zha (2006) employ an alternative definition of determinacy, requiring stationary—mean-square stable—solutions, to generate multiple solutions. As appendix A points out, this definition admits solutions in which inflation can exceed any finite bound with positive probability, a result that is ruled out by the standard definition in linear models. Of course, both with and without regime switching, there are many explosive solutions to (11).

Proposition 2. *Given $\alpha_i > p_{ii}$ for $i = 1, 2$, the following statements are equivalent:*

(A) *All the eigenvalues of M lie inside the unit circle.*
(B) $\alpha_i > 1$, *for some $i = 1, 2$, and the* long-run Taylor principle (LRTP)

$$(1 - \alpha_2) p_{11} + (1 - \alpha_1) p_{22} + \alpha_1 \alpha_2 > 1 \tag{14}$$

is satisfied.

The premise of proposition 2—that $\alpha_i > p_{ii}$ for all i—is unfamiliar and requires some discussion. If regime were fixed, the premise amounts to satisfying the Taylor principle in both regimes. But when regime can change, it is a much weaker requirement. The LRTP defines a hyperbola in (α_1, α_2)-space with asymptotes $\alpha_1 = p_{11}$ and $\alpha_2 = p_{22}$. The premise restricts the α's to the space containing the economically interesting portion of the hyperbola, in which monetary policy seeks to stabilize, rather than destablize, the economy.

Two eigenvalues inside the unit circle imply two linear restrictions that uniquely determine the regime-dependent expectations of inflation in (9). This is quite different from fixed regimes because with regime switching, when there is a determinate equilibrium, the solutions always come from "solving forward," even in regimes where monetary policy behavior is passive ($\alpha_i < 1$). This delivers solutions that are qualitatively different from those obtained with fixed regimes.

A range of monetary policy behavior is consistent with the LRTP: monetary policy can be mildly passive most of the time or very passive some of the time. To see this, suppose that regime 1 is active and regime 2 is passive and consider the limiting case that arises as α_1 becomes arbitrarily large. Driving $\alpha_1 \to \infty$ in the LRTP, (14), implies that $\alpha_2 > p_{22}$ is the lower bound for α_2 in a determinate equilibrium. For α_1 sufficiently large, a unique equilibrium can have α_2 arbitrarily close to 0 (a pegged nominal interest rate), so long as the regime in which this passive policy is realized is sufficiently short-lived ($p_{22} \to 0$). When regime 1 is an absorbing state ($p_{11} = 1$), the eigenvalues are α_1 and α_2/p_{22}. A unique equilibrium requires that $\alpha_1 > 1$ and $\alpha_2 > p_{22}$.[8] The general determinacy principle is that an active regime that is either very aggressive ($\alpha_1 \to \infty$) or very persistent ($p_{11} = 1$) imposes the weakest condition on behavior in the passive regime.

Alternatively, the passive regime can be extremely persistent ($p_{22} \to 1$), so long as α_2 is sufficiently close to, but still less than, 1. In this case, if the active regime has short duration, it is possible for the ergodic probability of the passive regime to be close to 1 (but less than 1), yet still deliver a determinate equilibrium.

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[8]When $p_{11} = 1$, the system is recursive, so the difference equation for inflation in state 1 is independent of state 2 and yields the usual fixed-regime solution for inflation. The second equation reduces to a difference equation in inflation in state 2 and a unique bounded solution to that equation requires $\alpha_2 > p_{22}$.

An interesting special case arises when both regimes are reflecting states. With $p_{11} = p_{22} = 0$, the eigenvalues reduce to $\lambda_k = \pm 1/\sqrt{\alpha_1 \alpha_2}$. When the α's are both positive and regime 1 is active, the lower bound on the passive policy (α_2) for a unique equilibrium is $\alpha_2 > 1/\alpha_1$. In this case, the economy spends equal amounts of time in the two regimes, but it changes regime every period with probability 1. This inequality reinforces the general principle that the more aggressive monetary policy is in active regimes, the more passive it can be in other regimes and still deliver determinacy.

Figure 1 uses the expressions for the eigenvalues in (12) and (13) to plot combinations of the policy-rule coefficients, α_1 and α_2, that deliver determinate equilibria for given transition probabilities. Light-shaded areas mark regions of the parameter space that imply the fixed-regime equilibrium is determinate. When regime can change, those regions expand to include the dark-shaded areas. The top two panels show that as the mean duration, given by $1/(1 - p_{ii})$, of each regime declines, the determinacy region expands. Asymmetric mean duration expands the determinacy region in favor of the parameter drawn from the more transient regime (α_2 in the southwest panel of the figure). As the mean durations of both regimes approach 1 period, the determinacy region expands dramatically along both the α_1 and α_2 dimensions, as the southeast panel shows. The figure and the LRTP make clear the hyperbolic relationship betweeen α_1 and α_2, for given (p_{11}, p_{22}).

2.3. **Solutions.** Having delineated the class of monetary policy processes that deliver a determinate equilibrium, we now find the minimum state variable (MSV) solution using the method of undetermined coefficients. We posit regime-dependent linear solutions of the form

$$\pi_{it} = a(s_t = i)r_t, \tag{15}$$

for $i = 1, 2$, where

$$a(s_t = i) = \begin{cases} a_1 \text{ for } s_t = 1 \\ a_2 \text{ for } s_t = 2 \end{cases}. \tag{16}$$

Expected inflation one step ahead depends on this period's realizations of regime and real interest rate, as well as on next period's expected solution

$$\begin{aligned} E_t \pi_{t+1} &= E[\pi_{t+1} | s_t, r_t] \\ &= \rho r_t E[a(s_{t+1}) | s_t, r_t], \end{aligned} \tag{17}$$

where we have used the independence of the random variables r and s. The posited solutions, together with (17), imply the following regime-dependent expectations

$$E[\pi_{t+1} | s_t = 1, r_t] = [p_{11}a_1 + (1 - p_{11})a_2]\rho r_t, \tag{18}$$

$$E[\pi_{t+1} | s_t = 2, r_t] = [(1 - p_{22})a_1 + p_{22}a_2]\rho r_t. \tag{19}$$

Substituting (18) and (19) into (6) for each $s_t = 1, 2$, we obtain a linear system in the unknown coefficients, (a_1, a_2)

$$A \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = b, \tag{20}$$

where

$$A = \begin{bmatrix} \alpha_1 - \rho p_{11} & -\rho(1 - p_{11}) \\ -\rho(1 - p_{22}) & \alpha_2 - \rho p_{22} \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{21}$$

The solutions are

$$a_1 = a_1^F \left(\frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right), \tag{22}$$

and

$$a_2 = a_2^F \left(\frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right), \tag{23}$$

where we have used the facts that $p_{12} = 1 - p_{11}$ and $p_{21} = 1 - p_{22}$, and we have defined the "fixed-regime" coefficients to be[9]

$$a_i^F = \frac{1}{\alpha_i - \rho p_{ii}}, \qquad i = 1, 2. \tag{24}$$

The limiting arguments applied to (14), together with the bounded real interest rate process, imply that in a determinate equilibrium, $\alpha_i > \rho p_{ii}$, so $a_i^F \geq 0$. a_i^F is strictly increasing in ρ, strictly decreasing in α_i, and strictly increasing in p_{ii}. It is straightforward to show that the volatility of inflation is smaller in the regime where policy is more active; that is, $a_1 < a_2$ if $\alpha_1 > \alpha_2$.

The a_1 and a_2 coefficients have the intuitive properties that they are strictly decreasing in both α_1 and α_2 and strictly increasing in ρ. More-active monetary policy raises the α's and decreases the inflation impacts of real interest rate shocks. Greater persistence in real interest rates amplifies the magnitude and therefore the impact of real-rate shocks on inflation. If $\alpha_1 > \alpha_2$, then as p_{11} rises (holding p_{22} fixed), the persistence of the more-active regime and the fraction of time the economy spends in the more-active regime both rise. This reduces the reaction of inflation to real-rate disturbances in both regimes. Of course, if $\alpha_1 > \alpha_2$ and p_{22} rises (holding p_{11} fixed), then both a_1 and a_2 rise.

When the real interest rate shock is serially uncorrelated ($\rho = 0$), the solutions collapse to their "fixed-regime" counterparts, $a_1 = 1/\alpha_1$ and $a_2 = 1/\alpha_2$. But there is an important difference. Determinacy of the fixed regime requires $\alpha_i > 1$ all i, so monetary policy always dampens the impacts of shocks on inflation. With regime switching, when $\alpha_1 > 1$ and $p_{22} < \alpha_2 < 1$, there can be a determinate equilibrium in which monetary policy in regime 2 *amplifies* the effects of shocks.

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[9]When regime is fixed at i, $p_{ii} = 1, p_{jj} = 0, i \neq j, i, j = 1, 2$ and the coefficients reduce to $a_i^F = 1/(\alpha_i - \rho)$.

In general, all policy parameters enter the solution. Expectations of policy behavior in regime 2 affects the equilibrium in regime 1 and vice versa. Let $D = 1 - \rho^2 p_{12} a_2^F p_{21} a_1^F$ denote the denominator common to (22) and (23). $D \in (0,1]$ and reaches its upper bound whenever regimes are absorbing states ($p_{12} = 0$ or $p_{21} = 0$). Values of D less than 1 scale up the coefficients relative to their "fixed-regime" counterparts. D achieves its minimum when regimes are reflecting states ($p_{12} = p_{21} = 1$). In that case, $D = 1 - 1/\alpha_1\alpha_2$, raising the variability of inflation by its maximum amount (given values for α_1 and α_2).

The numerators in the solutions report the two distinct effects that news about future real interest rates has on current inflation. Suppose the economy is in regime 1 and a higher real interest rate is realized. One effect is direct and raises inflation by an amount inversely related to α_1, just as it would if regime were fixed. A second effect works through expected inflation, $E[\pi_{t+1}|s_t = 1, r_t]$, which is the function given by (18), $(p_{11}a_1 + p_{12}a_2)\rho r_t$. The term $p_{12}a_2^F$ in (22) arises from the expectation that regime can change, with p_{12} the probability of changing from regime 1 to regime 2. The size of this effect is also inversely related to α_1 through the coefficient a_1^F. Both of these effects are tempered when the current policy regime is active ($\alpha_1 > 1$) or amplified when current policy is passive ($\alpha_1 < 1$).

Impacts that arise from expectations of possible future regimes are called *expectations formation effects*, as in Leeper and Zha (2003). These effects are present whenever agents' rational expectations of future regime change induce them to alter their expectations functions. Expectations formation effects are the difference between the impact of a shock when regime can change and the impact when regime is fixed forever.

The strength of expectations formation effects flowing from regime 2 to regime 1 depends on the probability of transitioning from regime 1 to regime 2, p_{12}, and on the policy behavior in and the persistence of regime 2, which are determined by α_2 and p_{22}. Expectations formation effects in regime 1 can be large if p_{12} is large, p_{22} is large, or α_2 is small. The only way to eliminate these effects is for regime 1 to be an absorbing state. In that case, $p_{11} = 1$ and the solution in that regime is $\pi_t = [1/(\alpha_1 - \rho)]r_t$, exactly the fixed-regime rule.

3. A Model of Inflation and Output Determination

This and the next sections report the implications of a regime-switching monetary policy process for determinacy and equilibrium dynamics in a model of inflation and output. We use a bare-bones model from the class of models with sticky prices that use Calvo's (1983) price-adjustment mechanism. Ours is a textbook version, as in Walsh (2003) and Woodford (2003), but the general insights extend to the variants being fit to data. There are several reasons to examine regime change in

a more complex model: it brings the analysis closer to models now being used to confront data, compute optimal policy, and conduct actual policy analysis at central banks; the model contains an explicit transmission mechanism for monetary policy— an endogenous real interest rate—which tempers some of the expectations formation effects found in the Fisherian model; it allows us to track how the possibility of regime change influences the dynamic impacts of aggregate demand and aggregate supply shocks on inflation and output.

3.1. **The Model.** The linearized equations describing private sector behavior are the consumption-Euler equation and aggregate supply relations

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u_t^D, \tag{25}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t^S, \tag{26}$$

where x_t is the output gap, u_t^D is an aggregate demand shock, and u_t^S is an aggregate supply shock. σ^{-1} represents the intertemporal elasticity of substitution, κ is a function of how frequently price adjustments occur, as in Calvo (1983), and of β, the discount factor. The slope of the supply curve is determined by $\kappa = (1-\omega)(1-\beta\omega)/\omega$, where $1 - \omega$ is the randomly selected fraction of firms that adjust prices. Prices are more flexible as $\omega \to 0$, which makes $\kappa \to \infty$. As a baseline, we set $\sigma = 1$, $\beta = .99$ and $\omega = .67$, so $\kappa = .17$. We interpret a model period as one quarter in calendar time.[10] Exogenous disturbances are autoregressive and mutually uncorrelated

$$u_t^D = \rho_D u_{t-1}^D + \varepsilon_t^D, \tag{27}$$
$$u_t^S = \rho_S u_{t-1}^S + \varepsilon_t^S, \tag{28}$$

where $|\rho_D| < 1$, $|\rho_S| < 1$, ε_t^D and ε_t^S are mean zero random variables with bounded supports, and $E[\varepsilon_t^D \varepsilon_s^S] = 0$ for all t and s. If shocks are $i.i.d.$, then regime switching is irrelevant to the dynamics, but not to the determinacy properties of the equilibrium.

As before, monetary policy is the source of regime switching and we assume a Taylor rule that sets the nominal interest rate according to

$$i_t = \alpha(s_t)\pi_t + \gamma(s_t)x_t, \tag{29}$$

where s_t evolves according to a Markov chain with transition matrix whose typical element is $p_{ij} = \Pr[s_t = j | s_{t-1} = i]$ for $i, j = 1, 2$. s_t is independent of u_t^D and u_t^S. As before, $\alpha(s_t)$ equals α_1 or α_2 and $\gamma(s_t)$ equals γ_1 or γ_2. We assume the steady state does not change across regimes.

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[10]The model is linearized around a steady state inflation rate of 0 to keep the analysis simple. In future work it is worthwhile to explore the implications of allowing the inflation target to fluctuate stochastically [Cogley and Sbordone (2005), Ireland (2006)] and to allow varying degrees of indexation to inflation [Ascari and Ropele (2005)].

3.2. **Fixed-Regime Equilibrium.** Intuition from the fixed-regime equilibrium carries over to a switching environment. Solutions are

$$\pi_t = \frac{\kappa}{\Delta^D} u_t^D + \frac{\sigma^{-1}\gamma + 1 - \rho_S}{\Delta^S} u_t^S, \tag{30}$$

$$x_t = \frac{1 - \beta\rho_D}{\Delta^D} u_t^D - \frac{\sigma^{-1}(\alpha - \rho_S)}{\Delta^S} u_t^S, \tag{31}$$

where $\Delta^Z = 1 + \sigma^{-1}(\alpha\kappa + \gamma) - \rho_Z[1 + \sigma^{-1}(\kappa + \beta\gamma) + \beta(1 - \rho_Z)]$, $Z = S, D$.

More-active monetary policy (higher α) reduces the elasticities of inflation and output to demand shocks. Supply shocks, however, present the monetary authority with a well-known tradeoff: a more-active policy stance reduces the elasticity of inflation with respect to supply shocks, but it raises the responsiveness of output. A stronger reaction of monetary policy to output (higher γ) reduces the elasticities of inflation and output to demand shocks. Higher γ reduces the elasticity of output to supply shocks and raises the responsiveness of inflation to supply shocks.

3.3. **The Long-Run Taylor Principle.** Turning back to the setup with regime change, this section describes how to derive restrictions on the monetary policy process that ensure the long-run Taylor principle is satisfied. Substituting the policy rule, (29), into (25) yields

$$x_t = E_t x_{t+1} - \sigma^{-1}(\alpha(s_t)\pi_t + \gamma(s_t)x_t - E_t\pi_{t+1}) + u_t^D. \tag{32}$$

The system to be solved consists of (26) and (32).

To specify the system whose eigenvalues determine whether there exists a unique bounded equilibrium, we follow the procedure for the Fisherian model. Let $\pi_{it} = \pi_t(s_t = i, u_t^D u_t^S)$ and $x_{it} = x_t(s_t = i, u_t^D, u_t^S)$, $i = 1, 2$, denote state-specific inflation and output. As appendix B describes, after defining the forecast errors

$$\eta_{1t+1}^\pi = \pi_{1t+1} - E_t\pi_{1t+1}, \qquad \eta_{2t+1}^\pi = \pi_{2t+1} - E_t\pi_{2t+1}, \tag{33}$$

$$\eta_{1t+1}^x = x_{1t+1} - E_t x_{1t+1}, \qquad \eta_{2t+1}^x = x_{2t+1} - E_t x_{2t+1}, \tag{34}$$

the model is cast in the form

$$AY_t = BY_{t-1} + A\eta_t + Cu_t, \tag{35}$$

where

$$Y_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix}, \qquad \eta_t = \begin{bmatrix} \eta_{1t}^\pi \\ \eta_{2t}^\pi \\ \eta_{1t}^x \\ \eta_{2t}^x \end{bmatrix}, \qquad u_t = \begin{bmatrix} u_t^S \\ u_t^D \end{bmatrix}, \tag{36}$$

and the matrices are defined in the appendix. A straightforward extension of proposition 1 (and its proof in appendix A) applies to this model: necessary and sufficient

conditions for the existence of a unique bounded solution to (35) is that all the generalized eigenvalues of (B, A) lie outside the unit circle. The eigenvectors associated with those eigenvalues generate four linear restrictions that determine the regime-dependent forecast errors for inflation and output. The eigenvalues of this system determine whether the monetary policy process satisfies the long-run Taylor principle. The model structure is such that analytical expressions for the eigenvalues are available, but they do not yield compact expressions.

Figure 2 illustrates that recurring regime change can dramatically expand the set of policy parameters that deliver a determinate equilibrium.[11] As long as one regime is active, the less persistent the other regime is, the smaller is the lower bound on the response of monetary policy to inflation. The bottom panels of the figure indicate that when regimes are transitory, a large negative response of policy to inflation is consistent with determinacy.[12] As in the Fisherian model, a determinate equilibrium can be produced by a policy process that is mildly passive most of the time or very passive some of the time.

In contrast to fixed regimes, recurring regime change makes determinacy of equilibrium depend on the policy process and all the parameters describing private behavior, (β, σ, κ), even when the Taylor rule does not respond to output. Because the current regime is not expected to prevail forever, parameters that affect intertemporal margins interact with expected policies to influence determinacy [figure 3]. Greater willingness of households to substitute intertemporally (lower σ) or greater ability of firms to adjust prices (lower ω) enhance substitution away from expected inflation, giving expected regime change a smaller role in decisions. This shrinks the determinacy region toward the flexible-price region in section 2.

3.4. **Solutions.** To solve the model, define the state of the economy at t as $\left(u_t^D, u_t^S, s_t\right)$. The method of undetermined coefficients delivers solutions as functions of this smallest set of state variables—the MSV solution. We posit solutions of the form

$$\pi_t = a^D(s_t)u_t^D + a^S(s_t)u_t^S, \tag{37}$$
$$x_t = b^D(s_t)u_t^D + b^S(s_t)u_t^S, \tag{38}$$

where

$$a^Z(s_t) = \left\{ \begin{array}{l} a_1^Z \text{ for } s_t = 1 \\ a_2^Z \text{ for } s_t = 2 \end{array} \right., \qquad b^Z(s_t) = \left\{ \begin{array}{l} b_1^Z \text{ for } s_t = 1 \\ b_2^Z \text{ for } s_t = 2 \end{array} \right., \qquad Z = D, S. \tag{39}$$

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[11]For simplicity, figures 2 and 3 are drawn setting $\gamma(s_t) = 0$, $s_t = 1, 2$, so in fixed regimes, the Taylor principle is $\alpha_1 > 1$ and $\alpha_2 > 1$.

[12] In the Fisherian model we restricted attention to cases where $\alpha_i > 0$ for $i = 1, 2$. This restriction focuses on policies of economic interest, though as these results indicate, it is not a necessary condition for determinacy in the new Keynesian model.

These posited solutions, along with their one-step-ahead expectations,

$$E\left[\pi_{t+1}\,|s_t=i\right] = p_{ii}\left(a_i^D\rho_D u_t^D + a_i^S\rho_S u_t^S\right) + p_{ij}\left(a_j^D\rho_D u_t^D + a_j^S\rho_S u_t^S\right), \quad (40)$$

$$E\left[x_{t+1}\,|s_t=i\right] = p_{ii}\left(b_i^D\rho_D u_t^D + b_i^S\rho_S u_t^S\right) + p_{ij}\left(b_j^D\rho_D u_t^D + b_j^S\rho_S u_t^S\right), \quad (41)$$

for $i,j = 1,2$, are substituted into (26) and (32) to form a system whose solution yields expressions for π and x as functions of the model parameters and the monetary policy process. Appendix B describes the systems of equations that are solved.

4. Some Practical Implications of Regime Switching

We turn now to the implications of monetary policy processes that empirical evidence suggests are relevant. In practice, obeying the Taylor principle is viewed as desirable because of its well-known stabilization properties and its ability to prevent fluctuations due to self-fulfilling expectations. However, no central bank systematically implements a policy with the primary goal of satisfying the Taylor principle on a period-by-period basis. Instead, central banks seem to have internalized Taylor's key prescription: on average, raise nominal interest rates more than one-for-one with inflation. But central banks also desire the flexibility to respond to developments that may entail a departure from the Taylor principle. Should such departures be of concern? Addressing this question requires a complete specification of the monetary policy process—the degree of the departure, given by regime-dependent values of the policy rule coefficients, and the duration of the departure, determined by the transition probabilities.

Two types of departures from the Taylor principle are of particular interest in describing Federal Reserve behavior. The first arises when private agents believe there is a small probability of returning to a persistent regime like the one that prevailed in the 1970s. This policy process reflects empirical work that finds U.S. monetary policy followed very different rules from 1960 to 1979 and after 1982. The second kind of departure occurs when central banks abandon their "business-as-usual" rule and do something different for brief periods of time. Examples include the October 1987 stock market crash, Asian and Russian financial crises in the 1990s, credit controls in 1980, sluggish job-market recoveries from recessions, and currency crises. These are events with small probability mass that recur and can entail a substantial deviation from the usual rule. We model these events as relatively short-lived excursions into passive policy behavior, though we recognize that this is, at best, a crude representation of the diversity of examples listed above.

Departures from a constant rule obeying the Taylor principle have two possible ramifications. The first is determinacy of equilibrium and the second is volatility arising from expectation formation effects. We examine these ramifications for the two types of departures from the Taylor principle.

4.1. **A Return to the 1970s?** Many observers of U.S. monetary policy fear that the Fed could revert to the policies of the 1970s. Such a fear is often behind arguments for adopting inflation targeting in the United States [Bernanke and Mishkin (1997), Bernanke, Laubach, Mishkin, and Posen (1999a), Mishkin (2004), Goodfriend (2005)]. The United States seems particularly susceptible to this kind of policy reversal because, in the absence of institutional reforms, the Fed relies on what Bernanke, Laubach, Mishkin, and Posen (1999b) call the "just trust us" approach, which relies more on the personal credibility of policy makers than on the credibility of the policy institution or the policymaking process.[13]

Three widely cited empirical studies report constant-coefficient estimates of Taylor rules for the United States [Clarida, Gali, and Gertler (2000), Taylor (1999a), Lubik and Schorfheide (2004)]. Each of these reports that U.S. monetary policy was passive through the 1960s and 1970s and active since 1982. Efforts to estimate Markov-switching versions of these rules frequently find analogous results [Favero and Monacelli (2005), Davig and Leeper (2006b)]. A literal interpretation of the switching results is that agents place substantial probability mass on a return to the inflationary times of the 1970s.

4.1.1. *Determinacy Regions for Previous Studies.* Previous studies posit that U.S. monetary policy unexpectedly shifted from a rule that allowed a large multiplicity of equilibria to one that delivered a determinate equilibrium. For example, Lubik and Schorfheide (2004) emphasize that in a model with a fixed policy rule, their estimate of Fed behavior from 1960-1979 leaves the equilibrium indeterminate and subject to self-fulfilling sunspot equilibria.[14] Since the early 1980s, however, Lubik and Schorfheide infer that their estimates imply a determinate equilibrium. In the latter period, for the mean of the posterior distribution they estimate $\alpha_1 = 2.19$ and $\gamma_1 = .3$, while for the earlier period the estimates are $\alpha_2 = .89$ and $\gamma_2 = .15$. Their maximum likelihood estimates contrast the fit of determinate to indeterminate equilibria under the maintained assumption that policy rules cannot change.

Central to Lubik and Schorheide's study is the logical inconsistency that Cooley, LeRoy, and Raymon (1982, 1984) observed about rational expectations policy experiments: although policy rules can *and do* change, agents in the model always believe

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[13]Fiscal policy in the United States represents a possible impetus for a change from an active to a passive monetary policy stance. As fiscal pressures build, it may be reasonable to expect some erosion of the much-vaunted independence of the Federal Reserve. A possible outcome is a shift to a policy that accommodates inflation as a source of fiscal financing. Sargent's (1999) learning environment offers a different rationale for how a return to the 1970s might arise. In his setup, time inconsistency and constant-gain learning combine to create incentives for policy to optimally choose to revert to an accommodative stance.

[14]Clarida, Gali, and Gertler (2000) also suggest this possibility.

such changes are impossible. Do Lubik and Schorfheide's inferences about determinacy of equilibrium stand up in an environment in which agents' expectations reflect the possibility that policy regime can change? An answer requires specifying values for β, κ, and σ, as well as the transition probabilities. Lubik and Schorfheide do not constrain the estimates of private parameters to be the same across regimes, so there is no straightforward method to choose single values for those parameters. Instead, we compare two sets of parameter values for κ and σ, which draw extreme values for κ and σ from Lubik and Schorfheide's estimated 90-percent probability intervals. The first set shrinks the region of determinacy in (p_{11}, p_{22})-space and the second set expands it. We assume $\beta = .99$. Shaded regiones in figure 4 report combinations of the transition probabilities, (p_{11}, p_{22}), that yield a determinate equilibrium.[15]

The figure appears to lend support to Lubik and Schorfheide's inference that inflation in the 1970s may have been driven by sunspots. After all, if the passive regime has an expected duration of more than 5 years ($p_{22} > .95$), then Lubik and Schorfheide's policy parameter estimates imply indeterminacy.

Carrying this argument forward, however, reveals an unappealing implication. Unless one is willing to maintain the implausible assumption that the post-1982 regime is an absorbing state ($p_{11} = 1$), the U.S. economy must still be in an indeterminate equilibrium.[16] Without assuming people place no probability mass on future passive policy, it is difficult to reconcile Lubik and Schorfheide's conclusion that the equilibrium switched from indeterminate to determinate with an environment of recurring regime change.

Alternatively, if the passive regime has an expected duration of less than 5 years, then a sufficiently persistent active regime yields a determinate equilibrium. The logic then implies that the 1970s fluctuations were not driven by sunspots; rather, they are the outcome of a determinate equilibrium with shocks whose impacts are amplified by passive monetary policy behavior.

Table 1 reports the volatility of inflation and output, conditional on exogenous shocks, in each regime relative to a fixed regime with active policy. The results use Lubik and Schorfheide's estimated policy parameters and the baseline calibration. Regimes are equally likely with expected duration of 5 years. Expectations formation effects are substantial, raising the relative inflation volatility from 9 to 15 percent, even when the prevailing regime is active.

Of more importance for assessing Lubik and Schorfheide's inferences, in the passive regime inflation is about $2\frac{1}{2}$ times more volatile than in the active regime. The change in output volatility is also substantial and depends on the source of disturbance,

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[15]The smaller region uses $\sigma = 1.04$ and $\kappa = 1.07$; the larger region uses $\sigma = 2.84$ and $\kappa = 0.27$.

[16]The reasoning is identical to that contained in footnote 8. Once the economy transits to the active, absorbing state, the equilibrium is identical to a fixed regime with $\alpha_1 = 2.19$.

rising for demand shocks and falling for supply shocks. Lubik and Schorfheide find that sunspot shocks help to account for the macroeconomic instability of the 1970s. This example suggests that indeterminacy may not be necessary to account for the observed shift in volatility in the post-war period.

Of course, determinacy of equilibrium depends on all the parameters of the model, so the 5-year duration for the passive monetary regime, which the figure suggests, is sensitive to the parameter settings. For example, when α_2 increases to .95, a value well inside Lubik and Schorfheide's 90-percent probability interval, determinacy requires the expected duration of the passive regime to be about 10 years, a highly plausible value. Modest changes in other parameters can also have substantial impacts on the determinacy regions. A satisfactory resolution to the question of whether aggregate fluctuations in the U.S. are driven by sunspots or are the outcome of a determinate equilibrium subject to various shocks requires estimation of a complete DSGE model with a switching monetary policy process.

A central bank that seeks to stabilize inflation and output should be concerned with the private sector's beliefs about possible future policy regimes. The possibility of prolonged episodes of passive policy introduces the potential for destabilizing sunspot fluctuations. Even if beliefs about alternative regimes do not create indeterminacy, the expectations formation effects can make it more difficult for monetary policy to achieve its goals, even when current policy is active. The next section illustrates that even very brief recurring regimes of passive policy can generate expectations formation effects that contribute importantly to aggregate volatility.

4.2. **Financial Crises and Business Cycles.** Periodically, monetary policy shifts its focus from price stability to other concerns. Two other concerns that recurrently come into the central bank's focus are financial stability and job creation. Episodes in which price stability is de-emphasized in favor of other objectives can last a few months or more than a year. Distinctive features of these episodes are that they recur fairly often and that they represent an important shift away from monetary policy's usual reaction to inflation and output. In the United States, since Greenspan became chairman of the Fed in the summer of 1987, the episodes include at least two stock market crashes, two foreign financial crises, and two "jobless recoveries"—an episode every three years, on average.[17]

Marshall (2001) carefully documents the financial crisis in late summer and fall of 1998. In August the Russian government devalued the rouble, defaulted on debt, and suspended payments by financial institutions to foreign creditors. These actions

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[17]We do not include the terrorist attacks of September 11, 2001 in this list because, although the Fed reacted sharply by pumping liquidity into the market and lowering the federal funds rate, within two months it had just as sharply withdrawn the liquidity. This event is probably best modeled as a sequence of additive shocks to the policy rule.

precipitated the near collapse of Long-Term Capital Management, a large hedge fund. The Fed reacted swiftly by cutting the federal funds rate by a total of 75 basis points over three moves. One of the policy moves arose from an unusual intermeeting conference call on October 15 and all the moves occurred against a backdrop of concern by Federal Open Market Committee members about inflation. In fact, until the August 18 FOMC meeting, which left the funds rate unchanged, the Committee concluded the risks to the outlook were tilted toward rising inflation. Marshall argues that the Fed's unusually rapid response signalled that the "policy rule had changed," with the purpose of discretely shifting private-sector beliefs to a lower likelihood of a liquidity crisis in the United States.

Rabanal (2004) presents a variety of evidence on time variation in Taylor rules. First, he reports estimates of Taylor rules with parameter drift that buttress Marshall's claim: during periods that Rabanal calls "high risk in the economy," the Fed's response to inflation declines appreciably. High-risk periods include financial crises.

Rabanal also estimates a two-state—recessions and expansions—Taylor rule to find that during recessions the Fed's reaction to inflation is weaker and its reaction to output is stronger than during expansions. Davig and Leeper's (2006b) estimates of (29) identify the "jobless recoveries" from the recessions of 1990-91 and 2000 as episodes of passive Fed behavior, with a weaker response to inflation and a stronger response to output than in the surrounding active episodes. Whereas Rabanal estimates the economy is three times more likely to be in an expansion than a recession, Davig and Leeper, using a longer time series beginning in the late 1940s, estimate that active and passive regimes are almost equally likely.

Table 2 reports that expectations formation effects from a passive regime can substantially raise the standard deviations of inflation and output in an active regime relative to their values in a fixed regime. The probabilities of transitioning to the passive regime are 5 percent and 2.5 percent ($p_{11} = .95$ and $p_{11} = .975$), which correspond to a financial crisis or stronger concern about job growth occurring every 5 or 10 years, on average. In the active and the fixed regimes, $\alpha_1 = \alpha = 1.5$ and $\gamma_1 = \gamma = .25$. Passive policy responds more strongly to output ($\gamma_2 = .5$), while both its response to inflation, α_2, and its persistence, p_{22}, take different values in the table.[18]

When the passive regime lasts only one period ($p_{22} = 0$), expectations formation effects are relatively small and intuition from fixed regimes directly applies: when regime 2 is more passive (lower α_2), expectations formation effects raise the volatility of inflation and output from demand shocks, raise the volatility of inflation from supply shocks, and lower the volatility of output from supply shocks. Fixed-regime

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[18]In the case of strict inflation targeting, $\gamma_1 = \gamma_2 = \gamma = 0$, the relative standard deviations of inflation are amplified, but the patterns are identical to those in the table.

intuition carries over because when the passive regime lasts only one period, it generates only minor expectations formation effects.

As the passive regime becomes more persistent (p_{22} rises), the monetary policy process becomes less active and the relative volatility of inflation rises monotonically across both types of shocks. Even when the expected duration of passive policy is only 2 quarters ($p_{22} = .5$), as it might be during some financial crises, if policy is very passive, inflation volatility can be 20 percent or more higher in the active state than in a fixed-regime setup. When the duration is one year ($p_{22} = .75$), as when the Fed kept interest rates low for extended periods during the two recent recoveries from recession, inflation can be 50 percent more volatile than in a fixed regime [see columns for $p_{11} = .95$].

Persistence in the passive regime changes the effects on relative output variability of increases in the degree to which policy is passive. The prospect of moving to a passive regime raises current and expected inflation in the active regime relative to a fixed regime. Although it starts at a higher level, in the long run the ergodic mean of inflation in the switching environment converges to the mean when regime is constant. With inflation expected to fall more rapidly in the active regime, the real interest rate rises more sharply. A higher real rate offsets the effects of a demand shock on output, but it reinforces the impacts of a supply shock. This shows up in table 2 as declining relative output variability in the demand columns and rising relative output variability in the supply columns, as the monetary policy process becomes more passive.

The table shows that plausible departures from the Taylor principle during episodes when the central bank's focus shifts from inflation stabilization to other concerns, can produce quantitatively important expectations formation effects that can make it more difficult for the central bank to achieve its stabilization objectives during normal periods.

5. Some Empirical Implications of Regime Change

Regime change also carries implications for empirical work on monetary policy. This section illustrates two pitfalls in interpreting time series generated by switching policies with theoretical models in which policy rules are time invariant.

5.1. **Qualitative Inferences from Estimated Policy Rules.** It is commonplace for empirical studies of monetary policy to split data samples into subperiods over which researchers believe a particular policy regime prevailed. This section illustrates the pitfalls of this procedure when actual time series are generated by recurring regime change. We imagine that a researcher has access to a long time series of data and seeks to estimate a monetary policy rule, which is then inserted into a conventional

new Keynesian model with a fixed policy regime. The researcher has extra-sample information that specifies when regime changes occurred. We assume this information is accurate, as are the equations describing private behavior.

We use the model with the baseline calibration of private parameters—$\beta = .99, \sigma = 1, \kappa = .17$—and the estimates of policy behavior and exogenous shocks that Lubik and Schorfheide (2004) report to generate a sample of data on $\{x_t, \pi_t, i_t\}$ of length 10,000 from the regime-switching new Keynesian model. We consider three scenarios: conditional on being in regime 1 (active monetary policy with $\alpha_1 = 2.19$ and $\gamma_1 = 0.30$); conditional on being in regime 2 (passive monetary policy with $\alpha_2 = 0.89$ and $\gamma_2 = 0.15$); recurring changes in policy regime between regime 1 and regime 2. Using these simulated data, the researcher estimates a VAR with a common set of identifying assumptions across samples: monetary policy affects aggregate demand directly, while its contemporaneous effects on aggregate supply operate through output via a Phillips curve.[19] With these restrictions, the model is just identified when the response of monetary policy to output, γ_i, is calibrated at its true value. No restrictions are imposed on lags in any equation. The estimated model is summarized by

$$\begin{aligned} x_t &= \delta i_t + u_t^D + lags \\ \pi_t &= \theta x_t + u_t^S + lags \\ i_t &= \alpha \pi_t + \bar{\gamma} x_t + u_t^{MP} + lags \end{aligned} \qquad (42)$$

where $\bar{\gamma}$ denotes the parameter that is fixed at its true value and the serially correlated shock, u_t^{MP}, has been added to the policy rule.[20]

Table 3 reports that the VAR accurately estimates the policy parameters in each of the three scenarios, with the estimated values of the response of monetary policy to inflation remarkably close to their theoretical values. Researchers who import the policy estimates into a new Keynesian model with a calibration of the discount factor, β, of about 0.99 will conclude that regime 1 yields a determinate equilibrium, regime 2 leaves the equilibrium indeterminate, and the full sample is consistent with a determinate equilibrium.[21]

In the simulated regime-switching model the equilibrium is determinate, so the qualitative inference drawn about regime 2 is incorrect. Using the full sample yields qualitatively accurate inferences because it brings information from both regimes to

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[19]These restrictions accurately represent the direct contemporaneous interactions among variables in the new Keynesian model, but they do not necessarily reflect all the contemporaneous interactions that operate through expectations.

[20]The shock processes are calibrated to be roughly consistent with Lubik and Schorfheide's (2004) estimates: standard deviations are $\sigma_D = 0.23, \sigma_S = 0.80, \sigma_{MP} = 0.20$ and the autoregressive parameters are $\rho_j = 0.75, j = D, S, MP$.

[21]Woodford (2003, Appendix C) proves that the equilibrium of this model is determinate if and only if $\alpha + \gamma(1 - \beta)/4\kappa > 1$.

bear, producing more accurate estimates of policy behavior in the long run. Splitting the sample into distinct regimes, in contrast, can distort inferences: conditioning on regime 2, for example, discards all observations in which policy behaved actively, thereby uniquely determining the equilibrium.

5.2. **Quantitative Predictions of the Impacts of Shocks.** To illustrate the potential expectations formation effects from a belief that policy might return to its passive behavior in the 1970s, we employ the baseline calibration with Lubik and Schorfheide's policy parameter estimates, along with the transition probabilities $p_{11} = .95$ and $p_{22} = .93$, which deliver a determinate equilibrium. These probabilities mean there is a 5 percent chance of returning to a passive policy rule. The active regime is expected to last 20 quarters, while the passive regime lasts 14 quarters, on average. We gauge the extent that expectations of a future passive regime affects the equilibrium in the active regime by contrasting responses of inflation and output to demand and supply disturbances in the active regime with switching to those in an equivalently active fixed regime.

Expectations formation effects from this policy process are substantial. Figure 5 shows that researchers predicting the impacts of exogenous disturbances assuming the policy rule is fixed will consistently underpredict inflation.[22] The underprediction can be more than 20 basis points following demand shocks and nearly 1 percentage point following supply disturbances. Output predictions depend on the source of the shock. A hump-shaped response of output in the switching environment means the fixed-regime model initially overpredicts and then underpredicts output. With supply shocks, the prediction errors are quite large. A constant-coefficient policy rule misses the initial decline in output by nearly 1 percentage point; the errors change sign after several periods when constant-coefficient predictions are about .3 percentage points too pessimistic.

6. Concluding Remarks

This paper offers a broader perspective on the Taylor principle and the range of unique bounded equilibria it supports by allowing policy regime to vary over time. Examples show that endowing conventional models with empirically relevant monetary policy switching processes can generate important expectations formation effects. These effects can alter the qualitative and quantitative predictions of standard models. Along the way, the paper develops a two-step solution method that obtains determinacy conditions and solutions for a rational expectations equilibrium. This

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[22]These are expected paths, computed taking draws from regime after the initial period.

method can be applied to a broad class of purely forward-looking rational expectations models with exogenous Markov switching in parameters and many discrete regimes.

The paper's results should be useful for both researchers and policy analysts using constant-coefficient policy rules in DSGE models. The choice of how to model deviations from such rules is potentially quite important. Under prevailing practice, that choice is made implicitly. That choice should be explicit, with careful consideration given to the characteristics of the deviation—how likely is it to recur? how long is it likely to last? what is the nature of policy behavior during the period of deviation? Some deviations are more naturally modeled as additive, exogenous errors to the policy rule. Some might be better modeled as systematic responses to an expanded information set for the policy authority. Others are best treated as recurring changes in rules mapping endogenous variables to policy choices, as in this paper.

Modeling policy as we do in this paper requires no more heroic assumptions than those routinely made in policy research. Largely as a matter of convenience, nearly all theoretical models assume—rather heroically—that future policy *is* current policy. When the current regime is an absorbing state, this assumption is reasonable. If, as seems more likely, alternative future policies are possible, then rational agents must have a probability distribution over those policies, and the properties of observed equilibria will depend critically agents' beliefs about those policies and their probabilities.

APPENDIX A. PROOF OF DETERMINACY IN FISHERIAN MODEL

Following the notation in section 2.1, $\pi_t(z_t) = (\pi_{1t}, \pi_{2t})'$ denotes the MSV solution, while $\widehat{\pi}_t = (\widehat{\pi}_{1t}, \widehat{\pi}_{2t})'$ denotes any other solution to (11). The associated systems, for $i = 1, 2$, are

$$\alpha_i \pi_{it} = p_{i1} E_t \pi_{1t+1} + p_{i2} E_t \pi_{2t+1} + r_t \tag{43}$$

and

$$\alpha_i \widehat{\pi}_{it} = p_{i1} E_t \widehat{\pi}_{1t+1} + p_{i2} E_t \widehat{\pi}_{2t+1} + r_t. \tag{44}$$

Let $x_{it} \equiv \widehat{\pi}_{it} - \pi_{it}$ be the difference between any other solution and the MSV solution. Subtracting (43) from (44) yields

$$\alpha_i x_{it} = p_{i1} E_t x_{1t+1} + p_{i2} E_t x_{2t+1}, \tag{45}$$

the system of interest for the present analysis. Bounded solutions for inflation correspond to bounded solutions for the process $\{x_t\}$.

Defining the matrix

$$M = \left[\begin{array}{cc} \alpha_1^{-1} & 0 \\ 0 & \alpha_2^{-1} \end{array} \right] \left[\begin{array}{cc} p_{11} & p_{12} \\ p_{21} & p_{22} \end{array} \right], \tag{46}$$

and letting $x_t = (x_{1t}, x_{2t})'$, write (45) as

$$x_t = M E_t x_{t+1}. \tag{47}$$

To establish determinacy, we must show that $E[x_{t+1} | \Omega_t] = 0$ so that, given $\alpha_i > 0$ for $i = 1, 2$, $x_{it} = 0$. This establishes that $\pi_{it} = \pi_t(z_t)$ for $i = 1, 2$, and the MSV solution is the unique bounded solution to the original system in (11).

For convenience, we reproduce the eigenvalues of M when $\alpha_i > 0$ for $i = 1, 2$

$$\lambda_1 = \frac{1}{2\alpha_1\alpha_2} \left(\alpha_2 p_{11} + \alpha_1 p_{22} + \sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1\alpha_2 p_{12} p_{21}} \right), \tag{48}$$

$$\lambda_2 = \frac{1}{2\alpha_1\alpha_2} \left(\alpha_2 p_{11} + \alpha_1 p_{22} - \sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1\alpha_2 p_{12} p_{21}} \right). \tag{49}$$

Note that the roots λ_1 and λ_2 are necessarily real, that $\lambda_1 > 0$, and that $\lambda_1 > \lambda_2$.

Proposition 1. *When $\alpha_i > 0$, for $i = 1, 2$, a necessary and sufficient condition for determinacy of equilibrium, defined as the existence of a unique bounded solution for $\{x_t\}$ in (47), is that all the eigenvalues of M lie inside the unit circle.*

Proof. (**Sufficiency**) Suppose there exists a vector of bounds $K = (K(1), K(2))'$, where $K(i) \geq 0$ for $i = 1, 2$ such that whenever $s_t = i$, $|x_{it}| \leq K(i)$. Then there exist bounds $K'(i) \geq 0$ where $|x_{it}| \leq K'(i)$ for all i and $K'(i)$ is defined by

$$K'(i) = \max_{x_{it}} \left| \alpha_i^{-1} [p_{i1} x_{1t} + p_{i2} x_{2t}] \right| \quad \text{s.t.} \quad |x_{it}| \leq K(i) \quad \text{for } i = 1, 2. \tag{50}$$

The solution to this maximization problem is to set each x_{it} equal to its upper bound, $x_{it} = K(i)$, so the vector of bounds evolves according to

$$K' = MK. \tag{51}$$

Repeating this argument, existence of the vector of bounds K' implies existence of a vector of bounds

$$K'' = MK' = M^2 K. \tag{52}$$

Continuing with this line of argument, it follows that if the vector of bounds K exists, then $M^n K$ is also a vector of bounds, for any n. If all the eigenvalues of M lie inside the unit circle, then $\lim_{n \to \infty} M^n = 0$ and the only bounded solution to (47) is $x_t = 0$, for all t.

(**Necessity**) Suppose, by way of contradiction, that one eigenvalue does not lie inside the unit circle; say $\lambda_1 \geq 1$, while $\lambda_2 < 1$. We now show that under these conditions there exist a continuum of solutions to (47). Diagonalize M by writing $M = V\Lambda V^{-1}$, and define $y_t = V^{-1} x_t$, so (47) becomes

$$y_t = \Lambda E[y_{t+1} | \Omega_t^{-s}] = \Lambda E_t y_{t+1}, \tag{53}$$

or

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} E_t y_{1t+1} \\ E_t y_{2t+1} \end{bmatrix}. \tag{54}$$

Bounded solutions for y_{it} are

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \gamma \lambda_1^{-t} \\ 0 \end{bmatrix}, \tag{55}$$

where γ is an arbitrary constant associated with the loose initial condition. In terms of the underlying $\{x_t\}$ process, $x_t = V y_t$, the solution is

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} \gamma v_{11} \lambda_1^{-t} \\ \gamma v_{21} \lambda_1^{-t} \end{bmatrix}, \tag{56}$$

where $(v_{11}, v_{21})'$ is the first column of V, the matrix of right eigenvectors. The solution in (56) shows that if one eigenvalue of M fails to lie inside the unit circle, then there exist a continuum of solutions to (47), indexed by the arbitrary constant γ.

The indeterminate solution also supports bounded sunspot equilibria. Consider the solution $y_{1t+1} = \lambda_1^{-1} y_{1t} + \phi_{t+1}$, where ϕ is any random variable with bounded support that satisfies $E_t \phi_{t+1} = 0$. Evidently, this solution, together with $y_{2t} = 0$, also satisfies (54) and produces bounded fluctuations in the underlying $\{x_t\}$ process. $\square$

The proof of necessity of proposition 1 illustrates that, in general, indeterminacy is a property that transmits across regimes. This occurs because, except in the case when the current (state-contingent) determinate regime is an absorbing state, the expectation of moving to the other (state-contingent) indeterminate regime makes

the current regime indeterminate also. If, for example, $p_{11} = 1$, so regime 1 is an absorbing state, then $(v_{11}, v_{21})' = (0, 1)'$ and the solution in (56) becomes $x_{1t} = 0$ and $x_{2t} = \gamma \lambda_1^{-t}$.

Definition 1. *The **long-run Taylor principle (LRTP)** is*

$$(1 - \alpha_2) p_{11} + (1 - \alpha_1) p_{22} + \alpha_1 \alpha_2 > 1. \tag{57}$$

For given p_{11} and p_{22}, the LRTP defines a hyperbola in (α_1, α_2) −space with the vertical asymptote where $\alpha_1 = p_{11}$ and the horizontal asymptote where $\alpha_2 = p_{22}$. As in figure 1, our analysis focuses on the hyperbola in the region of the parameter space where $\alpha_i > p_{ii}$, for $i = 1, 2$. This region captures the economically interesting set of monetary policy processes and has the intuitive implication that $\alpha_i > 1$ for some $i = 1, 2$ is a necessary condition for the LRTP to imply determinacy. For example, $\alpha_i < 0$, for $i = 1, 2$, can satisfy the LRTP, but is at odds with the way central banks set policy and does not result in all the eigenvalues of M being inside the unit circle, so fails to deliver a unique bounded equilibrium.

Lemma 1. *If $\alpha_i > p_{ii}$ for all $i = 1, 2$ and LRTP, then $\alpha_i > 1$ for some $i = 1, 2$.*

Proof. For $p_{ii} = 1$ for some i, then $\alpha_i > 1$ for some $i = 1, 2$ by the condition $\alpha_i > p_{ii}$ for all $i = 1, 2$. For $p_{ii} < 1$ for both i, take $\alpha_2 > p_{22} \geq 0$ and rewrite the LRTP as

$$\alpha_1 > \frac{1 - p_{11} - p_{22} + \alpha_2 p_{11}}{\alpha_2 - p_{22}}. \tag{58}$$

Note that the right side of (58), expressed as a function of α_2 and the transition probabilities, is monotonically decreasing in α_2. We now show that over the range $p_{22} < \alpha_2 < 1$, $\alpha_1 > 1$ for all $p_{11} \in [0, 1)$ and $p_{22} \in [0, 1)$. Letting $\alpha_2 \to 1$, (58) implies

$$\alpha_1 > \frac{1 - p_{22}}{1 - p_{22}} = 1. \tag{59}$$

Letting $\alpha_2 \to p_{22}$ implies the right side of (58) approaches ∞ for any $p_{11} \in [0, 1)$ and $p_{22} \in [0, 1)$. $\square$

Proposition 2. *Given $\alpha_i > p_{ii}$ for $i = 1, 2$, the following statements are equivalent:*

    (A) *All the eigenvalues of M lie inside the unit circle.*
    (B) *$\alpha_i > 1$, for some $i = 1, 2$, and the long-run Taylor principle (LRTP) is satisfied.*

Proof. **(Statement A implies statement B)**

We know $\lambda_1 > 0$, so the restriction on that root is $0 < \lambda_1 < 1$. Hence we seek the implications for $(\alpha_1, \alpha_2, p_{11}, p_{22})$ of the conditions that $\lambda_1 > 0$ and $\lambda_1 < 1$. Considering each case,

$\lambda_1 > 0$: This is true by inspection and imposes no additional restrictions on the policy process.

$\lambda_1 < 1$: This condition implies

$$\sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1 \alpha_2 p_{12} p_{21}} < 2\alpha_1 \alpha_2 - (\alpha_2 p_{11} + \alpha_1 p_{22}). \tag{60}$$

The restriction that $-1 < \lambda_2 < 1$, written as $\lambda_2 > -1$ and $\lambda_2 < 1$, carries further implications for the policy process that delivers a determinate equilibrium. Considering each case,

$\lambda_2 < 1$: This condition implies

$$-\sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1 \alpha_2 p_{12} p_{21}} < 2\alpha_1 \alpha_2 - (\alpha_2 p_{11} + \alpha_1 p_{22}). \tag{61}$$

$\lambda_2 > -1$: This condition implies

$$2\alpha_1 \alpha_2 + (\alpha_2 p_{11} + \alpha_1 p_{22}) > \sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1 \alpha_2 p_{12} p_{21}}. \tag{62}$$

Squaring both sides and simplifying yields

$$\alpha_1 \alpha_2 + p_{11}(1 + \alpha_2) + p_{22}(1 + \alpha_1) > 1 \tag{63}$$

Note that (60) and (61) together imply that

$$\left| \sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1 \alpha_2 p_{12} p_{21}} \right| < 2\alpha_1 \alpha_2 - (\alpha_2 p_{11} + \alpha_1 p_{22}), \tag{64}$$

so it must be the case that $2\alpha_1 \alpha_2 - (\alpha_2 p_{11} + \alpha_1 p_{22}) > 0$ and squaring both sides of (60) preserves the inequality. Doing this and rearranging yields the LRTP

$$\alpha_1 \alpha_2 + p_{11}(1 - \alpha_2) + p_{22}(1 - \alpha_1) > 1. \tag{65}$$

Since, as shown above, when $\lambda_1 < 1$,

$$\sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1 \alpha_2 p_{12} p_{21}} < 2\alpha_1 \alpha_2 - (\alpha_2 p_{11} + \alpha_1 p_{22}), \tag{66}$$

and

$$-\sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1 \alpha_2 p_{12} p_{21}} < \sqrt{(\alpha_2 p_{11} - \alpha_1 p_{22})^2 + 4\alpha_1 \alpha_2 p_{12} p_{21}}, \tag{67}$$

(61) also yields the LRTP

$$\alpha_1 \alpha_2 + p_{11}(1 - \alpha_2) + p_{22}(1 - \alpha_1) > 1. \tag{68}$$

It is straightforward to show that condition (68) implies condition (63). To see this, rewrite the conditions as

$$\alpha_1 \alpha_2 + p_{11} + p_{22} > 1 + \alpha_2 p_{11} + \alpha_1 p_{22} \tag{69}$$

$$\alpha_1 \alpha_2 + p_{11} + p_{22} > 1 - \alpha_2 p_{11} - \alpha_1 p_{22} \tag{70}$$

By the non-negativity of $(\alpha_1, \alpha_2, p_{11}, p_{22})$, it is clear that if (69) holds, then (70) also holds. Given $\alpha_i > p_{ii}$ for $i = 1, 2$ and LRTP, Lemma 1 implies that $\alpha_i > 1$, for some $i = 1, 2$.

(Statement B implies statement A)

Write the LRTP as

$$1 - \left(\frac{p_{11}}{\alpha_1} + \frac{p_{22}}{\alpha_2} \right) > \frac{1 - p_{11} - p_{22}}{\alpha_1 \alpha_2}, \tag{71}$$

and note the eigenvalues satisfy

$$\lambda_1 + \lambda_2 = \frac{p_{11}}{\alpha_1} + \frac{p_{22}}{\alpha_2} \geq 0, \tag{72}$$

since $\alpha_i > p_{ii}$ for $i = 1, 2$ and

$$\lambda_1 \lambda_2 = \frac{p_{11} + p_{22} - 1}{\alpha_1 \alpha_2}. \tag{73}$$

Substituting (72) and (73) into (71), yields

$$\lambda_1 + \lambda_2 - 1 < \lambda_1 \lambda_2. \tag{74}$$

Now (74) implies that $\lambda_1 \neq 1$ and $\lambda_2 \neq 1$ because under the maintained assumption of the LRTP, equality of the roots to unity implies the obvious contradiction that $\lambda_i < \lambda_i$.

Since $\lambda_1 > 0$, (74) is

$$1 - 1/\lambda_1 < \lambda_2(1 - 1/\lambda_1), \tag{75}$$

and (75) implies if $0 < \lambda_1 < 1$ then $\lambda_2 < 1$ or if $\lambda_1 > 1$ then $\lambda_2 > 1$. Thus, (75) states that both eigenvalues are either outside or inside the unit circle.

Now using the assumption that $\alpha_i > p_{ii}$ for $i = 1, 2$, expression (72) becomes

$$0 \leq \lambda_1 + \lambda_2 = \frac{p_{11}}{\alpha_1} + \frac{p_{22}}{\alpha_2} < 2. \tag{76}$$

imposing a restriction on λ_2. We now have that

$$0 < \lambda_1 < 1 \text{ and } |\lambda_2| < 1 \tag{77}$$

because the alternative of $\lambda_1 > 1$ and $\lambda_2 > 1$ violates (76). Clearly, if $0 < \lambda_1 < 1$ and $\lambda_2 \leq -1$, then (76) is violated, leading to $|\lambda_2| < 1$. □

Uniqueness of the bounded solution, which is established by proposition 1, need not imply there is a unique *stationary* solution, as Farmer, Waggoner, and Zha (2006) show. Those authors require a solution to be mean square stable, ensuring the existence of finite first and second moments, and argue that the long-run Taylor principle admits a continuum of solutions, including sunspots. Their solution, which allows lagged states to enter, is not bounded, as it can exceed any finite bound with positive

probability.[23] This implication of stationarity makes their solution at odds with the standard definition of determinacy applied to linear rational expectations models. Boundeness precludes lagged states from entering the solution.

APPENDIX B. DETERMINACY AND SOLUTION FOR THE NEW KEYNESIAN MODEL

The proof of determinacy and the solution method described in appendix A can be applied directly to any purely forward-looking linear model to show that the minimum state variable solution is the unique bounded solution. The equations of the model are

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t^S, \tag{78}$$

$$x_t = E_t x_{t+1} - \sigma^{-1}(\alpha(s_t)\pi_t + \gamma(s_t)x_t - E_t\pi_{t+1}) + u_t^D. \tag{79}$$

The state-contingent expectations are

$$E_t\pi_{t+1} = E[\pi_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1}E[\pi_{1t+1} | \Omega_t^{-s}] + p_{i2}E[\pi_{2t+1} | \Omega_t^{-s}], \tag{80}$$

$$E_t x_{t+1} = E[x_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1}E[x_{1t+1} | \Omega_t^{-s}] + p_{i2}E[x_{2t+1} | \Omega_t^{-s}]. \tag{81}$$

The model can be rewritten as

$$\pi_{it} = \beta \left(p_{i1}E_t\pi_{1t+1} + p_{i2}E_t\pi_{2t+1} \right) + \kappa x_{it} + u_t^S, \tag{82}$$

$$x_{it} = p_{i1}E_t x_{1t+1} + p_{i2}E_t x_{2t+1} - \tag{83}$$

$$\sigma^{-1}\left((\alpha_i\pi_{it} + \gamma_i x_{it}) - (p_{i1}E_t\pi_{1t+1} + p_{i2}E_t\pi_{2t+1}) \right) + u_t^D, \tag{84}$$

where π_{it} and x_{it} are inflation and output at t for $s_t = i$, where $i = 1, 2$. The information set $\Omega_t^{-s} = \{u_t^S, u_{t-1}^S, \ldots, u_t^D, u_{t-1}^D, \ldots s_{t-1}, s_{t-2}, \ldots\}$ denotes the agents' information set at t, not including the current regime, and $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$. All expectations in (78) and (79) are formed conditional on Ω_t. Define the state-contingent forecast errors

$$\eta_{1t+1}^\pi = \pi_{1t+1} - E_t\pi_{1t+1}, \qquad \eta_{2t+1}^\pi = \pi_{2t+1} - E_t\pi_{2t+1}, \tag{85}$$

$$\eta_{1t+1}^x = x_{1t+1} - E_t x_{1t+1}, \qquad \eta_{2t+1}^x = x_{2t+1} - E_t x_{2t+1}. \tag{86}$$

and use them to eliminate the conditional expectations in (82)-(83), yielding the system

$$AY_t = BY_{t-1} + A\eta_t + Cu_t, \tag{87}$$

---

[23]One aspect of Farmer, Waggoner, and Zha's indeterminate solution includes $x_{t+1} = \frac{\alpha_2}{p_{22}}x_t +$ *shocks*. $|\alpha_2/p_{22}| > 1$ is ruled out by boundedness, while $|\alpha_2/p_{22}| < 1$ produces a multiplicity of bounded solutions.

where

$$Y_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix}, \qquad \eta_t = \begin{bmatrix} \eta_{1t}^{\pi} \\ \eta_{2t}^{\pi} \\ \eta_{1t}^{x} \\ \eta_{2t}^{x} \end{bmatrix}, \qquad u_t = \begin{bmatrix} u_t^{S} \\ u_t^{D} \end{bmatrix}, \tag{88}$$

$$A = \begin{bmatrix} \beta \otimes \Pi & 0_{2\times 2} \\ \sigma^{-1} \otimes \Pi & \Pi \end{bmatrix}, \tag{89}$$

$$B = \begin{bmatrix} I_{2\times 2} & \vdots & -\kappa I_{2\times 2} \\ --- & \vdots & ---------- \\ \begin{matrix} \sigma^{-1}\alpha_1 & 0 \\ 0 & \sigma^{-1}\alpha_2 \end{matrix} & \vdots & \begin{matrix} 1 + \sigma^{-1}\gamma_1 & 0 \\ 0 & 1 + \sigma^{-1}\gamma_2 \end{matrix} \end{bmatrix}, \tag{90}$$

$$C = \begin{bmatrix} -1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}. \tag{91}$$

The roots of the system are the generalized eigenvalues of (B, A), where a unique, bounded equilibrium requires all four eigenvalues to lie inside the unit circle. Obtaining analytical restrictions on the roots that deliver determinacy are more complicated in the new Keynesian model because, as was highlighted in the text, the determinacy regions vary with private sector parameters. To establish determinacy, note that the model can be written so current state-contingent variables depend only on the expectations of future state-contingent variables, as in (11). Writing the model in this form highlights that the model has the same structure as the simple Fisherian model.

Solutions for the model are derived using the method of undetermined coefficients. However, given that the expectational errors in (87) are conditionally mean zero, standard methods for solving linear rational expectations models can be used to compute the solution. For example, Sims's (2001) method and corresponding gensys code produces a solution matching the method of undetermined coefficients solution. McCallum (2004) notes that in purely forward looking models, such as (87), the method of undetermined coefficients using the minimum set of state variables yields the unique, bounded solution.

In obtaining the solution, we assume supply and demand shocks are uncorrelated, so the coefficients on the demand shocks and those on the supply shocks can be solved separately. Coefficients on the supply shock come from solving for the undetermined

coefficients in

$$
\begin{bmatrix}
1 - \beta p_{11} \rho_S & -\beta \rho_S (1 - p_{11}) & -\kappa & 0 \\
-\beta \rho_S (1 - p_{22}) & 1 - \beta p_{22} \rho_S & 0 & -\kappa \\
\frac{1}{\sigma}(\alpha_1 - \rho_S p_{11}) & -\frac{\rho_S}{\sigma}(1 - p_{11}) & 1 + \sigma^{-1}\gamma_1 - p_{11}\rho_S & -\rho_S (1 - p_{11}) \\
-\frac{\rho_S}{\sigma}(1 - p_{22}) & \frac{1}{\sigma}(\alpha_2 - p_{22}\rho_S) & -\rho_S (1 - p_{22}) & 1 + \sigma^{-1}\gamma_2 - p_{22}\rho_S
\end{bmatrix}
\begin{bmatrix}
a_1^S \\
a_2^S \\
b_1^S \\
b_2^S
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
1 \\
1 \\
0 \\
0
\end{bmatrix}
$$

and those on demand shocks from solving for the coefficients in

$$
\begin{bmatrix}
1 - \beta p_{11} \rho_D & -\beta \rho_D (1 - p_{11}) & -\kappa & 0 \\
-\beta \rho_D (1 - p_{22}) & 1 - \beta p_{22} \rho_D & 0 & -\kappa \\
\frac{1}{\sigma}(\alpha_1 - \rho_D p_{11}) & -\frac{\rho_D}{\sigma}(1 - p_{11}) & 1 + \sigma^{-1}\gamma_1 - \rho_D p_{11} & -\rho_D (1 - p_{11}) \\
-\frac{\rho_D}{\sigma}(1 - p_{22}) & \frac{1}{\sigma}(\alpha_2 - p_{22}\rho_D) & -\rho_D (1 - p_{22}) & 1 + \sigma^{-1}\gamma_2 - p_{22}\rho_D
\end{bmatrix}
\begin{bmatrix}
a_1^D \\
a_2^D \\
b_1^D \\
b_2^D
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
0 \\
0 \\
1 \\
1
\end{bmatrix}.
$$

Analytical expressions for the coefficients are not easy to interpret, but are straightforward to compute. These coefficients are the impact elasticities of the various shocks on output and inflation.

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	Demand		Supply	
	Inflation	Output	Inflation	Output
Active Regime	1.152	0.936	1.090	1.022
Passive Regime	2.650	1.980	2.866	0.359

TABLE 1. **Standard deviation relative to fixed active regime.** Uses Lubik and Schorfheide's estimated policy parameters—$\alpha_1 = 2.19, \gamma_1 = .30, \alpha_2 = .89, \gamma_2 = .15$—and baseline parameters—$\beta = .99, \sigma = 1, \kappa = .17$. Transition probabilities are $p_{11} = p_{22} = .95$. Fixed active regime is $\alpha = 2.19, \gamma = .30$.

	$p_{11} = .95$				$p_{11} = .975$			
	Demand		Supply		Demand		Supply	
	Inflation	Output	Inflation	Output	Inflation	Output	Inflation	Output
$p_{22} = 0$								
$\alpha_2 = .5$	1.044	1.008	1.075	.995	1.022	1.004	1.037	.998
$\alpha_2 = .25$	1.060	1.011	1.092	.994	1.030	1.005	1.045	.997
$\alpha_2 = 0$	1.073	1.014	1.110	.992	1.037	1.007	1.054	.997
$p_{22} = .5$								
$\alpha_2 = .5$	1.084	.988	1.143	1.008	1.042	.993	1.071	1.004
$\alpha_2 = .25$	1.120	.983	1.185	1.010	1.059	.990	1.091	1.006
$\alpha_2 = 0$	1.165	.977	1.238	1.013	1.080	.987	1.115	1.007
$p_{22} = 2/3$								
$\alpha_2 = .5$	1.123	.961	1.209	1.025	1.061	.979	1.104	1.014
$\alpha_2 = .25$	1.188	.940	1.290	1.034	1.092	.968	1.142	1.018
$\alpha_2 = 0$	1.283	.910	1.408	1.048	1.135	.953	1.194	1.025
$p_{22} = .75$								
$\alpha_2 = .5$	1.162	.931	1.275	1.044	1.080	.963	1.137	1.024
$\alpha_2 = .25$	1.268	.886	1.412	1.066	1.129	.940	1.199	1.034
$\alpha_2 = 0$	1.454	.807	1.653	1.104	1.210	.903	1.302	1.052

TABLE 2. **Standard deviation in active regime 1 relative to fixed regime.** Active and fixed regimes set $\alpha_1 = \alpha = 1.5$ and $\gamma_1 = \gamma = .25$. Passive regime sets $\gamma_2 = .5$. Ergodic probability of active regime ranges from .83 $(p_{11} = .95, p_{22} = .75)$ to .98 $(p_{11} = .975, p_{22} = 0)$.

	α	$\bar{\gamma}$	δ	θ
Regime 1	2.182	0.30	-1.690	0.409
Regime 2	0.885	0.15	-0.750	1.675
Full Sample	1.375	0.225	-1.476	0.657

TABLE 3. **Estimated parameters from an identified VAR in (42) using simulated data from regime-switching new Keynesian model.** Regime 1 is conditional on remaining in regime with $\alpha_1 = 2.19$; Regime 2 is conditional on remaining in regime with $\alpha_2 = 0.89$; Full sample is recurring changes from regime 1 to regime 2. α is the estimated response of monetary policy to inflation; $\bar{\gamma}$ is the policy response to output, held fixed in estimation.
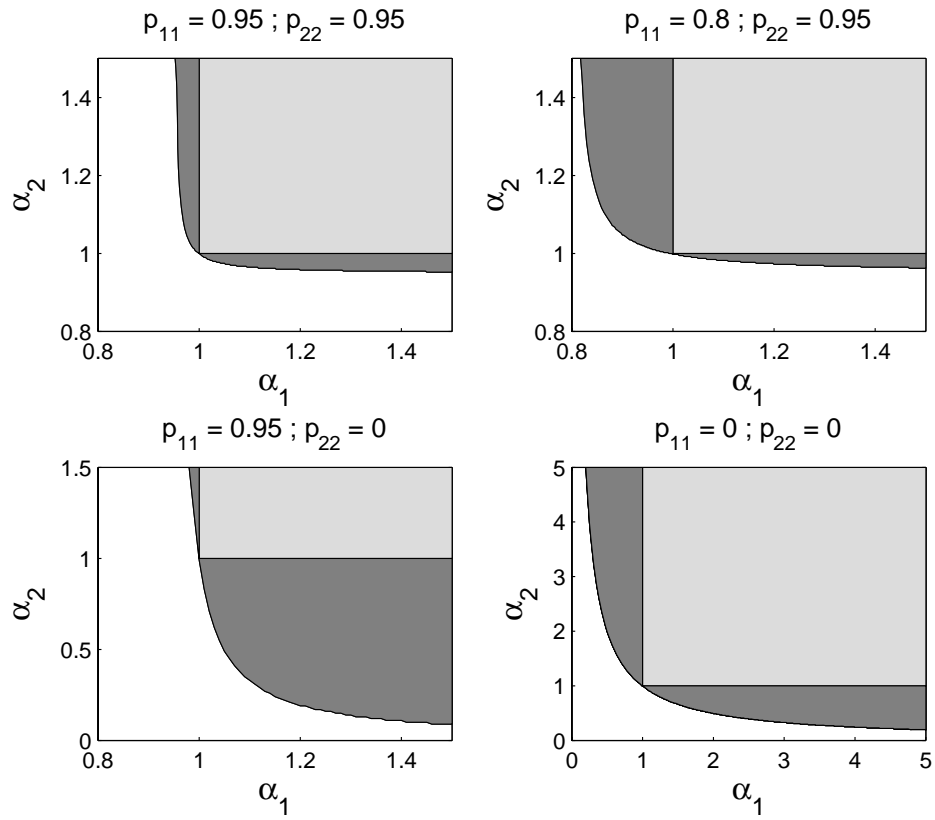
FIGURE 1. **Determinacy regions: Fisherian model.** Parameter combinations in the light-shaded regions imply a unique equilibrium in fixed-regime model; combinations in dark-shaded plus light-shaded regions imply a unique equilibrium in regime-switching model.
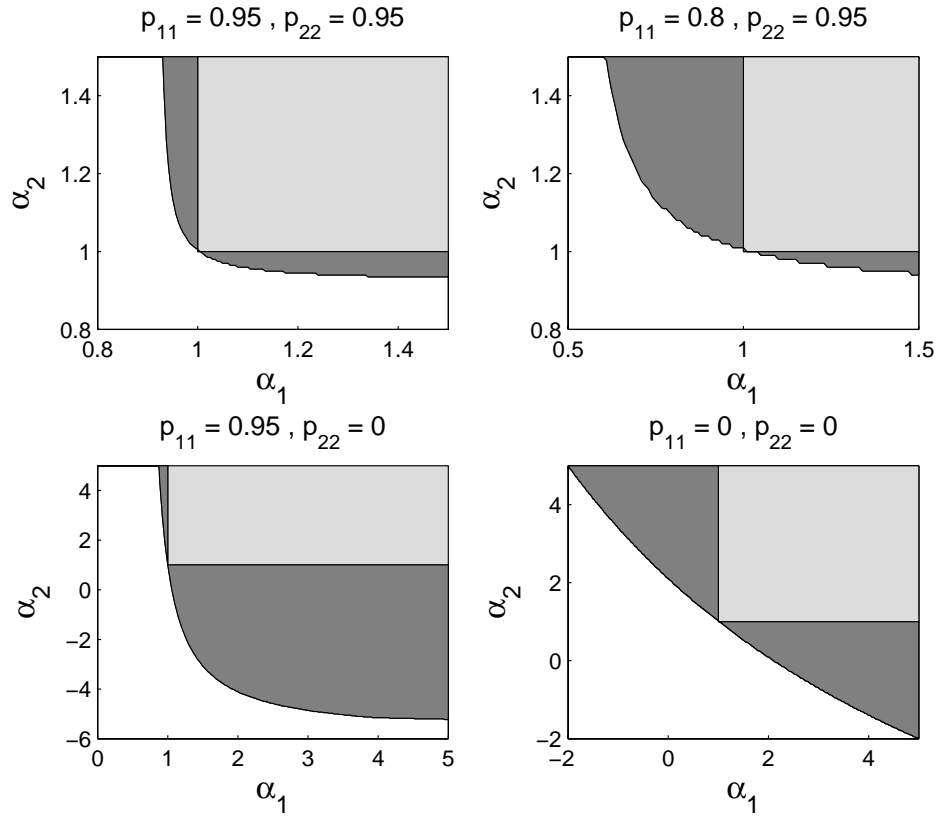
FIGURE 2. **Determinacy regions: New Keynesian model.** Parameter combinations in the light-shaded regions imply a unique equilibrium in fixed-regime model; combinations in dark-shaded plus light-shaded regions imply a unique equilibrium in regime-switching model.
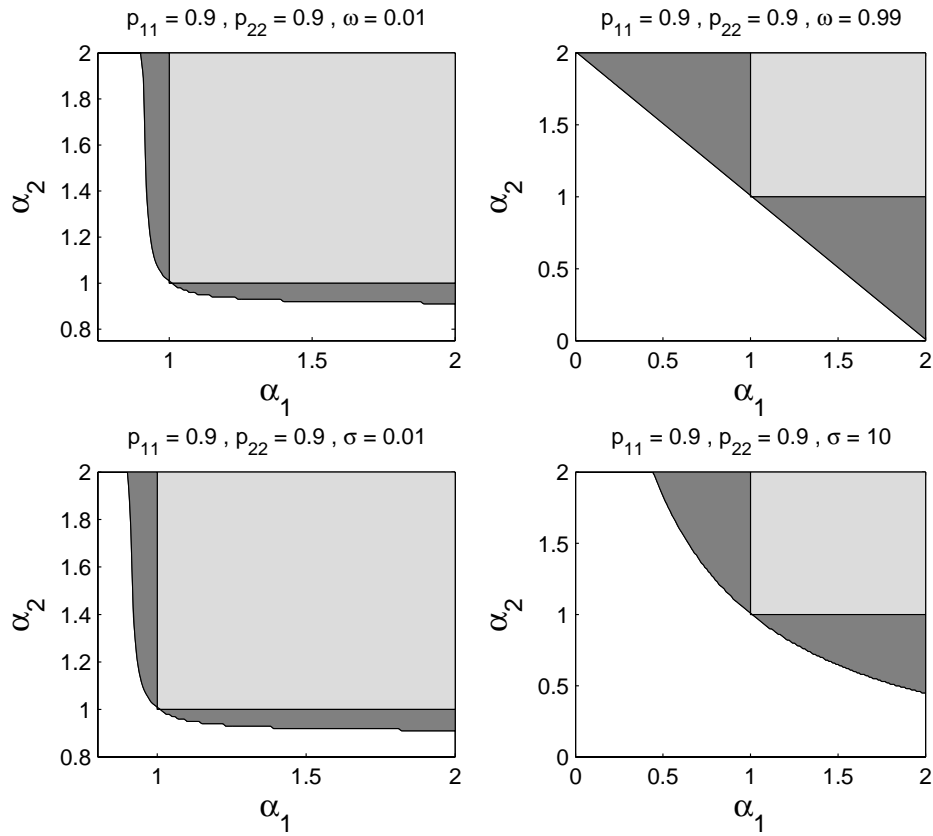
FIGURE 3. **Determinacy regions and private parameters: New Keynesian model.** Parameter combinations in the light-shaded regions imply a unique equilibrium in fixed-regime model; combinations in dark-shaded plus light-shaded regions imply a unique equilibrium in regime-switching model for various settings of ω and σ.
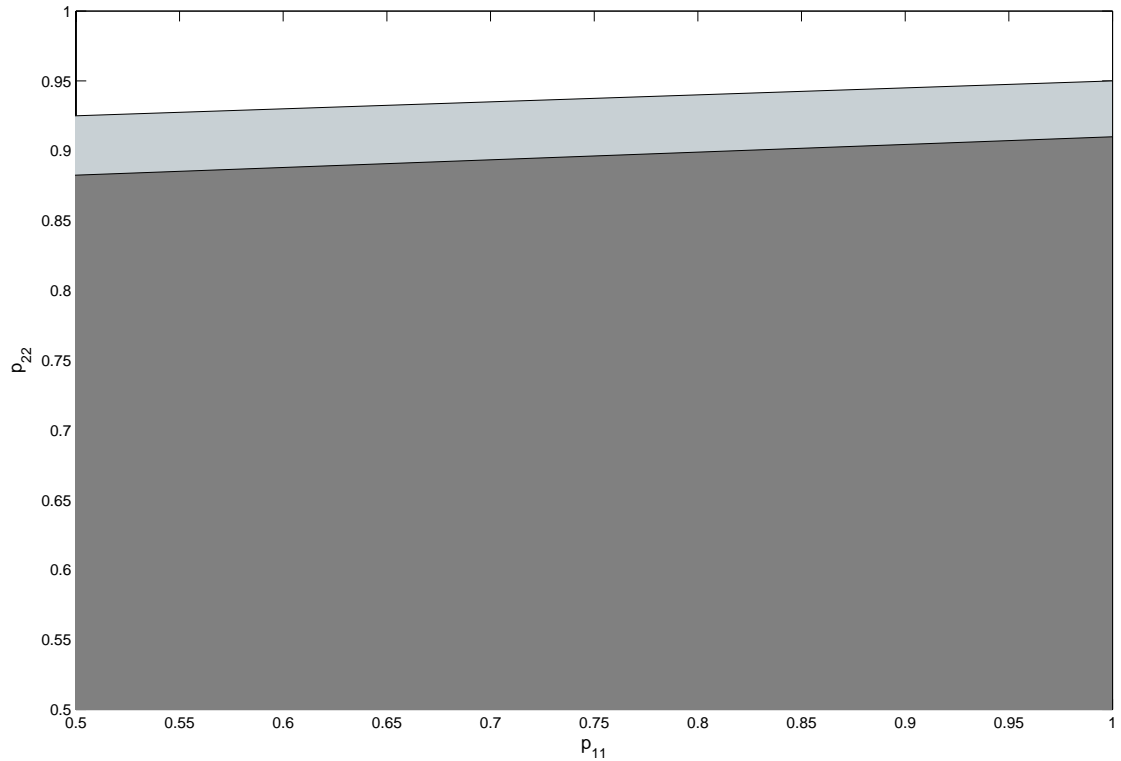
FIGURE 4. **Determinacy regions for Lubik and Schorfheide's estimates.** Shaded regions give (p_{11}, p_{22}) combinations that yield a determinate equilibrium. Dark region is for parameters implying high degree of flexibility and substitution ($\sigma = 1.04, \kappa = 1.07$); light region plus dark region for a low degree of flexibility and substitution ($\sigma = 2.84, \kappa = 0.27$); $\beta = .99$ in both regions. Using Lubik and Schorfheide's (2004) estimates: $\alpha_1 = 2.19, \gamma_1 = .30, \alpha_2 = .89, \gamma_2 = .15$.
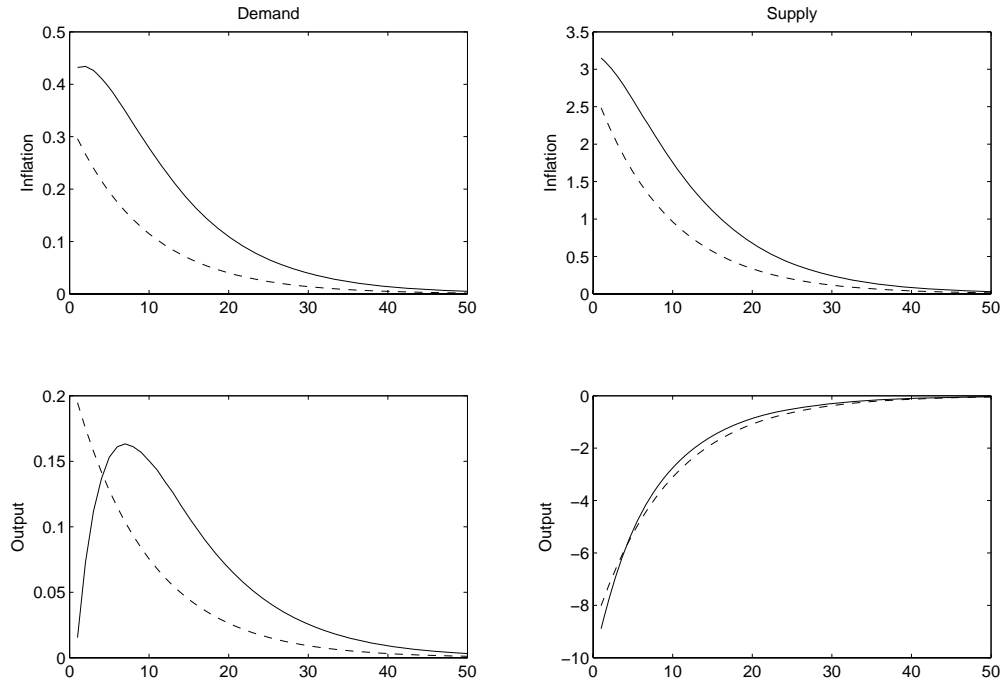
FIGURE 5. **Demand and supply shocks under Lubik and Schorfheide's estimates of policy parameters.** Solid line is conditional on active regime initially ($\alpha_1 = 2.19, \gamma_1 = .30$) when other regime is passive ($\alpha_2 = .89, \gamma_2 = .15$). Transition probabilities are $p_{11} = .95, p_{22} = .93$. Dashed line is fixed regime with $\alpha = \alpha_1, \gamma = \gamma_1$. Figures plot the mean responses from 50,000 draws of regime, beginning in the second period.