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PORTFOLIO DIVERSIFICATION, REAL INTEREST RATES, AND THE BALANCE OF PAYMENTS

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ABSTRACT

The paper shows that differences in real interest rates across countries can arise even with perfect competition and fully integrated international capital markets. Specifically, we find that factor returns will differ across countries which are identical except for differences in technological riskiness, overall productivity, or labor force size. We also show that differences across countries in technological riskiness, in risk aversion, in population size and in overall productivity will lead to a non-zero current account in the steady state. Higher technological riskiness, greater risk aversion, and a larger population should be associated with a current account surplus.

The analysis is carried out using a two-country Diamond overlapping-generations model in which technological uncertainty is reflected in factor returns.

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PORTFOLIO DIVERSIFICATION, REAL INTEREST RATES, AND THE BALANCE OF PAYMENTS

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This paper addresses two issues. First, we ask "Under what circumstances will the long run current account balance between two countries differ from zero?" In 1981 Buiter showed that a difference across countries in consumers' rate of time preference is sufficient to cause a steady-state current account deficit for the country whose consumers have a greater preference for current consumption. Here we extend Buiter's analysis to show that differences in technological riskiness, in risk aversion, in population size and in overall productivity will also lead to a non-zero current account in the steady state. More specifically, we find that a current account surplus should be associated with higher technological riskiness, with greater risk aversion, and with a larger population.

The second issue addressed in this paper is the origin of inter-country differences in real interest rates. Recent evidence (Mark 1985; Cumby and Mishkin 1986) shows that such differences exist and that they are strongly associated with deviations from open interest parity (Buckles 1985). The present paper shows how these differences can arise even with perfect competition and fully integrated international capital markets. Specifically, we find that factor returns will differ across countries which are identical except for differences in technological riskiness, overall productivity, or labor force size. We also note that when countries produce distinct goods there are additional reasons for factor returns to difference, including differences in consumer preferences such as rate of time preference or level of risk aversion.

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A positive relationship between capital risk and returns such as we find in our analysis is consistent with the analysis of many earlier researchers, such as Solnick (1974a), Grauer, Litzenberger, and Stehle (1976), de Macedo (1981), and Stulz (1981). These authors have little to say about wages and capital stocks, however, since their models take these variables as fixed. Grossman and Razin (1984) also study a model of trade in capital under nation-specific uncertainty though in that model the world's capital supply can move across countries. They find that the capital stock of the country with higher technological riskiness will be lower than the stock of the other country, as we do, but they conclude that there will be no associated differences in factor returns across countries.

The fact that equity returns could differ for countries which only differ in their overall productivity or in the size of their labor force is startling, since we are dealing with a general equilibrium situation in which capital markets are entirely integrated. This result undermines obliquely the condition usually considered sufficient for open interest parity to hold under open capital markets, namely that assets are "perfect substitutes", since it implies that "perfect subsitutability" is only a sufficient condition under the strictest possible interpretation: equal variability and perfect correlation of returns across countries. For example, our analysis implies that it is internally inconsistent to say that "the Fisher open hypothesis ...holds if and only if securities that are similar except for currency of denomination are perfect substitutes" (Boughton 1986, page 9), since the portfolio diversification motive makes assets imperfect substitutes automatically once they are denominated in different currencies.

The paper has four parts. Following this Introduction, Part II describes our model, which is a two-country Diamond (1965) overlapping-generations model with technological uncertainty. In Part III we present our main results, and in Part IV we conclude with some related observations.

PART II: THE MODEL

II.A. Autarchy

We will begin this section by explaining the production side of one overlapping-generations economy, follow that with a discussion of consumer behavior, and finish with a description of the steady-state equilibrium under autarchy.

Production

Output of each firm, where firms are indexed by i, is generated from inputs of labor and capital according to the stochastic production function

$$Q_t^i = F(K_t^i, L_t^i) + \alpha_t K_t^i \quad ,$$

where α_t is an i.i.d. random variable distributed over the interval $[-d,d]$, $0 < d < 1$, with mean zero and variance σ^2.[1] For specificity we will assume that $F(K,L)$ is Cobb-Douglas, with the form: $F(K,L) = DK^aL^{1-a}$, $0 < a < 1$, and refer to D as the "overall productivity parameter". Since $F(K,L)$ is linear-homogeneous, we can re-express production in terms of per-worker output, $q = Q/L$, and the capital-labor ratio, $k = K/L$:

$$q_t^i = f(k_t^i) + \alpha_t k_t^i \quad .^2$$

The exogenously determined labor force is fully employed in each period, as is the endogenously determined stock of capital. Factor markets are assumed to be perfectly competitive, in consequence of which we can aggregate firms' output and express total output in terms of total capital and labor:

$$Q = F(K,L) + \alpha K = Lq = L[f(k) + \alpha k].$$

Another implication of perfect competition in the factor markets is that labor and capital will always be paid their marginal product: wages will be $w = f(k) - kf'(k)$, and the actual return to capital will be $\underline{r} = f'(k) + \alpha$, with expectation $r = f'(k)$.

Consumption

During any period t a new generation of individuals is born which is n percent larger than the previous generation. Each member of this new generation, who will liver for two periods, works during period t earning the prevailing wage, w_t, consumes some (proper) fraction of his income, and invests the rest. The members of the older generation living in period t do not work, and consume their interest-augmented savings. Denoting per-worker savings of the young as s_t, we can see that per-worker output available for consumption in period $t, f(k_t) + \alpha_t k_t + k_t$, will be exhausted by consumption of the young, $w_t - s_t$, investment by the young, s_t, and per-worker consumption of the old, $(1 + r_t + \alpha_t)k_t$, where $k_t = s_{t-1}/(1+n)$. It is convenient to imagine that savings is used to purchase equities from firm managers, who in turn employ the income derived from equity sales as capital. The equities are denominated in terms of the single good and their stochastic return is identical to the return per unit of capital employed.

All individuals have the same utility function: $U(c_1) + \beta V(c_2)$, where $\beta = 1/(1+\rho)$, and ρ is the rate of time preference, so $0 < \beta \leq 1$. We assume that $U(\,)$ is twice continuously differentiable, with $U' > 0$, $U'' < 0$. To simplify the mathematics of uncertainty, we specify the following functional form for V( ):

$$V(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

and assume further that $\gamma = 1/2$.

The value of s_t, which determines generation t's entire (expected) lifetime consumption profile, is chosen by them to solve

$$\max_{s} E_t\{U(w_t - s_t) + \beta V[s_t(1 + r_{t+1} + \alpha_{t+1})]\} \quad ,$$

taking w_t, r_{t+1} and the distribution of α_{t+1} as given. The first-order condition for this problem implies:

$$U'(w_t - s_t) = E\{V'(1 + r_{t+1} + \alpha_{t+1})\}.$$

The consumers' desired level of savings will be determined implicitly as a function of w_t, r_{t+1}, and the system parameters σ^2, β, and γ: $s = s(w,r,\sigma^2,\beta,\gamma)$. A simple comparative statics exercise shows that $0 < s_w < 1$, $s_r > 0$, $s_\sigma < 0$, $s_\beta > 0$, and $s_\gamma < 0$ (where $s_w = \partial s/\partial w$, etc.).[3]

Both the unambiguously positive response of savings to r and its unambiguously negative response to σ^2 are driven by the assumption that $0 < \gamma < 1$. That is, $0 < \gamma < 1$ assures that the substitution effect of the change in expected returns dominates the income effects, and that the same is true for analogous "income" and "substitution" effects of a rise in σ^2. This less familiar "income effect" is the increase in savings required to achieve the same expected future utility, in light of which we can think of $d\sigma^2 > 0$ as an increase in the relative price of future expected utility, which could induce "substitution" away from it. With the utility function of this paper, substitution effects will dominate for interest rate and the variance of future consumption, or not at all (Sandmo 1970).[4,5]

The empirical evidence regarding the size of γ is mixed, so we assume $0 < \gamma < 1$ because it brings the model of this paper into consistency with those of its forebearers, such as Buiter (1981), Blanchard (1985), and Kole (1985), where $s_r > 0$ is assumed. It is usually not difficult to ascertain how the results presented here would differ if income effects dominated substitution effects.

Temporary Equilibrium

Three equilibrium conditions characterize the economy in each period:

$$U'(w_t - s_t) = E\{V'(1 + r_{t+1} + \alpha_{t+1})\}$$
$$r_{t+1} = E\{f'(k_t) + \alpha_{t+1}\} = f'[s_t/(1+n)]$$
$$w_t = f(k_t) - k_t f'(k_t).$$

The first two of these conditions comprise a system in the two endogenous variables s_t and r_{t+1}, which could be re-expressed as $s_t = s(w_t, r_{t+1}; \sigma^2, \beta, \gamma)$ and $r_{t+1} = r(s_t)$. We know that $s_r(\ ) > 0$ and $r'(\ ) < 0$, so the assumption that these two functions intersect is sufficient to assure an unique equilibrium, the location of which depends on w_t, σ^2, β, and γ. Henceforth we will discuss optimum savings only at its economy-wide equilibrium value, $\tilde{s}(w_t; \sigma^2, \beta, \gamma)$. The signs of the responses of $\tilde{s}_t$ to changes in w_t, σ^2, and the parameters of the utility function are unchanged from those of the partial equilibrium setting described above, though the magnitudes of the effects differ. For example, $\tilde{s}_\sigma$ is smaller (in absolute value) than s_σ because a decrease in savings increases the expected return to equities for the next period, mitigating the original response.[6]

Steady-State Equilibrium

This model will have no "steady state" in the normal sense, because it is stochastic: in particular, the path of the actual return to equities, $\underline{r}_t = r_t + \alpha_t$, is necessarily uncertain. The paths of w_t, k_t, and r_t can be determined with certainty, however, if we know the initial value of one of them. The fact that $f(k)$ is strictly increasing and twice continuously differentiable ensures that we can find functions $k(w)$ and $r(w)$, the latter of which is commonly known as the "factor-price frontier". A first-order difference equation in capital can be generated by using the familiar relations

$$k_t = \tilde{s}_t(w_{t-1})/(1+n), \tag{1a}$$

$$w_t = f(k_t) - k f'(k_t), \tag{1b}$$

since together these imply:

$$k_{t+1} = \frac{\tilde{s}[f(k_t) - k f'(k_t)]}{(1+n)}. \tag{2}$$

where the dependence of $\tilde{s}_t$ on σ^2, β, and γ has been suppressed. Equation (2) allows us to trace the behavior of k_t from some initial period through time indefinitely. The steady state of this system is a value k^* such that $k_{t+1} = k_t$. We can be sure that such a fixed point exists, since $k(w)$ is bounded and continuous (LaSalle, 1976). It is also unique.[7]

The response of steady-state equilibrium wages, capital stock and expected interest rates to changes in the parameters β, γ, and σ^2 can be ascertained by totally differentiating equation (1c) evaluated at k^*. These results are summarized in the following propositions, the proof of which can be found in Appendix A:

Proposition 1: *Given two otherwise identical economies under autarchy, the one in which (i) consumers have a higher rate of time preference, (ii) consumers are more risk averse, or (iii) output is subject to greater variability, will have the lowest capital stock and wages and the highest expected returns to capital .*

Proposition 2: *Under autarchy, economies in which production has higher overall productivity will have a higher capital stock and wages than otherwise identical countries.*

The mechanism that drives Part (i) of Proposition 1 is straightforward: a higher rate of time preference leads to a decline in savings at every income level, which is reflected in the capital stock, wages, and equity returns. This result is no more than an application of Buiter's Proposition 4 (1981) to in a more complex model.

In Proposition 1, Parts (ii) and (iii), we begin extending to a broader spectrum of system parameters Buiter's analysis of how differences in underlying economic tendencies affect autarchic and international equilibria. Part (ii) notes that a rise in risk aversion has a similar effect, in autarchy, to a rise in the rate of time preference. Once again, the driving factor is savings behavior, as it is in Proposition 1, Part (iii),where we begin to consider the effects of technology parameters as well as preferences: as discussed earlier, when σ^2 rises, consumers perceive future expected utility as more expensive in terms of current utility, and save less in consequence.

We continute this line of questioning in Proposition 2, which tells us the effects on the autarchy equilbrium of a rise in the overall productivity parameter D. The unambiguously positive effect of such a change on wages and savings is easy enough to understand. The effect of a rise in D on the expected returns to equities is ambiguous in sign because there are conflicting tendencies: the rise in D itself would tend to raise expected interest rates, but the concomitant rise in savings will tend to lower them.

II.B Two-Country Equilibrium

Suppose now that there is a "home" country and a "foreign" country, both of which produce the same good, and which freely trade both output and equities. (Unless otherwise specified, the word "domestic" will refer to the home country, while foreign values of endogenous and exogenous variables will be denoted by a superscript ^.)

Production

Each country has an underlying production function of the same type that we have assumed in earlier chapters. Denote foreign production technology as:

$$\widehat{Q} = G(\widehat{K},\widehat{L}) + \widehat{\alpha}\widehat{K} = \mathrm{D}\widehat{K}^{a}\widehat{L}^{1-a} + \widehat{\alpha}\widehat{K}.$$

Labor supplies , L and $\widehat{L}$, may differ across countries, as well as D and $\widehat{D}$, the overall productivity parameters, though we will assume throughout that the coefficient "a" and the rates of population growth, n, do not. We will represent the ratio $\widehat{L}/L$ as "φ". The random productivity shocks, α at home and $\widehat{\alpha}$ abroad, have mean zero, variances σ^2 and $\widehat{\sigma}^2$, respectively, and correlation coefficient η. Until recently, most models of trade under uncertainty assumed perfect correlation of uncertainty across countries (Batra 1975; Helpman and Razin 1978). We join two recent papers by Grossman and Razin (1984, 1985) in assuming that α and $\widehat{\alpha}$, are imperfectly correlated. As discussed later, their analysis provides an interesting counterpart to our own.

Consumption

Consumers must still choose a savings level, but now they must also choose the allocation of their savings between the assets of both countries. Savers will invest in "equities" which are denominated in terms of the output of the country from which they are purchased, and have stochastic returns corresponding identically to that country's return to capital. "π" will represent the share of domestic savings invested in home equities, and the share of foreign savings invested in domestic equities will be "$\widehat{\pi}$". Choice of π will contribute to determining the realized value, expected value, and variance of individuals' returns to savings as follows:

$$R_{t+1} = 1 + \pi_t(r_{t+1} + \alpha_{t+1}) + (1-\pi_t)(\hat{r}_{t+1} + \widehat{\alpha}_{t+1}), \tag{3a}$$

$$E\{R_{t+1}\} = 1 + \pi_t r_{t+1} + (1-\pi_t)\hat{r}_{t+1}, \tag{3b}$$

$$Var(R_{t+1}) = \pi_t^2\sigma^2 + (1-\pi_t)^2\widehat{\sigma}^2 + 2\pi_t(1-\pi_t)\sigma\widehat{\sigma}\eta . \tag{3c}$$

The optimization problem facing domestic residents,

$$\underset{s,\,\pi}{Max}\ U(w_t - s_t) + \beta E\{V(s_t R_{t+1})\},$$

has first-order conditions

$$U' = \beta E\{V' R_{t+1}\} \tag{4a}$$

and

$$E\{V'\}(r_{t+1} - \hat{r}_{t+1}) = E\{V'(\alpha_t - \widehat{\alpha}_t)\} . \tag{4b}$$

The interpretation of equation (4a), which describes consumers' optimal savings level, has already been discussed in Section II. Equation (4b) describes equilibrium portfolio shares. As equations (3b) and (3c) illustrate, a change in π_t will affect expected retirement utility through R_{t+1} and $Var(R_{t+1})$. Equilibrium condition (4b) states that at the margin, the change in expected utility from these two factors should be equal and opposite.

The solution value for π_t is independent of s_t, a property associated with the iso-elastic utility function. π_t can be decomposed into two terms, a "minimum variance portfolio share", π_m, and a "speculative portfolio share", π_s.[8] π_m minimizes portfolio variance, without regard to expected return:

$$\pi_m = \frac{\hat{\sigma}^2 + \sigma\hat{\sigma}\eta}{\sigma^2 + \hat{\sigma}^2 + \sigma\hat{\sigma}\eta} .$$

Since consumers care about return as well as risk, the actual solution value for π is:

$$\pi^* = \pi_m + \frac{(r-\hat{r})R_m\xi}{P(\gamma)} \equiv \pi_m + \pi_s \qquad (5)$$

where R_m is the expected return on the minimum variance portfolio, $P(\gamma) > 0$ for most plausible parameter values, and $P'(\gamma) > 0$. Equation (5) indicates that the optimum shares of domestic assets will be the sum of the minimum variance share and a term which is an increasing function of the divergence $(r - \hat{r})$, where the size of the response is affected by risk preference. Specifically, the more risk-averse the individual, the less he will allow return considerations to affect portfolio choices. π_s can be viewed as a share in a zero net worth "speculative portfolio", since it appears with the opposite sign in the expression for the optimal share of foreign securities, $1-\pi^* = (1-\pi_m) - \pi_s$.

Temporary Equilibrium

Ten conditions that characterize the temporary equilibrium: the four consumers' first-order conditions, corresponding to s_t, π_t, $\hat{s}_t$, and $\hat{\pi}_t$; the four factor-market clearing conditions, corresponding to w_t, r_{t+1}, $\hat{w}_t$, and $\hat{r}_{t+1}$; and the definition of domestic and foreign capital stocks:

$$k_{t+1} = \frac{\pi_t s_t + \hat{\pi}_t \hat{s}_t \varphi}{1+n} , \qquad \hat{k}_{t+1} = \frac{(1-\pi_t)s_t/\varphi + (1-\hat{\pi}_t)\hat{s}_t}{1+n} .$$

These show that, unless equilibrium is characterized by a corner solution where $\pi_t = 1 - \hat{\pi}_t = 1$, the domestic capital stock will be owned in part by foreigners, while the foreign capital stock will be owned in part by domestic residents. Thus domestic wages and equity returns are directly determined by the savings and portfolio behavior of residents of both countries. Anything that changes such behavior, such as a change in income or in relative risk aversion, will have direct effects on the capital stock of both countries.

The capital account, per domestic worker, equals capital inflow minus capital outflow:

$$\kappa_t \equiv [\hat{\pi}_t \hat{s}_t \varphi - (1-\pi_t)s_t] - \left(\frac{1}{1+n}\right)[\hat{\pi}_{t-1}\hat{s}_{t-1}\varphi - (1-\pi_{t-1})s_{t-1}]$$

The current account is the negative of the capital account, and the trade balance is the difference between the current account and net capital service inflow, or:

$$tb_t = -\kappa_t - \left(\frac{1}{1+n}\right)\{(\hat{r}_t+\hat{\alpha}_t)(1-\pi_{t-1})s_{t-1} - (r_t+\alpha_t)\hat{\pi}_{t-1}\hat{s}_{t-1}\varphi\}.$$

The balance of payments will automatically be in equilibrium whenever consumers satisfy their budget constraints.

Steady-State Equilibrium

The long-run steady-state values of k and $\hat{k}$ will be the solutions to the following system of nonlinear equations:

$$k(w,\hat{w}) = \frac{\pi^*(.)s^*\varphi + \hat{\pi}^*(.)\hat{s}^*}{1 + n}$$

$$\hat{k}(w,\hat{w}) = \frac{[1-\pi^*(\cdot)]s^*/\varphi + [1-\hat{\pi}^*(\cdot)]\hat{s}^*}{1 + n}$$

where

$$w^* = f[k(w^*,\hat{w}^*)] - k(w^*,\hat{w}^*)f'[k(w^*,\hat{w}^*)]$$

$$\hat{w}^* = g[\hat{k}(w^*,\hat{w}^*)] - \hat{k}(w^*,\hat{w}^*)g'[\hat{k}(w^*,\hat{w}^*)]$$

and the arguments in $s^*(.)$ and $\pi^*(.)$ are w^*, $\hat{w}^*$, r^*, $\hat{r}^*$, σ^2, $\hat{\sigma}^2$, η, D, $\hat{D}$, φ, n, and a. Since w and $\hat{w}$ are bounded, this system has a nonempty set of fixed points (LaSalle 1976). For the case ofsymmetric countries, we will assume that the solution $w^* = \hat{w}^*$ is one of these fixed points, and in fact is the unique fixed point. The condition for local stability of the system is:

$$0 < 1 - m\pi s_w - \hat{m}(1-\hat{\pi})\hat{s}_w < 1,$$

where

$$m \equiv {}^{\partial w}\!/_{\partial k} = {}^{-kf''}\!/_{(1+n)} > 0,$$

and $\widehat{m}$ is defined accordingly. We will assume that $m, \widehat{m} \leq 1$, which ensures that this stability condition is satisfied.

The steady-state capital account and trade balance will appear as follows:

$$\kappa^* = \left(\frac{n}{1+n}\right)\left(\widehat{\pi}^*\widehat{s}^*\varphi - (1-\pi^*)s^*\right) ,$$

$$tb^* = \left(\frac{1}{1+n}\right)\left(\left(r^* + \alpha_t - n\right)\widehat{\pi}^*\widehat{s}^*\varphi - \left(\widehat{r}^* + \widehat{\alpha}_t - n\right)(1-\pi^*)s^*\right) .$$

PART III: FACTOR PRICES AND THE BALANCE OF PAYMENTS UNDER FREE TRADE IN GOODS AND CAPITAL

III.A. Identical Countries

We will begin our analysis of this economic system at the steady-state equilibrium corresponding to synmmetry across countries: $\beta = \widehat{\beta}$, $\sigma^2 = \widehat{\sigma}^2$, $\varphi = 1$, etc. (The transition to the steady-state is analyzed in Osler 1987b.) Factor prices will be the same everywhere, implying equal domestic and foreign capital stocks. Savings will also be equal in the two countries, and all portfolios will be evenly divided between home and foreign equities. With so much symmetry, the fact that the (expected) value of each consituent element of the balance of payments will be zero can be proven with a simple glance at the expressions which describe them. Since individuals hold diversified portfolios both countries will be both importers and exporters of equities, though there will be no net capital flows.

III.B. Asymmetric Countries

Now we will analyze the two-country equilibria under isolated asymmetries between countries. We analyze specifically how differences in technology and utility parameters affect the steady-state equilibrium.

To find the comparative statics of this system we begin by reducing it as far as possible analytically, which leaves us with four equations in the four endogenous variables s, π, $\hat{s}$, and $\hat{\pi}$. We then totally differentiate the system, evaluated at the steady state, and find the comparative statics for those four variables, from which we calculate steady-state changes in the other endogenous variables, including capital stocks, wages, equity returns, and the balance of payments.

The general equations describing the capital stocks responses are:

$$(1+n)\frac{dk^*}{dz} = \left\{s^* \frac{d\pi^*}{dz} + \hat{s}^* \frac{d\hat{\pi}^*}{dz}\right\} - \left\{\pi^* \frac{ds^*}{dz} + \hat{\pi}^* \frac{d\hat{s}^*}{dz}\right\} + \frac{\partial k}{\partial z},$$

$$(1+n)\frac{d\hat{k}^*}{dz} = -\left\{s^* \frac{d\pi^*}{dz} + \hat{s}^* \frac{d\hat{\pi}^*}{dz}\right\} - \left\{\pi^* \frac{ds^*}{dz} + \hat{\pi}^* \frac{d\hat{s}^*}{dz}\right\} + \frac{\partial \hat{k}}{\partial z}.$$

where a superscript * now indicates the initial steady-state value of the associated variable.

Changes in capital stocks consist of a portfolio balance effect (the first term on the right-hand-side), a savings effect (the second term), and any direct effects, which in fact are associated only with changes in $\hat{L}/L$. The presence of the portfolio balance effect is important, because it is here that we begin to see the implications of embedding an explicit derivation of optimal international portfolios in a long-term model. Allowing the menu of national assets to include real equities implies that changes in optimal portfolio shares will affect the long-run levels of real factor returns, and welfare, in both countries.

The response of wages and expected returns to parameter changes can be summarized as follows:

$$\frac{dw^*}{dz} = m\frac{dk^*}{dz} + \frac{\partial w}{\partial z}, \qquad \frac{d\widehat{w}^*}{dz} = \widehat{m}\frac{d\hat{k}^*}{dz} + \frac{\partial \widehat{w}}{\partial z},$$

$$\frac{dr^*}{dz} = f''\frac{dk^*}{dz} + \frac{\partial r}{\partial z}, \qquad \frac{d\hat{r}^*}{dz} = g''\frac{d\hat{k}^*}{dz} + \frac{\partial \hat{r}}{\partial z}.$$

The responses of the capital account and the trade balance with respect to changes in technology and utility parameters are:

$$(1+n)\frac{d\kappa}{dz} = n\left\{s^*\left(\frac{d\pi^*}{dz} + \frac{d\hat{\pi}^*}{dz}\right) - \pi^*\left(\frac{ds^*}{dz} - \frac{d\hat{s}^*}{dz}\right)\right\}$$

$$\equiv n\Delta\kappa ,$$

$$(1+n)\frac{dtb^*}{dz} = s^*(1-\pi^*)\left(\frac{dr^*}{dz} - \frac{d\hat{r}^*}{dz}\right) + (r^* - n)\Delta\kappa .$$

Note that if factor returns do not differ in the new equilibrium, then the change in the trade balance will be propotional to the change in the capital account, and if $r^* > n$ net service income changes more than net capital outflow, so the current account and the trade balance must move in opposite directions. This is an example of the importance of considering service income when discussing long run changes in the current account and trade balance, a point also emphasized by Kole (1985).

Turning now to considering how differences across countries in specific parameters affects the equilibrium, we begin by considering the effects of a rise in the domestic rate of risk aversion. The change in preferences has no effect on portfolio allocations because domestic and foreign equities are still equally risky; in consequence, steady-state factor returns will not differ across countries. In Appendix B we show the following:

$$\frac{ds^*}{d\gamma} < 0, \quad \text{and} \quad \left|\frac{ds^*}{d\gamma}\right| > \frac{d\hat{s}^*}{d\gamma} \gtrless 0.$$

The economics behind these changes is that the preference-induced decline in domestic savings causes a rise in equity returns world-wide, and a corresponding decline in wages. The effects of these factor-price changes on foreign savings are contradictory, but total world savings definitely falls.

A rise in γ only affects the balance of payments through changes in savings levels:

$$(1+n)\frac{\mathrm{d}\kappa^*}{\mathrm{d}\gamma} \equiv n\Delta\kappa = n\pi^*\left(\frac{\mathrm{d}\hat{s}^*}{\mathrm{d}\gamma} - \frac{\mathrm{d}s^*}{\mathrm{d}\gamma}\right) > 0 \ ,$$

$$(1+n)\frac{\mathrm{d}tb^*}{\mathrm{d}\gamma} = (r^* - n)\Delta\kappa \gtrless 0 \text{ as } (r^* - n) \gtrless 0$$

Since $\partial s^*/\partial\gamma - \partial\hat{s}^*/\partial\gamma < 0$, an increase in risk aversion at home improves the capital account by decreasing domestic savings relative to foreign savings. Thus we have shown:

Proposition 3: *For otherwise identical countries under free trade in goods and equities, the country with the greater risk aversion will run a capital account deficit and also a trade deficit, but capital stocks and factor returns will not differ across countries.*

The rise in risk aversion at home reduces wages world wide, which illustrates how a change in one country's preferences can affect the other. If both countries were previously at "intertemporally efficient" equilibria, then such a change in preferences in one country would reduce foreign welfare. This outcome also illustrates the importance of allowing capital to be endogenous in long-run models: as we saw in Part II, with overall productivity unchanged wages could not ever move in parallel without a change in the worldwide supply of capital.

A decline in β has qualitatively identical effects as a rise in γ, which is not surprising since Buiter (1981) comes to these same conclusions in his simpler version of this model (see Appendix B for proof). As in our analysis of differences across countries in these variables under portfolio autarchy, their effects work entirely through changes in savings.

When one country's assets are more risky than the other's, then the contours of the international equilibrium are determined by portfolio allocations, rather than savings behavior. To see this, consider the following:

$$\frac{d\pi^*}{d\sigma^2} = \frac{d\hat{\pi}^*}{d\sigma^2} < 0 , \tag{6a}$$

$$\frac{ds^*}{d\sigma^2} < 0 , \qquad \frac{ds^*}{d\sigma^2} < \frac{d\hat{s}^*}{d\sigma^2} \gtrless 0 . \tag{6b}$$

(The comparative statics underlying these results are presented in Appendix B.) Relations (6a) shows that a rise in σ^2 relative to $\hat{\sigma}^2$ causes investors to shift their portfolios out of domestic assets. Relations (6b) must be explained with reference to changes in wages in the two countries. The portfolio shifts reduce domestic wages relative to foreign wages, and, consequently, reduce dome savings relative to foreign savings as well. The rise in σ^2 will also tend to reduce savings in both countries, since it raises portfolio risk. Foreign wages could rise or fall, since portfolio shifts will tend to raise foreign wages, while the savings decline will tend to reduce them; the direction of change of foreign savings is correspondingly ambiguous. Domestic wages will thus necessarily decline, since the portfolio and savings effects work in the same direction. It is clear that domestic savings will decline, that foreign savings will exceed domestic savings, and that total world savings will decline.

The portfolio shifts will definitely tend to reduce the home country's net capital inflow, while the increase in foreign savings relative to domestic savings will tend to raise it. That the balance of these effects favors the savings effect can be deterimined analytically, as shown in Appendix B. The effect of an increase in σ^2 on the trade balance depends on the size and sign of $r^* - n$. Since domestic interest rates rise more than foreign rates, the trade balance will rise along with the capital account under our maintained assumption that $r^* - n$.

Our analysis of steady-state equilibrium under differences in risk has shown the following:

Proposition 4: *Under free trade in goods and equities, returns to capital will be higher and wages and capital will be lower in the country which,* ceteribus paribus, *has higher technological riskiness. That country will run a deficit on capital account, and will also tend to run a trade deficit.*

This proposition is significant insofar as it presents yet another important economic factor which affects the balance of payments and other aspects of the international equilibrium. That differences in risk could have such effects is not surprising: as discussed in the introduction, numerous authors have noted that risk and return should be positively related across countries, as well as across assets. Our analysis is the first to approach the issue in a general equilibrium framework. Thus we can go beyond earlier work and conclude, for example, that higher technology risk will be reflected in a capital account deficit and lower wages, as well as in higher equity returns.

That a capital account deficit could be related to relative output riskiness has not been explicitly considered elsewhere in the international economics literature, so far as I know. Perhaps this is because it seems obvious how such a connection would work: for instance, it is commonplace in discussions of the self-imposed "constraints" facing many LDC's to note that unpredictable and severe government intervention in the economy increases economic risk and discourages international direct investment, forcing governments into the syndicated credit markets to maintain investment at "desired" levels. Yet our analysis shows that this is not the whole story, and that long-run changes in savings are important as well as portfolio shifts.

It is interesting to compare our conclusion that factor returns will differ when technological riskiness differs with a related result of Grossman and Razin (1984), who also analyze a model of free trade in goods and capital where technology is subject to nation-specific shocks. In the Grossman and Razin analysis, output is affected multiplicatively by the realization of country-specific productivity shocks, and "equities" are claims to a share of that output, rather than claims on the returns to one specific factor. It is the price of these "equities", rather than the returns to capital, that reflect differences in risk; the deterministic returns to capital are always equal across countries, while the equities of the high-risk country will be lower priced. With firm managers committed to maximizing firms' market values in the high-risk country, lower equity values reduce the marginal productivity of capital, and in this way higher risk becomes associated with a lower capital stock. Thus Grossman and Razin join us in concluding that a country with greater technological riskiness will have a lower capital stock, but find that factor returns will not differ.[9]

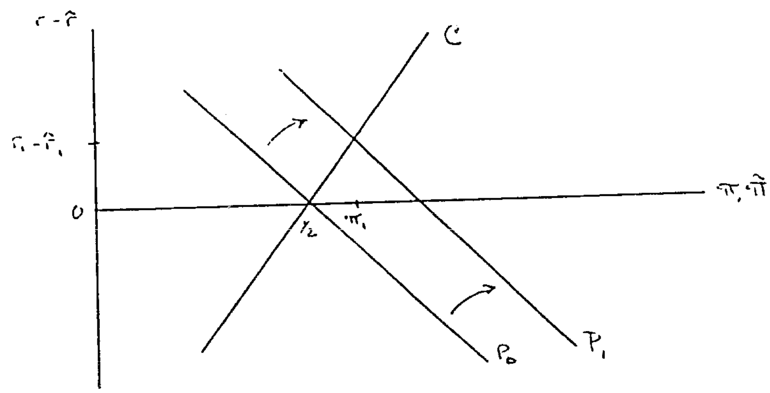
We next consider how differences in overall productivity affect the international steady-state equilibrium. In this case we will begin with a concise statement of these effects, and go on to show why they occur:

Proposition 5: *Under free trade in goods and equities, wages , the capital stock and also equity returns will be higher in a country which has higher overall productivity,* ceteribus paribus. *This country is unlikely to have a zero current account or trade balance, but these accounts could be either positive or negative.*

In Figure 1 we illustrate why domestic equity returns must exceed foreign returns if $D > \widehat{D}$. Here, C shows consumers' utility-maximizing portfolio shares as a function of differences between equity returns, and P shows differences in equity returns determined via production functions and portfolio shares. P_0 corresponds to symmetry across countries, and P_1 to the case we consider here, $D > \widehat{D}$. Clearly, the new equilibrium requires $\hat{r} > r$ and $\pi = \widehat{\pi} > 1/2$, from which it is straightforward to conclude that $w > \widehat{w}$ and $s > \hat{s}$.

A rise in D has two conflicting effects on the capital account: the portfolio shifts towards domestic assets will tend to raise it, while the increase in domestic savings relative to foreign savings will work in the opposite direction. Related ambiguities leave us uncertain about the fate of the trade balance, though it can be ascertained that if the capital account rises, and the portfolio effect dominates, then the trade balance will, also.

Figure 1.



That equilibrium factor prices could differ due to differences in overall productivity is startling from two points of view. First, microeconomic theory has accustomed us to the idea that, in an unrestricted factor market, the marginal product of that factor will be the same in all its uses. Here we find that capital at home will have a higher marginal product than capital abroad.

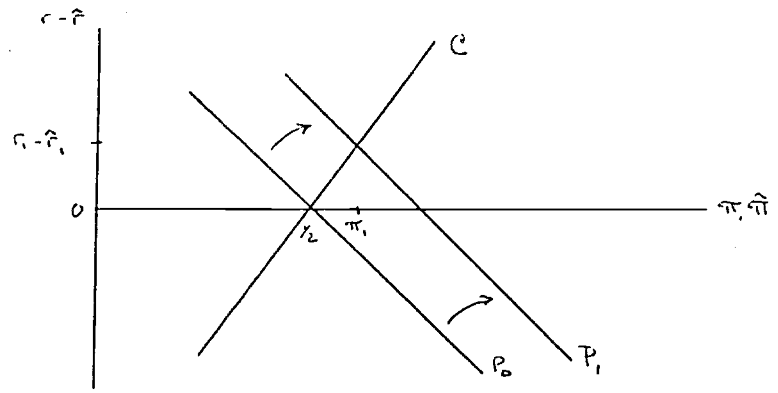
Second, in international economics we frequently find "perfect substitutability" of assets cited as a sufficient condition for open interest parity to hold when capital markets are fully integrated. Since all assets are subject to some risk when investors cross national boundaries, the closest realistic approximation to "perfect substitutability" is "equal variability" . Yet this result shows that the assumption of "equal variability" is not sufficient to ensure that open interest parity will hold unless asset returns are perfectly correlated, as well.

Another factor which will lead factor returns to differ in the absence of differences in equity variability is the size of the labor force.

Proposition 6: *For otherwise identical countries engaged in free trade in goods and equities, the country with the higher population will have higher equity returns, lower wages, a lower capital stock in absolute as well as per capita terms, and a capital account deficit.*

In Figure 2, which is interpreted similarly to Figure 1, we show that $L > \hat{L}$ requires $r > \hat{r}$ and $\pi = \hat{\pi} > 1/2$:

Figure 2.



The deficit on capital account is the outcome of two conflicting forces: the rise in domestic savings relative to foreign savings tends to cause a net capital outflow, while a shift in portfolio shares towards domestic equities works in the opposite direction. (That the savings effect dominates is shown in Appendix B.) There are also conflicting forces at work on the trade account, the balance of which cannot be determined analytically: the decline in the capital account would be associated with a trade deficit in the absence of changes in relative interest rates, but the rise in domestic rates relative to foreign rates tends to improve the trade balance, making its ultimate direction of change ambiguous.

Comparing once again our results with related conclusions of Grossman and Razin (1984), we find that in both analyses higher population is associated with a lower capital-labor ratio. The general motivation for this result is the same: if capital were evenly divided across countries, equity returns would be relatively low in the country with more workers, so some redistribution ensues. In their model, where factor returns never differ across countries, redistribution continues until $r = \hat{r}$, while in our model redistribution stops before that point, with r still less than $\hat{r}$, because individuals face a trade-off betweena portfolio variance and expected portfolio returns.

So far we have found balance of payments effects associated with differences across countries in all the parameters associated with our one-good model. Differences in factor returns have been found to be associated with differences in technology and in labor supplies, though not with differences in preferences. If we suppose that countries specialize in the production of unique goods, a number of additional causes for factor returns to differ come to light.

Such an adjustment to our model introduces a new variable, the relative price of foreign output in terms of domestic output: call this "p". If we assume further that the domestic capital stock consists entirely of home-country output, even though owned in part by foreigners (with a similar condition for the foreign capital stock), and ignore the possibility that labor force size could differ across countries, the capital stocks become:

$$k_{t+1} = \frac{\pi_t s_t + p_t \hat{\pi}_t \hat{s}_t}{1+n} , \qquad \hat{k}_{t+1} = \frac{(1-\pi_t) s_t / p_t + (1-\hat{\pi}_t)\hat{s}_t}{1+n} .$$

As is evident in these expressions, anything that causes a divergence of the terms of trade from unity will cause capital stocks to differ across countries. For example, it is now the case that differences across countries in consumers' risk aversion or rate of time preference will be reflected in differences in their equity returns, as well as in a capital account deficit, so long as import propensities are not 1/2 in both countries. Differences in import propensities themselves will cause factor returns to differ. (For extensive analysis of this model, see Osler 1987.) In this more complex setting, then, differences in preferences can be added to differences in technology in our table of factors that make "equal variability" an insufficient condition for open interest parity, and the list of those preference parameters is also expanded.

PART IV: DISCUSSION

In this paper we have analyzed how factor returns and the balance of payments are affected by free trade in equities and differences across countries in preferences and technology when technological risk is nation-specific. Since the salient results and their significance are summarized in the Introduction, we will focus here on the distinguishing features of our model.

First, the model examines the individuals' portfolio balance decisions, as well as their consumption decisions, in the consistent framework of utility maximization. This feature enables us to clarify the analysis of more traditional, Tobin-style portfolio balance models regarding the effects of individual system parameters, such as risk and risk aversion, on international equilibria.

With this model we ask with a broader range of questions than with many simpler portfolio balance models. We ask not only about capital flows and the balance of payments but also about wages, output and capital stocks, and how these, in turn, feed back into our conclusions about the balance of payments. That is, by employing a model which uses a rigourous approach to analyze portfolio balance as well as international trade concerns, we are able to explore the no-man's land between international trade and international finance.

Finally, our model is concerned with the long run, when capital stocks are fully responsive to savings behavior. This, along with the other attributes of the model outlined above, allow us to trace from one country to the next changes in savings preference, and to trace the effects of portfolio diversification, a very characteristic phenomenon of the present day.

APPENDIX A: COMPARATIVE STATICS OF AUTARCHY

$$\frac{ds^*}{d\beta} = \frac{U'}{\beta(A + Qm)\Theta} > 0 \qquad \frac{ds^*}{d\sigma^2} = \frac{Q'}{(A + Qm)\Theta} < 0$$

$$\frac{ds^*}{d\gamma} = \frac{-U' ln[s\overline{R}]}{(A + Qm)\Theta} < 0 \qquad \frac{ds^*}{dD} = \frac{-U''w + Qr}{(A + Qm)D\Theta} > 0$$

where

$$m \equiv -kf''/(1+n) > 0 \qquad Q \equiv (1-\gamma)\beta(s\overline{R})^{-\gamma}\xi > 0$$

$$\xi \equiv 1 + \gamma(1+\gamma)\sigma^2 / 2\overline{R}^2 > 0 \qquad Q' \equiv -\gamma(1-\gamma)\beta(s\overline{R})^{-\gamma} / \overline{R} < 0$$

$$A \equiv -U'' + U'/s > 0 \qquad \Theta \equiv 1 + U''m/(A + Qm) > 0$$

APPENDIX B: COMPARATIVE STATICS OF TRADE

$$\frac{ds^*}{d\beta} = \beta_1 + (\beta_1 + \beta_2)m\frac{(w_1 + w_2)}{2\Delta} > 0 \qquad \frac{ds^*}{d\gamma} = \gamma_1 + (\gamma_1 + \gamma_2)m\frac{(w_1 + w_2)}{2\Delta} < 0$$

$$\frac{ds^*}{d\beta} = \beta_2 + (\beta_1 + \beta_2)m\frac{(w_1 + w_2)}{2\Delta} \gtrless 0 \qquad \frac{ds^*}{d\gamma} = \gamma_2 + (\gamma_1 + \gamma_2)m\frac{(w_1 + w_2)}{2\Delta} \gtrless 0$$

$$\frac{d\pi^*}{d\beta} = \frac{d\hat{\pi}^*}{d\beta} = 0 \qquad \frac{d\pi^*}{d\gamma} = \frac{d\hat{\pi}^*}{d\gamma} = 0$$

$$\frac{d\kappa^*}{d\sigma^2} = \frac{2s^*}{1+n}[m(w_1 - w_2) + 1] > 0$$

$$\frac{ds^*}{d\sigma^2} = \frac{\sigma_1}{\Delta} + (w_1 - w_2)ms\sigma_2 > 0 \qquad \frac{ds^*}{d\varphi} = -\frac{d\hat{s}^*}{d\varphi} = \varphi_1 + 2\varphi_2(w_1 - w_2)ms > 0$$

$$\frac{ds^*}{d\sigma^2} = \frac{\sigma_1}{\Delta} - (w_1 - w_2)ms\sigma_2 \gtrless 0 \qquad \frac{d\pi^*}{d\varphi} = \frac{\partial\hat{\pi}^*}{\partial\varphi} = \varphi_2 < 0$$

$$\frac{d\pi^*}{d\sigma^2} = \frac{\partial\hat{\pi}^*}{\partial\sigma^2} = \sigma_2 < 0 \qquad \frac{d\kappa}{d\varphi} = s\left[\frac{1}{2} - 2\left(\frac{J'}{J+4J'}\right)\right] + \frac{ds^*}{d\varphi} > 0$$

$$\frac{ds^*}{dD} = D_1 + 2smD_3(w_1 - w_2) + (D_1 + D_2)m\frac{(w_1 + w_2)}{2\Delta} \quad > \quad 0$$

$$\frac{d\hat{s}^*}{dD} = D_2 - 2smD_3(w_1 - w_2) + (D_1 + D_2)m\frac{(w_1 + w_2)}{2\Delta} \quad \gtrless \quad 0$$

$$\frac{ds^*}{dD} - \frac{d\hat{s}^*}{dD} = D_1 - D_2 + 4smD_3(w_1 - w_2) \quad > \quad 0$$

$$\frac{d\pi^*}{dD} = \frac{d\hat{\pi}^*}{dD} = D_3 \quad > \quad 0$$

where

$$J \equiv -\frac{\gamma}{2}\frac{\partial Var(R)}{\partial \pi} \quad < \quad 0 \qquad D_1 \equiv \frac{\partial s}{\partial D} = \frac{-U''w}{(A + 2A')} + \frac{Q\pi r}{A} \quad > \quad 0$$

$$J' \equiv 2\hat{R}\xi f''k \quad < \quad 0 \qquad D_2 \equiv \frac{\partial \hat{s}}{\partial D} = \frac{Q\pi(-U''wm + Ar)}{A(A + 2A')} \quad > \quad 0$$

$$A' \equiv -Qf''k/2 \quad > \quad 0 \qquad D_3 \equiv \frac{\partial \pi}{\partial D} = \frac{\partial \hat{\pi}}{\partial D} = \frac{-R\xi r}{J(J + 2J')} \quad > \quad 0$$

$$\Delta \equiv AJ(A + 2A')(J + 2J') \quad > \quad 0 \qquad D_1 - D_2 = -U''w/DA \quad > \quad 0$$

$$w_1 \equiv \frac{\partial s}{\partial w} = \frac{-U''(A+Qm/2)}{A(A + Qm)} \quad > \quad 0 \qquad \sigma_1 \equiv \frac{\partial s}{\partial \sigma^2} = \frac{\partial \hat{s}}{\partial \sigma^2} = \frac{Q'\pi^2}{A + Qm} \quad < \quad 0$$

$$w_2 \equiv \frac{\partial \hat{s}}{\partial w} = \frac{U''Qm/2}{A(A + Qm)} \quad < \quad 0 \qquad \sigma_3 \equiv \frac{\partial \pi}{\partial \sigma^2} = \frac{\partial \hat{\pi}}{\partial \sigma^2} = \frac{\gamma}{2(J + 2J')} \quad < \quad 0$$

$$\beta_1 \equiv \frac{\partial s}{\partial \beta} = \frac{U'(A+Qm/2)}{\beta A(A + Qm)} \quad > \quad 0 \qquad \gamma_1 \equiv \frac{\partial s}{\partial \gamma} = \frac{-U'ln(s\bar{R})(A+Qm/2)}{A(A + Qm)} \quad < \quad 0$$

$$\beta_2 \equiv \frac{\partial \hat{s}}{\partial \beta} = \frac{-U'Qm/2}{\beta A(A + Qm)} \quad < \quad 0 \qquad \gamma_2 \equiv \frac{\partial \hat{s}}{\partial \gamma} = \frac{U'ln(s\bar{R})Qm/2}{A(A + Qm)} \quad > \quad 0$$

Partial derivatives refer to the effect of a parameter change in the first period after it occurs.

NOTES

1: The requirement that $0 < d < 1$ ensures that members of the older generation will always consume a positive amount. Since we have assumed $r \geq 0$, their retirement resources of $(1 + r + \alpha)s$ will be positive whenever $\alpha \geq 1$.

2: The choice of functional form for the production function deserves some comment. It is more common to analyze a linear homogenous function multiplied by a stochastic term with unit mean: $Q = \alpha F(K,L)$. Examples include Batra 1975; Mayer 1976; Baron and Forsythe 1979; Helpman and Razin 1978a and b; Grossman and Razin 1984, 1985. In this case the variance of the return to capital is $(f')^2\sigma^2$, implying that any change in per-worker capital stock affects not only the return to capital but also its variance. Since this is a complication with which this thesis is not concerned, our alternative formulation was adopted. Another advantage of the additive form is that it mimics reality more closely than the model with multiplicative uncertainty, since in actuality the return to capital does absorb most of the variance in output. The risk-sharing arrangements responsible for this asymmetry between labor and capital, which are the focus of the implicit contracts literature, (Bailey 1974) are not readily incorporated into this particular model.

3: These results are obtained from totally differentiating a second-order Taylor's expansion of the consumers' first-order condition for s at the point $\alpha = 0$ (the distribution of α is such that all higher-order moments are extremely small and can be safely disregarded).

4: The importance of γ in determining the effect of σ^2 on savings was originally noted for this model in an exact analysis, which lends support to our approach involving Taylor's series approximations (Sandmo, 1970).

5: Numerous studies have attempted to estimate this parameter directly. For example, Friend and Blume (1975) found that $\gamma > 2$, which was supported by Farber (1978), who found $\gamma > 2.5$ for the United Mine Workers, and contradicted recently by Hansen and Singleton (1983), who conclude that $0 < \gamma < 2$.

Other researchers have been concerned not with measuring parameters of a utility function, but instead with measuring the actual size and sign of the responses of personal savings (or consumption) to changes in expected returns. Here again, the evidence is mixed. Boskin (1978) found a significantly negative response of consumption to increases in the real, after-tax interest rate, but numerous subsequent authors have questioned his methodology and his conclusions (Howrey and Hymans 1978; Carlino 1982). Carlino finds that the direction of this relationship found empirically seems to depend on the interest rate used, and that working with the appropriate variable, after-tax real returns, there is a positive but insignificant effect of returns on consumption. Recent work by Hall (1985) also finds that the intertemporal elasticity of substitution is about zero.

Another line of research in this general area has tried to estimate the sign and size of the response of personal savings to changes in the riskiness of returns. Gylfason (1981) and Howrey and Hymans (1978) both have found that an increase in the standard deviation of inflation tends to raise savings in the U.S., implying, in the context of this thesis, that income effects dominate and we should consider $\gamma > 1$. However, in both of these studies, the interest elasticity of savings is also examined, with results that support the conclusion that $\gamma < 1$ and $\gamma = 1$, respectively.

6: Suppose γ were greater than unity. We know that consumers will wish to increase their level of savings in response to a decline in the expected return to assets and to a rise in the variability of asset returns. To ensure that the system is stable in the Walrasian sense, intratemporally, we must assume additionally that $|\partial s/\partial r| < |(\partial r/\partial s)^{-1}|$. When the variability of equity returns rises, savers' initial inclination to save more will be reinforced by the decline in expected returns that the resulting higher capital stock induces.

7: The uniqueness of the equilibrium can be ascertained by noting that $k'(w) > 0$ and $k''(w) > 0$ (where $k(w) = w^{-1}(k)$), while $\acute{s}_w > 0$, $\acute{s}_{ww} < 0$ everywhere.

8: The form of the solutions for π^* determined in this model are very close to those derived in a recent series of papers which attempted to analyze the optimal portfolio of currencies for risk-averse international investors when the purchasing power of currencies varies stochastically (Kouri and de Macedo 1978; Healy 1980; de Macedo 1981, 1982).

"Portfolio Diversification Across Currencies," (de Macedo 1982) analyzes the two-country case explicitly, and thus provides a useful point of comparison to the present model. The form of the minimum variance share is unchanged across the two papers, while the form of the speculative portfolio shares differs in insignificant ways. That these formula are so similar is not surprising, since in both models the random variables enter linearly into observed asset returns and only the first two moments of these returns' distributions are significant.

9: It seems to me that the present model's view of equities is more realistic, insofar as in our model as well as in reality returns to capital bear the lion's share of most output uncertainty, and "equity shares" are claims on those returns.

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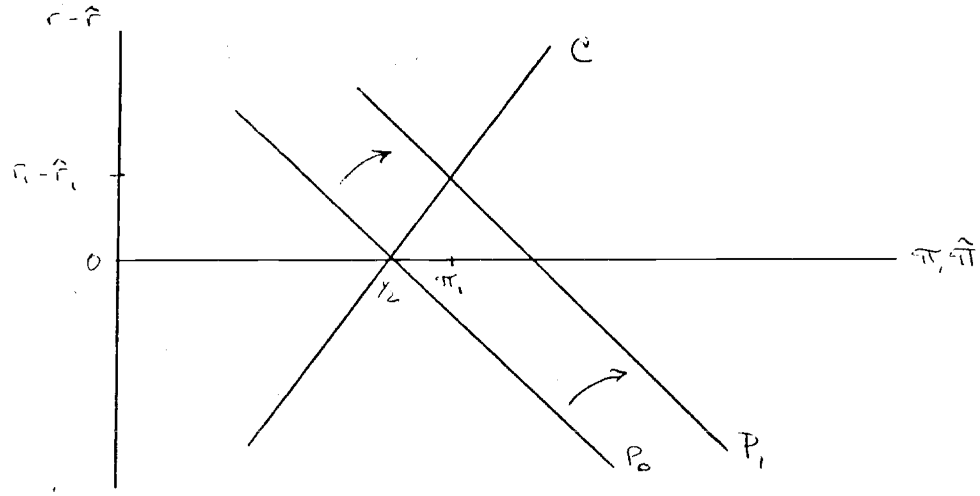

r - r̂
r₁ - r̂₁
0
C
P₀
P₁
π, π̂
π₁