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ARE UNEMPLOYMENT AND OUT OF THE LABOR FORCE BEHAVIORALLY DISTINCT LABOR FORCE STATES?

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ABSTRACT

This paper formulates and tests the hypothesis that the categories unemployed and out of the labor force are behaviorally distinct labor force states. Our empirical results indicate that they are. In the empirically relevant range the exit rate from unemployment to employment exceeds the exit rate from out of the labor force to employment. This evidence is shown to be consistent with a simple job search model of productive unemployment with log concave wage offer distributions. We prove that if unemployed workers receive job offers more frequently than workers out of the labor force, and if wage offer distributions are log concave, the exit rate from unemployment to employment exceeds the exit rate from out of the labor force to employment.

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1. Introduction

There is considerable controversy over the issue of whether or not the categories unemployed and out of the labor force are behaviorally distinct labor force states. This issue is particularly relevant in the study of the labor market dynamics of youth.

Given the range of nonmarket options available to many youths, and given practices of many state unemployment compensation programs which effectively limit the eligibility for unemployment compensation of most youths, it seems especially likely that there is no distinction between unemployment and out of the labor force status for young people. Clark and Summers (1982) and Ellwood (1982) have recently made this claim. They argue that the empirical distinction between reported "unemployment" and reported "out of the labor force" is so arbitrary that it is of little or no analytical value.

This point would seem to have some merit after examining the official Current Population Survey definition of unemployment which defines those individuals as unemployed "who had no employment during the survey week, were available for work, and (1) had engaged in any specific job seeking activity within the past four weeks, or (2) were waiting to be called back to a job from which they had been laid off, or (3) were waiting to report to a new wage or salary job scheduled to start within the following 30 days." Because there is no stipulation as to the quality or quantity of searches made within the month, the unemployment-out of the labor force distinction may be of little value in predicting employment probabilities for the nonemployed.

On the other hand, theoretical models suggest that being unemployed as opposed being to out of the labor force describes different behavior. For example, in search theory (e.g., Burdett and Mortensen (1978)) a key difference between unemployed individuals and those out of the labor force is that the former are at an interior point with respect to the amount of time they devote to search while the latter are at a corner and spend no time searching. In these models, separate behavioral equations characterize the reemployment probabilities from these two states.

In this paper we test the hypothesis that the classifications unemployed (u) and out of the labor force (o) are behaviorally meaningless distinctions. We reject this hypothesis. Distinct behavioral equations govern transitions from out of the labor force to employment (e) and from unemployment to employment.

The idea underlying our test is as follows: controlling for both observed and unobserved individual differences in explanatory variables, if the escape rate from unemployment to employment (h_{ue}) is the same as the escape rate from out of the labor force to employment (h_{oe}), the origin state (u or o) is irrelevant in determining the rate at which individuals leave nonemployment to enter employment. In a simple three state Markov model, this test is equivalent to testing the proposition that the two nonemployment states can be aggregated into a single state resulting in a two state Markov model for employment and nonemployment.

We find that in the empirically relevant range the exit rate from the state of unemployment to the employed state <u>exceeds</u> the exit rate from the state of out of the labor force to the employed state. This finding is consistent with versions of a traditional job search model in which the rate of arrival of wage offers for the unemployed exceeds the rate of arrival of wage offers for those out of the labor force. It is also consistent with versions of a job search model with a positive interest rate in which the mean of the wage offers for the unemployed exceeds the mean of the wage offers for those out of the labor force. Thus the empirical evidence reported in this paper is broadly consistent with versions of search theory in which unemployment is a state that facilitates the job search process.

The plan of the rest of this paper is as follows. In section 2, we exposit our test in a simple setting. In section 3, we describe the more general econometric specification used to perform our test. Section 4 presents empirical results and our interpretation of them. Section 5 contains concluding remarks. In an Appendix, we present a new sufficient condition for the exit rate from unemployment to be an increasing function of the rate of arrival of job offers.

2. Testing the Behavioral Distinction Between Unemployment and Out of the Labor Force

To motivate our test, we consider two cases.[1/] In the first case, the probability density function of employment spell durations in $g_e(t_e)$, where t_e denotes the length of the employment spell. The probability that a person terminating employment classifies himself as u or o is determined by tossing a coin that comes up u fraction π of the time and comes up o fraction $1-\pi$ of the time. Once acquired the person keeps these labels as long as he is nonemployed so that there is no direct switching between u and o states. The density function of duration times in nonemployment is $g_n(t_n)$. The exit rate, or hazard function, from nonemployment is defined as

$$h_n = \frac{g_n(t_n)}{1 - G_n(t_n)} ,$$

where $G_n(t_n)$ is the cumulative distribution function of nonemployment durations. The joint probability of unemployment classification and nonemployment duration t_n is

$$\pi \, g_n(t_n)$$

with associated hazard function

1/ To simplify the exposition we assume that agents are homogeneous throughout this section. This assumption is not essential and is not used in performing the empirical work reported in Section 4.

$$h_{ue} = h_n .$$

The joint probability of out of the labor force classification and nonemployment duration t_n is

$$(1 - \pi)g_n(t_n)$$

with associated hazard

$$h_{oe} = h_n,$$

so that

$$h_{oe} = h_{ue} . \tag{2.1}$$

The hazard rate will be the same for the two nonemployment states o and u.

In the second case considered here, individuals are allowed to switch their reported nonemployment status randomly. By this we mean that initial nonemployment classification is governed as before by a toss of a coin and that within a spell of nonemployment individuals switch randomly between o and u. The continuous time analogue of discrete time independent Bernoulli trials is an exponential waiting time model (Cox (1962)). Write the hazard for durations from o to u as h_{ou} and the hazard from u to o as h_{uo}. The density of time spent going from o to e (t_{oe}) is

$$h_{oe} \exp \left\{ - (h_{oe} + h_{ou})t_{oe} \right\}$$

while the density of time spent going from u to e (t_{ue}) is

$$h_{ue} \exp \left\{ -(h_{ue} + h_{uo}) t_{ue} \right\} .$$

Individuals may change between reported nonemployment states for any reason. All that is required for the origin state (o or u) to be irrelevant for characterizing transitions from nonemployment to employment is for $h_{oe} = h_{ue}$.

Note that in both cases, conditioning on o or u eliminates the classification probability parameter π. Thus the test advocated in this paper is valid even if individuals systematically report themselves as o or u and the reporting probabilities are functions of explanatory variables.

The condition $h_{oe} = h_{ue}$ is also the requirement that must be satisfied in a Markov model to aggregate o and u into a single state n, and for the resulting two state model for e and n to be a properly defined Markov model. To demonstrate this it is most convenient to work with the state probability representation of the three state Markov model (see, e.g., Tuma et. al. (1978)); Flinn and Heckman (1982a)). Define $P_j(t)$ as the probability that state j is occupied at time t and $\dot{P}_j(t)$ as the instantaneous rate of change of this probability. Then

$$\begin{bmatrix} \dot{P}_e(t) \\ \dot{P}_o(t) \\ \dot{P}_u(t) \end{bmatrix} = \begin{bmatrix} -(h_{eu} + h_{eo}) & h_{oe} & h_{ue} \\ h_{eo} & -(h_{oe} + h_{ou}) & h_{uo} \\ h_{eu} & h_{ou} & -(h_{ue} + h_{uo}) \end{bmatrix} \begin{bmatrix} P_e(t) \\ P_o(t) \\ P_u(t) \end{bmatrix}$$

or

$$\dot{P}^{(3)}(t) = AP^{(3)}(t)$$

in matrix notation where the superscript 3 indicates a three state model. Note that the rank of A is at most 2.

In order to aggregate o and u into a two state model defined in terms of n, we require that we be able to collapse the three state system into

$$\begin{bmatrix} \dot{P}_e(t) \\ \\ \dot{P}_n(t) \end{bmatrix} = \begin{bmatrix} -h_{en} & h_{ne} \\ \\ h_{en} & -h_{ne} \end{bmatrix} \begin{bmatrix} P_e(t) \\ \\ P_n(t) \end{bmatrix}$$

where $P_n(t) = P_o(t) + P_u(t)$. In matrix notation the two state model may be written as $\dot{P}^{(2)}(t) = BP^{(2)}(t)$. The rank of B is 1. For this to be an equivalent representation of the three state model, a necessary condition is that rank (A) = rank (B) = 1. A necessary and sufficient condition is that $h_{oe} = h_{ue} = h_{ne}$. Sufficiency may be checked by direct substitution into A.

This interpretation of our test is also informative in that it makes precise the sense in which o and u are irrelevant. Aggregating o and u into a single state for the purpose of statistical analysis does not alter the Markov property of the model. The rate at which individuals leave nonemployment to enter employment does not depend on which nonemployment state individuals occupy.

It is tempting to extend this type of reasoning to consider transitions from employment to the two nonemployment states. Thus it might be argued that if u and o are irrelevant distinctions, the rate of transition from e to u (h_{eu}) would be the same as the rate of transition from e to o (h_{eo}). This argument is correct only if the probability of exiting from employment to unemployment (η) equals the probability of exiting from employment to out of the labor force ($1-\eta$) so

$$\eta = 1 - \eta = \frac{1}{2} .$$

If $g_e(t_e)$ is the density of employment length durations with associated hazard rate $h_e(t_e)$, the hazard rate for transitions from e to u is

$$h_{eu} = \eta h_e$$

while the hazard rate for e to o transition is

$$h_{eo} = (1 - \eta)h_e \quad .$$

Obviously $h_{eu} + h_{eo} = h_e$ by the properties of conditional probabilities. But unless $\eta = 1-\eta = 1/2$, $h_{eu} \neq h_{eo}$. We have no theory of η. Even if reporting oneself as unemployed is strictly a matter of tossing a coin, nothing requires $\eta = 1/2$. However, if information is available on the fraction of employment spells that are immediately followed by unemployment, it is possible to obtain a consistent estimate of η,[1/] and to test the proportionality hypothesis that

$$h_{eu}/h_{eo} = \eta/1-\eta \quad . \tag{2.2}$$

[1/] Assuming η is constant in the population. If η is a function of observed and unobserved heterogeneity components, a more complicated but still straightforward estimation scheme is required.

3. Econometric Implementation of the Test

In order to test the hypothesis that unemployment and out of the labor force are not behaviorally distinct labor market states we first specify a parametric form for the hazard functions $(h_{jk}, j \neq k; j,k = e,o,u)$ on which the test is based. We have adopted a general functional form for the hazard functions in order to minimize the possibility of spuriously rejecting the two state model because of model misspecification. For a detailed consideration of the econometric issues which arise in the estimation of duration data models, the reader is referred to Flinn and Heckman (1982a). In this section we sketch the econometric specification employed in performing our proposed test.

Since a hazard function is a conditional probability density function, a requirement of any econometric specification is that for all possible values of the parameters and both observed and unobserved heterogeneity the hazard be nonnegative. The econometric specification adopted in this paper imposes nonnegativity.

A hazard function associated with a particular probability density[1/] is said to exhibit positive, no, or negative duration dependence according to whether

$$\frac{\partial h(t)}{\partial t} \gtrless 0 \quad .$$

1/ A hazard function is uniquely determined by the probability density functiom and *vice versa*.

For example, if $\partial h(t)/\partial t > 0$ for $t > 0$, the instantaneous conditional probability of exiting the state increases with the duration of the spell. In a job search model in which the reservation wage declines with the length of an unemployment spell, we expect to observe positive duration dependence in the hazard associated with unemployment to employment transitions. On the other hand, in a model that allows for specific human capital accumulation, we expect to observe negative duration dependence in the hazard associated with employment to unemployment transitions. The exponential distribution of duration times is the only distribution consistent with a hazard function exhibiting no duration dependence.

It is possible to specify hazard functions that exhibit all three types of duration dependence for different values of t. In labor economics it is especially important to allow for nonmonoticity of the hazard since many economic models have been developed that predict a nonmonotonic hazard (see, e.g., Jovanovic, 1979) Our econometric model allows for nonmonotonicity in the hazard in a simple and readily interpretable manner.

In our previous work Flinn and Heckman (1982a) we have demonstrated the importance of controlling for nonstationary in the environment. While most theoretical models assume stationarity, our econometric specification does not impose this frequently counterfactual assumption onto the data. We permit the hazard function associated with any transition to depend on time varying explanatory variables. Incorporating time-varying variables into our econometric model is computationally burdensome but proves to be essential in obtaining consistent parameter estimates (see Flinn and Heckman (1982a), Section IV).

The specific functional form for the hazard function that we employ in our empirical work is

$$h_{jk}(t_{jk}) = \exp\left\{ \underset{\sim}{X}(\tau_{jk} + t_{jk})' \underset{\sim}{\gamma}_{jk} + \varphi_{1,jk} t_{jk} + \varphi_{2,jk} t_{jk}^2 + v_{jk} \right\}$$

$$j,k = e,o,u; \quad j \neq k$$

where τ_{jk} is the calendar date at which the current spell began, $\underset{\sim}{\gamma}_{jk}$ is a parameter vector conformable with $\underset{\sim}{X}$, the linear and squared duration terms capture relatively general forms of duration dependence, and v_{jk} represents (a scalar measure of) the effect of unobserved individual differences on the state j to state k hazard. The hazard is nonnegative as required. Note that the time index on $\underset{\sim}{X}(\tau_{jk} + t_{jk})$ indicates that the instantaneous conditional probability of exiting from state j to state k after being in state j for duration t_{jk} at calendar time $\tau_{jk} + t_{jk}$ is a function of <u>current</u> values of $\underset{\sim}{X}$. For this specification, there exists no duration dependence in the j to k hazard if $\varphi_{1,jk} = \varphi_{2,jk} = 0$. If $\varphi_{2,jk} = 0$, we say that there is positive (negative) duration dependence in the j to k hazard if $\varphi_{1,jk} > 0$ $(\varphi_{1,jk} < 0)$. If $\varphi_{2,jk} \neq 0$ then the hazard need not be monotone.

We restrict the contribution of unobserved heterogeneity to the j to k hazard to be of the form

$$v_{jk} = c_{jk}\,\delta,$$

where the c_{jk} are parameters of the model and δ is an individual specific spell and time invariant heterogeneity component, the value of which is unobserved by the analyst. In the estimation procedure, we adopt a random effects specification and make an assumption concerning the form of $F(\delta)$, the cumulative distribution function of δ in the population. The parameters c_{jk} are identified up to a factor of proportionality.

4. Empirical Results And Interpretation

The sample used to perform the empirical work reported here is selected from the National Longitudinal Survey of Young Men. We follow 122 young men for thirty consecutive months from the time they graduate from high school. The small size of our sample is due to the stringent selection criteria imposed. To be included in the sample an individual must (1) be white; (2) have received a high school diploma in the spring or early summer of 1969; and (3) not have returned to school in the period beginning in the fall of 1969 and ending in December of 1971.

The sample was selected in this manner in an attempt to minimize the initial conditions problem discussed in the literature on applied stochastic processes (Heckman (1981), Flinn and Heckman (1982a)). By using individuals who have recently completed schooling, we have selected individuals with little or no previous labor market experience. The vast majority of individuals in our sample have not worked in full time jobs during high school. Because of this, we feel that the initial conditions problem can safely be ignored in deriving the maximum likelihood estimates presented here.

Unless duration times follow an exponential distribution, the distribution of the first spell observed during a sampling period will not have the same distribution as subsequent spells of the same type whose beginning and ending dates are observed. By constructing our sample in the manner indicated we claim that the first spell sampled follows the same distribution as subsequent spells of the same type.

Due to the small number of transitions between the nonemployment states ($u \rightarrow o$ and $o \rightarrow u$) we were not able to obtain estimates of the hazard functions associated with these transitions. In the three state model we estimate the parameters of the four hazards associated with the $e \rightarrow o$, $e \rightarrow u$, $u \rightarrow e$, and $o \rightarrow e$ transitions. In our test of the two versus three state model, we estimate three hazard functions by constraining the $u \rightarrow e$ and $o \rightarrow e$ hazards to be equal. Estimation is by maximum likelihood. The reader is referred to Flinn and Heckman (1982a) for details concerning the specification of the likelihood function and estimation procedure.

Table 1 presents estimates of the three state model estimated with observed and unobserved heterogeneity in the transition rates. The observable characteristics included are the duration of the spell, duration squared (to allow for nonmonotonic duration dependence), and whether the individual is married with spouse present (MSP) (1 if yes, 0 if no). The parameter c_{ij} is the factor loading for the state i to state j transition. The unobserved heterogeneity component δ is assumed to have a standard normal distribution.

The signs of the parameters are generally consistent with prior expectations. For example, currently married men have lower rates of transiting from employment to unemployment than do nonmarried men. The fact that the standard errors are so large relative to the magnitude of the parameters is to be expected given that we are attempting to estimate twenty parameters with so few degrees of freedom. Only the constant terms and the factor loading associated with the employment to out of the labor force transition are greater than twice their standard errors.

In Figure 1 the hazard functions from the two nonemployment states to

Table 1

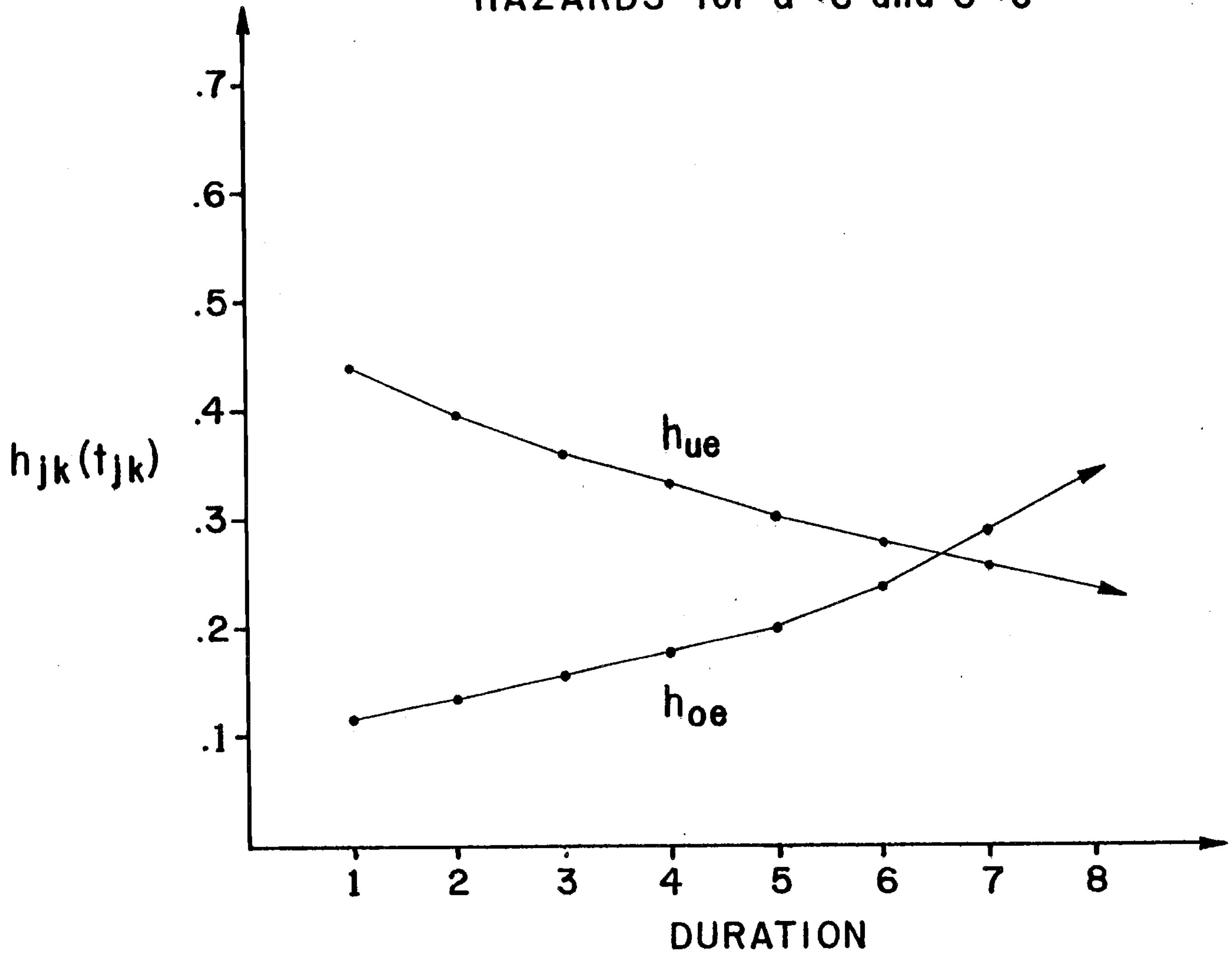
Parameter Estimates from the Three State Unrestricted Model

	From Employment to:		To Employment from:	
	Unemployment	OLF	Unemployment	OLF
Constant	-3.822 (9.778)[1]	-7.193 (2.768)	-.698 (3.782)	-2.384 (2.078)
Duration/10	.482 (.846)	.700 (.379)	-1.253 (1.530)	1.441 (.365)
$\text{Duration}^2/100$	-.240 (1.004)	-.019 (.030)	.481 (.547)	.208 (.084)
MSP	-.355 (.837)	.086 (.068)	-.065 (.193)	1.154 (.400)
C_{ij}	1.396 (3.336)	2.788 (1.025)	-.342 (1.633)	-1.866 (1.081)
$\mathcal{L} = -784.33$				

[1] Absolute value of asymptotic normal statistics in parentheses.

FIGURE 1

HAZARDS for u→e and o→e



employment are plotted for nonmarried (NM) young males who comprise the bulk of the sample. The hazard functions associated with the out of the labor force to employment transitions are monotonically increasing, while the hazards associated with the unemployment to employment transition are nonmonotonic. For nonmarried individuals $h_{ue} > h_{oe}$ for the first 6 months of the respective nonemployment spells, after which the inequality reverses. The vast bulk of nonemployment spells are completed in less than six months.

The estimates from the restricted three state model are given in Table 2. Let $\underset{\sim}{\theta}_{jk} = (\underset{\sim}{\gamma}'_{jk}\ \varphi_{1,jk}\ \varphi_{2,jk}\ c_{jk})$. The restrictions imposed are $\theta_{oe} = \theta_{ue}$, which constrains all parameters in the unemployment to employment and out of the labor force to employment transitions to equality. There are a total of five restrictions. Performing the likelihood ratio test on the restricted versus the unrestricted model, the value of the test statistic is 28.72 which is distributed $\chi^2(5)$. The critical value for a 5 percent significance level is 11.07. We are able to reject the null hypothesis of the equality of the parameters governing the two nonemployment states. These empirical results suggest that out of the labor force and unemployment are not artifical distinctions for this sample of young men. Because the simpler hypothesis (2.1) is rejected, it is unnecessary to test the more computationally demanding proportionality hypothesis (2.2) in order to reject the hypothesis that unemployment and out of the labor force are behaviorally distinct labor force states.

Table 2

Parameter Estimates from the Three State Restricted Model

	From Employment to: Unemployment	OLF	Nonemployment to Employment
Constant	-3.735 (9.934)[1]	-7.718 (2.596)	-.857 (4.756)
Tenure/10	.400 (.706)	.782 (.528)	-1.460 (1.790)
$Tenure^2/100$	-.220 (.940)	-.004 (.007)	.683 (1.116)
MSP	-.397 (.966)	.160 (.148)	.202 (.577)
C_{ij}	1.327 (4.195)	3.102 (1.078)	-.421 (1.894)

$\mathcal{L}$ = -.798.69 (Log Likelihood)

[1] Absolute value of asymptotic normal statistics in parentheses.

Within the framework of job search theory, the fact that over the empirically relevant range the exit rate from unemployment to employment exceeds the exit rate from out of the labor force to employment does not necessarily imply that the rate of arrival of job offers is higher for the unemployed. In the traditional infinite horizon stationary environment one state search model, increases in the rate of arrival of job offers increase the reservation wage and have an ambiguous effect on the exit rate from unemployment (see equations (A.2)-(A.4) in the Appendix). However, we demonstrate in the Appendix that if the wage offer distribution is log concave, higher arrival rates of wage offers imply higher exit rates from unemployment. The normal distribution is log concave. Other examples of log concave distributions are given in the Appendix.

Interpreting our findings within the framework of the traditional one state search model, and assuming log concavity of the wage offer distribution, our empirical evidence suggests that the rate of arrival of job offers is higher in the unemployment state than in the out of labor force state. Thus our evidence is consistent with the hypothesis of productive unemployment i.e. that being unemployed raises the rate of arrival of job offers. However, the fact that some individuals transit directly from out of the labor force to employment without first becoming unemployed suggests that job search activity occurs in both states, and that the difference between the two states is only a matter of degree of search activity.

An alternative interpretation of the evidence within the traditional one state search model is that searchers face a positive discount rate and that the unemployed face a wage offer distribution with a higher mean than do those individuals who are out of the labor force.[1],[2]

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[1] With a positive discount rate, unit translations of wage offer distribution produce less than unit changes in the reservation wage. *Ceteris paribus*, individuals searching from wage distributions with a higher mean will have higher exit rates from unemployment than will individuals searching from wage offer distributions with a lower mean.

[2] Note that this discussion is conducted within a traditional search model. Within the context of a general three state model, such as the one developed by Coleman and Heckman (1981), log concavity of the wage offer distribution is not sufficient to ensure that a higher rate of arrivals of wage offers in a state produces a higher exit rate from the state to employment. The conclusions in the text hold in a two state model. See the Appendix.

5. Conclusion

In this paper we have constructed a test of the proposition that the nonemployment states "unemployment" and "out of the labor force" are behaviorally indistinguishable. Our empirical results indicate that unemployment and out of the labor force are behaviorally distinct, so that in general it is not legitimate to aggregate the two states into a single nonemployment state when analyzing labor market dynamics. Our test is conducted using a flexible econometric model. We are confident that rejection of the two state (employment and nonemployment) model is not attributable to arbitrary functional form assumptions.[1/]

Rejecting the behavioral equivalence of unemployment and out of the labor force suggests that the task of building economic models that predict such a distinction is an empirically fruitful one. In Flinn and Heckman (1982b) and Coleman and Heckman (1981) we present a three state model of search unemployment that is consistent with the empirical evidence reported in this paper.

1/ Even after allowing for alternative distributions of the unobserved heterogeneity component δ, we overwhelmingly reject the two state model.

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<u>Appendix</u>

A SUFFICIENT CONDITION FOR THE EXIT RATE FROM UNEMPLOYMENT TO BE AN INCREASING FUNCTION OF THE RATE OF ARRIVAL OF JOB OFFERS.

Let wage offers arrive in accordance with a homogenous Poisson process with parameter λ. The discount rate is denoted by r, $r \in (0,1)$. $F(w)$ is the cdf of wage offers. The cost of search is c, $c \geq 0$. Accepted jobs last forever, the environment is stationary, and agents have infinite lifetimes. It is well known that in this model, the optimal search strategy has the reservation wage property if w has a finite absolute first moment (see Flinn and Heckman, (1982a)). The reservation wage R is the solution to the following equation

$$\text{(A.1)} \qquad c + r = \lambda/r \int_R^\infty (x-R)dF(x) \qquad \text{for} \quad R > 0.$$

If the R that solves (A.1) is negative, the agent doesn't search.

The exit rate from unemployment, h, is

$$h = \lambda(1-F(R))$$

(see, e.g., Flinn and Heckman, 1982a).

<u>Proposition:</u> In a one state infinite horizon continuous time job search model of unemployment with exponential arrival times of wage offers, a <u>sufficient</u> condition for an increase in the rate of arrival of wage offers to produce an increase in the exit rate from unemployment is that the wage offer distribution be log concave (i.e., that the hazard rate associated with the wage offer distribution be increasing in wages).

Proof: Assume F(w) is absolutely continuous and nondefective ($\lim_{w\to\infty} F(w) = 1$). The derivative of h with respect to λ is

$$\frac{\partial h}{\partial \lambda} = (1 - F(R)) - \lambda f(R)\,\frac{\partial R}{\partial \lambda} \qquad (A.2)$$

Differentiating (A.1) with respect to λ, assuming R > 0,

$$\frac{\partial R}{\partial \lambda} = \frac{1/r \int_R^{\infty} (1-F(x))dx}{1 + \lambda/r\ (1-F(r))} \qquad (A.3)$$

where we have used the well known fact (see, e.g., Ross, 1970) that

$$\int_R^{\infty} (x - R)dF(x) = \int_R^{\infty} (1-F(x))dx \quad .$$

Using (A.3) in (A.2) and rearranging terms we reach

$$\frac{\partial h}{\partial \lambda} = \frac{(1-F(R))}{1 + \lambda/r(1-F(R))} \left\{1 + \frac{\lambda}{r}\left[(1-F(R)) - \frac{f(R)}{1-F(R)} \int_R^{\infty} (1-F(x))dx\right]\right\}. \qquad (A.4)$$

A sufficient condition for (A.4) to be positive is that the term in brackets inside the braces be non-negative.

$$\left[(1 - F(R)) - \frac{f(R)}{1 - F(R)} \int_R^{\infty} (1 - F(x))dx\right] \geq 0 \ . \qquad (A.5)$$

It is convenient to characterize the wage offer distribution by the hazard g(u) which exists by virtue of the absolute continuity of F. We define

$$1 - F(x) = \exp\left(-\int_0^x g(u)du\right) \qquad (A.6)$$

It is convenient to work with $\Phi(R)$ which is defined as

$$\Phi(R) = \int_R^\infty (1 - F(x))dx = \int_R^\infty \exp(-\int_0^x g(u)du)dx \ .$$

Using this notation condition (A.5) may be rewritten as

$$\text{(A.5)}' \qquad - \Phi'(R) + \frac{\Phi''(R)}{\Phi'(R)} \Phi(R) \geq 0 \quad .$$

This condition is satisfied if $\ln \Phi(R)$ is concave in R since concavity requires

$$\text{(A.7)} \qquad \frac{\Phi''(R)}{\Phi(R)} - \left(\frac{\Phi'(R)}{\Phi(R)} \right)^2 \leq 0$$

and multiplication of (A.7) by $(\Phi(R))^2/\Phi'(R)$ produces (A.5)' since $\Phi'(R) < 0$.

By a theorem of Brascamp and Lieb (1976), as reported in Pratt (1981), a sufficient condition for $\ln \Phi(x)$ to be concave is that $\ln \Phi'(x)$ be concave, i.e., that

$$\ln \Phi'(x) = - \int_0^x g(u)du$$

be concave in x. Assuming $g(u)$ is differentiable, strict concavity requires that $g'(x) > 0$, i.e., that the hazard is increasing. This condition is satisfied if the log of the density of w is concave (see Barlow and Proschan, 1975). Examples of log concave densities include normal, exponential, LaPlace, and for certain parameter values (see Barlow and Proschan, 1975, p.79) truncated normal, Weibull and Gamma densities. A Cauchy distribution is <u>not</u> log concave nor is a log normal distribution.

Log concavity of the wage offer density is also a sufficient condition for the exit rate from unemployment to be an increasing function of the rate of arrival of job offers in a simple two state equilibrium model of labor market dynamics. The setup is essentially the same as in the one spell search model except that individuals are terminated from employment spells at an exogenously determined rate σ. This model is discussed at length and estimated in Flinn and Heckman (1982b). The reservation wage R_u in that model is given by the implicit function

$$R_u = -c + \frac{\lambda}{r+\sigma} \int_{R_u}^{\infty} (x-R_u)\,dF(x), \tag{A.8}$$

where $R_u = r\,V_u$ and V_u is the value of occupying the unemployment state. We could repeat the proof given above for the one spell search model to verify sufficiency of the log concavity condition for the two state model but the similarity between A.1 and A.8 is too obvious to warrant a detailed derivation.