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MATCHING EXTERNALITIES AND INVENTIVE PRODUCTIVITY

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Abstract

This paper generalizes and extends the labor market search and matching model of Berliant, Reed, and Wang (2006). In this model, the density of cities is determined endogenously, but the matching process becomes more efficient as density increases. As a result, workers become more selective in their matches, and this raises average productivity (the intensive margin). Despite being more selective, the search process is more rapid so that workers spend more time in productive matches (the extensive margin). The effect of an exogenous increase in land area on productivity depends on the sensitivity of the matching function and congestion costs to changes in density.

JEL Codes: O31 and R11

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Introduction

In Carlino, Chatterjee, and Hunt (2006), we argue that the inventive output of cities may be explained in part by the nature of worker interactions within firms. Our intuition follows from a robust correlation between *patent intensity* (patents per capita) and *job density* (jobs per square mile) that we report in regressions using a cross-section of 280 U.S. cities over the 1990s. All else equal, a doubling of job density results in a 20 percent increase in patenting per capita. The effect of job density on inventive productivity is about the same as the effect of city size (measured in jobs) that is more typically examined in the literature.

A formalization of this intuition is found in the labor market search model of Berliant, Reed, and Wang (2006), hereafter BRW. In their model, workers produce only in teams, which we can think of as firms. Workers are heterogeneous in their knowledge, and the productivity of firms depends on the combination of workers that agree to work together.

Using this set-up, BRW find that the presence of more workers has two effects on productivity. First, with more workers, the opportunity cost of searching for a good partner is lower because the average amount of time required to find an acceptable partner is lower. This is true even though workers are more selective about who they match with. As a result, the average productivity of matches is higher. In addition, since the average duration of search is lower, a higher proportion of the workforce is matched at any given time. Thus the model suggests implications for both the extensive and intensive margins.

In this paper, we develop a more general version of their model and derive a number of comparative static results. We extend BRW in several ways. We assume a more general production function for inventions. We also allow for consumption amenities that influence the equilibrium size of cities. Finally, we characterize the effect of an exogenous increase in land area on city population and inventive output.

The Model

Time is continuous. Workers live forever and discount consumption at the rate r. A measure N of these workers decides to live in the city because the increase in their income, plus utility derived from the available local public goods, exceeds the utility lost to the effects of congestion (including higher rents and taxes). Once located in the city, workers meet randomly and decide whether to match in order to engage in R&D that results in inventions. If the agents agree to match, the match may continue indefinitely, but all matches are subject to a constant dissolution rate π. Workers consume equal shares of the resulting output, and their resulting utility is linear in the output consumed.

The output generated in a match depends on the characteristics of the workers matched together. Workers are differentiated in terms of the variety of knowledge they possess. Agents' types are distributed uniformly along a circle of unit circumference. The most productive matches occur with an intermediate degree of heterogeneity. We focus on the properties of the unique symmetric stationary equilibrium of the game.

In steady state, a measure of agents $M \in [0, N]$ will already be matched and a corresponding measure $U = N - M$ will engage in search for a new partner. In a given match, workers produce inventions at the rate $A \cdot y(x)$. A is a scalar that reflects the overall productivity of the city.[1] Match-specific output depends on the Euclidean distance *x* between the type of an agent's ideal partner and location of his actual partner.

BRW assume $y(x)$ is a linear, decreasing function of *x*. Here, we assume that *expected* output $\tilde{y}(x)$ is continuous, differentiable, and strictly log concave. This implies that output falls more and more rapidly as we continue to increase *x*.[2] To simplify the analysis, we also assume that matches between identical agents are unproductive.[3] BRW make a comparable assumption, but they show that the results are qualitatively the same when this assumption is violated, or if, instead, it is assumed that output is maximized by matching among identical agents.

Unmatched workers engage in search for a new partner. The arrival rate of meetings between unmatched workers follows a Poisson process. In BRW, the *meeting hazard rate* is simply $\mu = \alpha \cdot U$.[4] The exogenous parameter α is a measure of the efficiency of the meeting technology, which in turn may depend on the time required to travel to meeting places, the depth of the labor market, or both.

Workers cannot recognize each other's type until after they meet, and this implies that not all meetings result in matches. The probability that a meeting results in a match will depend on the selectivity of workers, which in turn depends on the opportunity cost of declining to form a match in the current meeting. In a symmetric equilibrium, agents will agree to match with any agent whose type is sufficiently close to the ideal, i.e., where $x \le \delta$, where δ is endogenously determined. Agents choose the value of δ that equates the output from the *marginal* match with the flow value of rejecting the match and waiting for the next encounter: $Ay(\delta) = rV_U$. Let $\beta(\delta, U)$ denote the endogenous *match rate*, the product of the meeting rate μ times the probability that a meeting will result in a match.

Equilibrium

BRW prove the existence of a unique, symmetric, stationary equilibrium of the game for two situations: (1) a city of fixed population and (2) an "open" city where population is endogenously determined. We explore the latter equilibrium, taking into account our

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[1] In our empirical specification, *A* reflects the utilization of inputs other than labor plus any factors external to the firm.

[2] This assumption on production technology is sufficient to satisfy the conditions for assortative matching characterized in Shimer and Smith (2000).

[3] Let $\overline{d}$ denote the Euclidean distance from the agent's own type and his ideal partner. It is sufficient to assume there exists a value $\overline{x} \le \overline{d}$ such that $y(\overline{x}) = 0$.

[4] Note that the meeting technology exhibits increasing returns. BRW argue that the findings of empirical studies are consistent with this assumption, but they also point out that their results do not depend on this assumption.

assumption about the properties of expected output. Note that uniqueness follows from the fact that agents are distributed uniformly over the type space.[5]

We begin by computing the steady-state value functions holding population constant and assuming that agents choose symmetric strategies. The flow value of being matched to an agent whose type is of distance *x* from one's own is:

$$r\hat{V}_m(x,U,\delta) = Ay(x) + \pi\left[V_U(U,\delta) - \hat{V}_m\right],$$

which consists of the instantaneous rate of inventions generated by the current match plus the capital loss associated with the cessation of the match, discounted by the arrival rate of the exogenous separation shock.

The agent does not know the exact type of his next partner, so V_u is calculated as an expected value taking into account the agent's selectivity δ. The next match does not arrive immediately after the current one is dissolved, so V_u also reflects the expected amount of time before meeting a suitable partner, and this depends on both *U* and δ. More explicitly:

(1) $$rV_U(U,\delta) = \beta(\delta,U)\left[V_m(U,\delta) - V_U(U,\delta)\right],$$

which is simply the discounted present value of the capital gain resulting from the next match. Note that V_m is derived by integrating $\hat{V}_m$ over $x \in [0,\delta]$, holding constant both *U* and δ. Thus

(2) $$rV_m(U,\delta) = A\tilde{y}(\delta) + \pi\left[V_U(U,\delta) - V_m(U,\delta)\right],$$

where $\tilde{y}(\delta)$ is the expected value of output from matches when agents' selectivity is δ:

$$\tilde{y}(\delta) = \int_0^{\delta} y(x)\,f(x)dx \Big/ \left[1 - F(\delta)\right] = \frac{1}{\delta}\cdot\int_0^{\delta} y(x)\,f(x)dx,$$

where *f*(*x*) and $1 - F(\delta)$ are the pdf and cdf, respectively, of the distribution of worker locations. The equality with the right-hand side of the expression follows from our assumption that workers are distributed uniformly over a circle of unit circumference.

Substituting for the value functions in (1) and (2), we find that

(3) $$rV_u(U,\delta) = \frac{\beta A\tilde{y}(\delta)}{r + \pi + \beta}.$$

First Order Conditions. Agents that are currently matched enjoy the proceeds of the match until it is dissolved. Unmatched agents are engaged in search and must decide

[5] The closed city version of BRW can be mapped to a special case of the model in Burdett and Wright (1998), where agents' payoffs from prospective matches are symmetric. Proposition 2 of that paper shows that if the distribution over types is log-concave, there is only one steady-state equilibrium. Proposition 4 shows this holds even where the matching function exhibits increasing returns. Uniqueness is preserved in the open city version of the model because congestion costs are monotonically increasing in population.

whether to match with the unmatched agents they meet. They choose δ to maximize $rV_U(U,\delta)$. The associated first order condition is

(4)
$$[r+\pi+\beta]\beta\frac{\partial\tilde{y}}{\partial\delta}+[r+\pi]\tilde{y}\frac{\partial\beta}{\partial\delta}=0.$$

Holding constant *U* and the selectivity of other agents, becoming less selective (raising δ) increases the arrival rate of the next partner $(\partial\beta/\partial\delta>0)$ but reduces the expected income generated in the next match $(\partial\tilde{y}/\partial\delta<0)$.

Because agents' types are distributed uniformly over the unit circle and we assume that matches of identical agents are unproductive, $\beta(\delta,U)=4\delta\mu(U)$. This allows us to rewrite the first order condition:

(5)
$$\frac{\beta}{r+\pi+\beta}=\frac{y(\delta)}{\tilde{y}(\delta)}\equiv 1-\eta,$$

where $y(\delta)$ represents the output generated by a marginal match and η is the elasticity of *average* output with respect to increases in δ.

Lemma 1: There exists an interior solution to the worker's optimization problem characterized by the equality in (5).

Proof: Existence follows from the fact that $\tilde{y}(\delta)$, $\tilde{y}(\delta)$ and β are continuous and differentiable. If we take the limit as $\delta\rightarrow$ 0, holding *U* constant, the right-hand side of (5) converges to 1 while the left-hand side converges to 0. Conversely, if we take the limit as $\delta\rightarrow\bar{\delta}$, the right-hand side of (5) converges to 0, while the left-hand side converges to a value that is strictly greater than zero. Finally, note that the second order condition is simply $\left[r+\pi+\beta\right]\beta y'(\delta)/\delta<0$.■

The left-hand side of (5) reflects agents' discount rate and the rate of turnover between the states of being matched and unmatched. The first order condition implies that when the match rate β is relatively large, the ratio of marginal to average output is close to one. The following lemma shows that if $\tilde{y}(x)$ is log-concave, relatively large values of β are associated with relatively small values of δ. This establishes the intuition that, in equilibrium, a higher arrival rate of matches is associated with more selectivity (lower values of δ) even though $\partial\beta/\partial\delta\big|_U>0$.

Lemma 2: If $\tilde{y}(x)$ is strictly log-concave, η is strictly increasing in *x*.

Proof: This follows directly from the definition of $\tilde{y}(\delta)$, which is the expected value of output, conditional on the level of selectivity δ. Taking first and second derivatives of $Ln\left(\tilde{y}(\delta)\right)$, we have

$$Ln(\tilde{y}(\delta))'=\frac{1}{\delta}\left[\frac{y(\delta)}{\tilde{y}(\delta)}-1\right]=\frac{-\eta}{\delta}<0; \qquad Ln(\tilde{y}(\delta))''=\frac{1}{\delta^2}\left[\eta-(1-\eta)(\varepsilon-\eta)\right];$$

where $\varepsilon \equiv -\delta y'(\delta)/y(\delta)$ is the elasticity of *marginal* output with respect to agents' selectivity. Thus strict log concavity of expected output implies that

(6)
$$\frac{\eta}{1-\eta} < \varepsilon - \eta.$$

A necessary characteristic of the production function, then, is that marginal output falls proportionately more rapidly than average output as δ rises. In addition, the first order condition implies that $\eta \le 1$.[6] But these are sufficient conditions for our proof as

$$\frac{\partial \eta}{\partial \delta} \cdot \delta = \frac{\partial \left(1 - y(\delta)/\tilde{y}(\delta)\right)}{\partial \delta} = \frac{-y(\delta)}{\tilde{y}(\delta)} \cdot \left\{ \frac{\delta y'(\delta)}{y(\delta)} - \left[\frac{y(\delta)}{\tilde{y}(\delta)} - 1 \right] \right\} = (1-\eta)(\varepsilon - \eta). \blacksquare$$

In steady state the instantaneous measure of agents finding matches is just equal to the measure of agents whose matches are terminated: $\beta(U,\delta)U = \pi M$. Substituting for β and M, we can solve a quadratic equation in *U*. One root is positive: $U = \pi(\lambda - 1)/8\alpha\delta$ where $\lambda = \sqrt{1 + 16\alpha\delta N/\pi} \ge 1$. The corresponding match rate is $\beta(U,\delta) = \pi(\lambda - 1)/2$.

Equilibrium Population Mass. To close the model, we formalize the worker's participation constraint:

(7)
$$rV_u(U,\delta) + C \ge w \cdot N,$$

where agents take *U* and the selectivity of other agents as given.[7] *C* represents the utility derived from the available public goods, net of any lump sum taxes used to finance them. We assume $C \ge 0$. The last term, $w \cdot N$, represents the loss of utility that results from congestion, including the rise in costs that result as scarce local inputs are bid up or depreciate more rapidly.

Taken together, (3) and (5) imply that $rV_u(U,\delta) = Ay(\delta)$. In other words, the flow value of being an unmatched agent is just equal to the output generated by the marginal match. In equilibrium (7) is satisfied with equality, so $N = \left(Ay(\delta) + C\right)/w$. This in turn implies that

$$\beta(U,\delta) = \frac{\pi}{2}\left(\sqrt{1 + 16\alpha\delta\left(Ay(\delta) + C\right)/\pi w} - 1\right).$$

Comparative Static Results for Agents' Selectivity and Population

We can substitute the preceding expression for β into our first order condition and derive comparative static results based on a single equation.

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[6] We know there are values of δ where this condition holds because $\lim_{\delta \to 0} \tilde{y}(\delta) = y(0)$.

[7] The net benefit of living outside the city is normalized to zero.

Proposition 1: δ is decreasing in α, *A*, and *C*, but increasing in *w* and π.

Corollary: *N* is increasing in α, *A*, and *C*, but decreasing in *w* and π.

In words, agents become more selective in response to an increase in the efficiency of the matching technology, the average productivity of matches, or local public goods. Agents become less selective in response to an increase in congestion costs or the separation rate.

Proof: For simplicity, we present the comparative static results in terms of elasticities. We begin with the result for α, the arrival rate of meetings per unmatched worker:

$$\frac{\partial\delta}{\partial\alpha}\cdot\frac{\alpha}{\delta} = -\eta\left(\frac{\lambda+1}{2\lambda}\right)\Big/\left\{\eta\left(\frac{\lambda+1}{2\lambda}\right)(1-\phi)+(\varepsilon-\eta)\right\}<0,$$

where $\phi = -Ay'(\delta)\delta\big/\big(Ay(\delta)+C\big)$ is the elasticity of population with respect to changes in δ. We do not wish to rule out the possibility that $\phi > 1$, so we must verify that the denominator in the preceding expression is indeed positive. We begin by noting that $\varepsilon \geq \phi$ because we have assumed that $C \geq 0$. It is then sufficient to verify that

$$\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\varepsilon)+(\varepsilon-\eta)>0.$$

Dividing by η and rearranging terms we have

$$\frac{\varepsilon}{\eta}\left(1-\left(\frac{\lambda+1}{2\lambda}\right)\eta\right)-\left(1-\left(\frac{\lambda+1}{2\lambda}\right)\right)>0,$$

which is positive if $\tilde{y}(\delta)$ is strictly log concave.

Next, we consider the effect of increasing *w*, the coefficient on population that reflects utility lost to congestion:

$$\frac{\partial\delta}{\partial w}\cdot\frac{w}{\delta}=\frac{\partial\delta}{\partial\pi}\cdot\frac{\pi}{\delta}=\eta\left(\frac{\lambda+1}{2\lambda}\right)\Big/\left\{\eta\left(\frac{\lambda+1}{2\lambda}\right)(1-\phi)+(\varepsilon-\eta)\right\}>0.$$

This is also the expression for the elasticity of δ with respect to changes in the separation rate π. Now we consider an increase in *A*, the overall productivity of matches:

$$\frac{\partial\delta}{\partial A}\cdot\frac{A}{\delta}=-\eta\left(\frac{\lambda+1}{2\lambda}\right)\cdot\left(\frac{Ay(\delta)}{Ay(\delta)+C}\right)\Big/\left\{\eta\left(\frac{\lambda+1}{2\lambda}\right)(1-\phi)+(\varepsilon-\eta)\right\}<0,$$

where the extra term in the numerator is the elasticity of population with respect to *A*.

Finally, we consider the effect of an increase in public goods available in the city:

$$\frac{\partial\delta}{\partial C}\cdot\frac{C}{\delta}=-\eta\left(\frac{\lambda+1}{2\lambda}\right)\cdot\left(\frac{C}{Ay(\delta)+C}\right)\Big/\left\{\eta\left(\frac{\lambda+1}{2\lambda}\right)(1-\phi)+(\varepsilon-\eta)\right\}<0,$$

where the extra term in the numerator is the elasticity of population with respect to *C*.

The participation constraint (7) can be rewritten as $Ay(\delta)+C=wN$. Thus equilibrium population is inversely related to δ , which proves the corollary. ■

Comparative Static Results for Per Capita Output and Population Mass

Here we present the main results:

Proposition 2: Per capita output of inventions is increasing in α , *A*, and *C*, but decreasing in *w* and π .

Corollary: Per capita output of inventions is higher in cities with a larger population mass.

Proof: Per capita output is the product of the share of the population that is matched and the average productivity of matches:

$$\Phi \equiv \frac{M}{N}\cdot A\tilde{y}(\delta)=\frac{\beta}{\pi+\beta}\cdot A\tilde{y}(\delta)=\left(\frac{\lambda-1}{\lambda+1}\right)\cdot A\tilde{y}(\delta).$$

Again we present comparative static results in terms of elasticities, which take the form:

$$\frac{d\Phi}{dz}\cdot\frac{z}{\Phi\lambda}=\frac{z}{\lambda\left(\lambda^2-1\right)}\cdot\frac{\partial\lambda^2}{\partial z}+\left(1-\phi\right)\cdot\frac{\partial\delta}{\partial z}\cdot\frac{z}{\delta}-\eta\lambda\cdot\frac{\partial\delta}{\partial z}\cdot\frac{z}{\delta}.$$

The first two terms reflect the change in the share of agents matched at any given time $\left(M/N\right)$. These reflect changes in the intensive margin (holding *N* constant, does *M* rise or fall?) and the extensive margin (holding constant $\left(M/N\right)$, does *N* rise or fall?). The last term reflects the change in expected output per match, characterized in the previous proposition. Rearranging the expression, we have

$$\frac{d\Phi}{dz}\cdot\frac{z}{\Phi\lambda}=\frac{z}{\lambda\left(\lambda^2-1\right)}\cdot\frac{\partial\lambda^2}{\partial z}+\left\{\left(1-\phi\right)-\eta\lambda\right\}\cdot\frac{\partial\delta}{\partial z}\cdot\frac{z}{\delta}.$$

Consider first an increase in the arrival rate of meetings per unmatched worker α :

$$\frac{d\Phi}{d\alpha}\cdot\frac{\alpha}{\Phi\lambda}=1+\left\{\left(1-\phi\right)-\eta\lambda\right\}\cdot\frac{\partial\delta}{\partial\alpha}\cdot\frac{\alpha}{\delta}=\frac{\left(\frac{\lambda+1}{2}\right)\eta^2+\left(\varepsilon-\eta\right)}{\left(\frac{\lambda+1}{2\lambda}\right)\eta\left(1-\phi\right)+\left(\varepsilon-\eta\right)}>0.$$

Next consider an increase in the overall productivity of innovations *A*:

$$\frac{d\Phi}{dA}\cdot\frac{A}{\Phi\lambda}=\frac{Ay(\delta)}{Ay(\delta)+C}+\left\{(1-\phi)-\eta\lambda\right\}\cdot\frac{\partial\delta}{\partial A}\cdot\frac{A}{\delta}=\frac{\left(\frac{Ay(\delta)}{Ay(\delta)+C}\right)\left\{\left(\frac{\lambda+1}{2}\right)\eta^2+(\varepsilon-\eta)\right\}}{\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\phi)+(\varepsilon-\eta)}>0.$$

Consider the effect of an increase in public goods *C* available in the city:

$$\frac{d\Phi}{dC}\cdot\frac{C}{\Phi\lambda}=\frac{C}{Ay(\delta)+C}+\left\{(1-\phi)-\eta\lambda\right\}\cdot\frac{\partial\delta}{\partial C}\cdot\frac{C}{\delta}=\frac{\left(\frac{C}{Ay(\delta)+C}\right)\left\{\left(\frac{\lambda+1}{2}\right)\eta^2+(\varepsilon-\eta)\right\}}{\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\phi)+(\varepsilon-\eta)}>0.$$

Finally, consider the effect of an increase in the utility lost due to congestion as population increases *w* (the effect of an increase in the separation rate π takes on exactly the same expression):

$$\frac{d\Phi}{dw}\cdot\frac{w}{\Phi\lambda}=-1+\left\{(1-\phi)-\eta\lambda\right\}\cdot\frac{\partial\delta}{\partial w}\cdot\frac{w}{\delta}=\frac{-\left\{\left(\frac{\lambda+1}{2}\right)\eta^2+(\varepsilon-\eta)\right\}}{\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\phi)+(\varepsilon-\eta)}<0.$$

The corollary then follows from the previous corollary. ■

Comparative Static Results for Changes in Land Area

In BRW population varies, while the physical size of the city is fixed. In other words, in terms of the model, changes in population and density are the same.

But we also wish to consider the possibility that exogenous increases in land area might reduce density and therefore per capita output. This will occur if an insufficient number of workers respond to the reduction in congestion by moving into the city. To explore this possibility, we assume that the efficiency of the meeting rate parameter α and congestion coefficient *w* are decreasing in city land area *L*. We show the following:

Proposition 3: Increasing *L* decreases (increases) δ, and Φ if α is more (less) sensitive to changes in *L* than is *w*.

Proof: We derive the elasticity of per capita output with respect to changes in land area in the same manner as we did in Propositions 2 and 3:

$$\frac{d\Phi}{dL}\cdot\frac{L}{\Phi}=\frac{L}{\lambda}\left(\frac{\alpha'(L)}{\alpha(L)}-\frac{w'(L)}{w(L)}\right)\left\{2+\frac{(\lambda+1)(1-\eta)\eta^2}{\left(\frac{\lambda+1}{\lambda}\right)\eta(1-\phi)+2(\varepsilon-\eta)}\right\}.\ ■$$

In the model, increasing land area has two effects on workers' incentive to migrate to the city. The first effect is a reduction in the costs associated with congestion, which, all else equal, induces additional migration. The second is a reduction in expected income that results from waiting longer for the next meeting with an unmatched worker. All else equal, this would discourage workers from entering the city.

It turns out that, in the absence of a fully specified model for the supply of developed land, we cannot sign the effect a priori. If the reduction in congestion costs is sufficiently large relative to the decline in the efficiency of meeting technology, both density and per capita output rises. But if the reduction in congestion costs is relatively small, density and per capita output will fall.

Conclusion

This paper develops a generalized version of the labor market matching model of Berliant Reed and Wang (2006) that can explain why more dense cities (measured in jobs per square mile) would be more productive in the innovation process, as documented in Carlino, Chatterjee, and Hunt (2006).

We show that the results of BRW hold for more general functional forms as long as output is log concave with respect to deviations from the ideal match. We also show that the interior equilibrium remains unique. By explicitly distinguishing the mass of agents and land area, we are able to generate an additional result: an exogenous increase in available land increases per capita output whenever the meeting rate between unmatched workers is less sensitive to a decline in density than are the congestion costs borne by workers. This in turn implies the population of the city would rise.

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