Discussion paper

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The bond yield conundrum: alternative hypotheses and the state of the economy.[*]

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Abstract

We study the bond yield conundrum in a macro-finance framework. Building upon a flexible and non-structural macro-finance model, we test the hypothesis that the bond yield conundrum is connected to various sources of uncertainty in the financial markets. Moreover we explicitly test for the role of the state of the economy. Our findings give a richer description of the drivers of the term premium yet the conundrum remains. The results in this paper indicate that the underlying observable drivers of the term premium are not yet fully understood.

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1 Introduction

In his testimony to the senate on february 16 2005, Alan Greenspan used the word *conundrum* to describe the behavior of long-term interest rates. He used this term to coin the fact that long-term rates and short-term rates diverged. This divergence posed a conundrum for various reasons. First of all, the expectations hypothesis suggests that short and longer term rates should move together. Moreover energy prices were rising early 2005, while the federal fiscal position was deteriorating. All these factors were expected to lead to higher long-term interest rates yet this expectation remained unfulfilled.

The conundrum drew the attention of practitioners, academics and policy makers. Each seeking to explain this behavior. In academic circles, the conundrum was among others investigated by Rudebusch et al. (2006) and Backus and Wright (2007). To get a grip on the issue it is insightful to look at Figure 1 in which we have plotted the federal funds rate, the short-term interest rates and the long-term interest rates from 1981 onwards. We see that the bond yield conundrum is clearly indicated in the picture by the convergence of the long-term interest rates and the short-term interest rates. While the federal funds rate rose from 1 percent in June 2004 to 4.2 percent in December 2005, the rates on the 10-year U.S. Treasures remained fairly sTable and even diminished by 0.2 percent until a level of 4.5 percent over the same period.

As noted by some observers, such interest rate behavior is not unique, see for example Cochrane (2007). The conundrum lies in the combination of the different macro-economic ingredients. The mix of rising energy prices, the deteriorating fiscal budget, the robust economic expansion together with

the behavior of the interest rates is what made Greenspan use the word conundrum.

In this study we investigate this phenomenon by setting up a simple macro-finance model building upon the work of Rudebusch et al. (2006). We use a similar flexible macro-finance model with observable macro factors and no latent factors. We improve the model in terms of the fit by adapting the state variables. Within this framework we test some popular explanations while taking the state of the economy into account.

2 A general macro-finance framework

Our model is similar to one of the models used in Rudebusch et al. (2006) which these authors in turn borrowed from Bernanke et al. (2004). It is a macro-finance model with macro-economic factors as driving sources of variation and with the no-arbitrage assumption imposed. This type of models was put forward in a seminal article by Ang and Piazzesi (2003). The reader interested in a solid motivation is referred to that article. In all fairness, we should admit that this type of modeling has its own weaknesses too. Some important limitations are 1) the necessary low number of state variables, 2) the difficult to optimize likelihood function, 3) the implied homoskedastic yields, 4) the risk of overfitting. A good and in-depth discussion was written by Kim (2007). We discuss some of the major criticisms in the final section. This approach allows a comparison with previous literature on the bond yield conundrum.

The general model as described in Ang and Piazzesi (2003) consists of 4 key ingredients which we first briefly present and then combine in subsection 2.5.

2.1 State dynamics

Assume that we have observable macro economic variables F_t. The vector F_t follows a Gaussian VAR(p) process:[1]

$$F_t = \Phi_0 + \Phi_1 F_{t-1} + \cdots + \Phi_{t-p} + \theta u_t \qquad (1)$$

with $u_t \sim$ IID $N(O, \Omega)$ and Φ_i denoting the coefficientmatrix i with appropriate dimensions.

The dynamics can be rewritten in the following compact form:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \epsilon_t \qquad (2)$$

with $\epsilon_t = (u_t^o O_{(.)\times 1} u_t^u)'$ and Σ containing blocks of zeros to accommodate lags in F_t.

2.2 Short-rate equation

The one-period short rate r_t is taken to be an affine function of the state variables:

$$r_t = \delta_0 + \delta_1' X_t \qquad (3)$$

with δ_0 a scalar and δ_1 a $(.) \times 1$ vector.

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[1]In the more general case we would have $F_t = \text{vec}(f_t^o f_t^u)$ with f_t^o denoting the observable variables and f_t^u denoting the latent variables.

2.3 The price of risk

The market prices of risk arising because of uncertainty ϵ_t are denoted by Λ_t and is parametrized as an affine process:

$$\Lambda_t = \lambda_0 + \lambda_1 X_t. \tag{4}$$

With λ_0, λ_1 equal to 0 we have risk-neutral investors and no correction for risk. By allowing λ_0, λ_1 to take values different from 0, we can allow for constant or time-varying risk premia.

2.4 Pricing Kernel

The crucial assumption in this model is the assumption of no arbitrage which guarantees the existence of an equivalent martingale measure $\mathbb{Q}$ such that the price of any (non dividend paying) asset V_t satisfies $V_t = E_t^{\mathbb{Q}}(\exp(-r_t)V_{t+1})$. The Radon-Nikodym derivative is denoted by ξ_{t+1} so we have for any random variable Z_{t+1} that $E_t^{\mathbb{Q}} Z_{t+1} = E_t(\xi_{t+1} Z_{t+1})/\xi_t$. We assume that the Radon-Nikodym derivative follows a log-normal process:

$$\xi_{t+1} = \xi_t \exp(-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1}), \tag{5}$$

with Λ_t as in 2.3.

We define the pricing kernel m_{t+1} as

$$m_{t+1} = \exp(-r_t)\frac{\xi_{t+1}}{\xi_t}. \tag{6}$$

Substituting (3) and (5) into this expression yields the following expression for the pricing kernel:

$$m_{t+1} = \exp(-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} - \delta_0 - \delta_1'X_t). \tag{7}$$

2.5 Bond prices

The no arbitrage condition which links bonds of different maturities implies a stochastic discount and links the price of these through $E_t(m_{t+1}R_{t+1}) = 1$. Let p_t^n denote the price of an n-period zero coupon bond. For the one-period bond we have then $p_t^1 = E_t[m_{t+1}] = \exp(-r_t)$, substituting (3) for r_t yields $p_t^1 = \exp(-\delta_0 - \delta'X_t)$. We write $\bar{A}_1 = -\delta_0$ and $\bar{B}_1 = -\delta_1$. Through induction (see appendix) we obtain:

$$p_t^n = \exp(\bar{A}_n + \bar{B}_n'X_t) \tag{8}$$

For a derivation of the likelihood function to estimate this model we refer to Ang and Piazzesi (2003).

3 The model

The model we use is similar to the Bernanke-Reinhart-Sack model (BRS) used in Rudebusch et al. (2006) in the sense that our model is also a model with *observable* macro-economic state variables. All the variation is driven by observable macro-economic factors; there are no latent variables in our setup. This allows forecasting of the entire yield curve as function of the observable variables designated as underlying variables. This specification differs from most models in the macro-finance literature as it identifies the underlying factors that characterize the term structure by means of *observable* indicators of macroeconomic conditions and the stance of monetary

policy. The dynamics of the underlying factors are modeled with a vector autoregression in five observable variables:

- Real activity, which we obtained by the first principal component extracted from a set of industrial production indices[2].

- Monetary base, which we measure as the principal component extracted from a set of measures for the depository reserves.

- The Blue Chip survey of inflation expectations of the coming year as a measure of inflation expectations.

- The federal funds rate, to capture the current stance of the Federal Reserve.

- The rate on the Eurodollar futures contracts with four quarters to expiration.

The last three macro-economic variables are exactly the same as in Rudebusch et al. (2006). Finding a good measure of real productivity is difficult. Based on the expanding literature on the use of large datasets, see for example Marcellino et al. (2005) or Bernanke et al. (2005) , we prefer to extract a productivity measure from disaggregated indices instead of using the deviation of employment from the trend (using a Hodrick-Prescott filter) like in the original BRS formulation.

Following Ang and Piazzesi (2003), Bernanke et al. (2004) and Rudebusch et al. (2006) we estimate this model in two stages to reduce the number of parameters that have to be estimated.[3] In a first stage, we estimate

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[2]A detailed description of the variables can be found in the appendix.

[3]We sincerely thank Eric Swanson for providing us with the data and the code from his study with Glenn Rudebusch and Tao Wu. As reported in that study, the authors were able to improve significantly upon Bernanke et al. (2004) in terms of fitting the model.

a vector autoregression with the five macroeconomic variables and four lags over the 1984-2005 period. In the choice of the number of lags we follow Bernanke et al. (2004). In a second stage we estimate the risk loadings (see equation 4) with the vector autoregression coefficients fixed using nonlinear least squares. The model is estimated to closely match the 6-month, 1-year, 2-year, 3-year, 4-year, 5-year, 7-year and 10-year yields with equal weights on these maturities. This is important because putting a larger weight on the long end of the yield curve would favor our estimation results.

The results of the first stage estimation is graphically represented in Figure 2.

The impulse response functions reveal some interesting dynamics. Let us focus on the effects of production and especially depository reserves, a state variable which is a bit less common in these studies. A shock in production (first column) yields a hump shaped response function on the year-ahead Eurodollar rate and the federal funds rate, while the effect on the other state variables is closer to a geometric decay. The effect of a shock in the depository base (second column) gives more peculiar behavior. Such a shock induces some volatility on the Eurodollar rate in the short run, after which the Eurodollar rate returns to its earlier level. The effect on production and Blue Chip inflation expectations seems more long term.

Now to the estimated risk loadings which are presented in Table 1.

The risk loadings on some factors seem very large but one needs to keep in mind that the scaling of the different variables is not really comparable. Moreover if we look at Table 2, we see that the variables with the larger risk coefficients have relatively lower variance so the net effect on bond prices is not as large as a quick glance on Table 1 would suggest. To have a proper idea of the size of the impact of the different factors we should

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perform a variance decomposition of the long-term premium on the five stochastic shocks ϵ. However a drawback of our model is that the setup does not really allow for a proper decomposition. At those longer horizons the term premium is highly nonlinear and a direct computation of the variance decomposition becomes infeasible.[4] An approximation by the delta method is likely to yield poor approximations, see also Rudebusch et al. (2006).

How well does this model perform in terms of fit? Table 3 suggests that it performs significantly better then the results obtained by Rudebusch et al. (2006) with a root mean squared error which is on average about 20% lower. So use a broader measure of real activity and a measure of monetary base leads to a significant imporvement.

3.1 Towards explaining the Conundrum

Given the tight fit we obtained, we can focus our attention to the sample period of interest. We start out by plotting the 10-year zero-coupon US treasury yield curve along with yield curve implied by our model. This plot, shown in Figure 3, shows the actual yield curve in green, the model implied Treasury yield in blue, the model implied risk-neutral rate in red and the model implied term premium in cyan. The risk-neutral rate is the estimated yield curve under the hypothesis that risk is not priced i.e. where the prices of risk are always equal to zero. This corresponds to λ_0 and λ_1 restricted to zero (see equation 4). The state variables are in this case governed by the first stage VAR of which we have plotted the impulse response functions in Figure 2. The model implied Treasury yield is the estimated yield curve when the prices of risk are no longer restricted to zero, but are an affine

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[4]As noted in Rudebusch et al. (2006), this exercise amounts to minimizing a 10years $*$ 12months $=$ 120th-degree polynomial.

function of the macroeconomic variables. It follows that the term premium is the difference between those two lines and should be interpreted as the estimated term premium on the 10-year zero-coupon bond at each point in time.

Figure 3 displays the fit of our model. The blue and green line move very closely together which is consistent with the low RMSE's. Our model captures the downward trend *and* the high-frequency swings. One may be tempted to dismiss this result as merely the result of overfitting the data. We believe that this is not the case for two reasons. First of all, the optimization placed equal weight on all maturities considered and did not put too much weight on the long end. Secondly, our model does not contain flexible latent factors (level, slope, curvature) like most other macro-finance models, which are able to *absorb* a lot of variability. In our model the movements are entirely based on the variability in observable macro variables under the no arbitrage assumption. While this model is autoregressive and nonstructural with a large parameter set, the no arbitrage assumption puts some restrictions on the model. Figure 3 reveals for the ten year rates both the term premium and the risk-neutral yield curve have fallen over the sample, respectively with 230 and 330 basis points. Also the importance of the term premium has diminished over the sample. In Figure 4 we plot the residuals of our model.

Figure 4 reveals that despite the model's excellent fit, there are periods in which the fit was worse. To make this precise, while the average error was 62 basis points, the largest error (in absolute value) was 78 basispoints. There are several occasions in which the model fits the data poorly but these periods were briefer (1991) and milder (1997-1999). Moreover, if we look at *the relative size* of the residuals, the difference between 2004-2005

and earlier periods becomes really remarkable. Inspection of Figures 3 and 4 reveals that on average the residuals are about a twentieth of the level of the 10-year treasury yield while around 2005 the residuals amounted to nearly a seventh of the level of the yield curve. These results reveal the conundrum. Despite its excellent fit, the 2004-2005 period is difficult to explain with a general purpose macro-finance model. In the next section we take the analysis one step further by trying to identify factors that might be related to the residuals in our model.

4 Exploring the residuals

The macro-finance model presented above was not able to resolve the conundrum. This was clearly visible in Figure 4 where the residuals were relatively large during the conundrum period. The number of factors that we can consider within the macro-finance model is limited because fitting the model becomes increasingly more complex as the numbers of parameters increase. Since we only allowed for observable variables we needed to include variables which capture the bulk of the variation. In this section we look for variables outside our macro-finance model which may help explaining the conundrum.

There are three categories of variables we are going to consider. First of all we test for variables which were previously considered in the literature. Then we test for some additional variables loosely motivated by readings in press reports. Finally we also consider the importance of the state of the economy when identifying relevant variables. Conditioning on the state of the economy has been absent in the discussion of the bond yield conundrum so far. However, recent empirical studies on asset prices, for example

Boyd et al. (2005), have shown that the state of the economy matters a lot. Conditioning on the state of the economy may be important in so far that candidate explanatory variables might differ in importance along the business cycle.

We start out by univariate regressions in which we regress different explanatory variables on the residuals from our macro-finance model. Then we proceed with multivariate regressions in which we test for the best candidates obtained from the univariate regressions.

4.1 Regression analysis

Previous explanations for the bond yield conundrum based their choice of explanatory variables on a survey conducted by the firm Macroeconomic Advisors. This survey of market participants and business economists was held in early March 2005 and asked the respondents to provide their view on the low-level of long-term rates. The survey identified the following seven factors (the number between parentheses indicates a rough estimate of how much each factor was perceived to have lowered the bond yield in basis points): 1] demand by foreign central banks (21), 2] increased demand by pension funds (11), 3] reaching for yield (10), 4] minimal inflation risk (10), 5] greater transparency by the Fed (8), 6] excess global savings (8), 7] low economic growth volatility (7). The largest factor according to the respondents was the increased demand for US long-term securities by foreign central banks. The other six explanations also make sense. The second largest factor relates to an expected rise in the demand for long-term securities of pension funds to better match the duration of their assets to their liabilities. This is, however, difficult to quantify. Similarly the next two factors which re-

late to the risk appetite of investors are difficult to quantify. Also greater transparency of the Fed is difficult to incorporate in our analysis. There is a rich literature on central bank transparency and recently there were some indices developed for this purpose, see for example the study by Eijffinger and Geraats (2006). Unfortunately these indices are not entirely fit for our purposes as these variables are at a lower frequency.

The first group of explanatory variables is similar to those in Rudebusch et al. (2006). Two measures of financial market volatility: (1) The *Merrill Lynch Move Index* to measure the implied volatility in the longer-term U.S. treasury market. This index is a weighted average of a wide range of outstanding options on the 2-year, 5-year, 10-year and 30-year U.S. treasury securities with weights of 0.2, 0.2, 0.4, 0.2 respectively. (2) The *VIX measure* of implied volatility from options on the S&P 500 index. The first measure relates to uncertainty in the Treasury market, the second to uncertainty in the stock market.

Macroeconomic uncertainty is proxied by two variables. *Volatility of the growth rate of GDP* and *volatility of the core PCE deflator*.[5]

Additionally we explore the explanatory power of the following variables. The motivation for these was given by the macroeconomic conditions during the bond yield conundrum. A first variable is the *volatility of oil prices* as a proxy for uncertainty about energy prices. Oil prices were steeply rising in 2004-2005 and this rise was receiving increasing attention in the media. But rising energy prices tend to work against the behavior of the bond yields we observed in 2004-2005. This raises the question whether there was a role played by the volatility rather than the level of oil prices. A next variable

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[5]We measure volatility by calculating the rolling standard deviation. Details are provided in the appendix.

we consider is related to net government spending. We test for the impact of *the uncertainty around the growth rates of net government spending.* Government spending was very large in the period under scrutiny. With the fiscal situation of the U.S. steadily deteriorating, uncertainty related to the growth rate of spending might became more of a concern to investors in long-term bonds. Admittedly, this argument is a bit far fetched given the AAA rating of US treasuries but the 2010 turmoil in the European bond markets show that there is a link between the fiscal situation of a country and its long term financing through bonds -even for the U.S. We also test for uncertainty among the consumers by taking *volatility in consumer expectations* -as measured by the Michigan consumer survey- as an explanatory variable. Next we consider the U.S. credit market by taking both the *volatility of all outstanding consumer credit in the United States* and *the volatility of all loans by commercial banks* (industrial and commercial) as regressors. Finally we investigate whether there was an influence of the Chinese economy. The current chairman of the Federal Reserve, Benjamin Bernanke, once pointed to the role of the Chinese economy in a famous speech in which he introduced the term *global savings glut*, see Bernanke (2005). Instead of considering the level of the Chinese growth rates, we used the *volatility in these growth rates* to capture the uncertainty surrounding the Chinese growth path.

To have an idea on how these variables comove it is instructive to look at Table 4. The Table indicates that the variables are related little correlated and multicollinearity is unlikely to be a problem later on in our multivariate regressions. Only 8 out of the 55 correlations are larger than 0.4.

We regressed all these variables individually against the residuals, the results can be found in the Table below. A detailed description of the explanatory variables can be found in the appendix.

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The univariate regressions deliver four variables with explanatory power; the MOVE index, Oil price volatility, volatility of outstanding consumer credit and the GDP growth volatility. The volatility of the growth in the Chinese GDP and the volatility of loans by commercial banks are also statistically significant but seem to have low explanatory power.

Based on the univariate results, we have estimated multivariate regressions but we do not consider the explanatory variables with a t-value (in absolute value) below 2. The models we have estimated are the following:

$$\text{Resid}_t = \alpha + \sum_{i=1}^{6} \beta_i \text{var}_{it} + \epsilon_t \tag{9}$$

$$\text{Resid}_t = \alpha + \sum_{i=1}^{6} \beta_i \text{var}_{it} * \text{RP}_t + \sum_{i=1}^{6} \gamma_i \text{var}_{it} * (1 - \text{RP}_t) + \epsilon_t \tag{10}$$

The model represented by equation (9) is a multivariate regression with as independent variables var_i the six significant variables from the univariate regressions above: the MOVE index, Oil price volatility, GDP growth volatility, volatility of outstanding consumer credit, volatility of Chinese GDP growth, volatility of loans by banks. The model represented by equation (10) is a similar multivariate regression in which we interact each variable with a recession indicator and its complement, respectively. A similar specification was used in Basistha and Kurov (2008) to gauge the impact of the state of the economy. The recession probability, denoted by RP_t, was obtained from Chauvet and Piger (2008).

The results can be found in Table 6. Model 1 corresponds to regression equation 9 and model 2 corresponds to regression equation 10. Bull corresponds to RP_t and Bear $(1 - \text{RP}_t)$. Instead of simple dummy variables,

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these are recession probabilities. By interacting the variables with *Bear* or *Bull* we can gauge the effects of these variables across the business cycle. Given the time-series nature of the data, we used Newey-West errors. The reported t-statistics are based on these errors.

In the first multivariate specification, only three variables have explanatory power at the 5% confidence level, each with the expected sign. The signs on GDP growth volatility and the MOVE index are as expected and in line with the results of Rudebusch et al. (2006). Lower macroeconomic volatility is associated with lower yields on long-term Treasury securities. The same goes for the MOVE index. The negative coefficient on the volatility of oil prices indicates this uncertainty is inversely related to the long-term yields.

In the second specification (equation 10) we allow for different effects depending on the state of the economy. Surprisingly, the effects of the three important variables of regression (9) are only important in Bullish markets and are insignificant during a downturn. But, during Bearish markets the volatility of loans by commercial banks becomes significant. The coefficient on this variable is remarkably large in comparison with the univariate regressions. Taking the state of the economy into account certainly matters for this variable.

Now we are ready to answer the question on how much these explanatory variables can explain the remaining conundrum. We do this by decomposing the decline of bond yields between June 2004 and June 2005 according to the regression results presented in Table 6. June 2004 is chosen as the starting point because that month is most often cited as the beginning of the conundrum. Over this sample, the observed 10-year yields dropped by 77.7 basis points whereas the macro-finance model implied changes in

the risk-neutral yield and term premium of 17.99 and 14.31 basis points, respectively.

The first row in Table 7 shows the difference we seek to explain. The subsequent rows indicate to what extent this difference can be explained on the basis of our regressions. The results are disappointing. Only a little more than 14% can be explained. The unexplained part of the model implied residuals are larger than the observed drop in basis points. This is the crux of the conundrum. The macro economic environment, captured by the state variables in our macro-finance model, normally leads to higher long-term rates. Within our model this would be an increase of about 32.3 basis points. Instead we observed a decrease over the period under scrutiny. The alternative explanatory variables we proposed only explain a small part of the gap between what we observed and what the macro-finance model suggests.

5   Conclusion

In this paper we studied the bond yield conundrum. Building on the work of Rudebusch et al. (2006) we have tried to test some alternative hypotheses. Despite our improvements in terms of fit to the model by Bernanke et al. (2004) the 2005 conundrum is still a mystery. The conundrum is an interesting topic to study as it is a period for which the current generation of macro-finance models seem to have difficulties to grasp the behavior of the yield curve. The model we used in this paper is appealing because it does not use latent factors. It therefore allows us to relate the yield curve behavior to macro factors only. This task proved to be difficult. The likelihood function is highly nonlinear and not easy to handle. This problem was

explained in more detail by Kim (2007).

Although our decomposition of the results delivered less spectacular results than we hoped for, some interesting lessons can be drawn. Most importantly we feel that conditioning on the state of the economy is important. Variables which may be unimportant in general, may prove to be important in certain periods.

We have done an extensive study and at best we find variables which explain a modest part of the conundrum. We feel that further research should focus on putting more structure on the model. A particular interesting approach was recently proposed by Ang et al. (2005) where the authors introduce Taylor-rules in the macro-finance model. Another way may consist of introducing regime shifts. Bansal and Zhou (2002) provide strong evidence that a regime-switching model can deal with the violations of the expectations hypothesis. These regime switches are intimately related to the business cycle. Finally, one could also consider relaxing the Gaussian nature of most macro-finance models by incorporating heteroskedastic dynamics parameterized by discretized square-root processes as was suggested in Ang and Piazzesi (2003).

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A   Appendix: derivation of bond prices

Proof. Basis: The statement holds for the case $n = 0$ because rewriting p_t^1 in terms of $\bar{A}_t$ and $\bar{B}_t$ gives $p_t^1 = \exp(-\bar{A}_1 - \bar{B}_1'X_t)$. Inductive step: Assume that (8) holds. We start with the following identity for the stochastic discount factor:

$$p_t^{(n+1)} = E_t[m_{t+1}p_{t+1}^{(n)}]. \tag{11}$$

Using the identities (7), (8); substituting the compact form into the expression and using (3) gives:

$$
\begin{aligned}
p_t^{(n+1)} &= E_t\left[\exp\left\{-r_t - \tfrac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + \bar{A}_n + \bar{B}_n'X_{t+1}\right\}\right] \\
&= E_t\left[\exp\left\{-r_t - \tfrac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + \bar{A}_n + \bar{B}_n'(\mu + \Phi X_t + \Sigma\epsilon_{t+1})\right\}\right] \\
&= E_t\left[\exp\left\{-\delta_0 - \delta_1'X_t - \tfrac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} + \bar{A}_n + \bar{B}_n'(\mu + \Phi X_t + \Sigma\epsilon_{t+1})\right\}\right] \\
&= \exp\left[-\delta_0 + \bar{A}_n + \bar{B}_n'\mu + (\bar{B}_n'\phi - \delta_1')X_t - \tfrac{1}{2}\Lambda_t'\Lambda_t\right] \times E_t\left[\exp(-\Lambda_t' + \bar{B}_n'\Sigma)\epsilon_{t+1}\right]
\end{aligned}
\tag{12}
$$

Now recall that ϵ_t is assumed to be iid and normally distributed with $E[\epsilon_t] = 0$. Together with the assumptions on $\text{var}[\epsilon_t]$ and Λ_t, this implies $\Lambda_t'\Lambda_t = \Lambda_t'\text{var}(\epsilon_t)\Lambda_t$, which allows us to rewrite the last line of (12) as:

$$p_t^{(n+1)} = \exp\left\{-\delta_0 + \bar{A}_n + \bar{B}_n'(\mu - \Sigma\lambda_0) + \tfrac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n - \delta_1'X_t + \bar{B}_n'\phi X_t - \bar{B}_n'\Sigma\lambda_1 X_t\right\} \tag{13}$$

This final expression is of the form (8) with $\bar{A}_i$ and $\bar{B}_i$ recursively defined as:

$$
\begin{aligned}
\bar{A}_{n+1} &= -\delta_0 + \bar{A}_n + \bar{B}_n'(\mu - \Sigma\lambda_0) + \tfrac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n \\
\bar{B}_{n+1} &= \bar{B}_n'(\Phi - \Sigma\lambda_1) - \delta_1'
\end{aligned}
\tag{14}
$$

and $\bar{A}_1, \bar{B}_1$ as earlier. ∎

Because the continuously compounded yield on a y_t^n on an n-period zero-coupon bond is given by $-\log(p_t^n)/n$, we have that $y_t^n = -\frac{1}{n} \times (\bar{A}_n + \bar{B}_n{}' X_t)$. Define $A_n := -\bar{A}_n/n$, $B_n := -\bar{B}_n/n$ and we obtain:

$$y_t^n = A_n + B_n' X_t. \tag{15}$$

Since yields are affine functions of the state variables X_t, equation (15) can be viewed as the observation equation of a state-space system. Because lagged variables are state variables in our system (see 2.1), the affine form is maintained.

B   Appendix: Macro-economic variables

As mentioned in the text, three from the five state variables are identical to those uses by Bernanke et al. (2004). All data used for the new state variables in our macro-finance model can be obtained trough the FRED database. The series we used for the different variables are:

1. Production: This variable corresponds to the first principal component from the following set of variables: INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS; INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS; INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT; INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS; INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS; INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC); INDUSTRIAL PRODUCTION INDEX - FUELS. The idea is that using disaggregated series allows for cross-sectionally smoothing out noise in comparison with the use of an ag-

gregate index, see for example Stock and Watson (2008).

2. Monetary Base: `MONEY STOCK - M2; MONETARY BASE, ADJUSTED FOR RESERVE REQUIREMENT CHANGES(SA).`

The data used in the residual regressions where obtained through a variety of sources:

1. MOVE index: freely available on the web (monthly frequency).

2. Oil price volatility: Oil prices were obtained through Datastream (Spot Oil Price: West Texas Intermediate).

3. GDP growth volatility: The GDP growth data are available at quarterly frequency. We took these from Stock and Watson (2008). Growth rates are the log of the year-on-year differences. The volatility was obtained by taking the standard deviation over the past 24 months (quarterly frequency). Monthly observations were constructed by interpolation (cubic spline).

4. Volatility of outstanding consumer credit. Consumer credit data is available through the FRED database. The volatility was obtained by taking the standard deviation over the past 12 months (monthly frequency).

5. Macroeconomic data on the GDP of China was taken from a study by Rajaguru and Abeysinghe (2004). The volatility was obtained by taking the standard deviation over the past 24 months (quarterly frequency). Monthly observations were constructed by interpolation (cubic spline).

6. Volatility of loans by commercial banks. Consumer credit data is available through the FRED database. The volatility was obtained by taking the standard deviation over the past 12 months (monthly frequency).

7. Volatility of PCE inflation:: PCEinflation was obtained through the FRED database(Personal Consumption Expenditures: Chain-Type Price Index Less Food and Energy). The volatility measure was obtained by taking the standard deviation of prices over the past 24 months (monthly frequency).

8. VIX index: Obtained through Datastream.

9. Investor sentiment: Obtained from the website of Wurgler. The sentiment index and related issues are discussed in Baker and Wurgler (2007).

10. Volatility of net government spending. Net government spending was obtained through the FRED database. The volatility was obtained by taking the standard deviation over the past 24 months (quarterly frequency). Monthly observations were constructed by interpolation (cubic spline).

11. Volatility of expectations: The expectations are taken form the Michigan Survey. The volatility measure was obtained by taking the standard deviation of prices over the past 12 months (monthly frequency).

The recession probabilities we used, were taken from Chauvet and Piger (2008). The data are available on the website of Jeremy Piger.

C   Tables and figures

Constant Loadings	Matrix of factor loadings				
-156.87	0.37	-6.81	11.80	-3.59	16.34
629.34	-1.45	30.99	-49.94	11.83	-62.10
520.05	-1.23	22.10	-33.89	1.64	-49.57
-62.03	0.15	-2.41	3.15	1.67	4.98
40.70	-0.10	1.45	-1.97	0.40	-4.51

Table 1: Model Risk Factor Loadings

1.3247	-0.0057	-0.0121	0.0021	0.0585
0	0.1645	0.0038	-0.0072	-0.0190
0	0	0.0860	0.0197	0.0437
0	0	0	0.1503	0.1103
0	0	0	0	0.3857

Table 2: Cholesky-factored residual variance.

Maturity	RMSE our model	RMSE model by Rudebusch et al. (2006)
Six Months	20.59	29.52
One Year	20.99	33.35
Two Years	21.42	26.98
Three Years	22.94	26.58
Four Years	25.14	27.78
Five Years	27.27	30.03
Seven Years	31.22	34.45
Ten Years	33.76	39.30
Average	25.46	31.00

Table 3: Root Mean Squared Errors

	GDP growth	Oil price	PCE	MOVE	Net gov.sp.	cons. credit	expec-tations	VIX	Investor Sentiment	GDP China	Loans Com.Bank
GDP growth	1.000	0.183	0.289	0.094	0.019	-0.224	0.189	0.114	-0.019	0.165	-0.188
Oil price vol.	0.183	1.000	0.138	-0.347	-0.023	0.409	0.197	0.063	0.140	-0.117	-0.416
PCE	0.289	0.138	1.000	-0.193	-0.537	-0.062	0.500	-0.321	-0.460	0.244	0.360
MOVE	0.094	-0.347	-0.193	1.000	0.198	0.039	0.005	0.350	-0.0270	0.050	0.164
Net gov spending	0.019	-0.023	-0.537	0.198	1.000	0.077	-0.123	0.675	0.226	-0.411	-0.267
Consumer credit	-0.224	0.409	-0.062	0.039	0.077	1.000	-0.098	-0.114	0.220	-0.176	-0.186
Expectations	0.189	0.197	0.500	0.005	-0.123	-0.098	1.000	0.070	-0.236	0.227	0.330
VIX	0.114	0.063	-0.321	0.350	0.675	-0.114	0.070	1.000	0.231	-0.503	-0.079
Investor sentiment	-0.019	0.140	-0.459	-0.026	0.226	0.220	-0.236	0.231	1.000	-0.224	-0.213
GDP China	0.165	-0.117	0.244	0.050	-0.411	-0.176	0.227	-0.503	-0.2240	1.000	0.250
Loans comm. banks	-0.188	-0.416	0.360	0.164	-0.267	-0.186	0.330	-0.080	-0.213	0.250	1.000

Table 4: Correlations.

Variable	Estimated Coefficient	T-statistic	R^2
Merrill Lynch Move Index	0.0079	4.5424	0.1656
Oil price volatility	-0.0533	-3.9684	0.1184
GDP growth volatility	83.244	3.5634	0.0936
Volatility of outstanding consumer credit	-7.5765	-2.8740	0.0831
Volatility of the growth of GDP China	0.0206	2.2279	0.029
Volatility of loans by commercial banks	7.73	1.6439	0.0256
VIX index	0.6108	1.6412	0.0148
Volatility of the PCE inflation	-0.1137	-1.0834	0.0117
Investor sentiment	0.025	0.7087	0.0026
Volatility of net government spending	1.1306	-0.6493	0.0023
Volatility of expectations	-0.0036	-0.2935	0.0009

Table 5: Univariate regressions (the t-statistic between brackets), sample: January 1990-December 2005.

Variable	Model 1	Model 2
Merrill Lynch Move Index	0.0059	
	(3.5265)	
Merrill Lynch Move Index*Bull		0.0057
		(3.2510)
Merrill Lynch Move Index*Bear		0.0066
		(0.8027)
Oil price volatility	-0.0335	
	(-2.4891)	
Oil price volatility*Bull		-0.0501
		(-3.1168)
Oil price volatility*Bear		0.3342
		(1.6080)
GDP growth volatility	77.9410	
	(4.0295)	
GDP growth volatility*Bull		81.8979
		(3.6321)
GDP growth volatility*Bear		-754.0523
		(-1.5078)
Volatility of outstanding consumer credit	-3.4616	
	(-1.6074)	
Volatility of outstanding consumer credit*Bull		-3.9052
		(-1.4110)
Volatility of outstanding consumer credit*Bear		32.5494
		(1.6840)
Volatility of the growth of GDP China	0.0061	
	(0.6597)	
Volatility of the growth of GDP China*Bull		0.0128
		(1.3025)
Volatility of the growth of GDP China*Bear		-0.1195
		(-1.0618)
Volatility of loans by commercial banks	1.7969	
	(0.3952)	
Volatility of loans by commercial banks*Bull		-3.1693
		(-0.7106)
Volatility of loans by commercial banks*Bear		50.6434
		(2.5883)
R^2	0.3373	0.3811

Table 6: Multivariate regressions (t-statistic between brackets), sample:
January 1990-December 2005.

Variables	Model 1	Model 2
Change in model residuals	-110	-110
Change in implied volatility on long-term Treasury rates	-1.08	
Change in realized oil price volatility	-1.79	
Change in realized GDP growth volatility	-12.38	
Change in implied volatility on long-term Treasury rates*Bull		-1.10
Change in realized oil price volatility*Bull		-2.43
Change in realized GDP growth volatility*Bull		-13.16
Volatility of loans by commercial banks*Bear		1.30
Unexplained by above	-94.75	-94.61

Table 7: A decomposition of the Bond yield conundrum (June 2004 - June 2005).
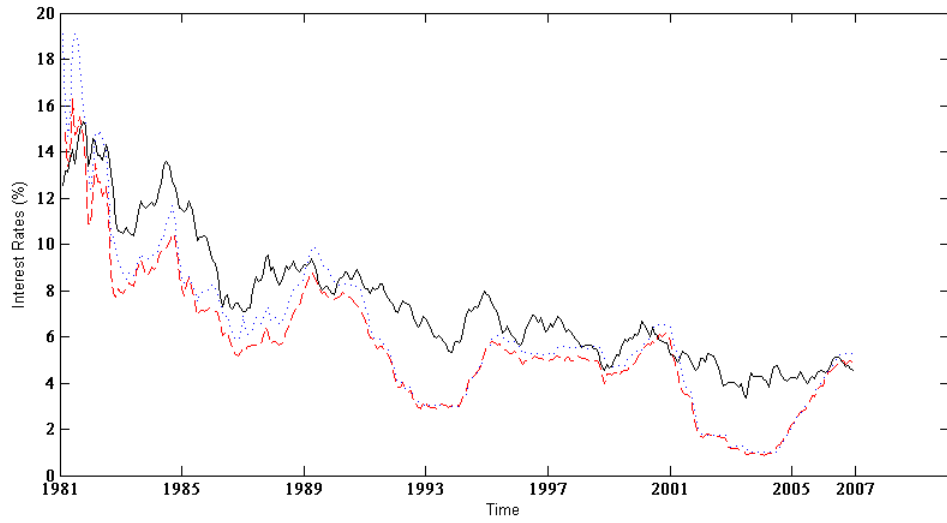
Figure 1: Plot of the federal funds rate - FFR (dots), short-term interest rates - 3M (dashed), long-term rates - 10Y (solid), monthly frequency: 1981-2006.
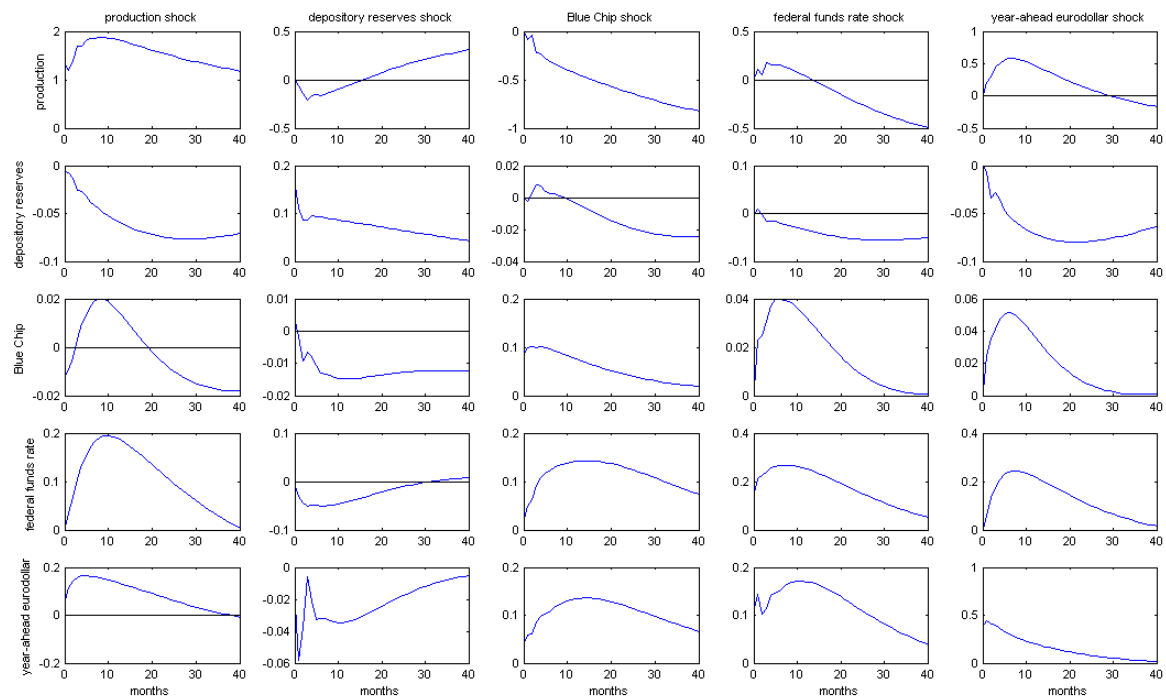
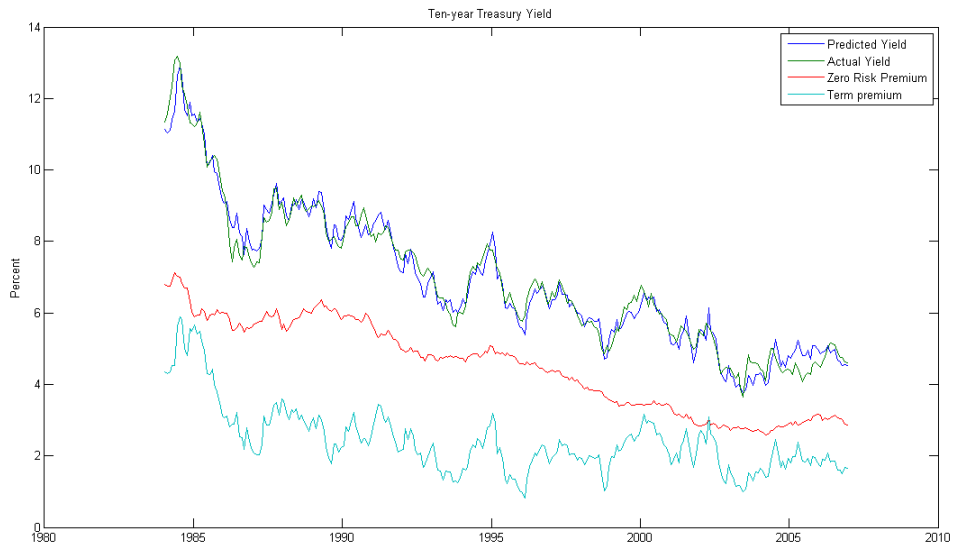Figure 2: Impulse response functions of the 1st stage estimation.
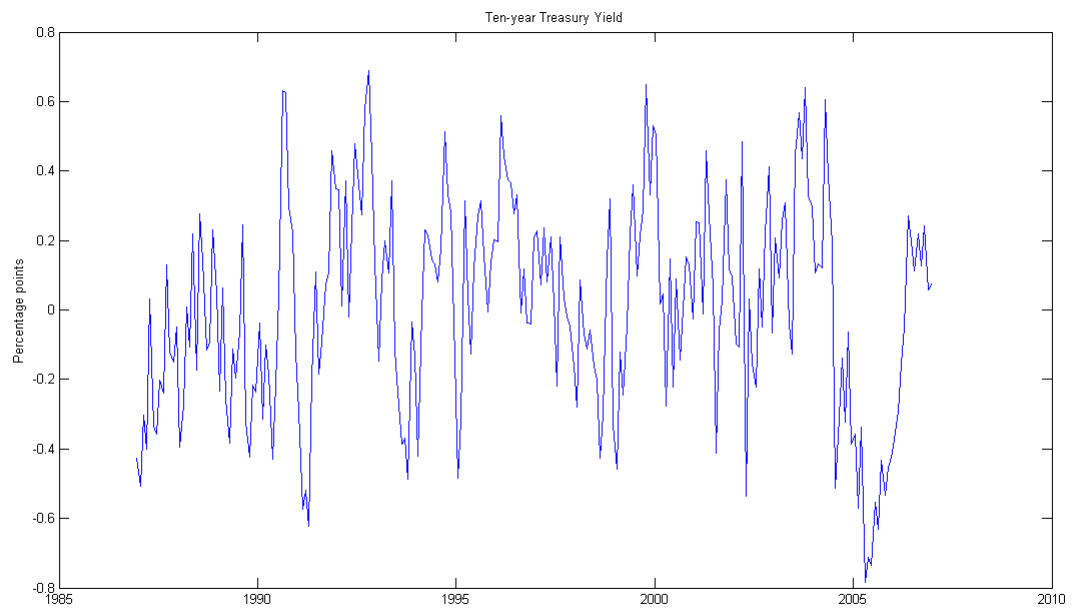
Figure 3: Long-term rates yield curve and model implied decomposition.

Figure 4: Model residuals

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