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Why Speed Doesn't Kill: Learning to Believe in Disinflation

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Abstract

Central bankers generally prefer to reduce inflation gradually. We show that a central bank may try to convince the private sector of its commitment to price stability by choosing to reduce inflation quickly. We call this "teaching by doing". We find that allowing for teaching by doing effects always speeds up the disinflation and leads to lower inflation persistence. So, we clarify why "speed" in the disinflation process does not necessarily "kill" in the sense of creating large output losses. This result also holds in an environment where private agents learn about the central bank's inflation target using a constant gain algorithm.

Keywords: learning, disinflation, credibility, sacrifice ratio

JEL Codes: E52, E58

1 Introduction

Central banks throughout the world have adopted long-run price stability as their primary goal. There is agreement among central bankers, academics and financial market representatives that low or zero inflation is the appropriate long-run goal of monetary policy. However, there is less agreement on what strategies should be adopted to achieve price stability.

The preference of central bankers - as expressed by King (1996) at the Kansas City Fed symposium on Achieving Price Stability at Jackson Hole - seems to be for a gradual timetable, with inflation targets consistently set below the public's inflation expectations.

Throughout, King (1996) emphasises the role of learning by central banks and the public. He shows how the optimal speed of disinflation depends crucially on whether the private sector immediately believes in the new low inflation regime or not. If they do, the best strategy is to disinflate quickly, since the output costs are zero. Of course, if expectations are slower to adapt, disinflation should be more gradual as well.

But the latter case is problematic, since the learning process implies that the learning parameter does not depend on the monetary regime. Put differently, the updating mechanism does not reflect the actual speed of disinflation, and thus it is not clear whether the private sector expectations mechanism is rational. Our suggestion in this paper is that, alternatively learning about the disinflation could be modelled using a two-period Bayesian set-up and along the lines of Evans and Honkapohja (2001). These cases are analysed in sections 3 and 5.

In his discussion of *endogenous learning* King (1996, p. 68) says that a central bank may try to convince the private sector of its commitment to price stability by choosing to reduce its inflation target towards zero quickly. King calls this "teaching by doing". Then the choice of a particular inflation rate influences the speed at which expectations adjust to price stability.

The problem however, with the King (1996) model is that the central bank decides on its optimal disinflation plan *given* those private sector expectations. Thus, although King calls this case teaching by doing, a more accurate description would be "doing without teaching".

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In sections 4 and 5 "teaching by doing" is modelled differently. We allow the central bank's "doing" to affect private sector learning. Of course, if the central bank recognises its potential for active "teaching" its incentive structure changes. More specific, it should realise that by disinflating faster, it can reduce the associated output costs by "teaching" the private sector that it means business. Thus, the dependence of private sector expectations on the actual inflation rate should be part of its optimisation problem.

This is in fact what we find: allowing for "teaching by doing" effects always speeds up the disinflation vis-à-vis the case where this effect is absent. So, in this paper we clarify why "speed" in the disinflation process does not necessarily "kill" in the sense of creating large output losses.

The remainder of the paper is organised as follows. Section 2 outlines King's model. In Section 3 we modify the King model with Bayesian learning. In Section 4 we look at "teaching by doing". Section 5 generalises the two-period model of Section 3 to a multi-period setting along the lines of Evans and Honkapohja (2001) . Our conclusions are given in Section 6. The appendices provide the derivations of the optimal monetary policy rules discussed in sections 4 and 5 of the paper.

2 The King Model of Disinflation Under Discretion

King (1996) discusses disinflation policy using a simple macroeconomic model, which combines nominal wage and price stickiness and slow adjustment of expectations to a new monetary policy regime. The model analyses the interaction between private sector expectations and the monetary regime, and in particular the speed at which the inflation target implicit in the latter converges to price stability. It features nominal rigidity and an optimising central bank that trades off price (inflation) versus output stabilisation.

More specifically, the model has three key equations aggregate supply, monetary policy preferences and inflation expectations. Aggregate supply exceeds the natural rate of output when inflation is higher than was expected by agents when nominal contracts were set. This is captured by a simple short-run Phillips curve[1]

$$z_t = \pi_t - \pi_t^{e} \tag{1}$$[2]

Here $\pi_t$is the rate of inflation, z_t is the output gap and π_t^e indicates the expectation of inflation as the subjective expectation (belief) of private agents. This belief does not necessarily coincide with rational expectations. The model is not restrictive as long as inflation expectations are in part influenced by past monetary policy (see e.g., Bomfim and Rudebusch (2000) and Yetman (2003)). [3]

The regime change is represented by a new inflation target π^*, which is announced to the public at the end of period $t-1$. The new target is lower than the initial steady state inflation rate, denoted by π_0.

The central bank's objective as of period t is to choose a sequence of current and future inflation rates $\{\pi_\tau\}_{\tau=t}^{\infty}$so as to minimise its intertemporal loss

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_\tau, z_\tau) \tag{2}$$[4]

[1]In their analysis of U.S. monetary policy experimentation in the 1960s, Cogley, Colacito and Sargent (2005) use a model similar to ours but with unemployment instead of output.

[2]For analytical convenience, we abstract from stochastic productivity shocks and the slope of the Phillips curve is set equal to unity.

[3]In the present paper - given expectations - the output costs of disinflation are constant and given by the slope of the Phillips curve. Here this parameter is normalised at unity. However, if we allow the output costs of disinflation to vary with the inflation rate, the central bank's incentives change substantially. Thus, one way of extending the model with state-contingent output costs of disinflation would be by means of a non-linear Phillips curve as discussed in Schaling (2004). For a preliminary analysis along those lines see Hoeberichts and Schaling (2006).

where

$$L\left(\pi_{\tau}, z_{\tau}\right) = \frac{1}{2}\left[a(\pi_{\tau} - \pi^{*})^{2} + (z_{\tau})^{2}\right]$$

E_t denotes expectations conditional on the central bank's information set at time t. The parameter $0 \leq a < \infty$is the relative weight on inflation stabilisation, while $0 < \delta \leq 1$is the discount factor.

The timing of events is such that the central bank chooses its disinflation policy after private sector inflation expectations are set. In the terminology of game theory, the private sector is a Stackelberg leader and the central bank is a Stackelberg follower.

King (1996) analyses two extreme cases of inflation formation: (1) a completely credible policy regime where private sector expectations adjust immediately to the inflation target (since the announcement is fully credible). This is the case of rational or model consistent expectations; (2) *exogenous learning*. In this case, the output costs of disinflation are non-trivial but depend solely on the mechanics of the inflation expectations, which in turn do not reflect the monetary regime.

The above statements can be analysed more precisely by explicitly considering the central bank's optimisation problem (where it takes inflation expectations as given, that is under discretion). The central bank's Lagrangian is

$$L = E_t\left\{\sum_{\tau=t}^{\infty} -\frac{\delta^{\tau-t}}{2}\left[a(\pi_{\tau} - \pi^{*})^{2} + (z_{\tau})^{2}\right] - \mu_t\left[z_{\tau} - \pi_{\tau} + \pi_{\tau}^{e}\right]\right\}$$

Defining z_t as the state variable, and $\pi_t$as the control variable, the first-order conditions are

$$\frac{\partial L}{\partial \pi_t} = E_t\left[-a\left(\pi_t - \pi^{*}\right) + \mu_t\right] = 0$$

$$\frac{\partial L}{\partial z_t} = E_t[-z_t - \mu_t] = 0$$

Combining these expressions we find the central bank's optimal inflation rate:

$$\pi_t = \frac{1}{1+a}\pi_t^{e} + \frac{a}{1+a}\pi^{*} \tag{3}$$

Of course, from **(E3)** it is clear that if expectations are slower to adapt, the disinflation should be more gradual as well. This can be easily seen from the simpler case where $\pi^{*} = 0$. Then we get $\pi_t = \frac{1}{1+a}\pi_t^{e}$; that is, the inflation rate should decline as a constant proportion of the exogenous expected inflation rate.

In general, expectations are affected both by the inflation target and by actual inflation performance. After experiencing high inflation for a long period of time, there may be good reasons for the private sector not to believe the disinflation policy fully (see also Bomfim and Rudebusch (2000) and Schaling (2003)). In light of this, King assumes that private sector inflation expectations follow a simple rule, that is a linear function of the inflation target and the lagged inflation rate

$$\pi_t^{e} = \rho\pi_{t-1} + (1-\rho)\pi^{*} \tag{4}$$

This is termed *endogenous learning*. The smaller is $0 \leq \rho \leq 1$ the faster is the learning process. Thus, ρ captures the credibility of the new regime. The closer is $\rho$to zero, the higher is the credibility of the regime change.[5] For a positive value of ρ expected inflation converges asymptotically to the inflation target. Then given this expectations mechanism, we can derive the central bank's optimal disinflation policy.

[5] Put differently, the lowerρ, the better inflation expectations are anchored at long horizons. For an empirical analysis for the U.S. examining observable measures of long-run inflation expectations, see Kiley (2008).

More specifically, substituting **(E4)** into **(E3),** then the central bank's optimal disinflation policy is()

$$\pi_t = \frac{1}{1+a}\left(\rho\pi_{t-1} + (1-\rho)\pi^*\right) + \frac{a}{1+a}\pi^{*6} \tag{5}$$

In the simpler case where $\pi^* = 0$, we get $\pi_t = \frac{\rho}{1+a}\pi_{t-1}$; that is the inflation rate should decline as a constant proportion of the exogenous expected (lagged) inflation rate.

Obviously, case 3 is a mixture of cases 1 and 2. Expectations do not adjust immediately (they depend on actual inflation experience, and hence on the policy choices made during the transition), but are not completely exogenous either. But case 3 is problematic, since the learning process implies that the learning parameter ρ does not depend on the monetary regime. Put differently, the updating mechanism does not reflect the actual speed of disinflation, and thus the private sector expectations mechanism is problematic

Our first suggestion is that, alternatively, case 3 could be modelled with the aid of Bayesian learning. Therefore in the next section of the paper we modify the model with Bayesian learning.

3 Disinflation in a two-period model with private sector learning

In general, an announcement by the central bank that in future the inflation target will be consistent with price stability, does not command immediate credibility. It takes time for the private sector to be convinced that the target will be chosen to be consistent with price stability. The private sector will try to *learn* about the true preferences of the central bank. Their pronouncements will not necessarily be taken at face value.

As pointed out by King (1996, p. 64) modelling learning is difficult, therefore consider first a two-period version of the King model of section 2 extended with learning. All other assumptions and parameters remain as per section 2. The idea is to bring inflation down from its initial level, π_0 say, to a situation of price stability where inflation is zero. Thus, the central bank has to disinflate the economy by π_0 percentage points. We assume that this "inflation stabilisation plan" has full credibility and that the only uncertainty is about its *timing*.[7] Thus, at the end of period 2 inflation has to be 0 under all scenarios, i.e.

$$\pi_2 = 0 \tag{6}$$

and this is believed by the private sector. The question now is how should the disinflation be spread over time?

One strategy is a *cold turkey* approach. In that case, the central bank disinflates the economy in period 1 by π_0 percentage points and does nothing in period 2.

The other strategy is a *gradualist* approach where the central bank inflates according to

$$\Delta\pi_{1,2} \begin{cases} -q\pi_0 \text{ in period } 1 \\ -(1-q)\,\pi_0 \text{ in period } 2 \end{cases} \tag{7}$$

where $0 < q < 1$ is the fraction of the disinflation that takes place in period 1.

At the start of period 1, under Bayesian learning wage setters assign a *prior* probability x_1 to the event that the central bank disinflates everything in one go ($x_1 = \Pr ob\langle\pi_1 = 0\rangle$), i.e. follows the cold turkey policy and $(1-x_1)$ to the complementary event that the central bank follows a gradualist policy ($(1-x_1) = \Pr ob\langle\pi_1 > 0\rangle$).

[6] Note that in case of a fully credible regime switch $\rho = 0$and we have $\pi_t = \pi^* = \pi_t^e$.

[7] This assumption will be relaxed in Section 5.

By observing monetary policy in period 1, wage setters learn something about the true nature of the policy. Wage setters' beliefs, x_1, are then revised according to Bayes' rule.[8] Period 2 nominal wages are then set on the basis of the *posterior* beliefs, x_2.

If wage setters observe either a *positive* ($\pi_1 > 0$) or a *zero* inflation rate ($\pi_1 = 0$) in period 1, *Bayes' rule*[9] suggests how to rationally update these prior beliefs

$$\begin{aligned} x_2 &= \Pr ob \langle q = 1 | \, \pi_1 \rangle = \frac{\Pr ob \langle \pi_1 = 0 \rangle \, . \Pr ob \langle \pi_1 | \, q = 1 \rangle}{\Pr ob \langle \pi_1 \rangle} \\ &= \frac{x_1 \Pr ob \langle \pi_1 | \, q = 1 \rangle}{\Pr ob \langle \pi_1 \rangle} \end{aligned} \tag{8}$$

Hence, the posterior probability that the central bank follows a cold turkey policy is given by the prior multiplied by the conditional probability of observing the policy π_1 given that the central bank follows a cold turkey policy, divided by the unconditional (*prior*) probability of observing the policy π_1.

Clearly, if the gradualist policy is followed in period 1, **(E8)** gives $x_2 = 0^{10}$, since a central bank that follows a cold turkey strategy would never have accommodated inflation expectations:

$$(\Pr ob \langle \pi_1 > 0 | \, q = 1 \rangle = 0)$$

Similarly, if the cold turkey strategy is followed in period 1, **(E8)** gives $x_2 = 1^{11}$, since a CB that follows a cold turkey strategy disinflates everything in period 1 with probability 1:

$$(\Pr ob \langle \pi_1 = 0 \, | q = 1 \rangle = 1).$$

Thus we have the following rational private learning process

$$x_2 = \begin{cases} 1 \text{ if } \pi_1 = 0 \\ 0 \text{ otherwise} \end{cases} \tag{9}$$

Since only a gradualist central bank leaves any inflation in the economy, and if it does at the rate $\pi_0 - q\pi_0 = (1 - q)\,\pi_0$, expected inflation at time 0 for period 1 is

$$E_0 \pi_1 = (1 - x_1)(1 - q)\,\pi_0 \tag{10}$$

Note that this expression can be interpreted also in the context of King's case of "endogenous learning". Using $\tilde{\rho}$ as shorthand for $(1 - x_1)(1 - q)$, **(E10)** can be written as

$$E_0 \pi_1 = \tilde{\rho} \pi_0 \; where \; 0 < \tilde{\rho} < 1 \tag{11}$$

Thus, here rational expectations can display some of the backward-looking characteristics of traditional adaptive expectations.[12]

[8] This is somewhat similar to the analysis by Huh et al. (2000) where agents update their prior assessment of the true inflation target in a (quasi) Bayesian way on the basis of the central bank's success or failure in reducing inflation over time.

[9] Bayes' rule is $\Pr ob \langle A \mid B \rangle = \frac{\Pr ob \langle A \rangle . \Pr ob \langle B | A \rangle}{\Pr ob \langle B \rangle}$.

[10]

$$x_2 = \frac{x_1 \Pr ob \langle \pi_1 > 0 \mid q = 1 \rangle}{\Pr ob \langle \pi_1 > 0 \rangle} = \frac{x_1 .0}{(1 - x_1)} = 0.$$

[11]

$$x_2 = \frac{x_1 \Pr ob \langle \pi_1 = 0 \mid q = 1 \rangle}{\Pr ob \langle \pi_1 = 0 \rangle} = \frac{x_1 .1}{x_1} = 1.$$

[12] More specific, in this context equation **(A2)** would read $E_0 \pi_1 = \rho \pi_0$.

Here we assume that the central bank has full knowledge of the process of private sector learning, or in other words we have what Gaspar, Smets and Vestin (2006) call "sophisticated central banking".

The central bank's Lagrangian (without discounting, that is, we have set $\delta = 1$) is

$$L = E_t \left\{ \sum_{\tau=1}^{2} -\frac{1}{2} \left[a(\pi_\tau - \pi^*)^2 + (z_\tau)^2 \right] - \mu_t \left[z_\tau - \pi_\tau + \pi_\tau^e \right] \right\} \tag{12}$$

In line with **(E11)** the optimal monetary policy is

$$\pi_1 = \frac{1}{1+a} E_0 \pi_1 \tag{13}$$

Substituting from equation **(E10)** yields

$$\pi_1 = \frac{(1-x_1)}{1+a} * (1-q)\, \pi_0 \tag{14}$$

Note that if $a \to \infty$ from **(E14)** it follows that the optimal period 1 inflation rate is zero. This means that in this case the central bank will follow a cold turkey strategy.

The expected cumulative output loss ("the sacrifice ratio") in the optimal transition is

$$\begin{aligned} CYL &= \sum_{t=1}^{2} E(z_t) = \sum_{t=1}^{2} (\pi_t - E_{t-1}\pi_t) = \\ -\frac{a(1-x_1)(1-q)}{1+a}\pi_0 &= -\frac{a\,\tilde{\rho}}{1+a}{\pi_0}^{13} \end{aligned} \tag{15}$$

From this expression we see that the sacrifice ratio is *lower* the *faster* the "speed of learning" $\tilde{\rho}$. From **(E11)** it follows that now the "speed of learning" $\tilde{\rho}$ - and hence the sacrifice ratio - *does* depend on the (private sector's expectations of the) monetary regime.[14] For example, if the prior probability that the central bank follows a cold turkey policy increases, the private sector will attach less weight to the past inflation rate, as a basis for forecasting next year's inflation.[15] That is, $\frac{\partial x}{\partial \tilde{\rho}} < 0$.

Equation **(E14)** suggests several things. Assuming that the central bank *cares* about output, $0 < a < \infty$, the first is that the central bank will *only* follow a cold turkey strategy ($\pi_1^* = 0$) if the private sector is convinced it will ($x_1 = 1$). So, in this case *beliefs* (or rather, priors) are self-fulfilling.

Next, if the private sector thinks the central bank might be *gradualist* ($x_1 < 1$) the central bank will indeed be gradualist. However, it will be *less gradual* than the private sector expects.

Hence, here beliefs are *partly* self-fulfilling. The reason is that the central bank not only cares about output, and hence about appeasing labour market participants, but also about *inflation* itself. Of course, the more it cares about inflation (the bigger a) the greater the incentive to "speed up" the disinflation. If the central bank only cared about output - that is, if $a$were equal to zero - then it would exactly accommodate the above expectations and follow the same timing. If it does care about inflation as well ($a > 0$), it will speed up things and disinflate *faster* than the private sector expects. Figure 3.1 illustrates.

How "gradualist" the central bank will be exactly, can be seen from equation **(E16)**.[16]

[14] For an application of linear updating rules in an empirical model of the US economy see Bomfim et al. (1997).

[15] Thus here the speed of learning is defined as the inverse of the weight attached to past inflation as a basis for forecasting future inflation.

[16] This equation has been obtained as follows. We start with the definition

$$\pi_1 = \pi_0 + \Delta\pi_1$$

Substituting for $\Delta\pi_1$ from equation**(7)** we get π_1=$(1-q)\,\pi_0$.In turn substitution for π_1 from **(13)** yields $\frac{1}{1+a}$ E $_0\pi_1$=$(1-q)^*\,\pi_0$.This can be rearranged as**(16)**.

$$(1-q)^* = \frac{(1-x_1)(1-q)}{(1+a)} \tag{16}$$

So the ex post degree of gradualism is always smaller than the ex ante degree. Of course the latter is the private sector's prior.[17] Thus, the central bank will always disinflate *faster* than the private sector thinks. This is reminiscent of section 2 (the King model) where the optimal disinflation strategy is also to accommodate partially inflation expectations

Finally, this section suggests that the central bank's optimal disinflation strategy is to be gradualist if (i) it cares about output and (ii) the prior that it *might* follow a gradualist strategy is non zero ($1-x_1>0$). Moreover, what is also interesting about this set-up is that if central bank statements influence private sector priors, such central bank talk is *not* cheap. This means that you can't communicate an inflation stabilisation programme without at the same time being constrained by your words.

4 Endogenous Inflation Persistence: Teaching by Doing

In this section we allow the central bank's "doing" to affect private sector learning. Of course, if the central bank recognises its potential for active "teaching" its incentive structure changes. More specifically, it should realise that by disinflating faster, it can reduce the associated output costs by "teaching" the private sector that it means business. All other assumptions remain as in section 2.

Now we allow the central bank's "doing" to affect private sector learning. Thus, he dependence of private sector expectations on the actual inflation rate – equation

$$\pi_t^e = \rho\pi_{t-1} \tag{17}$$

should be part of its optimisation problem. In what follows we refer to this as the case of "endogenous persistence".[18] We analyse this for the infinite horizon case (of which the two-period model would be a special case).[19]

Now, the central bank's problem is to choose $\{\pi_\tau\}_{\tau=t}^{\infty}$ so as to minimize

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \frac{1}{2}\left[a(\pi_\tau)^2 + (z_\tau)^2\right] \tag{18}$$

subject to **(E1)** and **(E6)**.

It is convenient to define $x_t = \pi_t^e = E_{t-1}^{PS}\pi_t$ as the state variable and $u_t = \pi_t$ as the control. We solve this problem by the method of Lagrange multipliers.[20] Introduce the Lagrange multiplier μ_t, and set to zero the derivatives of the Lagrangean expression:

$$L = E_t\left[\sum_{\tau=t}^{\infty}\left\{\frac{\delta^{\tau-t}}{2}\left[-a(u_\tau)^2 - (u_\tau - x_\tau)^2\right] - \delta^{\tau-t+1}\mu_{\tau+1}\left[x_{\tau+1} - \rho u_\tau\right]\right\}\right] \tag{19}$$

In Appendix A.1, it is shown that the first-order condition for this problem can be written as[21]

$$\pi_t = C_1 E_{t-1}^{PS}\pi_t \text{ where } \frac{1}{(1+a)+\delta\rho^2} < C_1 < \frac{\delta\rho^2+1}{(1+a)+\delta\rho^2} \tag{20}$$

[17] This is in fact equal to $(1-x_1)$. Now, $(1-x_1)$ - the ex-ante degree – is known to the private sector, but not$(1-q)^*$, the ex-post degree.

[18] The case where the central bank does not internalize this constraint is referred to as "naïve discretion".

[19] For an analysis in a two-period context see Hoeberichts and Schaling (2006).

[20] For a discussion of the relative merits of the methods of dynamic programming and Lagrange, see Schaling (2001). Svensson (1999) and Vestin (2006) solve optimal discretionary policy (which in their papers is also a dynamic optimisation problem) using dynamic programming.

[21] See Bullard and Schaling (2001) and Schaling (2002) for examples of the method of solving for the optimal policy.

The (initially undetermined) coefficient is given by

$$C_1 = \frac{1}{2}\left\{\left[\frac{(1+a)+\delta\rho^2}{\delta\rho^2}\right] - \sqrt{\frac{\left[(1+a)+\delta\rho^2\right]^2 - 4\delta\rho^2}{\delta^2\rho^4}}\right\} \tag{21}$$

From equation **(E21)** it can be seen that the optimal value of this coefficient is a nonlinear function of the central bank's weight on inflation stabilisation a, the discount factor δ and the extent to which inflation expectations depend on past inflation ρ in

$$\pi_t^e = \rho\pi_{t-1} + (1-\rho)\pi^* \tag{22}$$

that is the lack of credibility of the new regime (the closer is $\rho$to zero, the higher is the credibility of the regime change).

In Appendix A.1 we derive:

Proposition 1 *If the central bank's weight on inflation stabilisation is positive (that is if $a > 0$), optimal disinflation under "teaching by doing" is always faster than optimal disinflation under naïve discretion; that is faster than in the King model.*

Further, we show:

Proposition 2 *The higher a the lower the optimal value of the feedback parameter C_1.*

For the proof, see Appendix A.1. The argument is as follows. A central bank that is more concerned with inflation will be less concerned with output, and hence will accommodate inflation expectations to a lesser extent.

We can also derive a result in terms of the central bank's discount factor. In Appendix A.1 we verify:

Proposition 3 *The higher δ the lower the optimal value of the feedback parameter C_1.*

The intuition is that the higher δ, the more concerned the central bank is about the future, i.e. the longer is its policy horizon (conversely if this parameter is zero, the central bank only "lives for today"). Under a live-for-today policy, the central bank is not interested how monetary accommodation today affects inflation expectations for tomorrow. If it becomes more concerned about the future (higher δ) however, it will start paying attention to expected future "expectations invoices", and accommodate current inflation expectations to a lesser extent, hence the monetary accommodation coefficient C_1 falls.

As pointed out by Kiley (2008), with regard to inflation dynamics, the degree of anchoring of inflation expectations is central in most empirical and theoretical applications as inflation is a function of inflation expectations in most treatments. This is also true in this context. Let us therefore now look how the central bank responds to less faith in its inflation target, as proxied by a higher weight placed on past inflation by private agents in forecasting future inflation. It is easy to show:

Proposition 4 *The higher ρ the lower the optimal value of the feedback parameter C_1.*

The argument is that the higher ρ, the more leverage the central bank has over inflation expectations via past inflation. If the central bank cares about the future ($\delta \neq 0$), it will realize that it faces lower output costs of disinflation and hence needs less monetary accommodation.

Thus, the central bank's optimal choice of $\pi_t$is inversely related to the ex ante credibility of the regime change, or to the extent to which private sector inflation expectations are anchored at long horizons.

By substituting **(E6)** into **(E20)** we can derive the solution for the inflation process under teaching by doing

$$\pi_t = C_1 \rho \pi_{t-1} \tag{23}$$

It can be seen that the greater the parameter C_1, the greater the first-order autocorrelation in inflation (hence inflation persistence is now endogenous) Since this parameter is decreasing in the central bank's weight on inflation stabilisation a (see PROPOSITION 2), the greater the central bank's weight on inflation stabilisation, the smaller the first-order autocorrelation in inflation (a similar result as under discretion).[22]

Similarly, according to PROPOSITION 3, the higher the central bank's discount factor δ, the lower the optimal value of the parameter C_1. Thus, if the central bank becomes more concerned about the future (the longer its policy horizon and the higher δ), the lower the persistence of inflation.

It is interesting to contrast the persistence of inflation under "teaching by doing" with the case of naïve discretion (the King model from section 2). Then, according to equation **(6)** (where we have set $\pi^* = 0$) the optimal inflation rate should decline as a constant proportion of the exogenous expected inflation rate. Substituting **(6)** into **(5)** we find that inflation under "naïve discretion" (in the King model) obeys

$$\pi_t = \frac{\rho}{1+a} \pi_{t-1} \tag{24}$$

If we contrast equation **(E24)** with equation **(E23)** we can easily see that inflation persistence under teaching by doing is lower than inflation persistence under naïve discretion (the King model) if $C_1 < \frac{1}{1+a}$. In Appendix A.1 we prove that this condition is always satisfied if $a > 0$, that is, if the central bank's weight on inflation stabilisation is positive. Therefore we can finally state

Proposition 5 *If the central bank's weight on inflation stabilisation is positive (that is if $a > 0$), inflation persistence under '"teaching by doing"' (endogenous inflation persistence) is always lower than under naïve discretion (or doing without teaching as in the King model).*

5 Disinflation in a Multi-Period Model with Private Sector Learning

In Section 3 we assumed that that the central bank's' inflation stabilisation plan had full credibility and that the only uncertainty was about its timing. In this section we relax this assumption.

More specifically, in line with Molnár and Santoro (2007)[23] we assume that private agents do not know the inflation target and hence are in the dark about the exact process followed by inflation, but believe that inflation is a continuous invariant function of the inflation target only. This hypothesis implies that the conditional and unconditional expectation of inflation coincides, and are perceived by the private sector as a constant. All other assumptions and parameters remain as per section 2.

More specifically, suppose the private sector's forecasting function for inflation takes the same form as the rational expectations solution under full information (where $0 < a < \infty$), namely, equation

$$\pi^{RE} = \pi^* = 0 \tag{25}$$

The nature of imperfect information is such that, the private sector knows the correct functional form of the forecasting rule, that is

$$\pi_t^e = E_{t-1}^{PS} = \gamma \tag{26}$$

[22] From PROPOSITION 4 we know that the feedback parameter $C_1$is decreasing in the parameter ρ. Therefore, the dependence of the degree of inflation persistence on ρ is given by $\partial (C_1 \rho) / \partial \rho = (\partial C_1 / \partial \rho) \rho + C_1$, where the sign is ambiguous.

[23] They, however, focus on the case of constant gain learning in the context of a New-Keynesian model.

but not the actual value of γ, where γ is a time-invariant constant (which is in fact equal to $\gamma = \pi^* = 0$) because it doesn't know the central bank's inflation target.

We assume that private sector's expectations are formed according to the adaptive learning literature; in particular, agents' Perceived Law of Motion (PLM) is consistent with the Law of Motion that the central bank would implement under the assumption of rational expectations. In other words, inflation is assumed to be constant, and agents use a learning algorithm to find out this constant. We assume that private sector expectations evolve following a constant gain algorithm[24]:

$$E_{t-1}\pi_t = c_{t-1} = c_{t-2} + \underline{\kappa}\,(\pi_{t-1} - c_{t-2}) \;\; where\; \underline{\kappa} \in (0, 1) \tag{27}$$

As pointed out by Sargent (1999, p. 96), constant gain algorithms discount past observations. Other studies that use constant gain learning are Orphanides and Williams (2003), Milani (2005) and Gaspar, Smets and Vestin (2006).

The sequence of events is similar to section 1. First, the private sector sets E^{π}_{t-1t}. Then the central bank observes $E_{t-1}\pi_t$ and chooses π_t. In each period τ, the sequence of events is summarized as follows. See Table 1 in the appendix.

We remark that the learning rule **(E27)** is equivalent to the traditional adaptive expectations formula. Note also that it can be expressed as an exponentially weighted average of past inflation rates $c_t = \underline{\kappa} \sum\limits_{i=0}^{\infty} (1-\underline{\kappa})^i \, \pi_{t-i}$. In Appendix A.2 we show that the central bank's optimal monetary policy reaction function is $\pi_t = G_1 c_{t-1}$, where $G_1$is an (initially) undetermined coefficient and $0 \leq G_1 \leq 1$. Substituting this rule in **(E27)** we have

$$c_t = [1 + \underline{\kappa}\,(G_1 - 1)]\, c_{t-1} \tag{28}$$

This is an AR(1) process which is stationary if $|1 + \underline{\kappa}(G_1 - 1) < 1|$or $\underline{\kappa}\,(G_1 - 1) < 0$. Since $0 \leq \underline{\kappa} \leq 1$ and $0 \leq G_1 \leq 1$ this condition is always satisfied.

Setting $c_t$equal to its steady-state value $\overline{c}$, using **(E28)** it can be verified that $\overline{c} = [1 + \underline{\kappa}(G_1 - 1)]\, \overline{c}$ or $\overline{c} = 0$. So, the (asymptotic) mean of c_t is equal to its rational expectations value of 0. Thus, the private sector forecast is asymptotically unbiased.

As before we assume that the central bank has full knowledge of the process of private sector learning, or in other words we have what Gaspar, Smets and Vestin (2006) call 'sophisticated central banking'.

Again it is convenient to define$x_t = \pi^e_t = E_{t-1}\pi_t = c_{t-1}$and $u_t = \pi_t$ as the control, so that $z_t = u_t - x_t$. We solve this problem by the method of Lagrange multipliers. Introduce the Lagrange multiplier μ_t, and set to zero the derivatives of the Lagrangean expression:

$$L = E_t \left[\sum_{\tau=t}^{\infty} \left\{ \begin{array}{l} \frac{\delta^{\tau-t}}{2} \left[-a\,(u_\tau)^2 - (u_\tau - x_\tau)^2 \right] \\ -\delta^{\tau-t+1} \mu_{\tau+1} \left[x_{\tau+1} - (1-\underline{\kappa}) x_\tau - \underline{\kappa} u_\tau \right] \end{array} \right\} \right] \tag{29}$$

In Appendix A.2 it is shown that the first-order condition – or optimal feedback rule - for this problem can be written as

$$\pi_t = G_1 x = G_1 c_{t-1} where\; 0 \leq G_1 \leq 1 \tag{30}$$

Thus, as was the case in section 4 it is optimal to partially accommodate inflation expectations. Thus, as in Section 3 – where we use a two-period model - the central bank will always disinflate *faster* than the private sector thinks.

Note that substituting **(E30)** into **(E28)** yields $\pi_t = G_1 \{c_{t-2} + \underline{\kappa}\,(\pi_{t-1} - c_{t-2})\}$. Thus, adaptive learning can generate serial correlation in inflation though there is none in the fundamentals.

[24]Tesfaselassie and Schaling (2008) let the central bank learn using the Kalman filter, of which a constant gain algorithm is a special case. For some useful analytics on filtering see Sargent (1996, pp. 115-118).

Table 2 in the appendix computes the optimal value of the accommodation parameter $G_1$for different gain coefficients between 0 and 0.2.

In all computations we have used $a = 1$and $\delta = 0.9$. Coefficients between 0.015 and 0.025 are common in empirical studies adapting constant-gain learning as Orphanides and Williams (2003) and Milani (2005).

Table 2 shows that a higher gain parameter is associated with less monetary accommodation of inflation expectations. The intuition is similar to that in Section 3. The argument is that the higher $\underline{\kappa}$, the more leverage the central bank has over inflation expectations via past inflation. If it cares about the future (here $\delta = 0.9$), it will realise that it faces lower output costs of disinflation and hence needs less monetary accommodation.

6 Summary and Concluding Remarks

In this paper we have analysed disinflation in several environments. There are (at least) two important dimensions of this issue. The first is whether private sector expectations formation or updating reflect (expectations of) the new monetary regime (characterised by lower inflation). The second is whether the central bank properly internalises the fact that (rational) inflation expectations depend on past inflation outcomes. We have seen that the King (1996) model does not properly reflect these dimensions.

With respect to the second dimension we show that when the central bank realises that (ad hoc) inflation expectations depend on past inflation, it always speeds up the disinflation and in this way generates lower inflation persistence. So, we clarify why "speed" in the disinflation process does not necessarily "kill" in the sense of creating large output losses.

This "speed" result also holds in an environment where private agents rationally learn about the central bank's inflation target using a constant gain algorithm. Of course, in this case the first dimension – namely the fact that inflation expectations should reflect the monetary regime – is also properly addressed. In this case we also show that adaptive learning can generate serial correlation in inflation, though there is none in the fundamentals.

The results above were obtained in an environment with an "old-fashioned" Phillips curve. However, we expect results would broadly carry over into the New Keynesian environment. For an analysis about learning and uniqueness of rational expectations in such an environment, see Bullard and Schaling (2009).

7 APPENDIX A OPTIMAL DISINFLATION

7.1 Derivation of the First-Order Condition in Section 4

Now, the central bank's problem is to choose $\{\pi_\tau\}_{\tau=t}^{\infty}$ so as to maximise

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[-a(\pi_\tau)^2 \quad - \ (z_\tau)^2\right] \tag{A1}$$

subject to

$$z_t = \pi_t - E_{t-1}\pi_t \tag{A2}$$

and

$$E_{t-1}\pi_t = \rho\pi_{t-1} \tag{A3}$$

It is convenient to define $x_t = E_{t-1}\pi_t$ as the state variable and $u_t = \pi_t$ as the control, so the central bank's problem is to choose $\{u_\tau\}_{\tau=t}^{\infty}$. We solve this problem by the method of Lagrange multipliers.

Introduce the Lagrange multiplier μ_τ, and consider the Lagrangean expression[25]

$$L = E_t \left[\sum_{\tau=t}^{\infty} \left\{ \begin{array}{c} \frac{\delta^{\tau-t}}{2} \left[-a\left(u_\tau\right)^2 - \left(u_\tau - x_\tau\right)^2 \right] \\ -\delta^{\tau-t+1} \mu_{\tau+1} \left[x_{\tau+1} - \rho u_\tau \right] \end{array} \right\} \right] \tag{A4}$$

The central bank's first-order conditions take the form

$$\frac{\partial L}{\partial u_t} = -a(u_t) - (u_t - x_t) + \delta \rho E_t \mu_{t+1} = 0 \tag{A5}$$

$$\frac{\partial L}{\partial x_{t+1}} = \delta(u_{t+1}) - (x_{t+1}) + \delta E_t \mu_{t+1} = 0 \tag{A6}$$

Using the lag operator L (which operates on the time-subscript of a variable, not on the time at which the expectation is held) on **(A.6)** we obtain

$$(u_t - x_t) - \mu_t = 0 \tag{A7}$$

Next, we find an expression for $E_t \mu_{t+1}$. Leading **(A7)** by one period and taking expectations we get:

$$E_t \mu_{\ t+1} = (E_t u_{t+1} - E_t x_{t+1}) \tag{A8}$$

Substituting **(A.8)** into **(A.5)**, we can derive the Euler equation

$$-a(u_t) - (u_t - x_t) + \delta \rho \left(E_t u_{t+1} - E_t x_{t+1} \right) = 0 \tag{A9}$$

In the case of a policy of strict inflation reduction, the rule would be

$$u_t = 0 \tag{A10}$$

Similarly, in the case of full accommodation of expectations, the rule would be

$$u_t = x_t \tag{A11}$$

Thus, it appears that in case of flexible inflation targeting, the rule will be a linear combination of **(A.10)** and **(A.11)**, that is $u_t = c x_t$, where $0 \leq c \leq 1$. Or alternatively,

$$u_t = C_1 x_t \tag{A12}$$

which is equation (20) in the main text (where I have substituted $x_t = E_{t-1}\pi_t$ and $u_t = \pi_t$).Here the coefficient C_1 remains to be determined, and the prior is that $0 \leq C_1 \leq 1$. Now we identify the coefficient C_1.

Expectations for the state at period $t+1$ follow from the constraint in **(A.4)**. Combining the latter with the decision rule for u, we can write:

Expectations for the state at period follow from the constraint in (A.1). Combining the latter with the decision rule for , we can write:

$$E_t x_{t+1} = \rho C_1 x_t \tag{A13}$$

From (A.8) it follows that

$$E_t u_{t+1} = C_1 E_t x_{t+1} = C_1[\rho C_1 x_t] = \rho (C_1)^2 x_t \tag{A14}$$

[25] It is easy to convert the Lagrangean **(A.1)** into the standard form used by Schaling (2004) by setting $\tau = 0$in **(A.1)**. Then the central bank chooses the sequence $\{\pi_t\}_{t=0}^{\infty}$ rather than $\{\pi_\tau\}_{\tau=t}^{\infty}$.

Substituting (A.13) and (A.14) into the Euler equation (A.9) above, and equating constant terms and coefficients on the state variable yields the following expression for in terms of the structural parameters of the model

$$C_1 = \frac{1}{(1+a)+\delta\rho^2}\left[\delta\rho^2 C_1^2 + 1\right] \tag{A15}$$

Equation (A.15) implicitly defines the value of C_1. It can be written as $C_1 = F(C_1)$. Note that the function $F(C_1)$ on the RHS with domain $\langle 0,1\rangle$ is monotonically increasing in C_1, that ,

$$\lim_{C_1\to 0} F(C_1) = \frac{1}{(1+a)+\delta\rho^2}$$

$$\lim_{C_1\to 1} F(C_1) = \frac{\delta\rho^2+1}{(1+a)+\delta\rho^2}$$

We realise that there is a unique positive solution C_1 , which fulfills

$$\underline{C_1} < C_1 < \overline{C_1}$$

where

$$\underline{C_1} = \frac{1}{(1+a)+\delta\rho^2}$$

$$\overline{C_1} = \frac{\delta\rho^2+1}{(1+a)+\delta\rho^2} \tag{A16}$$

It can be solved analytically:

$$C_1 = \frac{1}{2}\left\{\left[\frac{(1+a)+\delta\rho^2}{\delta\rho^2}\right] - \sqrt{\frac{\left[(1+a)+\delta\rho^2\right]^2 - 4\delta\rho^2}{\delta^2\rho^4}}\right\} \tag{A17}$$

If we contrast equation (E24) with (E23), it is clear that disinflation under commitment (with teaching by doing) is faster than under naïve discretion if

$$C_1 < \frac{1}{1+a} \tag{A18}$$

We know that C_1 is implicitly defined by (A.15), which can be rewritten (decomposed) as

$$C_1 = \frac{1}{(1+a)}\left[\frac{(1+a)\delta\rho^2 C_1^2 + (1+a)}{(1+a)+\delta\rho^2}\right] \tag{A19}$$

From (A.19) we see that if the term inside the square brackets (hereafter [ ]) is equal to 1, $C_1 = \frac{1}{1+a}$ and optimal disinflation under commitment is equal to optimal disinflation under naïve discretion. It can be easily seen from (A.19) that if $(1+a)\,C_1^2 = 1$, the relevant term ([ ]) is equal to 1. Thus, we have [ ] < 1, and $C_1 = \frac{1}{1+a}$ if

$$(1+a)C_1^2 - 1 < 0 \Leftrightarrow -\sqrt{\frac{1}{1+a}} < C_1 < \sqrt{\frac{1}{1+a}} \tag{A20}$$

But, since from (A.16) we already know that $-\sqrt{\frac{1}{1+a}} < \underline{C}_1 < C_1$, the lower bound in (A.20) is not bonding so

$$(1+a)C_1^2 - 1 < 0 \Leftrightarrow C_1 < \frac{1}{1+a}$$

if

$$C_1 < \sqrt{\frac{1}{1+a}} \tag{A21}$$

We also know from (A.16) that $\underline{C}_1 < \bar{C}_1$, therefore, if we can prove that

$$\overline{C_1} \Leftrightarrow \frac{\delta\rho^2 + 1}{(1+a) + \delta\rho^2} < \sqrt{\frac{1}{1+a}} \tag{A22}$$

Then we have

$$-\sqrt{\frac{1}{1+a}} < \underline{C_1} < C_1 < \overline{C_1} < \sqrt{\frac{1}{1+a}}$$

and (A.20) is satisfied. Therefore, all that remains to be done is to show if and when the inequality (A.22) is satisfied . (A.22) can be rewritten as

$$\delta\rho^2 < \sqrt{(1+a)} \tag{A23}$$

As $0 < \delta \leq 1$ and $0 < \rho \leq 1$, the LHS of this inequality is bounded between 0 and 1. Since $0 \leq a < \infty$ the RHS is bounded between 1 and ∞. Therefore, we can now prove:

Proposition A1 If the central bank's weight on inflation stabilization is positive (that is if), optimal disinflation teaching by doing is always faster than optimal disinflation under naïve discretion.

Proof. If $a > 0$, condition (A.23) is satisfied. This implies that (A.20) holds. Next, if (A.20) is satisfied, (A.18) holds, that is, $C_1 < \frac{1}{1+a}$, so that disinflation under teaching by doing is faster than under naïve discretion. QED ■

Proposition A2 The higher a the lower the optimal value of the feedback parameter C_1.

Proof.

$$\frac{\partial F}{\partial a}=-\frac{\left(\delta\rho^{2}C_{1}^{2}+1\right)}{\left[(1+a)+\delta\rho\right]^{2}}<0$$

this implies that when a goes up, the function $F(C_1)$shifts downward. As a consequence, the equilibrium value of C_1 decreases. ■

Proposition A3 The higher δ the lower the optimal value of the feedback parameter C_1

Proof.

$$\frac{\partial F}{\partial \delta}=\frac{\rho^{2}\left[C_{1}^{2}(1+a)-1\right]}{\left[(1+a)+\delta\rho^{2}\right]^{2}}$$

Note that the numerator of this expression is negative if $C_1 < \sqrt{\frac{1}{1+a}}$. This condition always holds, since the inequality (A.22) is satisfied (see above). ■

Proposition A4 The higher ρ the lower the optimal value of the feedback parameter C_1 .

Proof.

$$\frac{\partial F}{\partial \rho}=\frac{2\delta\rho\left[C_{1}^{2}(1+a)-1\right]}{\left[(1+a)+\delta\rho^{2}\right]^{2}}$$

Note that the nominator of this expression is negative if

$$C_1<\sqrt{\frac{1}{1+a}}$$

For more details see PROPOSITION A3 above. ■

8 Derivation of the First-Order Condition in Section 5

Now, the central bank's problem is to choose

$$\{u_\tau\}_{\tau=t}^{\infty}=\{\pi_\tau\}_{\tau=t}^{\infty}$$

so as to maximize (2) subject to (1) and

$$E_{t-1}\pi_t=c_{t-1}=c_{t-2}+\underline{\kappa}(\pi_{t-1}-c_{t-2}) \tag{A24}$$

It is convenient to define $x_t = E_{t-1}\pi_1 = c_{t-1}$as the state variable. We solve this problem by the method of Lagrange multipliers. Introduce the Lagrange multiplier μ, and consider the Lagrangean expression

$$L=E_t\left[\sum_{\tau=t}^{\infty}\left\{\begin{array}{l}\dfrac{\delta^{\tau-t}}{2}\left[-a(u_\tau)^2-(u_\tau-x_\tau)^2\right]\\ -\delta^{\tau-t+1}\mu_{\tau+1}\left[x_{\tau+1}-\beta(1-\underline{\kappa})x_\tau-\rho u_\tau\right]\end{array}\right\}\right] \tag{A25}$$

Note that we have set up the constraint in such a way to make sure that if $\beta \to 0$(A.25) collapses to (A.4) – and thus all results are identical to those derived in Appendix A.1 above. Similarly, if $\beta \to 1$and $\rho = \underline{\kappa}$ we have the constraint (A.24) and thus are dealing with the derivation of the first-order condition in Section 5. The central bank's first-order conditions are (A.5) and

$$\frac{\partial L}{\partial x_{t+1}} = \delta(u_{t+1} - x_{t+1}) - \delta E_t \mu_{t+1} + \delta^2 \beta(1-\underline{\kappa})E_t \mu_{t+2} = 0 \tag{A26}$$

Lagging (A.26) by one period and dividing through by δ we obtain

$$(u_t - x_t) + \delta\beta(1-\underline{\kappa})E_t \mu_{t+1} = \mu_t \tag{A27}$$ [26]

To solve for the first-order conditions (A.5) and (A.27) we conjecture the following solution for $\mu(x)$

$$\mu(x) = A_1 x \tag{A28}$$

In the first step of the solution procedure we use we use (A.28) in (A.5) to yield

$$-a(u_t) - (u_t - x_t) + \delta\rho\lfloor A_1 E_t x_{t+1} \rfloor = 0 \tag{A29}$$

Using the constraint we get

$$-a(u_t) - (u_t - x_t) + \delta\rho\beta A_1(1-\underline{\kappa})x_t + \delta A_1 \rho^2 u_t = 0 \tag{A30}$$

Solving (A.30) for u gives $u = G_1 x$ in which

$$G_1 = -\left[\frac{1 + \delta\rho\beta A_1(1-\underline{\kappa})}{\delta A_1 \rho^2 - a - 1}\right] \tag{A31}$$

In the next step observe

$$E_t \mu_{t+1} = A_1 E_t x_{t+1} \; or$$
$$E_t \mu_{t+1} = A_1\{\beta(1-\underline{\kappa})x_t + \rho u_t\}$$

Using that $u = G_1 x$ yields

$$E_t \mu_{t+1} = A_1[\beta(1-\underline{\kappa}) + \rho G_1]x_t \tag{A32}$$

which is linear in x.

Substituting (A.32) in (A.27) yields

[26] Note that $\lim_{\beta \to 0} \quad (A.27) = (A.7)$

$$\mu_t = A_1 x_t + A_2 = \{(G_1 - 1) + \delta\beta(1-\underline{\kappa})A_1[\beta(1-\underline{\kappa} + \rho G_1)]\}x_t$$

equating coefficients on both sides gives

$$A_1 = \{(G_1 - 1) + \delta\beta(1-\underline{\kappa})A_1[\beta(1-\underline{\kappa} + \rho G_1)]\} \tag{A33}$$

Equations (A.31) and (A.33) are used to solve for $G_1$and A_1. We now focus on the solution for G_1, which is informative about the degree to which the central bank accommodates inflation expectations.

In order to do so, first rewrite (A.33) as

$$A_1 = \frac{G_1 - 1}{\{1 - \delta\beta(1-\underline{\kappa})[\beta(1-\underline{\kappa}) + \rho G_1]\}} \tag{A34}$$

Now we numerically solve for the (initially undetermined) coefficient G_1 in terms of the model's structural parameters.

The procedure is to first compute (A.34) for one set of structural parameters (including an ad hoc or guessed value for G_1). Then, we rewrite (A.31) as

$$G_1 + \frac{1 + \delta\rho\beta A_1(1-\underline{\kappa})}{\delta A_1 \rho^2 - a - 1} = 0 \tag{A35}$$

where $\beta = 1$ and $\rho = \underline{\kappa}$

Finally we plug the computed value of (A.34) in (A.35) and numerically solve for .

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Figure 1: Gradual Disinflation

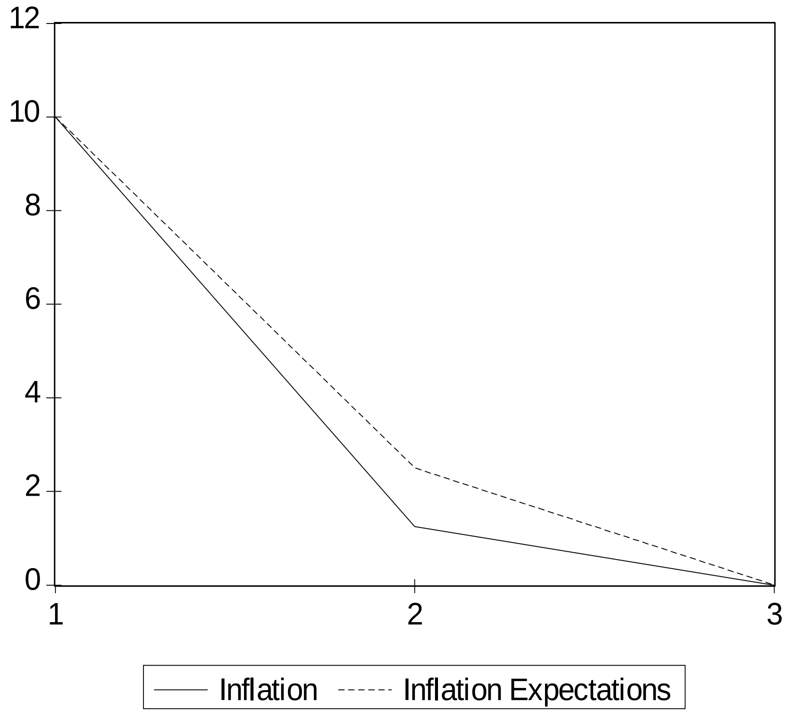

Table 1

Stage 1	Stage 2	Stage 3
The private sector sets $E_{\tau-1}\pi_\tau = c_{\tau-1} = c_{\tau-2} + \underline{\kappa}(\pi_{\tau-1} - c_{\tau-2})$	2a) Central bank chooses $\pi_\tau = \pi_\tau(E_{\tau-1}\pi_\tau)$ 2c) Inflation realises 2d) Private sector observes π_τ and forms an updated estimate $E_\tau\pi_{\tau+1} = c_\tau = c_{\tau-1} + \underline{\kappa}(\pi_\tau - c_{\tau-1})$	Back to stage 1 for time $\tau = t+1$

Table 2: Gain Parameter and Monetary Accommodation

$\underline{\kappa}$	G_1	$\underline{\kappa}$	G_1
0	0.500	0.025	0.456
0.015	0.471	0.1	0.400
0.020	0.463	0.2	0.387