Value at Risk for Large Portfolios

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Abstract

We argue that the practise of valuing the portfolio is important for the calculation of the VaR. In particular, the seller (buyer) of an asset does not face horizontal demand (supply) curves. We propose a partially new approach for incorporating this fact in the VaR and in an empirical illustration we compare it to a competing approach. We find substantial differences.

Key Words: Demand, Supply, Liquidity Risk, Limit Order Book, Bank, Sweden.
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1 Introduction

In this paper we address the question of how to properly assess the risk in large financial portfolios. In risk assessment it is usually assumed that the entire position can be sold at the market price (or mid-price), though one realizes that this can be a quite misleading valuation approach. The reason is that for large enough positions the seller (buyer) of an asset does not face a horizontal demand (supply) curve. Thus, there is an element of liquidity risk involved (see Malz, 2003, for a general discussion of liquidity risk) and this should reasonably be taken into account in risk assessment.

Here, the primary focus is on incorporating the liquidity risk in the Value at Risk (VaR) measure, which is the standard way of quantifying the risk of adverse price movements. VaR is defined as the maximum potential portfolio loss that will not be exceeded over a given time horizon for some small probability (see Jorion, 2007, for a survey). We emphasize, as argued by François-Heude and Van Wynendaele (2001) and others, that it is implicitly assumed that the liquidation occurs in one block at the end of the predefined holding period when calculating the VaR. The question of how to incorporate the liquidity risk into the VaR is a relatively old one and several alternative approaches have been proposed. Bangia, Diebold, Schuermann, and Stroughair (1999) were the first to account for it, with their spread based alternative. Ernst, Stange, and Kaserer (2009) evaluates some measures empirically.

Our proposed approach relies on essentially the same idea as in Giot and Grammig (2006) (GG hereafter). Rather than the mid-price at the end of the horizon they consider the average price per share that would be obtained upon immediate liquidation. Their VaR is volume dependent and it is based on the difference between the mid-price at the beginning of the horizon and the average price at the end of it. We argue that the relevant initial price is not the mid-price, but that the portfolio should be valued at the average price in the beginning of the period as well. We have assets traded on an order driven markets with a visible limit order book (LOB) (e.g., Gourieroux and Jasiak, 2001, ch. 14) in mind and the context is intra-day. Though frequently used on a (at least) daily basis, intra-day VaR's are of interest as well. For example, Dionne, Duchesne, and Pacurar (2008) argue that the investment horizon for very active agents on the market is typically less then one day.

When it comes to the modelling of the dynamics of the average prices the literature is quite scarce. The model employed in GG is of AR-GARCH type and it is essentially univariate. Other previous attempts include Gourieroux, Le Fol, and Meyer (1998) and Bowsher (2004). The former consider a factor model in transaction time, while the latter proposes a functional signal plus noise time series model in calender time. Our framework shares features with all three approaches and the resulting multivariate model allows for spatial (in the volume dimension) as well as serial correlation in the time dimension.

The paper is organized as follows. In Section 2 our VaR framework is presented. Section 3 gives some descriptive statistics of our data set consisting of high-frequency observations on the limit order book of Swedish banking stocks. In Section 4 we propose a time series model for the dynamics of the limit order book. Section 5 contains some empirical results including a comparison with the competing approach of GG.

2 The Liquidity adjusted VaR

The object of interest is the conditional VaR for the horizon T to $T+h$ for a univariate portfolio consisting of v_T shares of a financial asset. We will consider VaR's for both long and short portfolios. For the latter we borrow shares today and agree to return them at some future date. Thus, in that case v_T is negative. We do not allow for portfolio updating, so that $v_T = v_{T+i}$, $i = 1, ..., h$, and we denote the value of the portfolio at time point $t = T, ..., T+h$ by V_t. Following (Gourieroux and Jasiak, 2001, ch. 16) the VaR for the position v_T satisfies

$$\Pr\{V_{T+h} - V_T < -VaR^{1-\alpha}_{T,h} \mid \mathcal{F}_T\} = \alpha, \tag{1}$$

where $\mathcal{F}_T$ is the information available at time T. That is, with the (small) probability α the change in the value of the portfolio is less than $-VaR^{1-\alpha}_{T,h}$. In anticipation of what follows we note that the VaR depends on how we compute the values V_{T+h} and V_T. The approach typically adopted in the literature is to assume that the entire portfolio can be sold at one and the same price, e.g., the mid-price, $\tilde{P}_t$, $t = T, ..., T+h$ (say). This implies that the portfolio values V_T and V_{T+h} in (1) are approximated by $\tilde{V}_T = \tilde{P}_T v_T$ and $\tilde{V}_{T+h} = \tilde{P}_{T+h} v_T$, respectively. The corresponding approximative VaR for a long position then satisfies

$$\begin{aligned}\Pr\{\tilde{V}_{T+h} - \tilde{V}_T < -\widetilde{VaR}^{1-\alpha}_{T,h} \mid \mathcal{F}_T\} = \\ \Pr\{(\tilde{P}_{T+h} - \tilde{P}_T)v_T < -\widetilde{VaR}^{1-\alpha}_{T,h} \mid \mathcal{F}_T\}.\end{aligned} \tag{2}$$

For a short position the expression becomes $\Pr[(\tilde{P}_{T+h} - \tilde{P}_T)v_T > \widetilde{VaR}^{1-\alpha}_{T,h} \mid \mathcal{F}_T]$. The discussion below is for a long position, but it applies analogously for a short one. Now, for relatively small positions we expect the VaR as defined by (2) to provide a reasonable approximation. However, as argued in the introduction $\tilde{V}_T$ does not in general give the correct value of the portfolio. For example, assume that our position consists of 1000 shares and that at time $T+h$, 500 shares are demanded at the price 2 at the first level of the bid-side of the LOB, and that 1000 shares are demanded at price 1 at the second level. Whereas a marking to the mid-price approach would assign a value of, at least, 2000 we would actually obtain $500 \times 2 + (1000 - 500) \times 1 = 1500$ upon immediate liquidation. The average price per unit of sold volume for this transaction is 1.5 and it appears that this is the fair price to replace for $\tilde{P}_{T+h}$ in (2).

Generalizing, we define $\bar{P}_t(v)$ as the average price as a function of the volume, i.e. the average price per unit of volume that would result from immediately executing a market order of v shares. In the sequel we let superscripts a and b indicate whether the average price is for the ask or the bid side of the LOB. Figure 1 shows demand and supply schedules along with the corresponding average price curves for an observation of one of the stocks (SWB) in our data set.

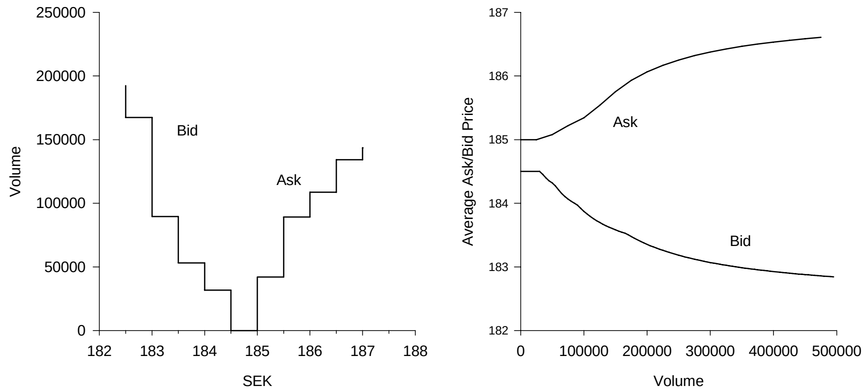

Figure 1: Supply and demand schedules (left) and average price curves (right) in SWB, August 1 at 10AM.

The question is then how to properly use $\bar{P}_t(v)$ to compute the relevant change in value and this is where we differ from GG. They consider a one-period setting and in their view the relevant change in the value of a position of size v_T is given by $\bar{P}^b_{T+1}(v_T)v_T - \tilde{P}_T v_T$, where $\tilde{P}_T = \left[\bar{P}^a_T(1) + \bar{P}^b_T(1)\right]/2$. They specify the dynamics of the log-returns, $p^{GG,v}_t = \ln(\bar{P}_t(v_T)/\tilde{P}_{t-1})$, on the location-scale form $p^{GG,v}_t = \mu^{GG,v}_t + \sigma^{GG,v}_t \varepsilon^{GG,v}_t$, where μ^{GG}_t and σ^{GG}_t are the conditional mean and standard deviation of $p^{GG,v}_t$, respectively, and $\varepsilon^{GG,v}_t$ is an iid random variable with zero mean and unit variance. Their VaR is[1]

$$VaR^{GG,1-\alpha}_{T,1} = -\tilde{P}_T v_T (\exp(\mu^{GG,v}_{T+1} + \sigma^{GG,v}_{T+1} q^{\eta}_{\alpha}) - 1), \tag{3}$$

where q^{η}_{α} is the αth quantile in the Student's t distribution with η degrees of freedom.

We argue that with the same motivation as we value the portfolio at the average price at the end of the period, we should also value it at the average price in the beginning of it. Thus the relevant one-period change in value is $\bar{P}^b_{T+1}(v_T)v_T - \bar{P}^b_T(v_T)v_T$. With the corresponding log-return dynamics

$$p^v_t = \mu^v_t + \sigma^v_t \varepsilon^v_t, \tag{4}$$

our VaR alternative is

$$VaR^{1-\alpha}_{T,1} = -\bar{P}^b_T(v_T)v_T(\exp(\mu^v_{T+1} + \sigma^v_{T+1} q_{\alpha}) - 1), \tag{5}$$

where q_{α} is the αth quantile of some suitable distribution.

For a horizon of h periods the VaR satisfies $\Pr\{[\bar{P}_{T+h}(v_T) - \bar{P}_T(v_T)]\, v_T \leq -VaR^{1-\alpha}_{T,h} \mid \mathcal{F}_T\}$. However, the dynamics of the h-period returns do not follow easily from that of the one-period returns (cf. Lönnbark, 2009). Note also that our VaR and the VaR in Giot and Grammig (2006) are related by

$$VaR^{1-\alpha}_{T,h} = VaR^{GG,1-\alpha}_{T,h} - (\bar{P}_T(v) - \tilde{P}_T)v_T.$$

[1] Actually, their VaR is the quantile of the distribution of the log-returns, but this is the implication for the VaR definition we use.

Hence, given one of the VaR's it is possible to obtain the other through an additive transformation that is known at time T. Note also that the difference between the two measures grows with an increasing volume.

The VaR in (5) implicitly assumes that we own the portfolio at T. If it is to be purchased at T we use $\bar{P}_T^a(v_T)$ for the initial price and the VaR becomes

$$VaR_{T,1}^{1-\alpha} = \bar{P}_T^a(v_T)v_T - \bar{P}_T^b(v_T)v_T \exp(\mu_{T+1}^v + \sigma_{T+1}^v q_\alpha). \tag{6}$$

We end this section by giving the corresponding VaR's for a short position. They are, respectively, given by

$$\begin{aligned} VaR_{T,1}^{1-\alpha} &= \bar{P}_T^a(v_T)v_T(\exp(\mu_{T+1}^v + \sigma_{T+1}^v q_{1-\alpha}) - 1), & (7)\\ VaR_{T,1}^{1-\alpha} &= \bar{P}_T^a(v_T)v_T \exp(\mu_{T+1}^v + \sigma_{T+1}^v q_{1-\alpha}) - \bar{P}_T^b(v_T)v_T. & (8) \end{aligned}$$

Note that $q_{1-\alpha}$ instead of q_α appears in (7) and (8).

3 Data and descriptives

Our dataset consists of time series for the four largest banks in Sweden (Nordea NRD, Skandinaviska Enskilda Banken SEB, Handelsbanken SHB, Swedbank SWB[2]) and covers the period May 3 – August 8, 2005.[3] Table 1 gives a few descriptive statistics for the trading patterns in the four banking stocks for the first trading month (21 days) of the data. The number of traded shares distributions are quite skewed with a long upper tail and the largest transactions in each month are quite large. The largest transaction was in SEB and amounted to about 1653 million SEK using the average price. This corresponds to about 17 percent of total transactions during the month. For the other stocks the corresponding percentages are about 4 percent. Trading is most frequent in NRD with about 900 daily transactions or about 2 per minute.

The sampling frequency is chosen to be 30 minutes, such that the records immediately preceding the given half-hour are chosen. The daily records cover 1000–1700, i.e. there are 15 observations during the day and the total time series length is $T = 936$ for SEB and SHB and $T = 861$ for NRD and SWB.

Table 1: Descriptive statistics for the number of traded shares and closing prices in individual transactions for the four banks in the first trading month.

	Nr of Traded Shares			Closing Price		
Bank	Mean	StDev	Max	Mean	StDev	n
NRD	9921.6	95424.5	7 383 816	67.7	0.42	19026
SEB	5341.5	$1.12 \cdot 10^5$	12 946 377	127.7	1.81	14325
SHB	3963.1	33227.2	1 831 705	161.1	2.40	10445
SWB	3644.2	22917.7	1 299 919	171.4	2.56	11468

[2] Föreningssparbanken in the sample period.

[3] For technical reasons the period June 7–10 is missing for all banks, and additionally May 27–June 1 for SWB and NRD.

Table 2: Cross correlations for log-returns (ask) in SHB across volume levels with $v = 200000$ as a base.

	Volumes (thousands)					
Lag	$1 \cdot 10^{-3}$	100	150	200	250	300
0	0.77	0.94	0.99	1.00	0.99	0.97
1	-0.01	-0.00	-0.03	-0.04	-0.04	-0.04
2	-0.02	-0.13	-0.13	-0.13	-0.13	-0.13
3	-0.07	-0.05	-0.06	-0.07	-0.07	-0.07
4	-0.03	0.00	0.00	0.01	0.01	0.01

Table 3: Parameter estimates and descriptive statistics for MA(1) models and their residuals of the ask/bid (a and b) average log-return series of the four banks at volume level $v = 200000$. p-values are used for the Ljung-Box statistics, LB.

Bank		MA(1)	t	LB_{10}	LB^2_{10}	Skew	Kurt
NRD	a	0.088	2.59	0.86	0.02	0.26	3.60
	b	0.141	4.15	0.95	0.03	-0.04	3.58
SEB	a	-0.030	-0.90	0.83	0.00	2.10	7.97
	b	-0.010	-0.32	0.82	0.00	-0.38	15.7
SHB	a	0.044	1.34	0.05	0.00	0.76	6.73
	b	0.063	1.94	0.60	0.00	-0.08	5.56
SWB	a	0.088	2.60	0.31	0.63	1.11	8.39
	b	0.018	0.54	0.63	0.00	-0.64	14.0

For the empirical modelling we can obtain time series of average prices for any chosen volume level. For the analyses reported later we have chosen five volume levels $v = 1, 100000(50000)300000$ and all results are based on log-returns $p_t^v = \ln(\bar{P}_t^v) - \ln(\bar{P}_{t-1}^v)$. As an illustration of the spatial/volume correlations within stocks we consider log-returns for the ask side of SHB, cf. Table 2. As expected from the smoothness of the average curve in Figure 1, we find that correlations between log-returns at the different volume levels are close to 1. Obviously, the correlations are weaker for lagged log-returns. The autocorrelation function closely matches the cross correlation, except for for the first volume level.

Based on the SWB series the autocorrelation functions suggest that MA(1) models will account for most of the serial correlation in the time series. Table 3 gives estimated models and some descriptive statistics for the residuals of the models. In all but one case there is significant autocorrelation in squared residuals, suggesting that ARCH effects are of major importance. For the ask series there is positive skewness and weak but negative for the bid series. For most series there is substantial kurtosis.

4 A time series model for the average price curves

We specify the dynamics of the average price curves in terms of log returns. Stock prices are widely taken to be random walks with drift and for returns various autoregressive and/or moving average extensions of the basic model seem to empirically surface. Based on some initial specification searches on the SWB stock we take as a reasonable model

$$\begin{aligned} p_t^{v_1} &= \alpha_{v_1} + \boldsymbol{\beta}'\mathbf{d}_t + \varepsilon_t^{v_1} + \theta_0 \varepsilon_{t-1}^{v_1} \\ p_t^{v_i} &= \alpha_{v_i} + \boldsymbol{\beta}'\mathbf{d}_t + \gamma_{v_i} p_{t-1}^{v_{i-1}} + \varepsilon_t^{v_i} + \theta_{v_i}\varepsilon_{t-1}^{v_i}, \qquad i = 2, \ldots, m, \end{aligned}$$

where $p_t^{v_i} = \log[\bar{P}_t(v_i)] - \log[\bar{P}_{t-1}(v_i)]$. The parameters γ_{v_i} and θ_{v_i} are volume dependent; $\gamma_{v_i} = \gamma_0 + \gamma_1 v_{i-1}$ and $\theta_{v_i} = \theta_0 + \theta_1 v_i$, $i = 2, \ldots, m$. The $\mathbf{d}_t$ is a vector of dummy variables to catch overnight impacts on the first observation of the day and time of day effects. In addition, the models of different volume levels may be correlated such that $E(\varepsilon_{t,v_i}\varepsilon_{s,v_j}) \neq 0$, for all v_i, v_j and also for $t \neq s$.

For all volume levels, $\mathbf{v} = \{v_1, v_2, ..., v_m\}$, we write

$$\begin{pmatrix} p_t^{v_1} \\ \vdots \\ p_t^{v_m} \end{pmatrix} = \begin{pmatrix} \alpha_{v_1} \\ \vdots \\ \alpha_{v_m} \end{pmatrix} + \boldsymbol{\beta}'\mathbf{d}_t\boldsymbol{\iota} + \begin{pmatrix} 0 & & \cdots & & 0 \\ \gamma_{v_2} & & & & \\ 0 & \gamma_{v3} & \ddots & & \vdots \\ \vdots & \ddots & \ddots & & \\ 0 & \cdots & 0 & \gamma_{v_m} & 0 \end{pmatrix} \begin{pmatrix} p_{t-1}^{v_1} \\ \vdots \\ p_{t-1}^{v_m} \end{pmatrix}$$

$$+ \begin{pmatrix} \varepsilon_t^{v_1} \\ \vdots \\ \varepsilon_t^{v_m} \end{pmatrix} + \begin{pmatrix} \theta_{v_1} & 0 & \cdots & 0 \\ 0 & \theta_{v_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \theta_{v_m} \end{pmatrix} \begin{pmatrix} \varepsilon_{t-1}^{v_1} \\ \vdots \\ \varepsilon_{t-1}^{v_m} \end{pmatrix}$$

or compactly

$$\mathbf{p}_t^{\mathbf{v}} = \boldsymbol{\alpha} + \boldsymbol{\beta}'\mathbf{d}_t\boldsymbol{\iota} + \boldsymbol{\Gamma}_{\mathbf{v}}\mathbf{p}_{t-1}^{\mathbf{v}} + \boldsymbol{\varepsilon}_t + \boldsymbol{\Theta}_{\mathbf{v}}\boldsymbol{\varepsilon}_{t-1}, \tag{9}$$

where $\boldsymbol{\iota}$ is a vector of ones and $\boldsymbol{\varepsilon}_t$ has zero mean and conditional covariance matrix $\boldsymbol{\Sigma}_t$. Thus, the model is of VARMAX type and has both a time series and volume/spatial dimension. The $\boldsymbol{\Sigma}_t$ may contain nonzero off-diagonal elements and is also indexed by t to allow for ARCH-effects. For the conditional variances we employ a version of the asymmetric GARCH specification of Glosten, Jagannathan, and Runkle (1993)

$$h_t^{v_i} = \omega_{v_i} + \delta h_t^{v_i} + \eta(\varepsilon_{t-1}^{v_i})^2 + \lambda(\varepsilon_{t-1}^{v_i})^2 \mathbf{1}(\varepsilon_{t-1}^{v_i} < 0), \tag{10}$$

where $\mathbf{1}(\cdot)$ is the indicator function. Note that ω_{v_i} is the only parameter that changes across v_i. As a full baseline model for $\boldsymbol{\Sigma}_t$ we consider (10) together with constant off-diagonal elements

$$\boldsymbol{\Sigma}_t = \boldsymbol{\Omega} + \delta \operatorname{diag}(\mathbf{h}_t^{\mathbf{v}}) + \eta \operatorname{diag}(\boldsymbol{\varepsilon}_{t-1}^{2,\mathbf{v}}) + \lambda \operatorname{diag}(\boldsymbol{\varepsilon}_{t-1}^{2-,\mathbf{v}}),$$

where $\mathbf{h}_t^{\mathbf{v}}$, $\boldsymbol{\varepsilon}_t^{2,\mathbf{v}}$ and $\boldsymbol{\varepsilon}_t^{2-,\mathbf{v}}$ have elements $h_t^{v_i}$, $(\varepsilon_t^{v_i})^2$ and $(\varepsilon_t^{v_i})^2\mathbf{1}(\varepsilon_t^{v_i} < 0)$, $i = 1, ..., m$, respectively. The diag$(\cdot)$ operator returns a matrix with the vector argument on the diagonal and zeros elsewhere. Hence, the conditional expectation and the conditional variance of the log returns are, respectively, given by

$$\begin{aligned} E(\mathbf{p}_t^{\mathbf{v}}|\mathcal{F}_{t-1}) &= \boldsymbol{\alpha} + \boldsymbol{\beta}'\mathbf{d}_t\boldsymbol{\iota} + \boldsymbol{\Gamma}_{\mathbf{v}}\mathbf{p}_{t-1}^{\mathbf{v}} + \boldsymbol{\Theta}_{\mathbf{v}}\boldsymbol{\varepsilon}_{t-1} \\ V(\mathbf{p}_t^{\mathbf{v}}|\mathcal{F}_{t-1}) &= \boldsymbol{\Sigma}_t. \end{aligned} \tag{11}$$

These expression are useful both for estimation and forecasting over time. From (9) it is straightforward to obtain the corresponding price levels as $\bar{P}_t^{v_i} = \bar{P}_{t-1}^{v_i}\exp(p_t^{v_i})$, $i = 1, ..., m$. The conditional expectation and variance of $\bar{P}_t^{v_i}$ may be obtained by taking first order expansions of the exponential function and (11).

With respect to the spatial aspects of the model note that this is an unusual context of observation availability for all volume levels. However, for low levels the volume curves are typically flat and for very large levels linear. Therefore, it appears reasonable to focus the modelling exercise on the intermediate levels, where the curvature is most pronounced. The way we choose v and m in the estimation phase impacts the precision of the estimates, but as our model is not able to predict in the volume direction, the choice is also practically related to the model's end use for VaR calculations.

4.1 Estimation

When it comes to predicting the VaR we use a multivariate version of a popular methodology known as filtered historical simulation (FHS) in the literature (e.g., Christoffersen, 2009). To explain the approach we first collect all model parameters in the vector $\boldsymbol{\psi}$ and consider the prediction error $\mathbf{e}_t = \mathbf{p}_t^{\mathbf{v}} - E_{\boldsymbol{\psi}}(\mathbf{p}_t^{\mathbf{v}}|\mathcal{F}_{t-1})$, where we subindex the expectation operator to emphasize that it is to be taken under $\boldsymbol{\psi}$. Assuming that the standardized prediction errors $\tilde{\mathbf{e}}_t = (\boldsymbol{\Sigma}_t^{1/2})^{-1}\mathbf{e}_t$, $t = 1, ..., T$, is an iid sequence we may approximate the conditional distribution of $\mathbf{p}_{T+1}^{\mathbf{v}}$ with the sequence $\mathbf{p}_{T+1,j}^{\mathbf{v},*} = E_{\boldsymbol{\psi}}(\mathbf{p}_{T+1}^{\mathbf{v}}|\mathcal{F}_T) + \boldsymbol{\Sigma}_{T+1}^{1/2}\tilde{\mathbf{e}}_j$, $j = 1, ..., T$. The predictors of the one-period VaR's are then trivially obtained from suitable empirical quantiles of the $\mathbf{p}_{T+1,j}^{\mathbf{v},*}$ sequences.

The FHS is a two-step procedure that in the first step estimates the underlying model parameters employing some estimator, $\hat{\boldsymbol{\psi}}$. In the second step it filters out the $\tilde{\mathbf{e}}_t$ sequence.

A natural choice for $\hat{\boldsymbol{\psi}}$ is the quasi maximum likelihood estimator. Given observations up til time T it involves finding the $\boldsymbol{\psi}$ that maximizes the log-likelihood function

$$\ln L = -\frac{1}{2}\sum\nolimits_{t=2}^{T}(\ln|\boldsymbol{\Sigma}_t| - \mathbf{e}_t'\boldsymbol{\Sigma}_t^{-1}\mathbf{e}_t).$$

For practical estimation we use the RATS 6.0 package and employ robust standard errors.

5 Empirical results

The empirical results are summarized in terms of VaR measures in Table 4 for the case when we own the portfolio at the horizon origin. Parameter estimates may be found in Table 5. The measures are calculated for the first post sample time period, i.e. 5PM of August 8 to 10AM of August 9, 2005. The numbers reported for a short position are throughout larger than the ones for the corresponding long position. This is a consequence of, at least, the asymmetry in average cost curves.

Figure 2 gives VaR's per share for SWB. With some exceptions, there is a modest growth in all measures. If we take the view that we own the portfolio at the horizon

Table 4: VaR estimates for $\alpha = 0.01$.

	NRD		SEB		SHB		SWB	
Volume	Short	Long	Short	Long	Short	Long	Short	Long
1	0.59	0.38	1.29	1.33	1.30	0.53	1.63	0.86
100 000	57 698	37 733	136 107	119 240	125 733	74 944	175 379	92 403
150 000	86 138	48 532	225 120	161 578	198 631	116 893	281 068	174 078
200 000	113 910	61 606	307 464	212 012	313 717	182 451	416 784	222 729
250 000	139 688	74 507	414 339	286 324	463 808	248 137	528 713	299 420
300 000	166 669	84 207	517 234	366 692	591 360	341 902	606 274	382 042

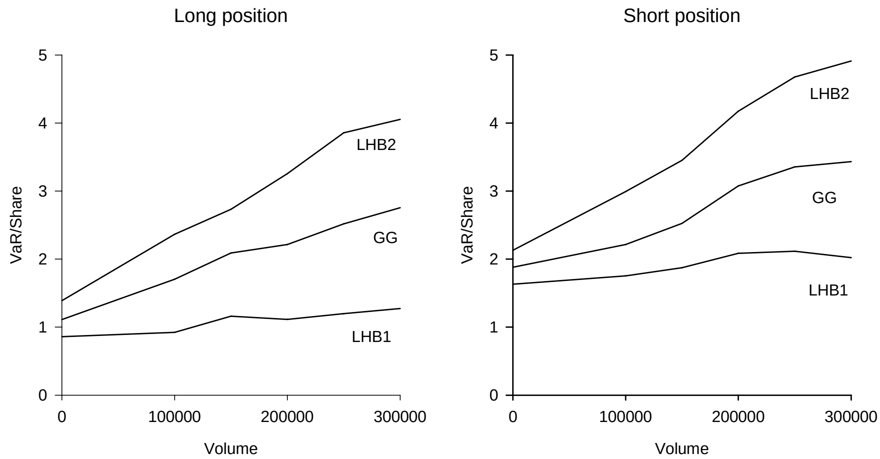

Figure 2: Value at Risk per share vs volume for long and short positions in the SWB stock. GG refers to the VaR as given by the approach in Giot and Grammig (2006). LHB1 and LHB2 are our VaR's for a portfolio owned and purchased at T, respectively.

origin, our VaR's are smaller than those calculated as in GG. If the portfolio is to be purchased, they are larger. Noteworthy is also that for the latter view our VaR's rise more sharply with volume. There is a growing difference between our VaR's and the ones as in GG, starting from one half of a tick (0.25 SEK) at volume 1 to exceededing 2 ticks for the largest position of $v = 300000$ shares. Obviously, these differences will have substantial consequences for how to set the required capital for large financial institutions.

Table 5: Estimates (bold facing indicates significance at 5 percent level). The Ljung-Box statistics are evaluated the volume $v = 200000$ level.

	NRD		SEB		SHB		SWB	
Param	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid
α_0	$-2.75 \cdot 10^{-5}$	$-2.20 \cdot 10^{-5}$	$3.84 \cdot 10^{-6}$	$1.51 \cdot 10^{-4}$	$-1.68 \cdot 10^{-5}$	$4.72 \cdot 10^{-5}$	$-6.72 \cdot 10^{-5}$	$5.43 \cdot 10^{-5}$
α_1	$-2.97 \cdot 10^{-5}$	$-3.21 \cdot 10^{-5}$	$1.10 \cdot 10^{-4}$	$\mathbf{1.62 \cdot 10^{-4}}$	$-3.88 \cdot 10^{-5}$	$4.43 \cdot 10^{-5}$	$-9.19 \cdot 10^{-5}$	$1.42 \cdot 10^{-5}$
α_2	$-3.27 \cdot 10^{-5}$	$-8.15 \cdot 10^{-5}$	$1.31 \cdot 10^{-4}$	$1.58 \cdot 10^{-4}$	$-3.47 \cdot 10^{-5}$	$3.89 \cdot 10^{-5}$	$-7.96 \cdot 10^{-5}$	$-1.70 \cdot 10^{-5}$
α_3	$-3.49 \cdot 10^{-5}$	$-3.93 \cdot 10^{-5}$	$1.40 \cdot 10^{-4}$	$1.48 \cdot 10^{-4}$	$-3.05 \cdot 10^{-5}$	$3.73 \cdot 10^{-5}$	$-6.93 \cdot 10^{-5}$	$\mathbf{-3.84 \cdot 10^{-5}}$
α_4	$-3.74 \cdot 10^{-5}$	$-5.16 \cdot 10^{-5}$	$1.58 \cdot 10^{-4}$	$1.55 \cdot 10^{-4}$	$-2.95 \cdot 10^{-5}$	$3.03 \cdot 10^{-5}$	$-6.21 \cdot 10^{-5}$	$\mathbf{-5.48 \cdot 10^{-5}}$
α_5	$-4.06 \cdot 10^{-5}$	$-6.06 \cdot 10^{-5}$	$1.76 \cdot 10^{-4}$	$1.72 \cdot 10^{-4}$	$-2.97 \cdot 10^{-5}$	$2.45 \cdot 10^{-5}$	$-5.79 \cdot 10^{-5}$	$\mathbf{-6.95 \cdot 10^{-5}}$
β_0	$\mathbf{2.27 \cdot 10^{-3}}$	$\mathbf{2.07 \cdot 10^{-3}}$	$\mathbf{1.96 \cdot 10^{-3}}$	$\mathbf{-5.55 \cdot 10^{-4}}$	$\mathbf{2.70 \cdot 10^{-3}}$	$5.55 \cdot 10^{-4}$	$\mathbf{4.13 \cdot 10^{-3}}$	$7.36 \cdot 10^{-5}$
β_{on}	$1.67 \cdot 10^{-6}$	$7.19 \cdot 10^{-6}$	$-3.88 \cdot 10^{-4}$	$-2.97 \cdot 10^{-5}$	$\mathbf{-3.68 \cdot 10^{-4}}$	$-1.59 \cdot 10^{-5}$	$\mathbf{-4.25 \cdot 10^{-4}}$	$1.09 \cdot 10^{-4}$
β_m	$-9.76 \cdot 10^{-5}$	$-6.64 \cdot 10^{-5}$	$-7.18 \cdot 10^{-5}$	$\mathbf{-8.49 \cdot 10^{-5}}$	$7.44 \cdot 10^{-5}$	$3.51 \cdot 10^{-6}$	$1.39 \cdot 10^{-4}$	$\mathbf{1.59 \cdot 10^{-4}}$
γ_0	0.0407	$-5.35 \cdot 10^{-3}$	-0.0136	**0.1071**	0.0217	**0.1867**	0.0331	**0.2763**
γ_1	$2.22 \cdot 10^{-7}$	$\mathbf{-2.15 \cdot 10^{-7}}$	$-7.88 \cdot 10^{-7}$	$\mathbf{-3.65 \cdot 10^{-7}}$	$\mathbf{6.92 \cdot 10^{-7}}$	$\mathbf{1.01 \cdot 10^{-6}}$	$-4.08 \cdot 10^{-7}$	$\mathbf{2.55 \cdot 10^{-6}}$
θ_0	**-0.1178**	**-0.0975**	-0.0612	**-0.1033**	0.0520	**-0.1818**	**-0.1440**	**-0.0818**
θ_1	$\mathbf{-1.13 \cdot 10^{-7}}$	$-6.49 \cdot 10^{-8}$	$8.32 \cdot 10^{-7}$	$\mathbf{3.81 \cdot 10^{-7}}$	$\mathbf{-1.41 \cdot 10^{-6}}$	$\mathbf{-1.26 \cdot 10^{-6}}$	$\mathbf{3.21 \cdot 10^{-7}}$	$\mathbf{-2.80 \cdot 10^{-6}}$
ω_0	$\mathbf{5.35 \cdot 10^{-6}}$	$\mathbf{5.51 \cdot 10^{-6}}$	$\mathbf{3.84 \cdot 10^{-6}}$	$\mathbf{6.03 \cdot 10^{-6}}$	$\mathbf{3.47 \cdot 10^{-6}}$	$\mathbf{3.39 \cdot 10^{-6}}$	$\mathbf{3.74 \cdot 10^{-6}}$	$\mathbf{3.72 \cdot 10^{-6}}$
ω_1	$\mathbf{4.91 \cdot 10^{-6}}$	$\mathbf{5.56 \cdot 10^{-6}}$	$\mathbf{3.71 \cdot 10^{-6}}$	$\mathbf{5.69 \cdot 10^{-6}}$	$\mathbf{2.98 \cdot 10^{-6}}$	$\mathbf{2.78 \cdot 10^{-6}}$	$\mathbf{3.67 \cdot 10^{-6}}$	$\mathbf{3.38 \cdot 10^{-6}}$
ω_2	$\mathbf{4.80 \cdot 10^{-6}}$	$\mathbf{5.77 \cdot 10^{-6}}$	$\mathbf{3.83 \cdot 10^{-6}}$	$\mathbf{6.11 \cdot 10^{-6}}$	$\mathbf{3.23 \cdot 10^{-6}}$	$\mathbf{2.92 \cdot 10^{-6}}$	$\mathbf{4.25 \cdot 10^{-6}}$	$\mathbf{3.79 \cdot 10^{-6}}$
ω_3	$\mathbf{4.72 \cdot 10^{-6}}$	$\mathbf{5.75 \cdot 10^{-6}}$	$\mathbf{4.19 \cdot 10^{-6}}$	$\mathbf{6.90 \cdot 10^{-6}}$	$\mathbf{3.57 \cdot 10^{-6}}$	$\mathbf{3.10 \cdot 10^{-6}}$	$\mathbf{4.83 \cdot 10^{-6}}$	$\mathbf{4.45 \cdot 10^{-6}}$
ω_4	$\mathbf{4.63 \cdot 10^{-6}}$	$\mathbf{5.54 \cdot 10^{-6}}$	$\mathbf{4.50 \cdot 10^{-6}}$	$\mathbf{7.51 \cdot 10^{-6}}$	$\mathbf{3.84 \cdot 10^{-6}}$	$\mathbf{3.39 \cdot 10^{-6}}$	$\mathbf{5.03 \cdot 10^{-6}}$	$\mathbf{5.01 \cdot 10^{-6}}$
ω_5	$\mathbf{4.55 \cdot 10^{-6}}$	$\mathbf{5.64 \cdot 10^{-6}}$	$\mathbf{4.81 \cdot 10^{-6}}$	$\mathbf{7.84 \cdot 10^{-6}}$	$\mathbf{4.06 \cdot 10^{-6}}$	$\mathbf{3.71 \cdot 10^{-6}}$	$\mathbf{5.03 \cdot 10^{-6}}$	$\mathbf{5.34 \cdot 10^{-6}}$
δ	$1.75 \cdot 10^{-4}$	$-3.16 \cdot 10^{-4}$	**0.3108**	**0.0159**	$\mathbf{1.94 \cdot 10^{-3}}$	$-1.02 \cdot 10^{-4}$	$\mathbf{1.51 \cdot 10^{-3}}$	$-6.27 \cdot 10^{-5}$
η	0.0348	**0.1069**	0.1507	**0.3102**	**0.1421**	0.0786	**0.1488**	0.1576
λ	0.0335	**0.0730**	0.0935	**0.1108**	$-5.07 \cdot 10^{-3}$	0.1801	-0.0996	0.0877
LB_{10}	6.01	4.12	4.39	9.63	20.3	3.90	11.6	18.8
LB^2_{10}	2.53	3.19	7.56	36.3	10.1	44.8	6.0	3.50
T	861	861	936	936	936	936	861	861

References

Bangia, A., Diebold, F. X., Schuermann, T., Stroughair, J. D., 1999. Modeling liquidity risk, with implications for traditional market risk measurement and management. Working paper 99-06, Wharton Financial Institutions Center.

Bowsher, C. G., 2004. Modelling the dynamics of cross-sectional price functions: An econometric analysis of the bid and ask curves of an automated exchange. Economics Papers 2004-W21, Economics Group, Nuffield College, University of Oxford.

Christoffersen, P., 2009. Value-at-risk models. In: Andersen, T. G., Davis, R. A., Kreiss, J.-P., Mikosch, T. (Eds.), Handbook of Financial Time Series. Springer Verlag, Berlin Heidelberg, forthcoming.

Dionne, G., Duchesne, P., Pacurar, M., 2008. Intraday value at risk (IVaR) using tick-by-tick data with application to the Toronto stock exchange. Working paper, HEC Montréal.

Ernst, C., Stange, S., Kaserer, C., 2009. Measuring market liquidity risk - which model works best? CEFS Working paper series 1, Center for Entrepreneurial and Financial Studies, Technische Universität München.

François-Heude, A., Van Wynendaele, P., 2001. Integrating liquidity risk in a parametric intraday VaR framework. Working paper.

Giot, P., Grammig, J., 2006. How large is liquidity risk in an automated auction market. Empirical Economics 30, 867–887.

Glosten, L. R., Jagannathan, R., Runkle, D. E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48, 1779–1801.

Gourieroux, C., Jasiak, J., 2001. Financial Econometrics. Princeton University Press, Princeton, New Jersey.

Gourieroux, C., Le Fol, G., Meyer, B., 1998. Etude du carnet d'ordres. Banque et Marchés 36, 5–20.

Jorion, P., 2007. Value at Risk: The New Benchmark for Managing Financial Risk, 3rd Edition. McGraw-Hill, New York.

Lönnbark, C., 2009. Uncertainty of multiple period risk predictors. Umeå Economic Studies 768, Department of Economics, Umeå University.

Malz, A., 2003. Liquidity risk: Current research and practise. RiskMetrics Journal 4, 35–72.