Welfare Measurement and Public Goods in a Second Best Economy*

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Abstract

This chapter concerns welfare measurement in economies, where the government raises revenue by means of distortionary taxation. A major issue is the treatment of (state-variable) public goods in the context of social accounting. Although the marginal value that the government attaches to a public good is model-specific (as it depends on the exact nature of the underlying decision-problem), the analysis explains how the direct resource cost of providing increments to the public good and the marginal cost of public funds can be used to measure this marginal value. The first part of the chapter is based on a representative-agent growth model with linear taxation, whereas the second part addresses a model with heterogeneous agents and nonlinear taxation. The latter model also provides a framework for analyzing redistribution in the context of social accounting, and enables me to compare the results with those that would follow in a first best resource allocation.

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1. Introduction

Earlier literature on social accounting has typically focused either on first best resource allocations, or resource allocations that - for one reason or another - are suboptimal from society's point of view in which case society as a whole has not made an optimal choice given its preferences and constraints. A basic message here is that the current value Hamiltonian underlying the economic system constitutes an exact welfare measure in utility terms if the resource allocation is first best, whereas it does not in general constitute an exact welfare measure if the resource allocation is suboptimal.[1] One purpose of the present chapter is to address the welfare measurement problem in an economy, where the first best is unattainable due to the necessity to raise public revenue by means of distortionary taxes. In such a second best economy, the government (or social planner) has made an optimal policy choice, although the set of policy instruments might not be flexible enough to implement the first best. Another purpose is to address the treatment of public consumption in the context of social accounting. I will also briefly consider redistribution, which is another important aspect of public policy.

Why are taxation, public consumption and redistribution interesting to analyze in the context of welfare measurement and social accounting? First, the public sector plays a crucial role for the allocation of resources in all developed countries by providing public services as well as by redistributing income among individuals and groups. As a consequence, public expenditure is likely to be of considerable importance for well-being, implying that the treatment of such expenditures in social accounting is a relevant issue. Second, if the public revenue is raised via distortionary taxation, there is an additional social cost associated with public expenditures, which ought to be relevant for welfare measurement. On the other hand, if the public revenue were raised via lump-sum taxation, there would be no 'extra' welfare cost of taxation and, therefore, no additional complications for social accounting of raising revenue for purposes of public provision and redistribution.

The chapter begins by analyzing the welfare measurement problem implicit in Chamley's (1985) dynamic second best economy, which is an extension of the Ramsey (1927) optimal

[1] As mentioned in Chapter 1 of this volume, the basic welfare economic foundation for social accounting in a first best economy originates from Weitzman (1976). For an overview of the literature dealing with social accounting in imperfect market economies, see Aronsson et al. (2004).

growth model in the sense of adding a public sector and assuming that the public revenue is raised by a linear, yet time-varying, labor income tax. The analysis carried out in this part of the chapter relies largely on Aronsson (1998, 2008), who examined how distortionary taxes affect the way in which welfare ought to be measured in a dynamic economy, as well as how public goods ought to be treated in social accounting if the public revenue is raised by distortionary taxes. A change of assumption I make here is that the public good will be assumed to be a state-variable (instead of a conventional flow-variable); an assumption motivated by the desire to add realism to the description of the economy.[2] Throughout this part of the chapter, I also compare the welfare measurement and valuation procedures of the second best economy with those that appear in the first best; the latter being a natural special case and, therefore, easy to discuss within the framework of the more general model. The final part of the chapter deals with welfare measurement – and, in particular, measurement of value of public consumption - in an economy with nonlinear taxation. The underlying framework here is the two-type optimal income tax model developed by Stern (1982) and Stiglitz (1982)[3] which is, in turn, a simplification of Mirrlees' (1971) optimal nonlinear tax model with a continuum of ability-types. Such an extension serves two purposes. First, it adds realism by relaxing the assumption (in the Chamley model) that the use of disortionary taxation is a consequence of restrictions on the set of policy instruments that the government is allowed to use; here, its use is, instead, a consequence of asymmetric information between the private sector and the government. Second, it allows me to provide a more close connection between social accounting and the literature on public good provision, as the policy rule for a public good and, as a consequence, the procedure for measuring its social value largely depends on the set of tax instruments that the government has at its disposal.

2. A Dynamic Representative Agent Model with Linear Taxation

The model developed in this section is, to a large extent, based on Chamley (1985), which implies that the consumption side of the economy is characterized by a representative consumer. As a consequence, we disregard any distributional objectives underlying public

[2] Typical textbook examples of public goods – or public good like variables – often include measures of environmental quality, national infrastructure and national defence; all of which struck me as better described as state-variables than conventional flow-variables.

[3] See also Pirttilä and Tuomala (2001) and Aronsson et al. (2009) for extensions of the two-type optimal income tax model to dynamic economies.

policy in this section. A multi-consumer economy along with distributional objectives is examined in Section 3 below.

2.1 Consumers

Following the main bulk of literature on social accounting, suppose that the economy is populated by a fixed number of identical consumers. Since this fixed number is of no concern here, it will be normalized to one. The preferences are described by a time-separable utility function. The instantaneous utility function at any time, *t*, takes the form

$$u(t) = u(c(t), z(t), G(t)) \tag{1}$$

where *c* is consumption of a privately provided good, *z* leisure and *G* consumption of a public good. The determination of the public good will be further discussed below. Leisure is defined as a (fixed) time endowment, $\bar{l}$, less the time spent in market work, *l*. The function $u(\cdot)$ is increasing in each argument and strictly concave. The consumer treats the public good as exogenous.

The consumer holds two assets; capital, k , and government bonds, b . These two assets are assumed to be perfect substitutes and have the same rate of return. If we define the composite asset $a = k + b$, the asset accumulation equation can be written as

$$\dot{a}(t) = r(t)a(t) + w_n(t)l(t) - c(t) \tag{2}$$

with $a(0) = a_0$, where w_n is the net wage rate and r the interest rate. The net wage rate is defined as $w_n(t) = w(t)[1-\tau(t)]$, where w is the gross wage rate and τ the tax rate[4].

The consumer chooses his/her consumption of the private good, *c*, and hours of work, *l*, at each instant to maximize the present value of future utility,

[4] Note that I abstract from capital income taxes in what follows. Since the model already contains one distortionary tax, adding another will not affect the principal findings below. See Chamley (1986) for a dynamic representative agent model with linear taxes on labor income and capital income.

$$U(0)=\int_0^\infty u(c(t),z(t),G(t))e^{-\theta t}dt\ ,$$

subject to equation (2), the initial condition as well as subject to a so called No Ponzi Game (NPG) condition, the purpose of which is to ensure that the present value of the asset is nonnegative at the terminal point. The parameter θ is the utility discount rate (i.e. the marginal rate of time preference). By using the first order conditions[5] $u_c(c(t),z(t))-\phi(t)=0$ and $-u_z(c(t),z(t))+\phi(t)w_n(t)=0$, in which ϕ is the marginal utility of wealth in current value terms, one can write the consumption and hours as functions of the net wage rate, the marginal utility of wealth and the public good, respectively (i.e. so called Frisch demand functions)

$$c(t)=c(w_n(t),\phi(t),G(t)) \tag{3}$$

$$l(t)=l(w_n(t),\phi(t),G(t))\ . \tag{4}$$

The marginal utility of wealth obeys, in turn, the differential equation

$$\dot{\phi}(t)-\theta\phi(t)=-\phi(t)r(t)\ . \tag{5}$$

Finally, by substituting equations (3) and (4) into the instantaneous direct utility function, we obtain the instantaneous indirect utility function defined conditional on the marginal utility of wealth

$$v(t)=v(w_n(t),\phi(t),G(t))=u(c(w_n(t),\phi(t),G(t)),\bar{l}-l(w_n(t),\phi(t),G(t)),G(t))\ . \tag{6}$$

Equations (3), (4), (5) and (6) will be used in the public decision-problem to be described below.

2.2 Firms

[5] Note that the current value Hamiltonian implied by the consumer’s decision-problem can be written as (if the time-indicator is suppressed)

$$\Im=u(c,z,G)+\phi\dot{a}\ .$$

The production side is characterized by identical competitive firms producing a homogenous good under constant returns to scale. Given these assumptions, the number of firms is, itself, not important for the analysis and will be normalized to one. Output is produced by labor and capital. The production function is given by $f(l(t),k(t))$, and the firm obeys the following first order conditions at each instant;

$$f_l(l,k) - w = 0 \tag{7}$$

$$f_k(l,k) - r = 0\,. \tag{8}$$

2.3 The Government

As mentioned above, the public good provided by the government is a state variable, and the accumulation of the public good will be described by the differential equation

$$\dot{G}(t) = g(t) - \delta G(t) \tag{9}$$

where $g(t)$ is the contribution to the public good at time *t* and δ the rate of depreciation.

Turning to the government budget constraint, note that the decision-variales facing the government are the income tax and the contribution to the public good at each instant. The stock of government bonds develops according to

$$\dot{b}(t) = r(t)b(t) + \rho g(t) - [w(t) - w_n(t)]l(t) \tag{10}$$

in which ρ is a fixed unit cost, measured in terms of private consumption, of providing the public good (i.e. the marginal rate of transformation between the public good and the private consumption good). Normally, one would normalize this cost to one; however, I will denote it by ρ here in order to keep track of its role in the welfare measures to be derived below. If we were to integrate equation (10) subject to an NPG condition implying that the present value of public wealth (i.e. the negative of b) is nonnegative at the terminal point, we obtain the intertemporal budget constraint of the government.

By combining equations (2), (10) and the zero profit condition, i.e. $f(l,k) - wl - rk = 0$, we can derive the resource constraint

$$\dot{k}(t) = f(l(t), k(t)) - c(t) - \rho g(t) \qquad (11)$$

meaning that output is used for private consumption, private net investments and contributions to the public good (i.e. public investments). As the present paper focuses on issues other than depreciation of the physical capital stock, I abstract from such depreciation here. This simplification is not important for the qualitative results presented below.

The decision-problem facing the government will be to choose the tax rate (or net wage rate) and contribution to the public good at each instant in order to maximize the present value of future utility facing the representative consumer, i.e.

$$\underset{w_n(t), g(t)}{Max} \int_0^{\infty} v(w_n(t), \phi(t), G(t)) e^{-\theta t} dt \,,$$

subject to the state-equations (5), (9), (10) and (11) as well as subject to the static first order conditions characterizing the private sector, i.e. equations (3), (4), (7) and (8). The reason why equation (5) appears as a state-equation constraint in the government's decision-problem is, of course, that the equation of motion for the private marginal utility of wealth is part of the necessary conditions for the consumer and, therefore, a constraint that the optimal tax and expenditure policy must fulfill. The resource allocation must also obey initial conditions for k and b as well as an NPG condition for b. As pointed out by Chamley (1985), the government does not face any explicit constraint on the initial marginal utility of wealth, $\phi(0)$.

The present value Hamiltonian associated with the public decision-problem can be written as (neglecting the time-indicator for notational convenience)

$$H_p = v(w_n, \phi, G) e^{-\theta t} + \lambda_p \dot{k} + \mu_p \dot{b} + \nu_p \dot{\phi} + \psi_p \dot{G} \qquad (12)$$

in which the subindex "*p*" attached to the Hamiltonian and the costate variables denotes 'present value'. One may, therefore, interpret λ_p, μ_p, ν_p and ψ_p, respectively, as the present value shadow price that the government attaches to the relevant state-variable (in this case, the stock of physical capital, the stock of government bonds, the private marginal utility of wealth and the public good, respectively). The first order conditions for the control variables are

$$\frac{\partial H_p(t)}{\partial w_n(t)} = 0 \text{ and } \frac{\partial H_p(t)}{\partial g(t)} = 0 \tag{13}$$

while the equations of motion for the present value costate variables become

$$\dot{\lambda}_p(t) = -\frac{\partial H_p(t)}{\partial k(t)},\ \dot{\mu}_p(t) = -\frac{\partial H_p(t)}{\partial b(t)},\ \dot{\nu}_p(t) = -\frac{\partial H_p(t)}{\partial \phi(t)} \text{ and}$$
$$\dot{\psi}_p(t) = -\frac{\partial H_p(t)}{\partial G(t)}. \tag{14}$$

We shall return to the interpretation of some of these conditions below, when the model set out here is used for purposes of welfare measurement.

2.4 Welfare Measurement in the Second Best Economy

In this subsection, I will derive a welfare measure for the second best economy set out above and, in particular, analyze the welfare properties of the Hamiltonian underlying this resource allocation. To do this in the simplest way possible, I just assume that a unique solution to the government's decision-problem exists[6]. Therefore, let

$$\left\{w_n^0(t), g^0(t)\right\}_0^\infty$$

denote the optimal paths for the government's control variables, where the superindex "0" is used to indicate 'second best optimal resource allocation', and let

$$V^0(t) = \int_t^\infty v(w_n^0(s), \phi^0(s), G^0(s))e^{-\theta(s-t)}ds \tag{15}$$

[6] For details regarding properties of the optimal solution, the reader is referred to Chamley (1985).

represent the associated optimal value function for the representative consumer at time *t*. As we are disregarding distributional concern in this section, equation (15) is also interpretable as a measure of social welfare.

Let me begin by establishing a relationship between the optimal value function in equation (15) and the Hamiltonian underlying the government's decision-problem. By totally differentiating the present value Hamiltonian in equation (12) with respect to time and using the first order conditions summarized by equations (13) and (14) gives

$$\frac{dH_p^0(t)}{dt} = -\theta v(w_n^0(t), \phi^0(t), G^0(t))e^{-\theta t}. \tag{16}$$

The intuition behind equation (16) is that the only non-autonomous time dependence of the optimal control problem facing the government originates from the utility discount factor. In other words, the effects of time via control, state and costate variables vanish as a consequence of optimization, which means that equation (16) follows directly from the dynamic Envelope Theorem. Therefore, by solving the differential equation (16) subject to the transversality condition[7] $\lim_{t\to\infty} H_p(t) = 0$ and then transforming the solution to current value (i.e. multiplying by $e^{\theta t}$), we obtain

$$\theta V^0(t) = H^0(t) \tag{17}$$

where (neglecting the time-indicator)

$$H^0 = v(w_n^0, \phi^0, G^0) + \lambda^0 \dot{k}^0 + \mu^0 \dot{b}^0 + \nu^0 \dot{\phi}^0 + \psi^0 \dot{G}^0$$

is the current value Hamiltonian evaluated at the second best optimum. This also means that the costate variables are measured in current value terms, i.e. $\lambda(t) = \lambda_p(t)e^{\theta t}$ and similarly for the other costate variables. Equation (17) was originally derived by Aronsson (1998) in a slightly different model than the one used here and establishes a second best analogue to

[7] This property of a well-behaved optimal control problem was derived by Michel (1982) for the situation where neither the instantaneous objective nor the instantaneous constraint functions depend explicitly on time (as in our case). See also Seierstad and Sydsaeter (1987) for an extension of this result to a situation where the instantaneous objective function and the instantaneous constraint functions may depend explicitly on time.

Weitzman's (1976) first best welfare measure. In summary, we have established the following result;

Proposition 1. *Within the given framework, and if the resource allocation is second best, then the current value Hamiltonian underlying the public decision-problem is proportional to the social welfare function.*

At the same time, although Proposition 1 establishes an analogue to Weitzman's (1976) Hamiltonian-based welfare measure, it is important to observe that the proposition does not mean that the principles for measuring welfare in the first best and second best are equivalent. To see this more clearly, note that the costate variable μ attached to the accumulation of government bonds is a measure of (the negative of) the marginal excess burden of taxation. In a first best resource allocation, μ would be equal to zero (as the first best, in this case, necessitates lump-sum taxation in order to raise the appropriate amount of revenue to finance the contribution to the public good). In addition, as a first best resource allocation would imply that the private marginal utility of wealth, ϕ, and the social value of capital, λ, are identical, one can show[8] that a first best resource allocation would also imply $\nu = 0$. Therefore, we have established the following corollary to Proposition 1;

Corollary 1. *In the special case where* $\mu(t) = \nu(t) = 0$ *and* $\phi(t) = \lambda(t)$ *for all t, in which the resource allocation is first best, the welfare measure in equation (17) reduces to read*

$$\theta V^*(t) = u(c^*(t), z^*(t), G^*(t)) + \lambda^*(t)\dot{k}^*(t) + \psi^*(t)\dot{G}^*(t)$$

where the superindex "" is used to denote 'first best'.*

The welfare measure in Corollary 1 is Weitzman's (1976) welfare measure for the model set out here. As such, it can also be derived from a conventional social planner problem, in which the planner chooses private consumption, hours of work and contribution to the public good in order to maximize the consumer's objective function, i.e. $U(0)$ above, subject to the

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[8] To see this, note that as $\phi(0)$ is free in the public decision-problem, the optimal resource allocation must obey the initial transversality condition $\nu(0) = 0$. Therefore, since $\mu(t) = 0$ and $\phi(t) = \lambda(t)$ for all *t* in the first best, we would also have $\partial H(t)/\partial\phi(t) = 0$ for all *t* in the first best, in which case $\nu(t) = 0$.

equations of motion for k and G (as well as the appropriate initial conditions). Taken together, Proposition 1 and Corollary 1 show how the necessity to collect public revenue by distortionary taxation will affect the welfare measure. This is summarized as follows;

Corollary 2. *Within the given framework, the necessity to raise public revenue by means of distortionary taxes affects the welfare measure mainly by (i) adding the marginal excess burden times the change in the stock of government bonds to the conventional Hamiltonian-based welfare measure and (ii) introducing a discrepancy between the private and social marginal value of capital.*

The first part of Corollary 2 is intuitive; as revenue collection is associated with a cost beyond the reduction in private consumption, this additional cost ought to affect the way in which welfare is measured. If $\mu < 0$, as one would normally expect, building up a stock of government bonds gives rise to a welfare cost in the sense that it necessitates more disortionary taxation in the future. To understand the second part of the corollary, note that the public decision-problem is formulated in terms of demand functions and an instantaneous indirect utility function, which are all defined conditional on the private marginal utility of wealth. If evaluated in the first best, $\partial H / \partial \phi = 0$ as the private and social cost benefit rules for private consumption and hours of work, respectively, would coincide, whereas $\partial H / \partial \phi$ is generally nonzero in the second best due to discrepancies between the private and social cost benefit rules. As a consequence, changes in ϕ influences the social welfare via the private decision-variables in the second best, which is why the government (acting as a benevolent social planner) attaches a marginal value (positive or negative) to ϕ.

Let me then turn to the measurement of value of an addition to the public good, i.e. the shadow price (or social value) of the public good in utility terms. To begin with, note from the final part of (14) that the equation of motion for the present value costate variable associated with the public good can be written as[9]

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[9] Note that

$$v_G(w_n^0, \phi^0, G^0) = u_G(c^0, z^0, G^0) + u_c(c^0, z^0, G^0)\frac{\partial c^0}{\partial G}$$
$$- u_z(c^0, z^0, G^0)\frac{\partial l^0}{\partial G}$$

$$\dot{\psi}_p^0(t) = -u_G(c^0(t), z^0(t), G^0(t))e^{-\theta t} - \Delta_p^0(t) + \delta\psi_p^0(t) \tag{18}$$

in which $\Delta_p^0(t)$ summarizes all indirect effects of G on the present value Hamiltonian that arise via the private consumption and hours of work, i.e. via equations (3) and (4). Solving the differential equation (18) subject to the transversality condition $\lim_{t\to\infty}\psi_p^0(t) = 0$ and then transforming the solution to current value (multiplying by $e^{\theta t}$) gives

$$\psi^0(t) = \int_t^\infty [u_G(c^0(s), z^0(s), G^0(s)) + \Delta^0(s)]e^{-(\theta+\delta)(s-t)}ds \tag{19}$$

where $\Delta^0(t) = \Delta_p^0(t)e^{\theta t}$. Note first that $\theta + \delta$ represents the discount rate used to calculate the present value of future marginal benefits of a contribution to the public good at time *t*. The intuition as to why the rate of depreciation of the public good ought to be added to the conventional utility discount rate is, of course, that the greater the depreciation, ceteris paribus, the less will be the effective size of past increments. Equation (19) means that the marginal value of an incremental public good at time *t* is equal to the present value of all future direct marginal benefits that this increment gives rise to for the consumer plus the present value of the indirect effects of the public good via the private consumption and hours of work. An important difference between the way in which the shadow price of the public good is measured in the second best and in the first best is that $\Delta = 0$ in the first best equilibrium. The intuition is that the indirect effects of *G* on the Hamiltonian, which are due to the effects of *G* on the conditional demand system given by equations (3) and (4), vanish in the first best as a consequence of optimization[10]. In the second best, on the other hand, these indirect effects generally remain, meaning that $\Delta \neq 0$.

However, if G is additively separable from the other goods in the utility function in equation (1), meaning that G does not directly influence the conditional demand system given by

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[10] To see this, recall once again that $\mu(t) = \nu(t) = 0$ and $\phi(t) = \lambda(t)$ in the first best, in which case one can show that

$$\Delta = [u_c - \phi]\frac{\partial c}{\partial G} + [-u_z + \phi w]\frac{\partial l}{\partial G} = 0 .$$

equations (3) and (4), the second best principle for measuring the shadow price of an incremental public good coincides with the corresponding principle that applies in the first best. This means that equation (19) reduces to read[11]

$$\psi^0(t) = \int_t^\infty U_G(G^0(s))e^{-(\theta+\delta)(s-t)}ds > 0 \,. \tag{20}$$

To be able to interpret the second best formula for public provision in a simple way, and since the social shadow price in the simplified case represented by equation (20) is unambiguously positive (as is the shadow price in the first best), I will throughout this section assume that $\psi^0(t)$ is unambiguously positive also in the general case represented by equation (19). This means that, if the second term on the right hand side contributes negatively to the shadow price, then this effect is not strong enough to dominate the positive direct effect. In this case, therefore, it is straight forward to interpret equation (19) as measuring the benefit side of an incremental public good.

Now, to go further, let me examine the policy rule for incremental contributions to the public good. Since the public decision-problem analyzed here is a second best problem in which the revenue is raised by distortionary taxation, it is clear that an intertemporal analogue to the standard Samuelson condition[12] - equating the direct marginal benefit facing the consumer derived above with the direct marginal cost – does not in general apply. Part of the intuition behind the failure of the Samuelson condition was outlined elegantly by Pigou (1947)[13], even before the Samuelson condition as we know is was formulated. He wrote;

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[11] Note that if the utility function is additively separable in the public good, then the marginal utility of the public good does not depend on *c* and *G*.

[12] The Samuelson condition is often formulated such that the marginal rate of substitution between the public good and private consumption should be equal to the marginal rate of transformation between the public good and the private consumption good. In the context of the model used here, an intertemporal analogue to the Samuelson condition can be written as

$$\frac{\int_t^\infty u_G(c^0(s), z^0(s), G^0(s))e^{-(\theta+\delta)(s-t)}ds}{\phi^0(t)} = \rho \,.$$

[13] Major early contributions to a more formalized theory of public good provision under linear taxation dates back to the early 1970s; see Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974).

“Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditures ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hand of the representative citizen” (1947, page 34).

The first order condition for the instantaneous contribution to the public good in the present model accords well with this intuition, at least in the special case where the shadow price of the public good is given by equation (20). This is so because with a separable public good, in which case the shadow price of an incremental public good is measured in the same general way as it would have been in the first best, the second best assumption will only modify the way in which the social cost of raising the additional tax revenue ought to be measured. By using the second part of conditions (13), i.e. $\partial H_p(t)/\partial g(t)=0$, we can derive

$$\psi^0(t)=[\lambda^0(t)-\mu^0(t)]\rho \, . \qquad (21)$$

The left hand side of equation (21) is the marginal benefit of an incremental public good, which was defined in equation (19) above, whereas the right hand side represents the marginal cost measured in terms of utility. The latter contains two parts; (i) the direct unit cost of providing the public good, ρ (which is interpretable as the marginal rate of transformation between the public good and the private consumption good), and (ii) the multiplier $\lambda-\mu$ which is interpretable in terms of the marginal cost of public funds. If defined in utility terms, this incorporates both the shadow price of the resource constraint, here represented by λ, and the marginal excess burden, here represented by $-\mu$. If we follow the convention in the literature in defining the marginal cost of public funds in real terms by dividing the multiplier $\lambda-\mu$ by the marginal utility of consumption, $m(t)=[\lambda(t)-\mu(t)]/\phi(t)$, equation (20) may be rewritten to read

$$\psi^0(t)=\phi^0(t)m^0(t)\rho \, . \qquad (22)$$

Therefore, and in accordance with Sandmo (1998), we may alternatively think of the marginal cost of public funds as the multiplier, $m(t)$, to be applied to the direct resource cost of providing the public good, ρ, in order to arrive at the socially relevant shadow price (in real terms) for the public sector. Equations (17) and (22) suggest the following result;

Proposition 2. *Within the given framework, and if the resource allocation is second best, the real accounting price of an addition to the public good at time t can be measured by the direct cost of providing the incremental public good,* ρ, *times the marginal cost of public funds,* $m(t)$.

Proposition 2 closely resembles a result derived by Aronsson (2008); the difference is that he focused on the accounting price of a flow-variable public good, whereas the Proposition above analyzes the accounting price of a state-variable public good. In a second best economy as the one addressed here, one would normally expect that $m(t) > 1$, whereas $m(t)$ would be equal to one in the first best. The latter follows because, in the first best, we have $\mu(t) = 0$ and $\lambda(t) = \phi(t)$ for all *t*. One may then immediately infer the following corollary to Proposition 2;

Corollary 3. *In the special case where the resource allocation is first best, the appropriate real accounting price of an addition to the public good is given by* ρ.

As a consequence, if the first best accounting price were (erroneously) applied in a second best economy, and if the marginal cost of public funds is greater then one, we would underestimate the real accounting price of an addition to the public good.

3. Heterogeneity

The analysis has so far focused on a representative-agent economy. Such a framework is problematic for at least two reasons. First, it completely disregards redistribution, which is arguably one of the most important tasks of the public sector in most real world economies. As a consequence, it is important to address redistribution also in the context of social accounting. Second, there is no apparent reason as to why the public sector in the model set out above should use distortionary taxes, other than that we have assumed that it must do so.

In fact, as we avoided redistribution and informational asymmetries completely in the previous sections, it ought to have been possible for the government of the model-economy to finance the public expenditures by a lump-sum tax. I will return to the issue of tax instruments and informational constraints below by analyzing welfare measurement in a model where ability is private information.

3.1 Briefly on Redistribution and Social Accounting

Very few earlier studies have examined how redistribution ought to be treated in the context of social accounting. Aronsson and Löfgren (1999) analyzed redistribution in the context of a first best economy, in which the government chooses the consumption paths for two individuals (or families) with infinite time-horizons in order to maximize a general social welfare function of the type

$$\int_0^\infty \omega(u^1(c^1(t)), u^2(c^2(t)))e^{-\theta t}dt \qquad (23)$$

subject to the aggregate resource constraint for the economy as a whole. In equation (23), the individual utility functions, $u^i(\cdot)$, have conventional properties (see above), whereas the aggregator function, $\omega(\cdot)$, is assumed to be increasing in the individual instantaneous utilities and concave. Their results show that the current value Hamiltonian associated with this decision-problem, if evaluated in the first best, constitutes an exact welfare measure (in a way analogous to the representative agent models used in earlier work). Furthermore, since the first best resource allocation equalizes the social marginal utility of consumption across agents, the only measure of private consumption that enters the linearized current value Hamiltonian will be the aggregate private consumption (although the consumption level, itself, may differ across consumers). Therefore, an approximation of real comprehensive NNP derived by linearizing the current value Hamiltonian does not depend on the distribution of consumption in the first best. Relaxing the first best assumption, Aronsson and Löfgren (1999) also show that if the actual distribution is not the outcome of an optimal policy choice, then the current value Hamiltonian does no longer constitute an exact welfare indicator. In other words, a suboptimal distribution gives rise to additional terms in the welfare measure in a way similar to market failures; see also Chapter 1 of this volume.

Aronsson (2008) extends the analysis to a second best economy in which the consumer differ with regards to their initial endowments, and where the government raises revenue by means of linear income taxation. The resulting second best resource allocation does not, in general, equalize the social marginal utility of consumption among individuals. A linearization of the current value Hamiltonian then suggests that the information about private consumption that ought to be part of real comprehensive NNP does not only reflect the aggregate private consumption (broadly defined to reflect the instantaneous utility function) as it typically does in the first best; it also depends on the distribution of the relevant aspects of private consumption among individuals. The intuition – which may be understood in terms of equation (23) above - is that if the government is unable to equalize the social marginal utility of consumption among individuals (i.e. unable to fully implement its distributional objectives), then this policy-failure ought to reflect the way in which the welfare effects of private consumption are measured. This is precisely the reason for why information about the distribution of private consumption becomes part of the linearized current value Hamiltonian in that case.

In the next subsection, I shall formally analyze the welfare measurement problem and – in particular the role public goods – in a second best economy with heterogeneous consumers. I will also discuss the role of redistribution more thoroughly. However, instead of using a model in which the government is restricted to using linear tax instruments, as in the previous section, I will consider a variant of the two-type optimal income tax model originally developed by Stern (1982) and Stiglitz (1982). To my knowledge, this model has not been used before in the context of social accounting. In such a framework, there are no restrictions on the set of policy instruments that necessitate using distortionary taxes for revenue collection and redistribution; instead, the use of distortionary taxes is, in this case, a consequence of optimization from the perspective of the government subject to the available information constraints. By comparison with the analysis carried out in the previous section, this extension also enables me to exemplify how the social value of an incremental public good depends on the tax instruments that the government has at its disposal.

3.2 A Model with Two Ability-Types and Asymmetric Information

Consider an economy with two types of consumers; a low-ability type (denoted by superindex 1) and a high-ability type (denoted by superindex 2). This distinction refers to productivity, meaning that the high-ability type faces a higher before tax wage rate than the low-ability type. The number of agents of each ability-type will be assumed to be fixed. Therefore, without any loss of generality, we normalize the number of agents of each ability-type to one at each point in time.

Consumers and firms

The consumers and firms behave in the same way as in Section 2. Ability-type i chooses his/her consumption of the private good, c^i, and hours of work, l^i, at each instant to maximize the present value of future utility,

$$U^i(0) = \int_0^\infty u(c^i(t), z^i(t), G(t)) e^{-\theta t} dt ,$$

subject to the asset accumulation equation

$$\dot{a}^i(t) = r(t)a^i(t) - \Phi(r(t)a^i(t)) + w^i(t)l^i(t) - T(w^i(t)l^i(t)) - c^i(t)$$

as well as subject to an initial condition and an NPG condition for the asset. In the asset accumulation equation, $T(\cdot)$ refers to the labor income tax and $\Phi(\cdot)$ to the capital income tax: both labor income and capital income are taxed according to nonlinear functions. The first order conditions include (neglecting the time indicator)

$$u_c(c^i, z^i, G) - \phi^i = 0 \tag{24}$$

$$-u_z(c^i, z^i, G) + \phi^i w^i [1 - T'(w^i l^i)] = 0 \tag{25}$$

$$\dot{\phi}^i - \theta\phi^i = -\phi^i r[1 - \Phi'(ra^i)]. \tag{26}$$

Turning to the production sector, I will also here assume identical competitive firms producing a homogenous good under constant returns to scale, and I will normalize the number of firms to one. The production function is written $f(l^1(t), l^2(t), k(t))$, and the first order conditions characterizing the firm become (again neglecting the time indicator)

$$f_{l^i}(l^1, l^2, k) - w^i = 0 \text{ for } i=1,2, \text{ and } f_k(l^1, l^2, k) - r = 0 . \tag{27}$$

The Government

To avoid complications not essential for the results to be derived below, the social objective function will be assumed to be a Utilitarian welfare function, i.e.

$$U(0) = \sum_i \int_0^\infty u(c^i(t), z^i(t), G) e^{-\theta t} dt . \tag{28}$$

As in other literature on the self-selection approach to optimal taxation, I assume that the government can observe income, although ability is private information. In addition, and also by analogy to earlier comparable literature, I assume that the government wants to redistribute from the high-ability to the low-ability type. Therefore, one would like to prevent the high-ability type from mimicking the low-ability type in order to gain from redistribution. This can be accomplished by imposing a self-selection constraint, which means that the high-ability type must (weakly) prefer the allocation intended for him/her over the allocation intended for the low-ability type. To formalize this aspect of the public decision-problem in the simplest possible way, and to avoid that each agent reveals his/her true type at the outset of infinite planning period (after which the government would be able to implement nondistortionary lump-sum taxation), I reinterpret each agent-type in terms of a continuum of perfectly altruistic consumers and assume that the ability of the two consumers living at the same time cannot be identified ex-ante by the government.[14]

As a consequence, the self-selection constraint that may bind at time t becomes

$$u^2(t) = u(c^2(t), z^2(t), G(t)) \geq u(c^1(t), \bar{l} - \varpi(t) l^1(t), G(t)) = \hat{u}^2(t) \tag{29}$$

[14] This simplification enables me to formulate the public decision-problem in a way similar to earlier literature on the self-selection approach to optimal income taxation in dynamic models; see, e.g., Brett (1997), Pirttilä and Tuomala (2001) and Aronsson et al. (2009).

where $\varpi = w^1 / w^2$ is the wage ratio (relative wage rate), implying that ϖl^1 is the hours of work that the mimicker must supply in order to reach the same income as the low-ability type. By using the first order conditions for the firm, it follows that the wage ratio can be written as a function of the hours of work by each ability-type and the capital stock, i.e. $\varpi = \varpi(l^1, l^2, k)$. The expression on the right hand side of the weak inequality is the utility of the mimicker, which is denoted by the hat. The mimicker enjoys the same consumption as the low-ability type, although the mimicker enjoys more leisure (as the mimicker is more productive than the low-ability type).

The accumulation equation for the public good and the resource constraint take the same forms as in the model discussed in Section 2, i.e.

$$\dot{G}(t) = g(t) - \delta G(t) \tag{30}$$

$$\dot{k}(t) = f(l^1(t), l^2(t), k(t)) - \sum_i c^i(t) - \rho g(t). \tag{31}$$

Before proceeding with the public decision-problem, two things are worth noticing. First, and by analogy to other literature on the self-selection approach to optimal labor income taxation, the labor income tax, $T(\cdot)$, can be used to implement any desired combination of work hours and private consumption for each ability-type. The reader may, in this case, think about a labor income tax function with ability-type specific slopes and intercepts. Therefore, it is more convenient to use l^1, c^1, l^2 and c^2 as direct decision-variables for the government than controlling them indirectly via the parameters of the function $T(\cdot)$. Second, as the government may transfer resources lump-sum over time via the tax system (recall that the general income tax functions have lump-sum components), we only need to consider one from the resource constraint and the government's budget constraint[15]. As I have chosen to use the resource constraint, this means that the costate variable associated with the resource constraint is interpretable in terms of the marginal cost of public funds. Therefore, and by contrast to the analysis carried out in Section 2, the second best problem is here formulated as a 'command optimum' problem, in which the social planner (or benevolent government) chooses quantities instead of tax rates. By comparing the first order conditions of this second best problem with

[15] See Atkinson and Sandmo (1980) and Pirttilä and Tuomala (2001).

those characterizing the private sector (see above), one can derive the marginal income tax structure that ought to be used in order to implement the second best resource allocation.

The second best resource allocation is derived by choosing l^1, c^1, l^2, c^2 and g at each instant to maximize the social welfare function in equation (28) subject to equations (29), (30) and (31) as well as subject to initial and terminal conditions. The present value Hamiltonian is given by (where the time indicator is suppressed)

$$H_p = \sum_i u(c^i, z^i, G)e^{-\theta t} + \lambda_p \dot{k} + \psi_p \dot{G}\,, \tag{32}$$

and the present value Lagrangean becomes $L_p = H_p + \eta_p[u^2 - \hat{u}^2]$. Notice that although the Lagrangean contains the self-selection constraint, which the second best resource allocation must obey (according to the assumptions made above), the Hamiltonian takes the same general form as its first best counterpart. I will return to this discussion below, when the appropriate procedure for measuring welfare is analyzed. The first order conditions include (where the time indicator has been suppressed once again)

$$\frac{\partial L_p}{\partial l^1} = -u_z^1 e^{-\theta t} + \eta_p \hat{u}_z^2[\varpi + \frac{\partial \varpi}{\partial l^1} l^1] + \lambda_p w^1 = 0 \tag{33}$$

$$\frac{\partial L_p}{\partial c^1} = u_c^1 e^{-\theta t} - \eta_p \hat{u}_c^2 - \lambda_p = 0 \tag{34}$$

$$\frac{\partial L_p}{\partial l^2} = -u_z^2 e^{-\theta t} + \eta_p[-u_z^2 + \hat{u}_z^2 \frac{\partial \varpi}{\partial l^2} l^1] + \lambda_p w^2 = 0 \tag{35}$$

$$\frac{\partial L_p}{\partial c^2} = u_c^2 e^{-\theta t} + \eta_p u_c^2 - \lambda_p = 0 \tag{36}$$

$$\frac{\partial L_p}{\partial g} = \psi_p - \lambda_p \rho = 0 \tag{37}$$

$$\dot{\lambda}_p = -\frac{\partial L_p}{\partial k} = -\lambda_p r - \eta_p \hat{u}_z^2 \frac{\partial \varpi}{\partial k} l^1 \tag{38}$$

$$\dot{\psi}_p = -\frac{\partial L_p}{\partial G} = -[u_G^1 + u_G^2]e^{-\theta t} - \eta_p[u_G^2 - \hat{u}_G^2] + \psi_p \delta \tag{39}$$

in which $u^i = u(c^i, z^i, G)$, and subindices attached to the instantaneous utility and production functions denote partial derivatives. Note also that $w^i = f_{l^i}(l^1, l^2, k)$ for i=1,2, and $r = f_k(l^1, l^2, k)$.

Welfare Measurement and Public Good Provision

The implications for optimal taxation (in a dynamic model) of a binding self-selection constraint have been discussed in other studies (e.g. Pirttilä and Tuomala 2001), and it would be beyond the purpose of this study to address implementation here. I will, instead, analyze the model from the point of view of welfare measurement. As in Section 2, the superindex "0" is used below to indicate the second best resource allocation.

As the government faces a Utilitarian objective function by assumption, the optimal value function – the welfare function we are about to analyze – may be written as

$$V^0(t) = \sum_i V^{i,0}(t) = \int_t^\infty \sum_i u(c^{i,0}(s), z^{i,0}(s), G^0(s)) e^{-\theta(s-t)} ds \, . \tag{40}$$

Now, by totally differentiating the Lagrangean with respect to time and using the first order conditions given by equations (33)-(39), we obtain

$$\frac{dL_p^0(t)}{dt} = -\theta \sum_i u(c^{i,0}(t), z^{i,0}(t), G^0(t)) e^{-\theta t} \, . \tag{41}$$

Solving the differential equation (41) subject to the transversality condition $\lim_{t\to\infty} L_p(t) = 0$, using that $L_p^0(t) = H_p^0(t)$ at the optimum and, finally, transforming the solution to current value (i.e. multiplying by $e^{\theta t}$), the welfare measure can be written in the now familiar way

$$\theta V^0(t) = H^0(t) \, . \tag{42}$$

Equation (42) is analogous to equation (17) and may be interpreted as follows;

Proposition 3. *In a second best economy with heterogeneous consumers and a binding self-selection constraint, the current value Hamiltonian underlying the public decision-problem is proportional to the social welfare function.*

It is interesting to compare Proposition 3 with Aronsson and Löfgren (1999), who derived a similar result in the context of a first best resource allocation: in an economy with heterogeneous consumers, they also found that the present value of social welfare is proportional to the current value Hamiltonian facing the social planner. Therefore, the basic intuition behind equation (42) is, of course, the same as that offered by them; the underlying decision-problem is time-autonomous (except for the direct dependence of time through the utility discount factor). At the same time, it is important to emphasize that equation (42) is not a first best welfare measure. This will become evident below, where equation (42) is used to address how the marginal value of the public good ought to be measured.

To simplify the notations, let $MRS^i_{G,c}(t) = u^i_G(t)/u^i_c(t)$ and $M\hat{R}S^2_{G,c}(t) = \hat{u}^2_G(t)/\hat{u}^2_c(t)$ denote the marginal rate of substitution between the public good and private consumption for ability-type *i* and the mimicker, respectively, at time *t*. One can then derive the following result with respect to the public good[16];

Proposition 4. *If the income tax is optimally chosen, the marginal utility value of an incremental public good at time t can be written as*

$$\psi^0(t) = \int_t^\infty \{\lambda^0(s)\sum_i MRS^{i,0}_{G,c}(s) + \eta^0(s)\hat{u}^{2,0}_c(s)[MRS^{1,0}_{G,c}(s) - M\hat{R}S^{2,0}_{G,c}(s)]\}e^{-(\theta+\delta)(s-t)}ds\,.$$

Proof: see the Appendix.

By analogy to the corresponding shadow price derived in Section 2, note first that the appropriate rate to discount the future adjusted marginal benefits of an increment to the public good at time *t* - where the adjustment refers to the effects via the self-selection constraint - is

[16] Public good provision in economies with nonlinear taxation has been addressed by, e.g., Christiansen (1981) and Boadway and Keen (1993). See also Pirtillä and Tuomala (2001) for an extension to a dynamic economy, in which the public good is a state variable.

given by the sum of the utility discount rate and the rate of depreciation of the public good. The first term on the right hand side of the formula in Proposition 4 reflects the sum of marginal rates of substitution between the public good and private consumption, i.e. the sum of the marginal willingness to pay by the consumers. Note that this sum is not only measured over consumers living at the same time; it is measured over time as well, since an increment to the public good at time *t*, ceteris paribus, will affect all future $G(s)$ for $s \in (t, \infty)$ and, therefore, also the future instantaneous utilities.

The second term on the right hand side is due to the self-selection constraint: it shows that a binding self-selection constraint at any future time affects the value that the government attaches to an increase in the contribution to the public good at time *t*. The intuition is that the public good constitutes an instrument by which the government can relax the self-selection constraint which, in turn, allows for more redistribution. How the government ought to modify its use of public provision due to the self-selection constraint depends, in turn, on how the marginal willingness to pay for the public good is related to the leisure choice[17]. If leisure is substitutable for the public good in the sense that the low-ability type is willing to pay more at the margin for the public good than the mimicker, i.e. $MRS^1_{G,c} > M\hat{R}S^2_{G,c}$, increased public provision leads to a greater utility gain for the low-ability type than it does for the mimicker. In this case, therefore, the government may relax the self-selection constraint by increasing the contribution to the public good at time *t*, which means that the government attaches a higher marginal value to the public good than it would otherwise have done. If, on the other hand, leisure is complementary with the public good in the sense that the mimicker is willing to pay more at the margin for the public good than the low-ability type, so $MRS^1_{G,c} < M\hat{R}S^2_{G,c}$, the government may, instead, relax the self-selection constraint by lowering its contribution to the public good, in which case it attaches a lower marginal value to the public good than it would otherwise have done. Finally, in the special case where leisure is weakly separable from the other goods in the utility function – which with the formulation set out above means that $MRS^1_{G,c} = M\hat{R}S^2_{G,c}$ for all *t* - the self-selection constraint would have no direct effect on the marginal value that the government attaches to the public good[18].

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[17] Recall that the only difference between the low-ability type and the mimicker is that the mimicker is more productive, and consumes more leisure, than the low-ability type.

[18] Note that, even if we were to add the assumption that leisure is weakly separable from the other goods in the utility function, a binding self-selection constraint will, nevertheless, indirectly affect the marginal value that the

However, although the marginal social value of the public good derived in Proposition 4 is measured in a different way than in the representative agent models analyzed in Section 2, the qualitative content of Proposition 2 applies here as well. This is seen from the first order condition for the contribution to the public good, i.e. equation (37), which implies $\psi^0(t) = \lambda^0(t)\rho$ for all *t*. In other words, the marginal social value of the public good reflects the product of the direct marginal cost of providing the public good and the marginal cost of public funds (here measured in utility terms). The difference between the model set out in this section and the model in Section 2 is, instead, that the marginal cost of public funds here reflects that revenue collection affects the self-selection constraint; not the necessity per se to use distortionary taxation as in Section 2. Note also that, in the special case when the self-selection constraint does not bind (in any period), we obtain the first best resource allocation, where $\phi^1 = \phi^2 = \lambda$, implying that the marginal social value of the public good can be elicited by using an intertemporal analogue to the conventional Samuelson condition, i.e. $\int_t^\infty \sum_i \phi^0(s) MRS_{G,c}^{i,0}(s) e^{-(\theta+\delta)(s-t)} ds = \phi^0(t)\rho$.

Briefly on Redistribution and Distributional Objectives

The distribution of resources between the consumers is implicit in the welfare measure given by equation (42). Despite that the underlying resource allocation is optimal from the perspective of society, note that the government is not able to fully implement its distributional objectives. With the objective function set out above, the (benevolent) government would have liked to equalize the marginal utility of consumption between the two ability-types at each instant; however, concern for the self-selection constraint prevents it from doing so. An interesting question is whether this inability ought to affect the way in which the comprehensive NNP is measured. Following much of the earlier literature on social accounting[19], I will here define the comprehensive NNP by using the linearized current value Hamiltonian, and then use that the current value Hamiltonian can be written as the sum of the linearized current value Hamiltonian and the consumer surplus.

government attaches to the public good, as a binding self-selection constraint means that the marginal utility of consumption differs between the consumers as well as differs from the shadow price of capital.

[19] An overview the literature is given by Aronsson et al. (2004).

Let $w_n^i = w^i(1-T'(w^i l^i))$ denote the marginal wage rate of ability-type *i*, and denote the 'exchange value of leisure' by $q^i = w_n^i z^i$. In addition, and to simplify the notations even further, let ϕ_m, c_m and q_m denote the average marginal utility of consumption, the average consumption and the average exchange value of leisure, respectively. Then, since $E[\phi c] = \phi_m c_m + \text{cov}(\phi, c)$ and $E[\phi q] = \phi_m q_m + \text{cov}(\phi, q)$, where $E[\cdot]$ denotes the mean operator, and if we define the consumer surplus[20] as $s = \sum_i [u(c^i, z^i, G) - u_c^i c^i - u_z^i z^i - u_G^i G]$, equation (42) may be rewritten as follows (suppressing the time indicator);

$$\theta V^0 = \phi_m^0 [C^0(1+\zeta_c^0) + Q^0(1+\zeta_q^0) + \sum_i \alpha^{i,0} MRS_{G,c}^{i,0} G^0 + \bar{m}^0 \dot{k}^0 + \bar{m}^0 \rho \dot{G}^0] \\ + s^0 \qquad (43)$$

in which I have used the short notations $\alpha^i = \phi^i / \phi_m$ and $\bar{m} = \lambda / \phi_m$, and where $C = \sum_i c^i$, $Q = \sum_i q^i$, $\zeta_c = \text{cov}(\phi, c)/(\phi_m c_m)$ and $\zeta_q = \text{cov}(\phi, q)/(\phi_m q_m)$. The first line on the right hand side of equation (43) is the linearized current value Hamiltonian, which is interpretable as the comprehensive NNP times the average marginal utility of consumption. The variables ζ_c and ζ_q are distributional characteristics, and reflect the correlations between, on the one hand, the marginal utility of consumption and, on the other, the private consumption and exchange value of leisure, respectively. These distributional characteristics would be absent in the first best, in which case equation (43) reduces to read (as the first best implies $\phi^1 = \phi^2 = \phi = \lambda$ and $\bar{m} = 1$)

$$\theta V^* = \phi^* [C^* + \sum_i w^{i,*} l^{i,*} + \sum_i MRS_{G,c}^{i,*} G^* + \dot{k}^* + \rho \dot{G}^*] + s^*. \qquad (44)$$

Note that the comprehensive NNP on the right hand side of equation (44) only comprises aggregate variables. The essence is that equation (44) is derived in a situation where the marginal utility of consumption has become equalized across agents (meaning that $\zeta_c = \zeta_q = 0$), whereas equation (43) is based on a second best resource allocation with a

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[20] For a thorough analysis of the role of the consumer surplus in the context of welfare measurement, see Li and Löfgren (2002).

binding self-selection constraint, in which case the government is not able to equalize the marginal utility of consumption.

A comparison between equations (43) and (44) suggests that, if the government is able to fully implement its distributional objectives – and given the form of the social welfare function assumed here – only the aggregate value of each private decision-variable (in this case the private consumption and leisure) ought to be part of comprehensive NNP. However, if the government is not able to fully implement its distributional objectives, then this policy failure ought to be recognized, which is why the 'accounting prices' for aggregate private consumption and the aggregate exchange value of leisure reflect distributional characteristics in the second best resource allocation.

4. Conclusion

This chapter has analyzed some aspects of welfare measurement in a second best economy, where the public revenue is raised by distortionary taxation, and much attention has been paid to the treatment of a state-variable public good. I would like to emphasize two general conclusions from the analysis carried out above. First, a second best analogue to Weitzman's (1976) Hamiltonian-based welfare measure contains information about the welfare cost of raising revenue. Second, the shadow price associated with an addition to the public good (i.e. the marginal benefit of an incremental public good) is, to some extent, model specific, and depends on the exact nature of the social decision-problem. This was exemplified by using two different models, i.e. an intertemporal representative agent model with linear taxation (where indirect effects of the public good affects the value attached to it by the government) and an intertemporal variant of the two-type optimal income tax model (in which the desire to relax the self-selection constraint affects the marginal value of the public good). In either case, however, an optimal choice from the perspective of the government implies that this shadow price is equal to the product of the direct cost of providing the public good and the marginal cost of public funds. In a sense, therefore, this product provides a static equivalent to the forward looking shadow price of a state-variable public good.

It is not difficult to imagine several possible, and very interesting, directions for future research. One would be to extend the analysis to a global economy, in which individual countries engage in tax competition for mobile capital. Such an extension appears well motivated due to the increased openness (and capital mobility) that has been observed during the latest 20 or 30 years. In addition, since many services typically provided by the public sector in real world economies have the character of private goods, another extension would be to analyze the role of public provision of private goods. Earlier research shows that such provision might be an important means of redistribution, which makes it relevant to address also in the context of welfare measurement. Finally, issues associated with the measurement of sustainable development have so far primarily been analyzed in an 'Environmental Economics context', which typically abstracts from other aspects of public policy. Yet another possible direction for future research is, therefore, to address the measurement of sustainability in an economy, where the public finance aspects addressed in this chapter are also considered. I leave these and other extensions for future research.

Appendix

Proof of Proposition 4

Note first that equation (39) can be written as

$$\dot{\psi}_p = -MRS^1_{G,c} u^1_c e^{-\theta t} - MRS^2_{G,c} u^2_c [e^{-\theta t} + \eta_p] + \eta_p \hat{u}^2_c M\hat{R}S^2_{G,c} + \psi_p \delta . \tag{A1}$$

By using equations (34) and (36) to obtain $u^1_c e^{-\theta t} = \eta_p \hat{u}^2_c + \lambda_p$ and $u^2_c [e^{-\theta t} + \eta_p] = \lambda_p$, substituting into equation (A1) and rearranging gives

$$\dot{\psi}_p = -\lambda_p \sum_i MRS^i_{G,c} - \eta_p \hat{u}^2_c [MRS^1_{G,c} - M\hat{R}S^2_{G,c}] + \psi_p \delta . \tag{A2}$$

Solving equation (A2) subject to the transversality condition $\lim_{t\to\infty} \psi_p(t) = 0$ and then transforming the solution to current value gives the expression in Proposition 4. ■

References

Aronsson, T. (1998) Welfare Measurement, Green Accounting and Distortionary Taxes. *Journal of Public Economics* **70**, 273-295.

Aronsson, T. (2008) Social Accounting and the Public Sector. *International Economic Review* **49**, 349-375.

Aronsson, T. and Löfgren, K-G. (1999) Welfare Equivalent NNP under Distributional Objectives. *Economics Letters* **63**, 239-243.

Aronsson, T., Löfgren, K-G. and Backlund, K. (2004) *Welfare Measurement in Imperfect Markets: A Growth Theoretical Approach*. Edward Elgar Publishing Limited.

Aronsson, T., Sjögren, T. and Dalin, T. (2009) Optimal Taxation and Redistribution in an OLG Model with Unemployment. *International Tax and Public Finance* **16**, 198-218.

Atkinson, A. and Stern, N. (1974) Pigou, Taxation and Public Goods. *Review of Economic Studies* **41**, 119-128.

Boadway, R., and M.J. Keen (1993) Public Goods, Self-Selection and Optimal Income Taxation. *International Economic Review* **34**, 463-478.

Brett, C. (1997) A Note on Nonlinear Taxation in an Overlapping Generations Model. Mimeo.

Chamely, C. (1985) Efficient Taxation in a Stylized Model of Intertemporal General Equilibrium. *International Economic Review* **26**, 451-468.

Chamely, C. (1986) Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica* **54**, 607-622.

Christiansen, V. (1981) Evaluation of Projects under Optimal Taxation. *Review of Economic Studies* **48**, 447-457.

Diamond, P. and Mirrlees, J. (1971) Optimal Taxation and Public Production: Tax Rules. *American Economic Review* **61**, 261-278.

Hartwick, J. (1990) Natural Resources, National Accounting and Economic Depreciation. *Journal of Public Economics* **43**, 291-304.

Li, C-Z. Löfgren, K-G. (2002) On the Choice of Metrics in Dynamic Welfare Analysis: Utility versus Money Measures. Umeå Economic Studies no 590.

Michel, P. (1982) On the Transversality Conditions in Infinite Horizon Optimal Control Problems. Econometrica 50, 975-985.

Mirrlees, J. A. (1971) An Exploration into the Theory of Optimum Income Taxation. *Review of Economic Studies* **38**, 175-208.

Pigou, A. C. (1947) *A Study in Public Finance*. MacMillan.

Pirttilä, J. and Tuomala, M. (2001) On Optimal Non-Linear Taxation and Public Good Provision in an Overlapping Generations Economy. *Journal of Public Economics* **79**, 485-501.

Sandmo, A. (1998) Redistribution and the Marginal Cost of Public Funds. *Journal of Public Economics* **70**, 365-382.

Seierstad, A. and Sydsaeter, K. (1987) *Optimal Control Theory with Economic Applications*. Amsterdam: North-Holland.

Stern, N.H. (1982) Optimum Taxation with Errors in Administration. *Journal of Public Economics* **17**, 181-211.

Stiglitz, J. E. (1982) Self-Selection and Pareto Efficient Taxation. *Journal of Public Economics* **17**, 213-240.

Stiglitz, J. and Dasgupta, P. (1971) Differetial Taxation, Public Goods, and Economic Efficiency. *Review of Economic Studies* **38**, 151-174.

Weitzman, M.L. (1976) On the Welfare Significance of National product in a Dynamic Economy. *The Quarterly Journal of Economics* **90**, 156-162.