On the Choice of Metrics in Dynamic Welfare Analysis: Utility versus Money Measures[1]

by Chuan-Zhong Li * and Karl-Gustaf Löfgren**

**Department of Economics, University of Dalarna, S-781 88 Borlänge, Sweden*

***Department of Economics, Umeå University, S-901 87 Umeå Sweden*

Abstract

This paper is concerned with the choice of metrics for social cost-benefit analysis and dynamic welfare comparisons. In a utility-theoretic framework, we show that there is always a money measure that can serve as a substitute for the maximized utility wealth. Thus, under the non-arbitrage course of discount rate, the choice between utility and money measures has no real effect on project evaluations. We also define a generalized comprehensive net national product measure with a consumer surplus term incorporated, which is completely consistent with the Weitzman foundation. It is shown that while a green (comprehensive) NNP growth simply reflects the income effect, the change in consumer surplus captures the welfare effect of relative price changes. We argue that the reason for green NNP to be a weak welfare indicator is not due to its choice of money metric per se but the ignorance of a consumer surplus term.

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1. Background

In recent years, we have witnessed a growing awareness of the interactions between social, economic and environmental issues. Social cost-benefit analysis has become the standard tool for project evaluations on which to base more informed public decisions. Efforts have also been made to construct the so-called “green net national product” as a welfare indicator of sustainable development where environmental pollution and natural resource depletion are taken into account. Conceptually, underlying these practices is optimal growth theory in which welfare is typically expressed in utility terms. However, since utility is simply a theoretical construct with no objective measurement unit, a money metric is used as measure in empirical applications.

Although a successful body of theory has been built up on these topics, it seems that several issues still remain unresolved. To start with, we show how a cost-benefit rule based on the net present value criterion can be justified according to the discounted utilitarian theory through the use of non-arbitrage condition between the utility and money discount rate. Next, we deal with the issue of whether or not money-metric green net national product can serve as a satisfactory indicator of social welfare. Since utility is in general a non-linear function of consumption, it is far from obvious that the properties of welfare analysis based on utility carry over automatically to money NNP. In this paper, we will take a fresh look at these issues using a growth theoretical framework with heterogeneous goods and services.

For social cost-benefit analysis of investment projects, the criterion function is usually the present discounted value of future profits in monetary terms[2] (Little and Mirrlees, 1974; Lind, 1982). Consider a development project involving mining in a wilderness area. If the present value of all future net benefits from production and environmental services is positive, as calculated with the discount rate in a perfect capital market, then the project can be regarded as socially desirable. Similarly, a tract of Swedish virgin forest should be preserved if the present value of its environmental services outperforms the forgone income from timber

[2] This is typically referred to as Fisher’s separation Theorem, which shows that under a perfect capital market, the present value of the project is an objective investment criterion, i. e. any project with a positive present value can be recommended independently of the preferences of the investor. See e.g. Johansson and Löfgren (1984), chapter 1.

harvests[3]. In this paper we take a fresh look at these questions. In particular, we show how a non-arbitrage course of discount rate contributes to this result, and to related results on welfare measurement in an economy where the capital market is perfect in the sense that one can lend and borrow at the same interest rate. Drawing on some recent ideas in Weitzman (2001), we stress the role of the non-arbitrage condition between the utility and money discount rates, that holds, along an optimal path as a key to the equivalence between utility and money measures.

The recent literature on dynamic welfare comparisons seems to focus on two separate but interrelated branches of research. One is to "green up" the national account system by incorporating the value of non-market goods and services such as natural resource stocks and environmental amenities. The idea is that such an augmented NNP would serve both as a better indicator of the overall macroeconomic performance and as a better measure of social welfare[4]. However, the story does not end there. The more intriguing strand of research, at least from a theoretical point of view, is the welfare significance of comprehensive net national product. Here it is assumed that all goods and services, including the non-market ones such as air and water quality, biodiversity, and even technical knowledge, are perfectly accounted for. In such an idealistic setting, one can focus sharply on the core theory of concern. We will consider the welfare significance of the comprehensive net national product with respect to two aspects. First, as a stationary equivalent measure of future income, does a larger (real) NNP at a given point in time imply a higher level of welfare? Second, does a NNP-growth over time indicate welfare improvement?

While the answer to the first question is affirmative, the answer to the second one is not so clear-cut. In addition to the complexity of the problem itself, there have been several controversies over the definition and interpretation of the NNP concept under first best conditions (Dasgupta and Mäler, 2000; Weitzman, 2001; Asheim and Weitzman, 2001). This paper attempts to reconcile some of these issues and to shed light on the welfare significance of net national product. By defining a new concept of the generalized comprehensive net national product (GCNNP), we examine the implications of exogenous NNP change and NNP growth over time as two special cases. Among other things, we show that the choice of metrics, either in utility or monetary terms, has no real effects on welfare measurement, provided that the money measure is complete and the correct discount rate is used. When NNP growth does not indicate a welfare improvement, we argue that it is due to the endogenous change in accounting prices rather than the choice of the metrics *per se*.

The remainder of the paper is structured as follows. Section 2 gives a brief description of the optimal multi-sector growth model used by Weitzman (1976, 2001, 2002). In section 3, we study the correspondence between maximized welfare and maximized money wealth, and establish a general dynamic cost-benefit rule for project evaluations. In section 4, we take a fresh look at the welfare significance of a static money NNP and NNP growth by means of a new concept of "generalized comprehensive net national product". Section 5 summarizes the findings of our study.

2. The Optimal Multi-Sector Growth Model

Let $\mathbf{C} = (C_1, C_2, ..., C_m)$ be a m-dimensional vector of consumption flows at a given time t, which is supposed to exhaust all possible goods and services that are relevant to social welfare or the standard of living of a representative individual. In addition to the usual market commodities, environmental services such as forest amenities, biodiversity and ecosystem functions, in flow terms, are also considered to be a part of the consumption vector. This means that the prices of these services are rental prices. The utilitarian measure of intertemporal welfare at time t can be expressed as

$$W(t) = \int_t^\infty U(\mathbf{C}(s)) \exp(-\theta(s-t)) dt \qquad (1)$$

where $U(\mathbf{C})$ is a given concave, non-decreasing, instantaneous utility function with continuous second order derivative defined for $\mathbf{C} \geq \mathbf{0}$, and θ is the utility rate of discount.

Let $\mathbf{K} = (K_1, K_2, ..., K_n)$ be a vector of capital goods, which is assumed to contain all types of capital goods in the economy including natural resources such as minerals, forests, air, water, and even human capital in the form of technological knowledge. Net investments are, by definition, the change in capital stocks, i.e. $I_i = \dot{K}_i$, $i = 1,2,...,n$, which in a vector form can be expressed as

[4] For a survey see Aronsson et. al. (1997)

$$\mathbf{I} = \dot{\mathbf{K}}, \text{ given } \mathbf{K}(0) = \mathbf{K}_0 > \mathbf{0} \tag{2}$$

At each point in time t, consumption $\mathbf{C}(t)$ and investment $\mathbf{I}(t)$ are allocated within the $m+n$ dimensional production possibility set $S(\mathbf{K}(t);\alpha)$, conditional on a governance parameter α, such that

$$(\mathbf{C}(t), \mathbf{I}(t)) \in S(\mathbf{K}(t);\alpha) \tag{3}$$

which is presumed to be strictly convex. The governance parameter may represent any premise that modifies the feasible set for consumption and investment allocations, such as a given property right regime, a given taxation system, or an inherent public infrastructure.

Conditional on a certain governance parameter, a social planner is assumed to maximize the current-value Hamiltonian at each point in time t

$$H(t) = U(\mathbf{C}(t)) + \mathbf{\Psi}(t)\mathbf{I}(t) \tag{4}$$

with respect to $\{\mathbf{C}(t), \mathbf{I}(t)\}$ subject to (3), where $\mathbf{\Psi}(t)$ is the n-dimensional vector of utility shadow prices of capital satisfying the following equation of motion

$$\dot{\mathbf{\Psi}} = \theta\mathbf{\Psi} - \nabla H_{\mathbf{K}}\Big|_{*(t)} \tag{5}$$

where the notion $\Big|_{*(t)}$ means evaluation along the optimal trajectory at time t. The maximized current-value Hamiltonian at time t is thus

$$H^*(t) \equiv U(\mathbf{C}^*(t)) + \mathbf{\Psi}(t)\mathbf{I}^*(t) \tag{6}$$

which can now be considered as a function of $\mathbf{K}(t)$ and $\mathbf{\Psi}(t)$, since $\mathbf{I}(t)$ and $\mathbf{C}(t)$ are already optimized out. Analogously to the equation of motion for capital in (5), the scalar utility shadow price of money or the marginal utility of income, $\lambda(t)$, satisfies[5]

$$\dot{\lambda}(t) = [\theta - r(t)]\lambda(t) \tag{7}$$

where $r(t)$ is the money interest rate at time t, a profile of which over time consists of a non-arbitrage course of discount rate. By solving the differential equation in (7), we obtain the following lemma, which will serve as a link between utility and money measures in the subsequent analysis.

[5] In other words, it follows an equation similar to the shadow price of real capital.

***Lemma 1**. Along the optimal growth path, the relationship between the marginal utility of income $\lambda(s)$, the utility discount rate θ, and the money interest rate $r(s)$ can be described by the following expression*

$$\lambda(s)\exp(-\theta(s-t)) = \exp(-\int_t^s r(\tau)d\tau) \tag{8}$$

where the initial marginal utility at time t is normalized to be $\lambda(t) = 1$.

Lemma 1, which is a consequence of capital and consumption being efficiently allocated over time, will be used frequently below.

3. Wealth Numeraire in Social Cost-Benefit Analysis

According to the seminal paper by Weitzman (1976), the maximized current-value Hamiltonian $H^*(t)$ can be expressed as the interest on wealth $W^*(t)$ in utility terms, such that

$$H^*(t) = \theta W^*(t) \tag{9}$$

at any time t. Moreover, this Hamiltonian expression can be interpreted as the stationary equivalent of future utility along the optimal growth path. In other words, a hypothetical constant flow of utility $H^*(t)$ from time t onwards would yield exactly the same level of wealth as the actual flows $U(\mathbf{C}^*(s))$ for $s \geq t$, i.e.

$$W^*(t) \equiv \int_t^\infty U(\mathbf{C}^*(s))\exp(-\theta(s-t))ds = \int_t^\infty H^*(t)\exp(-\theta(s-t))ds \tag{10}$$

In the literature, this maximized Hamiltonian has also been termed "net national product (NNP) in utility terms" or simply "utility NNP". From the definition, it follows that that utility NNP measures the annuity equivalent of future utility under the utility discount rate θ. In this sense, an increase in NNP at time t, which corresponds to a proportional increase in utility wealth from equation (9), would indicate a welfare improvement. Intuitively, we can imagine that a larger utility NNP or, equivalently, a higher level of wealth at time t, would enlarge the feasible set for reallocating future utilities $U(\mathbf{C}(s))$ for $s \geq t$. Such an "as if" reallocation implicitly accompanies the additive utilitarian framework represented by equation (1).

Although this theory is illuminating, nobody in everyday life measures income in utility terms, and for the obvious reason that utility is not observable in practice. In Weitzman's (1976) original contribution, he implicitly considered a kind of linear-in-aggregate consumption utility function, so that the maximized Hamiltonian also corresponds to real NNP with (aggregate) consumption as numeraire. However, for more general non-linear utility functional forms, the maximized Hamiltonian retains to be a welfare measure only in utility terms.

In a recent paper, Dasgupta and Mäler (2000) seem to reject the definition of utility NNP by arguing that the Hamiltonian as a welfare measure should not be confused with NNP. It seems that, in their minds, NNP should be strictly measured in monetary units. To provide a link between the utility and the money NNP, Hartwick (1990) and Mäler (1991), among others, have linearized the Hamiltonian to reach a monetary NNP measure. Weitzman (2000, 2002) shows how utility at time *t* can be money metricized to exactly reflect the current value of future welfare. Although this shows the fundamental importance of comprehensive NNP for welfare measurement, the transformation that achieves this will, however, vary over time, effectively making welfare comparisons over time impossible. In this paper, we attempt to establish the exact correspondence between utility NNP and money NNP, and to explore their properties for cost-benefit analysis and welfare comparisons. We start by deriving an intuitive and important relationship between the utility maximizing consumption path and the competitive consumption path[6].

Proposition 1. *If a time path* $\{\mathbf{C}^*(s), \mathbf{I}^*(s), \mathbf{K}^*(s)\}$ *for* $s \geq t$ *solves the dynamic optimization problem (1) - (3), with a maximal welfare* $W^*(t) = \int_t^\infty U(\mathbf{C}^*(s)) \exp(-\theta(s-t))ds$, *then it also maximizes the present value of the stream of future consumption* $\{\mathbf{C}^*(s)\}$, *i.e.*

$$M^*(t) \equiv \int_t^\infty \exp(-\int_t^s r(\tau)d\tau)\mathbf{P}^*(s)\mathbf{C}^*(s)ds \qquad (11)$$

[6] A similar theorem is proved by Heal and Kriström (2002) using a separating hyper-plane argument.

evaluated at efficiency prices $\mathbf{P}^*(s) = \nabla U(\mathbf{C}^*(s))/\lambda(s)$ *and discounted at the money interest rate* $r(s)$ *for* $s \geq t$.

Proof: Following Dixit et al. (1980) and Weitzman (2001), we know that the optimal growth path $\{\mathbf{C}^*(s), \mathbf{I}^*(s), \mathbf{K}^*(s)\}$ for $s \geq t$ satisfies the following competitive conditions (the time argument s for vector variables is omitted for notational ease)

$$\begin{aligned} U(\mathbf{C}) - \lambda(s)\mathbf{P}^*\mathbf{C} &\leq U(\mathbf{C}^*) - \lambda(s)\mathbf{P}^*\mathbf{C}^* \\ \mathbf{P}^*\mathbf{C} + \mathbf{Q}^*\mathbf{I} + \dot{\mathbf{Q}}^*\mathbf{K} - r(s)\mathbf{Q}^*\mathbf{K} &\leq \mathbf{P}^*\mathbf{C}^* + \mathbf{Q}^*\mathbf{I}^* + \dot{\mathbf{Q}}^*\mathbf{K}^* - r(s)\mathbf{Q}^*\mathbf{K}^* \end{aligned} \tag{12}$$

where $\mathbf{P}^* = \nabla U(\mathbf{C}^*)/\lambda$ and $\mathbf{Q}^* = \boldsymbol{\psi}^*/\lambda^*$ are the efficiency prices for consumption and investment, respectively. These conditions can be re-arranged to read

$$U(\mathbf{C}) - U(\mathbf{C}^*) \leq \lambda(s)P^*(\mathbf{C} - \mathbf{C}^*) \leq -\lambda(s)\{\mathbf{Q}^*(\mathbf{I} - \mathbf{I}^*) + (\dot{\mathbf{Q}}^* - r(s)\mathbf{Q}^*)(\mathbf{K} - \mathbf{K}^*)\} \tag{13}$$

evaluated at any point in time *s*. By integrating the discounted present value of the expressions in (13) from time t onward, we obtain

$$\begin{aligned} &\int_t^\infty [U(\mathbf{C}) - U(\mathbf{C}^*)]\exp(-\theta(s-t))ds \\ &\leq \int_t^\infty \mathbf{P}^*(\mathbf{C} - \mathbf{C}^*)\lambda(s)\exp(-\theta(s-t))ds \\ &\leq -\int_t^\infty [\mathbf{Q}^*(\mathbf{I} - \mathbf{I}^*) + (\dot{\mathbf{Q}}^* - r(s)\mathbf{Q}^*)(\mathbf{K} - \mathbf{K}^*)]\lambda(s)\exp(-\theta(s-t))ds \end{aligned} \tag{14}$$

Since

$$d[\lambda(s)\exp(-\theta(s-t))]/dt = -r(s)\exp(-\int_t^s r(\tau)d\tau) \tag{15}$$

according to the fundamental lemma, the integrand in the last integral in (14) can be shown to be an exact differential of $\mathbf{Q}^*(\mathbf{K} - \mathbf{K}^*)\lambda(s)\exp(-\theta(s-t))$ with respect to time s. The right-hand-side of the last inequality thus becomes

$$\begin{aligned} &-\int_t^\infty [\mathbf{Q}^*(\mathbf{I} - \mathbf{I}^*) + (\dot{\mathbf{Q}}^* - r(s)\mathbf{Q}^*)(\mathbf{K} - \mathbf{K}^*)]\lambda(s)\exp(-\theta(s-t))ds \\ &\quad = -\mathbf{Q}^*(\mathbf{K} - \mathbf{K}^*)\lambda(s)\exp(-\theta(s-t))\Big|_{s=t}^{\infty} \leq 0 \end{aligned} \tag{16}$$

in which we have used the initial condition $\mathbf{K}(t) = \mathbf{K}^*(t)$ and the transversality condition $\liminf_{s\to\infty}\{\mathbf{Q}^*(s)(\mathbf{K}(s) - \mathbf{K}^*(s))\lambda(s)\exp(-\theta(s-t))\} \geq 0$. Now combine (15) and (16) to obtain

$$\int_t^{\infty}\{U(\mathbf{C})-U(\mathbf{C}^*)\}\exp(-\theta(s-t))ds \le \int_t^{\infty}\lambda(s)\exp(-\theta(s-t))\mathbf{P}^*(\mathbf{C}-\mathbf{C}^*)ds \le 0 \tag{17}$$

which implies that

$$\int_t^{\infty}U(\mathbf{C})\exp(-\theta(s-t))ds \le \int_t^{\infty}U(\mathbf{C}^*)\exp(-\theta(s-t))ds \tag{18}$$

and

$$\int_t^{\infty}\exp(-\int_t^s r(\tau)d\tau)\mathbf{P}^*(s)\mathbf{C}(s)ds \le \int_t^{\infty}\exp(-\int_t^s r(\tau)d\tau)\mathbf{P}^*(s)\mathbf{C}^*(s)ds \tag{19}$$

by invoking the result stated in Lemma 1, Q.E.D.

The first thing to note from Proposition 1 is that the money wealth measure, $M^*(t)$ in (11), gives exactly the same preference orderings of growth paths as the utility welfare measure, $W^*(t)$, even though they are not defined in the same units of measurement. When one of these measures is maximized with respect to a feasible growth path, we know with certainty that the other measure is also at its maximum when evaluated at the efficiency prices and discounted by the non-arbitrage interest rate. This is an intuitive and rather powerful result in that money wealth under first best conditions that can be used as a theoretical alternative to utility in welfare analysis.

This means that Proposition 1 may also be envisioned as a consequence of the representative agent's lifetime consumption allocation problem, i.e. to maximize the dynamic welfare (1) under the lifetime budget constraint[7] $\int_t^{\infty}\exp(-\int_t^s r(\tau)d\tau)\mathbf{P}^*(s)\mathbf{C}(s)ds = M^*(t)$. This is technically an infinite multi-dimensional static maximization problem with respect to consumption, which can be solved using the standard Lagrangian method. The key result we can extract from such a reformulation is that the marginal effect of the as-if-fixed budget, $M^*(t)$, on the maximized welfare, $W^*(t)$, is simply $\partial W(t)/\partial M(t)\big|_{*(t)} = \lambda(t) > 0$, indicating that the two wealth measures go along with each other.

[7] To derive the intertemporal budget constraint one has to invoke a so called No-Ponzi game condition, which means that the present value of wealth asymptotically will remain non-negative.

The result in Proposition 1 may also help in reconciling the controversial definitions and interpretations of the NNP concept. As shown in equation (9), the maximized current-value Hamiltonian $H^*(t)$, or utility NNP, is proportional to utility wealth $W^*(t)$. Thus, utility NNP itself can serve as a wealth welfare measure. Up to this point, one may wonder whether a similar relationship exists between the money wealth $M^*(t)$ and the corresponding money NNP as defined by $Y^*(t) = \mathbf{P}^*(t)\mathbf{C}^*(t) + \mathbf{Q}^*(t)\mathbf{I}^*(t)$. If this is the case, then the money NNP may also be regarded as a proper welfare measure, since a higher NNP implies a higher money wealth and, in turn, a higher utility wealth. However, it is known today that this is not true, unless the money interest rate $r(t)$ is constant over time t. On this ground, Dasgupta and Mäler (2000) appear to reject the definition of NNP in utility terms. They argue that the maximized current-value Hamiltonian as a welfare measure should not be confused with (money) NNP. From the correspondence between the utility and money wealth measures established in Proposition 1, however, we can still see some hope of using some kind of money NNP as a welfare measure. After all, it is not the money metric itself that is the issue but rather the derived money interest rate that varies over time. Suppose that the money interest rate is constant with $r(s) = r$, then the properties concerning utility NNP would carry over to money NNP such that $Y^*(t) = rM^*(t)$, i.e. the interest income on money wealth[8].

Now, we address dynamic cost-benefit rules with special focus on the effect of a chosen welfare metric. Given that the governance parameter α sets a premise for the dynamic optimization problem, we are concerned about the welfare effect of a change in this parameter. Let us consider a *policy reform* with a small change $\partial\alpha$ in the parameter at time t, which may cause changes in consumption and investment from time t onwards. To fix ideas, one may envision the policy reform as a small public investment project, a minor change in the tax system, or a small reform in the regime of property rights. The aim of a cost-benefit analysis is to evaluate whether or not the resulting change on the stream of consumption and investment values over time is welfare improving.

According to the discounted utilitarian theory, the reform $\partial\alpha$ at time t is socially profitable if it can increase utility wealth, i.e. $\partial W^*(t)/\partial\alpha > 0$. However, since utility welfare is not

[8] Trivially this means that NNP will serve as an welfare indicator in a steady state.

directly observable by the social planner, it will prove useful to find a monetary alternative that can provide the same ranking order in project evaluations (cf Aronsson et al, 2002).

By invoking the Seierstad dynamic envelope theorem (cf. Aronsson et al. 1997, p68), we have[9]

$$\partial W^*(t)/\partial\alpha = \int_t^\infty \partial\hat{H}(\mathbf{C},\mathbf{I};\mathbf{\Psi})/\partial\alpha\Big|_{*(t)} \exp(-\theta(s-t))ds \tag{20}$$

where $\hat{H}(\mathbf{C},\mathbf{I};\mathbf{\Psi})$ denotes the maximized current-value Hamiltonian (6), now with $\mathbf{C}(s)$ and $\mathbf{I}(s)$ as its arguments. Its partial derivative with respect to the policy variable[10], α, evaluated along the optimal trajectory can now be expressed as

$$\partial\hat{H}(\mathbf{C},\mathbf{I};\mathbf{\Psi})/\partial\alpha\Big|_{*(t)} = \nabla U(\mathbf{C})\mathbf{C}_\alpha + \mathbf{\Psi}\mathbf{I}_\alpha = \lambda(s)\left(\mathbf{P}^*\mathbf{C}_a + \mathbf{Q}^*\mathbf{I}_a\right) \tag{21}$$

where $\mathbf{C}_\alpha = \partial\mathbf{C}[\mathbf{K}(\alpha,s),\alpha,s]/\partial\alpha$ and $\mathbf{I}_\alpha = \partial\mathbf{I}[\mathbf{C}(\mathbf{K}(\alpha,s),\alpha,s),\mathbf{K}(\alpha,s),\alpha,s]/\partial\alpha$ represent, respectively, the partial (direct) effects of the reform on consumption and investment at time s. By inserting the expression in (21) back into the integral in (20), we obtain

$$\begin{aligned}\partial W^*(t)/\partial\alpha &= \int_t^\infty \left(\mathbf{P}^*\mathbf{C}_\alpha + \mathbf{Q}^*\mathbf{I}_\alpha\right)\lambda(s)\exp(-\theta(s-t))ds \\ &= \lambda(t)\int_t^\infty \left(\mathbf{P}^*\mathbf{C}_\alpha + \mathbf{Q}^*\mathbf{I}_\alpha\right)\exp(-\int_t^s r(\tau)d\tau)ds \\ &= \lambda(t)\int_t^\infty \partial Y^*(s)/\partial\alpha \exp(-\int_t^s r(\tau)d\tau)ds \\ &= \lambda(t)\partial M^*(t)/\partial\alpha\end{aligned} \tag{22}$$

where the second equality follows from Lemma 1, the third equality is obtained by the definition of money NNP, and the last equality by the definition of money wealth. Since $\lambda(t) > 0$ is an arbitrary scale parameter, we can without loss of generality, normalize it to unity, and propose the following proposition.

Proposition 2. *The effect of a small policy reform, $\partial\alpha$, at time t on the utility wealth $W^*(t)$, is completely captured by the change in the present discounted value of future costs and benefits in money terms such that*

[9] The formal conditions for differentiability, due to Seierstad (1981), are given in Aronsson et al (1997, p68)

[10] Note that all variables are functions of α, but here we assume that α has a direct impact on the consumption and net investment. The indirect effects disappear from envelope properties of the optimal solution.

$$\partial W^*(t)/\partial\alpha = \lambda(t)\int_t^\infty \left(\mathbf{P}^*(s)\mathbf{C}_\alpha(s) + \mathbf{Q}^*(s)\mathbf{I}_\alpha(s)\right)\exp(-\int_t^s r(\tau)d\tau)ds \qquad (23)$$

Equivalently, when the marginal utility of income at time t is normalized to be unity i.e. $\lambda(t) = 1$*, we have*

$$\partial W^*(t)/\partial\alpha = \partial M^*(t)/\partial\alpha \qquad (24)$$

It is worth mentioning that the dynamic cost-benefit rule in (23)-(24) should be understood in its broadest meaning. Since $\mathbf{P}^*$ and $\mathbf{Q}^*$ are the respective *m*- and *n*-dimensional price vectors of consumption and investment goods, the integrand on the right-hand-side of equation (23) allows both trade-offs within and between the two categories of goods. Consider a development project for mining in a wilderness area, and assume, for the moment, that we know the correct prices of all the goods and services, including those for the environmental benefits and other externalities derived from an ideal contingent valuation study. On the consumption side, then, we may imagine a loss of scenic value, and on the capital-investment side, we may observe the value of a positive net investment in physical capital stocks but also negative "investments" in the mineral stock and environmental quality due to resource extraction and pollution. The dynamic cost-benefit rule presented here simply says that if all these gains and losses are aggregated with the ideal accounting prices at each point in time, then the sum of their present discounted values would reveal whether or not the development project is socially profitable. In other words, a generalized version of Fisher's Separation Theorem holds[11]. Once again, we see that the choice of metric between utility and money does not matter as long as the appropriate money interest rate, $r(t)$, is used. In fact, the conclusion about the dynamic cost-benefit rule is stronger than the correspondence between the utility wealth $W^*(t)$ and money wealth $M^*(t)$ as stated in Proposition 1. For the project evaluation case, as stated in Proposition 2, the partial effects $\partial W^*(t)/\partial\alpha$ and $\partial M^*(t)/\partial\alpha$ are exactly equal after normalization! The reason is that at the margin, prices, scaled by the marginal utility of money equals, marginal utility of consumption.

It is worth mentioning that the general dynamic cost-benefit rule derived in Proposition 2 can be easily modified for evaluating a small policy reform the effect of which extends only over

[11] The fundamental reason is that we started from a standard utilitarian framework, and that we assumed that first best principles are valid. We have also implicitly assumed that the intergenerational income distribution is correct in the initial equilibrium. See also Aronsson and Löfgren (1999). For cost benefit rules under externalities, see Johansson and Löfgren (1997) and Aronsson et al (1997).

a finite time period $[t,T]$. In this case, the partial derivative in (21) becomes identically zero for all $s > T$, so the modified version of the dynamic cost-benefit rule becomes

$$\begin{aligned}\partial W^*(t)/\partial\alpha &= \int_t^T \left(\mathbf{P}^*(s)\mathbf{C}_\alpha(s) + \mathbf{Q}^*(s)\mathbf{I}_\alpha(s)\right)\exp(-\int_t^s r(\tau)d\tau)ds \\ &= \int_t^T \left(\mathbf{P}^*(s)\mathbf{C}_\alpha(s) + (r(s)\mathbf{Q}^*(s) - \dot{\mathbf{Q}}^*(s))\mathbf{K}_\alpha(s)\right)\exp(-\int_t^s r(\tau)d\tau)ds \\ &\quad + \mathbf{Q}^*(T)\mathbf{K}_\alpha(T)\exp(-\int_t^T r(\tau)d\tau)\end{aligned} \tag{25}$$

in which we have taken advantage of the fact $\mathbf{I} = \dot{\mathbf{K}}$ and the integration-by-part formula. Note that while the term $\mathbf{P}^*(s)\mathbf{C}_\alpha(s)$ appeared on the second line in (25) reflects the partial effect of the policy reform at time *t* on the aggregated consumption value at time *s,* the term $(r(s)\mathbf{Q}^*(s) - \dot{\mathbf{Q}}^*(s))\mathbf{K}_\alpha(s)$ measures the change in the value of holding capital resulting from the reform. The last expression in (25) captures the effect of the reform on the present value of capital stocks at the end of the project period.

If the effect of a policy reform, $\partial\alpha$, prevails only instantly in the sense that $T - t \to 0$, then we get $\partial W^*(t)/\partial\alpha = \mathbf{P}^*(t)\mathbf{C}_\alpha(t) + \mathbf{Q}^*(t)\mathbf{I}_\alpha(t)$. In this case, the change in money NNP at a given time *t* can be used *on its own* to make social cost-benefit analysis, as suggested by Dasgupta, Kriström and Mäler (1995) and Dasgupta and Mäler (2000).

4. Static NNP, NNP-Growth and Welfare Significance

It has been long known that conventional (i.e. non-comprehensive or "non-green") NNP may not be a good indicator of welfare due to the omission of many components that are relevant to the standard of living, such as environmental benefits and other externalities. Given an incomplete national accounting, there is always a "welfare gap" between what NNP is able to capture and the true welfare level (Turner and Tschirhart, 1999). The recent efforts in green accounting to integrate environmental and natural resource values in an augmented NNP concept (Vincent, 2000; 2001) are a step in the right direction towards reducing the welfare gap[12]. In this paper, we will not deal with the details of green accounting practice as such, but will try to shed light on the welfare implications of NNP in the ideal case where all goods and services that are relevant to human welfare are accounted for. As argued by

Weitzman (2001), a study with a comprehensive national accounts as a departure point enables us to focus sharply on the conceptual issues, which are of our main concern here. We are now about to examine whether the choice between utility and money metrics affects welfare analysis, and whether money NNP can be regarded as a satisfactory measure for welfare and, if not, to suggest the correct monetary welfare measure.

When talking about the welfare significance of comprehensive net national product, it seems necessary to distinguish between two rather different issues. The first is whether static NNP is a satisfactory welfare indicator at a given point in time, and the second is whether NNP growth over time implies a welfare improvement. To deal with these issues, we will use an ingenious observation in Weitzman (2001). Conditional on the market prices along the first best path of the economy, we can represent consumer choice at time *t* as the solution to the following static optimization problem

$$\begin{aligned} &\underset{(\mathbf{C}(t),\mathbf{Z}(t))}{Max}\ H(t) = U\big(\mathbf{C}(t)\big) + \lambda^*(t)\mathbf{Z}(t) \\ &\textit{subject to } \mathbf{P}^*(t)\mathbf{C}(t) + \mathbf{Z}(t) = Y^*(t) \end{aligned} \tag{26}$$

where $Z(t) = \mathbf{Q}^*(t)\mathbf{I}(t)$ is the total aggregate money value of investments in the *n* capital stocks, $\lambda^*(t)$ is the "as-if" constant (at time *t*) marginal utility of income, and the money NNP $Y^*(t)$ is the relevant quasi-fixed consumer's income of the "as-if" one-period budget constraint. Since the objective function in (26) is quasi-linear, the solution for current consumption is simply $\mathbf{C}^*(t) = \mathbf{D}(\mathbf{P}^*(t), \lambda * (t))$, and the corresponding investment value is $Z^*(t) = Y^*(t) - \mathbf{P}^*(t)\mathbf{D}(\mathbf{P}^*(t), \lambda^*(t))$ (Varian, 1992; Weitzman, 2001). This means that, along the optimal path, we can represent the utility function in the following manner

$$U(\mathbf{C}^*(t)) = \int_{\mathbf{0}}^{\mathbf{C}^*(t)} \nabla U(\mathbf{C})d\mathbf{C} = \lambda^*(t)\left\{\mathbf{P}^*(t)\mathbf{C}^*(t) + \int_{P^*(t)}^{\tilde{P}} \mathbf{D}(\mathbf{P}, \lambda^*(t))d\mathbf{P}\right\} \tag{27}$$

where the $\mathbf{D}(\mathbf{P}, \lambda^*(t))$ is the *m*-dimensional short-run demand function with respect to the counterfactual prices $\mathbf{P}$ with a given marginal utility of income $\lambda^*(t)$. $\tilde{\mathbf{P}}$ denotes a vector of choke-off prices at which all consumptions would cease. Note that $\lambda^*(t)$ is treated as a constant in (26) and (27) for a given *t*, but that its dynamics over time obey equation (8).

[12] For welfare gaps between the comprehensive NNP in an imperfect market setting and first best case, see Aronsson and Löfgren (1998) and Aronsson et al (2002).

While the first equality in (27) simply follows the definition of a utility function, the second one is derived by integration-by-parts using the duality between direct and inverse demand functions $C = D(P, \lambda^*)$ and $P = P(C, \lambda^*)$. The last integral in equation (27) represents the standard Dupuit-Marshallian consumer surplus corresponding to the area to the left of the demand curve integrated from the actual to the choke-off prices. Thus, the maximized current-value Hamiltonian can be expressed as

$$\begin{aligned} H^*(t) &= U(\mathbf{C}^*(t)) + \mathbf{\Psi}^*(t)\mathbf{I}^*(t) = \lambda^*(t)\big(\mathbf{P}^*(t)\mathbf{C}^*(t) + \mathbf{Q}^*(t)\mathbf{I}^*(t) + CS^*(t)\big) \\ &= \lambda^*(t)\big(Y^*(t) + CS^*(t)\big) \end{aligned} \tag{28}$$

where $Y^*(t) \equiv \mathbf{P}^*(t)\mathbf{C}^*(t) + \mathbf{Q}^*(t)\mathbf{I}^*(t)$ is, by definition, the (nominal) money NNP at time *t*, and $CS^*(t) = \int_{\mathbf{P}^*(t)}^{\tilde{\mathbf{P}}} \mathbf{D}\big(\mathbf{P}, \lambda^*(t)\big) d\mathbf{P}$ is the net consumer surplus at time *t*.

Concerning the welfare significance of static NNP at a given time t, we examine the effect of a small exogenous change $\partial Y(t)$. From the third equality in (28), we can derive

$$\frac{\partial H^*(t)}{\partial Y(t)}\Big|_{*(t)} = \lambda^*(t) > 0 \tag{29}$$

where $\Big|_{*(t)}$ implies that the exogenous change in NNP is evaluated at its optimum $Y(t) = Y^*(t)$. The result in (29) indicates that the money NNP is indeed a static indicator of welfare. Since, conditional on a fixed marginal utility of income consumption would not change, $Z(t)$ would increase by the same amount as NNP according to the budget constraint for the optimization problem (26). This, in turn, signifies potential increases in production and consumption of goods and services in subsequent years, and thereby increases the present utility wealth[13]. More formally, it can be shown that

$$\frac{\partial H^*(t)}{\partial Y^*(t)} = \frac{\partial H^*(t)}{\partial Z(t)} \frac{\partial Z(t)}{\partial Y(t)}\Big|_{*(t)} = \lambda^*(t) \tag{30}$$

since $\partial Z^*(t) / \partial Y^*(t) = 1$. We summarize this result in the following proposition:

[13] In a full re-optimization to the new conditions also the marginal utility of income would change, implying that welfare will be further improved.

Proposition 3. *Money NNP is a satisfactory static welfare indicator in that any exogenous increase (decrease) of this measure at a given point in time, in terms of increased (decreased) value of net investment, always leads to a higher (lower) level of welfare.*

Without loss of generality, we may normalize the marginal utility of income at the given time t to be unity i.e. $\lambda^*(t) = 1$. This implies that, at the margin, money NNP exactly measures welfare in terms of the money-metric Hamiltonian. When it comes to NNP growth and its relationship with welfare improvement, things become more complicated. Firstly, whether the growth is due to an increase in consumption or in the general price level, and to what extent these two items contribute to growth in NNP will affect welfare interpretations. It is obvious that if the growth in NNP is purely attributed to inflation, without any change in consumption, then it would not have any real welfare effect. Secondly, growth in NNP may be interrelated with changes in several other variables, such as the marginal utility of income $\lambda^*(t)$, the relative prices $\mathbf{P}^*(t)$ and $\mathbf{Q}^*(t)$, and thereby the optimal mix of consumption and investment goods $\mathbf{C}^*(t)$ and $\mathbf{I}^*(t)$. This makes it difficult to use the formula $H^*(t) = \lambda^*(t)\big(Y^*(t) + CS^*(t)\big)$ to evaluate the welfare effect of a growth in NNP by taking a partial derivative as in the static case. In other words, for the NNP growth case, we can no longer treat the time derivatives of the marginal utility of income $\lambda^*(t)$, of the (nominal) net national product $Y^*(t)$, and of the consumer surplus $CS^*(t)$ as if they were independent of each other.

We are now ready to suggest a complete money measure of welfare, which we will term as the *Generalized Comprehensive Net National Product (GCNNP)*. In the same spirit as Weitzman (2001), we define an ideal consumer price index (CPI)

$$\pi(t) = \frac{\mathbf{P}(t;\mathbf{C}) \cdot \mathbf{C}}{\mathbf{P}(t_0;\mathbf{C}) \cdot \mathbf{C}} \tag{31}$$

as a measure of the price level at time t relative to that at time t_0. In the definition (31), $\mathbf{P}(t;\mathbf{C})$ and $\mathbf{P}(t_0;\mathbf{C})$ denote the imputed market-clearing prices that would be observed at the two points in time if the market basket of goods being consumed in the economy were $\mathbf{C}$. Since this measure is invariant to the choice of the market basket (Weitzman, 2001), we can, without loss of generality, choose the consumption $\mathbf{C}_0 = \mathbf{C}^*(t_0)$ and efficiency price

$\mathbf{P}^*(t_0) = \mathbf{P}(t_0;\mathbf{C}_0)$ at time t_0 as a benchmark so that $\pi(t) = \mathbf{P}(t;\mathbf{C}_0)\mathbf{C}_0 / \mathbf{P}(t_0;\mathbf{C}_0)\mathbf{C}_0$. Since the utility function is stationary, we have $\nabla U(\mathbf{C}_0) = \lambda^*(t_0)\mathbf{P}(t_0;\mathbf{C}_0) = \lambda^*(t)\mathbf{P}(t;\mathbf{C}_0)$, which implies that $\lambda^*(t_0)\mathbf{P}(t_0;\mathbf{C}_0)\cdot\mathbf{C}_0 = \lambda^*(t)\mathbf{P}(t;\mathbf{C}_0)\cdot\mathbf{C}_0$ such that

$$\lambda^*(t_0) = \pi(t)\lambda^*(t) \tag{32}$$

is a constant. The maximized current-value Hamiltonian at any time t can, according to equations (27) and (32) be expressed as

$$H^*(t) = \lambda^*(t)\pi(t)\left(\frac{Y^*(t) + CS^*(t)}{\pi(t)}\right) = \lambda^*(t_0)\left(Y_R^*(t) + CS_R^*(t)\right) \tag{33}$$

where $Y_R^*(t) = Y^*(t)/\pi(t) = \mathbf{P}_R^*(t)\mathbf{C}^*(t) + \mathbf{Q}_R^*(t)I^*(t)$ corresponds to the comprehensive NNP and $CS_R^*(t) = CS^*(t)/\pi(t)$ the consumer surplus both expressed *in real terms*. The real prices for consumption and investment goods are given by $\mathbf{P}_R^*(t) = \mathbf{P}^*(t)/\pi(t)$ and $\mathbf{Q}_R^*(t) = \mathbf{Q}^*(t)/\pi(t)$, and consumer surplus, *in real terms*, can be expressed as

$$CS_R(t) \equiv \left\{\int_{\mathbf{P}^*(t)}^{\tilde{\mathbf{P}}} \mathbf{D}(\mathbf{P},\lambda^*(t))d\mathbf{P}\right\}/\pi(t) = \int_{\mathbf{P}_R^*(t)}^{\tilde{\mathbf{P}}_R} \mathbf{D}(\mathbf{P}_R,\lambda^*(t_0))d\mathbf{P}_R \tag{34}$$

To understand the rationale behind this expression, let us recall that the short-run demand function $\mathbf{C}(t) = \mathbf{D}(\mathbf{P}(t),\lambda^*(t))$ is defined by a first-order condition $\nabla U(\mathbf{C}(t)) = \lambda^*(t)\mathbf{P}(t)$ which may also be written as $\nabla U(\mathbf{C}(t)) = \lambda^*(t)\pi(t)\cdot\mathbf{P}(t)/\pi(t) = \lambda^*(t_0)\mathbf{P}_R(t)$. Since the utility functional form is assumed to be time invariant, it is obvious that the demand function satisfies $\mathbf{D}(\mathbf{P}(t),\lambda^*(t)) = \mathbf{D}(\mathbf{P}_R(t),\lambda^*(t_0))$. This implies that the consumption demand function at any time t with respect to real prices $\mathbf{P}_R(t)$, with t_0 as the base year, is time-invariant as if the total disposable income were held constant at its year t_0 level. As a result, we may simply write the demand function as $\mathbf{D}_0(\mathbf{P}_R(t))$ for a given time t, and thereby the consumer surplus in real terms as

$$CS_R(t) \equiv \int_{\mathbf{P}_R^*(t)}^{\tilde{\mathbf{P}}_R} \mathbf{D}_0(\mathbf{P}_R)d\mathbf{P}_R \tag{35}$$

With these devices, we now define a real money metric welfare measure

$$H_R^*(t) \equiv \frac{H^*(t)}{\lambda(t_0)} = Y_R^*(t) + CS_R^*(t) \tag{36}$$

as the *Generalized Comprehensive Net National Product (GCNNP)*, which is expected to be a satisfactory measure both for static and dynamic welfare. In the comprehensive (or green) national accounting practice, the word "comprehensive" means that all relevant goods and services, including environmental benefits and other externalities, are accounted for. Thus, the comprehensiveness is interpreted in terms of the number of goods and services that are involved. However, it is not general or comprehensive enough to account for the total value of each good or service. The reason for this is that it is only the price of the last unit of the good or service, rather than the value for each previous unit, that is used in national accounting. If each unit of each commodity (as under perfect price discrimination) is valued at its marginal price, then the resulting net national product will be exactly the expression we defined in equation (36), that is the *Generalized Comprehensive Net National Product (GCNNP)*. It requires, in addition to the current green accounting practice, that the National Accounting Authorities can also report the consumer surplus measures for the relevant commodities. This may seem unrealistic, but Proposition 4 at least shows how Weitzman's seminal result can be extended to be valid in a money metric.

Proposition 4. *The Generalized Comprehensive Net National Product (GCNNP) in (36) is a stationary equivalent of the future value of consumption plus the consumer surplus in real money terms such that*

$$\int_t^\infty H_R^*(t)\exp(-\theta(s-t))ds = \int_t^\infty \left\{ \mathbf{P}_R^*(s)\mathbf{C}^*(s) + \int_{\mathbf{P}_R^*(t)}^{\tilde{\mathbf{P}}_R} \mathbf{D}_0(\mathbf{P}_R)d\mathbf{P}_R \right\} \exp(-\theta(s-t))ds \qquad (37)$$

or equivalently

$$H_R^*(t) = \theta M_R^*(t) \qquad (38)$$

where

$$M_R^*(t) = \int_t^\infty \left[\mathbf{P}_R^*(s)\mathbf{C}^*(s) + \int_{\mathbf{P}_R^*(t)}^{\tilde{\mathbf{P}}_R} \mathbf{D}_0(\mathbf{P}_R)d\mathbf{P}_R \right] \exp(-\theta(s-t))ds \qquad (39)$$

can be interpreted as the Generalized Wealth in real terms.

The proof follows from equations (27), (28) and the definitions from (32) to (36). It is now seen that, when the generalized welfare measure GCNNP is used, all properties from Weitzman's foundation carry over to the money NNP.

Now, consider an exogenous change in $Y_R^*(t)$. Since the partial derivative $\partial H_R(t)/\partial Y_R(t)\big|_{*(t)} = 1$, it is obvious that $Y_R^*(t)$ is a satisfactory static welfare measure. For an infinitesimal increase in time, we have $\dot{H}_R^*(t) = \dot{Y}_R^*(t) + dCS_R^*(t)/dt = \dot{Y}_R^*(t) + \dot{\mathbf{P}}_R^*(t)\mathbf{C}^*(t)$, from which it can be seen that the effect of relative price changes enters the picture[14]. For welfare comparisons over a discrete time interval or across countries, the GCNNP defined in (36) has to be used. For two different dates, t_1 and t_2 with $t_2 > t_1$, the welfare at t_2 can be said to higher than at date t_1, if and only if, $H_R^*(t_2) - H_R^*(t_1) = Y_R^*(t_2) - Y_R^*(t_1) + \int_{\mathbf{P}_R^*(t_1)}^{\mathbf{P}_R^*(t_2)} \mathbf{D}_0(\mathbf{P}_R)d\mathbf{P}_R > 0$. For details on such a dynamic welfare comparison, see Weitzman (2001) which contains a thorough exposition.

The intuition behind the presence of the consumer surplus term in the formula above is the following. The relative real prices have been changed over the time interval, which will alter the optimal mix of the consumption bundle. For example, with a rising real price for pears and a falling real price for apples, a representative individual would consume more apples and less pears than before. Even though the real expenditure would remain constant, welfare would have been changed unless the two consumption bundles before and after the change happened to lie exactly on the same indifference curve. Note that welfare is defined with respect to the consumption bundle rather than the aggregated real income or expenditure.

We have now shown that the choice of metrics between utility or real prices does not matter for welfare comparisons. Given the right rescaling parameter, it is theoretically possible to define a money-metric generalized comprehensive net national product measure (GCNNP), which can be used for general purpose welfare analysis. Even though ordinary comprehensive NNP can work as a substitute for an exogenous change, it is in general not an appropriate welfare measure for dynamic welfare comparisons due to the change in relative prices. Thus, we may characterize the ordinary real money NNP as a weak welfare indicator.

[14] In case that the price effect cancels, then the ordinary comprehensive NNP growth, over an infinitesimal time interval, would indicate welfare improvement. This was achieved by Asheim and Weitzman (2001) by defining CPI as a Divisia index (Allen , 1986, p 178).

5. Concluding Remarks

This paper attempts to shed light on two important issues in dynamic welfare analysis. One of them is to show how social cost-benefit analysis based on money measures can be justified by discounted utilitarian theory, and the other is to study the welfare significance of net national product. We have shown, among other things, that the choice of either utility or money welfare metrics has, under ideal circumstances, no real effect on the issues. Behind intertemporal welfare maximization in a utilitarian framework, there is always a money wealth measure that can serve as a substitute for the maximized utility wealth. The crucial assumption for this strong conclusion is that the correct accounting or efficiency prices and the right money interest rate are used.

Second, this money wealth measure is a perfect substitute as for a utility based intertemporal welfare measure in social cost-benefit analysis. This provides a theoretical justification for the conventional practice used in project evaluations, i.e. to transform all future costs and benefits into monetary measures and then discount to present values. Here we stress the use of the derived consumption money interest rate rather then the utility rate of discount.

Third, we have clearly distinguished two different interpretations of the welfare significance of the net national product. We have shown that comprehensive (money) net national product is a satisfactory static indicator of welfare in that an exogenous increase in NNP, in the form of new discoveries or beneficial gene mutations etc, always increases welfare.

Finally, we have, in the same spirit as Weitzman (2001), developed a Generalized Comprehensive Net National Product (GCNNP) in real terms as a general purpose welfare measure. For a special case with an exogenous NNP increase, we have shown that the welfare effect from using the GCNNP measure coincides with the use of ordinary real money NNP. However, for welfare comparisons over time, where the relative real prices may have been changed, then the consumer surplus-inclusive measure such as the GCNNP has to be used, as was anticipated in Weitzman (2001).

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