

Positional Concerns with Multiple Reference Points: Optimal Income Taxation and Public Goods in an OLG Model^{**}

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Abstract

This paper concerns optimal income taxation and provision of a state-variable public good under asymmetric information in a two-type overlapping generations model, where people care about their relative consumption. Each individual may compare his/her own current consumption with his/her own past consumption as well as with other people's current and past consumption. The appearance of positional concerns affects the policy choices via two channels: (i) the size of the average degree of positionality and (ii) positionality differences between the (mimicked) low-ability type and the mimicker. Under plausible empirical estimates, the marginal labor income tax rates become substantially larger, and the absolute value of the marginal capital income tax rate of the low-ability type becomes substantially smaller, compared to the conventional optimal income tax model. The extent by which the rule for public provision should be modified depends crucially on the preference elicitation format.

Keywords: Optimal income taxation, asymmetric information, public goods, relative consumption, status, positional goods.

JEL Classification: D62, H21, H23, H41

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1. Introduction

Since the late 1970s, literature dealing with public policy in economies where the consumers have positional preferences, i.e. relative consumption concerns, has gradually developed.¹ The importance of this literature has become more apparent over time, as corresponding empirical literature has grown. There is by now convincing empirical support for the idea that relative consumption comparisons are important from at least three independent economic sub-literatures: happiness research (e.g. Easterlin 2001; Blanchflower and Oswald 2005; Ferrer-i-Carbonell 2005; Luttmer 2005), questionnaire-based experiments² (e.g. Johansson-Stenman et al. 2002; Solnick and Hemenway 2005; Carlsson et al. 2007), and more recently from brain science (Fliessbach et al. 2007). There are also recent evolutionary models consistent with relative consumption concerns (Samuelson 2004; Rayo and Becker 2007). According to Rayo and Becker, selfish genes would prefer that the humans they belong to were motivated by their own current consumption relative to (i) their own past consumption, (ii) other people's current consumption, and (iii) other people's past consumption. The present paper takes these three types of consumption comparisons as a point of departure in a study of optimal income taxation and provision of public goods in a dynamic economy.

Earlier studies on optimal taxation and public good provision in economies where people make relative consumption comparisons often assume that the government uses linear tax instruments. Furthermore, almost all of them are based on static models, and have in common that they neglect capital income taxation. By relying on static models, earlier literature also neglects that the consumption comparisons may

¹ Earlier studies address a variety of issues such as optimal taxation, public good provision, social insurance, growth, environmental externalities, and stabilization policy; see e.g. Boskin and Sheshinski (1978), Layard (1980), Ng (1987), Tuomala (1990), Blomquist (1993), Corneo and Jeanne (1997, 2001), Ireland (2001), Brekke and Howarth (2002), Abel (2005), Aronsson and Johansson-Stenman (2008) and Wendner and Goulder (in press). Clark et al. (2008) provide a good overview of both the empirical evidence and economic implications of relative consumption concerns.

² There are also experimental results from the social preference literature suggesting that people dislike inequity generally and disadvantageous inequity in particular, which can be interpreted as concern about the relative outcome; see e.g. Fehr and Schmidt (1999) and Bolton and Ockefels (2000).

have an intertemporal dimension. In the present paper, we consider an overlapping generations (OLG) model with two ability-types and asymmetric information between the private sector and the government (an extension of the two-type optimal income tax model developed by Stern 1982 and Stiglitz 1982). The set of policy instruments consists of nonlinear taxes on labor income and capital income as well as a state-variable public good. Therefore, the tax instruments considered here are based on informational limitations and not on any other a priori restrictions. The overall purpose is to analyze how the appearance of positional preferences modifies the optimal income tax structure and public good provision, respectively, by comparison with the outcome of the standard two-type OLG model where people only care about their absolute consumption levels.

Only a few earlier studies have dealt with optimal nonlinear taxation in economies where people have positional preferences. To our knowledge, the first was a paper by Oswald (1983), who assumes a continuous ability-distribution and that each individual compares his/her own consumption with a reference point; the latter is interpretable as reflecting either jealousy or altruism. Oswald shows that allowing for jealousy/altruism affects the optimal tax structure in a complex way, and that several standard results of optimal tax theory (such as zero marginal tax rates at the ends of the skill-distribution and that differentiated commodity taxes should not be used with certain forms of separable preferences) may no longer apply. Furthermore, the results show that if the utility function is separable in the measure of reference consumption, then the marginal tax rates are higher in an economy with predominantly jealous people and lower in an economy with predominately altruistic people, compared with the standard model without social interaction. Tuomala (1990) uses a similar model, where the utility of each individual depends negatively on the average consumption of others, and generalizes some findings by Oswald beyond additive separability. In addition, he provides numerical simulations showing, for instance, that the optimal marginal tax rates may be substantially higher when taking positional concerns into account. Ireland (2001) also uses a model with a continuous ability-distribution and nonlinear taxation of labor income. He assumes that individuals signal their social status position which, in turn, necessitates using resources that could otherwise have been used for beneficial consumption. This constitutes an incentive for the government to intervene, meaning (again) that social interaction justifies the use of

distortionary taxation. Finally, Aronsson and Johansson-Stenman (2008) analyze a two-type model in which agents value their own consumption both in absolute terms and relative to a measure of reference consumption (the average consumption in the economy as a whole). The results show, among other things, how the redistributive and corrective roles of income taxation may interact, due to possible differences between agents with respect to the degree of positionality, as well as how positional preferences affect the optimal provision of public goods in an economy where the income tax is optimally chosen.

The present paper is also related to a small – yet growing – literature dealing with redistribution and/or provision of public goods under asymmetric information in dynamic economies. The seminal contribution here is a paper by Ordober and Phelps (1979). In a model with a continuum of ability-types, they show (among other things) that if leisure is separable from private consumption in terms of the utility function (so the marginal rate of substitution between present and future consumption does not depend on the leisure choice other than via income), then the marginal capital income tax rate should be zero for each ability-type. Pirttilä and Tuomala (2001), in a generalization of the model in Brett (1997), consider an OLG model with two ability-types and endogenous before-tax wage rates. Their results show that production inefficiency at the second best optimum (which is a consequence of the desire to relax the self-selection constraint) justifies capital income taxation, whereas the marginal labor income tax rates take the same general form as in Stiglitz (1982), i.e. a positive marginal labor income tax rate should be imposed on the low-ability type and a negative marginal labor income tax rate on the high-ability type. They also derive the optimality condition for a public good, which is assumed to be a state variable. A somewhat related argument for using capital income taxation is found by Aronsson et al. (in press); they show that the appearance of equilibrium unemployment may justify capital income taxation, as it implies intertemporal production inefficiency at the second-best optimum. Finally, Boadway et al. (2000) analyze nonlinear labor income taxation and proportional capital income taxation in a model where both ability and initial wealth are unobserved by the government. In their framework, the capital income tax is interpretable as an indirect instrument to tax wealth.

The present study makes at least three distinct contributions to the literature, all of which are related to the intertemporal aspects of the analysis. First, we are able to consider capital income taxation. As far as we know, the only previous study that analyzes capital income taxation under relative consumption concerns is Abel (2005). He considers optimal capital income taxation in an OLG model where all consumers of a given generation are identical, and where a linear capital income tax constitutes the only tax instrument.³ The present paper, in contrast, analyzes the remaining role for capital income taxation when the labor income tax has been chosen in an optimal way. As earlier research indicates that the capital income tax might be a useful tool for relaxing the self-selection constraint, as noted above, a natural question is whether this tax is also useful for purposes of internalizing positional externalities. We show (for a special case) that under plausible empirical estimates regarding relative consumption concerns, the marginal capital income tax rate implemented for the low-ability type may be substantially smaller in absolute value than would be predicted by a model without positional concerns. Moreover, we show that the well-known result of zero marginal capital income tax rates under leisure separability (Ordober and Phelps 1979) generalizes to our more general framework for a natural benchmark case.

Second, our study addresses public good provision in an economy with positional preferences. As far as we know, there are only two earlier studies in this area: Wendner and Goulder (in press) and Aronsson and Johansson-Stenman (2008). Both are based on static models, and the former also assumes linear tax instruments. Our contribution here is to address public good provision and positional preferences in a dynamic economy, where the public good is a state variable, i.e. a stock that accumulates over time both due to the instantaneous contributions and depreciation. This is both a theoretically relevant and practically important extension, not least because many environmental public goods have this particular character. The most obvious example is the global climate, where the quality of the atmosphere provides essential benefits to humanity. The quality observed at present is clearly not only affected by the actions taken today (such as the current public abatement activities); it

³ In Abel's study, the tax revenues are returned lump-sum to the old generation. The model also contains a social security system (based on lump-sum payments) with its own budget.

is also affected by the actions taken in previous periods (cf. Stern 2007). Our results here depend crucially on the preference elicitation format. If people's marginal willingness to pay for a public good is measured independently, i.e. without considering that other people also have to pay for increased public provision, then relative consumption concerns typically (for reasonable parameter values) work in the direction of increasing the optimal provision of the public good. However, this is not the case when a referendum format is used, so that people are asked for their marginal willingness to pay conditional on that all people will have to pay for increased public provision. Conditions are also presented for when a dynamic analogue of the conventional Samuelson rule applies.

Third, the dynamic framework also allows us to simultaneously consider relative consumption comparisons between people (within the same period) and over time – an issue not dealt with in earlier comparable literature. In the first part of this paper, we focus on between-people comparisons at a given point in time and show (among other things) that important results carry over in a natural way from the static setup of Aronsson and Johansson-Stenman (2008). For example, under plausible empirical parameter estimates, the marginal labor income tax rates become substantially larger compared to the conventional optimal income tax model. The same applies to the public good provision in the limiting case when the public good converges to a conventional flow variable. By adding consumption comparisons over time (while retaining the between-people comparisons within the same period), the policy rules become more complex. However, this additional complexity is due solely to the comparisons with the past consumption of *others*. If the only extension were to assume that people also compare their own current consumption with *their own* past consumption, then all results derived in the simpler framework with only between-people comparisons in each period would continue to hold. Moreover, for the most general model we can for a natural benchmark case derive optimality conditions for the marginal labor income tax rates, the marginal capital income tax rates, and the public good, where the relative consumption concerns give rise to the same qualitative effects as they do in the simpler framework.

The outline of the study is as follows: Section 2 presents the model and the outcome of private optimization based on a model where each individual compares his/her

consumption with the average consumption in that period. Section 3 characterizes the corresponding optimal tax and expenditure problem of the government, whereas Sections 4 presents the results in a format that aims to facilitate straightforward interpretations and comparisons with earlier literature. Section 5 presents the general model, which also encompasses comparisons with the individual's own past consumption and the mean value of the past consumption of others. Section 6 summarizes and concludes the paper.

2. Positional preferences, firms, and market equilibrium

2.1 The OLG framework and utility functions

Consider an OLG model where each agent lives for two periods. Following the convention in earlier literature, we assume that each individual works during the first period of life and does not work during the second. There are two types of individuals, where the low-ability type (type 1) is less productive than the high-ability type (type 2). The number of individuals of ability-type i who were born at the beginning of period t is denoted n_t^i . Each such individual cares about his/her consumption when young and when old, c_t^i and x_{t+1}^i ; his/her leisure when young, z_t^i , given by a time endowment, H , less the hours of work, l_t^i (when old, all available time is leisure); and the amount of the public good available when young and when old, G_t and G_{t+1} .

In addition, as the agents are assumed to have positional preferences, they also compare their own consumption with a measure of reference consumption. We follow earlier comparable literature in assuming that the private consumption good (the consumption of which is denoted c when young and x when old) is in part a positional good, whereas leisure and the publicly provided good are completely non-positional.⁴

⁴ As noted by Aronsson and Johansson-Stenman (2008), it is of course possible to extend the analysis by allowing people to care about their relative amount of leisure and their relative benefit from a publicly provided good. We leave this to future research. Our conjecture is that the qualitative insights will still hold as long as private consumption is more positional than leisure and the publicly provided good, which is consistent with the limited empirical evidence (Solnick and Hemenway 1998, 2005; Carlsson et al. 2007).

The determinants of the relevant measure of reference consumption at the individual level constitute, of course, also an empirical question; yet, there is not much information available. Our approach is to follow the recent contribution by Rayo and Becker (2007), who argue in the context of an evolutionary model of happiness that the reference point of the individual might be determined by three components: (i) the individual's own past consumption, (ii) other people's current consumption, and (iii) other people's past consumption. For pedagogical reasons, we start with a simpler model where each individual only compares his/her current consumption (when young and old, respectively) with other people's current consumption, while we provide the full model in Section 5. As is demonstrated there, the results derived in the simpler model continue to hold for the case when each individual also compares his/her current consumption with his/her own past consumption. Moreover, for a special case, the results from the simpler model carry over in a natural sense to the most general model, which also contains comparisons with other people's past consumption.

The preferences for relative consumption, or positional preferences, can of course still be modeled in many different ways. Here we follow the approach chosen by many earlier studies by letting the relative consumption be described by the *difference* between the individual's own consumption and the mean consumption in the economy as a whole, given by \bar{c}_t at time t ; cf. e.g. Akerlof (1997), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Bowles and Park (2005), and Carlsson et al. (2007).⁵ The utility function of ability-type i born in the beginning of period t can then be written as

$$\begin{aligned} U_t^i &= v_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, G_t, G_{t+1}) \\ &= u_t^i(c_t^i, z_t^i, x_{t+1}^i, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1}) \end{aligned} \quad (1)$$

⁵ Alternative approaches include ratio comparisons (Boskin and Sheshinski 1978; Layard 1980; Abel 2005; Wendner and Goulder, in press) and comparisons of the ordinal rank (Frank 1985; Hopkins and Kornienko 2004). Dupor and Liu (2003) consider a specific flexible functional form that includes the difference comparison and ratio comparison approaches as special cases.

The public good is a state variable governed by the difference equation

$$G_t = g_t + (1 - \xi)G_{t-1}, \quad (2)$$

where g_t is the addition provided by the government to the public good in period t and ξ is the rate of depreciation. Therefore, the traditional flow-variable public good appears as the special case where $\xi = 1$.

The utility function $v_t^i(\cdot)$ is increasing in each argument, implying that $u_t^i(\cdot)$ is decreasing in \bar{c}_t and \bar{c}_{t+1} (a property denoted “jealousy” by Dupor and Liu 2003) and increasing in the other arguments. Both $v_t^i(\cdot)$ and $u_t^i(\cdot)$ are assumed to be twice continuously differentiable in their respective arguments and strictly quasi-concave. The reference consumption levels in periods t and $t+1$ are measured by the average consumption among all people alive:

$$\bar{c}_t = \frac{n_t^1 c_t^1 + n_t^2 c_t^2 + n_{t-1}^1 x_t^1 + n_{t-1}^2 x_t^2}{N_t}, \quad (3)$$

$$\bar{c}_{t+1} = \frac{n_{t+1}^1 c_{t+1}^1 + n_{t+1}^2 c_{t+1}^2 + n_t^1 x_{t+1}^1 + n_t^2 x_{t+1}^2}{N_{t+1}}, \quad (4)$$

in which $N_t = n_t^1 + n_t^2 + n_{t-1}^1 + n_{t-1}^2$ and $N_{t+1} = n_{t+1}^1 + n_{t+1}^2 + n_t^1 + n_t^2$. This means that each individual compares his/her own consumption with the average consumption in each period. We also assume that each individual treats the reference levels, \bar{c}_t and \bar{c}_{t+1} , as exogenous.

The utility function in equation (1) is quite general and may vary both between ability-types and over time, and is furthermore not necessarily time-separable, meaning for example that the marginal rate of substitution between relative and absolute consumption when old is not necessarily independent of the consumption level when young. Thus, the model is flexible enough to encompass habit formation in private consumption. We will perform much of the analysis with the more general

utility formulation given by the second line of equation (1). This case resembles a classical externality problem e.g. in terms of pollution associated with private consumption. However, we will need the formulation on the first line when we relate the optimum tax and expenditure conditions to the extent that people care about relative consumption. The definition of such measures is the issue to which we turn next.

2.2 The degree of consumption positionality

Since much of the subsequent analysis is focused on relative consumption concerns, it is useful to introduce measures of the degree to which such concerns matter for each individual. By defining $\Delta_t^{i,c} = c_t^i - \bar{c}_t$ and $\Delta_t^{i,x} = x_{t+1}^i - \bar{c}_{t+1}$, we can rewrite the first part of equation (1) as

$$U_t^i = v_t^i(c_t^i, z_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_t^{i,x}, G_t, G_{t+1}).$$

We can then define the *degree of consumption positionality* (cf. e.g. Johansson-Stenman et al. 2002; Aronsson and Johansson-Stenman 2008) when young and old, respectively, based on the utility function in equation (1) as follows:

$$\alpha_t^{i,c} = \frac{v_{t,\Delta_t^c}^i}{v_{t,\Delta_t^c}^i + v_{t,c}^i}, \quad (5a)$$

$$\alpha_t^{i,x} = \frac{v_{t,\Delta_t^x}^i}{v_{t,\Delta_t^x}^i + v_{t,x}^i}, \quad (5b)$$

where $v_{t,c}^i \equiv \partial v_t^i / \partial c_t^i$ and similarly for the other variables. The term $\alpha_t^{i,c}$ can then be interpreted as the fraction of the overall utility increase from the last dollar spent in period t that is due to the increased relative consumption. For instance, if $\alpha_t^{i,c} = 0$, then relative consumption does not matter at all on the margin, whereas in the other extreme case where $\alpha_t^{i,c} = 1$, absolute consumption does not matter at all (i.e. all that matters is relative consumption). The interpretation of $\alpha_t^{i,x}$ is analogous except that this term reflects the degree of consumption positionality when old instead of when

young. From the assumptions about the utility functions, we have $0 < \alpha_t^{i,c}, \alpha_t^{i,x} < 1$. In addition, let us denote the *average degree of consumption positionality* in period t by

$$\bar{\alpha}_t = \sum_i \alpha_{t-1}^{i,x} \frac{n_{t-1}^i}{N_t} + \sum_i \alpha_t^{i,c} \frac{n_t^i}{N_t} \in [0, 1]. \quad (6)$$

In other words, $\bar{\alpha}_t$ reflects the average value of the degree of consumption positionality among the people alive in period t .

2.3 Individual optimization and market equilibrium

The individual budget constraint is given by⁶

$$w_t^i l_t^i - T_t(w_t^i l_t^i) - s_t^i = c_t^i, \quad (7a)$$

$$s_t^i (1 + r_{t+1}) - \Phi_{t+1}(s_t^i r_{t+1}) = x_{t+1}^i, \quad (7b)$$

where s_t^i is savings, r_{t+1} is the market interest rate, and $T_t(\cdot)$ and $\Phi_{t+1}(\cdot)$ denote the payments of labor income and capital income taxes, respectively. The first order conditions for the hours of work and savings can be written as

$$u_{t,c}^i w_t^i \left[1 - T_t'(w_t^i l_t^i) \right] - u_{t,z}^i = 0, \quad (8)$$

$$-u_{t,c}^i + u_{t,x}^i \left[1 + r_{t+1} \left(1 - \Phi_{t+1}'(s_t^i r_{t+1}) \right) \right] = 0, \quad (9)$$

⁶ As our model does not make a distinction between different types of commodities, we abstract from commodity taxation throughout the paper. This approach has also been taken in most of the earlier comparable literature (see the introduction). This does not reflect a belief that commodity taxation is unimportant in connection to positional preferences. However, there are several practical problems associated with such extensions. For example, different variants of the same group of commodities, such as cars, may be characterized by very different degrees of positionality. Moreover, the theoretical analysis would become considerably more complex, suggesting that commodity taxation warrants a paper of its own.

in which $u_{t,c}^i = \partial u_t^i / \partial c_t^i$, $u_{t,z}^i = \partial u_t^i / \partial z_t^i$ and $u_{t,x}^i = \partial u_t^i / \partial x_{t+1}^i$, and $T_t'(w_t^i l_t^i)$ and $\Phi_{t+1}'(s_t^i r_{t+1}^i)$ are the marginal labor income tax rate and the marginal capital income tax rate, respectively.

The production sector consists of identical competitive firms producing a homogenous good with constant returns to scale. Given these characteristics, the number of firms is not important and will be normalized to one for notational convenience. The production function is given by $F(L_t^1, L_t^2, K_t)$, where $L_t^i = n_t^i l_t^i$ is the total number of hours of work supplied by ability-type i in period t , and K_t is the capital stock in period t . The firm obeys the necessary conditions

$$F_{L_t^i}(L_t^1, L_t^2, K_t) - w_t^i = 0 \text{ for } i=1,2, \quad (10)$$

$$F_K(L_t^1, L_t^2, K_t) - r_t = 0, \quad (11)$$

where subindices attached to the production function denote partial derivatives.

3. The government's decision problem

3.1 Objective and constraints

We assume that the government faces a general social welfare function as follows:

$$W = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots), \quad (12)$$

which is increasing in each argument. Since the optimum conditions are expressed for any such social welfare function, they are necessary optimum conditions for a Pareto efficient allocation.⁷ A similar formulation is used by Pirttilä and Tuomala (2001),

⁷ All results obtained here that are independent of the social welfare function (i.e. basically all results that we comment on) could have been obtained by instead explicitly solving for the Pareto efficient allocation by maximizing the utility of one ability-type born in a certain period, while holding the utility constant for all other agents (the other ability-type born in the same period and both ability-types

although they in addition assume that the social welfare function is utilitarian within each generation.

The informational assumptions are conventional. The government is able to observe income, although ability is private information. As in most of the earlier literature on the self-selection approach to optimal taxation, we assume that the government wants to redistribute from the high-ability to the low-ability type.⁸ This means that the most interesting aspect of self-selection is to prevent the high-ability type from pretending to be a low-ability type. The self-selection constraint that may bind then becomes

$$\begin{aligned} U_t^2 &= u_t^2(c_t^2, z_t^2, x_{t+1}^2, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1}) \\ &\geq u_t^2(c_t^1, H - \phi_t l_t^1, x_{t+1}^1, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1}) = \hat{U}_t^2 \end{aligned} \quad (13)$$

where $\phi_t = w_t^1 / w_t^2$ is the wage ratio (relative wage rate) in period t . By using equation (10) for both ability-types, it is straightforward to show that ϕ_t can be written as a function of l_t^1 , l_t^2 , and K_t , i.e. $\phi_t = \phi(l_t^1, l_t^2, K_t)$. The expression on the right-hand side of the weak inequality in (13) is the utility of the mimicker. Although the mimicker enjoys the same consumption as the low-ability type in each period, he/she enjoys more leisure (as the mimicker is more productive than the low-ability type).⁹

Note that $T_t(\cdot)$ is a general labor income tax, which can be used to implement any desired combination of l_t^1 , c_t^1 , l_t^2 , and c_t^2 , given the savings chosen by each ability-type. Therefore, we will use l_t^1 , c_t^1 , l_t^2 , and c_t^2 , instead of the parameters of the labor income tax function, as direct decision variables in the optimal tax and expenditure

born in all other periods). The chosen strategy is motivated by convenience, as it simplifies the presentation.

⁸ This of course implies restrictions on the social welfare function beyond what is stated above.

⁹ Given the set of available policy instruments in our framework, it is possible for the government to control the present and future consumption as well as the hours of work of each ability-type (this is discussed more thoroughly below). As a consequence, in order to be a mimicker, the high-ability type must mimic the point chosen by the low-ability type on each tax function (both the labor income tax and the capital income tax), and thus consume equally much in both periods.

problem. Note also that the general capital income tax, $\Phi_{t+1}(\cdot)$, can be used to implement any desired combination of c_t^1 , x_{t+1}^1 , c_t^2 , x_{t+1}^2 , and K_{t+1} , given the labor income of each individual. Therefore, instead of choosing the parameters of the capital income tax function directly, we formulate the optimization problem such that x_{t+1}^1 , x_{t+1}^2 , and K_{t+1} are also used as direct decision variables. The resource constraint is given by

$$F(L_t^1, L_t^2, K_t) + K_t - \sum_{i=1}^2 [n_t^i c_t^i + n_{t-1}^i x_t^i] - K_{t+1} - g_t = 0. \quad (14)$$

Equation (14) means that output is used for private consumption, net investments, and public consumption.

Equations (2), (13), and (14) together constitute the set of restrictions facing the government. The Lagrangean is written as

$$\begin{aligned} \mathcal{L} = & W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots) + \sum_t \lambda_t [U_t^2 - \hat{U}_t^2] \\ & + \sum_t \gamma_t \left[F(L_t^1, L_t^2, K_t) + K_t - \sum_{i=1}^2 [n_t^i c_t^i + n_{t-1}^i x_t^i] - K_{t+1} - g_t \right]. \\ & + \sum_t \mu_t [g_t + (1 - \xi)G_{t-1} - G_t] \end{aligned} \quad (15)$$

For further use, let $\hat{u}_t^2 = u_t^2(c_t^1, H - \phi_t^1, x_{t+1}^1, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1})$. As the decision-problem facing the government is written,¹⁰ the direct decision-variables relevant for generation t are l_t^1 , c_t^1 , x_{t+1}^1 , l_t^2 , c_t^2 , x_{t+1}^2 , K_t , K_{t+1} , G_t , G_{t+1} , and g_t . The first-order conditions are presented in the appendix.

¹⁰ Note that there is a potential time inconsistency problem involved here since the government may have incentives to modify the second period taxation facing each generation once the individuals have revealed their true types. Although we acknowledge this potential problem, we follow earlier comparable literature by only considering situations where the government commits to its tax and expenditure policies. This approach is motivated by the observation that lack of commitment from the point of view of the government opens a spectrum of possibilities for modeling both public policy and the response by the private sector, which would be beyond the scope of this paper.

3.2 The positionality effect

Let us now turn to the welfare effect of an increase in the reference consumption. The derivative of the Lagrangean with respect to \bar{c}_t can be written as

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1, \bar{c}_t}^i + \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i u_{t, \bar{c}_t}^i \\ &+ \lambda_{t-1} [u_{t-1, \bar{c}_t}^2 - \hat{u}_{t-1, \bar{c}_t}^2] + \lambda_t [u_{t, \bar{c}_t}^2 - \hat{u}_{t, \bar{c}_t}^2] \end{aligned} \quad (16)$$

We will refer to this derivative as measuring the *positionality effect* in period t , since it reflects the overall welfare effects of a change in the level of reference consumption in period t , *ceteris paribus*. By using the first utility formulation in equation (1), i.e. the function $v_t^i(\cdot)$, this effect can be rewritten in terms of the individual degrees of consumption positionality. Let us use the short notation

$$\Gamma_t = \frac{\lambda_{t-1} \hat{u}_{t-1, x}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2, x} - \alpha_{t-1}^{1, x}] + \frac{\lambda_t \hat{u}_{t, c}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2, c} - \alpha_t^{1, c}]$$

for the positionality differences between the mimicker and the low-ability type (measured for both generations alive in period t), where $\Gamma_t > 0$ (< 0) if the mimicker is always, i.e. as both young and old, more (less) positional than the low-ability type.

We can then derive the following result:

Lemma 1. *The welfare effect of increased reference consumption in period t can be written as*

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= N_t \gamma_t \frac{\Gamma_t - \bar{\alpha}_t}{1 - \bar{\alpha}_t} \\ &= -\frac{\bar{\alpha}_t \gamma_t N_t}{1 - \bar{\alpha}_t} + \frac{1}{1 - \bar{\alpha}_t} \left[\lambda_{t-1} \hat{u}_{t-1, x}^2 \{ \hat{\alpha}_t^{2, x} - \alpha_t^{1, x} \} + \lambda_t \hat{u}_{t, c}^2 \{ \hat{\alpha}_t^{2, c} - \alpha_t^{1, c} \} \right] \end{aligned} \quad (17)$$

Therefore, increased reference consumption in period t reduces the welfare, so $\partial \mathcal{L} / \partial \bar{c}_t < 0$, if and only if $\Gamma_t < \bar{\alpha}_t$. A sufficient condition for this to hold is that

$\alpha_t^{1,c} \geq \hat{\alpha}_t^{2,c}$ and $\alpha_t^{1,x} \geq \hat{\alpha}_t^{2,x}$, meaning that the young and old low-ability type, respectively, is at least as positional as the corresponding mimicker in period t .

Proof: see Appendix.

Two mechanisms are worth noticing. First, in the absence of the self-selection constraint, i.e. if ability-type specific lump-sum taxes were possible to implement, an increase in the reference consumption would unambiguously decrease the welfare, since the reference consumption enters the utility function of each individual via the arguments $\Delta_t^{i,c} = c_t^i - \bar{c}_t$ and $\Delta_t^{i,x} = x_{t+1}^i - \bar{c}_{t+1}$. Thus, the reference consumption constitutes a negative externality for each ability-type in each period. This explains the first term in the second row of equation (17), which relates the positionality effect to the average degree of positionality without any reference to differences in the degree of positionality between ability-types. Second, if the low-ability type is more positional than the mimicker in both generations alive in period t (i.e. generations t and $t-1$), then an increase in the reference consumption means a larger utility loss for the low-ability type than for the mimicker; as such, it contributes to an additional welfare loss via the self-selection constraint. However, if the mimicker is more positional than the low-ability type, then an increase in the reference consumption contributes to relax the self-selection constraint, implying that the second term in the second row of equation (17) is positive; this mechanism will be discussed in more detail subsequently. In this case, the sign of $\partial \mathcal{L} / \partial \bar{c}_t$ can be either positive or negative depending on whether or not $\Gamma_t < \bar{\alpha}_t$.

4. Tax and expenditure results

In this section we present the optimality conditions for the marginal labor income tax rates, the marginal capital income tax rates, and the public good provision in a format that facilitates straightforward economic interpretations and comparisons with the benchmark case with no relative consumption concerns.

4.1 Labor Income Taxation

By defining the marginal rate of substitution between leisure and private consumption for ability-type i as

$$MRS_{z,c}^{i,t} = \frac{u_{t,z}^i}{u_{t,c}^i},$$

and similarly for the mimicker, we obtain the marginal labor income tax rate for the low-ability type by combining equations (8), (A1), and (A2), while the marginal labor income tax rate for the high-ability type is derived by combining equations (8), (A4), and (A5). We show in the appendix that

$$T_t'(w_t^1 l_t^1) = \frac{\lambda_t^*}{w_t^1 n_t^1} \left[MRS_{z,c}^{1,t} - \hat{MRS}_{z,c}^{2,t} \left[\phi_t + \frac{\partial \phi_t}{\partial l_t^1} l_t^1 \right] \right] - \frac{MRS_{z,c}^{1,t}}{\gamma_t w_t^1 N_t} \frac{\partial \mathcal{E}}{\partial \bar{c}_t}, \quad (18)$$

$$T_t'(w_t^2 l_t^2) = -\frac{\lambda_t^*}{w_t^2 n_t^2} \hat{MRS}_{z,c}^{2,t} \frac{\partial \phi_t}{\partial l_t^2} l_t^2 - \frac{MRS_{z,c}^{2,t}}{\gamma_t w_t^2 N_t} \frac{\partial \mathcal{E}}{\partial \bar{c}_t}, \quad (19)$$

where $\lambda_t^* = \lambda_t \hat{u}_{t,c}^2 / \gamma_t$. The marginal labor income tax rates in equations (18) and (19) are straightforward extensions of the results in a static model with a linear production technology by Aronsson and Johansson-Stenman (2008). However, there are two important differences. First, the gross wage rates are endogenous here, meaning that the wage ratio responds to a change in the hours of work. Second, a change in the reference consumption in period t , induced by a change in the hours of work supplied by the young generation, will affect the well-being of both the young and the old generation in that period.

The first part of each tax formula is analogous to results derived in earlier literature and is due to the self-selection constraint. With $MRS_{z,c}^{1,t} > \hat{MRS}_{z,c}^{2,t}$ (which applies if the preferences do not differ between ability-types), and if we assume (by analogy to earlier literature on optimal income taxation) that $\partial \phi_t / \partial l_t^1 < 0$, the contribution of the self-selection constraint is to increase the marginal labor income tax rate of the low-ability type. Similarly, if $\partial \phi_t / \partial l_t^2 > 0$ (also by analogy to earlier literature), the self-selection constraint contributes to decrease the marginal labor income tax rate of the

high-ability type. These effects are well understood from earlier research (Stiglitz 1982).

The final part of each formula reflects the relative consumption concerns. By combining Lemma 1 with equations (18) and (19), we obtain the following result:

Proposition 1. *If the young and old low-ability type, respectively, is at least as positional as the corresponding mimicker in period t , or if the positionality differences are sufficiently small so that $\Gamma_t < \bar{\alpha}_t$, then the positionality effect contributes to increase the marginal labor income tax rate facing each ability-type in period t , ceteris paribus.*

Note that the positionality effect discussed in Proposition 1 contains two parts: an externality-correcting component and a component that serves to relax the self-selection constraint. To see this more clearly, we will combine equations (17), (18), and (19) in order to decompose the positionality effect. Let us use the short notations

$$\sigma_t^1 = \frac{\lambda_t^*}{w_t^1 n_t^1} \left[MRS_{z,c}^{1,t} - \hat{MRS}_{z,c}^{2,t} \left[\phi_t + \frac{\partial \phi_t}{\partial l_t^1} l_t^1 \right] \right],$$

$$\sigma_t^2 = -\frac{\lambda_t^*}{w_t^2 n_t^2} \hat{MRS}_{z,c}^{2,t} \frac{\partial \phi_t}{\partial l_t^2} l_t^1,$$

where σ_t^1 and σ_t^2 reflect the optimal marginal labor income tax rates without relative consumption concerns, i.e. the first term on the right-hand side of equation (18) and (19), respectively. We can then rewrite the formulas for the marginal labor income tax rates such that the contribution of positionality is decomposed into two effects as follows:

Proposition 2. *The optimal marginal labor income tax rate for each ability-type can be written in the following additive form (for $i=1, 2$):*

$$T_t^i(w_t^i l_t^i) = \sigma_t^i + [1 - \sigma_t^i] \bar{\alpha}_t - [1 - \sigma_t^i] [1 - \bar{\alpha}_t] \frac{\Gamma_t}{1 - \Gamma_t}. \quad (20)$$

Proof: See Appendix.

Equation (20) is an intertemporal analogue to (and has the same general interpretation as) a corresponding tax formula derived by Aronsson and Johansson-Stenman (2008) in a static model. Note first that in the special case where the resource allocation is first best, meaning that $\lambda_t = 0$ for all t , we have $\sigma_t^1 = \sigma_t^2 = \Gamma_t = 0$, so $T_t'(w_t^1 l_t^1) = T_t'(w_t^2 l_t^2) = \bar{\alpha}_t$, which exemplifies a straightforward Pigouvian tax. The interpretation is that each individual is taxed for the negative positional externality that he/she imposes on other people.¹¹

Returning to our more general second-best model, the intuition is straightforward. The first term on the right-hand side of equation (20) is the tax expression that would follow without any positional concern. The second term measures the marginal external cost of consumption – as reflected by the average degree of positionality – although its contribution to the marginal labor income tax rates is modified by comparison with the first-best. Increased private consumption, associated with an increase in the hours of work, causes negative external costs here as well; for the low-ability type, however, these external costs are smaller than in the first-best provided that $\sigma_t^1 > 0$ (which is the case we discussed above). The intuition is that the fraction of an income increase that is already taxed away does not give rise to positional externalities. By analogy, if $\sigma_t^2 < 0$, then $1 - \sigma_t^2 > 1$ means that the government attaches a higher weight to the corrective part of the tax formula for the high-ability type than it does in the first-best. The reason is that the self-selection component in the formula for the high-ability type is a marginal subsidy, which strengthens the positional externality.

The third term on the right-hand side of equation (20) reflects self-selection effects of positional concerns. Suppose first that $\Gamma_t > 0$, in which case the mimicker is more positional than the low-ability type. This means that increased reference consumption gives rise to a larger utility loss for the mimicker than it does for the low-ability type. Therefore, the government may relax the self-selection constraint by implementing

¹¹ This special case also resembles the consumption tax derived by Dupor and Liu (2003) in a representative-agent model.

policies that lead to increased reference consumption. This provides an incentive for the government to implement a lower marginal labor income tax rate than it would otherwise have done, which means that the third term contributes to decrease the marginal labor income tax rate. Consequently, if Γ_t is positive and sufficiently large, then this effect may (at least theoretically) dominate the externality-correcting component, implying that relative consumption concerns contribute to reduce the marginal labor income tax rates. If on the other hand $\Gamma_t < 0$, then the opposite argument applies. The latter case also explains in greater detail why the positionality effect unambiguously contributes to increase the marginal labor income tax rates if the low-ability type is at least as positional as the mimicker.

Let us briefly discuss how the appearance of positional preferences may affect the marginal labor income tax rates according to the empirical evidence described above. Consider first the high-ability type, who would most likely (see Stiglitz 1982) face a negative marginal labor income tax rate in the absence of relative consumption concerns. To exemplify, suppose that $\sigma_t^2 = -0.1$ and $\Gamma_t = 0$. The latter is motivated by the lack of empirical evidence of positionality differences due to differences in leisure (remember that the mimicker and the low-ability type have the same consumption levels). In this case, if $\bar{\alpha}_t = 0.5$ (roughly consistent with Alpizar et al. 2005 and Carlsson et al. 2007), then the marginal labor income tax rate facing the high-ability type is 0.45, whereas $\bar{\alpha}_t = 0.8$ (more in line with Easterlin 1995 and Luttmer 2005) implies a marginal labor income tax rate of 0.78. These are clearly very dramatic differences compared to the negative rate of -0.1 that would apply in the absence of positional preferences.¹² For the low-ability type, the same assumptions would imply corresponding effects, although the changes in relative terms would seem less dramatic since the pure self-selection component, σ_t^1 , is most likely positive (see above and Stiglitz 1982).

4.2 Capital Income Taxation

¹² Similarly, with fixed before-tax wage rates so that $\sigma_t^2 = 0$, the corresponding marginal labor income tax rates for the high-ability type are 0.5 and 0.8.

Let us now turn to the marginal capital income tax structure. Define the marginal rate of substitution between consumption in periods t and $t+1$ for ability-type i

$$MRS_{c,x}^{i,t} = \frac{u_{t,c}^i}{u_{t,x}^i},$$

and similarly for the mimicker. The marginal capital income tax rate for the low-ability type can be derived by combining equations (9), (A2), (A3), and (A7), whereas the marginal capital income tax rate for the high-ability type can be derived by combining equations (9), (A5), (A6), and (A7). We show in the appendix that the marginal capital income tax rates can be written as

$$\begin{aligned} \Phi'_{t+1}(s_t^1 r_{t+1}) &= \frac{\lambda_t \hat{u}_{t,x}^2}{\gamma_{t+1} n_t^1 r_{t+1}} [MRS_{c,x}^{1,t} - M\hat{R}S_{c,x}^{2,t}] - \frac{\lambda_{t+1} \hat{u}_{t+1,z}^2 l_{t+1}^1}{\gamma_{t+1} r_{t+1}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}, \\ &+ \frac{1}{\gamma_{t+1} r_{t+1}} \left[\frac{\partial \mathcal{E}}{\partial \bar{c}_t} \frac{1}{N_t} - MRS_{c,x}^{1,t} \frac{\partial \mathcal{E}}{\partial \bar{c}_{t+1}} \frac{1}{N_{t+1}} \right], \end{aligned} \quad (21)$$

$$\begin{aligned} \Phi'_{t+1}(s_t^2 r_{t+1}) &= - \frac{\lambda_{t+1} \hat{u}_{t+1,z}^2 l_{t+1}^1}{\gamma_{t+1} r_{t+1}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}} \\ &+ \frac{1}{\gamma_{t+1} r_{t+1}} \left[\frac{\partial \mathcal{E}}{\partial \bar{c}_t} \frac{1}{N_t} - MRS_{c,x}^{2,t} \frac{\partial \mathcal{E}}{\partial \bar{c}_{t+1}} \frac{1}{N_{t+1}} \right]. \end{aligned} \quad (22)$$

Let us start by discussing the marginal capital income tax rate of the low-ability type. Note that the first row is due to the appearance of the self-selection constraints. The first term reflects the self-selection constraint in period t . It means that if the relative valuation of current consumption by the low-ability type exceeds (falls short of) the relative valuation by the mimicker, there is an incentive for the government to stimulate (discourage) the current consumption via a higher (lower) marginal capital income tax rate. As such, this incentive effect serves to relax the self-selection constraint by making mimicking less attractive. There is a similar purpose behind the second term in the first row, although this effect is associated with the self-selection constraint in period $t+1$. It arises here because the savings in period t determine the capital stock in period $t+1$. If an increase in the capital stock increases (decreases) the wage ratio, then mimicking becomes less (more) attractive, providing an incentive for

the government to stimulate (discourage) savings by choosing a lower (higher) marginal capital income tax rate than it would otherwise have done. Note also that the first row of the formula for the high-ability type is analogous to, and has the same interpretation as, the second term in the first row of the formula for the low-ability type. These effects are well understood from earlier research (Brett 1997; Pirttilä and Tuomala 2001).

The second row of each tax formula is novel and refers to the assumption that the private consumption good is, in part, a positional good. As the marginal capital income tax rates reflect a desired tradeoff between present and future consumption, each such term is decomposable into two parts. The intuition is, of course, that each individual values relative consumption both when young and old. By combining Lemma 1 with equations (21) and (22), we can derive the following result:

Proposition 3. *If the young and old low-ability type, respectively, is at least as positional as the corresponding mimicker in periods t and $t+1$, then the positionality effect in period t , $\partial \mathcal{L} / \partial \bar{c}_t < 0$, contributes to decrease the marginal capital income tax rates in period $t+1$, whereas the positionality effect in period $t+1$, $\partial \mathcal{L} / \partial \bar{c}_{t+1} < 0$, contributes to increase the marginal capital income tax rates in period $t+1$, ceteris paribus.*

The intuition behind Proposition 3 is straightforward. The positionality effect in period t means that an increase in the reference consumption in period t gives rise to a welfare loss. This provides an incentive for the government to choose lower marginal capital income tax rates than it would otherwise have done, which in turn stimulates savings and discourages consumption in period t . By analogy, the positionality effect in period $t+1$ means that an increase in the reference consumption in period $t+1$ results in a welfare loss. As a consequence, there is an incentive for the government to reduce the average consumption in period $t+1$, which means that the government chooses higher marginal capital income tax rates than it would otherwise have done. The relative sizes of these two effects determine whether the appearance of positional preferences constitutes an incentive to tax or subsidize the capital income at the margin, ceteris paribus.

So far, we have not used the decomposition of the positionality effect given by equation (17). In general, since two such effects are involved, this decomposition does not give results that are as easy to interpret as the corresponding expressions for the marginal labor income tax rates in Proposition 2. Nevertheless, it is instructive to combine equation (17) with equations (21) and (22) in the special case where the degree of positionality does not vary over time. Consider Proposition 4:

Proposition 4. *If the average degree of positionality as well as the positionality differences between the mimicker and the low-ability type remain constant over time, so $\bar{\alpha}_{t+1} = \bar{\alpha}_t = \bar{\alpha}$ and $\Gamma_{t+1} = \Gamma_t = \Gamma$, then the marginal capital income tax rates reduce to*

$$\Phi'_{t+1}(s^1_t r_{t+1}) = \frac{\lambda_t \hat{u}_{t,x}^2}{\gamma_{t+1} n_t^1 r_{t+1}} \left[MRS_{c,x}^{1,t} - M\hat{R}S_{c,x}^{2,t} \right] \frac{1 - \bar{\alpha}}{1 - \Gamma} - \frac{\lambda_{t+1} \hat{u}_{t+1,z}^2 l_{t+1}^1}{\gamma_{t+1} r_{t+1}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}, \quad (23)$$

$$\Phi'_{t+1}(s^2_t r_{t+1}) = - \frac{\lambda_{t+1} \hat{u}_{t+1,z}^2 l_{t+1}^1}{\gamma_{t+1} r_{t+1}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}. \quad (24)$$

Proof: See the Appendix.

Two aspects of Proposition 4 are worth emphasizing: First, there is no direct effect of positionality in the tax formulas. Second, there is no need to modify the effects of the self-selection constraint that are common in the two tax formulas (the term associated with the wage distribution). Therefore, in this special case, the appearance of positionality does not change the way in which we measure the marginal capital income tax rate of the high-ability type (compared with an economy without positional goods). The intuition is that under the conditions in the proposition, the current and future aspects of positionality cancel each other out to a large extent, suggesting that the incentives underlying capital formation are similar to those that would apply in economies without positional goods. However, this does not mean that the effect of positionality that still remains is unimportant.

Note that leisure is of course not generally weakly separable from private consumption. As a consequence, the low-ability type and the mimicker will differ with respect to the relative value attached to current consumption; the contribution of this difference to the marginal capital income tax rate of the low-ability type is still affected by concern for positionality. To interpret the “positionality-weight” $[1 - \bar{\alpha}]/[1 - \Gamma]$, consider first the situation where $MRS_{c,x}^{1,t} > \hat{MRS}_{c,x}^{2,t}$, meaning that the first term on the right-hand side of equation (23) contributes to increase the marginal capital income tax rate of the low-ability type. As such, this term works to increase the current (first period) consumption of the low-ability type and, as a consequence, also the reference consumption in period t . The expression $1 - \bar{\alpha}$ serves to modify this effect, as increased reference consumption gives rise to positional externalities. In other words, if we (for the moment) were to abstract from differences in the degree of positionality between the mimicker and the low-ability type, implying that $\Gamma = 0$, then the positionality-weight works to decrease the marginal capital income tax rate. This effect is counteracted (further strengthened) by $\Gamma > 0$ (< 0), as increased reference consumption in this case relaxes (tightens) the self-selection constraint in period t . The interpretation is analogous if $MRS_{c,x}^{1,t} < \hat{MRS}_{c,x}^{2,t}$.

Let us also discuss the marginal capital income tax rates in the light of the empirical evidence regarding relative consumption concerns described above. To simplify (as the appearance of positional preferences generally affects the structure of capital income taxation in a very complex way), we focus on the case illustrated in Proposition 4, in which positional concerns only affect the marginal capital income tax rate of the low-ability type. As with the marginal labor income tax rates, we concentrate the discussion on the contribution of the average degree of positionality by assuming that $\Gamma = 0$; the reason is again the lack of clear empirical evidence regarding differences in the degree of positionality across agent types. In this (highly simplified) case, equation (23) suggests that the absolute value of the marginal capital income tax rate may be substantially smaller than would be predicted in the absence of positional concerns. In fact, with the expression proportional to $1 - \bar{\alpha}$ held

constant,¹³ the positionality effect contributes to scale down the absolute value of the marginal capital income tax rate by a factor between 2 and 5.

It is worth emphasizing once again that there is no direct effect of positionality in equations (23) and (24) that is independent of the self-selection constraint. The following result is a direct consequence of Proposition 4:

Corollary 1. *Suppose that the average degree of positionality as well as the positionality differences between the mimicker and the low-ability type remain constant over time. Then, if leisure is weakly separable from private consumption in the sense that $U_t^i = q_t^i(f_t(c_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_t^{i,x}, G_t, G_{t+1}), z_t^i, G_t, G_{t+1})$ describes the utility function, and the wage ratio is constant, then both marginal capital income tax rates are zero.*

Proof: See Appendix.

Note that while the function $q_t^i(\cdot)$ may still vary across ability-types, the function $f_t(\cdot)$ is the same for both ability-types. Although the above result is based on assumptions that may not seem entirely realistic, it is nevertheless interesting from the perspective of comparison with earlier literature. Corollary 1 implies that the important result derived by Ordober and Phelps (1979), for when capital income taxation is not needed, carries over to our more general case that includes relative consumption concerns.

4.3 Public good provision

Define the marginal rate of substitution between public and private consumption for ability type i , when young and old respectively, in period t as

¹³ In reality, positional concerns of course give rise to indirect effects on the other terms as well. Therefore, this discussion only refers to the direct influence of the positionality effect.

$$MRS_{G,c}^{i,t} = \frac{u_{t,G_t}^i}{u_{t,c}^i},$$

$$MRS_{G,x}^{i,t} = \frac{u_{t-1,G_t}^i}{u_{t-1,x}^i},$$

and similarly for the mimicker. To shorten the formulas to be derived, we shall also use the short notations

$$MB_{t,G} = \sum_i n_t^i MRS_{G,c}^{i,t} n_t^i + \sum_i n_{t-1}^i MRS_{G,x}^{i,t}$$

$$\Omega_t = \lambda_t \hat{u}_{t,c}^2 \left[MRS_{G,c}^{1,t} - \hat{MRS}_{G,c}^{2,t} \right] + \lambda_{t-1} \hat{u}_{t-1,x}^2 \left[MRS_{G,x}^{1,t} - \hat{MRS}_{G,x}^{2,t} \right]$$

for the sum of the marginal willingness to pay for the public good (measured as the marginal rate of substitution between the public good and private consumption) among those alive in period t and the difference in the marginal value attached to the public good between the mimicker and the low-ability type (measured both for the young and old) in period t , respectively.

Consider first the special case with $\xi = 1$, in which the state-variable public good is equivalent to an atemporal control (or flow) variable, i.e. $G_t = g_t$. We can then combine the short notations above with equations (A2), (A3), (A5), (A6), and (A8) to derive¹⁴

$$MB_{t,G} + \Omega_t - \frac{MB_{t,G}}{N_t \gamma_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = 1, \quad (25)$$

which is analogous to the formula for public provision derived in a static model by Aronsson and Johansson-Stenman (2008). The right-hand side is the direct marginal cost of providing the public good, which is measured as the marginal rate of transformation between the public good and the private consumption good and is normalized to one. The left-hand side is interpretable as the marginal benefit of the public good adjusted for the influences of the self-selection constraint and positional

¹⁴ For thorough discussions of public good provision in economies with asymmetric information, although without relative consumption concerns, see Christiansen (1981) and Boadway and Keen (1993).

preferences, respectively. The main differences between a static model and the intertemporal model analyzed here are that the self-selection effect and the positionality effect relevant for public provision in period t reflect the incentives facing generations t and $t-1$, as the high-ability type in each of these generations may act as a mimicker in period t . Note also that if we combine Lemma 1 and equation (25), then the third term on the right-hand side is interpretable to mean that if the young and old low-ability type, respectively, in period t is at least as positional as the corresponding mimicker, so that $\partial\mathcal{L}/\partial\bar{c}_t < 0$, then the positionality effect in period t contributes to increased provision of the public good. The intuition is, of course, that if the private consumption is associated with a positional externality whereas the public good is not, it is welfare improving to increase the public good beyond the level that would be chosen without this externality, i.e. beyond the level that would be chosen if $\partial\mathcal{L}/\partial\bar{c}_t = 0$, *ceteris paribus*. The argument goes the other way around if $\partial\mathcal{L}/\partial\bar{c}_t > 0$, in which case the mimicker is more positional than the low-ability type and sufficiently so to offset the negative effect associated with the average degree of positionality.

Let us then turn to the case with a state variable public good, i.e. where $\xi < 1$. By solving the difference equation (A8) for μ_t , and then using $\mu_t = \gamma_t$ from equation (A9), we show in the appendix that

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} + \Omega_{t+\tau} - \frac{MB_{t+\tau,G}}{N_{t+\tau}\gamma_{t+\tau}} \frac{\partial\mathcal{L}}{\partial\bar{c}_{t+\tau}} \right] [1-\xi]^\tau = 1. \quad (26)$$

Equation (26) essentially combines the policy rule for a state-variable public good in an OLG model without positional preferences derived by Pirttilä and Tuomala (2001), with the policy rule for a control-variable public good summarized by equation (25). Again, the right-hand side is the direct marginal cost of a small increase in the contribution to the public good in period t , which is measured as the marginal rate of transformation between the public good and the private consumption good, whereas the left-hand side measures the marginal benefit of an increase in the contribution to the public good in period t adjusted for the influences of the self-selection constraint

and positional preferences, respectively. Note that this measure of adjusted marginal benefit is intertemporal as an increase in g_t , *ceteris paribus*, affects the utility of each ability-type, as well as the self-selection constraint and the welfare the government attaches to increased reference consumption, in all future periods.

In order to express the optimality condition in terms of individual degrees of positionality, we can substitute equation (17) into equation (26) to obtain:

Proposition 5. *The optimal provision of the public good is given by*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau, G} \frac{1-\Gamma_{t+\tau}}{1-\bar{\alpha}_{t+\tau}} + \Omega_{t+\tau} \right] [1-\xi]^{\tau} = 1. \quad (27)$$

By analogy to the expressions for the marginal income tax rates analyzed above, Proposition 5 uses the decomposition of the positionality effect into the influences of the average degree of positionality and positionality differences between the mimicker and the low-ability type. The interesting aspect here is the implication of positional preferences as captured by $(1-\Gamma_{t+\tau})/(1-\bar{\alpha}_{t+\tau})$, which is interpretable as the “positionality-weight” in period $t+\tau$: $\bar{\alpha}_{t+\tau}$ (the average degree of positionality) contributes to scale up the aggregate instantaneous marginal benefit and, therefore, increases the provision of the public good. The effect of $\Gamma_{t+\tau}$ (the measure of differences in the degree of positionality between the mimicker and the low-ability type) can be either positive or negative. Therefore, a sufficient (not necessary) condition for the positionality weight in period $t+\tau$ to scale up the aggregate instantaneous marginal benefit of the public good in that period is that $\Gamma_{t+\tau} \leq 0$, meaning that the low-ability type is at least as positional as the mimicker. We have more generally:

Proposition 6. *A necessary and sufficient condition for the joint impact of present and future positionality effects to increase the contribution to the public good in period t is that*

$$\sum_{\tau=0}^{\infty} MB_{t+\tau,G} \frac{\Gamma_{t+\tau} - \bar{\alpha}_{t+\tau}}{1 - \bar{\alpha}_{t+\tau}} [1 - \xi]^{\tau} < 0.$$

Hence, a sufficient condition is that the low-ability types are predominantly at least as positional as the mimickers in the sense that

$$\sum_{\tau=0}^{\infty} MB_{t+\tau,G} \frac{\Gamma_{t+\tau}}{1 - \bar{\alpha}_{t+\tau}} [1 - \xi]^{\tau} < 0.$$

Note that even though the second condition in Proposition 5 is much stronger than the first, it still does not require the low-ability types to be at least as positional as the mimickers in all periods.

Following Aronsson and Johansson-Stenman (2008), it is interesting to analyze whether there is some special case in which the second-best policy rule for the public good reduces to a first-best policy rule. To be able to address this issue more thoroughly, note first that individual benefits of the public good are measured by each individual's marginal willingness to pay for a small increment, *ceteris paribus*, i.e. while holding everything else fixed. At the same time, increased public provision typically comes together with other changes, notably that one's own as well as other people's taxes or charges are increased. In one frequently used method, the contingent valuation method, it is typically recommended (see Arrow et al. 1993) that a realistic payment vehicle is used when asking people about their maximum willingness to pay. One commonly used payment vehicle is to ask subjects how they would vote in a referendum where everybody would have to pay a certain amount, the same for all, through increased taxes (or charges) for the improvement. In the standard case where people do not care about relative consumption, this formulation has no important theoretical implication. Here, however, it does. To see this, let us define the marginal rate of substitution between the public good and private consumption at any time, t , conditional on the requirement that $c_t^i - \bar{c}_t$ and $x_{t+1}^i - \bar{c}_{t+1}$ remain constant, which would follow if the willingness to pay question were supplemented by the information that everybody has to pay the same amount for an incremental public good. With reference to equation (1), this measure of instantaneous marginal benefits can be written as:

$$CMRS_{G,c}^{i,t} = \frac{v_{t,G_t}^i}{v_{t,c}^i},$$

$$CMRS_{G,x}^{i,t} = \frac{v_{t-1,G_t}^i}{v_{t-1,x}^i}.$$

By using equations (5a) and (5b), we can write the original measures of instantaneous marginal willingness to pay in terms of these conditional measures as:

$$MRS_{G,c}^{i,t} = (1 - \alpha_t^{i,c}) CMRS_{G,c}^{i,t}, \quad (28)$$

$$MRS_{G,x}^{i,t} = (1 - \alpha_{t-1}^{i,x}) CMRS_{G,x}^{i,t}. \quad (29)$$

The measure of aggregate instantaneous marginal benefits then becomes

$$MB_{t,G} = (1 - \bar{\alpha}_t) CMB_{t,G} [1 + \Psi_t], \quad (30)$$

where $CMB_{t,G} = \sum_i n_t^i CMRS_{G,c}^{i,t} + \sum_i n_{t-1}^i CMRS_{G,x}^{i,t}$, and

$$\Psi_t = \text{cov} \left(\frac{1 - \alpha_t}{1 - \bar{\alpha}_t}, \frac{CMRS_{G,c}^t}{CMRS_{G,c}^t} \right)$$

is the (normalized) covariance between the degree of non-positionality, measured by $1 - \alpha_t$, and the marginal willingness to pay for the public good.

By substituting equation (30) into equation (27), we obtain

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} [CMB_{t,G} [1 + \Psi_t] [1 - \Gamma_{t+\tau}] + \Omega_{t+\tau}] [1 - \xi]^\tau = 1. \quad (31)$$

We are then able to present the following result that explains the conditions under which an intertemporal analogue to the Samuelson rule applies:

Proposition 7. *If (i) the degree of positionality is the same for both ability-types in all periods, as both young and old, (ii) leisure is weakly separable from private and public consumption in the sense that the utility function can be written to read $U_t^i = q_t^i(f_t(c_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_t^{i,x}, G_t, G_{t+1}), z_t^i)$ for all t , and (iii) the instantaneous marginal*

willingness to pay for the public good is always measured by using a payment vehicle where all individuals living at the same time have to pay the same amount, then the optimal provision of the public good is given by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} CMB_{t,G} [1-\xi]^\tau = 1.$$

Proof: see Appendix.

Given that all individuals living at the same time believe they have to pay the same amount at the margin (in which case there is no correlation between the marginal willingness to pay and the degree of positionality at the individual level), then a weighted sum over time of instantaneous marginal benefits should equal the marginal cost of an incremental public good. In other words, an intertemporal analogue to the traditional Samuelson rule applies. What is less clear, perhaps, is how we should apply this or any other policy rule in practice, as we would need information about the willingness to pay for the public good by future generations. However, before taking the discussion about implementation to any greater detail, it is important to know the point of departure. Note also that in the special case where $\xi = 1$, i.e. the case where the public good is a flow variable, Proposition 7 implies that the conventional Samuelson rule holds for each moment in time.

5. The general model with consumption comparisons over time

The analysis carried out in earlier sections is based on the assumption that the only measure of reference consumption at the individual level, in any period, is based on the average consumption in that particular period. As mentioned in Section 2, although this idea accords well with earlier literature on public policy and positional preferences, it neglects the possibility that agents also compare their own current consumption with both their own past consumption and that of other people. In this section, we will present and analyze the more general model that takes such comparisons into account. In all other respects, we make the same assumptions, e.g. with respect to the production sector and available policy instruments, as in the previous sections.

Following Rayo and Becker (2007), we will here thus assume that people care about three different kinds of relative consumption: their own current consumption compared to: (i) the current average consumption when young and when old, i.e. $c_t^i - \bar{c}_t$ and $x_{t+1}^i - \bar{c}_{t+1}$; (ii) their own consumption one period earlier, i.e. $x_{t+1}^i - c_t^i$; and (iii) the average consumption one period earlier when young and when old, i.e. $c_t^i - \bar{c}_{t-1}$ and $x_{t+1}^i - \bar{c}_t$.¹⁵ We can then rewrite equation (1) as follows:

$$\begin{aligned}
U_t^i &= V_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, x_{t+1}^i - c_t^i, c_t^i - \bar{c}_{t-1}, x_{t+1}^i - \bar{c}_t, G_t, G_{t+1}) \\
&= v_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, c_t^i - \bar{c}_{t-1}, x_{t+1}^i - \bar{c}_t, G_t, G_{t+1}) \quad . \quad (32) \\
&= u_t^i(c_t^i, z_t^i, x_{t+1}^i, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1})
\end{aligned}$$

The first line of equation (32) is expressed in terms of these five differences, as well as in terms of leisure and private and public consumption when young and old, respectively. However, since c_t^i and x_{t+1}^i are decision variables of the individual, we can without loss of generality rewrite this utility formulation as the "reduced form" function on the second line,¹⁶ although the partial derivatives will now have a more complex interpretation than on the first line. For example, $v_{t,c}^i$ reflects both the direct utility effect of increased absolute consumption when young and the (presumably negative) utility effect due to lower relative consumption when old compared to when being young. The third line is a more general formulation that corresponds to the second line of equation (1).

Therefore, the second line of equation (32) means that all results derived in Section 4 will continue to hold when people also make comparisons with their own past consumption. The only difference is in terms of the interpretations, where for example people's marginal willingness to pay for increased provision of the public good now takes into account that the money foregone will also change the reference

¹⁵ In Abel (1990), people also compare with the consumption level one period earlier. However, as Abel assumes that people are identical within each period, there is no point in distinguishing between the individual's own earlier consumption and that of others.

¹⁶ On the second line, the effect of $x_{t+1}^i - c_t^i$ on utility is embedded in the effects of c_t^i and x_{t+1}^i .

consumption in the next period (cf. Arrow and Dasgupta 2007). The same modification applies to the interpretations of the marginal rates of substitution underlying the marginal income tax structure. However, others' past consumption will, of course, give rise to positional externalities. The definition of useful measures of this kind of positionality is the task to which we turn next.

5.1 The degree of current versus intertemporal consumption positionality

With equation (32) at our disposal, the concept of "degree of positionality" can be given a broader interpretation than in Sections 2, 3, and 4, where the consumption comparisons only referred to other people's current consumption. Equation (32) allows us to distinguish between the current and intertemporal degree of positionality. If we use the short notations $\Delta_t^{i,c} = c_t^i - \bar{c}_t$ and $\Delta_t^{i,x} = x_{t+1}^i - \bar{c}_{t+1}$ (as we did before), and introduce the additional short notations $\delta_t^{i,c} = c_t^i - \bar{c}_{t-1}$ and $\delta_t^{i,x} = x_{t+1}^i - \bar{c}_t$, we can define the *degree of current consumption positionality* when young and old, respectively, as

$$\alpha_t^{i,c} = \frac{v_{t,\Delta_t^c}^i}{v_{t,\Delta_t^c}^i + v_{t,\delta_t^c}^i + v_{t,c}^i} \quad \text{and} \quad \alpha_t^{i,x} = \frac{v_{t,\Delta_t^x}^i}{v_{t,\Delta_t^x}^i + v_{t,\delta_t^x}^i + v_{t,x}^i}.$$

By analogy, we can define the *degree of intertemporal consumption positionality* when young and old, respectively, as

$$\beta_t^{i,c} = \frac{v_{t,\delta_t^c}^i}{v_{t,\Delta_t^c}^i + v_{t,\delta_t^c}^i + v_{t,c}^i} \quad \text{and} \quad \beta_t^{i,x} = \frac{v_{t,\delta_t^x}^i}{v_{t,\Delta_t^x}^i + v_{t,\delta_t^x}^i + v_{t,x}^i}.$$

As before, the variable $\alpha_t^{i,c}$ is interpreted as reflecting the fraction of the overall utility increase from an additional dollar spent in period t , when young, that is due to the increased consumption relative to the average consumption in period t , whereas $\alpha_t^{i,x}$ can be given a similar interpretation when old in period $t+1$. Similarly, $\beta_t^{i,c}$ and $\beta_t^{i,x}$ reflect the fraction of the overall utility increase from an additional dollar spent in period t and $t+1$ (i.e. when young and old), respectively, that is due to the increased

consumption relative to other people's past consumption. By analogy to the analysis carried out in previous sections, we assume (to begin with) that $0 < \alpha_t^{i,c}, \alpha_t^{i,x}, \beta_t^{i,c}, \beta_t^{i,x} < 1$ for all t . The average degree of current consumption positionality and the average degree of intertemporal consumption positionality become

$$\bar{\alpha}_t = \sum_i \alpha_{t-1}^{i,x} \frac{n_{t-1}^i}{N_t} + \sum_i \alpha_t^{i,c} \frac{n_t^i}{N_t} \in [0,1],$$

$$\bar{\beta}_t = \sum_i \beta_{t-1}^{i,x} \frac{n_{t-1}^i}{N_t} + \sum_i \beta_t^{i,c} \frac{n_t^i}{N_t} \in [0,1].$$

Note that $\bar{\alpha}_t$ and $\bar{\beta}_t$ are measured among those alive in period t .

5.2 The generalized positionality effect

Except that equation (1) is now replaced by equation (32), the Lagrangean takes the same general form as in Section 3. For the same reason as before, the derivative $\partial \mathcal{L} / \partial \bar{c}_t$ plays a key role in the formulas for the marginal income tax rates and contribution to the public good in period t . However, since the positionality concept discussed here has an intertemporal dimension, equation (17) no longer applies. To see this more clearly, and by analogy to the analysis carried out in Section 4, let

$$\Gamma_t = \frac{\lambda_{t-1} \hat{u}_{t-t,x}^2}{\gamma_t N_t} [\hat{\alpha}_{t-1}^{2,x} - \alpha_{t-1}^{1,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2,c} - \alpha_t^{1,c}],$$

$$\Lambda_t = \frac{\lambda_{t-1} \hat{u}_{t-t,x}^2}{\gamma_t N_t} [\hat{\beta}_{t-1}^{2,x} - \beta_{t-1}^{1,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} [\hat{\beta}_t^{2,c} - \beta_t^{1,c}]$$

represent differences in the current and intertemporal degree of positionality, respectively, between the mimicker and the low-ability type in period t . Then, by using the short notation

$$A_{t+k} = \frac{N_{t+k} \gamma_{t+k} [\Gamma_{t+k} - \bar{\alpha}_{t+k}]}{1 - \bar{\alpha}_{t+k}},$$

$$B_{t+k} = \frac{N_{t+k+1} \gamma_{t+k+1} [\Lambda_{t+k+1} - \bar{\beta}_{t+k+1}]}{1 - \bar{\alpha}_{t+k}},$$

we obtain (see Appendix)

$$\frac{\partial \mathcal{E}}{\partial \bar{c}_t} = A_t + B_t + \sum_{i=1}^{\infty} (A_{t+i} + B_{t+i}) \prod_{j=1}^i \frac{\bar{\beta}_{t+j}}{1 - \bar{\alpha}_{t+j-1}}. \quad (33)$$

We will refer to equation (33) as the *generalized positionality effect* in period t , as it provides a generalization of equation (17). There are two important differences between equations (17) and (33): First, the effects of the average degree of positionality and differences in the degree of positionality between the mimicker and the low-ability type, respectively, can be decomposed into two parts – the first and second terms of equation (33) – as the utility function in equations (32) distinguishes between current and intertemporal positionality. Therefore, an increase in the reference consumption in period t , *ceteris paribus*, directly affects the young and the old generation in period t due to the comparison with other people's current consumption (the first term), and also directly affects the young and old generations living in period $t+1$ due to the comparison with other people's past consumption (the second term). Second, a change in the reference consumption in period t gives rise to an intertemporal chain reaction, which is captured by the remaining term of equation (33). The intuition is that the intertemporal aspect of the consumption comparisons, i.e. that other people's past consumption affects the utility, means that the welfare effects of changes in the reference consumption are no longer time-separable (as they were in earlier sections). This is so because a change in the reference consumption today means behavioral adjustments in the future, which, in turn, influence the reference consumption relevant for future generations. The following results are analogous to the last part of Lemma 1:

Lemma 2. *If, from period t and onwards, the low-ability type is at least as positional as the mimicker on average in any of the following senses*

$$(i) \quad \frac{N_t \gamma_t \Gamma_t + N_{t+1} \gamma_{t+1} \Lambda_{t+1}}{1 - \bar{\alpha}_t} + \sum_{i=1}^{\infty} \frac{N_{t+i} \gamma_{t+i} \Gamma_{t+i} + N_{t+i+1} \gamma_{t+i+1} \Lambda_{t+i+1}}{1 - \bar{\alpha}_{t+i}} \prod_{j=1}^i \frac{\bar{\beta}_{t+j}}{1 - \bar{\alpha}_{t+j-1}} \leq 0,$$

$$(ii) \quad \Gamma_{t+k} < \bar{\alpha}_{t+k} \text{ and } \Lambda_{t+k+1} < \bar{\beta}_{t+k+1} \quad \forall k \geq 0,$$

$$(iii) \quad \Gamma_{t+k} < 0 \text{ and } \Lambda_{t+k+1} < 0 \quad \forall k \geq 0,$$

then increased reference consumption in period t reduces the welfare.

Given the assumption that the individual degrees of positionality (both in the current and intertemporal dimensions) are always between zero and one, (i) gives a sufficient condition for when increased reference consumption in period t leads to lower welfare. Yet, analogous to Lemma 1, condition (i) is not necessary, because the measures of the average degrees of positionality contribute to lower welfare as well. Condition (ii) is not necessary either, since $\partial \mathbb{E} / \partial \bar{c}_t$ can clearly be negative even if (ii) does not hold for some k . Note finally that condition (iii), which we refer to for its straightforward interpretation, is actually redundant since it implies condition (ii).

5.3 General optimality results

Let us next turn to the implications for the optimality tax and expenditure results of the more general setup. The main implication of the extension carried out in this section is that equation (33) replaces equation (17); therefore, the first-order conditions for the optimal tax and expenditure problem are still given by equations (A1)-(A9). It is straightforward to show that equations (18) and (19) still characterize the marginal labor income tax rates, equations (21) and (22) the marginal capital income tax rates, and equation (26) the provision of the public good. As a consequence, if we combine equations (18), (19), and (26) with Lemma 2, we obtain the following result:

Proposition 8. *If any of the conditions in Lemma 2 hold so that increased reference consumption leads to lower welfare, ceteris paribus, then the generalized positionality effect in period t contributes to increase the marginal labor income tax rates for both ability-types in period t . Furthermore, if Lemma 2 always applies (i.e. applies for all t) along the general equilibrium path, then the generalized positionality effects contribute to increase the provision of the public good in period t .*

The interpretation of Proposition 8 is straightforward. If the low-ability type is at least as positional as the mimicker on average, loosely speaking, and given the assumption that the individual degrees of positionality are always between zero and one, then the right-hand side of equation (33) is negative. The result then follows by recalling that

the generalized positionality effect in equation (33) replaces equation (17) in equations (18), (19), and (26), whereas the other terms remain as they were in Section 4. In addition, and for reasons similar to those discussed above, positional preferences may affect the marginal capital income tax rates in either direction, although the mechanisms are considerably more complex here than in Section 4.

5.4 Further results under more restrictive assumptions

To gain further insight into the consequences of positional preferences, let us follow the approach in Section 4 by considering the special case where the degree of positionality is constant over time. Therefore, suppose that $\bar{\alpha}_t = \bar{\alpha}$, $\bar{\beta}_t = \bar{\beta}$, $\Gamma_t = \Gamma$, and $\Lambda_t = \Lambda$ for all t . To simplify the calculations further (yet with little loss of generality), we also add the assumptions that the population is constant, that the wage ratio is fixed in each period (meaning that it does not change with the capital stock), and that the interest rate is fixed and equal to r ; the latter implies from equation (A7) that $\gamma_{t+k} = \gamma_t / (1+r)^k$. In this case, equation (33) reduces to the geometric series

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial \bar{c}_t} &= \frac{N\gamma_t}{1-\bar{\alpha}} \left[\Gamma - \bar{\alpha} + \frac{\Lambda - \bar{\beta}}{1+r} \right] \sum_{i=0}^{\infty} \left[\frac{\bar{\beta}}{(1-\bar{\alpha})(1+r)} \right]^i, \\ &= N\gamma_t \frac{\Gamma - \bar{\alpha} + (\Lambda - \bar{\beta})/(1+r)}{1 - \bar{\alpha} - \bar{\beta}/(1+r)} \end{aligned}$$

where in the last step we have implicitly assumed that $0 < \bar{\beta} < (1-\bar{\alpha})(1+r)$ so that the series converges. Define next the average degree of total consumption positionality and the difference between the total degree of consumption positionality between the low-ability type and the mimicker, respectively, in present value terms as

$$\begin{aligned} \rho &= \bar{\alpha} + \frac{\bar{\beta}}{1+r}, \\ \Upsilon &= \Gamma + \frac{\Lambda}{1+r}. \end{aligned}$$

We can then simplify even further to obtain

$$\frac{\partial \mathcal{E}}{\partial \bar{c}_t} = N\gamma_t \frac{\Upsilon - \rho}{1 - \rho}. \quad (34)$$

Equation (34) is analogous to equation (17); the main difference is that the concept of positionality is broader here, as it reflects the current and intertemporal degrees of positionality (the latter was absent in equation (17)). With equation (34) at our disposal, analogues to several results derived earlier – with the same general interpretation as given before – will follow immediately.

To see this, note that the marginal labor income tax structure takes the same form as in Proposition 2,

$$T'_t(w_t^i l_t^i) = \sigma_t^i + [1 - \sigma_t^i] \rho - [1 - \sigma_t^i][1 - \rho] \frac{\Upsilon}{1 - \Upsilon}, \quad (35)$$

for $i=1,2$, with the same general interpretation as before. The generalized positionality effect contributes to increase the marginal labor income tax rates in period t if the low-ability type is at least as positional as the mimicker in the sense that $\Upsilon \leq 0$; however, note that this condition reflects the total degree of consumption positionality and not just the current degree as in Section 4. Similarly, the capital income tax structure is analogous to that in Proposition 4 (with the exception that the wage ratio is constant), i.e.

$$\Phi'_{t+1}(s_t^1 r_{t+1}) = \frac{\lambda_t \hat{u}_{t,x}^2}{\gamma_{t+1} n_t^1 r_{t+1}} \left[MRS_{c,x}^{1,t} - M\hat{R}S_{c,x}^{2,t} \right] \frac{1 - \rho}{1 - \Upsilon}, \quad (36)$$

$$\Phi'_{t+1}(s_t^2 r_{t+1}) = 0. \quad (37)$$

Therefore, the analogue to the result derived by Ordovery and Phelps (1979) continues to apply here as well; if leisure is weakly separable from the other goods in the utility function (and since the wage ratio is constant by assumption), then the marginal capital income tax rate facing each ability-type is zero. In this case, the appearance of positional preferences does not lead to distortionary capital income taxation. The formula for optimal provision of the public good becomes

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1-\Upsilon}{1-\rho} + \Omega_{t+\tau} \right] [1-\xi]^{\tau} = 1, \quad (38)$$

with the same general interpretation as in the context of Proposition 6.

Note finally that our analysis also applies to another special case discussed by Rayo and Becker (2007); namely, when an increase in other people's past consumption, *ceteris paribus*, leads to higher utility for the individual so that $\beta < 0$. As long as $|\bar{\alpha}| \geq |\bar{\beta}|/(1+r) \in (0,1)$, i.e. $\rho \in (0,1)$, in which case equation (34) is well defined, all results discussed here have the same qualitative interpretations independently of whether an increase in other people's past consumption leads to lower (as we assumed above) or higher utility for each individual.

6. Conclusion

As far as we know, the present paper is the first to consider optimal nonlinear income taxation and public good provision in a second-best economy with asymmetric information, where people care about relative consumption, based on a dynamic (OLG) model. The model used is an extension of the standard optimal nonlinear income tax model with two ability-types. Our approach recognizes three mechanisms behind the positional concerns: each individual compares his/her current consumption with (i) his/her own past consumption, (ii) other people's current consumption, and (iii) other people's past consumption.

We began by analyzing the simple case where the comparison with other people's consumption is limited to their current consumption. This situation enabled us to derive several distinct results with respect to the consequences of positional preferences for the marginal income tax structure and public provision. Our results show that the more positional people are on average, *ceteris paribus*, the higher the marginal labor income tax rates. The intuition is that a higher marginal labor income tax rate reduces the hours of work and, therefore, the resources available for private consumption (if the public consumption is held constant). As a consequence, it also reduces the reference consumption by which people compare their own consumption.

However, the effect of positional preferences on the marginal labor income tax rates also depends on whether the (mimicked) low-ability type is more or less positional than the mimicker, as this will determine whether an increase or a decrease in the reference consumption works to relax the self-selection constraint. By using the (scarce) available empirical evidence, our model implies that the optimal marginal labor income tax rates are likely to be much higher than suggested by models without relative consumption comparisons.

The effects of positional preferences on the marginal capital income tax rates are ambiguous in general. This also accords very well with intuition, as the marginal capital income tax rates reflect a tradeoff between present and future consumption, and the consumers are allowed to be positional in both periods. However, in the special case where the degree of positionality is constant over time and across agent-types, plausible empirical estimates suggest that the marginal capital income tax rate of the low-ability type may be substantially smaller in absolute value than in the conventional optimal income tax model. In addition, if the degree of positionality is constant over time for all agent-types, we are able to reproduce the well-known result of Ordober and Phelps (1979), although the consumers have positional preferences in our framework; in other words, if leisure is weakly separable from the other goods in the utility function, and with constant relative wage rates, the marginal capital income tax rates should be zero.

As the public good in our model is a state variable, the effects of positional preferences are more complex than in the static model analyzed by Aronsson and Johansson-Stenman (2008). The reason is that the marginal benefit of an incremental contribution to the public good in period t is intertemporal (it reflects the present value of all future instantaneous marginal benefits), meaning that it is governed by the preferences of the current and all future generations. If an individual's marginal willingness to pay for the public good is measured by holding the contributions made by others constant, it follows that the more positional people are on average now and in the future, *ceteris paribus*, the larger the optimal public provision compared to the case where relative consumption comparisons are absent. However, it matters also here (as it does for the marginal income tax structure) whether the low-ability type is more or less positional than the mimicker (both at present and in the future), as this

determines whether an incremental contribution to the public good in period t relaxes or tightens the self-selection constraint. By analogy to Aronsson and Johansson-Stenman (2008), it is shown here that the adjustment of the formula for public provision implied by relative consumption concerns depends on whether each individual's marginal willingness to pay is elicited by holding everything else constant or by using a payment vehicle implying that each individual knows that other agents have to pay too.

Adding the intertemporal aspects of relative consumption comparisons, i.e. that each individual also compares his/her current consumption with his/her own and others' past consumption, gives a richer structure, as it enables us to distinguish between the current and intertemporal degrees of consumption positionality. It is first shown that comparisons with own past consumption do not affect the optimal policy rules, since such comparisons are internalized by each individual. However, comparisons with others' past consumption complicate the analysis and the interpretations considerably, as the welfare effects of a change in the reference consumption in period t effectively become dependent on the preferences of all future generations. Still, we were able to show that for the special case where the degrees of (current and intertemporal) consumption positionality are constant over time, and with some additional assumptions, many of the results derived earlier in the paper carry over to this more general framework. More specifically, the appearance of positional preferences will affect the marginal income tax structure and public provision in the same general way as in the simpler model, with the exception that the positionality concept is broader in the sense that each individual also makes intertemporal consumption comparisons. In other words, the results referred to above – which were derived in a model without intertemporal consumption comparisons – will under certain conditions continue to apply in a framework where each individual also compares his/her current consumption with other people's past consumption.

Finally, although the present paper in several respects generalizes the literature on optimal taxation and public expenditures when relative consumption matters, there are still many important aspects left to explore. Examples include public provision of private goods, heterogeneous relative consumption concerns (e.g. that people may compare themselves more with their own ability-type), a multi-country setting, and

the case where also relative leisure matters. We hope to address these issues in future research.

Appendix

First-order conditions

The first-order conditions for l_t^1 , c_t^1 , x_{t+1}^1 , l_t^2 , c_t^2 , x_{t+1}^2 , K_{t+1} , G_t and g_t are given by

$$-\frac{\partial W}{\partial(n_t^1 U_t^1)} n_t^1 u_{t,z}^1 + \lambda_t \hat{u}_{t,z}^2 \left[\phi_t + l_t^1 \frac{\partial \phi_t}{\partial l_t^1} \right] + \gamma_t n_t^1 w_t^1 = 0, \quad (\text{A1})$$

$$\frac{\partial W}{\partial(n_t^1 U_t^1)} n_t^1 u_{t,c}^1 - \lambda_t \hat{u}_{t,c}^2 - \gamma_t n_t^1 + \frac{n_t^1}{N_t} \frac{\partial \mathfrak{F}}{\partial \bar{c}_t} = 0, \quad (\text{A2})$$

$$\frac{\partial W}{\partial(n_t^1 U_t^1)} n_t^1 u_{t,x}^1 - \lambda_t \hat{u}_{t,x}^2 - \gamma_{t+1} n_t^1 + \frac{n_t^1}{N_{t+1}} \frac{\partial \mathfrak{F}}{\partial \bar{c}_{t+1}} = 0, \quad (\text{A3})$$

$$-\left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,z}^2 + \lambda_t \hat{u}_{t,z}^2 l_t^1 \frac{\partial \phi_t}{\partial l_t^2} + \gamma_t n_t^2 w_t^2 = 0, \quad (\text{A4})$$

$$\left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,c}^2 - \gamma_t n_t^2 + \frac{n_t^2}{N_t} \frac{\partial \mathfrak{F}}{\partial \bar{c}_t} = 0, \quad (\text{A5})$$

$$\left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,x}^2 - \gamma_{t+1} n_t^2 + \frac{n_t^2}{N_{t+1}} \frac{\partial \mathfrak{F}}{\partial \bar{c}_{t+1}} = 0, \quad (\text{A6})$$

$$\gamma_{t+1}(1+r_{t+1}) - \gamma_t + \lambda_{t+1} \hat{u}_{t+1,z}^2 l_{t+1}^1 \frac{\partial \phi_{t+1}}{\partial K_{t+1}} = 0, \quad (\text{A7})$$

$$\sum_{i=1}^2 \left[\frac{\partial W}{\partial(n_t^i U_t^i)} n_t^i u_{t,G_t}^i + \frac{\partial W}{\partial(n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1,G_t}^i \right] + \lambda_t \left[u_{t,G_t}^2 - \hat{u}_{t,G_t}^2 \right], \quad (\text{A8})$$

$$+ \lambda_{t-1} \left[u_{t-1,G_t}^2 - \hat{u}_{t-1,G_t}^2 \right] + \mu_{t+1}(1-\xi) - \mu_t = 0$$

$$-\gamma_t + \mu_t = 0, \quad (\text{A9})$$

where we have used $w_t^i = F_L(L_t^1, L_t^2, K_t)$ for $i=1,2$, and $r_t = F_K(L_t^1, L_t^2, K_t)$ from the first-order conditions of the firm.

Proof of Lemma 1

From equation (1) we have that $u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t}^i$, $u_{t,\bar{c}_t}^i = -v_{t,\Delta_t}^i$, $u_{t,x}^i = v_{t,x}^i + v_{t,\Delta_{t+1}}^i$ and $u_{t,\bar{c}_{t+1}}^i = -v_{t,\Delta_{t+1}}^i$, so

$$u_{t,\bar{c}_t}^i = -\alpha_t^i u_{t,c}^i, \quad (\text{A10})$$

$$u_{t,\bar{c}_{t+1}}^i = -\beta_t^i u_{t,x}^i. \quad (\text{A11})$$

Corresponding expressions hold for the mimicker. By combining equations (17), (A10), and (A11), and the corresponding expressions for the mimicker, we obtain

$$\begin{aligned} \frac{\partial \mathfrak{F}}{\partial \bar{c}_t} = & -\sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i u_{t-1}^i)} n_{t-1}^i \alpha_{t-1}^{i,x} u_{t-1,x}^i - \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i u_t^i)} n_t^i \alpha_t^{i,c} u_{t,c}^i \\ & - \lambda_{t-1} [\alpha_{t-1}^{2,x} u_{t-1,x}^2 - \hat{\alpha}_{t-1}^{2,x} \hat{u}_{t-1,x}^2] - \lambda_t [\alpha_t^{2,c} u_{t,c}^2 - \hat{\alpha}_t^{2,c} \hat{u}_{t,c}^2] \end{aligned} \quad (\text{A12})$$

Note that equations (19), (20), (22), and (23) imply

$$\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,c}^1 = \lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathfrak{F}}{\partial \bar{c}_t}, \quad (\text{A13})$$

$$\frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^2 u_{t,c}^2 = -\lambda_t u_{t,c}^2 + \gamma_t n_t^2 - \frac{n_t^2}{N_t} \frac{\partial \mathfrak{F}}{\partial \bar{c}_t}, \quad (\text{A14})$$

$$\frac{\partial W}{\partial (n_{t-1}^1 U_{t-1}^1)} n_{t-1}^1 u_{t-1,x}^1 = \lambda_{t-1} \hat{u}_{t-1,x}^2 + \gamma_t n_{t-1}^1 - \frac{n_{t-1}^1}{N_t} \frac{\partial \mathfrak{F}}{\partial \bar{c}_t}, \quad (\text{A15})$$

$$\frac{\partial W}{\partial (n_{t-1}^2 U_{t-1}^2)} n_{t-1}^2 u_{t-1,x}^2 = -\lambda_{t-1} v_{t-1,x}^2 + \gamma_t n_{t-1}^2 - \frac{n_{t-1}^2}{N_t} \frac{\partial \mathfrak{F}}{\partial \bar{c}_t}. \quad (\text{A16})$$

Substituting equations (A13)-(A16) and the definition of Γ_t into equation (A12) gives equation (17).

The Marginal Labor Income Tax Rates

Consider the tax formula for the low-ability type. By combining equations (A1) and (A2), we obtain

$$\frac{u_{t,z}^1}{u_{t,c}^1} \left[\lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathcal{F}}{\partial c_t} \right] = \lambda_t \hat{u}_{t,z}^2 \left[\phi_t + l_t^1 \frac{\partial \phi_t}{\partial l_t^1} \right] + \gamma_t n_t^1 w_t^1. \quad (\text{A17})$$

By substituting $T'(w_t^1 l_t^1) w_t^1 = w_t^1 - u_{t,z}^1 / u_{t,c}^1$ into equation (A8) and rearranging, we obtain equation (18). The marginal labor income tax rate of the high-ability type, equation (19), can be derived in a similar way.

To derive equation (20), we combine equations (17) and (18) to obtain

$$\begin{aligned} T'(w_t^1 l_t^1) &= \frac{\lambda_t^*}{w_t^1 n_t^1} \left[MRS_{z,c}^{1,t} - \hat{MRS}_{z,c}^{2,t} \left[\phi_t + \frac{\partial \phi_t}{\partial l_t^1} l_t^1 \right] \right] \\ &\quad - \frac{MRS_{z,c}^{1,t}}{\gamma_t w_t^1 N_t} \frac{1}{1 - \bar{\alpha}_t} \left[-\lambda_{t-1} (\alpha_{t-1}^{1,x} - \hat{\alpha}_{t-1}^{2,x}) \hat{u}_{t-1,x}^2 - \lambda_t (\alpha_t^{1,c} - \hat{\alpha}_t^{2,c}) \hat{u}_{t,c}^2 \right. \\ &\quad \left. - \gamma_t N_t \bar{\alpha}_t \right]. \quad (\text{A18}) \end{aligned}$$

Then, by using $MRS_{z,c}^{1,t} / w_t^1 = 1 - T'(w_t^1 l_t^1)$ and rearranging, we obtain equation (20) for the low-ability type. The marginal labor income tax rate for the high-ability type can be derived in a similar way.

The Marginal Capital Income Tax Rates

Let us consider the marginal capital income tax rate of the low-ability type. By combining equations (A2) and (A3), we obtain

$$MRS_{c,x}^1 \left[\lambda_t \hat{u}_{t,x}^2 - \gamma_{t+1} n_t^1 + \frac{n_t^1}{N_{t+1}} \frac{\partial \mathcal{F}}{\partial c_{t+1}} \right] = \lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathcal{F}}{\partial c_t}. \quad (\text{A19})$$

We then use equations (9) and (A7) to derive $MRS_{c,x}^{1,t} = 1 + r_{t+1} - r_{t+1} \Phi'(s_t^1 r_{t+1})$ and $\gamma_t = \gamma_{t+1} (1 + r_{t+1}) + \lambda_{t+1} u_{t+1,z}^2 l_{t+1}^1 [\partial \phi_{t+1} / \partial K_{t+1}]$, respectively. Substituting into equation (A19) and rearranging, we obtain equation (21). Equation (22) can be derived in a similar way.

To derive equations (23) and (24), let us substitute $[\partial \mathcal{L} / \partial \bar{c}_t] / [\gamma_t N_t] = [\Gamma_t - \bar{\alpha}_t] / [1 - \bar{\alpha}_t]$ as well as the corresponding expression for period $t+1$ into equations (21) and (22).

We shall also use the short notations

$$\delta_t^1 = \frac{\lambda_t \hat{u}_{t,x}^2}{\gamma_{t+1} n_t^1 r_{t+1}} \left[MRS_{c,x}^{1,t} - M\hat{R}S_{c,x}^{2,t} \right] - \frac{\lambda_{t+1} \hat{u}_{t+1,z}^2 l_{t+1}^1}{\gamma_{t+1} r_{t+1}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}},$$

$$\delta_t^2 = - \frac{\lambda_{t+1} \hat{u}_{t+1,z}^2 l_{t+1}^1}{\gamma_{t+1} r_{t+1}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}$$

to represent the self-selection terms in equations (21) and (22). The marginal capital income tax rates can then be rewritten as

$$\Phi'_{t+1}(s_t^i r_{t+1}) = \delta_t^i + \frac{\gamma_t}{\gamma_{t+1}} \frac{1}{r_{t+1}} \frac{\bar{\alpha}_t - \Gamma_t}{1 - \bar{\alpha}_t} - \frac{1 + r_{t+1}}{r_{t+1}} \frac{\bar{\alpha}_{t+1} - \Gamma_{t+1}}{1 - \Gamma_{t+1}}$$

$$+ \Phi'_{t+1}(s_t^i r_{t+1}) \frac{\bar{\alpha}_{t+1} - \Gamma_{t+1}}{1 - \Gamma_{t+1}}$$

for $i=1, 2$. Now, using $MRS_{c,x}^{i,t} = 1 + r_{t+1} - r_{t+1} \Phi'_{t+1}(s_t^i r_{t+1})$ and rearranging, we obtain

$$\Phi'_{t+1}(s_t^i r_{t+1}) = \delta_t^i \frac{1 - \bar{\alpha}_{t+1}}{1 - \Gamma_{t+1}} - \frac{\gamma_t}{\gamma_{t+1}} \frac{1}{r_{t+1}} \frac{\bar{\alpha}_t - \Gamma_t}{1 - \bar{\alpha}_t} \frac{1 - \bar{\alpha}_{t+1}}{1 - \Gamma_{t+1}}$$

$$+ \frac{1 + r_{t+1}}{r_{t+1}} \frac{\bar{\alpha}_{t+1} - \Gamma_{t+1}}{1 - \Gamma_{t+1}}. \quad (\text{A20})$$

Assuming $\rho_{t+1} = \rho_t = \rho$ and $\Gamma_{t+1} = \Gamma_t = \Gamma$, and then substituting into equation (A20), gives

$$\Phi'_{t+1}(s_t^i r_{t+1}) = \delta_t^i \frac{1 - \bar{\alpha}}{1 - \Gamma} + \frac{(1 + r_{t+1}) - \gamma_t / \gamma_{t+1}}{r_{t+1}} \frac{\bar{\alpha} - \Gamma}{1 - \Gamma}. \quad (\text{A21})$$

By substituting $(1 + r_{t+1}) - \gamma_t / \gamma_{t+1} = -[\lambda_{t+1} / \gamma_{t+1}] \hat{u}_{t+1,x}^2 l_{t+1}^1 [\partial \phi_{t+1} / \partial K_{t+1}]$ into equation (A21) and using the definition of δ_t^i , we obtain equations (23) and (24).

The proof of Corollary 1 follows from acknowledging that the mimicker and the low-ability type differ only with respect to preferences and the use of leisure. Given the separability assumption, and that the consumers share a common sub-utility function

$f_t(\cdot)$, it follows that $MRS_{c,x}^{1,t} = M\hat{R}S_{c,x}^{2,t}$. In addition, a constant wage ratio implies that $\partial\phi_{t+1}/\partial K_{t+1} = 0$. These conditions substituted into equations (23) and (24) imply Corollary 1.

Public Good Provision

Rewrite equation (A8) as

$$\sum_{i=1}^2 \left[\frac{\partial W}{\partial(n_t^i u_t^i)} n_t^i u_{t,c}^i MRS_{G,c}^{i,t} + \frac{\partial W}{\partial(n_{t-1}^i u_{t-1}^i)} n_{t-1}^i u_{t-1,x}^i MRS_{G,x}^{i,t} \right] + \lambda_t [u_{t,G}^2 - \hat{u}_{t,G}^2] + \lambda_{t-1} [u_{t-1,G}^2 - \hat{u}_{t-1,G}^2] + \mu_{t+1}(1 - \xi) - \mu_t = 0 \quad (\text{A22})$$

By substituting equations (A13)-(A16) into equation (A22), we obtain

$$\left[n_t^1 MRS_{G,c}^{1,t} + n_t^2 MRS_{G,c}^{2,t} + n_{t-1}^1 MRS_{G,x}^{1,t} + n_{t-1}^2 MRS_{G,x}^{2,t} \right] \left[1 - \frac{N_t}{\gamma_t} \frac{\partial \mathcal{F}}{\partial \bar{c}_t} \right] + \frac{\lambda_t}{\gamma_t} \hat{u}_{t,c}^2 [MRS_{G,c}^{1,t} - M\hat{R}S_{G,c}^{2,t}] + \frac{\lambda_{t-1}}{\gamma_t} \hat{u}_{t-1,x}^2 [MRS_{G,x}^{1,t} - M\hat{R}S_{G,x}^{2,t}] + \mu_{t+1} [1 - \xi] - \mu_t = 0 \quad (\text{A23})$$

By solving equation (A23) for μ_t , and then using $\mu_t = \gamma_t$ from equation (A9), we obtain equation (26).

Proposition 7 follows directly from equation (33). Since $f_t(\cdot)$ is the same for both ability-types, and the mimicker has the same absolute as well as relative consumption as the low-ability type, it follows that $M\hat{R}S_{G,c}^{2,t} = MRS_{G,c}^{1,t}$ and $M\hat{R}S_{G,x}^{2,t} = MRS_{G,x}^{1,t}$ so that $\Omega_t = 0$. It also follows that $\hat{\alpha}_t^{2,c} = \alpha_t^{1,c}$ and $\hat{\alpha}_t^{2,x} = \alpha_t^{1,x}$ so that $\Gamma_t = 0$. Note also that, although the function $f(\cdot)$ is the same for both ability-types, the function $q^i(\cdot)$ can still vary between agents. Furthermore, by assuming that the degree of positionality is the same for both ability-types, we have $\Psi = 0$. Substituting $\Omega_t = 0$, $\Gamma_t = 0$, and $\Psi = 0$ into equation (33) implies Proposition 7.

The generalized positionality effect

The derivative of the Lagrangean with respect to \bar{c}_t can in this more general case be written as

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1, \bar{c}_t}^i + \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i u_{t, \bar{c}_t}^i \\ &+ \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t+1}^i U_{t+1}^i)} n_{t+1}^i u_{t+1, \bar{c}_t}^i + \lambda_{t-1} \left[u_{t-1, \bar{c}_t}^2 - \hat{u}_{t-1, \bar{c}_t}^2 \right] \\ &+ \lambda_t \left[u_{t, \bar{c}_t}^2 - \hat{u}_{t, \bar{c}_t}^2 \right] + \lambda_{t+1} \left[u_{t+1, \bar{c}_t}^2 - \hat{u}_{t+1, \bar{c}_t}^2 \right] \end{aligned} \quad (\text{A25})$$

From equation (32) we have

$$u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t^c}^i + v_{t,\delta_t^c}^i = \frac{v_{t,\Delta_t^c}^i}{\alpha_t^{i,c}} = \frac{v_{t,\delta_t^c}^i}{\beta_t^{i,c}},$$

$$u_{t,x}^i = v_{t,x}^i + v_{t,\Delta_t^x}^i + v_{t,\delta_t^x}^i = \frac{v_{t,\Delta_t^x}^i}{\alpha_t^{i,x}} = \frac{v_{t,\delta_t^x}^i}{\beta_t^{i,x}},$$

$$u_{t,\bar{c}_t}^i = -v_{t,\Delta_t^c}^i - v_{t,\delta_t^x}^i,$$

$$u_{t,\bar{c}_{t-1}}^i = -v_{t,\delta_t^c}^i,$$

$$u_{t,\bar{c}_{t+1}}^i = -v_{t,\Delta_t^x}^i,$$

so

$$u_{t,\bar{c}_t}^i = -\alpha_t^{i,c} u_{t,c}^i - \beta_t^{i,x} u_{t,x}^i, \quad (\text{A26})$$

$$u_{t,\bar{c}_{t-1}}^i = -\beta_t^{i,c} u_{t,c}^i, \quad (\text{A27})$$

$$u_{t,\bar{c}_{t+1}}^i = -\alpha_t^{i,x} u_{t,x}^i, \quad (\text{A28})$$

which substituted into equation (A25) imply

$$\begin{aligned}
\frac{\partial \mathcal{E}}{\partial \bar{c}_t} &= -\sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i \alpha_{t-1}^{i,x} u_{t-1,x}^i \\
&\quad - \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i \left[\alpha_t^{i,c} u_{t,c}^i + \beta_t^{i,x} u_{t,x}^i \right] \\
&\quad + \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t+1}^i U_{t+1}^i)} n_{t+1}^i \beta_{t+1}^{i,c} u_{t+1,c}^i + \lambda_{t-1} \left[-\alpha_{t-1}^{2,x} u_{t-1,x}^2 + \hat{\alpha}_{t-1}^{2,x} u_{t-1,x}^2 \right] \\
&\quad + \lambda_t \left[-\alpha_t^{2,c} u_{t,c}^2 - \beta_t^{2,x} u_{t,x}^2 + \hat{\alpha}_t^{2,c} \hat{u}_{t,c}^2 + \hat{\beta}_t^{2,x} \hat{u}_{t,x}^2 \right] \\
&\quad + \lambda_{t+1} \left[-\beta_{t+1}^{2,c} u_{t+1,c}^2 + \hat{\beta}_{t+1}^{2,c} \hat{u}_{t+1,c}^2 \right]. \tag{A29}
\end{aligned}$$

By substituting equations (A13)-(A16) into equation (A29), and collecting terms, we obtain

$$\begin{aligned}
\frac{\partial \mathcal{E}}{\partial \bar{c}_t} &= \frac{\partial \mathcal{E}}{\partial \bar{c}_{t+1}} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t} - N_t \gamma_t \frac{\bar{\alpha}_t}{1-\bar{\alpha}_t} - N_{t+1} \gamma_{t+1} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t} \\
&\quad + \frac{\lambda_{t-1} \hat{u}_{t-1,x}^2}{1-\bar{\alpha}_t} \left[\hat{\alpha}_{t-1}^{2,x} - \alpha_{t-1}^{1,x} \right] + \frac{\lambda_t \hat{u}_{t,c}^2}{1-\bar{\alpha}_t} \left[\hat{\alpha}_t^{2,c} - \alpha_t^{1,c} \right] \\
&\quad + \frac{\lambda_t \hat{u}_{t,x}^2}{1-\bar{\alpha}_t} \left[\hat{\beta}_t^{2,x} - \beta_t^{1,x} \right] + \frac{\lambda_{t+1} \hat{u}_{t+1,c}^2}{1-\bar{\alpha}_t} \left[\hat{\beta}_{t+1}^{2,c} - \beta_{t+1}^{1,c} \right], \tag{A30} \\
&= \frac{1}{1-\bar{\alpha}_t} \left[\bar{\beta}_{t+1} \frac{\partial \mathcal{E}}{\partial \bar{c}_{t+1}} + N_t \gamma_t [\Gamma_t - \bar{\alpha}_t] + N_{t+1} \gamma_{t+1} [\Lambda_{t+1} - \bar{\beta}_{t+1}] \right]
\end{aligned}$$

where we have used the short notations Γ_t and Λ_t as defined earlier. Using the short notations

$$\begin{aligned}
A_t &= \frac{N_t \gamma_t [\Gamma_t - \bar{\alpha}_t]}{1-\bar{\alpha}_t}, \\
B_t &= \frac{N_{t+1} \gamma_{t+1} [\Lambda_{t+1} - \bar{\beta}_{t+1}]}{1-\bar{\alpha}_t}, \\
\varphi_t &= \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t},
\end{aligned}$$

the recursive equation (A30) can more conveniently be rewritten and expanded as follows:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= A_t + B_t + \varphi_t \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} = A_t + B_t + \varphi_t \left[A_{t+1} + B_{t+1} + \varphi_{t+1} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+2}} \right] \\
&= A_t + B_t + \varphi_t \left[A_{t+1} + B_{t+1} + \varphi_{t+1} \left[A_{t+2} + B_{t+2} + \varphi_{t+2} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+3}} \right] \right] \quad . \quad (A31) \\
&= A_t + B_t + [A_{t+1} + B_{t+1}] \varphi_t + [A_{t+2} + B_{t+2}] \varphi_t \varphi_{t+1} + [A_{t+3} + B_{t+3}] \varphi_t \varphi_{t+1} \varphi_{t+2} \dots \\
&= A_t + B_t + \sum_{i=1}^{\infty} [A_{t+i} + B_{t+i}] \prod_{j=1}^i \varphi_{t+j-1}
\end{aligned}$$

Substituting back $\varphi_t = \bar{\beta}_{t+1}/(1 - \bar{\alpha}_t)$ into equation (A31) implies equation (33).

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