Essays on Redistribution and Local Public Expenditures

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Abstract

This thesis consists of a summary and four papers. The first two papers are theoretical contributions within the area of optimal taxation and public expenditures under asymmetric information between the government and the private sector, and the last two are empirical contributions to the literature on local public expenditures.

Paper [I] concerns the optimal use of publicly provided private goods in an economy with equilibrium unemployment. The paper points out that imperfect competition in the labor market gives rise to additional policy incentives associated with the self-selection constraint, which motivates adjustments in the public provision of private goods. It also addresses employment related motives behind publicly provided private goods.

Paper [II] addresses optimal income and commodity taxation in a dynamic economy, where used durable goods are subject to second-hand trade. In our framework, the government is unable to directly control second-hand transactions via commodity taxation. We show how the appearance of a second-hand market affects the use of commodity taxation on the new durable goods as well as the use of income taxation.

Paper [III] relates the existence and size of the flypaper effect to observable municipal characteristics. The analysis is based on a political economy model, which implies that the effect of a change in the tax base on the majority voter’s tax share will be crucial for finding a flypaper effect. The empirical part is based on Swedish data on municipal expenditures and revenues for the period 1996-2004. The results show that the size of the flypaper effect varies among municipalities depending on the relative composition of grant and tax base.

In Paper [IV], the composition of municipal expenditures in Sweden is analyzed by estimating a demand system for local public services, in which tax revenue collection is treated as endogenous. The estimation is based on the QAIDS specification and uses panel data for the period 1998-2005 and for six local public services. The results show that the point estimates of the income elasticities are positive (with one exception), whereas the point estimates of the own-price elasticities are negative and less than one.

Key Words: optimal taxation, unemployment, durable goods, flypaper effects, intergovernmental relations, demand system, local government spending
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I arrived at the Department of Economics filled with expectations. The staff’s polite reception disappeared before long and was replaced by confusion around my assignment. I had been encouraged to study for a PhD, but my specific research task was lacking. Since then, the situation around my thesis has been characterized by vagueness and numerous assumptions. Clarifying my assignment felt nearly impossible; in fact, it was comparable to entering a far-away castle. I remember there being two assistants that were designated to elucidate the confusion regarding my undertaking, assistants that were always assigned to me during my stay at the Department. After a while it was decided that I should disrupt my writings and, instead, focus on a temporary job at the institution of higher education. This meant that I practically had to stay all night at campus. Later on the administration dispatched me to America in order to return to my learning and, one again, to clarify the confusion regarding my thesis. My obligation has slowly but surely become clearer; the fog has lifted around the fortress. The assistants have recently come with unconfirmed reports indicating that I may be close to the end of my work. According to this rumour, higher authorities within the administration are expected to prepare some kind of announcement. To my understanding, a grading committee is supposed to be near a decision on the subject of my educational rank. I look forward to verifying this information and I entertain a strong confidence that the castle soon will be taken.

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This thesis consists of a summary and the following four papers:


1 Introduction

This thesis consists of four papers. Papers [I] and [II] are theoretical contributions within the area of optimal taxation and public expenditures under asymmetric information between the government and the private sector. Paper [I] concerns the optimal use of publicly provided private goods in an economy with equilibrium unemployment, whereas Paper [II] analyzes optimal income and commodity taxation in a dynamic economy where used durable goods are subject to second-hand trade. Paper [III] and [IV] are empirical contributions to the literature on local public expenditures. Paper [III] relates the existence and size of the flypaper effect to observable municipal characteristics, whereas Paper [IV] analyzes the composition of local public services using Swedish municipality data.

This introductory chapter is organized as follows. Section 2 presents an overview of relevant earlier literature for the first part of the thesis and summarizes Papers [I] and [II]. Section 3 is concerned with the background to the second part of the thesis and summarizes Papers [III] and [IV].

2 Optimal Nonlinear Taxation and Provision of Public Services

There is a large literature dealing with the optimal use of nonlinear income taxation and supplemental commodity taxation.\(^1\) The underlying decision-problem is that of redistribution under asymmetric information: the government is unable to observe individual ability (in which case it is also unable to implement the first best resource allocation via ability-type specific lump-sum taxes).

To exemplify the policy incentives underlying the use of income taxation, consider the two-type optimal income tax model developed by Stern (1982) and Stiglitz (1982), where a distinction is made between a low-ability type and a high-ability type, which differ in productivity (measured in terms of the gross wage rate). Furthermore, assume that the government wants to redistribute from the high-ability type to the low-ability type. In this case, the government

\(^1\)Seminal contributions to the literature on optimal nonlinear taxation and redistribution have been made by Mirrlees (1971, 1976), Atkinson and Stiglitz (1976) and Atkinson (1977), Stern (1982), Stiglitz (1982) and Edwards et al. (1994).
must not redistribute in such a way that the high-ability type would like to pretend to be a low-ability type, i.e. become a mimicker, in order to gain from the redistribution. The self-selection constraint serves to prevent mimicking: the self-selection constraint that may bind in this specific example means that the high-ability type must (weakly) prefer the allocation intended for him/her over the allocation intended for the low-ability type. As a consequence, the government may use the tax system to make mimicking less attractive which, in turn, creates further room for redistribution. One way to relax the self-selection constraint is by imposing a marginal labor income tax on the low-ability type (the agent that might be mimicked in our example). By relaxing the self-selection constraint, the government may transfer more resources from the high-ability to the low-ability type without creating strong enough incentives for the high-ability type to become a mimicker.

For reasons similar to those described above, other tax instruments might be useful as a supplement to the labor income tax. If the high-ability type is the potential mimicker, differentiated commodity taxation may contribute to make mimicking less attractive by encouraging the consumption of goods that are substitutable for leisure and discouraging the consumption of goods that are complementary with leisure (Edwards et al., 1994). The intuition is that the mimicker consumes more leisure than the low-ability type (as the mimicker is more productive than the low-ability type and needs to supply fewer hours of work than the low-ability type to reach the income-consumption point intended for the low-ability type). Brett (1997) studies redistribution under asymmetric information in an intertemporal setting. He argues that the use of a non-zero marginal capital income tax on the low-ability type (who also is the mimicked agent in this framework) may contribute to relax the self-selection constraint, since the marginal rate of substitution between current and future consumption may differ between the low-ability type and the mimicker. This is discussed in more detail below.

Another possible mechanism for relaxing the self-selection constraint goes via the wage distribution. This mechanism requires, of course, that the gross wage rates are endogenous. A higher gross wage rate for the low-ability type, relative to the gross wage rate for the high-ability type, makes mimicking less attractive if the high-ability type is the mimicker. Naito (1999) shows, in the context of a static model, that the use of differentiated commodity taxation
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is also motivated if it contributes to relax the self-selection constraint via the wage distribution. Pirtilä and Tuomala (2001) argue that the use of marginal capital income taxes may also be justified for a similar reason. This is further discussed below.

2.1 Labor Market Distortions

Earlier studies on optimal nonlinear taxation often assume a competitive labor market. Allowing for imperfect competition in the labor markets will, in general, imply that the equilibrium is characterized by unemployment. Considering that many industrialized countries have experienced high unemployment rates for a long time, it is clearly relevant to consider imperfectly competitive labor markets in the context of optimal taxation and provision of public services. Note also that reducing unemployment is, by itself, a way of accomplishing redistribution.

Marceau and Boudway (1994) analyze minimum wages and unemployment insurance as additional policy instruments in an optimal tax and expenditure problem under asymmetric information. They conclude that a higher minimum wage may be welfare improving, if the higher expected utility for the low-ability type makes up for the tighter self-selection constraint (the high-ability type is assumed to act as mimicker). Another situation with imperfect competition in the labor market arises when trade-unions have an influence on the wage formation. Aronsson and Sjögren (2003) assume that the hourly gross wage rates are decided upon by monopoly unions and analyze optimal taxation and provision of public goods in a mixed tax problem with two ability-types. They show that union wage formation implies that a broader set of variables affects the distribution of gross wage rates than under competitive labor markets, and that the appearance of equilibrium unemployment provides an incentive for the government to use the policy instruments in order to increase the number of employed persons. This will modify the policy rules for the marginal income tax rates, the commodity taxes and the public good.

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2 An overview of theories of wage formation in unionized labor markets is given by Oswald (1985).
3 There is also a literature dealing with optimal tax progression under imperfect competition in the labor market (Fuest and Huber, 1997; Aronsson and Sjögren, 2004a,b).
4 There is a related literature dealing with different aspects of union wage setting and optimal linear income taxation in an open economy (see Boeters and Schneider, 1999; Koskela and Schöb, 2000).
Engström (2002) models optimal redistribution policy and unemployment by using a search unemployment model, where unemployed workers derive disutility from search (the model is further discussed in Pissarides, 2000).\textsuperscript{5} He finds that the policy maker may improve the welfare by reducing the risk of unemployment for the low-ability type.

### 2.2 Public Provision of Private Goods

Publicly provided private goods constitute another type of instrument that may be used for redistribution purposes. Public provision of private goods is typically analyzed as a supplement to nonlinear income taxation in the context of redistribution under asymmetric information.\textsuperscript{6} The early literature on public provision of private goods under asymmetric information assumes that the hourly gross wage rates are fixed (see e.g. Blomquist and Christiansen, 1995, 1998a,b; Broadway and Marchand, 1995; Broadway et al., 1998). Blomquist and Christiansen (1995) argue that the private goods suitable for public provision are those that cannot be supplemented by the private agents or resold and lack a close market substitute (such as education, health care and care for the elderly), i.e. the provision should not be equivalent to a transfers in cash. They examine situations where the low-ability type accepts the public provision scheme, whereas the high-ability type only accepts the public provision scheme under certain conditions. In order to discourage mimicking, an optimal level of public provision may imply that the low-ability type either overconsumes or underconsumes the publicly provided private good relative to the first-best rule.

Broadway and Marchand (1995) provide a rationale for public provision of private goods when individuals are allowed to supplement (top up) the publicly provided quantity by private consumption. They show that public provision of private goods may relax the self-selection constraint if the mimicker’s private purchase of the publicly provided good becomes crowded out (i.e. falls to zero), while the true ability-type’s private purchases are not. The intuition is that further increases in the public provision of the private good will restrict the

\textsuperscript{5}Engström (2003) introduces unemployment benefits in this analysis, whereas Engström (2007) deals with the importance of tax progression for the bargained wage.

\textsuperscript{6}There is also a literature which assumes other tax instruments when studying public provision of private goods. Munro (1991, 1992) analyzes public provision of private goods supplemented by linear taxes, whereas Balestrino (2000) studies public provision of private goods in combination with mixed taxation.
mimicker, which makes the mimicker worse off. Boadway and Marchand also find that if leisure is weakly separable from the other goods in the utility function, it is not possible to increase welfare via public provision of private goods. In this case, the mimicker and low-ability type become crowded out at the same point.⁷

Pirttilä and Tuomala (2002) extend the analysis by allowing the wage rates to be endogenous, and also by assuming that the publicly provided private good gives rise to an external effect in the production. In their model, the wage rate becomes a function of the publicly provided private good. This means that the publicly provided private good affects the self-selection constraint also via the wage distribution. Pirttilä and Tuomala show that if the wage rates depend on the publicly provided quantity of a private good, there will be a role for such public provision even if leisure is weakly separable from the other goods in terms of the utility function. The explanation is, as mentioned above, that a higher relative wage rate for the low-ability type implies that the mimicker needs to supply more hours of work in order to reach the same income as the low-ability type, which relaxes the self-selection constraint.

**Summary of Paper [I]** The purpose of this paper is to analyze public provision of a private good under equilibrium unemployment. We assume that the government is unable to observe individual ability. In addition to the publicly provided quantity of the private good, the set of policy instruments contains a nonlinear income tax and unemployment benefits. The paper contributes to the literature in primarily two ways. First, the paper points out that imperfect competition in the labor market gives rise to additional policy incentives associated with the self-selection constraint, which motivates adjustments in the public provision of private goods. Second, the paper also addresses employment motives behind publicly provided private goods.

We apply the self-selection approach developed by Stern (1982) and Stiglitz (1982), which means that the consumers are divided into two types: a low-ability type and a high-ability type. The wage rate of each ability type is determined

⁷Blomquist and Christiansen (1998a) use the same model as Boadway and Marchand when analyzing the conditions under which public provision schemes should allow for topping up. They show that it will be at least weakly preferable to allow for private purchases as a supplement to the public provision, instead of just having public provision, if the publicly provided private good is a substitute for leisure in terms of the utility function.
in a bargain between unions and firms in a right-to-manage framework. Union
wage formation distorts wage setting and leads to equilibrium unemployment.
The gross wage rate facing each ability-type will be a function of (among other
things) the publicly provided quantity of the private good.

Our results imply that public provision of private goods can be used for
redistribution purposes by influencing the self-selection constraint and by af-
flecting the number of employed persons. We show that union wage formation
implies that a broader set of variables affects the hourly wage rates than under
competitive labor markets, which is relevant from the perspective of relaxing
the self-selection constraint. In addition, the policy maker may also use the pub-
lic provision of the private good to increase the number of employed persons.
The intuition is that an increase in the number of employed persons leads to
higher tax revenues net of transfer payments, which relaxes the financial burden
on the public sector.

2.3 A Dynamic Economy

Earlier literature on redistribution under asymmetric information in dynamic
economies has mainly focused on whether capital income taxation ought to be
used as a supplement to the labor income tax. In a pioneering study, Ordover
and Phelps (1979) model a continuum of ability-types and find that if leisure is
separable from the other goods in the utility function, then the marginal capital
income tax rate should be zero for each ability-type.

More recent studies use two-type models (in an OLG context) when analyz-
ing optimal redistribution under asymmetric information in dynamic economies.
Brett (1997) shows, by relaxing the separability assumption made by Ordover
and Phelps (1979), that the marginal capital income tax rate of the low-ability
type (who is assumed to be the mimicked type) may be either positive or neg-
ative, depending on whether the marginal rate of substitution between current
and future consumption for the low-ability type exceeds, or falls short of, the
corresponding marginal rate of substitution for the mimicker. For instance, if
the low-ability type attaches a higher relative value to current consumption
than the mimicker (i.e. a higher marginal rate of substitution between the cur-
rent and future consumption), the government may discourage mimicking by
choosing a higher marginal income tax on capital for the low-ability type than
it would otherwise have done. Boadway et al. (2000) analyze nonlinear labor in-
come taxation and proportional capital income taxation in a dynamic economy with two-ability types, where the government neither can observe ability nor inherited wealth. In their framework, the capital income tax is interpretable as an indirect instrument to tax wealth. They find that unobservable inheritance makes it desirable to supplement labor income taxes by a proportional capital income tax in order to collect revenue in a more efficient way.⁸

Pirttilä and Tuomala (2001) extend the previous literature by analyzing marginal capital income taxation in an economy with endogenous gross wage rates. The endogenous gross wage rates result in additional mechanisms for relaxing the self-selection constraint. In addition to the motive for capital income taxation discussed by Brett (1997), they find that production inefficiency at the second best optimum (due to that the capital stock influences the wage distribution) justifies capital income taxation. Aronsson et al. (in press) study the case with trade-union wage formation and the appearance of equilibrium unemployment in the second best optimum. In their model, the marginal capital income taxation also affects the employment. They show that there is an employment-motive behind the use of labor income and capital income taxation. Part of the employment-motive for using capital income taxation is related to intertemporal production inefficiency at the second best optimum, as a change in the capital stock affects the number of employed persons.

### 2.3.1 Durable Goods and Second-hand Trade

The second paper in this thesis is the first study that incorporates a second-hand market into the framework for studying redistribution under asymmetric information. Much of the earlier research dealing with optimal taxation and durable goods is found in the area of environmental economics,⁹ whereas the literature concerning second-hand markets for durable goods has focused on the second-hand trade with cars. The latter is often based on the adverse selection approach developed by Akerlof (1970). Akerlof showed that if the

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⁸Tenhunen and Tuomala (2007) argue that if some individuals discount future utility in a way different than the social planner or behave myopic, there will also be a paternalistic motive for using capital income taxation when redistributing between generations.

⁹Barkan (2004) analyzes taxation of a pollution-generating durable good in an economy where the producers operate under imperfect competition. There is also a large literature on environmental policy in intertemporal models, in which the environmental quality has durable good properties.
owners of used cars know more about the quality of the car than a buyer, this will be an important source of inefficiency in the second-hand trade. Hendel and Lizzeri (1999) analyze a dynamic model and allow for interactions between the second-hand market for used cars and the market for new cars.\textsuperscript{10} They show that the interaction between the markets may lead to a higher price of new cars if there is adverse selection in the second-hand market. In contrast to Akerlof, they find that the second-hand market never shuts down (due to adverse selection) and that the distortions are lower than previously thought. Hendel and Lizzeri (2002) find that leasing contracts serve to ease the consequences of adverse selection by increasing the average quality of used durables good traded in second-hand markets.\textsuperscript{11} Gilligan (2004) finds empirical evidence for the predictions made by Hendel and Lizzeri (1999, 2002) when using data for used aircrafts in North America.

Summary of Paper [II] The purpose of this paper is to study optimal income and commodity taxation in a two-type OLG model, where used durable goods are traded in a second-hand market. Our paper is the first study that analyzes second-hand trade of used durable goods in the context of redistribution under asymmetric information.

In our model, each consumer generation lives for two periods. The consumer derives utility from the consumption of a nondurable good and a durable good as well as from leisure. New and used durable goods are assumed to be imperfect substitutes in consumption. Furthermore, the production structure is such that the gross wage rates are endogenous. The supply of the used durable good is constrained by the amount bought of the new durable good in the previous period. The set of policy instruments consists of nonlinear taxes on labor and capital income as well as a linear commodity tax on the new durable good. However, the government is unable to directly control second-hand transactions via commodity taxation. Instead, the government needs to rely on the other tax instruments in order to indirectly affect the price of the used durable good.

Our model shows that the marginal value that the government attaches to the used durable good depends on whether leisure is a complement or a substitute to the used durable good. It also depends on how a change in the

\textsuperscript{10}See also related studies, e.g. Anderson and Ginsburgh (1994), Sattler (1996), Sandfort (1997) and Porter and Sattler (1999).

\textsuperscript{11}See also Guha and Waldman (1996).
price of the used durable good influences the wage distribution. The results imply that if the government attaches a positive marginal value to the used durable good (i.e. the government finds it desirable to make the used durable good less scarce), it follows that the appearance of a second-hand market for the used durable good provides an incentive for the government to choose a lower commodity tax on the new durable good than it would otherwise have done. The intuition is that a lower commodity tax on the new durable good leads to a lower price of the used durable good in the present period and increases the supply of the used durable in the next period. Our results also show how the appearance of a second-hand market modifies the use of income taxation. The marginal labor income tax rates affect the labor supply and, therefore, indirectly also the price of the used durable good. To be more specific, the modification of the marginal labor income tax structure caused by the second-hand market depends on whether leisure is a complement or a substitute to the new and used durable good. The modification of the marginal capital income tax rates are ambiguous because a change in the capital income tax affects the current and future price of the used durable good in opposite directions.

3 Local Public Expenditures

The most frequently used model when analyzing local public expenditures is the "median voter" model. According to this model, the size of the public spending is decided upon using a majority rule in the locality. The result, originally derived by Bowen (1943) and explained in greater detail by Black (1958), implies that the voter who has the median preferred outcome will be the decisive. This result requires that the preferences of the voters are single peaked, that there is no strategic voting and that the choices are one dimensional.

A key problem in empirical studies has been the identification of the median voter.12 The most common approach is to identify the median voter as the voter with the median income (see e.g. Bradford and Oates 1971a,b; Borcherding and Deacon, 1972; Bergstrom and Goodman, 1973). Such an interpretation requires several assumptions, see e.g. Bergstrom and Goodman (1973). Wildasin (1986) points out that the person with the median income is not, in general, the median

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12In a pioneering study, Barr and Davis (1966) assume that the individuals differ in their ownership of property. As a result the individual with the median property value share becomes decisive.
Another problem with the median voter model is how to treat multidimensional issues. Applying the model to a system of public sector demand equations implies that the median voter generally will differ for different services. Extensions of the traditional median voter model (based on deterministic voting) to the multidimensional case gives a probabilistic voting model (see Enelow and Hinich, 1984). Probabilistic voter models consider elections in which candidates are unsure as to whether voters’ preferences will be on all or most issues. A fundamental question concerns the optimal strategy for the candidates given uncertainty about voter preferences. Craig and Inman (1986) propose a voter group decision model to solve the multidimensionality problem, where the allocation is a political compromise among voter groups. In this model, the political influence is either defined in terms of the age composition (Craig and Inman, 1986; Borge and Ratts, 1995) or by using the party composition in the local council (Renaud and Winden, 1991).

3.1 The Aggregate Local Public Expenditure

There is a large literature dealing with the aggregate local public expenditure. Since the beginning of the 1970s, it has been common to use median voter models (see e.g. Borcherding and Deacon, 1972; Bergstrom and Goodman, 1973). In these empirical studies, the median voter is defined as the voter with median income. Much of the attention focuses on how variables characterizing the median voter’s budget constraint, such as the median voter’s tax price and the median income, affect the demand for local public services. Borcherding and Deacon (1972) assume that the tax price is constant and find that the point estimate of the income elasticity is positive; between zero and one. Bergstrom and Goodman (1973) derive more rigorous demand equations for municipal public services, where the demand for local public spending also depends on the tax price (tax share).\textsuperscript{14} The main results from their study are that local public expenditures, in general, depend negatively on the tax price and positively on the median voter’s income. Bergstrom and Goodman also find that size of

\textsuperscript{13}For a test of the median voter model on Swedish data, see Aronsson and Wikström (1996).

\textsuperscript{14}Using the median voter approach when analyzing the determinants of the local public expenditures in Sweden, Aronsson and Wikström (1996) estimated the income elasticity to be 0.82 and the price elasticity to be 0.53 in absolute value.
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the jurisdiction may be important for the demand of local public expenditures, i.e. larger municipalities do not appear to benefit from economies of scale in the provision of public services. Other studies have extended the study of local public expenditure determination by including socio-economic indicators to account for the effects of social needs and political considerations (see Dunne and Smith, 1983; Dunne et al., 1984; etc).

3.1.1 The Flypaper Effect

According to the 'basic' median voter model, where the tax price is treated as fixed by the median voter, the source of public revenues should not matter for the local public expenditure decision.15 This is so because a transfer payment from the central to the local government is effectively equivalent to a transfer payment to the decisive voter. This result is questioned by an extensive empirical literature that finds that local public spending responds more to increased transfers from the central government than to a corresponding increase in the tax base, an empirical finding that is labelled the "flypaper effect".16

Several competing hypotheses aim to explain the flypaper effect.17 Niskanen (1968) and Romer and Rosenthal (1980) argue that disharmony of interest between voters and bureaucrats/politicians is the main reason for the flypaper effect. Similar for Niskanen’s model of bureaucratic behaviour and Romer and Rosenthal’s agenda setter model is the assumption that the official authority (represented by the bureaucrats in the former case and an agenda setting agent in the latter case) acts in its own interest and tries to maximize the local public budget. In contrast to the voters, the bureaucrats/agenda setters are assumed to have information on tax and other revenues as well as the cost of production. The control over the budget in combination with the behaviour of the bureaucrats/agenda setters increase the marginal propensity to spend out of lump-sum grants. Another often cited explanation for the flypaper effect is the "fiscal illusion" hypothesis (see Cournot et al., 1979; Oates, 1979). Fiscal illusion means, in the context of the flypaper effect, that the voters’ perceived effect of the local public budget is what really matters: a flypaper effect exists because the local officials are assumed to use lump-sum grants to deceive local voters into

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15See Bradford and Oates (1971a,b).
16The result was labelled the “flypaper effect” by Courant et al. (1979).
17A good survey is given by Bailey and Connolly (1998).
thinking that the cost of publicly provided services is less than it actually is.\textsuperscript{18} Fossett (1990) and Turnbull (1992) address the importance of including uncertainty and instability of grant revenue in the analysis. According to Fossett, it is not in the interest of local officials that the local tax rate fluctuates too much over time. The flypaper effect is, thereby, a result of risk aversion and caution by the local officials. A more recent explanation for the flypaper effect is presented by Hines and Thaler (1995). They argue that the flypaper effect is a result of resource fungibility, i.e. that different types of revenues are allocated into different "mental accounts."

The evidence for the existence of a flypaper effect is mixed. The early empirical literature dealing with the flypaper effect can be criticized for not separating lump-sum grants from matching grants (see King, 1984). Moffitt (1984) emphasizes the problem of ignoring the simultaneous determination of matching grants and public expenditures. He estimates the effect of piecewise-linear budget constraints created by grant formulas, where the subsidy rate is not constant over expenditures, and finds that the flypaper effect disappears. Recent studies addressing these problems are criticized for using an inappropriate functional form, e.g., studies that use a linear-in-variables functional form often find support for a flypaper effect, while studies based on logarithms of all variables do not (Becker, 1996; Worthington and Dollery, 1999). Hamilton (1983) contends that empirical support for the flypaper effect is affected by the omitted variable biases. Subsequent efforts to correct for omitted variables that may correlate with intergovernmental transfers come to different conclusions regarding the flypaper effect (Wyckoff, 1991; Knight, 2002; Gordon, 2004; Dahlberg et al., 2007). Empirical support for a flypaper effect is generally demonstrated for certain situations, rather than across the board. Moisio (2002) finds that the flypaper effect varies for different categories of expenditures; it is larger for 'education and culture' and 'comprehensive schools' compared to other local public services. Other studies find that the response in local public expenditures to a change in the lump-sum transfers from the central government is asymmetric, depending on whether the transfer increases or decreases (Gramlich, 1987; Gamkhar and Oates, 1996; Karlsson, 2006).

\textsuperscript{18}The empirical results for the fiscal illusion explanation are mixed. Dollery and Worthington (1995) and Borge (1995) find empirical support for the explanation, whereas Wyckoff (1991) reject’s the hypothesis.
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Summary of Paper [III]  The purpose of this paper is to analyze the determinants of the local public expenditures and, in particular, test for a flypaper effect. The paper focuses on an aspect of income heterogeneity, which is not previously discussed in the literature dealing with the flypaper effect. A political economy model is presented, where the voters are divided into low-income and high-income earners. This means that a change in the tax base, in addition to its effect on the local public budget, changes the income distribution: an effect which does not appear when transfer payment from the central government changes. The model implies that the effect on the majority voter’s tax share, caused by a change in the tax base, will be crucial for finding a flypaper effect. The intuition is that a change in the private income for any of the two voter groups affects the relative financial burden (tax share) of the majority voter group and, therefore, its willingness to use taxation as a marginal source of funds for public expenditures. Another implication of the model is that the spread of private income will be important for the local public expenditures. In the special case with a single ability-type in the local economy, there is no flypaper effect (i.e. if the two voter groups were identical).

The empirical part of the paper estimates a model of local public expenditures, which is related to the theoretical discussion, and tests for a flypaper effect. The regressions are based on panel data containing between 282 and 287 Swedish municipalities and nine years, 1996-2004. The data is particularly suited for studying the flypaper effect because transfers from the central to the local government in Sweden are, to a large extent, general grants during the study period. The empirical analysis finds evidence of flypaper effects. The benchmark specification, which corresponds to previous empirical literature, results in a point estimate of the flypaper effect of about three. This specification is rejected when it is tested against more general specifications where different aspects of income heterogeneity are considered. The results of the extended models imply that the size of the flypaper effect is larger in municipalities where a relatively high share of the residents are low-income earners and that the flypaper effect becomes larger as the proportion of grants in relation to the sum of grant and tax base increases.
3.2 The Composition of Local Public Expenditures

Previous studies dealing with the expenditure composition in the local public sector often assume that the municipality acts as if the policies are decided upon by a representative individual. Solving such a decision problem for the local government yields a system of estimable demand equations. Expenditure composition problems in the public sector are thereby treated in a way similar to consumer choices in the private sector. This approach was first applied by Deacon (1978). He estimates a demand system for local public services in which a given local public budget is allocated between the service sectors. The results indicate that the point estimates of the compensated own-price elasticities are all negative, and that the demand for local public services is homogenous of degree zero in local public income and all prices.

The model used by Deacon is consistent with the idea of two stages budgeting, where the local public revenues are collected in a first stage and allocated between the service sectors in a second stage. Borge and Rattso (1995) also estimate a demand system for local public services in which the local public revenues are predetermined (by the central government). They use a voter group decision model where different age groups compete for local public services, and allow for inertia in the adjustment process. They find that there is a systematic difference between the short- and long-run budget elasticities, and that the point estimates of the budget elasticities are all close to unity in the short run.

The point estimates of the compensated own-price elasticities are all negative and, in general, less than one in absolute value. Their result also suggests that the age composition is important for the allocation of local public services: due to mean reversion, age groups which are declining in relative size are able to resist reallocations and gain in terms of local public per-capita expenditures. In a related study, Aaberge and Langørgen (2003) treat user fees as a local tax instrument (all other sources of revenue are treated as exogenous). The estimation results imply that ‘social services’ is the service sector with the lowest point estimate of the budget elasticity (close to zero), whereas “infrastructure” is the sector with the highest point estimate of the budget elasticity.

A problem in the analysis of the demand for public services is the lack of

\footnote{Deacon estimates the demand system by using US data. He studies the demand for six local public services in the city of Seattle for the period 1921-1970.}

\footnote{Intergovernmental grants, local tax rates as well as the local tax base are assumed to be determined by the central government.}
appropriate measures of public output. The most common solution is to use expenditure as a proxy for output. The lack of information on service volumes also makes the identification of price effects problematic. A useful approach to get around the problem is to calculate the operating cost per unit of labor corrected for matching grants (see Ehrenberg 1973; Bahl et al. 1980). This "public employment approach" has been applied to Norwegian data by Borge and Rattso (1995) and Aaberge and Langorgan (2003).

**Summary of Paper [IV]** The purpose of this paper is to analyze the composition of municipal expenditures in Sweden. This is done by estimating a demand system for local public services, in which tax revenue collection is treated as endogenous. The local public expenditures are analyzed by using a 'community preference' model, where a representative agent allocates her/his resources between the consumption of private goods and public services. The representative agent’s demand for each good depends on the total per capita income, prices and municipal characteristics.

In contrast to earlier literature on composition of local public expenditures, this paper does not separate the local public budget process into two steps. Instead, the decision to collect revenue is analyzed simultaneously with the decision to allocate the resources between different local public services. This approach implies that also a composite private good should be included in the system of demand equations. Furthermore, this paper is the first study that analyzes the local public expenditure composition using Swedish data.

The empirical part of the paper uses panel data for the period 1998-2005 and 273 Swedish municipalities. The estimations are based on the QAIDS specification and the analysis presents results from estimating a demand system consisting of a composite private consumption good and six local public services (child care, social services, elderly care, comprehensive education, 'infrastructure and protection' and 'other expenditures'). The results show that the point estimates of all income elasticities except one are positive, and that none of them significantly exceeds one. Comparing the income elasticities between municipalities indicates only small differences. Furthermore, the own-price elasticities are negative and less than one in absolute value for all services. For purposes of comparison with previous studies, a demand system that is conditioned on the resources available to the local public sector is also estimated. The results
from this conditional demand system do not deviate much from related studies.

References


Summary


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Summary


Public Provision of Private Goods and Equilibrium Unemployment

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This paper deals with public provision of a private good in a two-type model with optimal nonlinear income taxation. We assume that the wage rates are determined by bargaining between unions and firms, meaning that the equilibrium is characterized by unemployment. The results show that imperfect competition in the labor market gives rise to additional policy incentives associated with the self-selection constraint, which may justify either more or less public provision than under perfect competition. The paper also addresses employment-related motives behind public provision of private goods.

Keywords: public provision of private goods, nonlinear taxation, unemployment

JEL classification: H 21, H 23, J 51

1. Introduction

This paper deals with public provision of private goods in an economy with optimal nonlinear income taxation and imperfect competition in the labor market. The basic idea is to use insights from the literature on optimal taxation under unemployment in order to learn more about how to use public provision of private goods for purposes of efficiency and redistribution. The analysis to be carried out in the paper relates to the self-selection approach to optimal taxation and is based on the two-type model developed by Stern (1982) and Stiglitz (1982).

Previous studies dealing with public provision of private goods in economies with nonlinear income taxation are typically based on the assumption of competitive markets, and the decision to publicly provide private goods is explained in the context of optimal redistribution under asymmetric information. Furthermore, earlier studies often rely on linear production technologies, implying that the wage rates are fixed. In such a framework,

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the usefulness of a publicly provided private good as a tool for redistribution depends on the structure of consumer preferences. If the utility function is nonseparable, then public provision of private goods will be welfare-improving under certain conditions by relaxing the self-selection constraint. The idea is to make mimicking less attractive, which, in turn, increases the choice set underlying redistributive policy. If, on the other hand, leisure is weakly separable from the other goods in terms of the utility function, it may not be possible to increase welfare via public provision of private goods.

Pirttilä and Tuomala (2002) extend the analysis by allowing the wage rates to be endogenous, as well as by assuming that the publicly provided private good gives rise to an external effect in the production; then public provision affects the self-selection constraint via the wage distribution. In this case, therefore, public provision of private goods may have a distinct role to play in the context of redistributive policy, even if leisure is weakly separable from the other goods.

In this paper, we relax the assumption that the labor market is competitive and, instead, assume that the wage rates are determined by bargaining between unions and firms. Therefore, the equilibrium is generally characterized by unemployment in our framework. This extension is important from at least two perspectives. First, labor unions are important institutions, at least in the European labor markets, suggesting that the introduction of equilibrium unemployment may provide additional realism to the study of public provision of private goods. Indeed, government involvement in the provision of private goods is often motivated by the desire to redistribute, and reducing unemployment is one means of accomplishing redistribution. Considering that many industrialized countries have experienced high unemployment rates for a long time, it is clearly relevant to extend the theory of public provision accordingly. Second, our framework provides a natural extension of the literature developed so far on optimal nonlinear taxation, redistribution, and public-good provision in economies with unemployment. This enables us to consider how a broader set of policy instruments can be combined for purposes of efficiency and redistribution, as well as explaining

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2 There is an earlier literature on public provision of private goods in economies with linear taxes (combined with uniform lump-sum taxes); see, e.g., Munro (1991, 1992). The welfare gains of in-kind transfers are in this case associated with tax-revenue effects. See also the related paper by Balestrino (2000) dealing with public provision of private goods under mixed taxation (a nonlinear income tax and linear commodity taxes). In his paper, therefore, the possible welfare gains of relaxing the self-selection constraint via public provision are analyzed simultaneously with tax revenue effects, since public provision affects the revenues from commodity taxation.

3 See Marceau and Boadway (1994) and Aronsson and Sjögren (2003). See also the related literature dealing with optimal tax progression under imperfect competition in the labor market; e.g., Fuest and Huber (1997) and Aronsson and Sjögren (2004a, 2004b).
how the results in previous studies dealing with public provision of private goods will be modified if equilibrium unemployment is incorporated into the analysis.

The paper contributes to the literature in primarily two ways. One is by showing how imperfect competition in the labor market gives rise to additional policy incentives associated with the self-selection constraint, which may justify either more or less public provision than under perfect competition. The intuition is that, with union wage formation, a broader set of variables affects the wage distribution than under perfect competition. Furthermore, since public provision of a private good influences employment – both directly via the quantity provided and indirectly via the requirement of budget balance for the government – the paper also addresses employment-related motives behind publicly provided private goods.

The outline of the paper is as follows. In section 2, we present the model and discuss the outcome of private optimization. Public provision of private goods is analyzed in section 3. Section 4 summarizes the results.

2. The Model

The optimal tax and expenditure problem analyzed in this paper is conventional in the sense that the government is assumed to act as Stackelberg leader, whereas the private sector (including the unions) acts as follower. This section deals with the behavior of the private sector by analyzing the outcome of optimization by consumers, firms, and unions, respectively, as well as by describing wage and employment formation. The optimization problem of the government will be described in section 3.

2.1. Consumers and Firms

There are two types of consumers participating in the labor market: a low-ability type (denoted by superindex 1 if employed) and a high-ability type (denoted by superindex 2 if employed). The distinction between ability types refers to productivity, which is here interpreted to mean that the high-ability type faces a higher before-tax wage rate than the low-ability type.

The consumers have identical preferences defined by the utility function \( u(c, h, q) \), where \( c \) is a normal private good and \( h \) leisure, whereas \( q \) will be referred to as a quasi-private good; it is a private good partly provided by the government. The function \( u(\cdot) \) is increasing in each argument and strictly quasiconcave. We characterize \( q \) as follows: \( q = g + z \), where \( g \) is the amount provided by the government, and \( z \) is the additional amount purchased by the individual himself/herself. The government provides the same amount of the quasi-private good to all households, and we assume that \( g \) cannot
be resold. For simplicity, the prices of both goods are normalized to one.
Leisure is defined as $h = H - l$, where $H$ is a time endowment and $l$ the hours of work. Finally, to simplify the analyses to be carried out below, we assume that $\lim_{q \to 0} u_q(c, h, q) = \lim_{q \to 0} u_q(c, h, q) \to \infty$, where the subindices denote partial derivatives, i.e., $u_c(\cdot) = \partial u(\cdot)/\partial c$ and $u_q(\cdot) = \partial u(\cdot)/\partial q$.

If employed, a consumer of ability type $i$ earns labor income $w_i l$, where $w_i$ is the wage rate. The after-tax income is denoted $B_i = w_i l - T(w_i l)$, where $T(\cdot)$ is an income-tax function. Since the optimal-tax and -expenditure problem will be formulated in terms of conditional indirect utility functions, it is convenient to start by solving the private optimization problem conditional on the hours of work. This means choosing $c_i$ and $z_i$ to maximize the utility subject to the budget constraint $B_i = c_i + z_i$ and the nonnegativity constraint $z_i \geq 0$, where $B_i$ is treated as a fixed income. Substituting the budget constraint into the utility function and maximizing with respect to $z_i$ gives the following Kuhn–Tucker conditions:

$$u_i^c - u_i^l \leq 0, \quad z_i (u_i^c - u_i^l) = 0$$

for $i = 1, 2$, where $u_i = u(c_i, h_i, q_i)$. If the nonnegativity constraint does not bind, equation (1) implicitly defines the conditional demand functions, $z_i = z(B_i, h_i, g)$ and $c_i = B_i - z(B_i, h_i, g)$. The conditional indirect utility function is written

$$v_i = v(B_i, h_i, g) = u(B_i - z(B_i, h_i, g), h_i, g + z(B_i, h_i, g))$$

and satisfies the properties

$$v_i^h = u_i^h, \quad v_i^v = u_i^v, \quad v_i^g = u_i^g = u_i^q.$$ 

With the conditional indirect utility function at our disposal, we can derive the hours of work by maximizing the conditional indirect utility function with respect to $l$ subject to the budget constraint $B_i = w_i l - T(w_i l)$. The first-order condition is given by

$$\frac{v_i^h}{v_i^l} = w_i (1 - T_i'),$$

where $T_i' = \partial T(y')/\partial y'$ is the marginal income tax rate, and $y' = w_i l$ measures the before-tax income. Conditional on the tax schedule, equation (4) implicitly defines the hours of work as a function of the wage rate and the publicly provided quantity of the quasi-private good, i.e., $l = l(w_i, g)$.

If unemployed, on the other hand, the consumer receives a fixed tax-free benefit $b$. The budget constraint becomes $b = c'' + z''$, where the superindex

4 In principle, the unemployment benefit could be allowed to differ depending on skill and/or some (exogenous) earnings history. To simplify the analysis, we have made no such distinctions here. Note also that, by our assumptions, the government controls the posttax income of the unemployed, an assumption that appears to be reasonable in the
\( u \) refers to “unemployed”. By analogy to the analysis carried out above, the conditional indirect utility function is written

\[
v^u = v(b, H, g) = u(b - z(b, H, g), H, g + z(b, H, g)). \tag{5}
\]

Consumers have identical preferences; therefore, equation (5) applies to unemployed individuals of both ability types.

Since the technology is characterized by decreasing returns to scale (see below), there will be pure profits in the equilibrium. Therefore, following previous studies on optimal taxation under union wage setting, such as Fuest and Huber (1997) and Aronsson and Sjögren (2004a, 2004b), we introduce a second category of consumers, called firm owners. To simplify the analysis, we assume that the firm owners do not work, implying that their only source of income is profits from the production, and we normalize the number of firm owners to one. The budget constraint of the firm owner is given by

\[
B^0 = (1 - s) \pi = c^0 + z^0,
\]

where \( \pi \) is the profit income and \( s \) the profit income tax rate, while the superindex 0 is used to denote the firm-owner type. The indirect utility function becomes

\[
v^0 = v(B^0, H, g) = u(B^0 - z(B^0, H, g), H, g + z(B^0, H, g)). \tag{6}
\]

Turning to the production side of the economy, identical firms use the same technology to produce both commodities. Labor is the only variable production factor. Since the number of firms is not important in the analysis, it will be normalized to one. The production function is written \( F(L^1, L^2) \), where \( L^i \) is the total employment of ability type \( i \), measured as the hours of work per employee, \( \bar{L} \), times the number of employed persons, \( N^i \). The function \( F(\cdot) \) is increasing in each argument and strictly concave. In addition, both types of labor are essential. Profit maximization implicitly defines the labor demand function for each ability type, \( L^i = L^i(w_1, w_2) \). Dividing by the hours of work per employee, we can write each labor demand function in terms of the number of employed persons:

\[
N^i = \bar{N}^i(w_1, w_2, \bar{L}) = L^i(w_1, w_2)/\bar{L} \quad \text{for} \ i = 1, 2.
\]

The profit function is written \( \pi = \pi(w_1, w_2) \).

In the context of this particular model, where the government effectively controls the posttax income of the other agent types (see below). One interpretation is that the fixed unemployment benefit constitutes a direct decision variable of the government; another is that the government controls the posttax income of the unemployed via the general income tax.

5 The firm-owner type is interpreted as the owners of any fixed production factor. Instead of introducing the firm owners, another alternative would have been to assume that the profit income is equally distributed among workers. It is not of importance for the qualitative results derived below which option is chosen. Introducing a firm-owner type is slightly more convenient from a computational point of view.
2.2. The Labor Market

Let $M_i$ denote the number of workers of ability type $i$, all of whom are organized by an ability-type-specific trade union. The individuals are assumed to enter into a specific union after the government decides upon public policy. In addition, we assume that wage setting is decentralized in the sense that the unions treat the policy variables of the government as exogenous. Given these characteristics, the number of unions facing each ability type is not important for the analysis. By analogy to the treatment of the production sector, therefore, the number of unions facing each ability type will be normalized to one. Each union acts as a Nash competitor towards the other union.

At present, there seems to be no standard approach for specifying the objective function of a union. Following Oswald (1993), we assume that the union members can be ranked according to (exogenous) seniority rules, meaning that there is a known layoff ordering within each union. Decisions within each union are taken by a majority vote, implying that the union member with median seniority is decisive. As a consequence, if the majority of union members are not at an immediate risk of becoming unemployed (which we assume here), the union is locally indifferent to the level of employment. The objective of the union will be the objective function facing the majority of its members; $u_i = u(c_i, h_i, q_i)$.

The wage rate of ability type $i$ is determined by bargaining between union $i$ and the firm in the context of the right-to-manage framework. If the parties fail to reach an agreement, all union members become unemployed and receive the fallback utility, $v^u = v(b, H, g)$, which is exogenous to the union.

6 In practice, union membership may reflect several factors such as ability, industry, and occupation. For simplicity, we have chosen to organize the individuals on the basis of their abilities. Another possibility would have been to assume that both ability types are organized by the same union. Such a change of assumption would allow us to consider union preferences for distributional equality; see Schneider (2005). Although the description of union behavior matters for the outcome of wage formation (both in terms of the qualitative properties of the wage equation and in terms of quantitative outcomes), it is not necessarily important for the policy rules derived below which option is chosen. As long as the equilibrium wage rates and employment levels can be written as general functions of the policy instruments, as in equations (11) and (12) below, the description of how the individuals are organized into unions will not affect the qualitative policy rules derived in section 3.

7 See Kaufman (2001) for an overview of models of union wage setting.

8 There is also a more formal argument behind our choice of labor market model; namely, that we would like to provide a framework suitable for studying public provision of private goods under unemployment. This rules out a description of the labor market where the government can effectively control all channels through which the wage rates influence the number of employed persons. Such a situation may arise if the union faces the common utilitarian objective; see Aronsson and Sjögren (2004a).
by the assumption of decentralized wage formation, whereas the fallback profit is zero. The outcome of the bargain for ability type \( i \) will be the wage rate that maximizes the Nash product

\[
\Omega^i = [v^i - v^u]^a \pi (1 - s)^{1-a}
\]

subject to

\[
B^i = w^i l^i - T(w^i),
\]

\[
l^i = l(w^i, g),
\]

where \( a^i \) is the bargaining power of union \( i \). The first-order condition can be simplified to read

\[
a^i v^i B^i (1 - T^i) \pi - (1 - a^i) N^i [v^i - v^u] = 0.
\]

The second-order condition is assumed to be fulfilled.

Since the optimal tax and expenditure problem below will be formulated such that \( B^i \) and \( l^i \), instead of the parameters of the labor income tax function, constitute the direct decision variables, it is convenient to rewrite equation (9) in order to eliminate \( 1 - T^i \). This is done by using \( v^i_B(1 - T^i) = v^i_B/w^i \) from equation (4), in which case we obtain

\[
a^i v^i_B w^i \pi - (1 - a^i) N^i [v^i - v^u] = 0.
\]

By observing that \( v^i \) is a function of \( B^i, l^i, \) and \( g \), while \( v^u \) is a function of \( b \) and \( g \), it is possible to write the bargained wage rate for ability type \( i \) as a function of \( B^i, l^i, b, g \), and \( w^j \) for \( j \neq i \). Then, combining the resulting wage equations for the two ability types, we can derive

\[
w^i = w^i(B^1, B^2, l^1, l^2, b, g)
\]

for \( i = 1, 2 \), where the parameter representing bargaining power has been suppressed. Substituting into the labor demand functions, we obtain reduced-form equations for the number of employed persons of each ability type:

\[
N^i = N^i(B^1, B^2, l^1, l^2, b, g)
\]

for \( i = 1, 2 \).

### 3. Public Provision of the Quasi-private Good

Let us begin by solving the optimal tax and expenditure problem conditional on the publicly provided quantity of the quasi-private good. The government is assumed to implement a Pareto-efficient policy in the sense of maximizing the utility of the employed low-ability type subject to minimum-utility restrictions for the employed high-ability type, the unemployed, and the firm owner. The set of policy instruments consists of the parameters of the
income-tax function $T(\cdot)$, the profit income tax rate $s$, the unemployment income $b$, and the publicly provided quantity of the quasi-private good, $g$ (which is held constant to begin with).

Note that, by choosing the parameters of the income-tax function, the government can induce any desired combination of consumption and work hours for each ability type. It is therefore convenient to formulate the optimization problem so that the government directly chooses $B_1^1, l_1^1, B_2^2, a n d l_2^2$ instead of choosing them indirectly via the parameters of $T(\cdot)$. Similarly, we use the consumption of the firm owner, instead of the profit income-tax rate, as a direct policy instrument. By using the private budget constraints and the objective function of the firm, it is straightforward to show that the government’s budget constraint can be written as

$$F(\cdot) - \sum_{i=1}^{2} N_i^{i_1} - \sum_{i=1}^{2} (M^{i_1} - N^{i_1})b - B^{0} - (M^{1} + M^{2} + 1)g = 0,$$

where $F(\cdot) = F(L_1^1, L_2^2)$, while $L_1^1(w, w_{-})$ and $L_2^2(w, w_{-})$ depend on $B_1^1, l_1^1, B_2^2, l_2^1, b$, and $g$ via the equations (11).

The informational assumptions made in this paper are standard and imply that the government can observe the income of all individuals, although it cannot observe ability. Following most previous studies on optimal nonlinear taxation, we assume that the aim of redistribution is to redistribute from the high-income earners to the low-income earners. As a consequence, we would like to avoid tempting the high-ability types to pretend to be low-ability types. The number of hours of work that the high-ability type needs to supply in order to reach the same income as the low-ability type is given by

$$\hat{l}_2^2 = w_1^{i_1}(B_1^1, B_2^2, l_1^1, l_2^2, b, g) w_2(B_1^1, B_2^2, l_1^1, l_2^1, b, g) l_1^1 = \phi l_1^1,$$

in which $\phi = w_{1}^{i_1}/w_{2}^{i_2}$ is the wage ratio. The self-selection constraint that may become binding can then be written as

$$v_2 = v(B_2^2, h^2, g) \geq v(B_1^1, H - \phi l_1^1, g) = \hat{v}_2,$$

where $\hat{v}_2$ denotes the utility of the mimicker. The Lagrangian corresponding to the optimal tax and expenditure problem becomes

$$\mathcal{L} = v_1 + \mu[v^2 - \hat{v}_2] + \beta[v^0 - \hat{v}_0] + \rho[v^{0} - \hat{v}_0] + \lambda[v^2 - \hat{v}_2]$$

$$+ \gamma \left[ F(\cdot) - \sum_{i=1}^{2} N_i^{i_1} - \sum_{i=1}^{2} (M^{i_1} - N^{i_1})b - B^{0} - (M^{1} + M^{2} + 1)g \right],$$

where $w^{i}$ and $N^{i}$ satisfy equations (11) and (12), respectively, while $\mu, \beta, \rho, \lambda,$ and $\gamma$ are Lagrange multipliers. The first-order conditions for $B_1^1, l_1^1, B_2^2, l_2^1, b,$
and $B^0$ can be written

$$B^1 : v_B^1 = \lambda \hat{\nu}_B^2 - \lambda \hat{\nu}_B^2 \frac{\partial \phi}{\partial B^1} \ell^1 + \gamma (N^1 - \eta_B^1),$$

(16)

$$l^1 : v^1_h = \lambda \hat{\nu}_h^2 \left( \phi + \frac{\partial \phi}{\partial l^1} l^1 \right) + \gamma (w^1 N^1 + \eta^1_l),$$

(17)

$$B^2 : (\mu + \lambda) v_B^2 = -\lambda \hat{\nu}_B^2 \frac{\partial \phi}{\partial B^2} \ell^1 + \gamma (N^2 - \eta_B^2),$$

(18)

$$l^2 : v^2_h = \lambda \hat{\nu}_h^2 \frac{\partial \phi}{\partial l^2} l^1 + \gamma (w^2 N^2 + \eta^2_l),$$

(19)

$$b : \beta v^u_b = -\lambda \hat{\nu}_l^2 \ell^1 \frac{\partial \phi}{\partial b} + \gamma (U - \eta_b),$$

(20)

$$B^0 : \rho v^0_B = \gamma,$$

(21)

where

$$\eta_B^1 = \sum_{i=1}^2 (b + T^i) \frac{\partial N^i}{\partial B^1}, \quad \eta^1_l = \sum_{i=1}^2 (b + T^i) \frac{\partial N^i}{\partial l^1},$$

$$\eta_B^2 = \sum_{i=1}^2 (b + T^i) \frac{\partial N^i}{\partial B^2}, \quad \eta^2_l = \sum_{i=1}^2 (b + T^i) \frac{\partial N^i}{\partial l^2},$$

$$\eta_b = \sum_{i=1}^2 (b + T^i) \frac{\partial N^i}{\partial b}, \quad U = \sum_{i=1}^2 (M^i - N^i).$$

The marginal income tax rates of the two employed ability types implicit in equations (16)–(19) and the determinants of the unemployment benefit have been characterized by Aronsson and Sjögren (2003) in the context of a similar model and will not be further discussed here.

Our purpose is to characterize the cost–benefit rule for public provision of the quasi-private good, given that the income tax and the unemployment income are optimally chosen. By differentiating the Lagrangian with respect to $g$, we have

$$\frac{dL}{dg} = v^1_g + (\mu + \lambda) v^2_g + \beta v^u_g - \lambda \hat{\nu}_g^2 \frac{\partial \phi}{\partial g} l^1 + \gamma \left( \eta_g - \sum_{i=1}^2 (b + T^i) \frac{\partial N^i}{\partial g} \right),$$

(22)

where

$$\eta_g = \sum_{i=1}^2 (b + T^i) \frac{\partial N^i}{\partial g}.$$
Proposition 1 Within the given framework, the welfare effect of increasing the public provision of the quasi-private good can be written

\[
\frac{dL}{dg} = \left[ v^1_g - v^1_B \right] + (\mu + \lambda) \left[ v^2_g - v^2_B \right] + \rho \left[ v^0_g - v^0_B \right] - \lambda \left[ \hat{v}^2_g - \hat{v}^2_B \right] + \beta \left[ v^0_g - v^0_B \right] + \lambda \hat{v}^2_B \Phi + \gamma \eta,
\] (23)

where

\[
\phi = \frac{\partial \Phi}{\partial g} - \frac{\partial \Phi}{\partial B_1} - \frac{\partial \Phi}{\partial B_2} - \frac{\partial \Phi}{\partial b},
\]

\[
\eta = \eta_g - \eta^1_B - \eta^2_B - \eta_b.
\]

Note first that the cost–benefit rule derived here is general in the sense of applying for any initial level of \( g \). To interpret Proposition 1, consider first the special case with exogenous wage rates and perfect competition in the labor market. Equation (23) then simplifies to read

\[
\frac{dL}{dg} = \left[ v^1_g - v^1_B \right] + (\mu + \lambda) \left[ v^2_g - v^2_B \right] + \rho \left[ v^0_g - v^0_B \right] - \lambda \left[ \hat{v}^2_g - \hat{v}^2_B \right].
\] (24)

Except for the expression \( \rho \left[ v^0_g - v^0_B \right] \), which is associated with the firm owner, equation (24) is a restatement of a result derived by Boadway and Marchand (1995). Notice that, since \( z^0 > 0 \), \( z^1 > 0 \), \( z^2 > 0 \), and \( \hat{z}^2 > 0 \) for \( g = 0 \), the welfare effect of introducing a small \( g \) is zero in this special case. However, as \( g \) continues to increase, the private purchases of the quasi-private good will be reduced and eventually go to zero. If the mimicker becomes crowded out, whereas the true ability types do not become crowded out, then public provision leads to higher welfare, in which case the results seem to suggest that we have an interior optimum for the publicly provided quantity of the quasi-private good\(^9\). In this situation, the first three terms on the right-hand side of equation (24) are equal to zero, whereas the fourth term is positive. The intuition is that increasing \( g \) above the point where the mimicker becomes crowded out relaxes the self-selection constraint.

Returning to the more general case with endogenous wage rates and unemployment, several additional effects appear in the cost–benefit rule. Note first that an increase in \( g \) influences the wage rates and, therefore, the self-selection constraint via the wage distribution. This has been pointed out by Pirttilä and Tuomala (2002), although the mechanism discussed in their study is an external effect in the production instead of union wage

\( ^9 \) Note that this argument implicitly assumes a well-behaved optimal tax and expenditure problem, where the solution satisfies the second-order conditions. Therefore, the interpretation should be combined with a warning to the reader, since optimal tax and expenditure problems (this one being no exception) may not be well behaved in that sense.
formation. In either case, however, the intuition is that if \( \frac{\partial \phi}{\partial g} > 0 \) (<0), then public provision of the quasi-private good increases (decreases) the wage ratio, which, in turn, makes mimicking less (more) attractive. Thus, this effect constitutes an incentive to choose a higher (lower) \( g \) than otherwise.

According to equation (23), public provision of the quasi-private good also constitutes a tool for influencing the number of employed persons; to our knowledge, this motive for public provision has been neglected in previous research. This so-called employment effect is measured by \( \eta_g \). The intuition is that an increase in the number of employed persons increases the tax revenues net of transfer payments, ceteris paribus, implying that the public expenditures become less costly to finance. To be more specific, each additional employee of ability type \( i \) gives rise to an increase in the tax revenues net of transfer payments equal to \( T_i + b \), which constitutes the welfare effect in real terms of a marginal increase in \( N_i \). Therefore, if an increase in \( g \) increases (decreases) the number of employed persons, ceteris paribus, so \( \eta_g > 0 \) (<0), then this employment effect implies an additional incentive (disincentive) to publicly provide the quasi-private good. This additional incentive (disincentive) can also be interpreted in terms of redistribution:

as long as \( v^i > v^u \) for \( i = 1, 2 \), higher employment implies redistribution in the sense of reducing the number of agents in unemployment (which is the group with the lowest utility among the labor-force participants)\(^{10}\). In addition, since higher employment increases the tax revenues net of transfer payments, more resources would (in principle) become available to support those who remain unemployed.

Another novelty is the appearance of budget-balance effects in the cost–benefit rule. The budget balance effects appear because an increase in \( g \) must be accompanied by an increase in the tax revenues, which, in turn, decreases the income available for private consumption, i.e., \( B^1, B^2, \) and \( b \).

\(^{10}\) This interpretation would, perhaps, become more apparent if the objective function and minimum-utility restrictions in the optimal tax and expenditure problem were replaced by a general social welfare function that reflected the size of each group. In the model set out above, the welfare increase associated with a marginal increase in the number of employed persons of ability type \( i \), \( \gamma(T_i + b) \), depends on the marginal cost of public funds and the increase in the tax revenues net of transfer payments. With a utilitarian social welfare function, on the other hand, the corresponding welfare increase becomes

\[ \gamma \left[ \frac{1}{\gamma} (v^i - v^u) + (T_i + b) \right]. \]

in which the private utility gain of being employed instead of unemployed is recognized.

\(^{11}\) The connection between public policy and employment described here differs from that discussed in the literature dealing with workfare, where transfer payments to people who might otherwise choose to be unemployed on a voluntary basis are conditioned on the participation in public work programs; see, e.g., Besley and Coate (1994) and Brett (1998). See also Bradebury (2004) for a literature review.
According to equation (23), $B^1$, $B^2$, and $b$ affect the cost–benefit rule for $g$ via two channels: (i) the wage ratio and (ii) the number of employed persons. These budget balance effects are absent in previous studies, which are based on the assumption of perfectly competitive labor markets. This is so because, if the labor market is competitive, then $b = 0$; in addition, neither $B^1$ nor $B^2$ influences the wage ratio\textsuperscript{12} or the number of employed persons under perfect competition. In an economy with union wage setting, on the other hand, a broader set of variables affect wage formation and, therefore, the wage ratio. If $\partial\phi/\partial B^1 > 0$ ($< 0$), then the decrease in the private income of the low-ability type – following the necessity to raise additional tax revenues – will contribute to increase (decrease) the wage differential, \textit{ceteris paribus}. Therefore, mimicking becomes more (less) attractive, meaning that the budget balance effect constitutes an incentive for the government to provide less (more) of the quasi-private good than it would otherwise have done. The effects of $B^2$ and $b$ on the wage ratio are interpretable in a similar way. The other channel through which $B^1$, $B^2$, and $b$ affect the cost–benefit rule is due to the relationship between private income and employment, here represented by $\eta^1_b$, $\eta^2_b$, and $\eta_b$, respectively. If $\eta^1_b > 0$ ($< 0$), then the decrease in the private income facing the low-ability type – following a higher $g$ – tends to decrease (increase) the number of employed persons, meaning that there is an extra cost (gain) in terms of lost (gained) employment of raising the additional tax revenues. Therefore, to avoid the decrease (stimulate an increase) in the number of employed persons, this budget balance effect works as an incentive to publicly provide less (more) of the quasi-private good than under full employment. The other budget balance effects associated with employment, $\eta^2_b$ and $\eta_b$, have similar interpretations.

In general, it is not clear whether the additional effects of public provision via the self-selection constraint and the employment effects tend to increase or to decrease the public provision relative to the situation with full employment. However, an interesting special case arises if the labor market of the high-ability type is competitive, whereas the wage rate of the low-ability type is determined by bargaining. To simplify the analysis, let us assume that $w^2$ is fixed. In this case, it is straightforward to show that, if the quasi-private good is complementary with leisure in the sense that $v_{bh} > 0$, then an increase in $g$ will unambiguously increase $w^1$. The basic intuition is that an increase in the public provision of the quasi-private good increases

\textsuperscript{12} Under perfect competition in the labor market, the wage ratio is defined as

$$\phi = w^1/w^2 = F_{1,2}(L^1, L^2)/F_{1,2}(L^1, L^2),$$

where $L^1 = M^1F$ and $M^1$ is a constant. This means that the effects of $l^1$ and $F$ fully capture the influence of taxation on the wage ratio.
the utility of being unemployed relative to the utility of being employed, *ceteris paribus*. This implies that the state of unemployment becomes relatively more attractive, which, in turn, provides the union with an incentive to bargain for a higher wage rate. Therefore, the government can alleviate the unemployment problem by providing a relatively low level of *g*. However, such a policy would also increase the pretax wage dispersion between the employed low-ability and high-ability types. In other words, the policies to reduce unemployment and counteract the wage inequality among the employed are not compatible. Given the unemployment problem among low-skilled workers in Europe, this situation exemplifies a possible conflict between basic policy objectives.

Weak separability between leisure and the other goods in the utility function means that the mimicker and the low-ability type become crowded out at the same point. As a consequence, in the special case with fixed wage rates and perfect competition, it may not be possible to justify public provision of private goods. However, by relaxing either of these two assumptions, public provision of private goods will have a distinct role to play, even if leisure is weakly separable from the other goods. Pirttilä and Tuomala (2002) consider a model with endogenous wage rates. By abstracting from the direct productivity effect emphasized in their study, which is not relevant in our case where the quasi-private good only enters the utility function (and not the production function), their result implies that if leisure is weakly separable from the other goods, then it is welfare improving to publicly provide the quasi-private good if this contributes to reduce the wage differential, i.e., $\partial \phi/\partial g > 0$. In our case, where the wage rates are determined by bargaining between unions and firms, we obtain the following result:

**Proposition 2** If $\Phi > 0$ and $\eta > 0$, then it is possible to increase the welfare via public provision of the quasi-private good.

Proposition 2 immediately follows from the assumptions on which the model is based. In other words, if the joint influence of *g*, $B^1$, $B^2$, and $b$ is to reduce the wage differential and/or to increase the number of employed persons, then public provision of the quasi-private good may be justified by arguments other than nonseparability. The reason is, of course, that private purchases of the quasi-private good neither reflect the effects via the wage ratio nor the changes in employment. The reason is, of course, that private purchases of the quasi-private good neither reflect the effects via the wage ratio nor the changes in employment. In addition, recall from the above discussion that $\Phi$ and $\eta$ do not only reflect the direct effects of *g* on the wage distribution and employment, respectively; they also reflect indirect effects via the requirements of budget balance. Thus, this paper points at several additional channels via which public provision of private goods may affect the welfare.
4. Summary

This paper concerns public provision of private goods under imperfect competition in the labor market. The analysis is based on a two-type model, where the wage rates are determined by bargaining between unions and firms. We would like to emphasize two aspects of publicly provided private goods. First, imperfect competition in the labor market gives rise to additional policy incentives associated with the self-selection constraint, which may justify either more or less public provision than under perfect competition. Second, since the equilibrium may be characterized by unemployment, public provision of private goods becomes a tool for influencing employment. Note also that these additional effects can be separated into direct effects of the publicly provided quantity of the private good, and indirect effects following from the necessity to raise additional tax revenues. Therefore, this paper points at a broader set of mechanisms behind public provision of private goods than previous studies.

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Optimal Taxation and Asymmetric Information in an Economy with Second-Hand Trade*

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Abstract
This paper concerns optimal income and commodity taxation in a two-type overlapping generations model, where used durable goods are traded in a second-hand market. As second-hand transactions are difficult to observe, we assume that the government is unable to directly control second-hand transactions via commodity taxation. A basic question is how the government in this case may use the second-hand market as a channel for relaxation of the self-selection constraint. We show how the appearance of a second-hand market for used durable goods affects the optimal use of labor income and capital income taxation as well as the optimal use of commodity taxation on new durable goods.

Key words: Optimal taxation, Intertemporal Choice, Durable Goods

JEL classification: D91, H21, H23

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1 Introduction

There is a large literature dealing with redistribution under asymmetric information, in which the optimal use of nonlinear income taxation and the optimal mix of income and commodity taxation are two important issues. Although this research has produced insights of considerable value, most studies are based on (at least) two important simplifications: the economy is described by a static model, and (in case of a mixed tax system) there is typically a separate tax instrument for each commodity (except the numeraire). A consequence of this reference-model is a tendency to abstract from durable goods and, in particular, that used durable goods might be traded in second-hand markets; transactions which for obvious reasons are difficult to observe (and tax) by the government. In real world market economies, this type of trade may refer to a variety of durable goods including vehicles\(^1\) such as boats and snowmobiles, kitchen equipment such as stoves, refrigerators and freezers, and gardening tools such as lawn-mowers: commodities of importance for the standard of living and, therefore, also relevant from the perspective of redistribution.

This paper concerns optimal income and commodity taxation in an overlapping generations (OLG) model with asymmetric information, where used durable goods are traded in a second-hand market.\(^2,3\) The argument that second-hand transactions are difficult to observe will be taken to its extreme point, as we assume that the government is unable to directly control second-hand transactions via commodity taxation. Therefore, a basic question is how the appearance of a second-hand market (where the transactions are untaxed) affects the optimal use of income taxation as well as the optimal use of other commodity taxes.

Our paper is based on, and tries to combine, insights from two (related)

\(^1\)The second-hand market for used cars, on the other hand, is not an equally good example, as governments often keep car registers (which include all cars in traffic).

\(^2\)The existing literature on optimal taxation and durable goods focuses on issues other than redistribution under asymmetric information. Much of the earlier research is found in the area of environmental economics; see, e.g., Runkel (2004) who analyzes taxation of a pollution-generating durable good in an economy where the producers operate under imperfect competition. There is also a large literature on environmental policy in intertemporal models (discussed elsewhere), in which the environmental quality has durable good properties.

\(^3\)A major issue in the literature on second-hand trade is the trade with used cars, and several studies are based on the adverse selection approach developed by Akerlof (1970). See, e.g, Hendel and Liizzeri (1999), Porter and Sattler (1999) and Gilligan (2004).
strands of literature dealing with redistribution under asymmetric information; (i) the study of mixed taxation and (ii) the study of optimal income taxation in dynamic economies. The former literature typically uses static models, and a major question is whether, and how, commodity taxes ought to supplement the income tax. Earlier research in this area shows that, if the redistribution is designed to benefit low-income agents (which is the common assumption), one may relax the self-selection constraint by encouraging the consumption of commodities which are substitutable for leisure, and discouraging the consumption of commodities which are complementary with leisure (e.g., Edwards et al. 1994). Furthermore, there is an incentive for the government to discourage the consumption of a commodity, if an increase in the consumer price of this particular commodity leads to less wage inequality, while the opposite policy incentive applies if an increase in the consumer price leads to more wage inequality (Naito 1999).

The literature dealing with redistribution under asymmetric information in dynamic economies has so far mainly focused on income taxation; more precisely, a major question is whether capital income taxation ought to supplement the labor income tax. The seminal contribution is a paper by Ordover and Phelps (1979), which is based on a model with a continuum of ability-types and fixed before tax wage rates. The results show that the marginal capital income tax rate should be zero for each ability-type, if leisure is separable from private consumption in the utility function. Brett (1997) considers an OLG framework with two ability-types (an extension of the two type model developed by Stern 1982 and Stiglitz 1982) and relaxes the separability assumption mentioned above. He finds that the marginal capital income tax rate of the low-ability type (who is the mimicked agent in his study) can be either positive or negative, whereas the marginal capital income tax rate of the high-ability type remains equal to zero. Pirttilä and Tuomala (2001) also use an OLG framework while adding endogenous gross wages rates. Their results show (in addition to mechanism emphasized by Brett) that intertemporal production inefficiency at

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4If leisure is weakly separable from the other goods in the utility function - and with fixed gross wage rates - a government that is able to use a nonlinear income tax has no need for differentiated commodity taxation; see Atkinson and Stiglitz (1976).
the second best optimum may justify capital income taxation\textsuperscript{5,6}, whereas the labor income tax structure corresponds to that derived by Stiglitz (1982)\textsuperscript{7} in a static model.

The present study is based on a two-type model, where one of the consumption goods is a durable good. We assume that the new and used units of the durable good - to be referred to as the new and used durable good, respectively, - are imperfect substitutes in consumption. In addition, as our model does not restrict the analysis to a linear technology, it follows that the before tax wage rates are endogenous. The government redistributes via nonlinear labor income and capital income taxation supplemented by a linear commodity tax on the new durable good. As indicated above, there is no commodity tax on the used durable good (when sold on the second-hand market), meaning that the government has fewer effective policy instruments than the number of variables it wishes to control\textsuperscript{5}.

Our study contributes to the literature in primarily two ways. First, and foremost, we add a durable private good and a second-hand market to the framework for studying redistribution under asymmetric information. For reasons mentioned above, this extension has practical relevance as well as adds a new theoretical dimension by comparison with earlier literature. Second, and to the best of our knowledge, there are no earlier studies analyzing the optimal combination of labor income taxation, capital income taxation and commodity taxation in a dynamic economy with asymmetric information. As a consequence, our paper also contributes to the study of mixed taxation more generally.

The results show how and why the second-hand market for the used durable

\textsuperscript{5}By a similar argument, Aronsson et al. (in press) show that the appearance of equilibrium unemployment (caused by non-competitive wage formation) may justify capital income taxation, as it implies intertemporal production inefficiency.

\textsuperscript{6}Other arguments for using capital income taxation are, e.g., that capital taxes may serve as an indirect instrument to tax unobservable wealth (Broadway et al. 2000) and paternalistic motives caused by differences in the utility discount rate between the consumers and the social planner (Tenhunen and Tuomala 2007).

\textsuperscript{7}This means of positive marginal labor income tax rate for the low-ability type and a negative marginal labor income tax rate for the high-ability type.

\textsuperscript{8}Although in contexts different from ours, certain aspects of optimal taxation in economies with 'too few' tax instruments have been addressed in earlier literature; e.g., national tax policies under transboundary environmental problems (Aronsson and Blomquist 2003, Aronsson et al. 2006) and alcohol taxation in economies with cross-border shopping (Christiansen 2003, Aronsson and Sjögren 2007).
good influences the incentives underlying the commodity tax implemented on the new durable good as well as the incentives underlying income taxation. The basic mechanism that our paper emphasizes is that a change in the second-hand price (which can be accomplished via tax policy) may contribute to relax the self-selection constraint. Therefore, the second-hand market constitutes in a sense an additional channel of relevance for redistribution. As the government is unable to directly tax the second-hand trade, the income taxes and the commodity tax on the new durable good will, in our framework, partly serve as (imperfect) instruments for influencing the second-hand market. As we will argue below, the underlying motive for modifying the commodity tax on the new durable good is similar to the arguments for commodity taxation described above: we ought to encourage (discourage) the consumption of commodities which are substitutable for (complementary with) leisure, with the addition that, in our case, the commodity tax on the new durable good will depend on whether leisure is complementary with, or substitutable for, both the new and used durable good. The modification of the income tax structure caused by the second-hand trade depends on (among other things) how the consumption of the used durable good correlates with the labor supply and savings behavior, which are the aspects of behavior targeted by income taxation.

The outline of the paper is as follows. Section 2 presents the model and the outcome of private optimization as well as describes the optimal tax and expenditure problem. The policy-outcome in terms of optimal taxation is discussed in Section 3. Section 4 provides a summary and concluding remarks.

2 The Model

This section describes the decision-problems facing the consumers, firms and the government. The outcome in terms of optimal taxation will be analyzed in the next section.

2.1 Consumers

Let us assume a closed economy, where each consumer generation lives for two periods. There are two types of consumers: a low-ability type (denoted by superindex 1) and a high-ability type (denoted by superindex 2). The distinction between ability-types refers to productivity, meaning that the high-ability type
is more productive and faces a higher gross wage rate than the low-ability type. To simplify the calculations, we assume that the number of consumers of each ability-type is constant across generations. Without loss of generality, therefore, we normalize the number of consumers of each ability-type and generation to one.

Consider a consumer of ability-type \( i \) born in the beginning of period \( t \), who is young in period \( t \) and old in period \( t+1 \). The consumer derives utility from the consumption of a nondurable good and a durable good when young and old, respectively, as well as from leisure. Let \( c_{1t}^i \) and \( c_{2t+1}^i \) denote the consumption of the nondurable good when young and old, respectively, and \( z_t^i \) the amount of leisure consumed when young. Leisure is, in turn, defined as \( z_t^i = h - l_t^i \), where \( h \) is the total time available in a given period and \( l_t^i \) the hours of work supplied when young. Following earlier related literature, we assume that the consumer does not work when old.

The durable good has a life-length of two time periods. When young, the consumer buys \( x_{1t}^i \) new units of the durable good and \( \eta_{1t}^i \) used units of the durable good. The used units are bought in a second-hand market and has a remaining life-length of one period. It is assumed that wear and tear means that new and used durable goods are imperfect substitutes. When the consumer becomes old, the amount of the new durable good bought in the previous period, \( x_{1t}^i \), becomes units of a used durable good, which can be sold in the second-hand market. The units sold when old will be denoted \( \chi_{2t+1}^i \).

Therefore, the utility function of ability-type \( i \) born in period \( t \) can be written as

\[
U_t^i = U \left( c_{1t}^i, x_{1t}^i, \eta_{1t}^i, z_t^i, c_{2t+1}^i, x_{1t}^i - \chi_{2t+1}^i \right),
\]

in which the consumption of leisure when old (which is fixed) has been suppressed. The function \( U(\cdot) \) is increasing in each argument and strictly concave. We assume that all goods are normal. Note also that the supply of the used durable good by the old consumer is constrained by the amount bought of the new durable good when young. Therefore, the expression \( x_{1t}^i - \chi_{2t+1}^i \) denotes the consumption of the used durable good when old.\(^9\)

\(^9\)As there are no bequests in the model, we assume that the old consumer does not buy the new durable good. In addition, note that the consumer is a net supplier of used durable goods when old.
Since the optimal tax problem set out below is formulated in terms of a conditional indirect utility function, it is convenient to start by solving the utility maximization problem conditional on the levels of leisure and saving, respectively. This decision-problem means choosing $c^i_{1t}$, $x^i_{1t}$, $\eta^i_{1t}$, $c^i_{2t+1}$ and $\chi^i_{2t+1}$ to maximize the utility function given by equation (1) subject the budget constraint

\begin{align*}
b^i_{1t} &= c^i_{1t} + q_{t,x}x^i_{1t} + q_{t,\eta}\eta^i_{1t} \\
b^i_{2t+1} &= c^i_{2t+1} - q_{t+1,\eta}\chi^i_{2t+1}
\end{align*}

where $b^i_{1t}$ and $b^i_{2t+1}$ are fixed budgets to be allocated for consumption in each period, and the nondurable good is used as the numeraire. The consumer price of the new durable good in period $t$, $q_{t,x}$, is given by $q_{t,x} = p_{t,x} + \tau_{t,x}$, where $p_{t,x}$ is the producer price and $\tau_{t,x}$ the commodity tax, while $q_{t,\eta}$ and $q_{t+1,\eta}$ are the (second-hand) prices of the used durable good in periods $t$ and $t+1$, respectively. Solving this conditional utility maximization problem gives the conditional demand functions

\begin{align*}
c^i_{1t} &= c_1 \left( b^i_{1t}, b^i_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z^i_{2t+1} \right) \\
x^i_{1t} &= x_1 \left( b^i_{1t}, b^i_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z^i_{2t+1} \right) \\
\eta^i_{1t} &= \eta_1 \left( b^i_{1t}, b^i_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z^i_{2t+1} \right) \\
c^i_{2t+1} &= c_2 \left( b^i_{1t}, b^i_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z^i_{2t+1} \right)
\end{align*}

and a conditional supply function

\begin{equation}
\chi^i_{2t+1} = \chi_2 \left( b^i_{1t}, b^i_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z^i_{2t+1} \right)
\end{equation}

By substituting equations (4) - (8) into (1), we obtain the conditional indirect utility function

\begin{equation}
V^i_{t} = V \left( b^i_{1t}, b^i_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z^i_{2t+1} \right)
\end{equation}

We can derive the hours of work and saving, respectively, by maximizing the conditional indirect utility function subject to the budget constraint
\[ b_{1t}^i = w_t^i l_t^i - T_t \left( y_t^i \right) - s_t^i \]  
\[ b_{2t+1}^i = (1 + r_{t+1}) s_t^i - \Phi_{t+1} \left( I_{t+1}^i \right) \]

where \( w_t^i \) is the gross wage rate, \( s_t^i \) saving, \( r_{t+1} \) the interest rate, \( y_t^i = w_t^i l_t^i \) the labor income and \( I_{t+1}^i = r_{t+1} s_t^i \) the capital income. The function \( T_t(\cdot) \) determines the labor income tax payment by the young generation in period \( t \) and \( \Phi_{t+1}(\cdot) \) the capital income tax payment by the old generation in period \( t+1 \). In the analysis below, we will use the following short notations for income tax payments and marginal income tax rates

\[ T_t^i = T_t \left( y_t^i \right), \quad T_{t,y}^i = \frac{\partial T_t \left( y_t^i \right)}{\partial y_t^i} \]
\[ \Phi_{t+1}^i = \Phi_{t+1} \left( I_{t+1}^i \right), \quad \Phi_{t+1,i}^i = \frac{\partial \Phi_{t+1} \left( I_{t+1}^i \right)}{\partial I_{t+1}^i} \]

The first order conditions for the hours of work and savings can then be written as

\[ \frac{\partial V_t^i}{\partial h_t^{i,1}} w_t^i (1 - T_t^i) - \frac{\partial V_t^i}{\partial z_t^i} = 0 \]  
\[ -\frac{\partial V_t^i}{\partial h_t^{i,1}} + \frac{\partial V_t^i}{\partial h_{2t+1}^i} \left[ 1 - r_{t+1} - r_{t+1} \Phi_{t+1,i}^i \right] = 0 \]

### 2.2 Production

There are two competitive private production sectors; sector \( c \) produces the nondurable good and sector \( x \) the durable good. In each sector, identical firms produce a homogenous good by using a constant returns to scale technology, and we abstract from possible entry into, and exit out of, the goods markets. Given these characteristics, the number of firms in each sector is not important and will be normalized to one.

The firm in sector \( j, j = c, x \), uses three variable production factors; labor of both ability-types, \( l_{t,j}^1 \) and \( l_{t,j}^2 \), and capital, \( K_{t,j} \). The production function is given by \( F_j \left( l_{t,j}^1, l_{t,j}^2, K_{t,j} \right) \), which is increasing and strictly concave
in each argument. Normalizing with respect to $l_{t,j}^i$, we have
\[ f_j(n_{t,j}, k_{t,j}) = F_j(l_{t,j}^1, l_{t,j}^2, K_{t,j})/l_{t,j}^1, \]
where \( n_{t,j} = l_{t,j}^2/l_{t,j}^1 \) and \( k_{t,j} = K_{t,j}/l_{t,j}^1 \). We assume that
labor and capital are perfectly mobile across sectors, meaning that the factor
prices will be the same in both sectors. The first order conditions can then be
written as

\[
w_t^1 = p_{t,j} f_j (n_{t,j}, k_{t,j}) - n_{t,j} p_{t,j} \frac{\partial f_j (n_{t,j}, k_{t,j})}{\partial n_{t,j}} \quad (14)
\]

\[ - k_{t,j} p_{t,j} f_j \frac{\partial f_j (n_{t,j}, k_{t,j})}{\partial k_{t,j}} \]

\[
w_t^2 = p_{t,j} \frac{\partial f_j (n_{t,j}, k_{t,j})}{\partial n_{t,j}} \quad (15)
\]

\[ r_t = p_{t,j} \frac{\partial f_j (n_{t,j}, k_{t,j})}{\partial k_{t,j}} \quad (16)
\]

for \( j = c, x \), where \( p_{t,j} \) is the producer price in sector \( j \) (recall that \( p_{t,c} = 1 \)
from normalization).

### 2.3 Equilibrium

The factor market equilibrium is defined by the equilibrium conditions

\[
l_t^i = \sum_j l_{t,j}^i \text{ for } i = 1, 2 \quad (17)
\]

\[
K_t = \sum_j K_{t,j} \quad (18)
\]

where \( K_t \) is the aggregate capital stock in period \( t \). Combining equations (14)-(18), we can solve for \( w_t^1, w_t^2, l_{t,e}^1, l_{t,e}^2, l_{t,e}^1, l_{t,e}^2, r_t, K_{t,e} \) and \( K_{t,x} \) as functions of
\( p_{t,x}, l_t^1, l_t^2 \) and \( K_t \). Substituting into the production function gives 'the
equilibrium supply function'

\[
S_{t,j} = S_j (p_{t,x}, l_t^1, l_t^2, K_t) \quad (19)
\]

for \( j = c, x \).

The equilibrium condition characterizing the market for the new durable
good in period \( t \) becomes
\[ S_{r,t} (\cdot) = \sum_i x_{1t} \]  

Equation (20) implicitly defines the equilibrium producer price of the new durable good, i.e.

\[ p_{t,x} = p \{ l_{1t}^1, l_{1t}^2, b_{1t}^1, b_{2t+1}^1, b_{2t+1}^2, b_{2t+1}^3, \tau_{r,x}, q_{t,q}, q_{t+1,q}, K_t \} \]  

Let us finally turn to the second-hand market, where the old generation sells the used durable good to the young generation. The equilibrium condition for the second-hand market in period \( t \) is given by

\[ \sum_{i=1}^{2} (\chi_{2t}^i - \eta_{1t}^i) = 0 \]  

Note also that Walras’ law implies that, as long as equations (20) and (22) hold, the market for the nondurable good will also be in equilibrium.

### 2.4 The Public Decision-Problem

The objective function facing the government is represented by a general social welfare function

\[ W = W(V_0^1, V_0^2, V_1^1, V_2^1, V_2^2, V_3^1, V_3^2, \ldots), \]  

which allows for a separate welfare weight attached to the utility function of each ability-type in each generation.

The government redistributes via income and commodity taxation. Note that \( T_t(\cdot) \) is a general labor income tax, which can be used to implement any desired combination of \( b_{1t}^1, b_{1t}^2, l_{1t}^2 \) and \( b_{2t}^2 \) (with the savings held constant). It is, therefore, convenient to follow the approach in earlier comparable literature by using \( l_{1t}^1, b_{1t}^1, l_{2t}^2 \) and \( b_{2t}^2 \), instead of the parameters of the function \( T_t(\cdot) \), as direct decision-variables facing the government. Similarly, the general capital income tax function \( \Phi_{t+1}(\cdot) \) can be used to implement any desired combination of \( b_{1t}^1, b_{2t+1}^1, b_{1t}^2 \) and \( b_{2t+1}^2 \) (with the labor income held constant); therefore, instead of using the parameters of the function \( \Phi_{t+1}(\cdot) \) as decision-variables in the optimal tax problem, we add \( b_{2t+1}^1 \) and \( b_{2t+1}^2 \) to the set of direct decision-variables mentioned above. Note that, as the government effectively controls the saving behavior, it also possesses perfect control of the capital stock.
If we use equations (20) and (22), the resource constraint can be written in terms of the numeraire good as\(^\text{10}\)

\[
S_{t,c} + K_t - K_{t+1} - \sum_i c_{1t}^i - \sum_i c_{2t}^i = 0
\]  

(24)

As in most earlier studies dealing with redistribution under asymmetric information, we assume that the government wants to redistribute from the high-ability type to the low-ability type. The government can observe the labor and capital income of each individual, although ability is private information. Therefore, the most interesting aspect of self-selection will be to prevent the high-ability type from pretending to be a low-ability type in order to gain from the redistribution policy. Note that the hours of work that the high-ability type needs to supply in order to reach the same labor income as the low-ability type is given by \(\bar{h}_t^1 = (w_t^1 / w_t^2) \bar{h}_t^1 = \phi_t \bar{h}_t^1\), where \(\phi_t = \phi(t_{t,x}, l_t^1, l_t^2, K_t)\) is the wage ratio (or relative wage rate) in period \(t\). Denoting the conditional indirect utility function of the mimicker by \(\bar{V}_t^2\), the self-selection constraint that may bind becomes

\[
V_t^2 = V (b_{1t}^2, b_{2t+1}^2, q_{1t}, q_{1t+1}, q_{2t}, \zeta_t) \geq V (b_{1t}^1, b_{2t+1}^1, q_{1t}, q_{1t+1}, q_{2t}, h - \phi_t \bar{h}_t^1) = \bar{V}_t^2
\]  

(25)

The expression on the left hand side of the weak inequality is the utility intended for the high-ability type, whereas the expression on the right hand side is the utility of the mimicker (in this case a high-ability type pretending to be a low-ability type). Note that the mimicker and low-ability type faces the same levels of before-tax and disposable income both when young and old\(^\text{11}\). However, the mimicker is more productive and, as a consequence, spends more time on leisure than the low-ability-type.

---

\(^{10}\)As the government is able to transfer resources lump-sum across generations in our framework and possesses perfect control of the capital stock, we only need to consider one from the resource constraint and the government budget constraint; see Atkinson and Sandmo (1980) and Pirttilä and Tuomala (2001). Note also that we have assumed that the nondurable good can be freely transformed into physical capital.

\(^{11}\)To be a mimicker, the high-ability type must mimic the point chosen on each tax function (both the labor income tax and the capital income tax) by the low-ability type.
Finally, note that the government uses that equation (21) determines the producer price of the new durable good, $p_{t,x}$, and that the equilibrium condition for the used durable good is given by equation (22).

The Lagrangean corresponding to the government’s optimization problem can be written as

$$\mathcal{L} = W + \sum_{t} \lambda_t [V_t^2 - \bar{V}_t^2] + \sum_{t} \gamma_t [S_{t,x} + K_t - K_{t+1} - \sum_i c_{1it} - \sum_i c_{2it}]$$

$$+ \sum_{t} \mu_t [\lambda_{2t} - \eta_{1t}]$$

where $\lambda_t$ and $\gamma_t$ are the Lagrange multipliers associated with the self-selection constraint and resource constraint, respectively, in period $t$. The constraints in the second row represent the equilibrium condition for the second-hand market for the used durable good in each period, and $\mu_t$ is the Lagrange multiplier associated with the equilibrium condition in period $t$. As we are focusing on the role of the second-hand market in the context of redistributive policy, this formulation is convenient as it enables us to identify the determinants of the shadow price that the government attaches to used durables. The additional (and artificial) direct decision-variable corresponding to this constraint is $q_{t,y}$. In summary, therefore, the decision-variables are $l_t^1, l_t^2, b_{1t}^1, b_{1t}^2, b_{2t}^1, b_{2t}^2, \tau_{t,x}, K_t$, and $q_{t,y}$ for all $t$. The first order conditions are presented in the Appendix.

Note finally that there is a potential time inconsistency problem. The reason is that when each consumer reveals his/her true type at the end of the first period, there may be an incentive for the government to change the structure of capital income taxation. Although this potential problem is recognized, we follow Pirttilä and Tuomala (2001) by assuming that the government can credibly commit to the announced tax and expenditure policies.
3 Optimal Taxation

The shadow price of the used durable good over the shadow price of the resource constraint, $\mu_t/\gamma_t$, will play an important role for the optimal tax structure. This variable represents the marginal value that the government attaches to increased supply of the used durable good measured in terms of tax revenues, i.e. the real marginal social value of the used durable good becoming less scarce. As increased supply means a lower price, ceteris paribus, one may also interpret $\mu_t/\gamma_t$ as indirectly reflecting the marginal cost that the government attaches to a price increase in the second-hand market.

It is instructive to begin by analyzing how $\mu_t/\gamma_t$ is determined. Let the symbol $^n -n$ indicate compensated demand and/or supply function. Recall from the properties of compensated supply and demand functions that

$$\frac{\partial \tilde{x}^n_t}{\partial q_{t,n}} > 0 \text{ and } \frac{\partial \tilde{\gamma}^n_t}{\partial q_{t,n}} < 0,$$

i.e. an increase in the price of the used durable good in period $t$, ceteris paribus, leads to increased compensated supply of, and decreased compensated demand for, the used durable good in period $t$. For purposes of interpretation, we also make the following assumptions;

$$\frac{\partial \tilde{x}^1_t}{\partial q_{t,n}} > 0, \quad \frac{\partial \tilde{x}^1_{t+1}}{\partial q_{t,n}} > 0, \quad \frac{\partial \tilde{x}^1_{t-1}}{\partial q_{t,n}} > 0 \text{ and } \frac{\partial \tilde{\gamma}^1_{t-1}}{\partial q_{t,n}} < 0,$$

To give the basic intuition behind the first and second parts of (26b), note that an increase in the price of the used durable good in period $t$ provides an incentive for the consumer to buy more of the new durable good in period $t$ (because new and used durable goods are substitutes). As a consequence, when the consumer becomes old (in period $t + 1$), he/she has more of the used durable good at his/her disposal and, therefore, more of the used durable good to supply in the second-hand market. The intuition behind the third and fourth parts of (26b) is that the higher $q_{t,n}$, ceteris paribus, the stronger will be the incentive to buy the new\footnote{This is interpretable to mean that the purchase of the new durable good contains a savings-component, as an increase in $q_{t,n}$ means a higher 'return' on the investment in the new durable good made at time $t - 1$.} (instead of the used) durable good in period $t - 1$ in order to supply the used durable good in period $t$. 

To simplify the notations, let $L_{\mu}$ denote the lag operator on $\mu$, so $L_{\mu}\mu_t = \mu_{t-1}$, and define

$$\sigma_t = \sum_i \left( \frac{\partial \tilde{\nu}^i_{2t}}{\partial q_{t,\eta}} - \frac{\partial \tilde{\nu}^i_{2t}}{\partial q_{t,\eta}} \right) + \sum_i \frac{\partial \tilde{\nu}^i_{2t+1}}{\partial q_{t,\eta}} L_{\mu}^{-1} - \sum_i \frac{\partial \tilde{\nu}^i_{1t}}{\partial q_{t,\eta}} L_{\mu} > 0,$$ \hspace{1cm} (26)

as measuring how an increase in $q_{t,\eta}$ affects the intertemporal net supply of the used durable good. This effect is, in turn, decomposable into three components. The first is a positive direct effect on the net supply in period $t$. The second and third components reflect that an increase in $q_{t,\eta}$ gives rise to intertemporal behavioral responses, as is leads to increased supply of the used durable good in period $t+1$ and reduced demand for the used durable good in period $t-1$ according to the assumptions made above.

Consider the following result:

**Proposition 1:** At the second best optimum, the shadow price of the used durable good over the shadow price of the government’s budget constraint can be written as

$$\frac{\mu_t}{\gamma_t} = \frac{1}{\sigma_t \gamma_t} \left[ \lambda_0 \frac{\partial \tilde{V}_{2t}}{\partial \tilde{q}_{1t}} [\tilde{q}_{1t}^2 - \tilde{q}_{1t}^2] + \lambda_{t-1} \frac{\partial \tilde{V}_{2t}}{\partial \tilde{q}_{2t}} [\tilde{q}_{2t}^2 - \tilde{q}_{2t}^2] \right]$$

$$- \frac{1}{\sigma_t \gamma_t} \sum_{k=0}^{t-1} \sum_i \frac{\partial \tilde{x}^i_{1t-k}}{\partial p_{t-i,\eta}} \left[ \rho_{t-k} \lambda_{t-k} \frac{\partial \tilde{p}_{t-k}}{\partial p_{t-k,\eta}} + \tau_{t-k,\eta} \gamma_{t-k} \right],$$

where

$$\rho_{t-k} = \frac{\mu_{t-k} \frac{\partial \tilde{V}_{2t-k}}{\partial \tilde{x}^i_{1t-k}}}{(1 + \frac{\partial p_{t-k,\eta}}{\partial p_{t-k,\eta}})} > 0 \text{ and } D_t = \frac{\partial S_{t,x}}{\partial p_{t,x}} - \sum_i \frac{\partial \tilde{x}^i_{1t}}{\partial q_{t,\eta}} > 0.$$

**Proof:** See the Appendix.

To interpret Proposition 1, note first that if the self-selection constraint does not bind (so $\lambda_{t-1} = \lambda_t = 0$), and if the commodity tax on the new durable good is always equal to zero (so $\tau_{t-1,x} = \tau_{t,x} = 0$), then $\mu_t/\gamma_t = 0$. This means that, in the absence of asymmetric information or other distortions, the constraint associated with the second-hand market becomes redundant.
With a binding self-selection constraint, on the other hand, the second-hand market is no longer redundant; instead, the second-hand market provides an additional channel via which the government may relax the self-selection constraint. The first row of the expression for $\mu_t / \gamma_t$ is directly related to the self-selection constraint; the two components reflect differences between the low-ability type and the mimicker with respect to the demand for the used durable good when young (the first term) and the supply of the used durable good when old (the second term). To understand why such differences in the consumption pattern may arise, recall that although the low-ability type and the mimicker face the same levels of before-tax and disposable income both when young and old, the mimicker is more productive and spends more time on leisure than the low-ability type. If leisure is substitutable for the used durable good in the sense that the young low-ability type consumes more of the used durable good than the young mimicker in period $t$, so $\bar{\eta}^2_t - \bar{\eta}^1_t > 0$, the first term on the right hand side contributes to increase the marginal value that the government attaches to the used durable good. As we will argue below, this mechanism provides an incentive for the government to try to reduce the price of the used durable good, i.e. making it less scarce, which contributes to relax the self-selection constraint via increased consumption of the used durable by the low-ability type. The argument goes the other way around if leisure is complementary with the used durable good in the sense that the young low-ability type consumes less of the used durable than the mimicker in period $t$.

The second term in the first row is an intertemporal analogue to the first term. If the old mimicker supplies more of the used durable good than the old low-ability type in period $t$, so $\bar{\chi}^2_{2t} - \bar{\chi}^1_{2t} > 0$, the second term on the right hand side contributes to increase the marginal value that the government attaches to the used durable good in period $t$. The intuition is based on the assumption that leisure is substitutable for the used durable good in the sense that the young low-ability type consumes more than the mimicker in period $t - 1$, in which case the mimicker (who bought more of the new durable good in period $t - 1$) is likely to be able to sell more of the used durable good in period $t$. Again, the argument goes the other way around if leisure is complementary with the used durable good in the sense that the young low-ability type consumes less of the used durable than the mimicker in period $t - 1$.

The first part of the second row is also related to the self-selection con-
Optimal Taxation and Second-Hand Trade

strained although for a different reason; the relative wage rate is endogenous and will, therefore, respond to a change in the price of the used durable good. There are two mechanisms involved (an atemporal and an intertemporal). First, as the new and used durable good are substitutes in consumption, we have \( \partial \bar{E}_{1t}/\partial q_{t,\eta} > 0 \), suggesting that an increase in \( q_{t,\eta} \) causes upward pressure on \( p_{t,x} \). If \( \partial \phi_1/\partial p_{t,x} > 0 \), it follows that an increase in the price of the used durable good in period \( t \) leads to an increase in the wage ratio in period \( t \), which makes mimicking less attractive and contributes to relax the self-selection constraint. Therefore, as a higher \( q_{t,\eta} \) makes mimicking less attractive, this provides an incentive for the government to attach a lower marginal value to the used durable good (or, equivalently, attach a lower marginal cost to an increase in the price of the used durable good) than it would otherwise have done. The argument will be the opposite if \( \partial \phi_1/\partial p_{t,x} < 0 \). Second, to interpret the relationship between \( q_{t,\eta} \) and \( \phi_{t-1} \), recall our assumption that an increase in \( q_{t,\eta} \) leads to increased compensated demand for the new durable good in period \( t - 1 \), i.e. \( \partial \bar{E}_{1t-1}/\partial q_{t,\eta} > 0 \) (otherwise, the compensated supply of the used durable good would not be upward sloping). This suggests that an increase in \( q_{t,\eta} \) also causes upward pressure on \( p_{t-1,x} \). Therefore, the interpretation of the relationship between \( q_{t,\eta} \) and \( \phi_{t-1} \) is analogous to the interpretation of the relationship between \( q_{t,\eta} \) and \( \phi_t \).

The second part of the second row contains tax base effects, as an increase in the price of the used durable good affects the demand for the new durable good and, therefore, also the tax revenues. As \( \partial \bar{E}_{1t-1}/\partial q_{t,\eta} > 0 \) and \( \partial \bar{E}_{1t}/\partial q_{t,\eta} > 0 \) by assumption, the tax base effects provide an argument for the government to attach a lower marginal value to the used durable good (i.e. it attaches a lower marginal cost to an increase in the price of the used durable good) than it would otherwise have done, as an increase in the price leads to increased tax revenues.

### 3.1 Commodity Taxation

Let us now turn to the commodity tax on the new durable good and, in particular, how the appearance of a second-hand market for the used durable good modifies the use of commodity taxation. For notational convenience, we introduce the short notation;
\[ H_t = \frac{\partial \hat{x}_t^1}{\partial q_{t,x}} + \frac{\partial \hat{x}_t^2}{\partial q_{t,x}} < 0 \]

Consider Proposition 2:

**Proposition 2:** At the second best optimum, the commodity tax on the new durable good can be written as

\[
\tau_{t,x} = \lambda_t \frac{\partial \hat{V}^2}{\partial \hat{q}_t} / \gamma_t H_t - \frac{\mu_t \lambda_t}{\gamma_t D_t} \frac{\partial \phi_t}{\partial q_{t,x}} + \frac{\mu_t}{\gamma_t} \left[ \sum_i \frac{\partial \hat{q}_i}{\partial q_{t,x}} - \sum_i \frac{\partial \phi_i}{\partial q_{t,x}} \right] \frac{1}{H_t}
\]

**Proof:** See the Appendix.

The first row of the formula in Proposition 2 reflects the self-selection constraint. If leisure is complementary with the new durable good in the sense that the young low-ability type consumes less than the young mimicker, so \( x_{t1}^1 - x_{t2}^2 < 0 \), then the first term on the right hand side contributes to increase the commodity tax on the new durable good. The intuition is that an increase in the consumer price of the new durable good will, in this case, cause a greater utility loss for the mimicker than it does for the low-ability type, which relaxes the self-selection constraint. Therefore, a higher commodity tax creates room for additional redistribution. By an analogous argument, if leisure is substitutable for the new durable good, so \( x_{t1}^1 - x_{t2}^2 > 0 \), the first term on the right hand side contributes to a lower commodity tax. This effect is well understood from earlier literature on redistribution under asymmetric information based on static models; e.g. Edwards et al. (1994).

The second term on the right hand side appears because a higher tax on the new durable good leads to a lower producer price which, in turn, affects the wage ratio. If \( \partial \phi_t / \partial p_{t,x} > 0 (< 0) \), this reduction in \( p_{t,x} \) makes mimicking more (less) attractive, as the mimicker needs to supply less (more) hours of work to reach the same labor income as the low-ability type. This provides an incentive for the government to implement a lower (higher) tax on the new durable good than it would otherwise have done. An analogous argument for commodity taxation is derived by Naito (1998) in the context of a static model.
The second row of the formula in Proposition 2 arises as the government attaches a nonzero marginal value to the used durable good. We will refer to the expression summarized by the second row as the 'direct effect' of the second-hand market, as its qualitative contribution is measured with the other terms (i.e. in the first row) held constant. This direct effect may either contribute to increase or decrease the commodity tax. To begin with, note that the expression within the square bracket in the second row is positive. First, as new and used durables are substitutes in consumption, we have $\partial \delta^1_t / \partial q_{t,x} > 0$. Second, as an increase in the consumer price of the new durable good in period $t$ reduces the consumption of the new durable good in period $t$, ceteris paribus, it will (according to our earlier assumptions) also contribute to reduce the supply of the used durable good in period $t + 1$, i.e. $\partial \tilde{\chi}_{t+1}^1 / \partial q_{t,x} < 0$. Therefore, the following result is a direct consequence of Proposition 2:

**Corollary 1:** If $\mu_t / \gamma_t > 0 \ (< 0)$, the direct effect of the second-hand market is to decrease (increase) the commodity tax on the new durable good.

The intuition is that the lower commodity tax on the new durable good contributes to reduce the price of the used durable good in period $t$ and increase the supply of the used durable good in period $t + 1$. This leads to higher welfare if $\mu_t / \gamma_t > 0$ (in which case it is desirable to make the used durable good less scarce) and lower welfare if $\mu_t / \gamma_t < 0$.

Note finally that the direct effect of the second-hand market is, in a sense, similar to the effect of the first term on the right hand side of the tax formula in Proposition 2, as the sign of each of them is dependent on whether leisure is complementary with, or substitutable for, the durable good. As such, they will under certain conditions reinforce each other. If leisure is substitutable for the new and used durable good, and if the latter effect is strong enough to imply that $\mu_t / \gamma_t > 0$, then these terms represent two sides of the same argument for subsidizing the new durable good. The analogous (joint) argument for taxation may apply if leisure is complementary with the new and used durable good.

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13Recall from Proposition 1 that $\tilde{\gamma}^1_t > \tilde{\delta}^2_t$ contributes to increase $\mu_t / \gamma_t$. 
3.2 Labor Income Taxation

As the government has fewer effective policy instruments than the number of variables it wishes to control, the appearance of a second-hand market is also important from the point of view of the income tax structure. Let us begin by characterizing the marginal labor income tax rates. Define

\[ MRS_{t,ab}^i = \frac{\partial V_t^i / \partial z_t^i}{\partial V_t^i / \partial \theta_{tt}}, \quad \text{and} \quad MRS_{t,ab}^2 = \frac{\partial V_t^2 / \partial z_t^2}{\partial V_t^2 / \partial \theta_{tt}}, \]

where \( \tilde{z}_t^i = h - \phi_i l_t^i \), to be the marginal rate of substitution between leisure and private income for ability-type \( i \) and the mimicker, respectively, in period \( t \). To simplify the expressions for the marginal labor income tax rates, we shall also use the short notation

\[ \Delta_{t,i} = \frac{\partial \phi_i}{\partial l_t^i} - \frac{1}{(1 + \partial p_{t, x}/\partial \tau_{t, x}) \partial p_{t, x} D_t} \frac{\partial S_t^{i,x} / \partial l_t^i + \partial \tilde{x}_t^{i,x} / \partial z_t^i}{D_t}, \]

(27)

where \( 1 + \partial p_{t, x}/\partial \tau_{t, x} > 0 \), for how an increase in the hours of work supplied by ability-type \( i \) affects the wage ratio (which involves a direct effect and an indirect effect via the producer price of the new durable good). Consider Proposition 3:

**Proposition 3:** At the second best optimum, the marginal labor income tax rates can be written as

\[ T^1_{t, y} = \frac{1}{u_t} \left\{ \lambda_{t, b} [MRS_{t, ab}^1 - MRS_{t, ab}^2 \phi_t] - \lambda_{t, b} \Delta_{t, i} + \tau_{t, s} \frac{\partial \tilde{x}_t^{i,x}}{\partial z_t^i} \right\} \]

\[ - \frac{\mu_t}{\gamma_t} \left\{ \frac{\partial \eta_{t, t}}{\partial z_t^i} - \frac{\partial \tilde{u}_{2hi}^1}{\partial z_t^i} L_{\mu}^{-1} \right\} \]

\[ T^2_{t, y} = \frac{1}{u_t} \left\{ -\lambda_{t, b} \Delta_{t, i} + \tau_{t, s} \frac{\partial \tilde{x}_t^{i,x}}{\partial z_t^i} \right\} \]

\[ - \frac{\mu_t}{\gamma_t} \left\{ \frac{\partial \eta_{t, t}}{\partial z_t^i} - \frac{\partial \tilde{u}_{2hi}^2}{\partial z_t^i} L_{\mu}^{-1} \right\} \]

where \( \lambda_{t, b} = [\lambda_t / \gamma_t] [\partial V_t^2 / \partial \beta_t]^2 > 0 \) and \( \lambda_{t, l} = [\lambda_t / \gamma_t] [\partial V_t^2 / \partial z_t^2]^2 > 0 \).
Proof: See the Appendix.

The marginal labor income tax rates in Proposition 3 reflect a combination of three underlying motives for influencing the hours of work; (i) a desire to relax the self-selection constraint via channels that do not directly depend on the second-hand market, (ii) a desire to offset distortions caused by the commodity tax, and (iii) a desire to use the second-hand market as an additional channel for redistribution. The latter follows because the government lacks a tax instrument by which it may directly control the second-hand trade (meaning that the use of the other tax instruments will be modified accordingly). We shall discuss each of these incentives below.

The first two motives mentioned above are captured by the first row in each tax formula in Proposition 3. In the formula for the low-ability type, the first term on the right hand side is positive and appears because the low-ability type needs to forego more leisure than the mimicker in order to accomplish a given increase the disposable income; in our model, this implies \( MRS^1_{t,ab} - MRS^2_{t,ab} \phi_t > 0 \). This incentive to tax the labor income of the low-ability type at the margin is well understood from earlier work by Stiglitz (1982), and the intuition is that a higher marginal labor income tax rate for the low-ability type makes mimicking less attractive which, in turn, contributes to relax the self-selection constraint.

The remaining two terms in the first row of the tax formula for the low-ability type have the same structure as their counterparts in the tax formula for the high-ability type. As we saw above, the variable \( \Delta^i_{t,1} \) is decomposible into two parts, both of which serve to relax the self-selection constraint in period \( t \) via the wage ratio (recall that \( \Delta^i_{t,1} \) is multiplied by \( \lambda_t \) in the tax formulas in Proposition 3). First, \( l^1_t \) and \( l^2_t \) directly affect the wage ratio; if \( \partial \phi_t / \partial l^1_t < 0 \) and \( \partial \phi_t / \partial l^2_t > 0 \), which is arguably realistic and in line with results derived by Stiglitz (1982), the direct effect of the hours of work on the wage ratio will contribute to increase the marginal labor income tax rate of the low-ability type and decrease the marginal labor income tax rate of the high-ability type. Second, in our framework, the hours of work also affect the wage ratio indirectly via the producer price of the new durable good, which is seen from the second term on the right hand side of equation (27). In general, as an increase in the hours of work\(^{14}\) leads to increased supply of the new durable good, whereas the...

\(^{14}\)Note that the marginal labor income tax rate for ability-type \( i \) is calculated by using the
demand for the new durable good may change in either direction, the sign of
the corresponding change in the producer price is ambiguous. To begin with,
suppose that the supply response dominates in the sense that an increase in the
hours of work leads to a lower producer price of the new durable good. In this
case, therefore, if $\partial \phi_t/\partial p_{1,t} > 0$ ($< 0$), the indirect effect of $l_t^i$ on $\phi_t$ constitutes
an incentive for the government to implement a higher (lower) marginal labor
income tax rate on ability-type $i$ than it would otherwise have done. The
intuition is that the resulting change in the producer price makes mimicking
less attractive, which contributes to relax the self-selection constraint. If, on
the other hand, an increase in the hours of work leads to a higher producer price
of the new durable good, the desire to relax the self-selection constraint implies
policy incentives opposite to those just described.

The final term in the first row of each tax formula appears because the
income tax might be used to offset distortions caused by the commodity tax.
To see this, note that the higher the optimal $\tau_{1,t}$, the higher (lower) will be the
marginal labor income tax rate facing ability-type $i$, if leisure is complementary
with (substitutable for) the new durable good in the sense that $\partial x_{1t}^i/\partial z_{1t}^i > 0$
($< 0$). For instance, if leisure is complementary with the new durable good, the
increase in the marginal income tax rate caused by the final term in the first
row leads to increased demand for leisure and, therefore, increased consumption
of the new durable. By analogy, if leisure is substitutable for the new durable
good, a lower marginal income tax rate can be used to increase the consumption
of the new durable. The intuition is that the government has no incentive to
distort the consumption of the new durable good per se (there is no externality
or other market failure associated with the new durable good); only to carry out
redistribution in the most efficient way. Another (yet equivalent) interpretation
is that the final term in the first row measures a tax base effect, suggesting
if leisure complementary with (substitutable for) the new durable good, then
the government will implement a higher (lower) marginal labor income tax rate
than it would otherwise have done in order to gain revenues from the commodity

\footnote{First order conditions for $l_t^i$ and $b_{1,t}^i$. Therefore, when we change the hours of work, the private
disposable income will also change in order to maintain budget balance for the government.
This also explains why the change in the demand for the new durable good (which, in part,
determines how an increase in the hours of work affects the producer price of the new durable
good) is measured in terms of the compensated demand; see the second part of the expression
for $\Delta_t^i$.}
optimal taxation and second-hand trade

The second row of each tax formula captures the effect of the second-hand market on the marginal labor income tax rate. This effect depends on (i) whether the marginal value that the government attaches to the used durable good is positive or negative, and (ii) whether leisure is complementary with, or substitutable for, the new and used durable goods. The latter means that the government may use the labor income tax structure in period $t$ to affect the price of the used durable good in periods $t$ and $t+1$.

Consider the role played by the two terms in the square bracket in the second row of each formula in Proposition 3. If leisure is complementary with the used durable good in the sense that $\partial y_{it}^2/\partial z_{it}^2 > 0$, and if $\mu_t/\gamma_t > 0$ ($< 0$), the first term within the square contributes to decrease (increase) the marginal labor income tax rate facing ability-type $i$. The intuition is that the government may use the marginal labor income tax rates in period $t$ to affect the price of the used durable good in period $t$; to be more specific, there is an incentive to reduce the price via a lower marginal labor income tax rate if $\mu_t/\gamma_t > 0$, and an incentive to increase the price via a higher marginal labor income tax rate if $\mu_t/\gamma_t < 0$. The policy incentives will be opposite to those just described if leisure is substitutable for the used durable good. By analogy, if leisure is complementary with the new durable good, increased consumption of leisure in period $t$ leads to increased supply of the used durable good in period $t+1$, so $\partial x_{2t+1}^i/\partial z_{it}^2 > 0$. In this case, therefore, and if $\mu_t/\gamma_t > 0$ ($< 0$), then the second term within the square bracket contributes to increase (decrease) the marginal labor income tax rate facing ability-type $i$, whereas opposite incentives apply if leisure is substitutable for the new durable good. The basic intuition is that the government can use the marginal income tax rates in period $t$ to affect the supply and, therefore, the price of the used durable good in period $t+1$.

As the contribution of the second-hand market to the labor income tax structure is complex, and depends on several components whose qualitative effects cannot be determined unambiguously, let us also discuss a specific example. Consistent with the idea that new durable goods are to a greater extent consumed by the high-ability type, while used durable goods are consumed to a greater extent by the low-ability type, suppose that leisure is complementary with the new durable good and substitutable for the used durable good in the sense that $\partial x_{2t+1}^i/\partial z_{it}^2 > 0$ and $\partial y_{it}^2/\partial z_{it}^2 < 0$. In addition, suppose that the
tendency to consume the used durable good by the low-ability type is strong enough to imply that the government attaches a positive marginal value to the used durable good. These additional assumptions mean that the second-hand market will contribute to higher marginal labor income tax rates for both ability-types. The intuition is that increased marginal labor income taxation will, in this case, unambiguously contribute to a lower price of the used durable good, which by our assumptions is in line with the distributional profile of the public policy.

3.3 Capital Income Taxation

Let us now turn to the marginal capital income tax rates. The main concern will be to analyze how the appearance of a second-hand market modifies the use of capital income taxation. This question has clear practical relevance, since the government (by assumption) is unable to control the second-hand market via a direct instrument.

To simplify as much as possible, let

\[ MRS_{t_{1:b_2}}^i = \frac{\partial V_t^i}{\partial \theta_{2t+1}^i} \]  
\[ MRS_{t_{1:b_2}}^{o1} = \frac{\partial V_t^o}{\partial \theta_{2t+1}^o} \]

denote the marginal rate of substitution between present and future disposable income for ability-type \( i \) and the mimicker, respectively, of generation \( t \). In addition, we shall use the following short notations for terms proportional to the marginal utility of private disposable income and leisure, respectively, facing the mimicker

\[ \lambda_{t,1}^* = \frac{\lambda_t^1}{\gamma_t^1} \frac{\partial V_t^2}{\partial \theta_{2t+1}^1} > 0, \lambda_{t,2}^* = \frac{\lambda_t^1}{\gamma_t^1} \frac{\partial V_t^2}{\partial \theta_{2t+1}^2} < 0, \]

\[ \lambda_{t+1,1}^* = \frac{\lambda_{t+1}^1}{\gamma_t^1} \frac{\partial V_{t+1}^2}{\partial \theta_{2t+1}^1} > 0 \]

and

\[ \lambda_{t+1,2}^* = \frac{\lambda_{t+1}^1}{\gamma_t^1} \frac{\partial V_{t+1}^2}{\partial \theta_{2t+1}^2} > 0 \]

as well as use the expression

\[ \lambda_{t+1,t+1}^* = \lambda_{t,1}^* \frac{\partial \phi_t}{\partial \theta_{2t}^1} \frac{\partial \phi_{t+1}}{\partial \theta_{2t+1}^1} + \lambda_{t+1,1}^* \frac{\partial \phi_{t+1}}{\partial \theta_{2t+1}^1} \frac{\partial \phi_{t+1}}{\partial \theta_{2t+1}^1} + \lambda_{t+1,2}^* \frac{\partial \phi_{t+1}}{\partial \theta_{2t+1}^2} \frac{\partial \phi_{t+1}}{\partial \theta_{2t+1}^2} \]

to denote how the self-selection constraints contribute to the marginal capital income tax rate of ability-type \( i \) via the wage ratio. Consider Proposition 4;
Proposition 4: At the second best optimum, the marginal capital income tax rates can be written as

\[
\Phi_{t,t}^1 = \frac{\tau}{\gamma_{t+1} \gamma_{t+2}} \left\{ \Lambda^*_{l,t+1} [MRS_{t,b_1,b_2}^1 - MRS_{t,b_1,b_2}^2] - \Lambda_{l,t+1} - \tau_{l,t+2} \frac{\partial x_{l,t}^1}{\partial \beta_{l,t}^1} \right\} \\
+ \frac{\mu_t}{\gamma_t} \frac{\partial \rho_{l,t}^1}{\partial \beta_{l,t}^1} - \frac{\partial \chi_{2l+1}^1 L_{\rho_{l,t}}^{-1}}{\partial \beta_{l,t}^1}
\]

\[
\Phi_{t,t}^2 = \frac{\tau}{\gamma_{t+1} \gamma_{t+2}} \left\{ -\Lambda^*_{l,t+2} - \tau_{l,t+2} \frac{\partial x_{l,t}^2}{\partial \beta_{l,t}^2} \right\} \\
+ \frac{\mu_t}{\gamma_t} \frac{\partial \rho_{l,t}^2}{\partial \beta_{l,t}^2} - \frac{\partial \chi_{2l+1}^2 L_{\rho_{l,t}}^{-1}}{\partial \beta_{l,t}^2}
\]

Proof: See the Appendix.

The marginal capital income tax rates are structured in a way similar to the marginal labor income tax rates. Therefore, and by analogy to the discussion in the previous subsection, the marginal capital income tax rates reflect a combination of three underlying motives for influencing savings and capital formation; (i) a desire to relax the self-selection constraint via channels that do not directly depend on the second-hand market, (ii) a desire to offset distortions caused by the commodity tax, and (iii) a desire to use the second-hand market as an additional channel for redistribution.

The first two motives are captured by the first row in each tax formula. Consider first the formula for the low-ability type, where the difference between the two marginal rates of substitution reflects a difference in relative valuation of current consumption between the low-ability type and the mimicker. If the low-ability type attaches a higher relative value to the current consumption than the mimicker, so \( MRS_{t,b_1,b_2}^1 > MRS_{t,b_1,b_2}^2 \), meaning that the relative valuation of current consumption decreases with the use of leisure, the government may relax the self-selection constraint by implementing a higher marginal capital income tax rate for the low-ability type than it would otherwise have done. An analogous motive for a lower marginal capital income tax rate applies if
\( MRS_{t,b_1b_2}^1 - MRS_{t,b_1b_2}^2 < 0 \). This argument for taxing or subsidizing the low-ability type at the margin is well understood from earlier research\(^{15}\); e.g. Brett (1997) and Pirttilä and Tuomala (2001).

The terms \( A_{t,t+1}^1 \) and \( A_{t,t+1}^2 \) are also associated with the self-selection constraint, although for another reason: the saving behavior of each ability-type in period \( t \) affects the wage ratio in periods \( t \) and \( t + 1 \). According to equation (28), three separate effects are distinguishable, of which only the second has been discussed in earlier literature (Pirttilä and Tuomala, 2001). To interpret how the first part of the expression for \( A_{t,t+1}^1 \) contributes to the marginal capital income tax rate facing ability-type \( i \), let us make the (reasonable) assumption that increased saving in period \( t \) reduces the demand for the new durable good in period \( t \) which, in turn, implies that the producer price of the durable good decreases. In this case, and if \( \partial \phi_t / \partial p_{t,x} > 0 \) \((< 0)\), a lower produce price contributes to decrease (increase) the wage ratio, which makes mimicking more (less) attractive. As a consequence, this provides an incentive for the government to relax the self-selection constraint by implementing a higher (lower) marginal capital income tax rate than it would otherwise have done. The interpretations of the second and third parts of \( A_{t,t+1}^1 \) are analogous with the exception that they refer to the wage ratio in period \( t + 1 \); the intuition is, of course, that a change in the saving behavior in period \( t \) affects the capital stock and, therefore, the wage ratio in period \( t + 1 \).

By analogy to the way in which the labor income tax is used, the government may also use the capital income tax structure to offset distortions caused by the commodity tax on the new durable good; an aspect captured by the final term in the first row. The intuition is that lower saving (caused by a higher marginal capital income tax rate) leads to increased consumption of the new durable good and contributes, therefore, to counteract the distortion caused by the commodity tax. Again, an alternative interpretation is in terms of a tax

\(^{15}\)Note also that the result derived by Ordover and Phelps (1979), explaining when the marginal capital income tax rate should be zero for each ability-type, applies as a special case here as well. To see this, let us assume, as they did, that the gross wage rates are fixed. For the purpose of the argument, we shall also assume away the use of commodity taxation as well as the second-hand market. In this case, and if leisure is weakly separable from the other goods in the utility function, meaning that the consumption pattern of the mimicker coincides with that of the low-ability type, so \( MRS_{t,b_1b_2}^1 = MRS_{t,b_1b_2}^2 \), Proposition 4 would imply that the marginal capital income tax rate is zero for each ability-type.
base effect.

The second row of each formula in Proposition 4 reflects the policy incentives created by the second-hand market. As our discussion serves to give the basic intuition behind the results, let us once again assume that increased saving in period $t$ means decreased consumption of durable goods in period $t$. There are two distinct effects in the second row of each tax formula. First, as increased saving by ability-type $i$ means reduced demand for the used durable good, i.e. $\partial \eta_{it}^1/\partial s_i^t < 0$, increased saving causes downward pressure on $q_{t, g}$. As a consequence, if $\mu_t/\gamma_t > 0$, this mechanism provides an incentive for the government to increase the saving via a lower marginal capital income tax rate. The intuition is that a decrease in the price of the used durable good is, in this case, associated with a social benefit; therefore, increased saving contributes to higher welfare via a lower price of the used durable. The policy incentive will be the opposite if $\mu_t/\gamma_t < 0$. Second, as increased saving in period $t$ means reduced consumption of the new durable good, it will also imply that the supply of the used durable good decreases in period $t+1$, i.e. $\partial \chi_{2t+1}^i/\partial s_i^t < 0$: in other words, increased saving in period $t$ causes upward pressure on $q_{t+1, g}$. If $\mu_t/\gamma_t > 0$, this is detrimental for welfare and provides incentive for the government to reduce savings via higher marginal capital income tax rates, meaning that the second term within the square bracket contributes to reduce the marginal capital income tax rates. Again, the opposite applies if $\mu_t/\gamma_t < 0$.

In summary, the effect of the second-hand market on the marginal capital income tax rates is ambiguous. The reason is that a change in the saving behavior in period $t$, ceteris paribus, tends to affect the prices of used durable goods in periods $t$ and $t+1$ in opposite directions. Therefore, whether the government wants to increase or decrease savings in order to relax the self-selection constraint via the second-hand market depends on the relative size of these two components.

4 Summary and Discussion

This paper deals with redistribution under asymmetric information by means of optimal income and commodity taxation in an OLG model with durable goods, where used durable goods are traded in a second-hand market. The type of transactions carried out in second-hand markets may, in terms of real world
market economies, involve a variety of used durables including vehicles such as boats and snowmobiles, kitchen equipment such as stoves, refrigerators and freezers, and gardening tools such as lawnmowers: transactions which for obvious reasons are difficult to observe by the government. The basic question is how the government may use the second-hand market as an additional mechanism for redistribution, even if it lacks a direct tax instrument to influence the market outcome. We assume that the set of tax instruments contains nonlinear taxes on labor income and capital income as well as a commodity tax on new durable goods.

In our framework, the government attaches a marginal value (positive or negative) to the used durable good, because its scarcity affects the possibilities of redistribution. If the government attaches a positive marginal value to the used durable good, it follows that the appearance of a second-hand market for the used durable good provides an incentive for the government to choose a lower commodity tax on the new durable good than it would otherwise have done. The intuition is that a decrease in the consumer price of the new durable good contributes to reduce the price of the used durable, as new and used durable goods are substitutes in consumption, and because increased consumption of the new durable good leads to increased supply of the used durable good in the next period. The effects of the second hand market on the marginal labor income tax rates depend on whether leisure is complementary with, or substitutable for, the new and used durable good. For instance, if leisure is substitutable for the used durable, this will provide an incentive for the government to increase the marginal labor income tax rates relative to the outcome without a second-hand market, as increased consumption of leisure, in this case, contributes to reduce the demand for and, therefore, the price of the used durable good. Note also that, if the government, instead, were to attach a negative marginal value to the used durable good, policy incentives opposite to those just described will apply. Finally, the effect of the second-hand market on the marginal capital income tax rates is ambiguous. The intuition is that a change in the saving behavior tends to affect the current and future prices of the used durable good in opposite directions; a result which holds independently of whether the government attaches a positive or negative marginal value to the used durable.

Future research in this area may take several directions, and we shall point of two of them. First, the assumption that the transactions in second-hand markets
are not observable is clearly a simplification; another alternative is, of course, that this information can be collected at a cost, in which case the government would be able to directly control (although imperfectly) the outcome of second-hand trade. Second, the contribution of the second-hand market to the tax structure is complex, and very few results hold unambiguously. Therefore, it would be interesting to complement the theoretical study set out here with numerical simulations, in which case the most likely qualitative effects may be assessed. We hope to carry out both these extensions in future research.
Appendix

The First Order Conditions

To simplify the analysis, let us first use equations (2) - (3) to rewrite the resource constraint, implying that the Lagrangean can be rewritten to read

\[
\mathcal{L} = W + \sum_{t=0}^{\infty} \lambda_t \left[ V_t^2 - \tilde{V}_t^2 \right] + \sum_{t=0}^{\infty} \mu_t \sum_{i=1}^{2} \left[ \chi_{i,t} - \eta_{i,t}^1 \right]
+ \sum_{t=0}^{\infty} \gamma_t \left[ S_{t,c} + p_{t,x} S_{t,x} + \tau_{t,x} x_{t+1} + K_t - K_{t+1} - \sum_{i} b_{i,t} - \sum_{i} b_{i,2t+1} \right]
\]

Define \( G_t = \sum_{i} \left[ \chi_{i,t} - \eta_{i,t}^1 \right] \). The first order conditions can then be written as

\[
b_{1,t}^1 : 0 = \frac{\partial W}{\partial V_t^1} \frac{\partial V_t^1}{\partial b_{1,t}^1} - \lambda_t \frac{\partial \tilde{V}_t^2}{\partial b_{1,t}^1} + \gamma_t \left[ \tau_{t,x} \frac{\partial x_{t+1}^1}{\partial b_{1,t}^1} - 1 \right]
+ \mu_t \frac{\partial G_t}{\partial b_{1,t}^1} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial b_{1,t}^1} + \frac{\partial \mathcal{L}}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial b_{1,t}^1} \tag{A.1}
\]

\[
t_1^1 : 0 = -\frac{\partial W}{\partial V_t^1} \frac{\partial V_t^1}{\partial t_1^1} + \lambda_t \frac{\partial \tilde{V}_t^2}{\partial t_1^1} \left( \phi_t + \mu_t \frac{\partial \phi_t}{\partial t_1^1} \right) + \gamma_t \left[ w_t^1 - \tau_{t,x} \frac{\partial x_{t+1}^1}{\partial t_1^1} \right]
+ \mu_t \frac{\partial G_t}{\partial t_1^1} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial t_1^1} + \frac{\partial \mathcal{L}}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial t_1^1} \tag{A.2}
\]

\[
b_{2,2t+1}^1 : 0 = \frac{\partial W}{\partial V_t^2} \frac{\partial V_t^2}{\partial b_{2,2t+1}^1} - \lambda_t \frac{\partial \tilde{V}_t^1}{\partial b_{2,2t+1}^1} + \gamma_t \tau_{t,x} \frac{\partial x_{t+1}^2}{\partial b_{2,2t+1}^1} - \gamma_{t+1}
+ \mu_t \frac{\partial G_t}{\partial b_{2,2t+1}^1} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial b_{2,2t+1}^1} + \frac{\partial \mathcal{L}}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial b_{2,2t+1}^1} \tag{A.3}
\]
\[ b_{2t}^2 : 0 = \left( \frac{\partial W}{\partial z_{2t}^2} + \lambda_t \right) \frac{\partial V_{2t}^2}{\partial z_{2t}^2} + \lambda_t \frac{\partial V_{2t}^2}{\partial z_{2t}^2} \frac{\partial \phi_t}{\partial z_{2t}} + \gamma_t \left[ \gamma_t - \frac{\partial x_{2t}^2}{\partial z_{2t}} \right] \\
+ \mu_t \frac{\partial G_t}{\partial z_{2t}^2} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial z_{2t}^2} + \frac{\partial L}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial z_{2t}^2} \tag{A.4} \]

\[ b_{2t+1}^2 : 0 = \left( \frac{\partial W}{\partial z_{2t+1}^2} + \lambda_t \right) \frac{\partial V_{2t+1}^2}{\partial z_{2t+1}^2} + \gamma_t \frac{\partial V_{2t+1}^2}{\partial z_{2t+1}^2} \frac{\partial x_{2t+1}^2}{\partial z_{2t+1}} - \gamma_{t+1} \\
+ \mu_t \frac{\partial G_t}{\partial z_{2t+1}^2} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial z_{2t+1}^2} + \frac{\partial L}{\partial p_{t+1,x}} \frac{\partial p_{t+1,x}}{\partial z_{2t+1}^2} \tag{A.5} \]

\[ K_t : 0 = \lambda_t \frac{\partial V_{2t}^2}{\partial z_{2t}^2} \frac{\partial \phi_t}{\partial K_t} + \gamma_t \left( 1 + r_t \right) - \gamma_{t-1} + \frac{\partial L}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial K_t} \tag{A.7} \]

\[ \tau_{t,x} : 0 = -x_{1t} \frac{\partial V_{1t}^1}{\partial \phi_t} \frac{\partial W}{\partial \phi_t} - x_{2t}^2 \left( \frac{\partial W}{\partial z_{2t}^2} + \lambda_t \right) \frac{\partial V_{2t}^2}{\partial z_{2t}^2} + \lambda_t x_{2t}^2 \frac{\partial \phi_t}{\partial \phi_t} \frac{\partial V_{2t}^2}{\partial \phi_t} \\
+ \gamma_t \left[ x_{1t} + x_{2t} + \tau_{t,x} \left( \frac{\partial \phi_t}{\partial q_{t,x}} + \frac{\partial x_{2t}^2}{\partial q_{t,x}} \right) \right] \\
+ \mu_t \frac{\partial G_t}{\partial q_{t,x}} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial q_{t,x}} + \frac{\partial L}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial \tau_{t,x}} \tag{A.8} \]
\[ q_{t,n} : 0 = -\eta_1 \frac{\partial V_1}{\partial b_t} \frac{\partial W}{\partial b_t} \frac{\partial V_2}{\partial b_t} - \eta_2 \left( \frac{\partial W}{\partial b_t} + \lambda_t \right) \frac{\partial V_2}{\partial b_t} + \lambda_2 \eta_2 \frac{\partial V^2}{\partial b_t} \\
+ \lambda_2 \frac{\partial V^2}{\partial b_t} \frac{\partial W}{\partial b_t} + \lambda_2 \left( \frac{\partial W}{\partial b_t} + \lambda_t \right) \frac{\partial V^2}{\partial b_t} - \lambda_2 \eta_2 \frac{\partial V^2}{\partial b_t} \\
+ \eta_2 \left[ x_1 \left( \frac{\partial x_1}{\partial q_{t,n}} + \frac{\partial x_2}{\partial q_{t,n}} \right) + \gamma \left( \frac{\partial x_1}{\partial q_{t,n}} + \frac{\partial x_2}{\partial q_{t,n}} \right) \right] \\
+ \mu \frac{\partial G_t}{\partial q_{t,n}} + \mu \frac{\partial G_{t+1}}{\partial q_{t,n}} + \frac{\partial L}{\partial b_{t-x}} \frac{\partial p_{t-x}}{\partial q_{t,n}} + \frac{\partial L}{\partial b_{t-x}} \frac{\partial p_{t-x}}{\partial q_{t,n}} \tag{A.9} \]

where

\[
\frac{\partial L}{\partial p_{t-x}} = -x_1 \left( \frac{\partial V^2}{\partial b_t} \frac{\partial W}{\partial b_t} \right) + x_2 \left( \frac{\partial W}{\partial b_t} + \lambda_t \right) \frac{\partial V^2}{\partial b_t} + \lambda_2 \frac{\partial V^2}{\partial b_t} \\
+ \gamma \left[ x_1 + x_2 + \mu \left( \frac{\partial x_1}{\partial b_{t-x}} + \frac{\partial x_2}{\partial b_{t-x}} \right) \right] \\
+ \mu \frac{\partial G_t}{\partial b_{t-x}} + \mu \frac{\partial G_{t+1}}{\partial b_{t-x}} + \lambda_2 \frac{\partial V^2}{\partial b_t} \frac{\partial \phi_t}{\partial b_{t-x}} \tag{A.10} \]

To simplify the expression for \( \frac{\partial L}{\partial p_{t-x}} \), let us define

\[
A_t = -x_1 \left( \frac{\partial V^2}{\partial b_t} \frac{\partial W}{\partial b_t} \right) + x_2 \left( \frac{\partial W}{\partial b_t} + \lambda_t \right) \frac{\partial V^2}{\partial b_t} + \lambda_2 \frac{\partial V^2}{\partial b_t} \\
+ \gamma \left[ x_1 + x_2 + \mu \left( \frac{\partial x_1}{\partial b_{t-x}} + \frac{\partial x_2}{\partial b_{t-x}} \right) \right] \\
+ \mu \frac{\partial G_t}{\partial b_{t-x}} + \mu \frac{\partial G_{t+1}}{\partial b_{t-x}} \tag{A.11} \]

which can be used to rewrite equation (A.8) as

\[
0 = A_t + \left( A_t + \lambda \frac{\partial V^2}{\partial b_{t-x}} \frac{\partial \phi_t}{\partial b_{t-x}} \right) \frac{\partial p_{t-x}}{\partial q_{t,n}} = 0 \tag{A.12} \]

Adding and subtracting \( \lambda \left( \frac{\partial V^2}{\partial b_{t-x}} \right) l_1 \left( \frac{\partial \phi_t}{\partial b_{t-x}} \right) \) in equation (A.12), while using that the expression within the parenthesis in equation (A.12) equals \( \frac{\partial L}{\partial p_{t-x}} \), we obtain
\[ \frac{\partial L}{\partial p_{t,s}} = \frac{\lambda_t \hat{V}_t^2 / \partial x_t^2}{(1 + \partial p_{s,x} / \partial \tau_t,s)} \frac{\partial \phi_t}{\partial p_{t,s}} \]  

(A.13)

**Proof of Proposition 1**

Use equations (A.1), (A.3), (A.4) and (A.6) to derive expressions for \((\partial W / \partial V_t^1)(\partial V_t^1 / \partial b_{1t})\), \((\partial W / \partial V_{t-1}^1)(\partial V_{t-1}^1 / \partial b_{2t})\), \((\partial W / \partial V_t^2 + \lambda_t)(\partial V_t^2 / \partial b_{1t})\) and \((\partial W / \partial V_{t-1}^2 + \lambda_{t-1})(\partial V_{t-1}^2 / \partial b_{2t})\), respectively, and substitute into equation (A.9). Then, by using

\[ \frac{\partial G_t}{\partial q_{t,n}} = \sum_i \frac{\partial \chi_{2t}^i}{\partial q_{t,n}} - \frac{\partial \eta_{1t}^i}{\partial q_{t,n}} \]

\[ \frac{\partial G_{t-1}}{\partial q_{t,n}} = -\sum_i \frac{\partial \eta_{1t-1}^i}{\partial q_{t,n}} \]

\[ \frac{\partial G_{t+1}}{\partial q_{t,n}} = \sum_i \frac{\partial \chi_{2t+1}^i}{\partial q_{t,n}} \]

\[ \frac{\partial G_t}{\partial b_{1t}^i} = \frac{\partial \eta_{1t}^i}{\partial b_{1t}^i} \]

\[ \frac{\partial G_t}{\partial b_{2t}^i} = \frac{\partial \chi_{2t}^i}{\partial b_{2t}^i} \]

\[ \frac{\partial G_{t+1}}{\partial b_{1t}^i} = \frac{\partial \chi_{2t+1}^i}{\partial b_{1t}^i} \]

\[ \frac{\partial G_{t-1}}{\partial b_{2t}^i} = -\frac{\partial \eta_{1t-1}^i}{\partial b_{2t}^i} \]

together with the Slutsky condition, we have

\[ -\lambda_t \frac{\partial \hat{V}_t^2 / \partial b_{1t}^i}{\partial \tau_t,s} [\eta_{1t}^i - \hat{\eta}_{1t}^i] + \lambda_{t-1} \frac{\partial \hat{V}_{t-1}^2 / \partial b_{2t}^i}{\partial \tau_{t-1,s}} [\chi_{2t}^i - \hat{\chi}_{2t}^i] + \gamma_{t-1} \frac{\partial \hat{\eta}_{r-1,s} / \partial q_{t,n}}{\partial q_{t,n}} + \gamma_{t-1} \frac{\partial \hat{\eta}_{r-1,s} / \partial q_{t,n}}{\partial q_{t,n}} \]

\[ + \mu_t [-\sum_i \frac{\partial \hat{\eta}_{1t-1}^i / \partial b_{1t}^i}{\partial \tau_t,s} L_{1t} + \sum_i \frac{\partial \hat{\chi}_{2t}^i / \partial q_{t,n}}{\partial q_{t,n}} + \sum_i \frac{\partial \hat{\eta}_{r-1,s} / \partial q_{t,n}}{\partial q_{t,n}} + \sum_i \frac{\partial \hat{\eta}_{r-1,s} / \partial q_{t,n}}{\partial q_{t,n}}] \]

\[ + \sum_i \frac{\partial \hat{\eta}_{1t-1}^i / \partial b_{1t}^i}{\partial \tau_t,s} L_{1t} \]

\[ = 0 \]  

(A.14)
Rearranging gives the formula in Proposition 1.■

Proof of Proposition 2

Use equations (A.1) and (A.4) to derive expressions for \( (\partial W/\partial V_1^2)(\partial V_1^2/\partial h_{1t}) \) and \( (\partial W/\partial V_t^2 + \lambda_t)(\partial V_t^2/\partial h_{1t}) \), respectively, and substitute into equation (A.8). Then, by using the derivatives of \( G_t \) and \( G_{t+1} \) derived above together with

\[
\begin{align*}
\frac{\partial G_t}{\partial q_{t,x}} &= -\frac{\partial \eta_{1t}}{\partial q_{t,x}} - \frac{\partial \eta_{2t}}{\partial q_{t,x}} \\
\frac{\partial G_{t+1}}{\partial q_{t,x}} &= \frac{\partial \chi_{2t+1}}{\partial q_{t,x}} + \frac{\partial \chi_{2t+1}}{\partial q_{t,x}}
\end{align*}
\]

and the Slutsky condition, we have

\[
0 = \left( x_{1t}^2 - x_{1t}^1 \right) \frac{\lambda_t}{\gamma_t} \frac{\partial \bar{V}_t^2}{\partial h_{1t}} + \tau_{t,x} \left( \frac{\partial \bar{V}_t^1}{\partial q_{t,x}} + \frac{\partial \bar{V}_t^2}{\partial q_{t,x}} \right) + \frac{\mu_t}{\gamma_t} \left( \frac{\partial \eta_{1t}}{\partial q_{t,x}} - \frac{\partial \eta_{2t}}{\partial q_{t,x}} + \frac{\partial \chi_{2t+1}}{\partial q_{t,x}} + \frac{\partial \chi_{2t+1}}{\partial q_{t,x}} \right) + \frac{1}{\gamma_t} \frac{\partial \mathcal{L}}{\partial p_{t,x}} \left( \frac{\partial p_{t,x}}{\partial \tau_{t,x}} + x_{1t}^1 \frac{\partial p_{t,x}}{\partial \theta_{1t}} + x_{2t}^1 \frac{\partial p_{t,x}}{\partial \theta_{1t}} \right) \tag{A.15}
\]

To evaluate the terms within the parenthesis in the third row, we differentiate equation (20) to obtain

\[
\begin{align*}
\frac{\partial p_{t,x}}{\partial \tau_{t,x}} &= \frac{\sum_i \partial x_{1i}/\partial q_{t,x}}{\partial S_{t,x}/\partial p_{t,x} - \sum_i \partial x_{1i}/\partial q_{t,x}} \tag{A.16} \\
\frac{\partial p_{t,x}}{\partial \theta_{1t}} &= \frac{\partial x_{1i}/\partial h_{1t}}{\partial S_{t,x}/\partial p_{t,x} - \sum_i \partial x_{1i}/\partial q_{t,x}} \tag{A.17}
\end{align*}
\]

Substituting equations (A.16) and (A.17) into equation (A.15), we obtain the formula in Proposition 2.■
Proof of Proposition 3

Let us begin with the marginal labor income tax rate of the low-ability type. By combining equations (A.1) and (A.2), while using equation (A.13), we have

\[
MRS_{t,z}^1 \left[ \lambda_t \frac{\partial V^2}{\partial b_t^1} - \gamma \left( \tau x_{it} \frac{\partial x_{it}^1}{\partial b_t^1} - 1 \right) - \mu_t \frac{\partial G_t}{\partial b_t^1} - \mu_{t+1} \frac{\partial G_{t+1}}{\partial b_t^1} \right] \\
- \frac{\lambda_t \frac{\partial V^2}{\partial z_t^1}}{1 + \frac{\partial p_{t,x}}{\partial \tau}} \frac{\partial \phi_t}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial b_t^1} \\
= \lambda_t \frac{\partial V^2}{\partial z_t^1} \left[ \phi_t + \frac{1}{\lambda_t} \frac{\partial \phi_t}{\partial z_t^1} \right] + \gamma \left( \frac{\partial w_t^1}{\partial z_t^1} - \frac{\partial x_{it}^1}{\partial z_t^1} \right) \\
+ \mu_t \frac{\partial G_t}{\partial b_t^1} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial b_t^1} \frac{\partial \phi_t}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial b_t^1} \tag{A.18}
\]

Then, by substituting \( w_t^1 - MRS_{t,z}^1 = w_t^1 T_{t,y}^1 \) from equation (12) into equation (A.18), while using

\[
\frac{\partial p_{t,x}}{\partial b_t^1} = - \frac{\partial S_{t,x}/\partial b_t^1 + \partial x_{it}^1/\partial z_t^1}{D_t} \\
\frac{\partial G_t}{\partial b_t^1} = - \frac{\partial \eta_t^1}{\partial z_t^1} \\
\frac{\partial G_{t+1}}{\partial b_t^1} = - \frac{\partial \chi_{t+1}^1}{\partial z_t^1} \\
\frac{\partial G_{t+1}}{\partial b_t^1} = \frac{\partial \chi_{t+1}^1}{\partial b_t^1} \\
\frac{\partial x_{it}^1}{\partial z_t^1} = \frac{\partial x_{it}^1}{\partial z_t^1} - \frac{\partial x_{it}^1}{\partial b_t^1} MRS_{t,z}^1 \\
\frac{\partial \eta_t^1}{\partial z_t^1} = \frac{\partial \eta_t^1}{\partial z_t^1} - \frac{\partial \eta_t^1}{\partial b_t^1} MRS_{t,z}^1 \\
\frac{\partial \chi_{t+1}^1}{\partial z_t^1} = \frac{\partial \chi_{t+1}^1}{\partial z_t^1} - \frac{\partial \chi_{t+1}^1}{\partial b_t^1} MRS_{t,z}^1
\]
gives the expression for the marginal labor income tax rate of the low-ability type in Proposition 3. The expression for the marginal labor income tax rate of the high-ability type can be derived in an analogous way. ■

Proof of Proposition 4

Consider the marginal capital income tax rate of the low-ability type. By combining equations (A.1) and (A.3), we can derive

\[
\begin{align*}
MRS_{t, b_1 b_2}^1 & = \lambda_t \frac{\partial \hat{V}_t^2}{\partial b_{1t}} - \gamma_{1t} \frac{\partial x_{1t}}{\partial b_{1t}} + \gamma_{1t} - \mu_t \frac{\partial G_{t+1}}{\partial b_{1t}} - \mu_{t+1} \frac{\partial G_{t+1}}{\partial b_{1t}} - \frac{\partial \mathcal{L}}{\partial p_{t, x}} \frac{\partial p_{t, x}}{\partial b_{1t}} \\
& = \lambda_t \frac{\partial \hat{V}_t^2}{\partial b_{2t+1}} - \gamma_{1t} \frac{\partial x_{1t}}{\partial b_{2t+1}} + \gamma_{t+1} - \mu_t \frac{\partial G_{t+1}}{\partial b_{2t+1}} - \mu_{t+1} \frac{\partial G_{t+1}}{\partial b_{2t+1}} - \frac{\partial \mathcal{L}}{\partial p_{t, x}} \frac{\partial p_{t, x}}{\partial b_{2t+1}} \\
& \quad \text{(A.19)}
\end{align*}
\]

Next, rewrite equation (A.7) to read

\[
\begin{align*}
\frac{\gamma_{t+1}}{\gamma_{t+1}} & = \frac{\lambda_{t+1}}{\gamma_{t+1}} \frac{\partial \hat{V}_{t+1}^2}{\partial x_{1t+1}} \frac{\partial x_{1t+1}}{\partial K_{t+1}} + (1 + r_{t+1}) + \frac{1}{\gamma_{t+1}} \frac{\partial \mathcal{L}}{\partial p_{t+1, x}} \frac{\partial p_{t+1, x}}{\partial K_{t+1}} \\
& \quad \text{(A.20)}
\end{align*}
\]

By substituting equation (A.20) into equation (A.19), while using equation (13), we have
\[
\Phi_{t,1}^1 = \frac{\lambda_t}{\gamma_{t+1} r_{t+1}} \frac{\partial \tilde{V}^2}{\partial b_{t+1}} \left( MRS_{t,b_1b_2}^1 - \tilde{MRS}_{t,b_1b_2}^2 \right) \\
+ \frac{\gamma_{t+1}^2}{\gamma_t r_{t+1}} \left( \frac{\partial x^1_t}{\partial b_{t+1}} - \frac{\partial x^1_t}{\partial b_{t+1}} MRS^1_{t,b_1b_2} \right) \\
- \lambda_{t+1} \frac{\partial \tilde{V}^2}{\partial \phi_{t+1}} \left( \frac{\partial \phi_{t+1}}{\partial b_{t+1}} + \frac{1}{p_{t+1} \theta_{t+1,x}} \frac{\partial \mathcal{L}}{\partial \phi_{t+1}} \right) \\
+ \frac{\mu_{t+1}}{\gamma_{t+1} r_{t+1}} \left( \frac{\partial G_{t+1}}{\partial b_{t+1}} - \frac{\partial G_t}{\partial b_{t+1}} MRS^1_{t,b_1b_2} \right) \\
+ \frac{G_{t+1}}{\gamma_{t+1} r_{t+1}} \left( \frac{\partial G_{t+1}}{\partial b_{t+1}} - \frac{\partial G_t}{\partial b_{t+1}} MRS^1_{t,b_1b_2} \right) \\
+ \frac{1}{\gamma_{t+1} r_{t+1}} \frac{\partial \mathcal{L}}{\partial \phi_{t+1}} \left( \frac{\partial p_{t,x}}{\partial b_{t+1}} - \frac{\partial p_{t,x}}{\partial b_{t+1}} MRS^1_{t,b_1b_2} \right) \\
- \frac{1}{p_{t+1} \theta_{t+1,x}} \frac{\partial \mathcal{L}}{\partial \phi_{t+1}} \left( \frac{\partial p_{t,x}}{\partial b_{t+1}} - \frac{\partial p_{t,x}}{\partial b_{t+1}} MRS^1_{t,b_1b_2} \right) \\
(A.21)
\]

Then, substituting

\[
\frac{\partial G_t}{\partial b_{t+1}} = -\frac{\partial \eta^1_{1t}}{\partial b_{t+1}} \\
\frac{\partial G_t}{\partial b_{2t+1}} = -\frac{\partial \eta^1_{2t}}{\partial b_{2t+1}} \\
\frac{\partial G_{t+1}}{\partial b_{t+1}} = \frac{\partial \eta^1_{1t+1}}{\partial b_{t+1}} \\
\frac{\partial G_{t+1}}{\partial b_{2t+1}} = \frac{\partial \eta^1_{2t+1}}{\partial b_{2t+1}} \\
\frac{\partial x^1_t}{\partial s^t} = \frac{\partial x^1_t}{\partial b_{2t+1}} MRS^1_{t,b_1b_2} - \frac{\partial x^1_t}{\partial b_{t+1}} \\
\frac{\partial \eta^1_{1t}}{\partial s^t} = \frac{\partial \eta^1_{1t}}{\partial b_{2t+1}} MRS^1_{t,b_1b_2} - \frac{\partial \eta^1_{1t}}{\partial b_{t+1}}
\]
\[
\frac{\partial \chi_{2t+1}^1}{\partial s^1_t} = \frac{\partial \chi_{2t+1}^2}{\partial b_{2t+1}^1} MRS_{t,b_t}^1 - \frac{\partial \chi_{2t+1}^1}{\partial b_{1t}^1}
\]

\[
\frac{\partial p_{2,t}}{{\partial b}_{2t+1}^1} = \frac{\partial x_{2t}^1}{\partial b_{2t+1}^1} \frac{D_t}{D_t}
\]

into equation (A.21) and rearranging gives the expression for the marginal capital income tax rate of the low-ability type in Proposition 4. The expression for the marginal capital income tax rate of the high-ability type can be derived in an analogous way.
References


III
Income Heterogeneity and the Flypaper Effect

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Abstract

The purpose of this paper is to analyze the determinants of the local public expenditures and, in particular, try to explain the so called ‘flypaper effect’. The analysis uses a political economy model to relate the existence and size of the flypaper effect to observable municipal characteristics such as the average tax base, income dispersion and whether or not a change in the average tax base affects the tax share of the majority voter. The empirical part of the study is based on Swedish data on municipal expenditures and revenues for the period 1996-2004. The results show that the size of the flypaper effect varies among municipalities depending on the relative composition of grant and tax base.

Key words: Income Dispersion, Local Public Expenditures, Intergovernmental Relations

JEL classification: D31, H72, H77

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1 Introduction

This paper addresses the question of whether the source of local public revenue matters for the size of local public expenditures. To be more specific, does an increase in the tax revenues (due to an increase in the tax base) affect the expenditures in the same way as a transfer payment (e.g., grant) of the same size from the central government? According to the basic median voter model, where the tax price is treated as exogenous from the perspective of the median voter, the source of public revenues does not matter in this respect.\(^1\) This is so because a transfer payment from the central to the local government is effectively equivalent to a transfer payment to the decisive voter. However, there is an extensive empirical literature that finds that local public spending responds more to increased transfers from the central government than to a corresponding increase in the tax base, an empirical finding that has been labelled the "flypaper effect".\(^2\) This paper investigates the determinants of municipal expenditures in Sweden and, in particular, how the composition of grants and tax base as well as the distribution of private income contribute to the flypaper effect. The empirical application is based on panel data for the period 1996-2004.

Several competing hypotheses aim to explain the flypaper effect: fiscal illusion (Courant et al., 1979; Oates, 1979), budget maximizing by government agents (Romer and Rosenthal, 1980), uncertainty and risk aversion (Fosset, 1990; Turnbull, 1992), mental accounting (Hines and Thaler, 1995) etc.\(^3\) None of these studies model the flypaper effect explicitly in a theoretical context. This paper focuses, instead, on income heterogeneity, and the regression equation will be related to a background theoretical model, where voters differ with respect to income.

The evidence for the existence of a flypaper effect is ambiguous. The early empirical literature dealing with the flypaper effect was criticized for not separating lump-sum grants from matching grants. Moffit (1984) emphasized the problem of ignoring the simultaneous determination of matching grants and public expenditures. Recent studies addressing these problems have been criticized for using an inappropriate functional form, e.g., studies that use a linear-

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\(^1\)See Bradford and Oates (1971a,b).
\(^2\)The result was labelled the “flypaper effect” by Courant, Gramlich and Rubinfeld (1979) following Arthur Okun’s observation that “money sticks where it hits”.
\(^3\)A good survey is given by Bailey and Connolly (1998).
in-variables functional form often found support for a flypaper effect, while studies based on logarithms of all variables found no support (Becker, 1996; Worthington and Dollery, 1999). Hamilton (1983) contends that empirical support for the flypaper effect is affected by the omitted variable biases. Subsequent efforts to correct for omitted variables that may correlate with intergovernmental transfers came to different conclusions regarding the flypaper effect (Knight, 2002; Gordon, 2004; Dahlberg et al., 2007). Empirical support for a flypaper effect has generally been demonstrated for certain situations, rather than across the board. For example, the effect has been shown to occur for different kinds of municipalities and different categories of expenditures (Moisio, 2002). Other studies have found that the response in local public expenditures to a change in the lump-sum transfers from the central government is asymmetric, depending on whether the transfer increases or decreases (Gramlich, 1987; Gamkhar and Oates, 1996; Karlsson, 2006).

This paper presents a simple political model for local public expenditures, where the voters are divided into two groups: low-income and high-income earners. The results imply that the source of marginal revenue (i.e. whether it originates from an increase in the transfers or an increase in the tax base) will matter for the local public expenditure decision, and that the effect on the majority voters’ tax share, caused by a change in the tax base, will be crucial for finding a flypaper effect. The intuition is that a change in the private income for any of the two voter groups affects the relative financial burden (tax share) of the majority voter group and, therefore, its willingness to use taxation as a marginal source of funds for public expenditures. In addition to income levels of the majority voters, the model emphasizes that the spread of private income will be important for local public expenditures.⁴

The empirical part of the paper estimates a model of local public expenditures from Swedish municipalities and tests for a flypaper effect. The local characteristics that affect the flypaper effect according to the theoretical model, such as the average tax base, the distribution of private income and the relative number of low income earners, will also be considered in the regressions. The data is particularly suited for studying the flypaper effect because transfers from

⁴Using US data Todo-Rovira (1991) found that income dispersion among voters matters when explaining local public expenditures. King (1984) also found that heterogeneity in income matters for the public expenditure decision.
the central to the local government in Sweden are, to a large extent, general grants. The use of general grants eliminates (at least in principle) the risk that grants and expenditures are determined simultaneously. The empirical analysis finds evidence of flypaper effects in Sweden during the study period.

The outline of the paper is as follows. In Section 2, a political model for local public expenditures is presented and discussed. The empirical part of the study is carried out in Section 3. Section 4 summarizes the results and concludes.

2 A Political Model for Local Public Expenditures

Consider a municipality that consists of two types of immobile residents who differ in ability; a low-ability type and a high-ability type. Let \( n^i \) denote the number of individuals of ability-type \( i \). Ability-type \( i \) derives utility from the consumption of a private good, \( c^i \), and from the consumption of public services, \( g \). The consumption of private goods is determined by the net income, \( c^i = w^i(1 - t) \), where \( w^i \) is the gross income and \( t \) is the income tax rate, while the consumption of the public service is determined via the public expenditure decision. We assume that the utility function is separable in \( c \) and \( g \), meaning that ability-type \( i \)'s utility can be written as

\[
u^i = v(c^i) + z(g) \tag{1}\]

where the functions \( v(\cdot) \) and \( z(\cdot) \) are assumed to be increasing and strictly concave in their respective arguments.

The local government raises revenues by using the proportional labour income tax and receives a lump-sum transfer, \( b \), from the central government. If ability-type \( i \) is the majority voter, the resource allocation will be determined as if a local planner solves

\[
\max_{t, g} \quad n^i[v(c^i) + z(g)] \tag{2}
\]

subject to the private budget constraint defined above and the local public budget constraint

\[
(n^iw^i + n^jw^j)t + (n^i + n^j)b = g \tag{3}
\]

for \( j \neq i \). The Lagrangian can be written

\[
\mathcal{L} = n^i[v(c^i) + z(g)] + \lambda[(n^iw^i + n^jw^j)t + (n^i + n^j)b - g] \tag{4}
\]
where $\lambda$ is the Lagrange multiplier. The first-order conditions become

$$\lambda : (n^i w^i + n^j w^j) t + (n^i + n^j) b - g = 0$$  \hspace{1cm} (5)$$

$$t : -n^i w^i v_c^i + (n^i w^i + n^j w^j) \lambda = 0$$  \hspace{1cm} (6)$$

$$g : n^i z_g - \lambda = 0$$  \hspace{1cm} (7)$$

The flypaper effect will be evaluated by examining whether an increase in the tax base affects the municipal expenditures in the same way as an increase in the transfer payment from the central government. The tax base and the transfer payment are considered exogenous from the local government’s point of view.

At the same time, note that a change in the tax base may, in addition to its effect on the municipality’s budget, imply a change in the income distribution among the residents in the municipality. This additional effect (previously not discussed in the flypaper literature) does not appear when the transfer payment changes. As a point of reference in the theoretical analysis, I will measure the flypaper effect by comparing the effect on the local public expenditures following an increase in the transfer payment with the effect of a spread-preserving increase in the mean private income. Define the mean private income (or average tax base) and the wage differential between the high-ability type and the low-ability type as $\bar{w} = (n^i w^i + n^j w^j)/(n^i + n^j)$ and $\bar{s} = w^i - w^j$, respectively. For further use, note also that $w^i$ and $w^j$ can be written in terms of $\bar{w}$ and $\bar{s}$:

$$w^i = \bar{w} - \frac{n^j}{(n^i + n^j)} \bar{s}$$  \hspace{1cm} (8)$$

$$w^j = \bar{w} + \frac{n^i}{(n^i + n^j)} \bar{s}$$  \hspace{1cm} (9)$$

Differentiating the first-order conditions with respect to $\bar{w}$ and $\bar{s}$, holding $\bar{s}$ constant and denoting the Hessian determinant by $|\bar{H}|$, yields

$$\frac{\partial g}{\partial \bar{w}} = -[(n^i + n^j) t \rho + \zeta \delta]/|\bar{H}|$$  \hspace{1cm} (10)$$

$$\frac{\partial g}{\partial \bar{s}} = -(n^i + n^j) \rho/|\bar{H}|$$  \hspace{1cm} (11)$$
where

\[
\rho = n^i (\bar{w} - \frac{n^j}{(n^i + n^j)} \bar{s})^2 v_{cc}^i
\]

\[
\zeta = (n^i + n^j) \tilde{w}
\]

\[
\delta = n^i v_c^i + n^i (\bar{w} - \frac{n^j}{(n^i + n^j)} \bar{s}) (1 - t) v_{cc}^i - (n^i + n^j) \lambda
\]

\[
|\mathcal{B}| = -(n^i z_{gg} \zeta^2 + \rho) > 0
\]

We are now ready to analyze if the source of local public revenue matters for the local public expenditures.

### 2.1 Implications of Income Heterogeneity

Equations (6), (8), (9), (10) and (11) together imply

\[
\frac{\partial g/\partial b}{\partial g/\partial \bar{w}} = \frac{1}{t + (1 - t) \varphi^i}
\]

(12)

where

\[
\varphi^i = \frac{(n^i + n^j) \bar{w} + n^i \frac{\bar{s}}{n}}{(n^i + n^j) \bar{w} - n^j \bar{s}}
\]

\[
\epsilon^i = \frac{v_{cc}^i c^i v_c^i}{v_c^i}
\]

The magnitude of the expression on the right hand side of equation (12) depends on local characteristics, \((\bar{w}, \bar{s}, n^i, n^j)\). A flypaper effect exists if a transfer payment from the central government stimulates the municipal expenditures more than an increase of the same size in the tax base, i.e. \((\partial g/\partial b)/(\partial g/\partial \bar{w}) > 1\).

The opposite effect, \((\partial g/\partial b)/(\partial g/\partial \bar{w}) < 1\), will be referred to as a 'negative' flypaper effect. The elasticity of marginal utility with respect to consumption, \(\epsilon^i\), reflects the curvature of the utility function (the relation between \(\epsilon^i\) and the flypaper effect is analyzed in Appendix).

Equation (12) implies that \((\partial g/\partial b)/(\partial g/\partial \bar{w}) = 1\) if, and only if, \(\varphi^i = 1\). Let us begin by considering two special cases when this will occur. First, if \(\bar{s} = 0\) \((\bar{w} = w^i = w^j)\), the majority voter will represent a single ability-type in

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5This elasticity is also known as the coefficient of relative risk aversion, defined as \(\epsilon^i = |\epsilon^i|\), when the utility function is used to describe attitudes towards risk.
the local economy and the result corresponds to that of a one-consumer model. The marginal valuation of private consumption is, in this case, equal to the marginal valuation of public consumption, i.e. \( \lambda / \nu_1 \equiv 1 \). Second, the source of marginal revenue does not matter for the public expenditures when the elasticity of marginal utility with respect to private consumption, \( \varepsilon^1 \), equals minus one.\(^6\) This result is further discussed in the Appendix.

Returning to the general model, note that a flypaper effect can only exist if \( \varphi^i < 1 \). For \( |\varepsilon^i| < 1 \), this requires that \( w^d < w^i \, (\delta > 0) \). A spread-preserving increase in the mean tax base implies that the tax share of the majority voters increases if the majority voters earn a low income. As a relatively larger financial burden falls on the majority voters, taxation becomes a less attractive source of marginal funds for the municipality from the view of the majority voters. In this case, differentiation of equation (12) shows that the flypaper effect decreases, as the mean tax base, \( \bar{w} \), increases. Under the same conditions, an increase in the number of the low-income earners, \( n^i \), reduces the size of the flypaper effect, whereas an increase in the spread of income, \( \delta \), increases the size of the flypaper effect, ceteris paribus.\(^7\)

Let us also consider the situation where the high-income earners are the majority voters. If \( |\varepsilon^i| < 1 \), this means that \( \varphi^i > 1 \). In this case, we find a 'negative' flypaper effect, i.e. \( (\partial g / \partial h) / (\partial g / \partial \bar{w}) < 1 \). The intuition is analogous to that given above. On the other hand, if \( |\varepsilon^i| > 1 \), and the high-income earners are in majority, we have \( (\partial g / \partial h) / (\partial g / \partial \bar{w}) > 1 \), indicating that a flypaper effect exists.\(^8\)

As we indicated above, it is not entirely clear from earlier empirical literature how the flypaper effect should be measured. What happens if we do not condition on income spread, or if we were to control for spread in such a way that the relative private income remains unaffected when the average tax base increases?

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\(^6\)Consider a Cobb-Douglas form of \( v(e^i) \), i.e., \( v^i = (1/(1 - a)) \times (e^i)^{1-a} \). If \( a = |\varepsilon^i| \) approaches 1, then the utility function approaches a logarithmic utility function, for which \( (\partial g / \partial h) / (\partial g / \partial \bar{w}) = 1 \).

\(^7\)The Cobb-Douglas form of \( v(e^i) \) mentioned in footnote 6 yields \( |\varepsilon^i| = a \). Differentiation of equation (12) with respect to \( a \) shows that the flypaper effect decreases as the marginal utility of private income decreases (i.e. as \( a \) increases), if the low-ability type is decisive.

\(^8\)The majority of estimates in the literature estimating the coefficient of relative risk aversion are in the interval 0.5 – 2. Blanchard and Fischer (1989) refer to the empirical evidence based on consumption choices over time, where the estimates vary substantially, but usually lie around or above unity.
Income Heterogeneity and the Flypaper Effect

If we were to increase the majority voters’ income without controlling for income spread (i.e. increase $w^i$ with $w^j$ held constant), and then compare the resulting change in $g$ with the effect of an additional transfer payment of the same size, we would expect to find a flypaper effect, independent of the ability-type in majority (at least if $|r^i| < 1$ and the derivative $\partial e^i/\partial \bar{w}$ is sufficiently small). By analogy, an increase in the minority voters’ tax share makes taxable income a more attractive source of marginal funds for the municipality. In this case we would, therefore, expect to find a ’negative’ flypaper effect. Finally, if the tax share of each voter group remains unaffected, then $(\partial g/\partial b)/(\partial g/\partial \bar{w}) = 1$, and we are back in the traditional representative agent model.

3 Empirical Analysis

The empirical part of the paper begins with a presentation of some institutional characteristics related to the intergovernmental transfer system as well as a description of the data. The empirical model and the estimation results follow.

3.1 Data and Institutional Characteristics

The empirical analysis is based on panel data containing between 282 and 287 Swedish municipalities and nine years (1996-2004). The data were obtained from Statistics Sweden and the Swedish Labour Market Board. Swedish municipalities collect tax revenues by using a proportional income tax. This tax is the most important source of funds for the majority of municipalities, followed by user fees and general grants. The study chooses this particular period due to a significant policy change in 1993 regarding transfers from the central government to the municipalities. The Swedish intergovernmental transfer system is built around formula-based general grants which, in theory, eliminate the risk that transfers and expenditures are determined simultaneously. The policy re-

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9 The number of Swedish municipalities increased during the study period from 288 (1996-1998) to 289 (1999-2002) to 290 (2003). Three large municipalities are excluded from the study (Malmö, Göteborg and Gotland) due to partly joint expenditures between the municipality and the region. In addition, seven observations are excluded due to missing values in the dependent variable (see Greene, 2003).

10 During the study period (1996-2004), the tax revenues as a share of the total revenues decreased from 68.2 % to 67.4 % for the municipalities while the unconditional transfers decreased from 11.6 % to 9.4 % on average.
form implemented in 1993 replaced most matching grants by a system of general grants.\textsuperscript{11} The current system contains two parts. First, a transfer from the central government to the municipalities, which is interpretable as a general grant based on the number of residents in the municipality. Second, the system also contains an element of revenue sharing in the sense that resources are redistributed among municipalities such that municipalities with high taxable income or low structural costs compensate municipalities with low taxable income or high structural costs.\textsuperscript{12} Therefore, for the former type of municipality, our measure of total grant may be negative.

Let us now turn to the variables to be used in the estimations. The municipality’s total net operating expenditure, $q$, is the dependent variable. This includes the sum of expenditure on child care, elderly care, compulsory education etc.\textsuperscript{13} The expenditure variable is adjusted by Statistics Sweden to ensure that it measures the running expenditures net of fees and matching grants. Variables characterizing a municipality’s revenue, beside the exogenous grant variable, $b$, and the tax base, $\bar{w}$, are therefore not considered in the analysis. All monetary variables are expressed in real per capita terms using the Swedish CPI (2004 is the base year).

The measure of income spread used in Section 2 is operationalized by using the standard deviation of private income, which is estimated annually for each municipality.\textsuperscript{14} As discussed in the theoretical section above, the existence of a flypaper effect depends on the relative size of the majority voters’ private

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\textsuperscript{11}In addition to the general grants, a smaller amount of matching grants was also used by the central government during the period of study.

\textsuperscript{12}A minor reform was implemented in 1996, when the equalization part of the system was made financially neutral from the view of the central government (see Law Proposition 1995/96:64). The calculation of the grant formulas was subject to minor modifications in 1998 (see Law Proposition 1998/99:89).

\textsuperscript{13}The municipalities’ business activities are excluded from the analysis.

\textsuperscript{14}Data on the income distribution in each municipality is collected by Statistics Sweden. The following intervals are defined for the period 1996-1997: 0, 0.1-39.9, 40-59.9, 60-79.9, 80-99.9, 100-119.9, 120-139.9, 140-159.9, 160-179.9, 180-199.9, 200-219.9, 220-239.9, 240-259.9, 260-299.9, 300-399.9, 400+, and the following for 1998-2004: 0, 0.1-39.9, 40-79.9, 80-119.9, 120-159.9, 160-199.9, 200-239.9, 240-279.9, 280-319.9, 320-359.9, 360-399.9, 400-499.9, 500-599.9, 600+. The data refers to the number of residents in each interval. The income is reported in current prices (thousands of SEK). The standard deviation is estimated for each municipality and each year by using the mean income in the municipality and applying a lognormal density function to the income distribution.
Income Heterogeneity and the Flypaper Effect

income. This will be taken into account by introducing a variable measuring the relative number of low-income earners in each municipality. A low-income earner is defined as a citizen earning less than 200,000 SEK per year, adjusted by the Swedish CPI (based year 2004).\textsuperscript{15} This definition implies that the share of low-income earners in all municipalities decreases over time, on average, from 72\% in 1996 to 57\% in 2004, due to economic growth and inflation. Therefore, in order to adjust for economic growth and inflation, the annual share of low-income earners in the municipality is related to the annual share of low-income earners in the country. A dummy variable, \( d \), equals one if the share of low-income earners in the municipality is larger than the share of low-income earners in the country.\textsuperscript{16}

Following the literature on the determinants of municipal expenditures, the estimations will include relevant local characteristics such as population density, age structure of the population, political preferences and political strength. The population density, \( Dens \), is measured by the number of residents per square kilometre. The classification of the age variables corresponds to the formulas used when calculating the structural cost due to the age structure in the municipalities. Political preferences are controlled by including the share of the seats in the municipal parliament occupied by members of either the Social Democratic Party or the Left Party, \( Left \). Political strength is represented by a Herfindahl index, \( Herf \), i.e. the sum of the squared shares of each party in the local parliament. Finally, a variable measuring the unemployment rate, \( Unemp \), is also included as a control variable.\textsuperscript{17} National income tax and expenditure policy are not included in the analysis.\textsuperscript{18} Descriptive and summary

\textsuperscript{15}This threshold earnings level is chosen because the interval 0-199.9 is given for the entire period (see previous footnote). The number of residents earning less than 200,000 SEK per year, adjusted by the CPI (2004 is the base year), is calculated by using a lognormal distribution. Use of this threshold income level implies that low-income earners are exempt from paying income tax to the central government. Measured in 2004 prices, earnings that exceeded 249,000 in 1996 or 308,000 in 2004 were subject to this additional income tax.

\textsuperscript{16}This is the case for 1469 observations out of 2566.

\textsuperscript{17}Other control variables were also tested, but these had no effect on the qualitative results discussed below. For example, alternative classifications of the age variables, a version of the Left-variable where the Environmental party is included and a version of the Unemployment-variable which includes persons in vocational training.

\textsuperscript{18}The models to be estimated contain municipality specific fixed effects and period specific effects. As is common in earlier comparable literature, we abstract from the influences of central government taxation and expenditures on the municipal expenditures (see description
statistics for these variables are presented in Tables A1-A2 in the Appendix.

3.2 Specification of the Empirical Model

We will first consider a benchmark version of the model where the flypaper effect is measured in the same way as in the earlier empirical literature. In this specification, the municipalities’ expenditures, \( g_i \), are explained solely by the grants, \( b \), the sum of the grant and the tax base, \( \theta \), and by the standard determinants of local government expenditures, \( x \). The estimating equation is given by

\[
g_{it} = \beta_0 + \beta_1 b_{it} + \beta_2 \theta_{it} + \beta_{11} x_{it} + \mu_i + \gamma_t + \varepsilon_{it} \tag{13}
\]

where subindex \( i \) refers to municipality and subindex \( t \) to time period. The parameter \( \mu_i \) is a municipality specific effect, \( \gamma_t \) a year specific effect, and \( \varepsilon_{it} \) is an error term. In order to be able to test the existence of a flypaper effect, the variable \( \theta \) is defined as the sum of the general grant and the tax base, i.e. \( \theta = b + \bar{w} \). According to the benchmark model, an increase in the grant affects the municipal expenditures in the same way as an increase in the tax base if \( \beta_1 = 0 \); a flypaper effect exists if \( \beta_1 > 0 \).

The first extension of the benchmark model means that the standard deviation of private income, \( \hat{s} \), is part of the set of explanatory variables.

\[
g_{it} = \beta_0 + \beta_1 b_{it} + \beta_2 \theta_{it} + \beta_3 \hat{s}_{it} + \beta_4 \hat{s}_{it} b_{it} + \beta_5 \hat{s}_{it} \theta_{it} + \beta_{11} x_{it} + \mu_i + \gamma_t + \varepsilon_{it} \tag{14}
\]

The standard deviation of private income enters the regression both as a separate effect and as an interaction effect with \( b \) and \( \theta \), respectively. These variables will also be present in the other extensions discussed below.

The second extension aims to test whether the flypaper effect is larger in municipalities with a higher share of low-income earners than in other municipalities. This will be done by using the dummy variable, \( d \), distinguishing
municipalities with a relatively high share of low-income earners, both as a separate effect and as interaction effects with $b$ and $\theta$.

\begin{equation}
    g_{it} = \beta_0 + \beta_1 b_{it} + \beta_2 \theta_{it} + \beta_3 \tilde{s}_{it} + \beta_4 \tilde{b}_{it} + \beta_5 \tilde{s}_{it} \theta_{it} + \beta_6 d_{it} + \beta_7 d_{it} b_{it} + \beta_8 d_{it} \theta_{it} + \beta_9 x_{it} + \mu_i + \gamma_t + \varepsilon_{it}
    \tag{15}
\end{equation}

Finally, the third extension aims to estimate a flypaper effect that captures non-linear properties in the revenue variables. This allows the flypaper effect to vary between municipalities depending on the relative composition of grants and taxable income. The composition of grants and tax base in the municipality is closely related to the relative number of low-income earners. More specifically, this extension means that the square of the grant and the square of the variable measuring the sum of grants and taxable income are added to the regressors.\(^{19}\)

\begin{equation}
    g_{it} = \beta_0 + \beta_1 b_{it} + \beta_2 \theta_{it} + \beta_3 \tilde{s}_{it} + \beta_4 \tilde{b}_{it} + \beta_5 \tilde{s}_{it} \theta_{it} + \beta_6 \tilde{b}_{it}^2 + \beta_7 \tilde{\theta}_{it}^2 + \beta_8 x_{it} + \mu_i + \gamma_t + \varepsilon_{it}
    \tag{16}
\end{equation}

The inclusion of the quadratic effects results in a less restrictive model compared to the linear specifications. The null hypotheses to be tested in the models presented above, which imply that the local public spending does not depend on the source of marginal revenue, are summarized in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>(\frac{\partial g}{\partial b}/\frac{\partial g}{\partial \hat{w}})</th>
<th>Null Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>[I]</td>
<td>(\beta_1 + \beta_2)</td>
<td>(\beta_1 = 0)</td>
</tr>
<tr>
<td>[II]</td>
<td>(\beta_1 + \beta_2 + (\beta_3 + \beta_4) \tilde{s}_{it})</td>
<td>(\beta_1 = \beta_4 = 0)</td>
</tr>
<tr>
<td>[III]</td>
<td>(\beta_1 + \beta_2 + (\beta_3 + \beta_4) \tilde{s}<em>{it} + (\beta_3 + \beta_4) d</em>{it})</td>
<td>(\beta_1 = \beta_4 = \beta_7 = 0)</td>
</tr>
<tr>
<td>[IV]</td>
<td>(\beta_1 + \beta_2 + (\beta_3 + \beta_4) \tilde{s}<em>{it} + (\beta_3 + \beta_4) d</em>{it} + 2\beta_8 \tilde{b}<em>{it} + 2\beta_8 \tilde{\theta}</em>{it})</td>
<td>(\beta_1 = \beta_4 = \beta_9 = 0)</td>
</tr>
</tbody>
</table>

Following most other earlier studies on local public expenditure determination, we assume that the tax base can be treated as exogenous in the regres-

\(^{19}\)The means are subtracted from the variables, before taking the square, in order to reduce the multicollinearity between the linear and quadratic terms, i.e. \(\tilde{b}_{it}^2 = (b_{it} - \bar{b})^2\) and \(\tilde{\theta}_{it}^2 = (\theta_{it} - \bar{\theta})^2\). The short period of study motivates the use of the global means.
sion. To test this assumption, this paper instruments for the tax base by using the indicators of industry structure in each municipality or time-lags of the tax base as instrumental variables. However, the potential instruments failed the validity test thus precluding their use in this study.

3.3 Results

The estimation results are presented in Table 2. First, we present several specification tests. A Hausman test implies that a fixed effects model is preferred to a random effects model in all specifications estimated. A Bhargava et al. modified Durbin-Watson (DW) test examines whether the residuals are serially correlated. Two null hypotheses are both rejected in all models estimated: (1) that the residuals are serially independent and (2) that the residuals form a random walk. Moreover, a modified Wald test for groupwise heteroskedasticity in the fixed effects model (denoted 'Htest' in Table 2; see Greene, 2003) rejects the null of homoskedasticity. The existence of serial correlation and groupwise heteroskedasticity in the fixed effects model implies that the estimates of the standard errors need to be corrected. This is done using White’s estimator of the variance-covariance matrix. All tests are reported in Table 2.

The first column refers to the benchmark model. As can be seen from Table 2, the grant variable and the sum of grants and tax base significantly

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20 The assumption of exogeneity has been questioned. For example, empirical studies based on Swedish data imply that a change in taxation affects the labour supply (Blomquist, 1983; Aromsson and Palme, 1998), and to some extent migratory behaviour (Westerlund and Wyzan, 1995).

21 Hansen’s J-test rejects the joint null hypothesis that the instruments are uncorrelated with the error term. This test is applicable in the presence of serial correlation and heteroskedasticity.

22 For the municipalities included in the regressions, the grant revenue can be either positive or negative. Using the entire sample increases estimate precision. Nevertheless, the qualitative results discussed below hold even when municipalities with non-positive grant revenues are excluded.

23 The part of the general grants that compensate for structural cost differences depends on local characteristics. The compensation for structural cost differences may, as a consequence, be correlated with potentially omitted variables. It is, therefore, important to be aware of the possibility that unobservable factors simultaneously affect the equalization of structural cost differences and expenditures. Estimations have, therefore, also been made for the situation where the part of the grant variable that equalizes structural cost differences has been excluded. The estimation results reported in Table 2 are robust to these changes.

24 The critical values can be found in Bhargava et al. (1982).
Income Heterogeneity and the Flypaper Effect

affect the municipal expenditures. The estimate and t-value of the parameter associated with the grant variable supports the existence of a flypaper effect. The point estimate of the flypaper effect becomes \((\partial y/\partial b)/(\partial y/\partial \hat{w}) = 2.88\), which is somewhat smaller than in previous studies.\(^{25}\)

Turning to the effects of local characteristics (the vector \(x\)), the results are qualitatively similar to those of earlier comparable literature on local public expenditures. Political preferences have significant effects on the expenditures, but the political strength in the municipal parliament does not. The results suggest that a municipality represented by a large share of socialist parties are associated with higher public expenditures than other municipalities, ceteris paribus. Regarding other municipal characteristics, recall that the municipalities are responsible for providing comprehensive education and elderly care. We find that the higher the share of individuals in the age corresponding to upper secondary education (age 16-18), and the higher the share of the oldest residents in the municipality (age 75-), the higher the per capita expenditures. The results also indicate that the per capita expenditure decreases as the population density in the municipality increases, which indicates that the municipalities benefit from economics of scale.

The estimation results for equation (14) are presented in the second column. The standard deviation of private income has neither a significant effect on the local public expenditures, nor the estimated magnitude of the flypaper effect. The parameter estimate of the flypaper effect is nearly unaffected as the spread of income increases, whereas an \(F\)-test rejects the null that \(\beta_1 = \beta_4 = 0\) (\(\chi^2 = 97.71\)).

The third column\(^{26}\) refers to the model where the municipalities with a high share of low-income earners are distinguished by a dummy variable \((d = 1\) if the share of low-income earners in the municipality is larger than the share of low-income earners in the country). As can be seen from Table 2, the flypaper effect is larger for municipalities with a higher share of low-income earners, \((\partial y/\partial b)/(\partial y/\partial \hat{w}) = 3.66\), than for the other municipalities, \((\partial y/\partial b)/(\partial y/\partial \hat{w}) = \)

\(^{25}\)Based on Swedish data collected during a period with specific grants (before 1993), Aronsson and Wikström (1995) simulated the effect of replacing a system of specific grants with a system of general grants and predicted that the flypaper effect would be about 6 in a new system of general grants.

\(^{26}\)The qualitative results in models [I1] and [IV] are unaffected if the standard deviation, \(\hat{\sigma}_x\), is excluded.
2.60. An $F$-test rejects the null that $\beta_1 = \beta_4 = \beta_7 = 0$ ($\chi^2 = 37.79$) and a Likelihood ratio-test rejects the null hypothesis that the dummy variable does not explain local public expenditures.

<table>
<thead>
<tr>
<th></th>
<th>[I]</th>
<th>[II]</th>
<th>[III]</th>
<th>[IV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.354$^{***}$</td>
<td>0.365$^{***}$</td>
<td>0.240$^{***}$</td>
<td>0.267$^{***}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.188$^{***}$</td>
<td>0.167$^{***}$</td>
<td>0.150$^{***}$</td>
<td>0.124$^{***}$</td>
</tr>
<tr>
<td>$s$</td>
<td>(12.07)</td>
<td>(9.56)</td>
<td>(8.61)</td>
<td>(6.78)</td>
</tr>
<tr>
<td>$\tilde{s}$</td>
<td>$2.21 \times 10^{-8}$</td>
<td>$1.53 \times 10^{-8}$</td>
<td>$-7.44 \times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>$\tilde{s}_b$</td>
<td>$-3.49 \times 10^{-12}$</td>
<td>$-2.73 \times 10^{-12}$</td>
<td>$1.85 \times 10^{-12}$</td>
<td></td>
</tr>
<tr>
<td>$\tilde{s}_\theta$</td>
<td>$-2.11 \times 10^{-13}$</td>
<td>$-1.56 \times 10^{-13}$</td>
<td>$4.68 \times 10^{-13}$</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td></td>
<td></td>
<td>$-1793.7^{***}$</td>
<td></td>
</tr>
<tr>
<td>$db$</td>
<td>0.173$^{***}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d\theta$</td>
<td>0.005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{b}^2$</td>
<td></td>
<td>$1.77 \times 10^{-5}^{***}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\theta}^2$</td>
<td></td>
<td>$-3.77 \times 10^{-7}^{**}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Continued on next page.)
Income Heterogeneity and the Flypaper Effect

Table 2. (Continued).

<table>
<thead>
<tr>
<th></th>
<th>[I]</th>
<th>[II]</th>
<th>[III]</th>
<th>[IV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 0-6</td>
<td>90.93</td>
<td>69.75</td>
<td>85.89</td>
<td>83.16</td>
</tr>
<tr>
<td></td>
<td>(0.84)</td>
<td>(0.65)</td>
<td>(0.79)</td>
<td>(0.72)</td>
</tr>
<tr>
<td>Age 7-15</td>
<td>96.97</td>
<td>81.32</td>
<td>113.6</td>
<td>231.8**</td>
</tr>
<tr>
<td></td>
<td>(1.16)</td>
<td>(0.97)</td>
<td>(1.33)</td>
<td>(2.42)</td>
</tr>
<tr>
<td>Age 16-18</td>
<td>732.4***</td>
<td>715.27***</td>
<td>691.8***</td>
<td>726.1***</td>
</tr>
<tr>
<td></td>
<td>(5.05)</td>
<td>(4.93)</td>
<td>(4.74)</td>
<td>(4.91)</td>
</tr>
<tr>
<td>Age 65-74</td>
<td>-256.3***</td>
<td>-223.6**</td>
<td>-179.7*</td>
<td>-184.0**</td>
</tr>
<tr>
<td></td>
<td>(-2.86)</td>
<td>(-2.47)</td>
<td>(-1.94)</td>
<td>(-2.00)</td>
</tr>
<tr>
<td>Age 75-</td>
<td>314.5***</td>
<td>205.9***</td>
<td>316.8***</td>
<td>199.3***</td>
</tr>
<tr>
<td></td>
<td>(2.84)</td>
<td>(2.75)</td>
<td>(2.78)</td>
<td>(1.69)</td>
</tr>
<tr>
<td>Dens</td>
<td>-9.45×10^{4***}</td>
<td>-7.70×10^{4***}</td>
<td>-8.35×10^{4***}</td>
<td>-9.69×10^{4***}</td>
</tr>
<tr>
<td></td>
<td>(-3.79)</td>
<td>(-2.93)</td>
<td>(-3.15)</td>
<td>(-3.75)</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.878</td>
<td>0.879</td>
<td>0.880</td>
<td>0.881</td>
</tr>
<tr>
<td>F, Listed variable</td>
<td>27.02</td>
<td>21.92</td>
<td>18.12</td>
<td>17.85</td>
</tr>
<tr>
<td>F, Individual effects</td>
<td>842.92</td>
<td>786.57</td>
<td>698.38</td>
<td>758.90</td>
</tr>
<tr>
<td>F, Time effects</td>
<td>37.26</td>
<td>38.07</td>
<td>36.59</td>
<td>34.33</td>
</tr>
<tr>
<td>χ^2 Fixed vs Random</td>
<td>148.27</td>
<td>120.90</td>
<td>137.60</td>
<td>38.27</td>
</tr>
<tr>
<td>Hestest</td>
<td>1.0×10^{30}</td>
<td>9.8×10^{29}</td>
<td>19553.1</td>
<td>1.3×10^{30}</td>
</tr>
<tr>
<td>DW</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>1.22</td>
</tr>
<tr>
<td>LR Extensions vs [I]</td>
<td>14.08</td>
<td>36.34</td>
<td>93.00</td>
<td></td>
</tr>
<tr>
<td>LR Extensions vs [II]</td>
<td>22.26</td>
<td>78.92</td>
<td></td>
<td></td>
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<tr>
<td>No. of observations</td>
<td>2566</td>
<td>2566</td>
<td>2566</td>
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<tr>
<td>No. of groups</td>
<td>287</td>
<td>287</td>
<td>287</td>
<td>287</td>
</tr>
</tbody>
</table>

Note: t-values in parentheses (obtained by using White’s estimator of the variance-covariance matrix). The regressions in Table 2 include municipality specific effects and period specific effects. ***, **, and * denote significance at the 1, 5 and 10 percent level, respectively.

Finally, the estimation of the model including the square of b and θ, respectively, is presented in the fourth column. The quadratic effects are significant. The advantage of including the quadratic effects is that they allow for a more flexible interpretation of the flypaper effect, meaning that the size of the flypaper effect varies among municipalities depending on the relative composition of grant and tax base. A large proportion of grant in relation to the sum of grants and tax base corresponds to a relatively high number of low-income earners. An F-test rejects the null that β₁ = β₂ = β₃ = 0 (χ² = 45.69) whereas Likelihood ratio-tests reject that the first two models fit the data better than the quadratic model. The flypaper effect is illustrated graphically in Figure 1 by plotting it against the ratio between grants and the sum of grants and tax base for each municipality and each year.\textsuperscript{27}

\textsuperscript{27} An illustration of the flypaper effect by plotting it against the relative number of low-income earners in the municipality gives the same picture as Figure 1.
According to Figure 1, the size of the flypaper effect becomes larger as the grants increase relative to the sum of grant and tax base. The point estimate of the flypaper effect is about eight for the municipality with the highest share of grants in relation to the sum of grants and tax base, whereas it takes a negative value for the municipalities with the lowest share of grants.\(^{28}\)

A functional form that is linear-in-variables is most frequently used in the empirical literature dealing explicitly with the flypaper effect. Likelihood ratio-tests reject the linear model against the more general quadratic alternative (see above). However, some researchers argue in favour of using a logarithmic functional form (Becker, 1996; Worthington and Dollery, 1999). A Pe-test, developed by MacKinnon, White, and Davidson (1983), implies that the quadratic model (where the underlying explanatory variables are not transformed into logarithms) is preferred to a logarithmic specification.\(^{29}\) It is difficult to compare results from the logarithmic model with results from the models discussed above because the parameters in the logarithmic model refer to elasticities rather than

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\(^{28}\)The negative values in Figure 1 can, most likely, be explained by a poor fit between the data and the functional form in the tails.

\(^{29}\)A Box-Cox transformation rejects the functional form of the benchmark model, both when it is linear-in-variables and when it is logarithmic in all variables. However, the estimates of the transformed parameters in the Box-Cox model are closer to result in a model that is linear-in-variables, and the \(\chi^2\)-statistic is closer not to reject this functional form.
marginal effects. The results from estimating a logarithmic version of the model are, nevertheless, presented in the Appendix.

4 Conclusion

The model for local public expenditures considered in this paper implies that the income distribution in the municipality matters for the majority voters’ marginal valuation of different sources of public revenue. Specifically, this paper found that the average tax base, the income spread in the municipality and the share of low-income earners all contribute to the existence and size of the flypaper effect. The model implies that the effect on the majority voters’ tax share, caused by a change in the tax base, will be crucial for finding a flypaper effect. In the case of a single ability-type in the local economy, there is no flypaper effect, and the results correspond to a representative agent model.

The empirical analysis is based on panel data containing between 282 and 287 Swedish municipalities and nine years, 1996-2004. The benchmark model, which corresponds to previous empirical literature, results in a point estimate of the flypaper effect of about three. However, Likelihood ratio-tests reject the benchmark specification when it is tested against more general models. The first extension of the benchmark model introduces the standard deviation of private income and finds that this measure of income dispersion does not seem to affect the size of the flypaper effect. The second extension aims to examine whether the size of the flypaper effect is larger in municipalities where the share of low-income earners is relatively high. This is examined by using a dummy variable-approach. The results imply that the size of the flypaper effect is larger in municipalities where a relatively high share of the residents are low-income earners. Finally, the model that includes the quadratic effects allows for a more flexible interpretation of the flypaper effect, meaning that the size of the flypaper effect varies among municipalities depending on the relative composition of grant and tax base. The estimation results illustrate that the flypaper effect becomes larger as the proportion of grants in relation to the sum of grant and tax base increases.
Appendix

e^i and the Flypaper Effect

To begin with, combine equations (6) and (7). This gives

$$\psi^i z_g - v^i_c = 0$$

where

$$\psi^i = (n^i w^i + n^j w^j) / w^i$$

Rewriting equation (11) in terms of $e^i$ gives that $\partial g / \partial b = 0$ when $e^i = 0$, and $\partial g / \partial b > 0$ when $|e^i| > 0$. The underlying mechanism is seen in equation (A1) and is illustrated in Figure 2(I), which for simplicity assumes a linear relationship between $\partial g / \partial b$ and $|e^i|$. Equation (A1) implies that $\partial g / \partial b = 0$ when $v^i_c$ is constant, i.e. when $e^i = 0$, because if $v^i_c$ is constant, $g$ must remain constant as well for equation (A1) to apply. An increase in $|e^i|$ changes the curvature of the utility function (with respect to private consumption) and makes it possible to increase $g$ by increasing $b$.

Besides affecting the marginal utility of private consumption (i.e. the second term on the left hand side of equation (A1)), an increase in $\bar{w}$ will also have indirect effects on the local public expenditures via the tax share of each ability-type; an effect which arises via $\psi^i$ in equation (A1). A change in $e^i$, therefore, results in a larger effect on $\partial g / \partial \bar{w}$ relative to the effect on $\partial g / \partial b$.

Rewriting equation (10) in terms of $e^i$ gives that $\partial g / \partial \bar{w} \neq 0$ when $e^i = 0$. In the situation where the low-income earners are the majority voters we find that $\partial g / \partial \bar{w} < 0$ when $e^i = 0$, because an increase in $\bar{w}$ will increase the tax share of the low-income earners (i.e. decrease $\psi^i$ in equation (A1)) and, therefore, has to be balanced by a decrease in $g$. Now, by increasing $|e^i|$, the effect via the tax share dominates the effect on the marginal utility of private consumption until $|e^i| = \kappa > 0$, when $\partial g / \partial \bar{w} = 0$ (see Figure 2(II)). The relation between $e^i$ and the flypaper effect is illustrated in Figure 2(IV). If the low-income earners are in majority, a flypaper effect will exist when $\kappa < |e^i| < 1$, and a 'negative' flypaper effect will exist when $|e^i| > 1$.

The marginal effects on spending of $\bar{w}$ and $b$ will be the same if, and only if, $|e^i| = 1$. To see this, start by taking the derivative of the local public budget
constraint in equation (3) with respect to \( b \) and \( \tilde{w} \), respectively,

\[
\frac{\partial g}{\partial b} = (n^i + n^j)(1 + \tilde{w}\frac{\partial t}{\partial b}) \\
\frac{\partial g}{\partial \tilde{w}} = (n^i + n^j)(t + \tilde{w}\frac{\partial t}{\partial \tilde{w}})
\]

To derive the expressions for \( \partial t/\partial \tilde{w} \) and \( \partial t/\partial b \), respectively, we begin by substituting equation (7) into equation (6), which gives a modified first-order condition for the tax rate:

\[
\Omega_t = -(\tilde{w} - \frac{n^i}{n^i + n^j})v^i_t + (n^i + n^j)\tilde{w}z_g = 0
\]

Differentiating with respect to \( t, \tilde{w} \) and \( b \), while using that \( g = t(n^i + n^j)\tilde{w} + (n^i + n^j)b \) from the local public budget constraint, we have

\[
\frac{\partial t}{\partial \tilde{w}} = -((n^i + n^j)z_g + t(n^i + n^j)^2\tilde{w}z_{gg} - v^i_t - v^i_c\epsilon^t)/\Omega_{tt} \\
\frac{\partial t}{\partial b} = -((n^i + n^j)^2\tilde{w}z_{gg})/\Omega_{tt}
\]

Substituting equations (A5)-(A6) into equations (A2) and (A3), and rearranging gives (rewrite \( v^i_c \) in terms of \( \epsilon^t \) by using that \( \epsilon^t = (v^i_c/v^i_e)\epsilon^t \))

\[
\frac{\partial g}{\partial b} = [(n^i + n^j)(w^j)^2(v^i_t/v^i_e)^2]/\Omega_{tt} \\
\frac{\partial g}{\partial \tilde{w}} = [(n^i + n^j)(w^j)^2(v^i_t/v^i_e)^2(\epsilon^t - (1 - \epsilon^t)) + \tilde{w}(n^i + n^j)v^i_c(1 + \epsilon^t)]/\Omega_{tt}
\]

Note that the indirect effects on the local government expenditures via the tax shares of both ability-types disappear as \( \epsilon^t = -1 \), and so does the flypaper effect, i.e. \( (\partial g/\partial b)/(\partial g/\partial \tilde{w}) = 1 \).

Finally, let us also consider the situation where the high-income earners are in the majority (see Figure 2(III)). In this case we find that \( \partial g/\partial \tilde{w} > 0 \) when \( \epsilon^t = 0 \), because an increase in \( \tilde{w} \) will increase the tax share of the high-income earners and, therefore, has to be balanced by an increase in \( g \). As a consequence, an increase in \( \epsilon^t \) results in a smaller effect on \( \partial g/\partial \tilde{w} \) relative to the effect on \( \partial g/\partial b \). A flypaper effect will exist when \( |\epsilon^t| > 1 \), and a ‘negative’ flypaper effect
will exist when $0 < |\epsilon^i| < 1$ (see Figure 2(V)). The marginal effects of $\bar{w}$ and $b$ on spending will be the same if, and only if, $|\epsilon^i| = 1$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The relation between $\epsilon^i$ and the Flypaper Effect}
\end{figure}
**Descriptive Statistics**

*Table A1. Description of the variables*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>Total net operational expenditures (SEK per capita).</td>
</tr>
<tr>
<td>b</td>
<td>Total unconditional grants to the municipality (SEK per capita).</td>
</tr>
<tr>
<td>w</td>
<td>Taxable income in the municipality (SEK per capita).</td>
</tr>
<tr>
<td>s</td>
<td>The standard deviation from the mean income in the municipality (SEK).</td>
</tr>
<tr>
<td>d</td>
<td>A dummy variable that equals one if the share of low-income earners in the municipality is larger than the share of low-income earners in the country. A low-income earner is defined as a citizen earning less than 200,000 SEK per year. The earnings are adjusted by the Swedish CPI (2004 is the base year).</td>
</tr>
<tr>
<td>Left</td>
<td>The percentage of the seats in the municipal parliament held by Social Democrats and members of the Left Party.</td>
</tr>
<tr>
<td>Herf</td>
<td>A Herfindahl index measuring political strength in the municipal parliament.</td>
</tr>
<tr>
<td>Unempl</td>
<td>The percentage of unemployed.</td>
</tr>
<tr>
<td>Age 0-6</td>
<td>The percentage of residents aged 6 or younger.</td>
</tr>
<tr>
<td>Age 7-15</td>
<td>The percentage of residents aged 7-15.</td>
</tr>
<tr>
<td>Age 16-18</td>
<td>The percentage of residents aged 16-18.</td>
</tr>
<tr>
<td>Age 65-74</td>
<td>The percentage of residents aged 65-74.</td>
</tr>
<tr>
<td>Age 75+</td>
<td>The percentage of residents aged 75 or older.</td>
</tr>
<tr>
<td>Dens</td>
<td>The population density, residents per square kilometre. This variable is divided by the factor ten thousand for computational purposes.</td>
</tr>
</tbody>
</table>

*Note: g, b, w and s are adjusted by the Swedish CPI (2004 is base year). The income variable used for calculating s and d is based on the population aged 16 and above.*
Table A2. Summary statistics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>29199.8</td>
<td>3487.6</td>
<td>36838.9</td>
<td>3871.0</td>
<td>33042.5</td>
<td>4482.8</td>
</tr>
<tr>
<td>b</td>
<td>5733.7</td>
<td>3884.6</td>
<td>7287.3</td>
<td>5502.4</td>
<td>6795.4</td>
<td>4557.8</td>
</tr>
<tr>
<td>w</td>
<td>97666.6</td>
<td>12851.1</td>
<td>128842.1</td>
<td>17212.2</td>
<td>111336.7</td>
<td>17815.2</td>
</tr>
<tr>
<td>$s^{a)}$</td>
<td>$5.71 \times 10^6$</td>
<td>$3.15 \times 10^6$</td>
<td>$4.46 \times 10^8$</td>
<td>$7.56 \times 10^9$</td>
<td>$3.49 \times 10^8$</td>
<td>$5.90 \times 10^9$</td>
</tr>
<tr>
<td>Left</td>
<td>50.77</td>
<td>11.94</td>
<td>46.82</td>
<td>11.19</td>
<td>47.70</td>
<td>11.73</td>
</tr>
<tr>
<td>Herf</td>
<td>29.82</td>
<td>5.77</td>
<td>25.22</td>
<td>4.64</td>
<td>25.96</td>
<td>5.29</td>
</tr>
<tr>
<td>Unempl</td>
<td>7.23</td>
<td>1.94</td>
<td>3.92</td>
<td>1.12</td>
<td>4.62</td>
<td>2.00</td>
</tr>
<tr>
<td>Age0-6</td>
<td>9.08</td>
<td>0.95</td>
<td>7.07</td>
<td>1.09</td>
<td>7.67</td>
<td>1.22</td>
</tr>
<tr>
<td>Age7-15</td>
<td>11.53</td>
<td>1.03</td>
<td>12.10</td>
<td>1.11</td>
<td>12.32</td>
<td>1.17</td>
</tr>
<tr>
<td>Age16-18</td>
<td>3.66</td>
<td>0.38</td>
<td>4.12</td>
<td>0.37</td>
<td>3.76</td>
<td>0.40</td>
</tr>
<tr>
<td>Age65-74</td>
<td>9.51</td>
<td>1.72</td>
<td>9.48</td>
<td>1.57</td>
<td>9.35</td>
<td>1.64</td>
</tr>
<tr>
<td>Age75-</td>
<td>9.18</td>
<td>2.21</td>
<td>9.78</td>
<td>2.25</td>
<td>9.58</td>
<td>2.26</td>
</tr>
<tr>
<td>Dens</td>
<td>0.0114</td>
<td>0.0389</td>
<td>0.0118</td>
<td>0.0411</td>
<td>0.0116</td>
<td>0.401</td>
</tr>
</tbody>
</table>

\(^{a)}\) The extreme values are created by poor fit (in a few municipalities) between the income distribution and the lognormal density function, which is used when the standard deviation is calculated. Even if some of the observations show high values, this is not considered as sufficient for excluding the observations. Nevertheless, omitting the extreme values from the analysis had no effect on the qualitative results discussed in Section 3.

The Logarithmic Estimation Results

Studies that use a functional form that is linear-in-variables often find support for a flypaper effect, while studies that use a functional form that is logarithmic in all variables do not find any support (Becker, 1996; Worthington and Dollery, 1999). The reader should note that a logarithmic estimation yields coefficients equal to elasticities and not marginal effects on spending. In order to compare the results, the estimated elasticities are often converted into marginal effects.

The marginal effects on spending by an increase in either \( b \) or \( \theta \) are given by

\[
\frac{\partial g}{\partial b} = \beta_1 \log \times \frac{g}{b}
\]

\[
\frac{\partial g}{\partial \theta} = \beta_2 \log \times \frac{g}{\theta}
\]

Does the use of a logarithmic functional form alter the results discussed in the empirical part of this paper? The model to be estimated is written.\(^{30}\)

\(^{30}\)The observations where the municipality’s total grant revenue is non-positive are excluded in order to make a logarithmic estimation possible. The sample will, therefore, be reduced by 123 observations. This leaves us with an unbalanced panel containing between 265 and 282 municipalities. Furthermore, the standard deviation of private income in each municipality for each year, \( s \), is excluded due to collinearity problems.
\[ \ln g_{it} = \beta_0 + \beta_1 \ln b_{it} + \beta_2 \ln \theta_{it} + \beta_3 \ln x_{it} + \mu_i + \gamma_t + \varepsilon_{it} \quad (A10) \]

An extended version of equation (A10) which aims to test whether the flypaper effect is larger for municipalities with a higher share of low-income earners than for other municipalities is estimated.

\[ \ln g_{it} = \beta_0 + \beta_1 \ln b_{it} + \beta_2 \ln \theta_{it} + \beta_3 d_{it} + \beta_4 d_{it} \ln b_{it} \quad (A11) \]

\[ + \beta_5 d_{it} \ln \theta_{it} + \beta_6 \ln x_{it} + \mu_i + \gamma_t + \varepsilon_{it} \]

The estimation results are presented in Table A3. The first column in Table A3 refers to the estimation for a simplified version of equation (A11) where \( \beta_3 = \beta_4 = \beta_5 = 0 \). The null hypothesis (\( \beta_1 = 0 \)) means that the elasticity of the grant variable equals the elasticity of \( \theta \). Following Becker (1996), and calculating the marginal effects at the means, the point estimate of the flypaper effect in this model becomes \( (\partial g/\partial b)/(\partial g/\partial w) \) = 1.30. The estimate of the marginal effect of grants is less than the estimate reported in Section 3. This results in a smaller flypaper effect than in the benchmark model. The \( t \)-value of the grant coefficient is significant at the 1 percent level.

The results of estimating equation (A11) are reported in the second column. The results imply that the flypaper effect is larger for municipalities with a relatively high share of low-income earners, \( (\partial g/\partial b)/(\partial g/\partial w) \) = 2.22, than for the other municipalities, \( (\partial g/\partial b)/(\partial g/\partial w) \) = 1.26. The qualitative result in this model is, therefore, equivalent to the result presented in Section 3 (even if the size of the flypaper effect is smaller in the logarithmic model). An \( F \)-test rejects the null that \( \beta_1 = \beta_4 = 0 \) (\( \chi^2 = 13.82 \)).
Table A3. Estimation results (logarithmic specification).

<table>
<thead>
<tr>
<th></th>
<th>$[I]$</th>
<th>$[III]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0.013***</td>
<td>0.011***</td>
</tr>
<tr>
<td></td>
<td>(3.80)</td>
<td>(3.49)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.674***</td>
<td>0.675***</td>
</tr>
<tr>
<td></td>
<td>(10.80)</td>
<td>(10.39)</td>
</tr>
<tr>
<td>d</td>
<td>-0.115</td>
<td>(-0.54)</td>
</tr>
<tr>
<td>db</td>
<td>0.039***</td>
<td>(3.63)</td>
</tr>
<tr>
<td></td>
<td>(-0.92)</td>
<td></td>
</tr>
<tr>
<td>Left</td>
<td>0.013</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(1.16)</td>
</tr>
<tr>
<td>Herf</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(0.87)</td>
<td>(0.94)</td>
</tr>
<tr>
<td>Unempl</td>
<td>0.017***</td>
<td>0.017***</td>
</tr>
<tr>
<td></td>
<td>(2.76)</td>
<td>(2.79)</td>
</tr>
<tr>
<td>Age 0-6</td>
<td>-3.26×10^{-4}</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(-0.02)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>Age 7-15</td>
<td>0.091**</td>
<td>0.083**</td>
</tr>
<tr>
<td></td>
<td>(2.50)</td>
<td>(2.28)</td>
</tr>
<tr>
<td>Age 16-18</td>
<td>0.070***</td>
<td>0.062***</td>
</tr>
<tr>
<td></td>
<td>(4.23)</td>
<td>(3.75)</td>
</tr>
<tr>
<td>Age 65-74</td>
<td>-0.021</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>(-0.80)</td>
<td>(-0.47)</td>
</tr>
<tr>
<td>Age 75-</td>
<td>0.035</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>(1.06)</td>
<td>(1.18)</td>
</tr>
<tr>
<td>Dens</td>
<td>-7.62×10^{-3}</td>
<td>-3.05×10^{-3}</td>
</tr>
<tr>
<td></td>
<td>(-0.38)</td>
<td>(-0.15)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.877</td>
<td>0.878</td>
</tr>
<tr>
<td>F, Listed variable</td>
<td>15.63</td>
<td>13.76</td>
</tr>
<tr>
<td>F, Individual effects</td>
<td>781.87</td>
<td>680.72</td>
</tr>
<tr>
<td>F, Time effects</td>
<td>27.23</td>
<td>28.55</td>
</tr>
<tr>
<td>$\chi^2$ Fixed vs Random</td>
<td>44.87</td>
<td>167.77</td>
</tr>
<tr>
<td>Hottest</td>
<td>1.0×10^{25}</td>
<td>5.0×10^{28}</td>
</tr>
<tr>
<td>DW</td>
<td>1.15</td>
<td>1.16</td>
</tr>
<tr>
<td>LR</td>
<td>20.17</td>
<td>20.17</td>
</tr>
<tr>
<td>No. of observations</td>
<td>2443</td>
<td>2443</td>
</tr>
<tr>
<td>No. of groups</td>
<td>282</td>
<td>282</td>
</tr>
</tbody>
</table>

Note: $t$-values in parentheses (obtained by using White’s estimator of the variance-covariance matrix). The regressions in Table 1 include municipality specific effects and period specific effects. *** , ** and * denote significance at the 1, 5 and 10 percent level, respectively.
References


[22] Law Proposition 1995/96:64, Ett nytt utjämningssystem för kommuner och landsting, m.m. (In Swedish)


Income Heterogeneity and the Flypaper Effect


The Demand for Local Public Services in Sweden

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February 2008

Abstract
This paper analyzes the composition of municipal expenditures in Sweden by estimating a demand system for local public services, in which tax revenue collection is treated as endogenous. The estimation is based on the QAIDS specification. The empirical application uses panel data for the period 1998-2005 and for six local public services. The results show that the point estimates of all income elasticities except one are positive, and that none of them significantly exceeds one. Furthermore, the point estimates of the own-price elasticities are negative and less than one in absolute value for all services.

Key words: Demand system, local public finance, local government spending

JEL classification: D12, H71, H72

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*The author would like to thank Thomas Aronsson, Jörgen Helleström, Tomas Sjögren and Magnus Wikström for helpful comments and suggestions. A research grant from the Wallander-Hedelius Foundation is gratefully acknowledged.
1 Introduction

In the literature on local public expenditure determination, the composition of expenditures has received less attention than the size. Research commonly focuses on either the determinants of the aggregate expenditure or on the individual components. An understanding of the determinants of the expenditure composition in local jurisdictions is especially important for the central government for two reasons: (1) the design of national policies that redistribute revenues via an intergovernmental transfer system, and (2) the development of national policies whose objective is to steer local jurisdictions in a specific policy direction. This paper, which focuses on Swedish municipalities, simultaneously analyzes tax revenue collection and how the expenditures are allocated between different local public services. The empirical application is based on panel data for the period 1998-2005.

Previous studies dealing with the expenditure composition in the local public sector often assume that the municipality acts as if the policies are decided upon by a representative individual: solving such a decision problem for the local government yields a system of estimable demand equations. With this approach expenditure composition problems in the public sector are treated similar to consumer choices in the private sector. This approach was first applied by Deacon (1978), who extended earlier work in the field (e.g. Barr and Davies 1966; Borcharding and Deacon 1972; and Bergstrom and Goodman 1973)\(^1\) by estimating a system of demand equations using US data on local public consumption expenditure (in the city of Seattle). He found that the local public sector allocation obeys two basic properties from traditional consumer theory: negative (compensated) own-price effects and zero degree homogeneity of demand in income and all prices. Other studies have extended the application of demand models to the pattern of consumption expenditures of local governments by including socio-economic indicators to account for differences in social needs and political considerations across local jurisdictions (see Dunne and Smith, 1983; Dunne et al., 1984; Borge and Ratts, 1995; Aaberge and Langorgen, 2003).

Earlier research on the composition of local public expenditures typically focuses on conditional demand systems, in which a given local public budget is

\(^1\)These studies focused either on the aggregate public spending or individual components of the total.
allocated between the service sectors (Deacon, 1978; Borge and Rattso, 1995). Such a framework is consistent with the idea of two-stage budgeting, where the local public revenues are collected in a first stage and allocated between the service sectors in a second stage. Analyzing the second step in such a process, therefore, presupposes that the revenues (or at least most of them) are predetermined. An alternative approach would be to consider the decision to collect revenue simultaneous with the decision to allocate resources between different local public services.

In order to fully understand the priorities made by the local public sector, the benchmark model in this paper does not separate the local public budget process into two steps. Instead, the benchmark model takes a more comprehensive approach by analyzing a complete system of demand equations, including private consumption. In order to compare results with previous studies, I also estimate a demand system that is conditioned on the resources available to the local public sector. Furthermore, this paper is the first study that analyzes the local public expenditure composition using Swedish data.

The approach where a municipality acts "as if" there is a representative individual that solves the resource allocation problem is referred to as the "community preference" model in the literature. It is typically assumed that the single individual allocates her/his resources between consumption of private goods and public services.² The present paper uses such a model for local public expenditures, where a representative agent derives utility from private and public consumption. The representative agent’s demand for each good depends on, e.g., the total per capita income, prices and municipal characteristics. The demand system is specified as a "quadratic almost ideal" demand system (QAIDS),³ which is an extension to Deaton and Muellbauer’s (1980) ’almost ideal’ demand system (AIDS). The advantage with the QAIDS specification (which is estimated in terms of expenditure shares), compared to the AIDS formulation, is that it recognizes the possibility that the expenditure shares are nonlinear functions of income.⁴

²The community preference model is used in numerous studies (a review of the early empirical studies is provided by Grunich, 1977). A good survey is given by Wildasen (1986).
³See Banks et al. (1997).
⁴Previous studies on local public expenditure decisions using Swedish data (e.g. Aronsson and Wijkström, 1995; Aronsson et al., 2000; Witterhals, 2007) find that income effects are nonlinear.
In Sweden, the municipalities are responsible for providing services such as child care, comprehensive education and elderly care. The municipalities collect tax revenues by using a proportional income tax. Other important sources of revenue are user fees and grants from the central government. The empirical part of this paper presents results from estimating a complete demand system consisting of a composite private consumption good and six local public services (child care, social services, elderly care, comprehensive education, ‘infrastructure and protection’ and ‘other expenditures’). The results show that the point estimates of all income elasticities except one are positive, and that none of them significantly exceed one. Furthermore, the results show that the point estimates of the own-price elasticities are negative and less than one in absolute value for all services. The results from the demand system that is conditioned on the size of the total public expenditure are similar to related studies based on Norwegian data (Borge and Rattso, 1995; Aaberge and Langørgen, 2003).

The outline of the paper is as follows. Section 2 presents and discusses a representative agent model for local public expenditures. Section 3 contains a description of the data, the empirical model and the estimation method as well as a discussion of the results. Section 4 summarizes the results and concludes.

2 A Representative Agent Model for Local Public Expenditures

Earlier research on the composition of local public expenditures often focuses on a demand system for local public services defined conditional on the revenue available to the local public sector, i.e. the total resources available for local public expenditures (Deacon, 1978; Borge and Rattso, 1995). This approach implies that the local public sector allocates a given resource, $c$, between a number of local public services, $g_1, \ldots, g_n$. The desired allocation of public consumption in this setting is given by maximizing the utility function

$$u = u(g_1, \ldots, g_n, c; \theta)$$

subject to the budget constraint

$$\sum_{j=1}^{n} p_j g_j = \hat{e}$$
where $c$ is private consumption, $\theta$ a vector of characteristics describing the community, $p_j$ (discussed below) the unit price of service $j$ net of matching grants and $e$ the total local government spending. The outcome of this problem is a demand system conditioned on $e$ and $c$, i.e.

$$g_j = f_j(p_1, ..., p_n, e, c; \theta) \quad j = 1, ..., n$$

(3)

The demand system given in equations (3) will be referred to as the 'conditional demand system'. The conditional demand system addresses how a given local public budget is allocated between local public services.

Let us now turn to the more comprehensive model (the benchmark model), which also recognizes how the municipality raises revenue. This model will be referred to as the 'complete demand system'. The representative agent still derives utility from the consumption of a private good $c$, and a bundle of public services, $g_1, ..., g_n$. The local government raises revenues by using a proportional income tax, $\tau$, matching grants, $w_1, ..., w_n$, and a lump-sum transfer from the central government, $z$. The allocation of private and public consumption preferred by the representative agent is defined by maximization of the utility function

$$u = u(g_1, ..., g_n, c; \theta)$$

(4)

subject to the private and local public budget constraints

$$c = y(1 - \tau)$$

(5a)

$$\sum_{j=1}^{n} p_j g_j = \tau y + z$$

(5b)

where the price of the private consumption good has been normalized to one and $y$ is the representative agent’s gross income. The matching grant to service sector $j$ is part of $p_j$. By solving equation (5a) for $\tau$ and substituting into equation (5b), we can write the budget constraint of the representative agent as

$$c = x - \sum_{j=1}^{n} p_j g_j$$

(6)

where the representative agent’s effective income (total income), $x$, is defined as $x = y + z$. Maximizing the utility function with respect to $c$ and $g_1, ..., g_n$,
subject to the budget constraints in equation (6), defines the representative agent’s demand for the local public services and the private good as functions of prices, total per capita income and municipal characteristics

\[ g_j = f_j(p_1, ..., p_n, x; \theta), \quad j = 1, ..., n \] (7a)

\[ c = f_{n+1}(p_1, ..., p_n, x; \theta) \] (7b)

Both the conditional and the complete demand system represent demand systems where the regular demand restrictions apply for the set of consumer goods (i.e., adding up, homogeneity and Slutsky symmetry).

A problem in the analysis of the demand for public services is the lack of appropriate measures of public output. The most common solution is to use expenditure as a proxy for output. The lack of information on service volumes also makes the identification of price effects problematic. A useful approach to this problem is to calculate the operating cost per unit of labor corrected for matching grants (see Ehrenberg 1973; Bahl et al. 1980).\(^5\) Following Ehrenberg and Bahl et al., this paper assumes a labor-intensive Leontief production function for each public service.\(^6\) Nonlabor expenditures (i.e. materials and capital) are assumed to be proportional to the labor input. Separating the expenditures into price and volume components gives

\[ e_j = (w_j + r_j)n_j = p_j^*n_j \] (8)

where \(e_j\) is the expenditure in service sector \(j\), \(w\) the wage rate, \(r\) the cost of nonlabor input per unit of labor and \(n\) the labor input. The net price, \(p\), is obtained by subtracting the matching grant per unit of labor

\[ p_j = p_j^* - (\bar{w}_j/n_j) = (e_j - \bar{w}_j)/n_j \] (9)

The net price per unit of labor reflects the wage rate, the cost of nonlabor inputs and the matching grants (the net price is further discussed in the next section).\(^7\)

---

\(^5\)This "public employment approach" is applied to Norwegian data by Borge and Rattsø (1995) and Aaberge and Langorgen (2003).

\(^6\)It follows, by their approach, that labor is considered to be homogenous within each service producing sector.

\(^7\)Another useful approach to identify prices is to calculate the decisive voter’s tax price, which is often measured as the tax share of the median income consumer in the locality (see Bergstrom and Goodman, 1973). The problem with extending a median voter model to a
3 Empirical Analysis

The empirical part of the paper begins with a description of the data. I will then present the empirical model and the estimation results.

3.1 Data and some Institutional Characteristics

The sample consists of a panel containing 273 Swedish municipalities over an eight year period (1998-2005). The data were obtained from Statistics Sweden, the Swedish Labour Market Board and the Swedish Association of Local Authorities and Regions. As mentioned above, Swedish municipalities collect tax revenues by using a proportional income tax. This tax is the most important source of funds for the majority of municipalities, followed by user fees and general grants. The Swedish intergovernmental transfer system is built around formula-based general grants. A significant policy reform in 1993 replaced the previous system of matching grants with a system of general grants. However, a small number of matching grants was also used by the central government during the period of study.

Let us now turn to the variables to be used in the estimations. The local public services examined are: child care, social services, elderly care, comprehensive education, ‘infrastructure and protection’ (called ‘infra’ in the tables) and ‘other expenditures’. Together, these six services account for the entire operating expenditure in the municipalities, \( \hat{c} \). The local public expenditures system of public sector demand equations is that the median individual generally will differ for different services. Extensions of the traditional median voter model (based on deterministic voting) to the multidimensional case gives a probabilistic voting model (see Enelow and Hinich, 1984, for an overview of probabilistic voting). Craig and Inman (1986) propose a voter group decision model to solve the multidimensionality problem, where the allocation is a political compromise among voter groups.

8 The number of Swedish municipalities increased during the study period from 288 (1998) to 289 (1999-2002) to 290 (2003-2005). Three large municipalities are excluded from the study (Malmö, Göteborg and Gotland) due to partly joint expenditures between the municipality and the region during part of the study period. In addition, twelve municipalities are excluded due to missing values. This leaves a balanced panel with 273 municipalities.

9 The service ‘infrastructure and protection’ includes expenditures on construction (roads, parking lots, parks etc.), fire services, environmental protection, community protection, tourism and expenditures for promoting local business.

10 The variable measuring ‘other expenditures’ includes expenditures on central administration, cultural services, leisure activities and expenditures for special events.

11 The municipalities’ business activities are not considered as operating costs. The reason
are adjusted by Statistics Sweden to ensure that it measures the operating expenditure net of fees and net of services consumed by other municipalities.\textsuperscript{12} The total per capita income, $x$, is computed as the sum of the average tax base in the municipality\textsuperscript{13} and the per capita lump-sum transfer from the central government, $z$.

The demand systems in equations (3) and (7a)-(7b) will be estimated by using the QAIDS specification (the specification is further discussed in the next subsection). The data available describes service expenditures and not quantities, which suggests that the demand equations should be estimated in terms of expenditure shares. It is important to distinguish between the conditional demand system and the complete demand system when the models are operationalized. The expenditure shares in the conditional demand system, $\hat{s}_j$, are defined as local public budget shares, which means that they are computed as the expenditure on each service, $j$, divided by the total local public expenditures, i.e. $\hat{s}_j = e_j/\hat{c}$.\textsuperscript{14} The private consumption, $c$, is defined as the total real per capita income net of local public consumption.\textsuperscript{15} The expenditure shares in the complete demand system, $s_j$, are defined in terms of total income, meaning that they are computed as the expenditure associated with each service, $j$, divided by the total per capita income, i.e. $s_j = e_j/x$. The trends of the expenditure shares (both in terms of the local public budget and in terms of income) during the period of study are shown in Table A4 in the Appendix.

The price of each public service, $p_j$, is measured as the operating cost per unit of labor\textsuperscript{16} corrected for matching grants, i.e. $p_j = (e_j - zw_j)/n_j$.\textsuperscript{17} It is reasonable to assume that the labor input, to a large extent, is determined by

\begin{itemize}
\item is that the business activities are financed by fees and charges and not via the local public budget.
\item The operating expenditure financed by fees is considered as private consumption by Statistics Sweden and therefore is not included in this measure of public expenditures.
\item An alternative would be to measure the private income net of the taxes paid to the central and regional governments. Changing the income measure in this way does not alter any of the main qualitative results.
\item See Deacon (1978) and Borge and Rattsø (1995).
\item The per capita private consumption is calculated by using the budget constraint given by equation (6). The variable is adjusted by the CPI (2005 is base year).
\item The hours worked within a service sector is converted into units of labor by the Swedish Association of Local Authorities and Regions. As a result, one unit of labor corresponds to a full-time job.
\item 2005 is the base year.
\end{itemize}
The wage rate. The development of the wage rate in different service sectors is expected to be the main source of relative price shift. Higher wage rates or smaller matching grants increase the prices of public services, which are paid by the representative agent in the municipality.

There exists a number of specific grants. Some of them are matching while others are not. The data describing the specific grants is an aggregate of all the specific grants to each service in each municipality, which leaves two possible approaches; (1) treating all of them as matching grants or (2) treating all of them as lump-sum grants. As it turned out, both these approaches give similar qualitative results. In the main text I will treat all the specific grants as matching.\footnote{Earlier studies based on Swedish data for the current study period give no guidance as to whether the specific grants should be treated as matching or lump-sum grants. A policy reform implemented in 2002 meant that the majority of the specific grants to the child care sector became lump-sum grants. The specific grants to the child care sector will, therefore, be treated as block grants for the years 2002-2005. Estimating a model where all the specific grants are treated as block grants does not lead to any important changes in the qualitative results discussed below (the results are available from the author upon request). This suggests that the variation in the price variables is driven mainly by factors other than the specific grants.} The trends of the relative prices during the period of study are shown in Table A3 in the Appendix.

Following earlier literature on the determinants of municipal expenditures, the models also include relevant local characteristics such as population density, age structure of the population, political preferences and political strength. The age variables correspond to the percentage of residents aged 6 or younger, \( CH \), residents aged 7-15, \( YO \), and residents aged 75 or older, \( EL \). The population density, \( DENS \), and the size of the population, \( POP \), are included in the analysis to detect possible scale effects.\footnote{See Borge and Rattso (1993).} The density is measured by the number of residents per square kilometer. Political preferences are controlled for by including the share of the seats in the municipal parliament occupied by members of either the Social Democratic Party or the Left Party, \( LEFT \). Political strength is represented by a Herfindahl index, \( HERF \), i.e. the sum of the squared shares of each party in the local parliament. Finally, a variable measuring the unemployment rate, \( UNEMP \), is also included as a control variable. Summary statistics for these variables are presented in Table A1-A2 in the Appendix.
3.2 Specification of the Empirical Model

I apply the QAIDS developed by Banks et al. (1997), implying that the conditional demand system in equations (3) is written as follows

\[
\hat{s}_{jt}^i = \hat{\alpha}_j + \sum_{k=1}^n \hat{\gamma}_{jk} \ln p_{kt}^i + \hat{\beta}_j \ln [\hat{e}_{jt}^i / a^i(p_r)] \\
+ \left(\hat{\lambda}_j / b^i(p_r)\right) \times \left(\ln [\hat{e}_{jt}^i / a^i(p_r)]\right)^2 \\
+ \hat{\kappa}_j \ln c^i_t + \hat{\theta}_j \ln \theta^i_t + \hat{\phi}_j^i + \hat{\pi}_{jt} + \hat{\delta}_{jt}
\]

for \( j = 1, ..., n - 1 \)

where superscript \( i \) refers to municipality, subscript \( j \) to service sector, subscript \( t \) to time period, \( \varphi \) is a municipality-specific effect, \( \pi \) is a year-specific effect, and \( u \) is an error term. The public service 'other expenditures' is excluded when estimating the conditional demand system. The excluded equation follows by adding-up. The functions \( \ln a^i(p_r) \) and \( \ln b^i(p_r) \) are defined by

\[
\ln a^i(p_r) = \sum_{j=1}^n \hat{\alpha}_j \ln p_{jt}^i + \frac{1}{2} \sum_{j=1}^n \sum_{k=1}^n \hat{\gamma}_{jk} \ln p_{jt}^i \ln p_{kt}^i \\
\ln b^i(p_r) = \sum_{j=1}^n \hat{\beta}_j \ln p_{jt}^i
\]

The QAIDS is a quadratic extension to Deaton and Muellbauer’s (1980) almost ideal demand system. The advantage with the QAIDS specification, compared to the AIDS formulation, is that it recognizes the possibility that expenditures are non-linear functions of income (or total expenditure).

The second specification that will be estimated is the complete demand system given by the equations (7a)-(7b)
\[ s_{jt} = \alpha_j + \sum_{k=1}^{n} \gamma_{jk} \ln p_{kt} + \beta_j \ln [x_t/a'(p_t)] \]
\[ + (\lambda_j/b'(p_t)) \times (\ln [x_t/a'(p_t)])^2 \]
\[ + \eta_j \ln \theta_j^t + \varphi_j^t + \pi_{jt} + v_{jt} \]

for \( j = 1, \ldots, n \)

The composite private consumer good is excluded when estimating the complete demand system. The excluded equation follows by adding-up.

Blundell et al. (1993) simplify the estimations by using the Stone price index, \( \ln P_t = \sum_{j=1}^{n} s_{jt} \ln p_{jt} \), instead of the translog form price index, \( \ln a'(p_t) \), and by using unity instead of the price aggregator, \( \ln b'(p_t) \). However, Matsuda (2006) finds that Laspeyres’s price index, \( \ln P_t = \sum_{j=1}^{n} s_{jt} \ln p_{jt} \), where \( s_{jt} \) is the budget share for service \( j \) in the base period \( t \), performs better than the Stone price index.\(^{20}\) According to Matsuda, the formulation of the price index is of great importance (especially if the time series turns out to be nonstationary). This paper follows Blundell et al. (1993) by replacing the price aggregator \( \ln b'(p_t) \) with unity, and Matsuda (2006) by using Laspeyres’s price index\(^{21}\) (based on unity in 2005) instead of \( \ln a'(p_t) \).\(^{22}\)

Income is a potentially endogenous regressor, because a change in the municipal policy may affect the labor supply and, to some extent, also the migratory behaviour (for a similar reason, the total resources available for public consumption and the consumption of the composite private good are potentially endogenous regressors in the conditional demand system).\(^{23}\) This suggests the use of an instrumental variables regression (IV) where the total income variables, \( \ln (x_t/P_t) \) and \( \ln (\hat{x}_t/P_t) \), are instrumented in the complete demand system. Similarly, the total expenditure variables, \( \ln (\hat{e}_t/P_t) \) and \( \ln (\hat{e}_t/P_t) \),

\(^{20}\)See also Moschini (1995).
\(^{21}\)2005 is used as the base year.
\(^{22}\)Replacing Laspeyres’s price index with Stone’s price index does not alter the qualitative results discussed in the next subsection.
\(^{23}\)Empirical studies based on Swedish data finds evidence that a change in taxation affects the labor supply (see Blomquist, 1983, and Aronsson and Palme, 1998). The relation between taxation and migratory behaviour, based on Swedish data, is analyzed by Westerlund and Wyzan (1995).
and the private consumption, Inc, are instrumented in the conditional demand system.24 The share of residents with college education, \textit{COLLEGE}, and the average age of women giving birth to their first child, \textit{FBORN}, are used as instrumental variables. A lower rate of college education and a lower average age of women giving birth to their first child are both expected to result in a lower income in the municipality. The lag of these variables as well as lagged exogenous variables25 will also be used as instruments. Each system of demand equations is estimated by using 3SLS. This approach estimates the demand equations simultaneously rather than as separate equations, which makes it possible to analyze cross-price effects and test for symmetry.

\footnote{The lags of the endogenous variables would be valid as instruments in the absence of serial correlation. However, it turns out that the variables are serial correlated.}

\footnote{The one and two-period lag of the exogenous variables DENS, POP, LEFT, HERF and UNEMP are used as instruments.}
3.3 Results

The unconstrained parameter estimates for both the complete and the conditional demand system are reported in Tables B1-B2 in the Appendix. The diagnostics for the first-stage regression imply that the chosen instruments are significant and valid. The conventional test for instrument significance is the $F$-statistic of the joint significance of the instruments in the first-stage regression. However, this test is not valid when there are multiple endogenous regressors (see Baum et al., 2003). In these cases, Stock and Yogo (2002) suggest using the Cragg-Donald $F$-statistic (the authors also compute the critical values).\(^{26}\) The estimate of the Cragg-Donald $F$-statistic rejects the null of weak instruments for both the complete demand system ($F = 10.96$) and for the conditional demand system ($F = 10.41$).\(^{27}\) Besides significance, the instruments also need to be exogenous, i.e., there shall be no direct effect of the instruments on the dependent variable (other than through their effect on the endogenous regressor). The validity of the instruments is tested by using Hansen’s $J$-test.\(^{28}\) In 8 out of 11 cases, the estimates of Hansen’s $J$ cannot reject the null hypothesis that the instruments are uncorrelated with the error term at the five percent level (see Tables B1-B2 in the Appendix).\(^{29}\)

The interpretation of the estimated income and price parameters, i.e., the estimated $\beta$s, $\lambda$s and $\gamma$s, will be discussed in terms of income and price elasticities. The formula\(^{30}\) for the income elasticity in the complete demand system

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\(^{26}\)The Cragg-Donald $F$-statistic is originally a test of under-identification.

\(^{27}\)The critical value for two endogenous variables, at the five percent level and allowing for a maximum relative bias of 10 percent compared to OLS, is 10.84. This level of maximum bias relative to OLS corresponds to the rule of thumb that, for the case of a single endogenous regressor, instruments are weak if the first-stage $F$-statistic is less than ten. The corresponding critical value for three endogenous variables is 10.14. Even if the null hypothesis is rejected for both demand systems, the set of instruments does not seem to be very strong. Estimating the demand systems by OLS, instead of by IV, does not change the qualitative results further discussed, neither in terms of point estimates of the elasticities nor in terms of $t$-values (one exception is that the $t$-value for the budget-elasticity for ‘infra’ in the conditional demand system turns significant, which suggests that ‘infra’ is a luxury good in the local public budget).

\(^{28}\)This test is applicable in the presence of serial correlation and heteroskedasticity.

\(^{29}\)Estimations where COLLEGE and FBORN are used as including instruments (only the lag of these variables and lagged exogenous variables are used as excluding instruments), do not alter the qualitative results discussed in this section.

\(^{30}\)The formulas for the income and price elasticities are derived in Blundell et al. (1993).
The Demand for Local Public Services in Sweden

is given by

$$E^i_{jk} = \left( \beta_j + 2 \lambda_j \ln m^i_j / s^i_{jk} \right) / (s^i_{jk} / s^i_j) + 1$$  \hspace{1cm} (14)

where \( m^i_j = x^i_j / P_t \). The formula for the uncompensated price elasticity of service \( j \) with respect to the price of service \( k \) is given by

$$E^i_{jkt} = \left( \gamma_{jk} / s^i_{jk} \right) - \left( \beta_j + 2 \lambda_j \ln m^i_j (s^i_{kt} / s^i_{jk}) - h_{jk} \right)$$  \hspace{1cm} (15)

where \( h_{jk} = 1 \) if \( j = k \) and \( h_{jk} = 0 \) if \( j \neq k \). The compensated price elasticities then become

$$S^i_{jkt} = E^i_{jkt} + E^j_{jkt} s^j_{kt}$$  \hspace{1cm} (16)

The corresponding elasticities in the conditional demand system, to be called \( \hat{E}^i_{jkt}, \hat{E}^i_{jkt}, \) and \( \hat{S}^i_{jkt} \), respectively, are measured in the same general way. The income elasticity will be denoted 'budget elasticity' in the conditional demand system (where \( \hat{m}^i_j = \hat{c}^i_j / P_t \)). Note that the budget elasticity measures the percentage change in the quantity demanded if the real public budget increases by one percent. The price elasticities referring to the conditional demand system reflect increased costs of local services within a given local public budget.

The Conditional Demand System

The elasticities associated with the conditional demand system are presented in Table 1. Child care and elderly care are the services with the lowest point estimate of the budget elasticity, 0.80, whereas 'infra' is the service with the highest point estimate, 2.09.\(^{31}\) Using that the sum of the weighted budget elasticities (where \( \hat{E}^i_{jkt} \) is weighted by \( \hat{s}^i_{jkt} \)) is equal to one\(^{32}\) gives a point estimate of the budget elasticity for the excluded service, 'other expenditures' (i.e. expenditures on cultural services, leisure activities, central administration and special events), equal to 1.09. The significance of the parameter estimates of the budget elasticities are tested with t-tests. None of the t-tests reject the null hypothesis that the budget elasticity is equal to one at the five percent significance level. This suggests that the budget elasticities are interpretable as if all

\(^{31}\) The elasticities in the conditional demand system are evaluated at the mean of the data (the budget shares are evaluated at the base year).

\(^{32}\) This is true by definition and easy to obtain by using \( \sum_j (\partial \hat{E}_{jkt} / \partial \hat{m}_j) = 0 \).
services are normal goods (at the mean of the data) in the local public budget. 'Infra' is close to being considered a luxury good, but the null hypothesis that the elasticity is one cannot be rejected at the five percent level ($t = 1.81$).

Comparing the point estimates of the budget elasticities between budget-quartiles (the municipalities are sorted with respect to the size of the total per capita budget) implies only small differences in the point estimates (the budget elasticities for each budget-quartile is presented in Table B3 in the Appendix). The coefficients of the conditional demand system reveal that the nonlinear effects of the total expenditure is significant (at the five percent level) in three out of five cases, which supports the use of QAIDS.

All the compensated own-price elasticities resulting from the conditional demand system are negative, suggesting that the negativity condition is fulfilled. The compensated own-price elasticities vary between -0.57 for education and -0.86 for social service. All the compensated cross-price elasticities are positive and relatively small (typically much smaller than the own-price elasticities). On the other hand, the uncompensated cross-price elasticities are in general negative, suggesting that the negative income effect dominates the positive substitution effect.

There are no previous studies, based on Swedish data, dealing with a conditional demand system for local public services. However, the Swedish local public decision structure resembles the Norwegian one in many respects (e.g. the organization of the public sector and the services provided by the local governments). Earlier research on the composition of local public expenditures in Norway are based on models resembling the conditional demand system (Borge and Ratté, 1995; Aaberge and Langørge, 2003).33 Consistent with the Norwegian study by Aaberge and Langørge, 'infra' is a sector with a relatively high point estimate of the budget elasticity. Also the budget elasticities for child care, elderly care and 'other expenditures'34 resemble the corresponding budget elasticities in the study by Aaberge and Langørge. Our point estimate of

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33The system of financing the local public services in Norway is more centralized than in Sweden. The maximum tax rate that the Norwegian municipalities are allowed to use is set by the central government. Municipalities that set a higher tax rate than the one decided upon by the central government are expected to be penalized via the grant system.

34The point estimate of budget elasticity for 'other expenditure' (i.e. expenditures on cultural services, leisure activities, central administration and special events) is compared with the point estimates of the budget elasticities for 'administration' and 'cultural services' (which have the same point estimates) estimated by Aaberge and Langørge (2003).
the budget elasticity of education, 1.01, is somewhat higher than the estimates found on Norwegian data (Borge and Rattso, 1995; Aaberge and Langørgen, 2003) and earlier studies based on US data (Feldstein, 1975; Ladd, 1975). Borge and Rattso (1995) find that local public services and private consumption cannot be treated as separable. In the present study, a t-test shows that public services and private consumption cannot be treated as separable in the equation for social services (see Table B1 in the Appendix). Furthermore, a Likelihood ratio-test rejects a restricted version of the conditional demand system where private consumption is excluded, compared to the alternative where private consumption is included ($\chi^2_{[5]} = 32.49$). This suggests that private consumption is not separable from the consumption of public services.

Table 1. Elasticities referring to the Conditional Demand System

<table>
<thead>
<tr>
<th>Commodity j</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>1.04</td>
<td>0.80</td>
<td>1.01</td>
<td>2.09</td>
<td></td>
</tr>
<tr>
<td>(-0.72)</td>
<td>(0.07)</td>
<td>(-1.26)</td>
<td>(0.07)</td>
<td>(1.81)</td>
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</tr>
</tbody>
</table>

B. Compensated Price Elasticities:

<table>
<thead>
<tr>
<th>Commodity k</th>
<th>Commodity j</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>0.73</td>
<td>0.05</td>
<td>0.25</td>
<td>0.08</td>
<td>0.13</td>
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</tr>
<tr>
<td>(-32.60)</td>
<td>(1.88)</td>
<td>(2.02)</td>
<td>(0.59)</td>
<td>(4.82)</td>
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<td></td>
</tr>
<tr>
<td>Social</td>
<td>0.10</td>
<td>-0.86</td>
<td>0.39</td>
<td>0.03</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>(1.80)</td>
<td>(-115.6)</td>
<td>(1.45)</td>
<td>(0.09)</td>
<td>(2.71)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eld</td>
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<td>0.25</td>
<td>-0.59</td>
<td>0.29</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>(2.09)</td>
<td>(1.64)</td>
<td>(5.43)</td>
<td>(14.91)</td>
<td>(3.66)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edu</td>
<td>0.28</td>
<td>0.23</td>
<td>0.33</td>
<td>-0.57</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>(4.30)</td>
<td>(1.61)</td>
<td>(29.96)</td>
<td>(-36.25)</td>
<td>(3.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Infra</td>
<td>0.08</td>
<td>0.05</td>
<td>0.34</td>
<td>0.05</td>
<td>-0.81</td>
<td></td>
</tr>
<tr>
<td>(1.82)</td>
<td>(5.01)</td>
<td>(1.49)</td>
<td>(0.18)</td>
<td>(-75.91)</td>
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</tr>
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</table>

(Continued on next page).

35The critical value, at the five percent significant level, is 11.07.
Table 1. (Continued)

C. Uncompensated Price Elasticities:

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<tr>
<th>Commodity $k$</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>-0.83</td>
<td>-0.07</td>
<td>0.16</td>
<td>-0.04</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>(-15.49)</td>
<td>(-1.26)</td>
<td>(1.10)</td>
<td>(-0.26)</td>
<td>(-1.73)</td>
</tr>
<tr>
<td>Social</td>
<td>0.05</td>
<td>-0.92</td>
<td>0.35</td>
<td>-0.03</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(0.73)</td>
<td>(-26.04)</td>
<td>(1.24)</td>
<td>(-0.11)</td>
<td>(-1.40)</td>
</tr>
<tr>
<td>Eld</td>
<td>-0.06</td>
<td>-0.08</td>
<td>-0.85</td>
<td>-0.03</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(-2.06)</td>
<td>(-2.00)</td>
<td>(-14.58)</td>
<td>(-0.53)</td>
<td>(-1.53)</td>
</tr>
<tr>
<td>Edu</td>
<td>-0.01</td>
<td>-0.14</td>
<td>0.05</td>
<td>-0.93</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>(-0.30)</td>
<td>(-2.02)</td>
<td>(0.74)</td>
<td>(-15.16)</td>
<td>(-3.16)</td>
</tr>
<tr>
<td>Infra</td>
<td>0.02</td>
<td>-0.02</td>
<td>0.28</td>
<td>-0.02</td>
<td>-0.95</td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(-0.56)</td>
<td>(1.19)</td>
<td>(-0.09)</td>
<td>(-20.26)</td>
</tr>
</tbody>
</table>

Note: $t$-values in parentheses. Under the null hypotheses it is assumed that the budget elasticities are equal to one and that the price elasticities are equal to zero.

The Complete Demand System

The elasticities in Table 2 refer to the complete demand system. The complete demand system recognizes that the local public expenditure composition is determined simultaneously with the collection of local public revenues. Social service has the lowest point estimate of the income elasticity, -0.51. For all other services, the point estimate of the income elasticity is positive. 'Infra' is the service with the highest point estimate of the income elasticity, 1.37. As can be seen from Table 2, $t$-tests can reject the null hypothesis that the income elasticity is equal to one for social services, elderly care and education. The null hypothesis that the income elasticity is equal to zero is rejected for all services except for social services ($t = -1.41$), which is close to being considered an inferior good. The $t$-tests imply that all services, except for social services, are normal goods at the mean of the data. Elderly care and education are necessities if measured at the mean of the data, meaning that the estimates of the income elasticity for these services are significantly positive and significantly less than one.

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36 The elasticities in the complete demand system are evaluated at the mean of the data (the income shares are evaluated at the base year).
Comparing the income elasticities between income-quartiles (the municipalities are sorted with respect to the size of the total per capita income) indicates only small differences (see Table B3 in the Appendix). The parameter estimates of the complete demand system show that the nonlinear effect of the total income is significant in five out of six cases at the five percent level (see Table B2 in the Appendix), which motivates the use of the QAIDS.

The compensated own-price elasticities implied by the complete demand system are negative for all services. The lowest absolute value is found for education, 0.75, while the highest value is found for social services, 0.89. In general, the compensated cross-price effects are positive and quite small. The point estimates of the uncompensated cross-price elasticities differ in sign across services and are relatively small in absolute value.

It is not easy to relate the results in Table 2 to earlier Swedish studies, as these have either focused on the determinants of aggregate local public spending (Aronsson and Wikström, 1996; Dahlberg and Jakobsson, 2000),37 or individual components of total spending (Ahlin and Johansson, 2001).38 In general, the income elasticities are smaller than the corresponding budget elasticities in the conditional demand system (especially for social services). All the own-price elasticities in the complete demand system are higher in absolute value compared to the own-price elasticities in the conditional demand system. Finally, both demand systems imply that the cross-price elasticities are relatively small (at least by comparison with the own-price elasticities).

What will happen to the share of income spent on municipal services, as the total per capita income increases in the municipality? The marginal effect that corresponds to this decision is given by

\[
\frac{\partial s^i_u}{\partial m^i} = \sum_{j=1}^{6} \frac{\partial s^j}{\partial m^i} = \sum_{j=1}^{6}(\beta_j + 2\lambda_j \ln m^i)/m^i
\]  

\(17\)

37 Using the median voter approach when analyzing the determinants of the total local public expenditures in Sweden, Aronsson and Wikström (1996) estimated the income elasticity to be 0.82 and the price elasticity to be 0.53 in absolute value. Dahlberg and Jakobsson (2000) also analyze the total local public expenditures in Sweden and found income elasticities in the interval 0.47-1.30 and price elasticities in the interval 0.67-1.48 in absolute value.

38 Ahlin and Johansson (2001) estimated the income elasticity for local school expenditures in Sweden to be 0.73-1.13 by using a method based on survey data.
where the summation represents the six local public services included in the regression of the complete demand system. Calculating the marginal effect (as an average for all municipalities) implies that the total local public budget, as a share of the total income, decreases by 0.09 percentage points when the real total per capita income increases by 1000 SEK. Comparing the marginal effect between income-quartiles indicates only small differences.

Table 2. Elasticities referring to the Complete Demand System

A. Income Elasticities:

<table>
<thead>
<tr>
<th>Commodity j</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.83</td>
<td>-0.51</td>
<td>0.43</td>
<td>0.49</td>
<td>1.37</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>(-0.91)</td>
<td>(-4.16)</td>
<td>(-4.40)</td>
<td>(-5.00)</td>
<td>(0.79)</td>
<td>(0.80)</td>
</tr>
</tbody>
</table>

B. Compensated Price Elasticities:

<table>
<thead>
<tr>
<th>Commodity k</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>-0.77</td>
<td>0.02</td>
<td>0.18</td>
<td>0.20</td>
<td>0.06</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(-60.12)</td>
<td>(0.75)</td>
<td>(5.98)</td>
<td>(6.78)</td>
<td>(2.01)</td>
<td>(7.97)</td>
</tr>
<tr>
<td>Social</td>
<td>0.03</td>
<td>-0.89</td>
<td>0.29</td>
<td>0.32</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(2.88)</td>
<td>(-119.8)</td>
<td>(4.66)</td>
<td>(5.17)</td>
<td>(3.96)</td>
<td>(0.76)</td>
</tr>
<tr>
<td>Eld</td>
<td>0.02</td>
<td>-0.07</td>
<td>-0.79</td>
<td>0.11</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(1.13)</td>
<td>(-1.53)</td>
<td>(-67.82)</td>
<td>(11.44)</td>
<td>(2.67)</td>
<td>(3.28)</td>
</tr>
<tr>
<td>Edu</td>
<td>0.12</td>
<td>-0.05</td>
<td>0.17</td>
<td>-0.75</td>
<td>0.05</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>(4.95)</td>
<td>(-0.99)</td>
<td>(10.16)</td>
<td>(-67.61)</td>
<td>(0.82)</td>
<td>(6.93)</td>
</tr>
<tr>
<td>Infra</td>
<td>0.01</td>
<td>0.00</td>
<td>0.23</td>
<td>0.28</td>
<td>-0.83</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(1.30)</td>
<td>(0.33)</td>
<td>(4.64)</td>
<td>(5.49)</td>
<td>(-71.29)</td>
<td>(5.12)</td>
</tr>
<tr>
<td>Other</td>
<td>0.03</td>
<td>0.02</td>
<td>0.21</td>
<td>0.26</td>
<td>0.06</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td>(3.50)</td>
<td>(1.27)</td>
<td>(4.87)</td>
<td>(5.94)</td>
<td>(2.79)</td>
<td>(-61.42)</td>
</tr>
</tbody>
</table>

(Continued on next page).
Table 2. (Continued)

C. Uncompensated Price Elasticities:

<table>
<thead>
<tr>
<th>Commodity k</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>-0.80</td>
<td>0.04</td>
<td>0.16</td>
<td>0.18</td>
<td>0.02</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(-53.48)</td>
<td>(1.39)</td>
<td>(4.85)</td>
<td>(5.61)</td>
<td>(0.61)</td>
<td>(5.73)</td>
</tr>
<tr>
<td>Social</td>
<td>0.02</td>
<td>-0.88</td>
<td>0.28</td>
<td>0.31</td>
<td>0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(1.27)</td>
<td>(-95.14)</td>
<td>(4.41)</td>
<td>(4.92)</td>
<td>(1.29)</td>
<td>(-1.22)</td>
</tr>
<tr>
<td>Eld</td>
<td>-0.05</td>
<td>-0.02</td>
<td>-0.82</td>
<td>0.07</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(-2.87)</td>
<td>(-0.66)</td>
<td>(-48.91)</td>
<td>(4.36)</td>
<td>(0.63)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>Edu</td>
<td>0.04</td>
<td>0.00</td>
<td>0.08</td>
<td>-0.79</td>
<td>-0.08</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(1.86)</td>
<td>(0.06)</td>
<td>(4.56)</td>
<td>(-49.75)</td>
<td>(-1.67)</td>
<td>(4.72)</td>
</tr>
<tr>
<td>Infra</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.23</td>
<td>0.27</td>
<td>-0.86</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-0.52)</td>
<td>(1.32)</td>
<td>(4.27)</td>
<td>(5.11)</td>
<td>(-62.67)</td>
<td>(1.53)</td>
</tr>
<tr>
<td>Other</td>
<td>0.02</td>
<td>0.03</td>
<td>0.20</td>
<td>0.25</td>
<td>0.03</td>
<td>-0.88</td>
</tr>
<tr>
<td></td>
<td>(1.32)</td>
<td>(1.89)</td>
<td>(4.37)</td>
<td>(5.41)</td>
<td>(1.27)</td>
<td>(-56.01)</td>
</tr>
</tbody>
</table>

Note: t-values in parentheses. Under the null hypotheses it is assumed that the income elasticities are equal to one and that the price elasticities are equal to zero.

Turning to the effects of local characteristics (the vector $\theta$) implied by the complete demand system (see Table B2 in the Appendix), the results are qualitatively similar to those of earlier comparable literature on local public expenditures.\(^{39}\) Recall that the municipalities are responsible for providing child care, comprehensive education and elderly care. We find that the higher the share of individuals in the age corresponding to child care (0-6), primary education (7-15), and the oldest residents in the municipality (75+), respectively, the higher the expenditure share in the corresponding service sector. Political preferences have significant effects on the expenditures: the results suggest that a municipality represented by a large share of left wing seats in the local parliament, ceteris paribus, is associated with higher expenditure shares on elderly care and ‘infra’. A higher rate of unemployment contributes to a higher expenditure share on education.\(^{40}\)

Finally, the results have been used for testing the general restrictions of the demand system. Tests of homogeneity and symmetry are given in Tables B1-B2 in the Appendix. A Likelihood ratio-test rejects a restricted demand system

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39 Replacing the local characteristics with the one-period lag of the local characteristics does not alter the qualitatively results.

40 Many municipalities offer comprehensive education also to adults.
(where all the restrictions are imposed) against the unconstrained alternative for both the complete demand system ($\chi^2_{[20]} = 397.09$) and the conditional demand system ($\chi^2_{[19]} = 344.19$).\footnote{The critical value, at the five percent significance level, is 38.89 when testing the complete demand system, and 30.14 when testing the conditional demand system.} However, even if homogeneity and Slutsky symmetry are imposed, this does not alter the main findings discussed above. The estimation results from the restricted versions of the demand systems, and a discussion of the restrictions, are presented in the Appendix C.

4 Conclusion

This paper, which concerns the determinants of municipal expenditures in Sweden, simultaneously analyzes tax revenue collection and how the resources are allocated between different local public services. In order to understand the priorities made by municipalities, a complete demand system is required, implying that private consumption is also considered in the model. The empirical study is based on panel data for the period 1998-2005 and contains results from estimating a demand system comprising a composite private consumption good and six local public services. The local characteristics that affect the allocation of local public consumption according to the theoretical model, such as total per capita income, prices, and municipal characteristics, are considered in the regressions. The results show that the point estimates of all income elasticities except one are positive, and that none of them significantly exceed one. Among the results, it is also found that the own-price elasticities are negative and less than one in absolute value for all services.

For purposes of comparison, we also estimate a demand system that is conditioned on the total revenue that can be used for local public consumption. The results from this conditional demand system are similar to related studies based on Norwegian data (Borge and Rattsø, 1995; Aaberge and Læringen, 2003).
Appendix A: Summary Statistics

Table A1. Description of the variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_j$</td>
<td>The expenditure on service $j$ as a share of the total income in the municipality.</td>
</tr>
<tr>
<td>$\delta_j$</td>
<td>The expenditure on service $j$ as a share of the local public budget.</td>
</tr>
<tr>
<td>$p_j$</td>
<td>The price of the local public service $j$.</td>
</tr>
<tr>
<td>$x$</td>
<td>Total income, the sum of the tax base and the general grants (SEK per capita).</td>
</tr>
<tr>
<td>$\hat{e}$</td>
<td>Total budget (SEK per capita).</td>
</tr>
<tr>
<td>$e$</td>
<td>The consumption of the composite private good (SEK per capita).</td>
</tr>
<tr>
<td>CH</td>
<td>The percentage of residents aged 6 or younger.</td>
</tr>
<tr>
<td>YO</td>
<td>The percentage of residents aged 7-15.</td>
</tr>
<tr>
<td>EL</td>
<td>The percentage of residents aged 75 or older.</td>
</tr>
<tr>
<td>DENS</td>
<td>The population density, residents per square kilometer.</td>
</tr>
<tr>
<td>POP</td>
<td>The population size, number of persons.</td>
</tr>
<tr>
<td>LEFT</td>
<td>The percentage of Social Democrats and Left Party members in municipal parliament.</td>
</tr>
<tr>
<td>HERF</td>
<td>A Herfindahl index (in percent) measuring political strength in the municipal parliament.</td>
</tr>
<tr>
<td>UNEMP</td>
<td>The percentage of unemployed.</td>
</tr>
<tr>
<td>FBORN</td>
<td>The average age of women giving birth to their first child.</td>
</tr>
<tr>
<td>COLLEGE</td>
<td>The percentage of residents with college education.</td>
</tr>
</tbody>
</table>

Table A2. Summary statistics, 1998-2005

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std.dv.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{m}$</td>
<td>36109</td>
<td>4820</td>
<td>22261</td>
<td>54203</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>127777</td>
<td>15020</td>
<td>94217</td>
<td>219500</td>
</tr>
<tr>
<td>$c^{(a)}$</td>
<td>91911</td>
<td>13547</td>
<td>63569</td>
<td>180656</td>
</tr>
<tr>
<td>CH</td>
<td>7.32</td>
<td>1.07</td>
<td>4.71</td>
<td>11.77</td>
</tr>
<tr>
<td>YO</td>
<td>12.41</td>
<td>1.13</td>
<td>6.81</td>
<td>16.43</td>
</tr>
<tr>
<td>EL</td>
<td>9.70</td>
<td>2.23</td>
<td>2.85</td>
<td>16.03</td>
</tr>
<tr>
<td>DENS</td>
<td>105.7</td>
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<td>4107.0</td>
</tr>
<tr>
<td>POP</td>
<td>29244</td>
<td>51739</td>
<td>2553</td>
<td>771038</td>
</tr>
<tr>
<td>LEFT</td>
<td>46.93</td>
<td>11.28</td>
<td>11.11</td>
<td>77.42</td>
</tr>
<tr>
<td>HERF</td>
<td>24.81</td>
<td>4.36</td>
<td>16.60</td>
<td>45.47</td>
</tr>
<tr>
<td>UNEMP</td>
<td>3.98</td>
<td>1.44</td>
<td>0.9</td>
<td>10.9</td>
</tr>
<tr>
<td>FBORN</td>
<td>27.43</td>
<td>1.24</td>
<td>23.02</td>
<td>33.03</td>
</tr>
<tr>
<td>COLLEGE</td>
<td>13.89</td>
<td>5.12</td>
<td>6.72</td>
<td>39.71</td>
</tr>
</tbody>
</table>

Note: The figures are unweighted averages over the estimation period using a sample of 273 municipalities (out of 290). The variables $\bar{m}$ and $\bar{m}$ are deflated by Laspeyre’s price index. 

* The private consumption is deflated by the Swedish CPI(2005−100).
**Table A3. Development of relative prices and real total per capita income**

<table>
<thead>
<tr>
<th>Year</th>
<th>P</th>
<th>Pcap</th>
<th>Pcap</th>
<th>Pcap</th>
<th>Pcap</th>
<th>Pcap</th>
<th>Pcap</th>
<th>x/P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>1999</td>
<td>101.03</td>
<td>101.90</td>
<td>105.13</td>
<td>99.03</td>
<td>100.28</td>
<td>98.06</td>
<td>100.27</td>
<td>103.04</td>
</tr>
<tr>
<td>2000</td>
<td>101.90</td>
<td>102.86</td>
<td>100.82</td>
<td>99.67</td>
<td>100.43</td>
<td>96.54</td>
<td>99.08</td>
<td>107.47</td>
</tr>
<tr>
<td>2001</td>
<td>103.05</td>
<td>103.54</td>
<td>97.85</td>
<td>100.20</td>
<td>100.40</td>
<td>92.73</td>
<td>101.12</td>
<td>112.87</td>
</tr>
<tr>
<td>2002</td>
<td>104.52</td>
<td>107.14</td>
<td>104.54</td>
<td>100.02</td>
<td>99.39</td>
<td>90.22</td>
<td>100.91</td>
<td>117.69</td>
</tr>
<tr>
<td>2003</td>
<td>106.01</td>
<td>108.19</td>
<td>102.24</td>
<td>100.06</td>
<td>99.64</td>
<td>88.82</td>
<td>100.97</td>
<td>122.14</td>
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<tr>
<td>2004</td>
<td>106.99</td>
<td>109.83</td>
<td>101.42</td>
<td>99.91</td>
<td>99.51</td>
<td>90.44</td>
<td>100.08</td>
<td>126.62</td>
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<tr>
<td>2005</td>
<td>107.90</td>
<td>109.80</td>
<td>100.16</td>
<td>99.23</td>
<td>100.38</td>
<td>91.29</td>
<td>99.05</td>
<td>129.73</td>
</tr>
</tbody>
</table>

*Note:* The figures are unweighted averages over the estimation period using a sample of 273 municipalities (out of 290). The price index P is the Laspeyre’s price index.

**Table A4. The Expenditure Shares**

A. In terms of Total Public Expenditures

<table>
<thead>
<tr>
<th></th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average budget shares (multiplied by 100) 1998-2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>11.7</td>
<td>6.2</td>
<td>29.4</td>
<td>35.2</td>
<td>7.6</td>
<td>10.0</td>
</tr>
<tr>
<td>1999</td>
<td>11.3</td>
<td>5.9</td>
<td>30.2</td>
<td>36.0</td>
<td>7.3</td>
<td>9.4</td>
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<td>5.9</td>
<td>31.0</td>
<td>36.6</td>
<td>7.0</td>
<td>8.7</td>
</tr>
<tr>
<td>2001</td>
<td>10.7</td>
<td>5.6</td>
<td>31.5</td>
<td>36.7</td>
<td>7.0</td>
<td>8.5</td>
</tr>
<tr>
<td>2002</td>
<td>11.4</td>
<td>5.6</td>
<td>31.6</td>
<td>36.2</td>
<td>6.9</td>
<td>8.3</td>
</tr>
<tr>
<td>2003</td>
<td>11.6</td>
<td>5.7</td>
<td>31.8</td>
<td>35.9</td>
<td>6.8</td>
<td>8.1</td>
</tr>
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<td>2004</td>
<td>11.7</td>
<td>5.8</td>
<td>31.7</td>
<td>36.1</td>
<td>6.8</td>
<td>8.0</td>
</tr>
<tr>
<td>2005</td>
<td>12.0</td>
<td>5.8</td>
<td>31.6</td>
<td>35.9</td>
<td>6.8</td>
<td>7.9</td>
</tr>
</tbody>
</table>

Cross-section variation in the budget shares, 2005

|        |        |        |        |        |        |        |
|        | Min    | 1.4    | 13.4   | 26.4   | 2.5    | 3.9    |
|        | Max    | 21.2   | 11.1   | 42.3   | 48.2   | 14.9   | 13.0   |
|        | Stdev  | 2.8    | 1.9    | 4.8    | 3.4    | 1.7    | 1.5    |

(Continued on next page).
Table A4. (Continued)

B. In terms of Total Income

<table>
<thead>
<tr>
<th></th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
<th>Private</th>
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<tr>
<td>1998</td>
<td>3.3</td>
<td>1.7</td>
<td>8.6</td>
<td>10.1</td>
<td>2.2</td>
<td>2.9</td>
<td>71.3</td>
</tr>
<tr>
<td>1999</td>
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<td>1.7</td>
<td>8.9</td>
<td>10.4</td>
<td>2.2</td>
<td>2.7</td>
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<td>3.0</td>
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<td>8.9</td>
<td>10.4</td>
<td>2.0</td>
<td>2.5</td>
<td>71.5</td>
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<td>1.6</td>
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</tr>
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<td>2002</td>
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<td>1.6</td>
<td>9.2</td>
<td>10.3</td>
<td>2.0</td>
<td>2.4</td>
<td>71.4</td>
</tr>
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<td>2003</td>
<td>3.3</td>
<td>1.6</td>
<td>9.2</td>
<td>10.2</td>
<td>2.0</td>
<td>2.3</td>
<td>71.4</td>
</tr>
<tr>
<td>2004</td>
<td>3.2</td>
<td>1.6</td>
<td>8.9</td>
<td>10.0</td>
<td>1.9</td>
<td>2.2</td>
<td>72.1</td>
</tr>
<tr>
<td>2005</td>
<td>3.2</td>
<td>1.6</td>
<td>8.8</td>
<td>9.9</td>
<td>1.9</td>
<td>2.2</td>
<td>72.4</td>
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Cross-section variation in the budget shares, 2005

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<th></th>
<th>Min</th>
<th>Max</th>
<th>Stdev</th>
</tr>
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<td>1.9</td>
<td>3.0</td>
<td>1.2</td>
<td>0.6</td>
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</table>

Note: The figures are unweighted averages over the estimation period using a sample of 273 municipalities (out of 290).
Appendix B: Estimation Results

Table B1. The Parameter Estimates from the Conditional DS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnm</td>
<td>-0.090*</td>
<td>0.369***</td>
<td>0.127</td>
<td>0.011</td>
<td>-0.089</td>
</tr>
<tr>
<td>(lnm)²</td>
<td>(-1.73)</td>
<td>(7.29)</td>
<td>(1.57)</td>
<td>(0.14)</td>
<td>(-1.37)</td>
</tr>
<tr>
<td>lnP_child</td>
<td>0.018**</td>
<td>-0.004</td>
<td>-0.003</td>
<td>-0.011***</td>
<td>-0.005</td>
</tr>
<tr>
<td>lnP_social</td>
<td>0.000</td>
<td>0.005***</td>
<td>-0.002***</td>
<td>-0.003***</td>
<td>0.001*</td>
</tr>
<tr>
<td>lnP_hrus</td>
<td>-0.008***</td>
<td>-0.005**</td>
<td>0.028***</td>
<td>-0.010**</td>
<td>-0.004</td>
</tr>
<tr>
<td>lnP_other</td>
<td>(-3.57)</td>
<td>(-2.18)</td>
<td>(8.19)</td>
<td>(-3.09)</td>
<td>(-1.35)</td>
</tr>
<tr>
<td>lnC</td>
<td>-0.002</td>
<td>-0.008**</td>
<td>-0.003</td>
<td>0.025***</td>
<td>-0.016***</td>
</tr>
<tr>
<td>lnCH</td>
<td>-0.002***</td>
<td>-0.001*</td>
<td>-0.005***</td>
<td>-0.001*</td>
<td>0.009***</td>
</tr>
<tr>
<td>lnYO</td>
<td>(-4.02)</td>
<td>(-1.70)</td>
<td>(-5.15)</td>
<td>(-1.68)</td>
<td>(11.95)</td>
</tr>
<tr>
<td>lnPOP</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.004**</td>
<td>-0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>lnDENS</td>
<td>-0.002</td>
<td>-0.053***</td>
<td>0.004</td>
<td>-0.012</td>
<td>0.003</td>
</tr>
<tr>
<td>lnEL</td>
<td>(0.12)</td>
<td>(-2.85)</td>
<td>(0.14)</td>
<td>(-0.43)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>lnHERF</td>
<td>0.039***</td>
<td>-0.034***</td>
<td>0.041***</td>
<td>-0.040***</td>
<td>-0.010</td>
</tr>
<tr>
<td>lnUNEMP</td>
<td>-0.009</td>
<td>-0.058***</td>
<td>0.015</td>
<td>0.069***</td>
<td>-0.050***</td>
</tr>
<tr>
<td>lnUNEMP</td>
<td>(-6.38)</td>
<td>(-5.66)</td>
<td>(4.36)</td>
<td>(-4.48)</td>
<td>(-1.25)</td>
</tr>
<tr>
<td>lnEL</td>
<td>(-0.65)</td>
<td>(-3.37)</td>
<td>(7.55)</td>
<td>(-2.87)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>lnPOP</td>
<td>0.036***</td>
<td>-0.100***</td>
<td>-0.033</td>
<td>0.043**</td>
<td>0.025</td>
</tr>
<tr>
<td>lnDENS</td>
<td>-0.003</td>
<td>-0.001</td>
<td>-0.021***</td>
<td>-0.002</td>
<td>0.017***</td>
</tr>
<tr>
<td>lnLEF</td>
<td>(0.58)</td>
<td>(-2.37)</td>
<td>(2.94)</td>
<td>(-2.29)</td>
<td>(2.51)</td>
</tr>
<tr>
<td>lnHERF</td>
<td>0.004</td>
<td>-0.003</td>
<td>0.011***</td>
<td>-0.012***</td>
<td>0.006</td>
</tr>
<tr>
<td>lnUNEMP</td>
<td>(1.36)</td>
<td>(-1.02)</td>
<td>(2.49)</td>
<td>(-2.88)</td>
<td>(1.56)</td>
</tr>
<tr>
<td>lnUNEMP</td>
<td>-0.002</td>
<td>0.002</td>
<td>-0.016***</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>lnUNEMP</td>
<td>(-0.65)</td>
<td>(0.76)</td>
<td>(-3.86)</td>
<td>(0.41)</td>
<td>(0.95)</td>
</tr>
<tr>
<td>lnUNEMP</td>
<td>(1.13)</td>
<td>(-0.10)</td>
<td>(-0.83)</td>
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<td>(-0.17)</td>
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<tr>
<td>R²</td>
<td>0.94</td>
<td>0.91</td>
<td>0.96</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
<td>Listed variable, $\chi^2_{[17]}$</td>
<td>170.34***</td>
<td>37.82***</td>
<td>475.49***</td>
<td>670.58***</td>
<td>54.58***</td>
</tr>
<tr>
<td>Muni. effects, $\chi^2_{[272]}$</td>
<td>22.68***</td>
<td>29.01***</td>
<td>33.30***</td>
<td>20.45***</td>
<td>15.57***</td>
</tr>
<tr>
<td>Time effects, $\chi^2_{[5]}$</td>
<td>20.91***</td>
<td>4.69***</td>
<td>13.70***</td>
<td>8.44***</td>
<td>3.12***</td>
</tr>
<tr>
<td>Hansen J, p-value</td>
<td>0.11</td>
<td>0.13</td>
<td>0.01***</td>
<td>0.28</td>
<td>0.06</td>
</tr>
<tr>
<td>Homogeneity, t-value</td>
<td>0.49</td>
<td>-1.68**</td>
<td>0.79</td>
<td>-0.11</td>
<td>-1.43</td>
</tr>
<tr>
<td>No. of observations</td>
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<td>2184</td>
<td>2184</td>
<td>2184</td>
<td>2184</td>
</tr>
<tr>
<td>No. of municipalities</td>
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<td>273</td>
<td>273</td>
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</tbody>
</table>

Symmetry: $\gamma_{ij} = \gamma_{ji}$, $\chi^2_{[10]} = 2.78$; $\lambda_i = \epsilon \beta_i$, $\chi^2_{[2]} = 9.96$

Note: t-values in parentheses. The regressions in Table B1 include municipality specific effects and period specific effects. "***", "**", and * denote significance at the 1, 5 and 10 percent level, respectively.
Table B2. The Parameter Estimates from the Complete DS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln \mu$</td>
<td>-0.015**</td>
<td>0.064***</td>
<td>0.048***</td>
<td>-0.002</td>
<td>-0.032***</td>
<td>-0.041***</td>
</tr>
<tr>
<td>($\ln \mu$)$^2$</td>
<td>(1.58)</td>
<td>(8.26)</td>
<td>(3.07)</td>
<td>(-0.16)</td>
<td>(-2.70)</td>
<td>(-4.43)</td>
</tr>
<tr>
<td>$\ln P_{\text{child}}$</td>
<td>0.000***</td>
<td>-0.004***</td>
<td>-0.004***</td>
<td>-0.002**</td>
<td>0.009**</td>
<td>0.002***</td>
</tr>
<tr>
<td>$\ln P_{\text{social}}$</td>
<td>(0.80)</td>
<td>(-6.94)</td>
<td>(-3.87)</td>
<td>(-2.15)</td>
<td>(2.01)</td>
<td>(3.08)</td>
</tr>
<tr>
<td>$\ln P_{\text{edu}}$</td>
<td>0.000***</td>
<td>0.000</td>
<td>0.002***</td>
<td>0.003***</td>
<td>0.000</td>
<td>0.003***</td>
</tr>
<tr>
<td>$\ln P_{\text{infra}}$</td>
<td>(15.06)</td>
<td>(0.96)</td>
<td>(2.85)</td>
<td>(4.08)</td>
<td>(0.75)</td>
<td>(6.20)</td>
</tr>
<tr>
<td>$\ln P_{\text{other}}$</td>
<td>0.000</td>
<td>0.001***</td>
<td>0.000</td>
<td>-0.001***</td>
<td>0.001***</td>
<td>-0.000</td>
</tr>
<tr>
<td>$\ln P_{\text{CH}}$</td>
<td>(1.31)</td>
<td>(12.41)</td>
<td>(0.05)</td>
<td>(-2.45)</td>
<td>(2.79)</td>
<td>(-0.99)</td>
</tr>
<tr>
<td>$\ln P_{\text{YO}}$</td>
<td>-0.002***</td>
<td>-0.000</td>
<td>0.011***</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>$\ln P_{\text{EL}}$</td>
<td>(-3.04)</td>
<td>(-0.79)</td>
<td>(10.92)</td>
<td>(0.97)</td>
<td>(0.67)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>$\ln P_{\text{POP}}$</td>
<td>0.011***</td>
<td>-0.000</td>
<td>0.004***</td>
<td>0.015***</td>
<td>-0.002*</td>
<td>0.004***</td>
</tr>
<tr>
<td>$\ln DENS$</td>
<td>(1.80)</td>
<td>(-0.04)</td>
<td>(2.82)</td>
<td>(14.11)</td>
<td>(-1.65)</td>
<td>(4.80)</td>
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<tr>
<td>$\ln LEFT$</td>
<td>-0.000***</td>
<td>-0.000</td>
<td>-0.001***</td>
<td>0.000*</td>
<td>0.003***</td>
<td>0.000***</td>
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<tr>
<td>$\ln HERF$</td>
<td>(-3.21)</td>
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<td>(-2.64)</td>
<td>(1.77)</td>
<td>(13.04)</td>
<td>(2.86)</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.002***</td>
<td>0.001*</td>
<td>0.003***</td>
</tr>
<tr>
<td>$\ln P_{\text{CH}}$</td>
<td>(1.00)</td>
<td>(0.94)</td>
<td>(0.22)</td>
<td>(4.57)</td>
<td>(1.65)</td>
<td>(9.09)</td>
</tr>
<tr>
<td>$\ln P_{\text{YO}}$</td>
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<td>-0.007***</td>
<td>0.011***</td>
<td>-0.007***</td>
<td>-0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>$\ln P_{\text{EL}}$</td>
<td>0.005***</td>
<td>-0.011***</td>
<td>0.010***</td>
<td>0.038***</td>
<td>-0.009***</td>
<td>0.008***</td>
</tr>
<tr>
<td>$\ln P_{\text{POP}}$</td>
<td>(2.10)</td>
<td>(-4.83)</td>
<td>(2.30)</td>
<td>(9.63)</td>
<td>(-2.61)</td>
<td>(3.03)</td>
</tr>
<tr>
<td>$\ln DENS$</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.026***</td>
<td>-0.002</td>
<td>0.003</td>
<td>-0.005***</td>
</tr>
<tr>
<td>$\ln LEFT$</td>
<td>(-0.34)</td>
<td>(-0.78)</td>
<td>(7.30)</td>
<td>(-0.54)</td>
<td>(1.00)</td>
<td>(-2.47)</td>
</tr>
<tr>
<td>$\ln HERF$</td>
<td>0.010***</td>
<td>-0.019***</td>
<td>-0.004</td>
<td>0.027***</td>
<td>0.016***</td>
<td>0.018***</td>
</tr>
<tr>
<td>$\ln UNEMP$</td>
<td>(3.19)</td>
<td>(-6.12)</td>
<td>(-0.60)</td>
<td>(4.94)</td>
<td>(3.42)</td>
<td>(4.99)</td>
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<tr>
<td>$R^2$</td>
<td>0.83</td>
<td>0.89</td>
<td>0.97</td>
<td>0.94</td>
<td>0.84</td>
<td>0.88</td>
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Symmetry: $\gamma_{ij} = \gamma_{ji}$; $\chi^2_{[16]} = 4.22$; $\lambda_i = \epsilon_{ij}$; $\chi^2_{[2]} = 11.61$

Note: t-values in parentheses. The regressions in Table B2 include municipality specific effects and period specific effects. *** and ** denote significance at the 1, 5 and 10 percent level, respectively.
Table B3. The Sum of the Budget and Income Elasticities

<table>
<thead>
<tr>
<th>Commodity j</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
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</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td>0.79</td>
<td>1.04</td>
<td>0.79</td>
<td>1.01</td>
<td>2.15</td>
</tr>
<tr>
<td>(−5.03)</td>
<td>(0.32)</td>
<td>(−5.93)</td>
<td>(11.49)</td>
<td>(3.96)</td>
<td></td>
</tr>
<tr>
<td>1st quartile</td>
<td>0.81</td>
<td>1.14</td>
<td>0.78</td>
<td>1.01</td>
<td>2.21</td>
</tr>
<tr>
<td>(−5.19)</td>
<td>(2.38)</td>
<td>(−5.02)</td>
<td>(10.03)</td>
<td>(4.54)</td>
<td></td>
</tr>
<tr>
<td>2nd quartile</td>
<td>0.80</td>
<td>1.07</td>
<td>0.79</td>
<td>1.01</td>
<td>2.20</td>
</tr>
<tr>
<td>(−4.41)</td>
<td>(2.55)</td>
<td>(−5.42)</td>
<td>(10.99)</td>
<td>(3.88)</td>
<td></td>
</tr>
<tr>
<td>3rd quartile</td>
<td>0.79</td>
<td>1.02</td>
<td>0.79</td>
<td>1.01</td>
<td>2.16</td>
</tr>
<tr>
<td>(−5.83)</td>
<td>(1.04)</td>
<td>(−7.50)</td>
<td>(12.02)</td>
<td>(3.98)</td>
<td></td>
</tr>
<tr>
<td>4th quartile</td>
<td>0.76</td>
<td>0.91</td>
<td>0.81</td>
<td>1.01</td>
<td>2.05</td>
</tr>
<tr>
<td>(−6.18)</td>
<td>(−0.92)</td>
<td>(−9.45)</td>
<td>(14.37)</td>
<td>(3.87)</td>
<td></td>
</tr>
</tbody>
</table>

Income Elasticities Resulting from the Complete Demand System

<table>
<thead>
<tr>
<th>Commodity j</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td>0.82</td>
<td>-0.67</td>
<td>0.39</td>
<td>0.48</td>
<td>1.40</td>
<td>1.26</td>
</tr>
<tr>
<td>(−6.72)</td>
<td>(−2.67)</td>
<td>(−3.39)</td>
<td>(−7.06)</td>
<td>(3.02)</td>
<td>(3.14)</td>
<td></td>
</tr>
<tr>
<td>1st quartile</td>
<td>0.82</td>
<td>-0.57</td>
<td>0.45</td>
<td>0.51</td>
<td>1.36</td>
<td>1.23</td>
</tr>
<tr>
<td>(−7.68)</td>
<td>(−3.22)</td>
<td>(−5.09)</td>
<td>(−9.89)</td>
<td>(3.24)</td>
<td>(4.93)</td>
<td></td>
</tr>
<tr>
<td>2nd quartile</td>
<td>0.82</td>
<td>-0.61</td>
<td>0.41</td>
<td>0.49</td>
<td>1.38</td>
<td>1.24</td>
</tr>
<tr>
<td>(−7.07)</td>
<td>(−3.14)</td>
<td>(−3.67)</td>
<td>(−9.83)</td>
<td>(3.34)</td>
<td>(4.09)</td>
<td></td>
</tr>
<tr>
<td>3rd quartile</td>
<td>0.83</td>
<td>-0.66</td>
<td>0.39</td>
<td>0.49</td>
<td>1.41</td>
<td>1.26</td>
</tr>
<tr>
<td>(−6.92)</td>
<td>(−2.94)</td>
<td>(−3.71)</td>
<td>(−8.14)</td>
<td>(3.25)</td>
<td>(4.01)</td>
<td></td>
</tr>
<tr>
<td>4th quartile</td>
<td>0.83</td>
<td>-0.85</td>
<td>0.30</td>
<td>0.45</td>
<td>1.46</td>
<td>1.31</td>
</tr>
<tr>
<td>(−6.01)</td>
<td>(−2.20)</td>
<td>(−3.02)</td>
<td>(−5.57)</td>
<td>(2.94)</td>
<td>(2.70)</td>
<td></td>
</tr>
</tbody>
</table>

Note: t-values in parentheses. It is assumed that the budget and income elasticities are equal to one under the null hypothesis.

Appendix C: The Restricted Models

Since homogeneity in all prices ($\sum_{k=1}^{n} \gamma_{jk} = 0$) is a within-equation restriction, it is sufficient to test for it by estimating each equation separately. According to the t-statistic in Table B2, homogeneity is rejected at the five percent level for all service sectors except social services.

Given the standard Slutsky symmetry restriction on the price parameters ($\gamma_{jk} = \gamma_{kj}$), a second condition is required aiming to satisfy proportionality between the parameters associated with ln$m$ and $(lnm)^2$ (i.e. $\lambda_j = \epsilon \beta_j$, where
\(\epsilon = 0\) implies that the demand system is linear in income).\(^{12}\) The symmetry restrictions are tested by using a Wald \(\chi^2\)-test. According to the \(\chi^2\)-statistics in Tables B1-B2, all the symmetry restrictions are rejected at the five percent level.

The income and price elasticities corresponding to the restricted versions (homogeneity and Slutsky symmetry are imposed) of the models are presented in Tables C1-C2. The results imply that neither the relative size of the income (or budget) elasticities, nor the negativity of the own-price elasticities, are altered when the restrictions are imposed. Most of the cross-price elasticities have the same sign. None of the cross-price elasticities that are significantly negative in the unrestricted models become significantly positive, or vice versa, in the restricted model.

Table C1. Elasticities in the Restricted Conditional Model

A. Budget Elasticities:

<table>
<thead>
<tr>
<th>Commodity (j)</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Child})</td>
<td>0.92</td>
<td>0.20</td>
<td>0.91</td>
<td>0.98</td>
<td>1.37</td>
</tr>
<tr>
<td>(\text{Soci})</td>
<td>(-0.67)</td>
<td>(-3.42)</td>
<td>(-1.28)</td>
<td>(-0.41)</td>
<td>(1.41)</td>
</tr>
</tbody>
</table>

B. Compensated Price Elasticities:

<table>
<thead>
<tr>
<th>Commodity (k)</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Child})</td>
<td>-0.75</td>
<td>0.05</td>
<td>0.16</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>(\text{Social})</td>
<td>(-77.85)</td>
<td>(2.26)</td>
<td>(3.25)</td>
<td>(2.25)</td>
<td>(5.46)</td>
</tr>
<tr>
<td>(\text{Eld})</td>
<td>0.07</td>
<td>-0.86</td>
<td>0.20</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>(\text{Edu})</td>
<td>(3.32)</td>
<td>(-127.1)</td>
<td>(1.74)</td>
<td>(0.82)</td>
<td>(6.90)</td>
</tr>
<tr>
<td>(\text{Infra})</td>
<td>0.24</td>
<td>0.05</td>
<td>-0.60</td>
<td>0.29</td>
<td>0.36</td>
</tr>
<tr>
<td>(\text{Edu})</td>
<td>(6.92)</td>
<td>(0.58)</td>
<td>(-84.93)</td>
<td>(41.07)</td>
<td>(4.50)</td>
</tr>
<tr>
<td>(\text{Infra})</td>
<td>0.28</td>
<td>0.03</td>
<td>0.32</td>
<td>-0.57</td>
<td>0.45</td>
</tr>
<tr>
<td>(\text{Infra})</td>
<td>(6.98)</td>
<td>(0.31)</td>
<td>(40.58)</td>
<td>(-97.79)</td>
<td>(4.95)</td>
</tr>
<tr>
<td>(\text{Infra})</td>
<td>0.06</td>
<td>0.06</td>
<td>0.17</td>
<td>0.10</td>
<td>-0.80</td>
</tr>
<tr>
<td>(\text{Infra})</td>
<td>(3.60)</td>
<td>(7.49)</td>
<td>(1.83)</td>
<td>(1.00)</td>
<td>(-90.33)</td>
</tr>
</tbody>
</table>

(Continued on next page).

\(^{12}\)This restriction is required in order to satisfy the integrability conditions of demand theory (see Blundell et al., 1993).
Table C1. (Continued)

C. Uncompensated Price Elasticities:

<table>
<thead>
<tr>
<th>Commodity k</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infrastructure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>-0.86</td>
<td>0.03</td>
<td>0.05</td>
<td>0.01</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(-46.17)</td>
<td>(2.77)</td>
<td>(0.88)</td>
<td>(0.09)</td>
<td>(-3.61)</td>
</tr>
<tr>
<td>Social</td>
<td>0.02</td>
<td>-0.87</td>
<td>0.14</td>
<td>0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.73)</td>
<td>(-57.98)</td>
<td>(1.24)</td>
<td>(0.35)</td>
<td>(-1.26)</td>
</tr>
<tr>
<td>Eld</td>
<td>-0.05</td>
<td>-0.02</td>
<td>-0.88</td>
<td>-0.02</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(-5.05)</td>
<td>(-2.02)</td>
<td>(-40.00)</td>
<td>(-0.79)</td>
<td>(-7.11)</td>
</tr>
<tr>
<td>Edu</td>
<td>-0.06</td>
<td>-0.04</td>
<td>-0.01</td>
<td>-0.92</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(-5.44)</td>
<td>(-4.71)</td>
<td>(-0.40)</td>
<td>(-44.85)</td>
<td>(-4.08)</td>
</tr>
<tr>
<td>Infra</td>
<td>-0.00</td>
<td>0.04</td>
<td>0.11</td>
<td>0.04</td>
<td>-0.89</td>
</tr>
<tr>
<td></td>
<td>(-0.03)</td>
<td>(3.29)</td>
<td>(1.13)</td>
<td>(0.35)</td>
<td>(-45.69)</td>
</tr>
</tbody>
</table>

Note: t-values in parentheses. Under the null hypotheses it is assumed that the budget elasticities are equal to one and that the price elasticities are equal to zero.

Table C2. Elasticities in the Restricted Complete Model

A. Income Elasticities:

<table>
<thead>
<tr>
<th>Commodity k</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infrastructure</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.99</td>
<td>-0.36</td>
<td>0.59</td>
<td>0.66</td>
<td>1.40</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>(-0.05)</td>
<td>(-3.81)</td>
<td>(-3.20)</td>
<td>(-3.30)</td>
<td>(0.87)</td>
<td>(2.17)</td>
</tr>
</tbody>
</table>

B. Compensated Price Elasticities:

<table>
<thead>
<tr>
<th>Commodity k</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infrastructure</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>-0.82</td>
<td>0.00</td>
<td>0.09</td>
<td>0.11</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(-84.13)</td>
<td>(0.20)</td>
<td>(3.40)</td>
<td>(3.82)</td>
<td>(0.46)</td>
<td>(4.13)</td>
</tr>
<tr>
<td>Social</td>
<td>0.02</td>
<td>-0.89</td>
<td>0.21</td>
<td>0.21</td>
<td>0.01</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(1.81)</td>
<td>(-127.7)</td>
<td>(3.40)</td>
<td>(3.41)</td>
<td>(1.94)</td>
<td>(-0.68)</td>
</tr>
<tr>
<td>Eld</td>
<td>0.02</td>
<td>-0.05</td>
<td>-0.81</td>
<td>0.06</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.95)</td>
<td>(-1.55)</td>
<td>(-109.0)</td>
<td>(11.41)</td>
<td>(1.20)</td>
<td>(2.85)</td>
</tr>
<tr>
<td>Edu</td>
<td>0.05</td>
<td>-0.09</td>
<td>0.09</td>
<td>-0.83</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(2.44)</td>
<td>(-2.51)</td>
<td>(9.26)</td>
<td>(-138.6)</td>
<td>(2.37)</td>
<td>(4.64)</td>
</tr>
<tr>
<td>Infra</td>
<td>-0.00</td>
<td>-0.01</td>
<td>0.16</td>
<td>0.18</td>
<td>-0.85</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-0.20)</td>
<td>(-1.42)</td>
<td>(3.28)</td>
<td>(3.57)</td>
<td>(-85.92)</td>
<td>(3.42)</td>
</tr>
<tr>
<td>Other</td>
<td>0.02</td>
<td>-0.01</td>
<td>0.14</td>
<td>0.16</td>
<td>0.03</td>
<td>-0.89</td>
</tr>
<tr>
<td></td>
<td>(2.75)</td>
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<td>(3.35)</td>
<td>(3.71)</td>
<td>(4.23)</td>
<td>(-78.08)</td>
</tr>
</tbody>
</table>

(Continued on next page).
### Table C2. (Continued)

C. Uncompensated Price Elasticities:

<table>
<thead>
<tr>
<th>Commodity $k$</th>
<th>Child</th>
<th>Social</th>
<th>Eld</th>
<th>Edu</th>
<th>Infra</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>-0.86</td>
<td>0.01</td>
<td>0.07</td>
<td>0.09</td>
<td>-0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(-72.69)</td>
<td>(1.87)</td>
<td>(2.34)</td>
<td>(2.73)</td>
<td>(-4.55)</td>
<td>(-1.17)</td>
</tr>
<tr>
<td>Social</td>
<td>0.00</td>
<td>-0.89</td>
<td>0.20</td>
<td>0.20</td>
<td>-0.01</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(-101.7)</td>
<td>(3.15)</td>
<td>(3.16)</td>
<td>(-0.86)</td>
<td>(-2.94)</td>
</tr>
<tr>
<td>Eld</td>
<td>-0.07</td>
<td>-0.02</td>
<td>-0.87</td>
<td>0.01</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
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<td>(-62.92)</td>
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<td>(-4.87)</td>
</tr>
<tr>
<td>Edu</td>
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<td>-0.05</td>
<td>-0.00</td>
<td>-0.90</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
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<td>(-5.50)</td>
<td>(-0.26)</td>
<td>(-74.81)</td>
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<td>(-1.66)</td>
</tr>
<tr>
<td>Infra</td>
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<td>0.02</td>
<td>0.15</td>
<td>0.17</td>
<td>-0.87</td>
<td>-0.01</td>
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<td>(2.91)</td>
<td>(3.20)</td>
<td>(-71.81)</td>
<td>(-1.09)</td>
</tr>
<tr>
<td>Other</td>
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<td>0.15</td>
<td>0.00</td>
<td>-0.93</td>
</tr>
<tr>
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<td>(0.12)</td>
<td>(2.86)</td>
<td>(3.21)</td>
<td>(0.13)</td>
<td>(-69.95)</td>
</tr>
</tbody>
</table>

*Note: t-values in parentheses. Under the null hypothesis it is assumed that the income elasticities are equal to one and that the price elasticities are equal to zero.*
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