Extreme-Value Characteristics in Daily Time Series of Swedish Stock Returns*

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Abstract

The paper studies Swedish stock series using extreme-value theoretical approaches. In a univariate setting support is found for the Fréchet family of distributions for minima and maxima. Pairs of return series are found to be asymptotically independent throughout. The results render support for joint modelling based on flexible moment specifications or, e.g., copulas.

Key Words: Value-at-Risk, minimum/maximum return, crossing.

JEL Classification: C14, C22, C32, G11, G12.

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1. Introduction

This paper studies empirically the extreme-value behavior of Swedish stock returns. One intention is to characterize the type of limiting extreme-value distribution for minima and maxima of the series. A second objective is to study the dependence between extreme returns of different stocks. Both objectives have clear ties to value-at-risk and to protecting a portfolio against extreme risk.

Methodologywise we employ standard tools for the univariate analysis of individual return series (e.g., Longin, 2000, Tsay, 2002, ch. 7). We give an account of the framework in Section 2. Also in Section 2, we focus on bivariate returns and give a brief account of the recently proposed approach of Poon, Rockinger and Tawn (2002). Both the univariate and bivariate approaches are essentially based on sequences of independent and identically distributed random variables. In the univariate setting a weak dependence in the return series still yields consistent estimation. For the bivariate case, e.g., Poon et al. (2002) employ corrections for conditional heteroskedasticity.

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The included time series are defined and described in Section 3. The stocks used for the study are those that underlied standardized stock options at the Stockholm stock exchange (Stockholmsbörsen) in the spring of 2002. Byström (2001) appears to be the only study to use extreme-value theoretical approaches to study aspects of the Swedish stock market, in his case an index. In Section 4 we report the empirical results. Some concluding comments are given in the final section.

2. The Extreme-Value Approach

In this section we start by giving some of the essential steps in an extreme-value approach to the value at risk, VaR, in a univariate context. Full length accounts of this research area can be found in, e.g., Longin (2000) or in Tsay (2002, ch. 7), who builds on Longin. Later, we discuss the bivariate case which enables us to consider associations between extremes. The adopted approach builds on Poon et al. (2002).

2.1 Single Series

There is a time series of daily returns r_t of length T. Returns are here and in the sequel defined to be percent day-to-day changes in stock prices. To estimate unknown parameters and probabilities we consider sub-periods of length n of the return series. Assuming independence between returns and a common distribution F, the distribution for the smallest value r_1 among n returns has the distribution function

$$F_{r_1}(x) = 1 - [1 - F(x)]^n .$$

This holds since if $r_1 > x$ then all $r_i > x$ and the probability for this event is $1 - F_{r_1}(x) = [1 - F(x)]^n$.

In each such sub-period of length n there is a minimum value that we now denote r_{ni} with i indicating the sub-period. There is g such sub-periods and minimum values in the series, so that the total time series length is $T \geq ng$. In case $ng < T$ we drop the first few observations of the first sub-period so that all sub-periods are of equal length n.

Assume that r_{ni} follows a limiting generalized extreme-value distribution for minimum values. The density has the form (for k different from zero)

$$f(r_{ni}) = \frac{1}{\alpha}\left[1 + \frac{k(r_{ni} - \beta)}{\alpha}\right]^{1/k-1} \exp\left\{-\left[1 + \frac{k(r_{ni} - \beta)}{\alpha}\right]\right\},$$

where $1 + k(r_{ni} - \beta)/\alpha > 0$ and α, β and k are unknown parameters. The k is a shape parameter that mirrors the tail behavior of the distribution. In financial time series there appears to be some support for $k < 0$, in which case the distribution is in the Fréchet family and has negative skewness. The β is a location and α is a scale parameter.

The corresponding density for maximum values is of a related type:

$$f(r_{ni}) = \frac{1}{\alpha}\left[1 - \frac{k(r_{ni} - \beta)}{\alpha}\right]^{1/k-1} \exp\left\{-\left[1 - \frac{k(r_{ni} - \beta)}{\alpha}\right]\right\}.$$

For maximum values we expect positive skewness.

The maximum likelihood estimator maximizes the log-likelihood function

$$\ln L(\alpha, \beta, k) = \sum_{i=1}^{g} \ln f(r_{ni})$$

with respect to the unknown parameters (k, β, α). If we estimate the parameters of the minimum density that density is used, while for maximum values the maximum density is used. Using simple numerical techniques it is found in practise that estimation is both very fast and numerically stable.

For the case of minimum values we can obtain a quantile r^* corresponding to a small probability p^* from the minimum distribution corresponding to the generalized extreme-value density, above, on the form

$$r^*_{\min} = \beta - \frac{\alpha}{k}\left\{1 - [1 - \ln(1 - p^*)]^k\right\}.$$

For the maximum distribution we have

$$r^*_{\max} = \beta + \frac{\alpha}{k}\left\{1 - [1 - \ln(1 - p^*)]^k\right\}.$$

We may then use

$$p^* = \Pr(r_{ni} \leq r^*) = 1 - [1 - \Pr(r_t \leq r^*)]^n = 1 - (1 - p)^n$$

to relate the probability p of the return series to the one for minima and sub-periods, i.e. p^*. Using this relationship, for a given small probability p, the value-at-risk, VaR, of holding a long position in the asset underlying the return r_t is

$$\mathrm{VaR}^- = \beta - \frac{\alpha}{k}\left\{1 - [1 - n\ln(1 - p)]^k\right\}$$

for k different from zero. The VaR of holding a short position is of related form. The $(1-p)$th quantile of the return is

$$\mathrm{VaR}^+ = \beta + \frac{\alpha}{k}\left\{1 - [1 - n\ln(1 - p)]^k\right\}.$$

In this case p is a small probability that corresponds to the risk of holding a short position.

For multi-period VaR, Tsay (2002, ch. 7) gives $\mathrm{Var}(h) = h^{-k}\mathrm{VaR}$, where h is the time distance and k is the shape parameter of the extreme-value distribution. Obviously, there are also other approaches to obtaining VaR measures (e.g., Gourieroux and Jasiak, 2002).

One very useful aspect in the robustness sense of the extreme-value approach is that no marginal distribution needs to be specified for the $\{r_t\}$ sequence. A second benefit is that the approach remains valid even when there is weak dependence in the $\{r_t\}$ sequence.

2.2 Bivariate Series

We first briefly consider the multivariate distribution of extreme returns. The account builds on Longin (2000) and takes multivariate minimal return to be a vector of univariate minimal returns over some time period. The F is a q-dimensional limiting extreme-value distribution, if and only if (i) the univariate marginal distributions $F^1, F^2, \ldots, F^q$ are extreme-value distributions, and (ii) there is a dependence function d, which satisfies the condition

$$F(r^1, \ldots, r^q) = 1 - \left[F^1(r^1) \cdots F^q(r^q)\right]^{d(r^q - r^1, \ldots, r^q - r^{q-1})}.$$

The dependence function has to be specified. For two extremes r^i and r^j, Longin suggests the use of a measure due to Tiago de Oliviera (1973)

$$d(r^i - r^j) = \rho_{ij} \frac{\max(1, \exp(r^i - r^j))}{1 + \exp(r^j - r^i)} + (1 - \rho_{ij}).$$

The ρ measure can be obtained for minimum and maximum values and compared to direct correlations between returns. In the bivariate and multivariate cases it appears that estimation will be more complicated.

Next, we consider the steps of the bivariate methodology of Poon et al. (2002). In this, the entire time series is utilized and rather than using sub-period maxima/minima we now consider up- and down-crossing of threshold levels.

First, we standardize the bivariate returns r_{1t} and r_{2t} to unit Fréchet marginals s_{1t} and s_{2t} by the transformations

$$s_{1t} = -1/\ln F_1(r_{1t}) \qquad \text{and} \qquad s_{2t} = -1/\ln F_2(r_{2t}),$$

where F_1 and F_2 are estimated as empirical distribution functions. The dependence structure between the upper tails of s_1 and s_2 is the same as for r_1 and r_2.

The standardized variables are on a common scale and extreme events of the form $s_1 > c$ and $s_2 > c$ are equally likely. The c is a threshold for up-crossings. We say that s_1 and s_2 are perfectly dependent if $\Pr(s_1 > c | s_2 > c) = 1$, they are exactly independent when $\Pr(s_1 > c | s_2 > c) = \Pr(s_1 > c)$, which tends to zero as $c \to \infty$.

Define

$$\chi = \lim_{c \to \infty} \Pr(s_1 > c | s_2 > c), \qquad \chi \in [0, 1].$$

We have asymptotic independence if $\chi = 0$ and asymptotic dependence for $\chi > 0$. A related measure is due to Coles, Heffernan and Tawn (1999):

$$\bar{\chi} = \lim_{c \to \infty} \frac{2 \ln \Pr(s_1 > c)}{\ln \Pr(s_1 > c, s_2 > c)} - 1, \qquad \bar{\chi} \in (-1, 1].$$

For the bivariate normal case $\bar{\chi}$ equals the correlation coefficient. Otherwise, the sign of $\bar{\chi}$ corresponds to the type of association in the extremes.

To estimate $\bar{\chi}$, univariate extreme-value techniques are used. Let $z = \min(s_1, s_2)$ and sort to get $z_{(1)} < \ldots < z_{(T)}$. The estimator and its variance estimator are

$$\begin{aligned} \bar{\chi}^* &= \frac{2}{n_c} \left[\sum_{j=1}^{n_c} \ln \left(\frac{\bar{z}_{(j)}}{c} \right) \right] - 1 \\ \mathrm{Var}(\bar{\chi}^*) &= \left(\bar{\chi}^* + 1\right)^2 / n_c, \end{aligned}$$

where n_c is the number of z-values exceeding c and these z-values are labeled $\bar{z}$. If $\bar{\chi}^*$ is significantly smaller than one, we conclude that the variables are asymptotically independent and take $\chi = 0$. If we cannot reject $\bar{\chi} = 1$ then we estimate χ. The estimator and its variance estimator are

$$\begin{aligned}\hat{\chi} &= cn_c/T \\ \mathrm{Var}(\hat{\chi}) &= c^2 n_c(T - n_c)/T^3.\end{aligned}$$

A difficulty that appears to have no very satisfactory solution yet is the determination of the threshold level c. With the relatively large number of series to be analyzed in this paper we adopt a simple solution of setting c equal to the 95 percent order statistic in the $\{z_{(i)}\}$ sequence. Poon et al. (2002) found an estimate of 2 percent, which gives a smaller n_c. Longin and Solnik (2002) used a range of c between 0 to 10 percentage points away from the sample mean. By a Monte Carlo based approach they find that on average the threshold should be placed such that 4-6 percent of the observations fall above (or below for minima) the threshold.

Another recent approach to non-constant correlation in a parametric setting is to use copulas.[1] The copula connection to bivariate extremes is discussed by Coles et al. (1999).

For bivariate and multivariate cases the VaR is to be determined using

$$p = \Pr(\mathbf{a}'\mathbf{r}_t \leq v) = \Pr\left(\frac{\mathbf{a}'\mathbf{r}_t - e}{s} \leq \frac{v - e}{s}\right) = F\left(\frac{v - e}{s}\right),$$

where $\mathbf{a}$ is the allocation vector across the m returns in the vector $\mathbf{r}$. Then $v = e + sF^{-1}(p)$, where $e = \mathbf{a}'\mathrm{E}(\mathbf{r}_t)$ and $s = \mathbf{a}'\mathrm{Var}(\mathbf{r}_t)\mathbf{a}$. For, e.g., the normal distribution case this has a well-known solution (e.g., Gourieroux and Jasiak, 2001, ch. 16). In the extreme-value context, Longin (2000) gives an ad hoc solution and Poon et al. (2002) give bounds for the p probability. Using these bounds Poon et al. (2002) found that portfolio risk may be overestimated if asymptotic independence is not accounted for. For bivariate and multivariate extreme-value situations the evaluation currently requires parametric specification. We abstain from doing such assumptions and then give no full VaR results. Approximate and partial VaR measures can be obtained when we empirically find that asymptotic independence prevails. Then $\Pr(ar_1 + (1 - a)r_2 \leq v | r_1 > c, r_2 > c)$ can be numerically evaluated using tail approximations.

3. The Stock Series

All stock series for which standardized derivatives exist at the Stockholmsbörsen stock exchange in Stockholm are studied. Return is defined as the one-day relative change (in percent) in the price. The time series are given in Table 1 with average returns and standard deviations over the full time series length. We note that the mean returns are positive and significantly different from zero in most cases. Note also that there is considerable variation in time series lengths.

[1] A copula is a function C such that for known marginal distributions $F(x)$ and $G(y)$, the bivariate distribution function $H(x, y) = C[F(x), G(y)]$ is well defined. This too is emerging as a practical tool. Note, however, that it apparently requires a much more tightly specified model setup.

Table 1: Stock series with descriptive statistics for daily returns in percent, period start (year, month, day all series end at 020307) and time series length, source: Stockholmsbörsen AB).

	Period	Descriptives		
Stock series	Start	Mean	Std dev	T
ABB Ltd	990622	-0.07	2.83	683
Allgon	880527	0.17	3.92	3454
Assa Abloy	941108	0.20	2.64	1838
Astra Zeneca	990406	0.06	1.89	736
Atlas Copco A	870102	0.08	2.07	3810
Autoliv SDB	970502	0.02	2.14	1216
Avesta Polarit	010130	0.22	2.26	276
Boss Media	990624	0.21	5.70	681
Electrolux B	870102	0.05	2.09	3810
Eniro	001010	0.05	2.87	353
Ericsson B	870102	0.13	2.72	3810
Europolitan	940527	0.15	3.02	1954
Föreningssparbanken	950601	0.09	2.14	1692
Gambro B	910718	0.04	2.05	2671
H&M B	870325	0.13	2.30	3751
Holmen B	870325	0.07	2.65	3751
Investor B	880418	0.07	2.19	3488
Kinnevik B	921112	0.08	2.53	2338
MTG B	990503	0.13	3.66	717
Nokia	870320	0.17	3.22	3754
Nordea	971208	0.07	2.48	1063
Pharmacia C	000403	0.01	2.23	483
Sandvik AB	870902	0.07	1.96	3643
SCA B	870102	0.05	1.96	3809
Scania B	960401	0.03	1.95	1485
SEB A	870102	0.07	3.24	3810
Securitas	920213	0.14	2.25	2527
SHB A	870323	0.08	2.33	3755
Skandia	870102	0.08	2.72	3810
Skanska	870318	0.05	2.01	3756
SKF B	870102	0.05	2.18	3810
Song Networks	000316	-0.48	8.68	495
SSAB A	890703	0.06	2.34	3184
Stora Enso Ser R	981229	0.11	2.67	801
Tele2 B	960514	0.13	2.85	1457
Telia	000613	-0.13	3.11	437
Tieto Enator	990709	0.04	3.78	671
Trelleborg B	870324	0.06	2.42	3752
Volvo B	870102	0.05	1.98	3810
WM-data	870401	0.12	2.67	3746

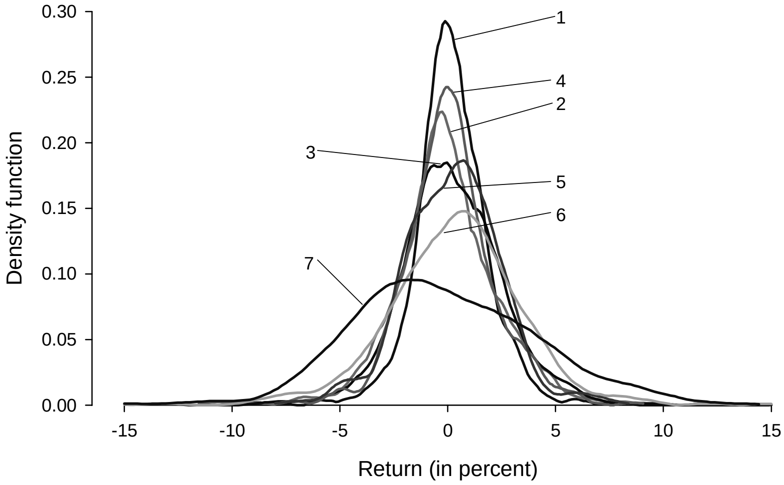

Figure 1: Estimated densities for two-year periods of Ericsson B 1987-01-01 – 2002-03-07 (kernel estimation, first sub-period is indicated by 1 and contains 810 observations, the later ones (indicated 2 to 7) 500 observations each).

We have employed the following pre-handling of the stock series: (i) splits and emissions are fully accounted for and (ii) a few missing observations in the price series are replaced by interpolated values.

4. Empirical Results

4.1 Single Series

The empirical results are based on sub-period lengths of $n = 21$ days, i.e. corresponding to one month of trading. As there is some variation in time series lengths T, the number of sub-periods g varies from series to series. Note also that all sub-periods are of equal length implying shorter total series lengths. We start by giving some rather detailed results for Ericsson B, before giving results for all other series in a more compact manner.

Figure 1 exhibits nonparametrically estimated density functions for Ericsson B in sub-periods. It appears that there is substantial variation in the shapes of the densities. First, the densities vary between positive and negative skewness. Second, the variation (spread) varies substantially. Third, the mean return fluctuates around zero. Finally, it appears that there is a temporal pattern with more negative skewness and larger spreads in later periods.

Figure 2 shows graphically the main ingredients of the extreme value approach for the Ericsson B series. Roughly, the minimum density appears to be a reflection of the maximum density. The parameter estimates are given in Table 2. The minimum density has $\hat{k} = -0.30$ and the maximum density $\hat{k} = -0.24$. Both are significantly different from zero, so that the Fréchet distribution cannot be rejected.

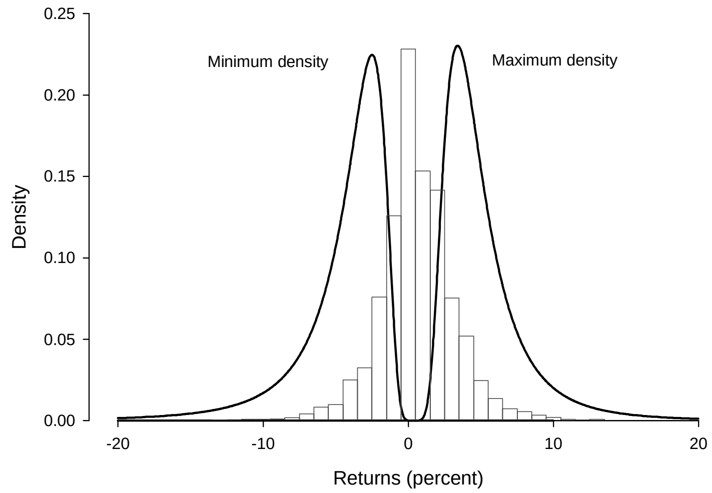

Figure 2: Histogram of full Ericsson B series, 1987-01-01 – 2002-03-07 with estimated minimum and maximum density functions.

From the estimates we may calculate the values at risk in percent as corresponding to a probability, say, $p = 0.05$:

$$\begin{aligned} \mathrm{VaR}^- &= -2.919 - \frac{1.729}{-0.296}\left\{1 - [1 - 21 \cdot \ln(1 - 0.05)]^{-0.296}\right\} = -1.7825 \\ \mathrm{VaR}^+ &= 3.666 + \frac{1.686}{-0.237}\left\{1 - [1 - 21 \cdot \ln(1 - 0.05)]^{-0.237}\right\} = 2.5344. \end{aligned}$$

For a one month horizon we get the VaR(21) measures as -4.3894 and 5.2148, respectively. For an investment of 100 000 SEK, the $\mathrm{VaR}^-(21)$ is 100000 SEK$\cdot$0.043984 = 4398 SEK and $\mathrm{VaR}^+(21) = 100000$ SEK$\cdot$0.052148 = 5215 SEK.

In Figure 3 we exhibit the minimum and maximum returns within months over successive months. It appears that there are larger extreme observations towards the end of the sequence, and that the black-Monday effect for this series is quite small.

Table 2 presents the parameter estimates of the minimum and maximum densities for each of the 40 stock series. Interestingly, we find a common result of $k \leq 0$ in all but 4 of the totally 80 cases. Hence, the Fréchet family receives support. The k estimates are significantly different from zero in close to 50 percent of the cases, and when significant this occurs for both the minimum and maximum densities. In a few instances the estimation algorithm diverged indicating a flat log-likelihood function or that $1 + k(r_{ni} - \beta)/\alpha > 0$ is violated for the minimum case. Using instead a leaner Gumbel ($k = 0$) specification gives convergence in these instances as well.

Table 2: Estimates and standard errors of generalized extreme-value distributions for minima and maxima.

	Minimum						Maximum					
	k		β		α		k		β		α	
Stock series	Est	se	Est	se	Est	se	Est	se	Est	se	Est	se
ABB	-0.285	0.222	-3.514	0.398	2.002	0.356	-0.245	0.192	3.319	0.351	1.678	0.282
Allgon	-0.233	0.083	-4.087	0.189	2.116	0.151	-0.266	0.045	5.143	0.266	2.793	0.196
Assa Abloy	-0.006	0.083	-3.622	0.150	1.555	0.137	-0.151	0.133	4.554	0.216	1.790	0.191
Astra Zeneca	-0.097	0.248	-2.636	0.206	1.136	0.223	-0.124	0.179	2.947	0.238	1.179	0.161
Atlas Copco	-0.083	0.050	-2.705	0.103	1.236	0.077	-0.201	0.066	3.135	0.117	1.380	0.091
Autoliv	-0.307	0.117	-2.704	0.194	1.173	0.151	-0.083	0.122	3.247	0.247	1.600	0.176
Avesta Polarit	-0.582	0.765	-2.907	0.232	0.795	0.303	-0.623	0.730	2.983	0.241	0.805	0.295
Boss Media	-0.288	0.176	-7.289	0.486	2.442	0.451	0.119	0.218	9.684	0.726	3.758	0.592
Electrolux	-0.131	0.068	-2.627	0.105	1.276	0.087	-0.153	0.063	2.978	0.128	1.526	0.101
Eniro[a]							-0.286	0.431	4.497	0.412	1.526	0.414
Ericsson	-0.296	0.090	-2.919	0.139	1.729	0.134	-0.237	0.085	3.666	0.138	1.686	0.125
Europolitan[a]	-0.090	0.125	-3.789	0.189	1.719	0.183						
Föreningssparbanken	-0.033	0.137	-3.065	0.158	1.195	0.114	0.056	0.093	3.621	0.176	1.481	0.165
Gambro	-0.151	0.086	-2.553	0.121	1.212	0.100	-0.243	0.087	2.804	0.142	1.363	0.113
H&M	-0.342	0.072	-2.535	0.117	1.354	0.102	-0.207	0.067	3.343	0.146	1.657	0.117
Holmen	-0.322	0.061	-3.007	0.120	1.376	0.093	-0.167	0.042	3.590	0.171	1.909	0.104
Investor	-0.267	0.086	-2.531	0.109	1.253	0.097	-0.315	0.073	2.890	0.116	1.328	0.099
Kinnevik	-0.227	0.064	-3.070	0.142	1.315	0.124	-0.153	0.062	3.882	0.181	1.605	0.112
MTG	-0.215	0.135	-4.545	0.449	2.189	0.345	-0.365	0.314	5.423	0.379	2.069	0.416
Nokia	-0.320	0.084	-3.893	0.155	1.906	0.156	-0.103	0.055	4.991	0.246	2.681	0.148
Nordea[a]	-0.053	0.179	-3.500	0.216	1.337	0.191						
Pharmacia	-0.042	0.391	-3.882	0.349	1.609	0.403	-0.179	0.192	3.335	0.305	1.154	0.259
Sandvik	-0.170	0.060	-2.465	0.101	1.181	0.007	-0.142	0.055	2.879	0.114	1.326	0.090
SCA	-0.147	0.073	-2.495	0.093	1.097	0.077	-0.137	0.054	2.987	0.115	1.347	0.094
Scania	-0.168	0.102	-2.126	0.172	1.205	0.126	-0.320	0.100	2.317	0.197	1.374	0.185
SEB	-0.355	0.063	-2.934	0.132	1.580	0.115	-0.297	0.054	3.421	0.150	1.689	0.117
Securitas	-0.036	0.086	-3.024	0.129	1.250	0.101	-0.160	0.092	3.627	0.177	1.730	0.143
SHB	-0.162	0.055	-2.634	0.115	1.259	0.079	-0.246	0.055	3.133	0.128	1.485	0.110
Skandia	-0.125	0.080	-3.198	0.138	1.635	0.110	-0.249	0.075	3.623	0.156	1.843	0.139
Skanska	-0.239	0.059	-2.366	0.097	1.114	0.074	-0.206	0.061	2.768	0.120	1.393	0.102
SKF	-0.151	0.075	-2.802	0.115	1.401	0.096	-0.093	0.078	3.426	0.118	1.408	0.092
Song Networks	-0.243	0.313	-9.992	1.291	4.688	1.158	-0.174	0.366	11.13	1.560	6.771	2.743
SSAB	-0.117	0.057	-2.876	0.125	1.332	0.083	-0.232	0.059	3.302	0.168	1.708	0.119
Stora Enso	-0.176	0.145	-3.210	0.263	1.417	0.200	-0.139	0.126	4.645	0.325	1.711	0.317
Tele2	-0.124	0.138	-3.649	0.217	1.638	0.188	-0.129	0.077	4.281	0.277	1.923	0.211
Telia	-0.218	0.387	-4.341	0.536	1.601	0.303	-0.201	0.487	4.559	0.480	1.992	0.475
Tieto Enator	-0.517	0.382	-4.697	0.411	2.122	0.540	0.037	0.346	5.315	0.512	2.923	0.677
Trelleborg	-0.297	0.081	-2.651	0.113	1.294	0.089	-0.211	0.061	3.252	0.146	1.626	0.113
Volvo	-0.217	0.056	-2.537	0.093	1.046	0.063	-0.144	0.059	2.993	0.108	1.247	0.082
WM-data	-0.118	0.041	-3.524	0.152	1.731	0.113	0.039	0.053	4.198	0.210	2.542	0.153

[a] Missing cells for nonconverged iterative estimation.

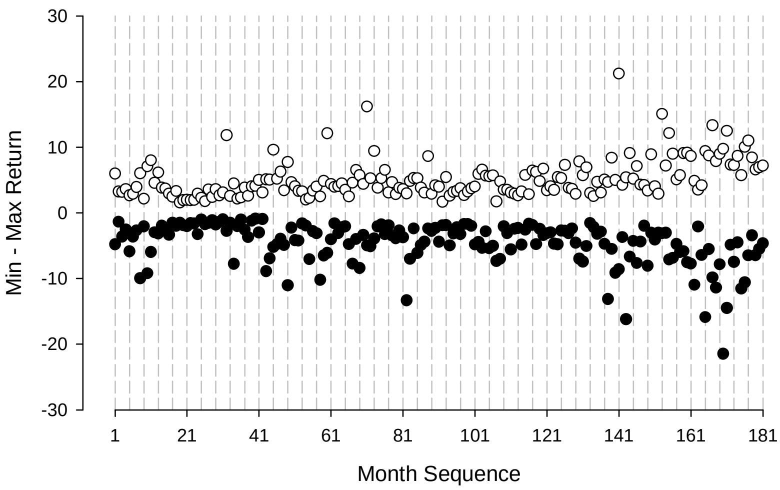

Figure 3: Minimum and maximum returns within months against months for Ericsson B, 1987-01-01 – 2002-03-07.

4.2 Bivariate Results

The examination starts with a description of the correlation structure in the 40 stock series. From the correlation structure we obtain the factor structure that may enable us to group, e.g., positively correlated stocks.

The correlations between the returns of the stock series of Table 1 are given in Table 3. The correlations are in all cases positive and usually well below 0.5. A notable stock with very small correlations with other stocks is SHB. To proceed, we next employ explorative factor analysis with Varimax rotation. By this we hope to see whether there is an underlying factor structure and a pattern in the factor loadings that may ease the interpretation of co-dependencies between return series. The use of factor analysis in financial settings is discussed in, e.g., Campbell, Lo and MacKinley (1997) and Press (1982).

For this purpose we leave out stock series shorter than 1000 observations, i.e. those with less than about four years of observations. To select the number of factors we employ Kaiser's rule. By this criterion we find six factors that explain 49.8 percent of the total variation in the resulting 30 stock return series. The first factor explains 27.8 percent of the variation in the series, while the other five factors add 3.5 – 6.5 percent each. The estimated factor loadings are given in Table 4, while Figure 4 presents the factor structure in a graphical form. For the figure we only depict factor loadings exceeding 0.5 in absolute value.

The first factor is made up of stocks for manufacturing companies. The second factor consists of telecom companies. The third appears difficult to cathegorize, though it contains three Wallenberg-sphere companies. Factor four is comprised of three banks and the sixth factor is made up solely by the remaining bank, SHB. The fifth factor is made up of two security

Table 3: Correlations between return series. Pairwise missing observations are excluded. (The orders vertically and horizontally are the same.)

Stock																		
ABB Ltd	1.0																	
Allgon	.19	1.0																
Assa Abloy	.21	.18	1.0															
Astra Zeneca	.13	.01	.10	1.0														
Atlas Copco A	.36	.17	.14	.05	1.0													
Autoliv SDB	.25	.28	.22	.16	.34	1.0												
Avesta Polarit	.16	.19	.29	.14	.21	.28	1.0											
Boss Media	.25	.25	.22	.01	.26	.20	.22	1.0										
Electrolux B	.29	.17	.17	.08	.46	.32	.31	.16	1.0									
Eniro	.25	.18	.31	.09	.25	.19	.21	.23	.25	1.0								
Europolitan B	.20	.26	.22	.03	.12	.18	.17	.30	.16	.33	1.0							
Föreningssparbanken	.28	.21	.17	.10	.29	.26	.32	.17	.30	.27	.16	1.0						
Gambro B	.15	.14	.11	.13	.28	.18	.25	.10	.25	.11	.14	.22	1.0					
H & M B	.19	.17	.24	.08	.20	.28	.27	.21	.24	.21	.26	.29	.19	1.0				
Holmen B	.21	.13	.13	.10	.29	.25	.27	.16	.27	.22	.16	.25	.27	.15	1.0			
Investor B	.39	.16	.25	.25	.27	.37	.27	.27	.29	.32	.31	.36	.41	.22	.28	1.0		
Kinnevik B	.31	.24	.20	.03	.25	.26	.24	.34	.27	.25	.31	.29	.21	.26	.25	.33	1.0	
MTG B	.21	.26	.28	.00	.21	.20	.20	.36	.17	.30	.37	.20	.08	.33	.17	.32	.43	1.0
Nokia	.32	.25	.23	.00	.17	.26	.17	.39	.17	.31	.37	.28	.16	.19	.17	.24	.30	.40
Nordea	.29	.20	.24	.16	.26	.21	.24	.15	.26	.30	.21	.43	.20	.24	.22	.35	.27	.13
Pharmacia C	.13	.05	.06	.41	.07	.03	.11	.05	.09	.06	.02	.07	.13	.06	.12	.10	.01	.08
Sandvik AB	.38	.13	.18	.07	.40	.33	.35	.20	.30	.28	.16	.35	.28	.18	.26	.29	.26	.17
SCA B	.20	.18	.14	.09	.43	.30	.28	.14	.44	.20	.17	.25	.32	.20	.38	.30	.24	.15
Scania B	.18	.22	.16	.06	.28	.24	.17	.16	.25	.24	.15	.20	.11	.21	.30	.43	.24	.15
SEB A	.33	.14	.20	.11	.23	.29	.34	.23	.24	.27	.19	.49	.17	.19	.17	.20	.19	.26
Securitas	.12	.13	.28	.11	.15	.20	.19	.15	.18	.21	.17	.15	.12	.19	.08	.20	.19	.17
SHB A	.05	.04	.00	.01	.01	.03	.14	.05	.04	.09	.02	.03	.02	.06	.03	.01	.01	.08
SKF B	.28	.19	.16	.02	.46	.30	.24	.25	.42	.25	.18	.25	.25	.21	.24	.23	.25	.24
Skandia	.36	.23	.26	.16	.38	.36	.28	.36	.39	.28	.32	.36	.25	.28	.23	.31	.37	.33
Skanska	.23	.18	.20	.11	.28	.27	.27	.14	.33	.31	.18	.32	.24	.20	.25	.25	.23	.09
Song Networks	.26	.29	.25	.05	.30	.26	.17	.35	.21	.17	.38	.28	.18	.21	.19	.34	.33	.33
SSAB A	.26	.18	.18	.09	.40	.28	.30	.12	.34	.29	.15	.22	.29	.19	.30	.28	.24	.15
Stora Enso Ser R	.24	.13	.11	.08	.40	.24	.24	.17	.28	.15	.03	.17	.18	.18	.47	.21	.15	.10
Tele 2 B	.31	.39	.31	.04	.28	.29	.18	.36	.30	.31	.52	.28	.19	.35	.28	.47	.50	.46
Telia	.31	.22	.31	.08	.33	.22	.21	.28	.31	.27	.50	.37	.17	.39	.24	.41	.46	.36
Tieto Enator	.19	.26	.26	.02	.14	.15	.23	.28	.11	.23	.37	.17	.09	.23	.12	.33	.34	.36
Trelleborg B	.20	.19	.18	.14	.29	.27	.30	.20	.35	.21	.16	.28	.32	.21	.26	.28	.25	.19
WM-data	.26	.17	.23	.01	.15	.26	.23	.23	.16	.28	.31	.23	.12	.17	.08	.17	.26	.34
Volvo B	.32	.16	.17	.10	.40	.34	.21	.21	.44	.22	.23	.25	.30	.22	.27	.31	.28	.20
Ericsson B	.32	.29	.25	.00	.36	.33	.26	.41	.37	.29	.39	.26	.32	.27	.26	.35	.37	.41

Table 3: Continued.

Stock																						
Nokia	1.0																					
Nordea	.26	1.0																				
Pharmacia	.01	.14	1.0																			
Sandvik	.13	.30	.09	1.0																		
SCA	.17	.23	.09	.27	1.0																	
Scania	.25	.20	.07	.28	.26	1.0																
SEB	.12	.47	.05	.16	.25	.24	1.0															
Securitas	.18	.22	.02	.16	.14	.16	.10	1.0														
SHB	.03	.07	.02	.00	.01	.02	.01	.03	1.0													
SKF B	.19	.25	.02	.30	.42	.27	.24	.14	.04	1.0												
Skandia	.29	.34	.04	.26	.36	.29	.30	.27	.01	.36	1.0											
Skanska	.14	.27	.10	.23	.31	.23	.25	.13	.01	.30	.30	1.0										
Song Netw	.39	.18	.05	.25	.23	.28	.33	.17	.02	.22	.40	.16	1.0									
SSAB	.18	.23	.10	.28	.39	.29	.20	.18	.02	.38	.27	.30	.24	1.0								
Stora Enso	.15	.16	.12	.39	.45	.11	.18	.08	.01	.33	.17	.22	.15	.31	1.0							
Tele 2	.50	.31	.02	.30	.27	.28	.35	.28	.02	.31	.49	.29	.40	.26	.12	1.0						
Telia	.45	.39	.08	.33	.29	.23	.37	.26	.08	.29	.46	.24	.31	.24	.24	.51	1.0					
Tieto Enat	.43	.14	.03	.14	.13	.14	.20	.18	.03	.18	.31	.12	.23	.14	.11	.38	.31	1.0				
Trelleborg	.14	.23	.07	.24	.35	.30	.21	.15	.06	.32	.27	.37	.23	.36	.22	.30	.24	.13	1.0			
WM-data	.23	.26	.02	.13	.13	.20	.11	.17	.01	.13	.25	.13	.35	.14	.09	.41	.39	.51	.08	1.0		
Volvo	.17	.31	.05	.26	.43	.28	.28	.18	.03	.40	.40	.31	.20	.37	.17	.33	.30	.17	.34	.15	1.0	
Ericsson	.40	.28	.01	.25	.36	.27	.25	.20	.00	.35	.46	.27	.42	.31	.17	.51	.42	.45	.29	.27	.41	1.0

oriented companies.

The given correlations are based on the two basic assumptions that (i) they are constant over time, and (ii) they are constant across the ranges of the returns. Both assumptions have been criticized in the literature. For example, Tsay (2000, ch. 9) discusses time-varying correlation models. The correlations between minima, between maxima and between minima and maxima in return series are illustrated in Longin (2000). As we abstain from employing more tightly model-based approaches as, e.g., GARCH models for volatility, we also abstain from using time-varying correlation models here. In a practical sense employing GARCH and variable correlation models is, however, perfectly feasible.

Results for bivariate associations between extremum values are given for the pairs of return series that are included in Figure 4 and also in the category Most traded on the A-list of Stockholmsbörsen. Europolitan, Kinnevik and Tele 2 are therefore not retained. Figure 5 illustrates the type of outcome such an exercise may produce by the pair Ericsson B and SEB A. The ρ estimate (cf. Tiago de Oliviera, 1973) between maxima of Ericsson B and SEB B is 0.499, while the Pearson correlation is 0.32, etc. The correlation between all returns is 0.25. We find indications for a variable correlation across the range of returns.

Figure 6 gives the corresponding test statistics $\bar{\chi}$, which is closely related to correlations, for varying cut-offs in maximum as well as minimum directions. In no case is there a significant test result, and we conclude that the series are asymptotically independent. Note that the figure is produced under no corrections for conditional heteroskedasticity. For the combination Ericsson down and SEB up the test outcome is 0.238 with an upper limit 0.977 of its confidence interval. This is relative close to one, which would have indicated

Table 4: Varimax rotated loadings in factor analysis. Series shorter than 1000 return observations are excluded. Loadings smaller than 0.2 in absolute value are excluded.

	Component					
	1	2	3	4	5	6
Allgon		0.48				
Assa Abloy		0.22			0.65	
Atlas Copco A	0.69					
Autoliv SDB	0.39	0.22			0.31	
Electrolux B	0.68					
Ericsson B	0.43	0.59				
Europolitan B		0.73				
Föreningssparbanken			0.24	0.74		
Gambro B	0.26		0.58			
H & M B		0.28		0.20	0.30	-0.22
Holmen B	0.27		0.57			
Investor B		0.29	0.68			
Kinnevik B		0.52	0.27			
Nokia		0.66				
Nordea				0.67		
Sandvik AB	0.30		0.42	0.23	0.24	
SCA B	0.65		0.27			
Scania B			0.51		0.20	
SEB A	0.21			0.79		
Securitas					0.75	
SHB A						0.89
SKF	0.69					
Skandia	0.47	0.42		0.24	0.22	
Skanska	0.39		0.26	0.29		
SSAB A	0.54		0.34			
Tele2 B		0.75	0.20			
Trelleborg B	0.41		0.43			
WM-data		0.48			0.31	
Volvo B	0.62					

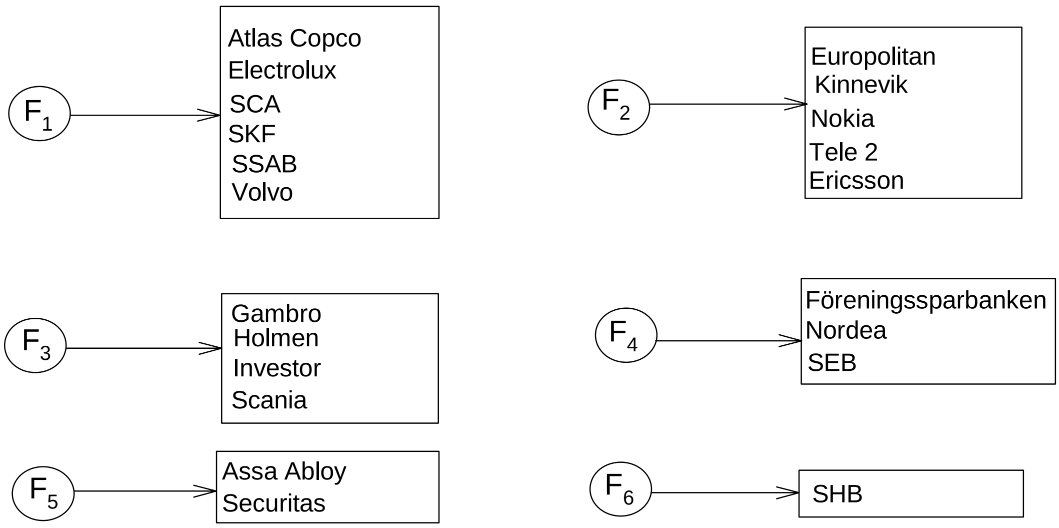

Figure 4: Factor loading structure (loadings exceeding 0.5 in absolute value).

asymptotic dependence.

Table 5 gives the estimates of $\bar{\chi}$ for the four quadrants indicated in Figure 6 for all series. In all cases $\bar{\chi}^*$ is significantly smaller than one, and hence the variables are taken to be asymptotically independent. We may then also take $\chi = 0$. The results of Poon et al. (2002) indicate that heteroskedasticity filtered $\bar{\chi}^*$ estimates are smaller than the reported unfiltered ones. This indicates that the reported $\bar{\chi}^*$ are rather overestimates than underestimates and therefore gives added supported for the asymptotic independence finding.

The table indicates by an asterix when $\bar{\chi}^*$ is significantly different from zero. The result of more likely crashes than upswings as indicated by Longin and Solnik (2001) and others from a smaller measure in the Max/Max quadrant than in the Min/Min does not receive a coherent support. The overall ratio between the $\bar{\chi}^*$s for upswings and crashes is not significantly different from one. Obviously, there are some differences between individual stocks. Size ranking $\bar{\chi}^*$ over the four quadrants and then studying the distribution across stocks reveals almost uniform distributions with roughly 0.25 proportions for ranking within quadrants as well as between quadrants. In addition, the signs of $\bar{\chi}^*$ are in most cases positive though of rather small size.

5. Conclusions

We find strong support for the Fréchet family for the minima and maxima of the univariate stock return series. Based on this distribution value-at-risk and related measures can be obtained without specifying a model directly for the return series.

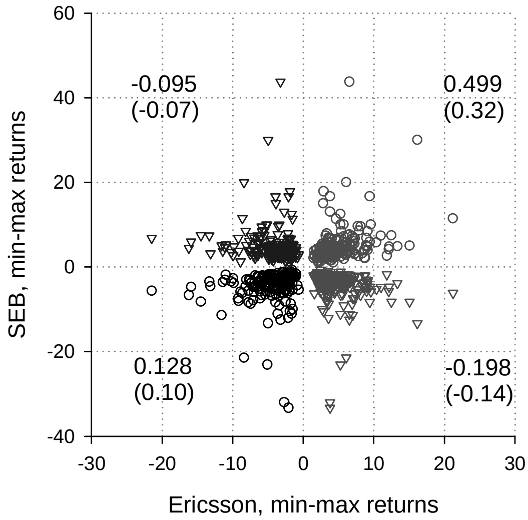

Figure 5: Relationships between extremes of Ericsson B and SEB A using monthly minima and maxima. The first number of each quadrant is the ρ ad hoc estimate of Longin and Tiago de Oliviera and the one in parenthesis is the conventional correlation coefficient.

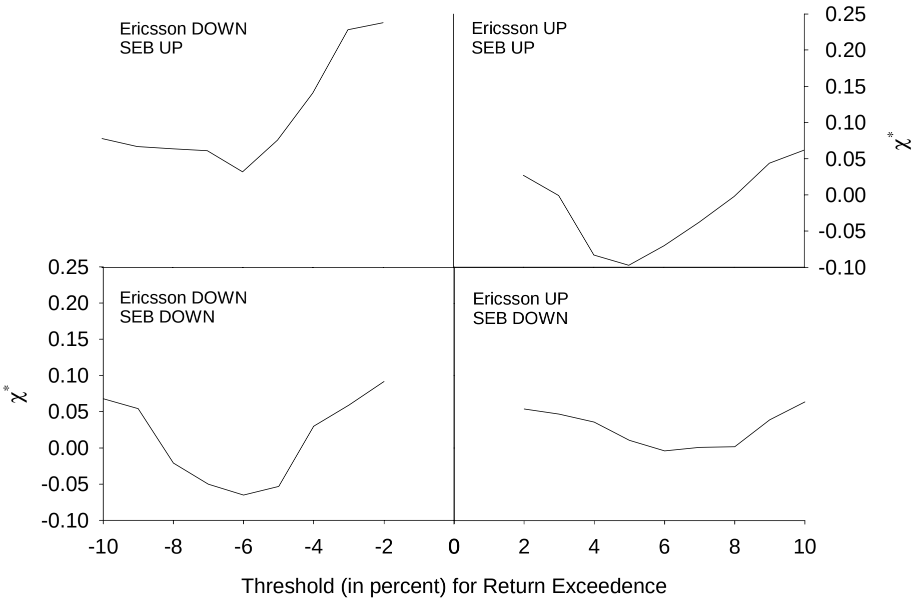

Figure 6: Test statistic $\bar{\chi}^*$ between extreme returns in Ericsson B and SEB A against threshold in percent. First quadrant corresponds to up-crossings in both series, the third to down-crossings in both, etc.

Table 5: Estimates $\bar{\chi}^*$ for pairs of series. Asterix indicates significant difference from zero at the 0.05 level.

	Max/Max	Max/Min	Min/Max	Min/Min	n
Ericsson vs					
SEB	-0.097	0.011	0.076	-0.053	3810
Atlas Copco	0.270 *	0.349 *	0.192 *	0.298 *	3810
Electorlux	0.176 *	0.258 *	0.223 *	0.104	3810
SCA	-0.007	0.042	0.068	-0.099	3809
SKF	0.036	0.038	0.149	0.094	3810
SSAB	0.121	0.127	0.119	0.150	3184
Volvo	0.194 *	0.025	-0.045	0.243 *	3810
Nokia	0.264 *	0.187 *	0.182 *	0.211 *	3754
Gambro	0.245 *	-0.013	-0.071	0.079	2671
Holmen	0.052	-0.011	-0.136	0.076	3751
Investor	0.121	0.108	0.096	0.128	3488
Scania	0.049	0.034	0.041	-0.053	1485
Föreningssparbanken	0.078	0.058	-0.042	0.045	1692
Nordea	0.031	-0.027	0.321	0.112	1063
Assa Abloy	0.168	0.100	0.213	0.091	1838
Securitas	-0.050	-0.092	-0.088	-0.060	2527
SHB	0.156	0.081	0.162	0.152	3755
SEB vs					
Atlas Copco	0.029	0.029	0.030	0.184 *	3810
Electrolux	0.059	-0.002	0.042	-0.025	3810
SCA	0.025	0.280 *	0.214 *	0.159	3809
SKF	-0.030	-0.022	-0.116	-0.065	3810
SSAB	0.059	-0.055	-0.066	-0.041	3184
Volvo	0.014	-0.123	-0.084	0.017	3810
Nokia	0.078	0.092	0.168 *	0.004	3754
Gambro	0.382 *	0.270 *	0.126	0.324 *	2671
Holmen	-0.089	-0.021	0.043	0.099	3751
Investor	0.042	0.051	0.149	-0.004	3488
Scania	0.051	0.008	-0.062	0.100	1485
Föreningssparbanken	-0.105	0.150	0.146	0.036	1692
Nordea	-0.046	-0.047	-0.135	-0.125	1063
Assa Abloy	0.086	0.088	0.233	0.012	1838
Securitas	-0.092	0.241 *	0.148	-0.024	2527
SHB	-0.099	0.049	0.155	-0.131	3755

Table 5: Continued.

	Max/Max	Max/Min	Min/Max	Min/Min	n
Atlas Copco vs					
Electrolux	0.071	-0.027	0.173 *	0.091	3810
SCA	0.129	-0.083	-0.006	0.038	3809
SKF	0.145	0.178 *	0.077	-0.017	3810
SSAB	0.140	0.003	-0.018	0.080	3184
Volvo	0.063	-0.027	-0.026	0.035	3810
Nokia	-0.064	0.253 *	0.160	-0.049	3754
Gambro	-0.038	-0.058	-0.099	-0.053	2671
Holmen	0.109	0.090	0.125	0.025	3751
Investor	-0.005	0.172	0.132	0.004	3488
Scania	0.258	-0.097	0.028	0.052	1485
Föreningssparbanken	0.160	0.000	0.032	-0.028	1692
Nordea	-0.101	-0.057	0.062	0.027	1063
Assa Abloy	0.016	0.257	0.039	-0.063	1838
Securitas	-0.013	0.038	0.006	0.014	2527
SHB	-0.110	0.031	0.059	0.002	3755
Electrolux vs					
SCA	-0.069	-0.008	0.101	0.062	3809
SKF	0.058	0.039	0.017	-0.021	3810
SSAB	-0.016	0.023	0.052	-0.001	3184
Volvo	0.221 *	0.156	0.101	0.256 *	3810
Nokia	0.081	0.118	0.119	0.106	3754
Gambro	-0.003	0.102	0.189	0.057	2671
Holmen	0.155	0.007	0.025	0.095	3751
Investor	-0.024	0.122	0.042	0.008	3488
Scania	0.174	0.051	-0.048	-0.095	1485
Föreningssparbanken	-0.075	-0.059	0.078	0.035	1692
Nordea	-0.016	-0.018	-0.036	0.011	1063
Assa Abloy	0.215	0.264 *	0.063	0.437 *	1838
Securitas	0.043	0.161	0.117	0.206	2527
SHB	0.166	0.167	0.258 *	0.068	3755
SCA vs					
SKF	0.069	0.193 *	0.164	0.070	3810
SSAB	-0.126	0.278 *	0.177	-0.068	3184
Volvo	-0.078	0.048	-0.068	-0.006	3810
Nokia	0.091	-0.004	0.057	0.144	3754
Gambro	0.179	0.302 *	0.007	0.043	2671
Holmen	0.178 *	0.048	0.092	0.027	3751
Investor	0.118	0.134	0.093	0.179 *	3488
Scania	0.337 *	0.045	0.168	0.192	1485
Föreningssparbanken	0.341 *	0.278 *	0.344 *	0.267	1692
Nordea	0.005	-0.096	-0.100	0.107	1063
Assa Abloy	-0.038	-0.033	0.029	-0.061	1838
Securitas	0.132	-0.010	0.091	0.109	2527
SHB	0.094	-0.101	-0.012	-0.034	3755

Table 5: Continued.

	Max/Max	Max/Min	Min/Max	Min/Min	n
SKF vs					
SSAB	0.096	0.111	0.169	0.119	3184
Volvo	0.054	0.102	-0.011	-0.108	3810
Nokia	0.070	-0.009	0.142	0.059	3754
Gambro	-0.038	0.063	-0.089	0.148	2671
Holmen	0.065	0.056	0.046	0.091	3751
Investor	0.126	0.040	0.095	0.085	3488
Scania	0.062	0.173	0.077	-0.026	1485
Föreningssparbanken	0.087	0.052	0.276 *	0.251	1692
Nordea	0.031	-0.033	0.092	-0.130	1063
Assa Abloy	0.031	0.003	-0.047	0.130	1838
Securitas	0.034	0.108	0.052	0.054	2527
SHB	0.079	0.016	0.025	0.118	3755
SSAB vs					
Volvo	0.241 *	0.126	0.127	-0.124	3810
Nokia	-0.203 *	-0.225 *	-0.154 *	-0.073	3754
Gambro	0.156	0.142	0.096	0.067	2671
Holmen	0.357 *	-0.046	0.167	0.073	3751
Investor	0.067	0.064	0.074	0.037	3488
Scania	0.217	0.050	0.106	0.051	1485
Föreningssparbanken	0.252	0.172	-0.023	-0.097	1692
Nordea	-0.060	-0.098	0.418 *	0.234	1063
Assa Abloy	0.252	0.163	0.029	0.095	1838
Securitas	-0.018	0.134	0.166	0.130	2527
SHB	-0.086	-0.034	-0.048	0.088	3755
Volvo vs					
Nokia	0.083	0.119	0.093	0.189 *	3754
Gambro	-0.078	0.099	0.067	-0.100	2671
Holmen	0.123	-0.023	0.058	0.141	3751
Investor	0.155	-0.024	-0.002	0.236 *	3488
Scania	0.043	0.007	0.052	0.004	1485
Föreningssparbanken	0.024	-0.273 *	-0.199 *	0.008	1692
Nordea	0.020	0.078	-0.011	0.113	1063
Assa Abloy	0.342 *	-0.045	0.080	0.294 *	1838
Securitas	0.061	0.168	0.307 *	0.030	2527
SHB	0.092	0.101	0.063	0.068	3755
Nokia vs					
Gambro	0.119	0.048	-0.067	0.099	2671
Holmen	-0.051	-0.058	-0.052	-0.037	3751
Investor	0.080	0.078	0.022	0.057	3488
Scania	0.208	0.414 *	0.477 *	0.044	1485
Föreningssparbanken	-0.034	0.262	0.181	0.231	1692
Nordea	-0.124	0.263	0.165	0.164	1063
Assa Abloy	0.324 *	0.224	0.169	0.225	1838
Securitas	0.202	0.148	0.104	0.136	2527
SHB	0.002	0.039	0.103	0.011	3755

Table 5: Continued.

	Max/Max	Max/Min	Min/Max	Min/Min	n
Gambro vs					
Holmen	0.308 *	0.214 *	0.440 *	0.321 *	3751
Investor	0.109	0.091	0.374 *	0.341 *	3488
Scania	-0.182	0.057	0.042	-0.052	1485
Föreningssparbanken	0.011	0.132	0.208	0.236	1692
Nordea	0.257	0.125	0.033	0.049	1063
Assa Abloy	0.103	-0.006	0.152	-0.016	1838
Securitas	0.053	0.137	-0.017	0.167	2527
SHB	0.157	0.120	0.195 *	-0.048	3755
Holmen vs					
Investor	0.199 *	0.053	-0.013	0.252 *	3488
Scania	-0.059	-0.082	0.029	0.078	1485
Föreningssparbanken	0.148	0.173	0.073	0.113	1692
Nordea	0.216	-0.092	-0.095	0.141	1063
Assa Abloy	0.283 *	0.088	0.146	0.343 *	1838
Securitas	-0.016	0.094	0.014	-0.003	2527
SHB	0.112	0.309 *	0.307 *	0.129	3755
Investor vs					
Scania	0.020	0.051	0.185	0.269	1485
Föreningssparbanken	-0.140	0.059	0.015	-0.021	1692
Nordea	-0.075	-0.096	0.124	0.033	1063
Assa Abloy	0.220	0.028	-0.018	0.363 *	1838
Securitas	0.243 *	0.038	0.104	0.199	2527
SHB	0.086	0.050	0.114	0.231 *	3755
Scania vs					
Föreningssparbanken	-0.118	-0.048	-0.115	0.063	1692
Nordea	0.113	0.028	0.134	0.261	1063
Assa Abloy	-0.063	-0.108	0.133	0.016	1838
Securitas	0.103	-0.060	0.016	0.252 *	2527
SHB	0.093	0.048	0.135	0.043	3755
Föreningssparbanken vs					
Nordea	0.174	0.390 *	0.394 *	0.204	1063
Assa Abloy	-0.124	-0.052	-0.163	-0.052	1838
Securitas	0.140	0.063	0.099	0.147	2527
SHB	0.129	0.010	0.058	0.013	3755
Nordea vs					
Assa Abloy	0.087	0.001	0.088	0.010	1838
Securitas	0.208	0.226 *	0.221 *	0.198	2527
SHB	0.111	0.284 *	0.100	0.266 *	3755
Assa Abloy vs					
Securitas	0.238 *	0.278 *	0.446 *	0.053	2527
SHB	0.088	0.010	-0.008	0.056	3755
Securitas vs					
SHB	0.047	0.103	0.129	0.111	3755

There are significant (Pearson) correlations between a number of the pairs of return series. The results from the testing against asymptotic independence indicate that the correlations vanish for large returns. The latter is based on truncated distributions while the former is not. Longin and Solnik (2001) discuss the case of a correlated bivariate normal variable and give some reasons for this type of disparity in results. The finding that series are frequently asymptotically independent has an interesting financial implication. If series are asymptotically independent portfolio risk may be overestimated.

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