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AN ANALYSIS OF THE STABILIZING AND WELFARE EFFECTS OF INTERVENTION IN SPOT AND FUTURES MARKETS

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ABSTRACT

This paper analyzes the effects of three alternative rules on the long-run distributions of both the spot and futures prices in a single commodity market, in which the key behavioral relationships are derived from the optimizing behavior of producers and speculators. The rules considered include: (i) leaning against the wind in the spot market; (ii) utility maximizing speculative behavior by the stabilization authority in the futures market; (iii) leaning against the wind in the futures market. Since the underlying model is sufficiently complex to preclude analytical solutions, the analysis makes extensive use of simulation methods. As a general proposition we find that intervention in the futures market is not as effective in stabilizing either the spot price of the futures price as is intervention in the spot market. Indeed, Rule (iii), while stabilizing the futures price may actually destabilize the spot price. Furthermore, the analogous type of rule undertaken in the spot market will always stabilize the futures price to a greater degree than it does the spot price. The welfare implications of these rules are also discussed. Our analysis shows how these can generate rather different distributions of welfare gains, including the overall benefits.

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1. INTRODUCTION

The theory of commodity price stabilization has grown into a voluminous literature. Many of the early studies analyze the question in terms of a buffer stock stabilization scheme. The typical approach is to determine the allocation of the welfare benefits to different groups in the economy following the introduction of such a scheme, with the stabilization authorities continually intervening so as to ensure the exact stabilization of the market price at some appropriate level (usually taken to be the arithmetic mean).[1/]

Of course the maintenance of a buffer stock scheme which stabilizes prices perfectly is extremely expensive and accordingly is not very practicable. In reality some sort of partial price stabilization scheme is more likely to be adopted. In the literature, two alternative aspects have been considered. The first is the intervention in the commodity market by means of some specified intervention rule.[2/] The second is the consideration of alternative institutional arrangements, such as futures markets, and the role they may play in stabilizing the spot price.[3/] Until recently, these latter models have suffered from an important deficiency. Specifically, they ignored the fact that any change in the environment--such as the introduction of a futures market--alters the structure of the model and this needs to be taken into account in assessing the effects of the institutional change on the equilibrium behavior of the market. To do this requires the analysis to be developed from the underlying microeconomic optimizing behavior.

In this paper, we analyze the effects of intervention by a stabilization authority in both the spot and futures market. The effects on the properties of both the spot and futures markets are considered. We also investigate the effects of such action on the welfare of the various agents in the economy. The model we employ is an extension of one developed by Turnovsky (1983) and refined by Turnovsky and Campbell (1985).[4/] In this model, the key behavioral relationships are derived from the optimizing behavior of producers and speculators. As a result of this behavior the coefficients of the relevant demand and supply functions become endogenous and change with the degree of intervention by the authority.

Three kinds of intervention rules are considered. The first of these postulates the stabilization authority to intervene in the spot market in accordance with a rule of leaning against the wind; i.e., it buys the commodity when the price is low, relative to its equilibrium, and sells otherwise. The second rule assumes that the authority enters the futures market as a private speculator, taking a long or short position, depending upon whether the futures price exceeds or is less than the expected spot price. The final rule is one of leaning against the wind in the futures market; i.e., buying futures contracts when their price is low and selling when they are high. Such a rule was suggested some years ago by Houthakker (1967).

The model underlying the analysis is sufficiently complex to render analytical solution to be infeasible. We therefore analyze the questions using simulation methods. While these techniques can yield only specific conclusions, relevant to the specific chosen parameter values, when conducted over comprehensive sets of parameter values, as we do here, they do provide an overall picture of what the likely effects of the various stabilization rules will be. They also have the advantage of indicating the likely orders of magnitudes of these effects and providing some insight as to the crucial parameters upon which these depend. Given the complexity of the model, this type of sensitivity analysis proves to be intractable, without resorting to numerical solutions.

The remainder of the paper is as follows. Section 2 discusses the model, with its solution being given in Section 3. The welfare measures are reported in Section 4. Section 5 discusses the simulation procedures. The effects of the three rules upon the distribution of the spot and futures prices together with their welfare effects are considered in Sections 6-8. The final section contains the general conclusions to emerge from our analysis.

2. REVIEW OF FRAMEWORK

This section summarizes the basic framework, developed in greater detail by Turnovsky (1983) and Turnovsky and Campbell (1985). There are four groups of participants in the market: (a) firms; (b) private speculators; (c) consumers; (d) the public stabilization authority. The behavior of each group is considered in turn and the corresponding aggregate market relationships are then derived.

A. Producers

We assume that there are n_p identical producers. The representative firm is assumed to be perfectly competitive and to produce its output subject to a quadratic cost function. In addition, at time t-1, when the production decision must be made, the firm can enter a contract to deliver a fixed specified quantity of output at an agreed contract price at time t.[5/] The remainder is sold on the spot market at whatever the random price may turn out to be. Its profit in period t, π^p_{it}, is therefore given by

$$\pi^p_{it} = P_t[y_{it} - z_{i,t-1}] + P^f_{t-1} z_{i,t-1} - \frac{1}{2} c\bar{y}^2_{it} \qquad (1)$$

where

$\bar{y}_{it}$ = planned output, chosen by the representative firm, and upon which costs are incurred,[6/]

y_{it} = actual output of the firm,

$z_{i,t-1}$ = firm's net position in futures contracts entered into at time t-1,

P_t = spot price of output at time t, taken as parameterically given to the firm,

P^f_{t-1} = one period futures price prevailing at time t-1, for delivery at time t.

If $z_{i,t-1} > 0$ the firm is selling futures contracts, while $z_{i,t-1} < 0$ denotes a purchase of futures contracts.

The firm is assumed to make its production decisions for time t at t-1, before the actual spot price P_t is known. Because of random fluctuations in production decisions, assumed to be beyond the control of the firm, actual and planned outputs are related by[7/]

$$y_{it} = \bar{y}_{it} + v_{it} \qquad (2)$$

where v_{it} is an additive random variable, having zero mean and finite variance. Combining equations (1) and (2), the profit of the representative firm is

$$\pi^P_{it} = P_t[\bar{y}_{it} + v_{it} - z_{i,t-1}] + P^f_{t-1}\, z_{i,t-1} - \frac{1}{2}\, c\bar{y}^2_{it} \qquad (1')$$

To preserve linearity of the model, we shall assume that the firm maximizes the following one-period function of expected profit and its variance

$$V^P_{it} \equiv E_{t-1}(\pi^P_{it}) - \frac{1}{2}\, \alpha\ \text{var}_{t-1}(\pi^P_{t-1}) \qquad (3)$$

where

$E_{t-1}(\pi^P_{t-1})$ = conditional expectation of profit for time t, formed at time t-1,

$\text{var}_{t-1}(\pi^P_{t-1}) = E_{t-1}[\pi^P_{it} - E_{t-1}(\pi^P_{it})]^2$ = conditional variance of profit for time t, formed at time t-1.

Under certain conditions, the parameter α describes the coefficient of absolute risk aversion. For a risk averse producer $\alpha > 0$, while $\alpha = 0$ corresponds to risk neutrality.[8/]

From (1') we drive

$$E_{t-1}(\pi^P_{it}) = P^*_{t,t-1}[\bar{y}_{it} - z_{i,t-1}] + E_{t-1}(P_t\, v_{it}) + P^f_{t-1}\, z_{i,t-1} - \frac{1}{2}\, c\bar{y}^2_{it} \qquad (4a)$$

$$\text{var}_{t-1}(\pi^P_t) = \sigma^2_p(t,t-1)[\bar{y}_{it} - z_{i,t-1}]^2 + \text{var}_{t-1}(P_t\, v_{it})$$
$$+ 2(\bar{y}_{it} - z_{i,t-1})\text{cov}_{t-1}(P_t, P_t v_{it}) \qquad (4b)$$

where var_{t-1}, cov_{t-1} denote the conditional variance and covariance, respectively, while $P^*_{t,t-1}$, $\sigma^2_p(t,t-1)$, denote the one period conditional mean and variance of the spot price for time t, formed at time t-1. Substituting these two expressions into (3) and maximizing V^p_{it} with respect to $\bar{y}_{it}$, $z_{i,t-1}$, we derive the following expressions for the optimal planned output and position in the futures market.

$$\bar{y}_{it} = \frac{P^f_{t-1}}{c} \tag{5a}$$

$$z_{i,t-1} = \frac{P^f_{t-1} - P^*_{t,t-1} + \alpha \, \mathrm{cov}_{t-1}(P_t, P_t v_{it})}{\alpha \sigma^2_p(t,t-1)} + \frac{P^f_{t-1}}{c} \tag{5b}$$

The implications of these two pairs of equations for producers have been discussed elsewhere.[9/]

B. Speculators

There are n_s identical speculators. The representative, denoted by j, holds a certain quantity of the inventories of the commodity in anticipation of price changes. We shall let $i_{j,t-1}$ denote the net position in the commodity by the speculator entered at time t-1. If $i_{j,t-1} > 0$, the speculator holds positive stocks of the commodity for speculative purposes, while $i_{j,t-1} < 0$ denotes that he is holding the commodity short. In addition, we assume he can speculate in the purchase and sales of futures contracts. We shall let $x_{j,t-1}$ denote the net position of the representative speculator in the futures market entered at time t-1, with $x_{j,t-1} > 0$ denoting sales and $x_{j,t-1} < 0$ denoting purchases of futures contracts, respectively. The profit of the representative speculator over the period (t-1, t) is postulated to be

$$\pi^s_{jt} \equiv i_{j,t-1}(P_t - P_{t-1}) + x_{j,t-1}(P^f_{t-1} - P_t) - \frac{1}{2} d i^2_{j,t-1} \tag{6}$$

where the quadratic term denotes the costs associated with trading in inventories.[10/] While the assumption of quadratic costs is a gross simplification, introduced to preserve linearity, the requirement $d > 0$ is necessary for a well defined inventory demand function, in the presence of a futures market, to exist.

Analogous to firms, the objective function of the representative speculator is to maximize

$$V^s_{jt} \equiv i_{j,t-1}(P^*_{t,t-1} - P_{t-1}) + x_{j,t-1}(P^f_{t-1} - P^*_{t,t-1})$$
$$- \frac{1}{2} di^2_{j,t-1} - \frac{1}{2} \beta\sigma^2_p(t,t-1)(i_{j,t-1} - \phi x_{j,t-1})^2 \qquad (7)$$

The parameter β measures the degree of risk aversion and need bear no particular relation to α. Maximizing (7) with respect to $i_{j,t-1}$, $x_{j,t-1}$ yields the following solutions for speculators

$$i_{j,t-1} = \frac{1}{d}(P^f_{t-1} - P_{t-1}) \qquad (8a)$$

$$x_{j,t-1} = \frac{1}{d}(P^f_{t-1} - P_{t-1}) + \frac{1}{\beta\sigma^2_p(t,t-1)}(P^f_{t-1} - P^*_{t,t-1}) \qquad (8b)$$

These equations are parallel to (5a) and (5b) derived above for the firm and have been discussed previously.11/

The assumption of a single period decision horizon is adopted to preserve simplicity. However, given the specifications of the profit functions in (1) and (6), (which involve only the current values of the decision variables), identical demand functions are obtained with a multiperiod objective function.12/

C. Aggregate Market Relationships

The agents collectively trade in the final goods market and in the futures market. These shall therefore be considered in turn.

The third group of market participants, consumers, are assumed to purchase only final goods. It is not necessary to derive their demand functions from underlying utility maximization. Instead, it suffices simply to postulate some aggregate demand function, D_t, which most conveniently can be taken to be linear

$$D_t = A - aP_t + u_t \qquad (9)$$

Summing the optimal production plans (5), over the identical firms leads to the aggregate supply function

$$S_t = n_p(\bar{y}_{it} + v_{it})$$
$$= \frac{n_p}{c} P^f_{t-1} + n_p v_{it} \qquad (10)$$

Since the n_p firms are assumed to be identical they also are assumed to contribute equally to the aggregate supply disturbance, v_t, so that

$$n_p \, v_{it} = v_t \tag{11}$$

Furthermore, for notational convenience letting c/n_p be denoted by c, leads to the aggregate supply function

$$S_t = \frac{1}{c} P^f_{t-1} + v_t \tag{12}$$

The aggregate demand and aggregate supply disturbances are assumed to be additive, normally independently distributed over time, to have zero means and finite covariances, and to be uncorrelated

$$E(u_t) = E(v_t) = 0 \tag{13a}$$

$$E(u_t^2) = \sigma_u^2 \; ; \; E(v_t^2) = \sigma_v^2 \; ; \; E(u_t v_t) = 0 \tag{13b}$$

Likewise, summing over the demand functions for the individual speculators leads to the aggregate inventory demand function I_{t-1},

$$I_{t-1} = \frac{n_s}{d} (P^f_{t-1} - P_{t-1})$$

$$= \frac{1}{d} (P^f_{t-1} - P_{t-1}) \tag{14}$$

where we have redefined d/n_s as d.

The specification of the output market is completed by the introduction of the market clearing condition

$$D_t + I_t + G_t = S_t + I_{t-1} \tag{15}$$

where G_t denotes the position taken by the stabilization authority in the market. If $G_t > 0$, the authority is a net purchaser of the commodity, adding to its inventory; if $G_t < 0$, it is a net seller, running down its inventory.

Aggregating over the representative firm's position in the futures market, (5b), leads to the following aggregate net supply function for futures contracts by firms

$$Z_{t-1} = \frac{n_p (P^f_{t-1} - P^*_{t,t-1}) + \alpha \, cov_{t-1}(P_t, P_t v_{it})}{\alpha \sigma_p^2(t,t-1)} + \frac{n_p}{c} P^f_{t-1} \tag{16}$$

where Z_{t-1} represents the aggregate. In the rational expectations equilibrium we consider, the current spot price deviates from what was previously anticipated by an amount which is linearly proportional to the aggregate demand and supply disturbances. Thus we may write

$$P_t = P^*_{t,t-1} + \rho_1 u_t + \rho_2 v_t \tag{17}$$

where ρ_1, ρ_2, are determined below. Using (11), (13) and (17), as well as the fact that u_t, v_t are normal and uncorrelated, we find that

$$\operatorname{cov}_{t-1}(P_t, P_t v_{it}) = \frac{\rho_2 P^*_{t,t-1} \sigma_v^2}{n_p} \tag{18}$$

Also, it is evident from (26a) below that the probability distribution of P_t is stationary so that the one period variance $\sigma_p^2(t,t-1)$ is independent of t and shall simply be noted by $\sigma_p^2(1)$. Thus substituting (18) into (16) and redefining α/n_p as α, we may write (16) as

$$Z_{t-1} = \frac{1}{\alpha\sigma_p^2(1)} [P^f_{t-1} - P^*_{t,t-1}(1-\alpha\rho_2\sigma_v^2)] + \frac{P^f_{t-1}}{c} \tag{19a}$$

Likewise, summing (8b) over the futures positions for individual speculators, leads to the aggregate position

$$X_{t-1} = \frac{1}{d} (P^f_{t-1} - P_{t-1}) + \frac{1}{\beta\sigma_p^2(1)}(P^f_{t-1} - P^*_{t,t-1}) \tag{19b}$$

where analogously we have defined d and β.

Equilibrium in the futures market is described by

$$Z_{t-1} + X_{t-1} = H_{t-1} \tag{20}$$

where H_{t-1} denotes the net position taken by the stabilization authority in the futures market. If $H_{t-1} > 0$, it is taking a long position; if $H_{t-1} < 0$ it is selling the contracts short.

Substituting the relevant demand and supply functions into the goods market and futures market equilibrium conditions yields the following pair of stochastic difference equations in the spot and futures prices

$$A - aP_t + \frac{1}{d} (P^f_t - P_t) + G_t = \frac{1}{c} P^f_{t-1} + \frac{1}{d} (P^f_{t-1} - P_{t-1}) - e_t \tag{21a}$$

$$\frac{1}{c} P^f_{t-1} + \frac{1}{d} (P^f_{t-1} - P_{t-1}) + \frac{k}{\sigma_p^2(1)} (P^f_{t-1} - P^*_{t,t-1}) + \frac{\rho_2 \sigma_v^2}{\sigma_p^2(1)} P^*_{t,t-1} = H_{t-1} \qquad (21b)$$

where

$$k \equiv (\frac{1}{\alpha} + \frac{1}{\beta})$$

$$e_t \equiv u_t - v_t$$

$$E(e_t) = 0 \ , \ \sigma_e^2 = \sigma_u^2 + \sigma_v^2$$

The solutions for P_t and P^f_t depend upon the specific forms for the intervention rules for spot and futures market intervention and to these rules we now turn.

D. Specification of Intervention Rules

The following three intervention rules are considered:13/

Rule 1: $$G_t = -\lambda(P_t - \overline{P}) \qquad (22a)$$

This rule asserts that the stabilization authority buys the commodity when the current spot price is below its long-run equilibrium and sells when this situation is reversed. It describes a type of "leaning against the wind" policy commonly encountered in the stabilization literature in general.14/ The case of a buffer stock stabilization scheme, when the intervention of the authority maintains P_t at $\overline{P}$, is obtained by letting $\lambda \rightarrow \infty$.15/

Rule 2: $$H_{t-1} = \nu(P^f_{t-1} - P^*_{t,t-1}) \qquad \nu > 0 \qquad (22b)$$

Rule 2 is analogous to the demand function for futures contracts by a pure private speculator. This form of intervention therefore assumes that the stabilization authority is behaving as if it were a pure private utility maximizing speculator.

Rule 3: $$H_{t-1} = -\mu(P^f_{t-1} - \overline{P}^f) \qquad \mu > 0 \qquad (22c)$$

The form of intervention suggested by Rule 3 is the analogue to Rule 1 applied to the futures market. It asserts that the stabilization authority sells futures contracts when their price is above the long run mean and buys them when they are below the mean. This is a formalization of the type of rule suggested by Houthakker (1967).16/

3. SOLUTION FOR SPOT AND FUTURES PRICES

Since the objective is to assess the impacts of the various intervention schemes on the market and its participants, as a benchmark, we begin with the case where there is no intervention by the authority in either the spot or futures markets. Setting $G_t = H_{t-1} = 0$ in (21a) and (21b), we obtain

$$A - aP_t + \frac{1}{d}(P^f_t - P_t) = \frac{1}{c}P^f_{t-1} + \frac{1}{d}(P^f_{t-1} - P_{t-1}) - e_t \qquad (21a')$$

$$\frac{1}{c}P^f_{t-1} + \frac{1}{d}(P^f_{t-1} - P_{t-1}) + \frac{k}{\sigma^2_p(1)}(P^f_{t-1} - P^*_{t,t-1}) + \frac{\rho_2\sigma^2_v}{\sigma^2_p(1)}P^*_{t,t-1} = 0 \qquad (21b')$$

We define the long-run average spot price, attained when expectations are realized ($P^*_{t+1,t} = P^*_{t,t-1} = P_t = P_{t-1}$) by $\overline{P}$ and the long-run futures price by $\overline{P}^f$. It follows from (21a') and (21b') that

$$\overline{P} = \frac{A}{a_1 + b} \qquad (23a)$$

$$\overline{P}^f = \left(\frac{\frac{k}{\sigma^2_p(1)} - \frac{\rho_2\sigma^2_v}{\sigma^2_p(1)} + \frac{1}{d}}{\frac{k}{\sigma^2_p(1)} + \frac{1}{c} + \frac{1}{d}}\right)\overline{P} \qquad (23b)$$

where for notational convenience we let

$$a_1 \equiv a + \frac{1/cd}{\frac{k}{\sigma^2_p(1)} + \frac{1}{c} + \frac{1}{d}} \qquad (24a)$$

$$b \equiv \frac{k/c\sigma^2_p(1)}{\frac{k}{\sigma^2_p(1)} + \frac{1}{c} + \frac{1}{d}} - \frac{d}{c}\phi \qquad (24b)$$

$$\phi = \frac{\rho_2\sigma^2_v/d\sigma^2_p(1)}{\frac{k}{\sigma^2_p(1)} + \frac{1}{c} + \frac{1}{d}} \qquad (24c)$$

Observe from (23b) that in the absence of supply disturbances, the long-run average futures price is in general below the long-run average spot price. This is necessary in order to induce producers to hedge permanently part of their output. In the presence of supply disturbances, however, the negative correlation between them and the spot price reduces the incentive for firms to hedge by selling futures contracts, thereby reducing their supply and raising the futures price. Indeed, it is now possible for the equilibrium futures price to exceed the spot price. The long-run bias will disappear if either $\alpha \to 0$, $\beta \to 0$.[17/]

Next, we define the following variables in deviation form, $p_t \equiv P_t - \overline{P}$, $p^*_{t+1,t} \equiv P^*_{t+1,t} - \overline{P}$, $p^f_t \equiv P^f_t - \overline{P}^f$. Substituting for P^f_{t-1} from the futures market equilibrium condition, the following difference equation is obtained for the spot price, expressed in deviation form

$$-a_1 p_t + \omega(p^*_{t+1,t} - p_t) - \phi(p^*_{t+1,t} - p^*_{t,t-1}) = bp^*_{t,t-1} + \omega(p^*_{t,t-1} - p_{t-1}) - e_t \tag{25}$$

where a_1, b and ϕ are defined in (24a), (24b), and (24c), and

$$\omega = \frac{k/d\sigma_p^2(1)}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d}} \tag{24d}$$

Equation (25) describes the behavior of the current market clearing price in terms of its conditional expectations. Using standard solution procedures, the stable solution for p_t (i.e., the rational expectations equilibrium) is given by the stochastic difference equation[18/]

$$p_t = rp_{t-1} + \frac{e_t}{a_1 + \omega(1-r) + \phi r} \tag{26a}$$

where r is the smaller root (which is real and lies in the range $0 < r < 1$) of the quadratic equation

$$\omega(1-r)^2 + \phi r(1-r) = (a_1+b)r \tag{26b}$$

Taking conditional expectations of (26a), and subtracting, we can immediately show that ρ_1, ρ_2 appearing in (17) are defined by

$$-\rho_1 = \rho_2 = \frac{-1}{a_1 + \omega(1-r) + \phi r} \equiv \rho$$

The one period and asymptotic variances of the spot price are therefore given by

$$\sigma_p^2(1) = \frac{\sigma_e^2}{[a_1 + \omega(1-r) + \phi r]^2} = \rho^2\sigma_e^2 \tag{27a}$$

$$\sigma_p^2 = \frac{\sigma_e^2}{(1-r^2)[a_1 + \omega(1-r) + \phi r]^2} = \frac{\rho^2 \sigma_e^2}{1-r^2} \tag{27b}$$

where r is the smaller root of (26b) and a_1, b, ω and ϕ are given by (24a)-(24d).

From the expressions (24a)-(24d) it is seen that the parameters a_1, b, ϕ, ω are endogenous functions of the one period spot price variance $\sigma_p^2(1)$, and therefore of σ_p^2 and r. As a consequence, equations (26b) and (27b) define a pair of highly nonlinear relationships between σ_p^2 and r. As pointed out in a related context by McCafferty and Driskill (1980), this nonlinearity is the cause of possible nonuniqueness and nonexistence problems pertaining to rational expectations equilibria. That is, when we take the definitions of a_1, b, ϕ and ω, given by (24a)-(24d) into account, it is possible that there is in fact no value of r lying in the range $0 < r < 1$; or alternatively, there may be a multiplicity of such roots. This question is considered further below.

Using the steady state relationship (23b) to write (21b') in deviation form, and taking expectations of (25), the current futures price can be expressed in terms of the past spot price as

$$P_{t-1}^f = \left[\frac{r\left(\frac{k}{\sigma_p^2(1)} - \frac{\rho\sigma_v^2}{\sigma_p^2(1)}\right) + \frac{1}{d}}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d}}\right] P_{t-1} \equiv \eta P_{t-1}$$

Thus given (25), the evolution of P_{t-1}^f is determined. From this it follows that the asymptotic variance of the futures price, σ_f^2, is

$$\sigma_f^2 = \eta^2 \sigma_p^2$$

and a similar relationship holds between the corresponding one period variances. Consequently, in the case of demand disturbances $\eta^2 < 1$ and the futures price has less variance than the spot price, both in the short run and in the long run. In the case of supply disturbances, however, it is possible for $\eta^2 > 1$, in which case the futures price may actually exhibit more variability.

The solution for the spot and futures prices, derived in the absence of intervention, may thus be summarized as follows[19/]

$$\overline{P} = \frac{A}{a_1 + b} \tag{28a}$$

$$\overline{P}^f = \left(\frac{\dfrac{k}{\sigma_p^2(1)} - \dfrac{\rho\sigma_v^2}{\sigma_p^2(1)} + \dfrac{1}{d}}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}} \right) \overline{P} \tag{28b}$$

$$\sigma_p^2 = \frac{\sigma_p^2(1)}{1 - r^2} \tag{28c}$$

$$\sigma_p^2(1) = \rho^2 \sigma_e^2 \tag{28d}$$

$$\sigma_f^2 = \left(\frac{r\left(\dfrac{k}{\sigma_p^2(1)} - \dfrac{\rho\sigma_v^2}{\sigma_p^2(1)}\right) + \dfrac{1}{d}}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}} \right)^2 \sigma_p^2 \tag{28e}$$

$$\rho = - \frac{1}{a_1 + \omega(1-r) + \phi r} \tag{28f}$$

$$\omega(1-r)^2 + \phi r(1-r) = (a_1+b_1)r \tag{28g}$$

$$a_1 = a + \frac{1/cd}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}} \tag{28h}$$

$$b = \frac{\dfrac{k}{c\sigma_p^2(1)}}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}} - \frac{d}{c}\,\phi \tag{28i}$$

$$\omega = \frac{\dfrac{k}{d\sigma_p^2(1)}}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}} \tag{28j}$$

$$\phi = \frac{\dfrac{\rho\sigma_v^2}{d\sigma_p^2(1)}}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}} \tag{28k}$$

These 11 equations determine between them the solutions for the 11 quantities $\overline{P}$, $\overline{P}^f$, σ_p^2, $\sigma_p^2(1)$, σ_f^2, r, ρ, a_1, b, ω, and ϕ. It is possible, by substitution, to reduce the number of variables, although for expositional purposes it is convenient to lay out the structure of the solution in full.[20/]

We turn now to the solutions obtained for the three alternative forms of intervention rules. It turns out that the forms of the solution in each case can be expressed in terms of relatively minor adjustments to the basic solution.

Rule 1

Substituting (22a) into the spot market clearing condition, the solution consists of the following equations: (28b), (28c), (28d), (28e), (28f), (28g), (28i), (28j), (28k), together with

$$\overline{P} = \frac{A}{a_1 - \lambda + b} \qquad (28a')$$

$$a_1 = a + \lambda + \frac{1/cd}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d}} \qquad (28h')$$

which again determine the 11 endogenous variables. In deriving this solution, the critical thing to observe is that this form of intervention leads to a transitional dynamic equation in the spot price, identical to that obtained in the absence of intervention ((26) above), with the difference being that the coefficient a (the slope of the demand function) is increased to $(a + \lambda)$; see (28h'). Combining (28a') with (28h') and likewise (28a) with (28g), we see that in either case, the steady-state solution for $\overline{P}$ can be written as

$$\overline{P} = \frac{A}{a + \frac{1/cd}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d}} + b}$$

In particular, $\overline{P}$ does not depend upon the intervention parameter λ through the effect of the latter on a, as during the transitional path. The mean spot and futures prices do, however, depend indirectly on λ via the effect which operates through the one period variance of the spot price $\sigma_p^2(1)$.

Rule 2

Substituting (22b) into the futures market clearing condition (21b) we find that this form of futures market intervention is equivalent to replacing the coefficient $k/\sigma_p^2(1)$, which describes the degree of private speculation, by $[\frac{k}{\sigma_p^2(1)} + \nu]$. It is therefore equivalent to a reduction in the degree of risk aversion of the private sector. The entire solution is of the same form as (28a)-(28i), the only difference being that $k/\sigma_p^2(1)$ is now replaced by $k/\sigma_p^2(1) + \nu$. In this case, the mean spot and future prices are now directly influenced by the intervention parameter ν, as well as indirectly through the variance $\sigma_p^2(1)$, as in Rule 1.

Rule 3

Substituting the third rule (22c) into the futures market equilibrium condition (21b), it turns out that the modifications introduced by this rule to the basic solution (28a)-(28i) are somewhat more substantial than in the other cases. We now obtain

$$\bar{P} = \frac{A}{a + \dfrac{\dfrac{1}{c}\left[\dfrac{k}{\sigma_p^2(1)} - \dfrac{\rho\sigma_v^2}{\sigma_p^2(1)} + \dfrac{1}{d}\right]}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}}} \qquad (28a'')$$

$$\bar{P}^f = \left(\frac{\dfrac{k}{\sigma_p^2(1)} - \dfrac{\rho\sigma_v^2}{\sigma_p^2(1)} + \dfrac{1}{d}}{\dfrac{k}{\sigma_p^2(1)} + \dfrac{1}{c} + \dfrac{1}{d}}\right)\bar{P} \qquad (28b')$$

$$\sigma_p^2 = \frac{\sigma_p^2(1)}{1-r^2} \qquad (28c)$$

$$\sigma_p^2(1) = \rho^2\sigma_e^2 \qquad (28d)$$

$$\sigma_f^2 = \left(\frac{r\left(\frac{k}{\sigma_p^2(1)} - \frac{\rho\sigma_v^2}{\sigma_p^2(1)}\right) + \frac{1}{d}}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d} + \mu} \right)^2 \sigma_p^2 \tag{28e'}$$

$$\rho = -\frac{1}{a_1 + \omega(1-r) + \phi r} \tag{28f}$$

$$\omega(1-r)^2 + \phi r(1-r) + \psi = (a_1+b_1)r \tag{28g'}$$

$$a_1 = a + \frac{\frac{1}{d}\left(\frac{1}{c} + \mu\right)}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d} + \mu} \tag{28h''}$$

$$b = \frac{\frac{k}{c\,\sigma_p^2(1)}}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d} + \mu} - \frac{d}{c}\phi \tag{28i'}$$

$$\omega = \frac{\frac{k}{d\sigma_p^2(1)}}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d} + \mu} \tag{28j'}$$

$$\phi = \frac{\frac{\rho\sigma_v^2}{d\sigma_p^2(1)}}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d} + \mu} \tag{28k'}$$

$$\psi = \frac{\frac{\mu}{d}}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d} + \mu} \tag{28l}$$

In contrast to the analogous rule for spot market intervention, Rule 1, this form of intervention in the futures market has direct effects on all three parameters (a_1,b,ω). While an increase in μ raises a_1, which can be shown to be a stabilizing effect on the spot price, it also reduces b and ω, and these changes can be shown to be destabilizing influences. Also, the increase in μ can be shown to be equivalent to an increase in a and therefore to be stabilizing. In addition, there are the indirect effects of these changes operating through $\sigma_p^2(1)$.

4. WELFARE MEASURES

In order to assess the distribution of welfare gains or losses arising from government intervention, it is necessary to introduce welfare criteria for the respective agents in the market. These include the three private agents (firms, speculators, and consumers) as well as the profits accruing to the stabilization authority. The traditional approach in the stabilization literature has been to evaluate welfare gains and losses in terms of changes in producer's and consumer's surplus. The limitations of these measures are well known, and while it can be shown that the qualitative implications they yield are generally acceptable, it is preferable to carry out the welfare analysis in terms of the underlying functions.

The underlying welfare maximizing approach of the present study allows us to move away from surplus measures, at least in the cases of firms and speculators, to welfare measures based on means and variances of the profits accruing to the respective groups. As we have simply assumed an aggregate demand function, without resorting to underlying utility maximization, we have retained the traditional consumer's surplus measure as an indicator of welfare gains or losses to consumers. As we shall see below, it is impossible to make any kind of general qualitative assessment of the welfare gains, without resorting to numerical methods. The analytical complexities, present in the price variance comparisons, are simply compounded.

We turn now to the expressions for the welfare gains for the three groups in the economy. These are evaluated at long-run equilibria, by taking asymptotic expectations of the welfare expressions. In the case of producers and speculators, the derivations of the resulting expressions are tedious and are not reported.[21/]

A. Producers

The welfare function for the representative firm is given by (4). The analogous expression for total firms is therefore

$$V_t^p = E_{t-1}(\Pi_t^p) - \frac{1}{2}\alpha\, \mathrm{var}_{t-1}(\Pi_t^p) \tag{29}$$

where $E_{t-1}(\Pi_t^p)$, $\mathrm{var}_{t-1}(\Pi_t^p)$, denote one period means and variance of aggregate producer profits. One issue associated with the use of (29) for the purposes of making welfare comparisons with other agents in the economy is the fact that embodied in the expression $\mathrm{var}_{t-1}(\Pi_t^p)$ are terms involving fourth moments (and squares of the second moments) of the distribution of the spot price. These are not present in the consumer surplus measure, which is analogous to expected profits. The measure (29) thus attributes greater weight to stability thereby perhaps giving a distorted comparison with consumer welfare, as measured by consumer surplus. To obtain the most complete picture of overall welfare effects, it is useful to consider the effects of the futures market on:

(i) the asymptotic expected profit of the firm, $E(\Pi^p)$;

(ii) the asymptotic one-period variance of profit of firms, $\mathrm{var}\ \Pi^p(1)$;

(iii) the asymptotic welfare of firms as defined by the asymptotic expectation of the utility function, $E(V^p)$.

The resulting welfare expressions are

$$E[\Pi^p(I)] = \frac{(\bar{P}^f(I)^2 + (rx_1 + x_2)^2\sigma_p^2(I)}{2c} + \frac{[\bar{P}^f(I) - \bar{P}(I)]^2 + [r(x_1-1) + x_2]^2\sigma_p^2(I)}{\alpha\sigma_p^2(1;I)} - \frac{\sigma_v^2}{a_1 + \omega(1-r)} \tag{30a}$$

$$\frac{-\sigma_v^2}{\sigma_p^2(1;I)[a_1 + \omega(1-r) + \phi r]}\left[[\bar{P}^f(I) - \bar{P}(I)]\bar{P}(I) + r[r(x_1-1) + x_2]\sigma_p^2(I)\right]$$

$$\mathrm{Var}[\Pi^p(1;I)] = \frac{[\bar{P}^f(I) - \bar{P}(I)]^2 + [r(x_1-1) + x_2]^2\sigma_p^2(I)}{\alpha^2\sigma_p^2(1;I)} + 2\sigma_v^2\sigma_p^2(I)(1-r^2) \tag{30b}$$

$$E[V^p(I)] = E[\Pi^p(I)] - \frac{1}{2}\alpha \, Var[\Pi^p(1;I)] \tag{30c}$$

where I = 1, 2, 3 refers to the three intervention rules and I = 0 denotes no intervention.[22/] The expressions for P(I), $\bar{P}^f$, σ_p^2, $\sigma_p^2(1)$, σ_f^2, r, a_1, ω, φ, are derived from (28). In the case of no intervention or intervention by Rule 1, x_1 and x_2 are given by

$$x_1 = \frac{\frac{k}{\sigma_p^2(1)} - \frac{\rho\sigma_v^2}{\sigma_p^2(1)}}{\frac{k}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d}} \; ; \; x_2 = \frac{\frac{1}{d}}{\frac{1}{\sigma_p^2(1)} + \frac{1}{c} + \frac{1}{d}} \tag{31}$$

In the case of Rule 2, x_1 and x_2 are given by (31) with $k/\sigma_p^2(1)$ being replaced by $k/\sigma_p^2(1) + \nu$. In the case of Rule 3, the numerators in (31) are unchanged; the denominators are now replaced by $(k/\sigma_p^2(1) + 1/c + 1/d + \mu)$.

The gains or losses to firms are obtained by comparing (30) in the case where there is no intervention with the corresponding expressions obtained under the three forms of intervention rules. The long run impacts of intervention on these welfare measures are therefore

$$\Delta Z = Z(I) - Z(0) \qquad Z = E(\Pi^p)\ ,\ Var\ \Pi^p(1)\ ,\ E(V^p)$$
$$I = 1,\ 2,\ 3$$

B. Speculators

The procedure for deriving the welfare gains or losses to speculators is directly analogous to that for firms. The welfare function of the representative speculator is given by (7) with an analogous expression applying in the aggregate. Again it is useful to focus on (i) asymptotic expected speculative profits; (ii) asymptotic one-period variance of speculative profits; (iii) asymptotic welfare as specified by the utility function. These expressions are:

$$E[\Pi^s(I)] = \left[\frac{1}{2d} + \frac{1}{\beta\sigma_p^2(1;I)}\right](\bar{P}^f(I) - \bar{P}(I))^2 + \left[\frac{(rx_1 + x_2 - 1)^2}{2d} + \frac{[r(x_1-1) + x_2]^2}{\beta\sigma_p^2(1;I)}\right]\sigma_p^2(I) \tag{32a}$$

$$\text{var}[\Pi^s(1;I)] = \frac{[\overline{P}^f(I) - \overline{P}(I)]^2 + [r(x_1-1) + x_2]^2\sigma_p^2(I)}{\beta^2\sigma_p^2(1;I)} \tag{32b}$$

$$E[V^s(I)] = E[\Pi^s(I)] - \frac{1}{2}\beta\ \text{var}[\Pi^s(1;I)] \tag{32c}$$

The long-run expected profits of the futures markets on these quantities are therefore

$$\Delta Z \equiv Z(F) - Z(N);\ Z = E[\Pi^s],\ \text{var}[\Pi^s(1)],\ V^s \tag{33}$$

C. Consumers

As noted above, when considering the gains or losses received by consumers through the introduction of a futures market, we resort to the traditional surplus measure. The gain in consumer surplus (CS) from intervention in the futures market is given by

$$\int_{P_t(I)}^{P_t(0)} D(P')dP'$$

With a linear demand function, and taking expected values, the expected gains to consumers from the introduction of a futures market are

$$\Delta E(CS) = \frac{1}{2} E\{[P_t(0) - P_t(I)][D(P_t(0)) + D(P_t(I))]\}\ . \tag{34}$$

Using the solution (25) for the spot price in deviation form, (34) can be expressed as

$$\Delta E(CS) = \frac{1}{2}[\overline{P}(0) - \overline{P}(I)][2A - a(\overline{P}(0) + \overline{P}(I))]$$

$$+ \frac{1}{2} a[\sigma_p^2(I) - \sigma_p^2(0)] + \sigma_u^2\left[\left(\frac{1}{a_1 + \omega(1-r)}\right)_I - \left(\frac{1}{a_1 + \omega(1-r)}\right)_0\right] \tag{35}$$

where the subscripts I, 0 identify the intervention regime. One interesting feature about (35), that is different from the usual surplus measures which now abound in the literature, is that it consists of two components. The first, and less familiar, is due to the shift in the long-run average price. The second is the contribution to the welfare gains from the stabilization of the spot price, and this of course is familiar from previous stabilization literature.

D. Stabilization Authority

The gains to the stabilization authority from intervention, denoted by ΔG, are straightforward, namely

Rule 1: $E[\lambda(P_t - \bar{P})P_t] = \lambda\sigma_p^2$ (36a)

Rule 2: $E[\nu(P^f_{t-1} - P^*_{t,t-1})^2] = \nu[(\bar{P}^f - \bar{P})^2 + [r(x_1-1) + x_2]^2\sigma_p^2]$ (36b)

Rule 3: $E[\mu(P^f_{t-1} - P^*_{t,t-1})(P^f_{t-1} - \bar{P}^f)] = \mu(rx_1 + x_2)(r(x_1-1) + x_2)\sigma_p^2$ (26c)

E. Total Welfare Gains

In our welfare results below, two measures of both total private and overall welfare gains are presented. The first measure of total private gains is the sum of the expected changes in the profits of producers and speculators, together with the expected change in consumer surplus, namely[23/]

$$\Delta E(\Pi) = E(\Pi^P) + \Delta E(\Pi^S) + \Delta E(CS) \quad (37a)$$

The second is the corresponding sum of the changes in expected utility

$$\Delta E(V) = \Delta E(V^P) + \Delta E(V^S) + \Delta E(CS) \quad (37b)$$

The corresponding expressions for overall welfare gains are these expressions, together with the return to the stabilization authority

$$\Delta E(\Pi^T) = \Delta E(\Pi) + \Delta G \quad (38a)$$

$$\Delta E(V^T) = \Delta E(V) + \Delta G \quad (38b)$$

5. SIMULATION PROCEDURE

Numerical solutions for the non-intervention case and the three intervention rules were obtained by assigning numerical values to the basic parameters-- a, c, d, α and β,[24/] and the intervention parameters. The resulting expressions for P, σ_p^2 were then used to obtain the corresponding numerical values for the welfare gains. The range of parameters considered is given in Table 1. As A and σ_e^2, act only as scale parameters in the solutions for the price distribution, they were assigned constant values set arbitrarily at 10 and 1, respectively. The case where the fluctuations in e_t are due to fluctuations in demand is specified by $\sigma_e^2 = \sigma_u^2 = 1$, $\sigma_v^2 = 0$; while for supply fluctuations $\sigma_e^2 = \sigma_v^2 = 1$, $\sigma_u^2 = 0$. The results are more sensitive to the other parameters including the intervention parameters, which were therefore allowed to vary over a wider range.

As demand and supply characteristics of commodities tend to vary, not only between different commodities, but also across countries for a given commodity, we try to cover a wide range of slopes for the relevant demand and supply functions. This of course will also provide some insight into the generality of any particular result which may arise. Indeed, given the small number of parameters involved, it is possible to approach generality in the results without resorting to a prohibitive number of simulations.

The values assigned to the parameters a, c and d are self-explanatory. They allow the implied slopes of the demand and supply functions to span a wide range. For example, the slope of the demand function ranges from 0.1 to 10, which is an increase of 100-fold. The rate of increase in the marginal cost of holding inventories, d, ranges between 0.2 and 4, which is a factor of 20.

The choice of values for α and β, the coefficients or risk aversion, is not so straightforward. While the degree of risk aversion is likely to vary between producers and speculators and across industries, the literature is unclear as to what reasonable values of α and β should be, although there is some agreement that the values are probably small. The consensus among authors would seem to be that these risk parameters should lie in the range 0.0001 to 0.1; see Freund (1956), Peck (1975), King and Robison (1981), Buccola (1981), Rolfo (1980), Kramer and Pope (1981), Yassour, Zilberman, and Rausser (1981). We consider values of the risk parameters α and β between 0.001 and 0.1. We find that for values of $\alpha, \beta < 0.001$ the results are difficult to distinguish from the case of risk neutrality which is one of the special cases considered by Turnovsky (1983). It would therefore seem reasonable to conclude that if α and β are extremely small, the special cases of $\alpha = \beta = 0$ are good proxies. The upper bound of 0.1 is probably too high. Nevertheless it enables us to conduct sensitivity analysis as the risk parameters change from the more reasonable values of 0.01.

The simulation procedure adopted was as follows:

(i) solve for the non-intervention solution (σ_p^2, $\sigma^{2f}p$, r, $\overline{P}$ and $\overline{P}^f$), for given parameters;

(ii) then solve for the intervention solutions for each of the three rules;

(iii) use solutions from (i) and (ii) to compare variances of the spot price and futures price and to compute welfare gains and losses.

Taking into account different combinations of the parameters set out in Table 1 and a search for multiple feasible roots set out in the next section, the total number of simulations carried out exceeded 4300.

The large number of simulations carried out enables us to draw some fairly general conclusions about the behavior of a futures market subject to intervention by a stabilization authority. Obviously it is impossible to present the results of all experiments. In order to get some idea of the dispersion of results, a number of representative cases are presented. For purposes of exposition, 10 cases are presented in detail (for each intervention rule). Parameter values for each case are presented in Table 2. Table 3 contains solutions applicable to the parameters in Table 2, assuming no intervention.[25/] The solutions provide a point of comparison with the intervention solutions. For each rule we have considered 4 values of the intervention parameter, 0.5, 1, 10, 100, thereby covering a wide dispension of degrees of intervention. However, results for only 0.5, 10 are presented in the tables.[26/] Before turning to a discussion of each of the three rules, we briefly consider the question of non-existence and multiple solutions.

The rational expectations procedure adopted in this paper is consistent with the possibility that multiple solutions may exist.[27/] Of particular interest is the possibility that intervention by a stabilization authority may result in non-existence or the creation of multiple feasible solutions. The numerical framework employed in this paper provides a convenient framework for analyzing the existence or otherwise of multiple solutions.

The algorithm used to solve for intervention solutions provides only one solution for each unknown and does not provide an indication as to the uniqueness of such solutions. Solutions are obtained by an iterative procedure which requires the provision of a set of initial conditions. For feasibility we require

$$0 < r_1, r_2 < 1 \; .$$

Initial conditions were therefore set in the range 0 to 1 with a grid search being carried out with alternative initial conditions within the range. No examples of multiple feasible solutions were found in the presence of intervention. Furthermore, numerous examples of nonfeasible solutions, after intervention, were found for particular initial conditions, but a unique feasible solution could always be found by using alternative initial conditions. The absence of multiple feasible solutions and the fact that intervention does not lead to existence problems is of some importance in a practical policy context.

6. RULE 1: $G_t = -\lambda(P_t - \overline{P})$

The results pertaining to Rule 1, where the stabilization authorities are assumed to intervene in the spot market, are set out in Tables 4 and 5. The solutions for the means and variances of the spot and futures prices are given in Table 4. The percentage differences refer to the differences between the intervention solutions and the corresponding nonintervention solutions presented in Table 3. The welfare results are reported in Table 5. In both cases, the effects for demand disturbances and supply disturbances are given separately. The values of the intervention parameters $\lambda = 0.5$, $\lambda = 10$ proxy "low" and "high" degrees of intervention, respectively.

A. Effects on Long-Run Distributions of Prices

As one would expect, intervention in the spot market by means of Rule 1 results in the stabilization of both the spot and futures prices. Interestingly, very large reductions in the variances of both the spot and futures prices can be achieved for a relatively low degree of intervention; see, e.g., $\lambda = 0.5$, Cases 1 and 3. But it is also possible for negligible stabilization to be achieved for $\lambda = 0.5$. Examples of this are given in Cases 2 and 8, which have in common a high value of a, but very different degrees of risk aversion. For relatively intensive intervention, the effect of a high a is swamped and it can be seen from Table 4 that for $\lambda = 10$ a high degree of stabilization of both the spot and futures price can be achieved for all parameter combinations.

It is of interest to note that while significant reductions in both variances can be achieved, the futures price is always stabilized to a greater degree than the spot price--even though the intervention occurs directly in the spot market. The reason for this can be seen from (28e). The intervention impinges on the variance of the futures price in two ways. First, it reduces the variance of the spot price. Secondly, by reducing the one-period variance of the spot price, it increases both the numerator and the denominator of η, the coefficient of σ_p^2. It can be shown that on balance the effect on the denominator dominates, so that η falls, leading to a relatively greater stabilization of the futures market.

In terms of variance reduction it would appear that for particular parameter combinations, little can be gained by increasing the degree of intervention beyond $\lambda = 1$. This is so, for example, for Cases 1, 3, and 10, which are typified by very low values of a. Furthermore, comparing the reductions in the variances in Tables 4A and 4B it is seen that intervention in accordance with Rule 1 is approximately equally effective in stabilizing for demand disturbances as it is for supply disturbances

The qualitative effect of intervention by means of Rule 1 on the long-run mean prices depends critically upon the source of origin of the stochastic disturbances.

In the case of demand shocks, the mean spot price is always lowered, while the mean futures price is always raised. The reason for this is seen from (28). The intervention reduces both the long-run and the one period variance of the spot price. While this tends to reduce a_1, there is a more than offsetting increase in b, so that $a_1 + b$ rises, thereby lowering the long-run mean spot price $\overline{P}$. At the same time, the coefficient of $\overline{P}$ in (28b) is increased, by a more than offsetting amount and the long-run mean futures price tends to rise. In the case of supply disturbances, the intervention may cause the long-run mean prices to either rise or fall. The reason is that whereas a_1 will still fall, b may now either rise or fall, rendering the net effect on the mean spot price $\overline{P}$ indeterminate. However, in all cases, the spot and futures prices move in opposite ways (as they do for demand disturbances).

In comparison with the reductions in the variances, which we showed could be significant, the changes in the mean spot and futures prices are relatively small. The greatest reduction in the spot price achieved for all parameter sets was around 20 percent. The changes in the futures price are all much smaller, the maximum being less than 6%.

In concluding our comments on the price effects, the following observations may be made. Whereas, the intervention appears to stabilize both demand and supply disturbances equally effectively, the intervention has a relatively greater impact on mean prices when disturbances originate with demand than when they are due to supply shocks. Secondly, reductions in the mean prices are fairly insensitive to varying degrees of intervention. Thirdly, as intervention increases, the mean spot and futures prices converge to a common value for a particular set of parameter values. Indeed, for $\lambda = 100$, (not reported) the spot and futures prices are identical for each particular set of parameters. Finally, the reason why intervention has such a large impact on both variances but a relatively small impact on both mean prices can be explained by the fact that intervention by means of Rule 1 has only an <u>indirect</u> effect on the means prices, whereas it impacts on the variances <u>directly</u>.

B. Welfare Effects

Since the welfare criteria introduced in Section 4 involve the asymptotic means and variances the prices, the results we have just been considering are the key inputs in determining the welfare effects within the economy of the intervention rule. The welfare of the various agents shall be discussed in turn.

Producers:

(i) The welfare of producers as measured by expected profit, can either increase or decrease with intervention. For most parameter sets, when disturbances originate with demand, expected profit tends to fall with intervention, while for supply disturbances, expected profit tends to rise. These findings reflect the stabilizing effect of intervention on the price level and are consistent with the usual results of the buffer stock stabilization literature. The exceptions to this pattern are marginal, except Parameter Set 1, where expected profit fall quite sharply with intervention in the presence of supply disturbances. This particular result is a consequence of the fall in the spot price, together with the only modest increase in the futures price.

(ii) Intervention tends to stabilize the profit of firms (i.e., reduce its one period variance) in the presence of demand disturbances and generally, but not always, destabilize it in the presence of supply shocks. The reduction in the variance of the spot and futures prices are obviously stabilizing. But the variance of profit also depends upon $(\overline{P}^f - \overline{P})^2$. This tends to be reduced by intervention in the case of demand disturbances and to be increased in the case of supply shocks. In the former case, the stabilizing effects of the reduced variance of price is reinforced; in the latter case it is offset.

(iii) The welfare of producers, as measured by $E(V^P)$ generally falls in the case of demand disturbances and rises in the case of supply disturbances. The losses in the former case are less than those of expected profit. This is because intervention tends to stabilize producer profit in that case. Likewise, the gains in the latter case are less than those of expected profit. This is due to the destabilizing effect of intervention on profit associated with supply disturbances.

Consumers:

Consumers gain from intervention when the stochastic fluctuations originate with demand. They may either gain or lose when the fluctuations are due to supply disturbances. They usually lose when the fluctuations are due to supply disturbances, althouth, in the latter case they can also make significant gains. The magnitude of the effects to consumers reflect the behavious of the mean and variance of the spot price. The reduction in the mean price tends to benefit them, while the reduction in the variance tends to harm them. The significant gains experienced for Parameter Set 3 are due primarily to the large reduction in the mean spot price (18% and 12% for demand and supply disturbances respectively) which occurs in that case. Since the reductions in the means through intervention are smaller under supply disturbances, the benefits to consumers are smaller in that case. It is also possible that increased intervention turns consumer gains into losses; see, e.g., Parameter Set 1 for $\lambda = 0{,}5$, $\lambda = 10$. This is because in the presence of supply disturbances, intervention can actually raise the mean spot price, thereby adversely affecting consumers.

Speculators:

(i) The expected profits of speculators always decline with intervention when disturbances originate with demand. The same applies in the case of supply disturbances for all but Parameter Set 1. The reduction in the variance of the spot price, resulting from intervention, tends to reduce the expected profit of speculators. But their expected profit also depends positively upon $(\overline{P}^f - \overline{P})^2$; see (32a). As noted, this tends to decline with intervention in the case of demand disturbances, so that the expected profit of speculators is reduced unambiguously in that case. However, it tends to be increased in the case of supply shocks, thereby offsetting the losses from more stable spot price. Indeed, in the case of Parameter Set 1, this effect can dominate leading to gains to speculators.

(ii) The one period variance of speculative profits are almost always reduced through intervention, in the case of demand disturbances and usually increase in the case of supply disturbances. Exceptions in both cases do exist. The reason involves the effects of intervention on the variance of the spot price and the divergence between the long-run mean spot and futures prices.

(iii) The welfare of speculators as measured by their overall utility function $E(V^S)$ is always reduced by intervention in the case of demand disturbances and usually falls in the case of supply disturbances. Parameter Set 1 is the exception for all degrees of intervention.

Total Private Welfare:

(i) Total private welfare effects, as measured by $\Delta E(\Pi)$ are almost always positive for both disturbances. Exceptions are Parameter Set 5 for demand disturbances and Parameter Set 8 for supply disturbances, when small losses are incurred when intervention is weak. These losses, however, are converted to gains with more intensive intervention ($\lambda > 1$). The overall private gains are larger in the case of supply disturbances than for demand disturbances.

(ii) Total private welfare effects as measured by $\Delta E(V)$ are always positive in the case of demand disturbances, although there may be losses in the case of supply disturbances, through the increases in the variances in producer and speculator profits. More intensive intervention does not eliminate the losses as measured in this way, and indeed may intensify them.

Government:

The government always ensures itself an expected profit from intervening in accordance with Rule 1. For a given degree of intervention, their gains are marginally higher for supply disturbances, than they are for demand disturbances.

For very high degrees of intervention ($\lambda = 100$) gains become small. This of course is due to the fact that although λ gets large, the subsequent decline in the variance of the spot price more than offsets this. The relationship between the intervention parameter λ, and gains to the government is nonlinear. For example, for Parameter Set 2 gains to the government are increased as λ increases from 0.5 to 1 to 10, before subsequently declining. There is therefore an optimal degree of intervention form the viewpoint of maximizing government revenue.[28/]

Total Welfare:

(i) Total welfare as measured by expected profits is always positive. Total welfare as measured by $\Delta E(V^T)$ is always positive in the case of demand disturbances and with the exception of Parameter Set 1, in the case of supply disturbances as well. In this case the losses to producers are not compensated by the gains to the other agents in the economy.

7. RULE 2: $H_{t-1} = \nu(P^f_{t-1} - P^*_{t,t-1})$

We consider now Rule 2, whereby the stabilization authority behaves as a utility maximizing agent. In contrast to Rule 1, we find that in general, stabilization is virtually ineffective for all degrees of intervention in terms of variance reduction and changes in the mean price. Indeed, Rule 2 is so ineffective that only 2 of the 10 representative cases are worth reporting. Both of these cases (Parameter Sets 1 and 3) are associated with high risk aversion and inelastic demand. These are set out in Tables 6 and 7. The effect of Rule 2 on the distribution of prices and welfare for all other parameter sets is virtually zero.

A. Effects on Long-Run Distributions of Prices

Intervention via Rule 2 has the effect of stabilizing the spot and futures prices. In the case of demand disturbances, the mean spot price is reduced, while in the case of supply disturbances it can be either reduced or increased. Likewise, the mean futures price is increased in the case of demand disturbances, while it can be either decreased or increased in the case of supply disturbances. In most cases, the mean futures price is less than the mean spot price, although this is not the case of supply disturbances with Parameter Set 1.

An interesting comparison of Rule 2 with Rule 1 is that the former has a relatively greater impact on the mean than on the variance of the spot price. For example, with Parameter Set 3, setting $\nu = 10$ we find that for supply disturbances a 9% reduction in the mean is associated with a .9% reduction in the variance. With $\lambda = 10$ in Rule 1, we find that a 10.6% reduction in the mean is associated with a 99% reduction in the variance. In all cases, the change in both mean prices are always greater under Rule 1 than under Rule 2 (for equivalent values of the intervention parameters). Parameter Set 3, with $\nu = 100$ (not reported) represents the greatest impact obtained for all Rule 2 simulations. While the effects on the mean spot and futures prices are comparable in magnitude than under Rule 1, the effects on the corresponding variances are considerably less.

In summary, we see that in terms of price and variance reduction, Rule 2 is substantially inferior to Rule 1. Indeed, in most cases, intervention by means of such a rule will have a negligible impact on the market.

B. Welfare Effects

Since the welfare effects operate through the means and variances of the prices, the welfare effects essentially mirror them. In particular, in all but Parameter Sets 1 and 3, the effects are negligible. For the other two sets, the effects can be summarized as follows

(i) The expected profit of producers declines with intervention, if disturbances originate with demand; it increases in the case of supply disturbances. Intervention in accordance with Rule 2 stabilizes profit in the case of demand disturbances, but destabilizes it for supply disturbances. Welfare of firms, as measured by $E(V^f)$, declines in the case of demand disturbances, but increases in the case of supply disturbances.

(ii) Consumers gain from intervention with demand disturbances; they may either gain or lose in the case of supply disturbances.

(iii) The expected profit of speculators declines with intervention, although they are stabilized as well. However, overall, the welfare of speculators declines.

(iv) Private welfare, as measured by $E(\Pi)$ declines with intervention in the case of demand disturbances; in the case of supply disturbances it may either rise or fall. By contrast, private welfare, as measued by $E(V)$ increases with intervention in the case of demand disturbances; for supply disturbances it may go either way.

(v) The government gains from intervention.

(vi) The overall gains from intervention are with one exception always positive. The exception is in the case of supply disturbances, where the measure $E(\Pi^T)$ is employed. In Parameter Set 1, the losses to consumers and speculators may dominate the gains to other agents.

8. RULE 3: $H_{t-1} = -\mu(P^f_{t-1} - \overline{P}^f)$

Representative solutions for intervention via Rule 3 are set out in Tables 8 and 9. Only for parameter sets 1, 3, 4, and 10 did this rule have uniformly substantial effects on the asymptotic distributions of prices. For parameter sets 5 and 6 significant effects could be generated for high degrees of intervention. For all other other parameter sets the effects are negligible. Note that as $\mu \to \infty$, $\sigma^2 \to 0$. However, since for parameter sets 2, 7, 8, 9, the benchmark (no intervention) values of σ_f^2 are so small, there is little opportunity for the present rule to stabilize the futures prices in these cases.

A. Effects on Long-Run Distributions of Prices

Being symmetric with Rule 1, one would expect that intervention with Rule 3 to stabilize both the spot and futures prices (when effective). However, as can be seen from the solutions given in Table 8, even though such a rule always stabilizes the futures price (albeit by a negligible amount) it can destabilize the spot price; e.g., parameter set 3 for $\mu = 10$. The reason for this is that one effect of this form of intervention is to reduce the slope of the supply curve and inventory demand function and both of these are destabilizing influences. In the case of parameter set 3, when $\mu = 10$, these effects dominate all the other stabilizing influences of the intervention. It is interesting to note that this case of destabilization of the spot price is associated with inelastic consumer demand and very high degrees of risk aversion for both producers and speculators. In practice, since the empirical evidence suggests that farmers are not as risk averse as the coefficients chosen for this parameter sets, the destabilization of the spot price may in fact not be such a problem.

Despite the fact that high degrees of intervention in the futures market can destabilize the spot price, futures market intervention via Rule 3 always reduces the mean spot price. As for the other two rules, the mean futures price is increased after intervention. In general, changes in both mean prices are less than the corresponding changes achieved under Rule 1 and in some cases even less than for Rule 2; see, e.g., parameter sets 1 and 3. Similarly, reductions in both variances are relatively small. We noted that for Rule 1 a considerable degree of spot and futures market stabilization could be achieved for relatively low degrees of intervention. In contrast, intervention in the futures market can achieve substantial stabilization in the futures market, but relatively little stabilization of the spot price. In fact, as futures market stabilization increases, spot price stabilization declines, until in particular cases, we get a reveral as noted above and the spot price becomes destabilized. Furthermore, significant reductions in the variability of the futures price can be achieved only for high degrees of intervention.

B. Welfare Effects

The allocation of the welfare effects resulting from intervention in accordance with Rule 3 can be summarized as follows:

(i) If disturbances originate with demand, the expected profit of producers always decreases with a low degree of intervention and increases with a high degree of intervention. The response in the case of supply disturbances is slightly less clear cut, although this same pattern generally applies in that case as well. A low degree of intervention always stabilizes producer profits, a high degree destabilizes them. The welfare of producers, as measured by $E(V^f)$, follows that of expected profit.

(ii) Consumers always gain from intervention with demand disturbances; they may either gain or lose in the case of supply disturbances.

(iii) The effect of intervention on the expected profit of speculators is somewhat mixed. In the case of demand disturbances it usually declines if the degree of intervention is low and to increase if the degree of intervention is high. In the case of supply disturbances, expected profit is increased, especially if the degree of intervention is high. For either demand or supply disturbances a low degree of intervention stabilizes speculator profit, while a high degree of intervention destabilizes them. The welfare of speculators, as measured by $E(V^s)$, generally, but not always, follows that of $E(\Pi^s)$.

(iv) Private welfare, as measured by $E(\Pi)$, is increased when the degree of intervention is high ($\mu = 10$). In the case of low degree of intervention, its effects are much less sure and private welfare losses may be incurred. The same comments apply when private welfare is measured by $E(V)$.

(v) The government tends to gain when the degree of intervention is low and to lose when it is high.

(vi) The total welfare gains from intervention are generally positive or low degrees of intervention, but negative for high degrees of intervention.

These general results merit some additional comments. First, the gains to consumers and producers under Rule 3 are always less than the gains achieved under Rule 1, with the corresponding losses being greater. Similarly, total private and overall total welfare gains are always lessfor Rule 3 than Rule 1, and indeed, as noted above, it is possible to get negative gains under the former.

Second, for Rules 1 and 2 it was found that speculators always lose from intervention. A major difference for Rule 3 is that speculators can in fact gain. Indeed, as the degree of intervention increases, so does the chance that speculators' welfare will be positive; it will always be so for $\mu = 100$. Where speculators lose under Rule 3, the losses are in fact less than the corresponding losses achieved under Rule 1. Therefore speculators will always be better off under Rule 3 than under Rule 1. However, the same general conclusion cannot be drawn for comparisons with Rule 2, as the latter was so ineffective in producing non-zero gains that it is possible for Rule 3 to yield very small losses for speculators, compared with the zero losses/gains under Rule 2.

A major divergence from the first two rules is that under Rule 3, the stabilization authority may in fact lose. We can see from the expression for the gains to the stabilization authority that whether or not gains or losses are achieved depends essentially upon the relative sizes of x_1 and x_2, defined in (31). In the expression for these gains, (3bc), $r(x_1-1)$ will always be negative. If x_2 is close to unity, then the chances for gains will be increased. However, for large values of μ, x_2 will tend to decline thus enhancing the possibility of losses. In fact, from the simulation results we can conclude that the stabilization authority tends to lose for $\mu = 10$ and <u>always</u> does so for $\mu = 100$.

Finally, we may note that whereas for Rules 1 and 2 total welfare was greater than private welfare, this is not so for Rule 3. On the contrary, as the degree of intervention increases under Rule 3, society as a whole tends to lose, this being always the case for $\mu = 100$. This of course is due to the losses being incurred by the stabilization authority in that case, which more than offset the possible private gains.

9. FINAL COMMENTS

In this paper we have analyzed the effects of three alternative intervention rules on the long-run distributions of both the spot and futures prices, together with the welfare implications which stem from these effects. The results for each rule have been summarized at the appropriate part of the text, with some comparisons being made. Space limitations preclude further detailed discussion here, and we conclude with some general observations.

As a general proposition we find that intervention in the futures market is not as effective in stabilizing either the spot price or the futures price as is intervention in the spot market. Indeed, the plausible rule of buying futures contracts when their price is low and selling when they are high, while stabilizing the future price, may actually destabilize the spot price. Furthermore, the analogous type of rule undertaken in the spot market will always stabilize the futures price to a greater degree than it does the spot price. The basic reason for these findings is due to the impact that intervention has on the behavioral relationships of the private sector. It underscores the need to develop the analysis in terms of a utility maximizing framework.

The various rules we have considered can generate rather different distributions of welfare gains, including the overall benefits. The first rule, of intervening in the spot market has the greatest effect. The allocation of benefits depends in part upon the sources of the disturbances. With demand shocks, consumers and government gain at the expense of speculators and producers. With supply disturbances producers and the government gain at the expense of speculators and consumers. Much the same emerges (although to a smaller degree) with Rule 2. In both these cases the gains to society from intervention will generally (but not always) be positive.

In the case of Rule 3, however, the benefits depend crucially upon the degree of intervention. However, a high degree of intervention, which may be needed in order to generate significant stabilizing effects on the spot price, may in fact generate overall welfare losses. In particular, it is likely to lead to losses by the stabilization authority and will therefore not be favored by them.

As a final point, we may note that our analysis treats the numbers of producers and speculators as being fixed exogenously. An important possibility is that the introduction of price stabilization through intervention will influence the number of participants in the market. An interesting extension of this analysis, therefore, would be to allow the numbers of agents to be endogenously determined through free entry or exit from the market.

Table 1

Parameter Values

A	10				
σ_e^2	1				
σ_u^2	1	0			
σ_v^2	0	1			
a	0.10	0.50	1.00	2.00	10.00
$\frac{1}{c}$	0.10	0.50	1.00	2.00	10.00
$\frac{1}{d}$	0.20	1.00	4.00		
α	0.001	0.01	0.1		
β	0.001	0.01	0.1		

Table 2

Representative Parameter Sets

	1	2	3	4	5	6	7	8	9	10
a	0.1	10	0.1	0.5	2	1	2	10	2	0.1
$\frac{1}{c}$	0.5	2	0.5	0.5	1	2	10	1	10	0.5
$\frac{1}{d}$	1	1	0.2	0.2	1	1	4	4	0.2	1
α	0.1	0.1	0.1	0.01	0.01	0.01	0.01	0.01	0.001	0.001
β	0.1	0.1	0.1	0.1	0.1	0.01	0.01	0.01	0.001	0.01

Table 3

Non-Intervention Solutions

Parameter Set	1	2	3	4	5	6	7	8	9	10
A. Demand Disturbances										
σ_p^2	3.291	0.0084	15.756	0.938	0.134	0.326	0.0392	0.0061	0.207	3.226
$\overline{P}$	17.465	0.833	21.016	10.018	3.335	3.340	0.835	0.909	0.834	16.683
σ_p^{2f}	0.765	0	0.754	0.138	0.0059	0.0142	0.0017	0.0003	0.0001	0.711
$\overline{P}^f$	16.507	0.833	15.797	9.982	3.331	3.330	0.833	0.909	0.833	16.664
B. Supply Disturbances										
σ_p^2	3.338	0.0084	15.837	0.939	0.134	0.326	0.0392	0.0061	0.207	3.227
$\overline{P}$	16.503	0.833	18.567	9.978	3.331	3.334	0.834	0.909	0.834	16.663
σ_p^{2f}	0.819	0	0.813	0.139	0.0059	0.0143	0.0017	0.0003	0.0001	0.712
$\overline{P}^f$	16.699	0.836	16.287	10.022	3.338	3.333	0.833	0.909	0.833	16.668

Table 4

Effects on Distributions of Prices: Rule 1

A. Demand Disturbances

		Parameter Sets									
		1	2	3	4	5	6	7	8	9	10
	σ_p^2	.7627	.0077	1.708	.3904	.0945	.1940	.0322	.0056	.1375	.7579
	% Diff	-77	-8	-89	-58	-30	-40	-18	-8	-34	-77
	σ_p^{2f}	.1063	0	.034	.0387	.0034	.0069	.0013	.0003	.00005	.1014
	% Diff	-86	0	-96	-72	-42	-51	-24	0	-50	-86
$\lambda = .5$	$\bar{P}$	16.8894	.8334	17.2358	10.0080	3.3343	3.3375	.8344	.9091	.8338	16.6708
	% Diff	-3.3	0	-18.0	-.1	-.01	-.08	-.02	0	-.04	-.07
	$\bar{P}^f$	16.6221	.8328	16.5528	9.9920	3.3315	3.3313	.8331	.9091	.8332	16.6658
	% Diff	0.7	0	4.79	0.1	0.02	0.04	0	0	0	.01
	σ_p^2	.0083	.0023	.0094	.0077	.006	.0071	.0043	.0019	.0067	.0083
	% Diff	-99	-73	-100	-99	-96	-98	-89	-67	-97	-100
	σ_p^{2f}	.0001	0	0	0	0	0	.0001	0	0	.0001
	% Diff	-99	-100	-100	-100	-100	-100	-94	-100	-100	-100
$\lambda = 10$	$\bar{P}$	16.6695	.8334	16.6699	10.0002	3.3334	3.3335	.8335	.9091	.8334	16.666
	% Diff	-4.55	0	-20.68	-.18	-.04	-.20	-.16	0	-.08	-.09
	$\bar{P}^f$	16.6661	.8332	16.6660	9.9998	3.3332	3.3333	.8333	.9091	.8333	16.666
	% Diff	0.96	0.05	5.50	0.18	0.79	0.10	0.02	0	0.01	0.02

B. Supply Disturbances

		Parameter Sets									
		1	2	3	4	5	6	7	8	9	10
	σ_p^2	.7676	.0077	1.7100	.3907	.0946	.1940	.0323	.0056	.1375	.7580
	% Diff	-77	-9	-89	-58	-30	-40	-18	-8	-34	-77
	σ_p^{2f}	.1124	0	.0358	.0390	.00337	.0069	.00133	.00026	.00003	.1015
	% Diff	-86	-100	-96	-72	-43	-52	-22	-13	-70	-86
$\lambda = .5$	$\bar{P}$	16.3528	.8328	16.3128	9.9812	3.3312	3.3327	.8338	.9091	.8337	16.6605
	% Diff	-.91	0	-12	.034	.006	-.04	-.024	0	-.024	-.012
	$\bar{P}^f$	16.7294	.8358	16.7253	10.0189	3.3376	3.3337	.8332	.9094	.8333	16.6678
	% Diff	.18	-.012	2.7	-.033	-.012	.021	0	0	.012	.0018
	σ_p^2	.0083	.0023	.0094	.0077	.0060	.0071	.0043	.0019	.0067	.0083
	% Diff	-99	-73	-99	-99	-96	-98	-89	-69	-97	-99
	σ_p^{2f}	.0001	0	0	.0001	.00003	.00003	.0001	.00004	0	.0001
	% Diff	-99.99	100	100	-99.93	-99.49	-99.79	-94.15	-87	100	-99.99
$\lambda = 10$	$\bar{P}$	16.6068	.8330	16.6028	9.9962	3.3326	3.3326	.8333	.9091	.8333	16.6656
	% Diff	.63	.02	-10.6	.18	.05	-.04	-.08	0	-.07	.02
	$\bar{P}^f$	16.6787	.8343	16.6795	10.0038	3.3348	3.3337	.8333	.9093	.8333	16.6669
	% Diff	-.12	-.19	2.4	-.18	-.1	.02	.012	-.01	.012	-.004

Table 5

Welfare Effects: Rule 1

A. Demand Disturbances

$\lambda = .5$

	Parameter Sets									
	1	2	3	4	5	6	7	8	9	10
A. Producers										
$\Delta E(\Pi^P)$	-1.6444	0.00003	-9.0216	-.0658	-.0019	-.0015	-.0024	-.00002	-.0014	-.2391
$\Delta[\text{var } \Pi^P(1)]$	-24.3307	0.0004	-149.5554	-9.1431	-.3272	-1.3514	-.0527	-.00001	-1.2895	-105.7742
$\Delta E(V^P)$	-.4279	0.00001	-1.5438	-.0201	-.0002	-.0048	-.0022	-.00002	-.0007	-.1862
B. Consumers										
$\Delta E(CS)$	5.4456	.00041	32.4760	0.2158	0.0179	0.0782	0.0127	0.0006	0.0167	0.7501
C. Speculators										
$\Delta E(\Pi^S)$	-3.1450	-.0002	-18.4657	-.0963	-.1120	-.0516	-.0087	-.0005	-.0080	-.3121
$\Delta[\text{var } \Pi^S(1)]$	-24.3307	0.0004	-149.5554	-.0914	-.0033	-1.3514	-.0527	-.00001	-1.2895	-1.0577
$\Delta E(V^S)$	-1.9284	-.0003	-10.9879	-.0917	-.0110	-.0449	-.0085	-.0005	-.0074	-.3068
D. Total Private Welfare										
$\Delta E(\Pi)$	0.6564	0.0002	4.9886	0.0536	-.096	0.0150	0.0016	0.00008	0.0073	0.1989
$\Delta E(V)$	3.0895	0.0012	19.9443	0.1040	0.0067	0.0285	0.0020	0.00008	0.0086	0.2571
E. Government	0.3814	0.0039	0.8539	0.1952	0.0473	0.0970	0.0161	0.0028	0.0688	0.3789
F. Total Welfare										
$\Delta E(\Pi^T)$	1.0378	0.0042	5.8425	0.2488	-.0487	0.1120	0.0177	0.0029	0.0761	0.5778
$\Delta E(V^T)$	3.4709	0.0040	20.7982	0.2992	0.0054	0.1255	0.0181	0.0029	0.0174	0.6360

$\lambda = 10$

	1	2	3	4	5	6	7	8	9	10
A. Producers										
$\Delta E(\Pi^P)$	-2.3685	0.0004	-11.0190	-.1074	-.0058	-.0251	-.0115	-.0001	-.0018	-.2951
$\Delta[\text{var } \Pi^P(1)]$	-34.9676	-.0026	-178.8472	-16.2364	-1.1234	-3.3503	-.5057	-.00006	-2.4155	-144.0074
$\Delta E[V^P)$	-.6201	0.0005	-2.0767	-.0262	-.0002	-.0083	-.0090	-.0001	-.0006	-.2231
B. Consumers										
$\Delta E(CS)$	7.9624	0.0133	38.2890	0.6649	0.1569	0.3601	0.1036	0.0123	0.1785	1.4663
C. Speculators										
$\Delta E(\Pi^S)$	-4.3906	-.0028	-21.5696	-.1919	-.0405	-.1324	-.0477	-.0046	-.0218	-.4648
$\Delta[\text{var } \Pi^S(1)]$	-34.9676	-.0026	-178.8472	-.1624	-.0112	-3.3503	-.5057	-.00006	-2.4155	-1.4401
$\Delta E(V^S)$	-2.6423	-.0027	-12.6272	-.1838	-.0399	0.1157	-.0452	-.0046	-.0206	-.4576
D. Total Private Welfare										
$\Delta E(\Pi)$	1.2033	0.0109	5.7004	0.3656	0.1106	0.2026	0.0444	0.0076	0.1549	0.7064
$\Delta E(V)$	4.7000	0.0111	23.5851	0.4549	0.1168	0.2361	0.0494	0.0076	0.1585	0.7856
E. Government	0.0829	0.0228	0.0944	0.0771	0.0601	0.0705	0.0426	0.0186	0.0672	0.0829
F. Total Welfare										
$\Delta E(\Pi^T)$	1.2862	0.0337	5.7948	0.4427	0.1707	0.2731	0.0870	0.0262	0.2221	0.7893
$\Delta E(V^T)$	4.7829	0.0339	23.6795	0.5320	0.1769	0.3066	0.0920	0.0262	0.2257	0.8685

Table 5

Welfare Effects: Rule 1

B. Supply Disturbances

λ = .5

	Parameter Sets 1	2	3	4	5	6	7	8	9	10
A. Producers										
$\Delta E(\Pi^P)$	-2.7444	.00381	9.6925	.2765	.0550	.1220	.0163	.0029	.0841	.5978
$\Delta[\text{var } \Pi^P(1)]$	15.04794	.00093	-54.2022	14.6024	.6130	-.2440	-.0655	.0003	-1.0559	65.5656
$\Delta E(V^P)$	-3.4968	.00376	12.4027	.2035	.0519	.1232	.0166	.0029	.0846	.5650
B. Consumers										
$\Delta E(CS)$	1.1301	-.0036	17.9053	-.1543	-.0402	-.0570	-.0056	-.0026	-.0683	-.1068
C. Speculators										
$\Delta E(\Pi^S)$	1.6911	-.00006	-4.0033	-.0709	-.0102	-.0380	-.0086	-.0005	-.0077	-.2941
$\Delta[\text{var } \Pi^S(1)]$	19.1727	.00238	-26.7348	.1551	.0069	.0053	-.0524	.0012	-.9165	.6929
$\Delta E(V^S)$	.7325	-.00018	-2.6665	-.0787	-.0105	-.0380	-.0084	-.0005	-.0072	-.2976
D. Total Private Welfare										
$\Delta E(\Pi)$	.0768	.00015	23.5945	.0513	.0046	.0270	.0071	-.0002	.0081	.1969
$\Delta E(V)$	-1.6342	-.00002	27.6458	-.0295	.0012	.0282	.0026	-.0002	.0091	.1606
E. Government										
G.	.3838	.00385	.8550	.1954	.0473	.0970	.0161	.0028	.0688	.3790
F. Total Welfare										
$\Delta E(\Pi^T)$	.4606	.0040	24.4495	.2467	.0519	.1240	.0232	.0026	.0769	.5759
$\Delta E(V^T)$	-1.2504	.00383	28.5008	.1659	.0485	.1252	.0187	.0026	.0779	.5396

λ = 10

	1	2	3	4	5	6	7	8	9	10
A. Producers										
$\Delta E(\Pi^P)$	-3.9532	.0415	7.3380	.8133	.2793	.4443	.1226	.0333	.3730	1.3213
$\Delta[\text{var } \Pi^P(1)]$	55.28	.0143	-2.6510	48.97	3.7840	1.2638	-.1981	.0045	-2.5842	1.92
$\Delta E(V^P)$	-6.7170	.0408	7.4705	.5685	.2604	.4379	.1236	.0333	.3743	1.2253
B. Consumers										
$\Delta E(CS)$	-1.0293	-.0309	15.4017	-.3251	-.1335	-.1497	-.0294	-.0211	-.1957	-.1867
C. Speculators										
$\Delta E(\Pi^S)$	5.6348	.00007	1.3503	-.1247	-.0353	-.0801	-.0440	-.0045	-.0216	-.4302
$\Delta[\text{var } \Pi^S(1)]$	60.730	.0265	28.16	.5057	.0403	1.8734	-.1315	.0125	-2.1832	1.9704
$\Delta E(V^S)$	2.5983	-.0013	-.0577	-.1500	-.0373	-.0895	-.0434	-.0046	-.0205	-.4401
D. Total Private Welfare										
$\Delta E(\Pi)$	.6523	.0107	24.09	.3635	.1105	.2145	.0492	.0077	.1557	.7044
$\Delta E(V)$	-5.148	.0086	22.8145	.0934	.0896	.1987	.0508	.0076	.1581	.5985
E. Government										
G.	.0829	.0228	.0944	.0771	.0601	.0705	.0426	.0186	.0672	.0829
F. Total Welfare										
$\Delta E(\Pi^T)$	.7352	.0335	24.1844	.4406	.1706	.2850	.0918	.0263	.2229	.7873
$\Delta E(V^T)$	-5.0651	.0314	22.9089	.1705	.1497	.2692	.0934	.0262	.2253	.6814

Table 6

Effects on Distributions of Prices: Rule 2

A. Demand Disturbances

		Parameter Sets 1	3
$\nu = .5$	σ_p^2	3.277	15.734
	% Diff	-.1	-.1
	σ_p^{2f}	.7616	.7360
	% Diff	-.4	-2
	$\overline{P}$	17.4199	19.9707
	% Diff	-.26	-4.97
	$\overline{P}^f$	16.5160	16.0059
	% Diff	0.05	1.32
$\nu = 10$	σ_p^2	3.2488	15.6852
	% Diff	-1	-.5
	σ_p^{2f}	.7343	.6919
	% Diff	-4	-8
	$\overline{P}$	17.0344	17.2646
	% Diff	-2.46	-17.85
	$\overline{P}^f$	16.5931	16.5471
	% Diff	0.52	4.74

B. Supply Disturbances

		Parameter Sets 1	3
$\nu = .5$	σ_p^2	3.3307	15.7916
	% Diff	-.222	-.272
	σ_p^{2f}	.8122	.7813
	% Diff	-.83	-3.8
	$\overline{P}$	16.5104	18.1421
	% Diff	.042	-2.29
	$\overline{P}^f$	16.6979	16.3720
	% Diff	-.009	.523
$\nu = 10$	σ_p^2	3.2730	15.6945
	% Diff	-1.95	-.885
	σ_p^{2f}	.7578	.7007
	% Diff	-7.47	-13.76
	$\overline{P}$	16.5810	16.9508
	% Diff	.47	-8.71
	$\overline{P}^f$	16.6838	16.6094
	% Diff	-.093	1.98

Table 7

Welfare Effects Rule 2

A. Demand Disturbances

	ν = .5 Parameter Sets 1	ν = .5 Parameter Sets 3		ν = 10 Parameter Sets 1	ν = 10 Parameter Sets 3
A. Producers			A. Producers		
$\Delta E(\Pi^P)$	-.3031	-5.8725	$\Delta E(\Pi^P)$	-2.043	-11.5065
$\Delta[\text{var } \Pi^P(1)]$	-3.7655	-75.3054	$\Delta[\text{var } \Pi^P(1)]$	-27.4769	-175.5806
$\Delta E(V^P)$	-.1148	-2.1073	$\Delta E(V^P)$	-.6691	-2.7275
B. Consumers			B. Consumers		
$\Delta E(CS)$	0.3735	8.3171	$\Delta E(CS)$	3.5804	30.3564
C. Speculators			C. Speculators		
$\Delta E(\Pi^S)$	-.4260	-8.6778	$\Delta E(\Pi^S)$	-3.1007	-20.2136
$\Delta[\text{var } \Pi^S(1)]$	-3.7655	-75.3054	$\Delta[\text{var } \Pi^S(1)]$	-27.4769	-175.5806
$\Delta E(V^S)$	-.2377	-4.9126	$\Delta E(V^S)$	-1.7268	-11.4346
D. Total Private Welfare			D. Total Private Welfare		
$\Delta E(\Pi)$	-.3556	-6.2332	$\Delta E(\Pi)$	-1.5633	-1.3637
$\Delta E(V)$	0.0210	1.2972	$\Delta E(V^S)$	1.1845	16.1943
E. Government	0.4104	7.8656	E. Government	1.9555	5.1514
F. Total Welfare			F. Total Welfare		
$\Delta E(\Pi^T)$	0.0548	1.6324	$\Delta E(\Pi^T)$	0.3922	3.7877
$\Delta E(V^T)$	0.4314	9.1628	$\Delta E(V^T)$	3.1400	21.3457

B. Supply Disturbances

	ν = .5 Parameter Sets 1	ν = .5 Parameter Sets 3		ν = 10 Parameter Sets 1	ν = 10 Parameter Sets 3
A. Producers			A. Producers		
$\Delta E(\Pi^P)$	0.0502	1.9314	$\Delta E(\Pi^P)$	0.6379	8.6114
$\Delta[\text{var } \Pi^P(1)]$	-.2088	-13.6422	$\Delta[\text{var } \Pi^P(1)]$	-1.7260	-34.0911
$\Delta E(V^P)$	0.0606	2.6135	$\Delta E(V^P)$	0.7242	10.3159
B. Consumers			B. Consumers		
$\Delta E(CS)$	-.0588	3.4678	$\Delta E(CS)$	-.6509	13.2856
C. Speculators			C. Speculators		
$\Delta E(\Pi^S)$	-.0181	-1.5437	$\Delta E(\Pi^S)$	-.1463	-3.8160
$\Delta[\text{var } \Pi^S(1)]$	-.1814	-13.4447	$\Delta[\text{var } \Pi^S(1)]$	-1.4784	-33.3667
$\Delta E(V^S)$	-.0091	-.8714	$\Delta E(V^S)$	-.0724	-2.1477
D. Total Private Welfare			D. Total Private Welfare		
$\Delta E(\Pi)$	-.0267	3.8555	$\Delta E(\Pi)$	-.1593	18.081
$\Delta E(V)$	-.0073	5.2099	$\Delta E(V)$	.0009	21.4538
E. Government			E. Government		
G.	0.0249	1.5856	G.	.1383	1.1788
F. Total Welfare			F. Total Welfare		
$\Delta E(\Pi^T)$	-.0018	5.4411	$\Delta E(\Pi^T)$	-.021	19.2598
$\Delta E(V^T)$	.0176	6.7955	$\Delta E(V^T)$	.1392	22.6326

Table 8

Effects on Distributions of Prices: Rule 3

A. Demand Disturbances

		1	3	4	10
$\mu = 0.5$	σ_p^2	3.2351	15.630	.937	3.225
	% Diff	-1.4	-1	-.1	-.02
	σ_p^{2f}	.7208	.627	.137	.7105
	% Diff	-6	-17	-.4	-.1
	$\overline{P}$	17.4408	20.9146	10.0180	16.6825
	% Diff	-.14	-.48	0	0
	$\overline{P}^f$	16.5118	15.8171	9.9820	16.6635
	% Diff	0.03	0.13	0.001	0
$\mu = 10$	σ_p^2	2.971	16.640	.927	3.210
	% Diff	-9	+6	-1	-.5
	σ_p^{2f}	.372	.112	.127	.696
	% Diff	-51	-85	-8	-2
	$\overline{P}$	17.2348	20.3371	10.0177	16.682
	% Diff	-1.32	-3.23	-.004	-.001
	$\overline{P}^f$	16.5530	15.9326	9.9823	16.663
	% Diff	0.29	0.86	0.004	0

B. Supply Disturbances

		1	3	4	6	10
$\mu = .5$	σ_p^2	3.2870	15.6797	.9387		3.2257
	% Diff	-1.53	-.978	-.064		-.025
	σ_p^{2f}	.7712	.6864	.1387		.7115
	% Diff	-5.84	-15.52	-.431		-.112
	$\overline{P}$	16.4917	18.5008	9.9778		16.6625
	% Diff	-.071	-.357	0		0
	$\overline{P}^f$	16.7017	16.2998	10.0222		16.6675
	% Diff	.014	.080	0		0
$\mu = 10$	σ_p^2	2.9801	16.4552	.9283	.3257	3.2112
	% Diff	-10.72	3.92	-1.17	-.061	-.474
	σ_p^{2f}	.3952	.1339	.1283	.0141	.6969
	% Diff	-51.75	-83.52	-7.9	-1.4	-2.16
	$\overline{P}$	16.4052	18.1030	9.9778	3.3341	16.6625
	% Diff	-.595	-2.5	0	0	0
	$\overline{P}^f$	16.7190	16.3794	10.0222	3.3330	16.6675
	% Diff	.117	.568	0	0	0

Table 9

Welfare Effects: Rule 3

A. Demand Disturbances

$\mu = .5$

	Parameter Sets 1	3	4	10
A. <u>Producers</u>				
$\Delta E(\Pi^P)$	-.0782	-.2087	-.0020	-.0006
$\Delta[\text{var } \Pi^P(1)]$	-1.0673	-3.3740	-.2310	-.4699
$\Delta E(V^P)$	-.0249	-.0400	-.0008	-.0003
B. <u>Consumers</u>				
$\Delta E(CS)$	0.2246	0.8500	0.0005	0.0006
C. <u>Speculators</u>				
$\Delta E(\Pi^S)$	-.1216	-.4251	-.00003	0.0001
$\Delta[\text{var } \Pi^S(1)]$	-1.0673	-3.3740	-.0023	-.0047
$\Delta E(V^S)$	-.0683	-.2564	0.00009	0.0001
D. <u>Total Private Welfare</u>				
$\Delta E(\Pi)$	0.0248	0.2162	-.0015	0.0001
$\Delta E(V)$	0.1314	0.5536	-.0002	0.0004
E. <u>Government</u>	0.0054	-.0467	0.0003	0.0001
F. <u>Total Welfare</u>				
$\Delta E(\Pi^T)$	0.0302	0.1645	-.0012	0.0002
$\Delta E(V^T)$	0.1368	0.5069	0.0001	0.0005

$\mu = 10$

	Parameter Sets 1	3	4	6	10
A. <u>Producers</u>					
$\Delta E(\Pi^P)$	0.6146	0.9913	0.0585	0.0006	0.1149
$\Delta[\text{var } \Pi^P(1)]$	3.3278	0.7452	5.9216	0.0746	118.8300
$\Delta E(V^P)$	0.4482	0.9540	0.0289	0.0002	0.0555
B. <u>Consumers</u>					
$\Delta E(CS)$	2.1667	5.8053	0.0099	0.0004	0.0109
C. <u>Speculators</u>					
$\Delta E(\Pi^S)$	0.2884	-.2688	0.0115	0.0010	0.0165
$\Delta[\text{var } \Pi^S(1)]$	3.3278	0.7452	0.0592	0.0746	1.1883
$\Delta E(V^S)$	0.1220	-.3061	0.0085	0.0007	0.0106
D. <u>Total Private Welfare</u>					
$\Delta E(\Pi)$	3.0697	6.5278	0.0799	0.002	0.1424
$\Delta E(V)$	2.7369	6.4532	0.0473	0.0013	0.0770
E. <u>Government</u>	-2.8936	-5.7417	-.0797	-.0018	-.1470
F. <u>Total Welfare</u>					
$\Delta E(\Pi^T)$	0.1761	0.7861	0.0002	0.0002	-.0046
$\Delta E(V^T)$	-.1567	0.7115	-.0324	-.0005	-.0700

Table 9

Welfare Effects: Rule 3

B. Supply Disturbanaces

$\mu = .5$

	Parameter Sets 1	3	4	10
A. <u>Producers</u>				
$\Delta E(\Pi^P)$	-.13305	.2517	-.0007	-.00043
$\Delta[\text{var } \Pi^P(1)]$	-.32815	-2.6851	-.1550	-1.4694
$\Delta E(V^P)$	-.11665	.38596	.00005	.00030
B. <u>Consumers</u>				
$\Delta E(CS)$	.0951	.5324	-.00024	.00004
C. <u>Speculators</u>				
$\Delta E(\Pi^S)$	.00120	-.1888	.00014	.0001
$\Delta[\text{var } \Pi^S(1)]$	-.13518	-1.8642	-.00153	-.0147
$\Delta E[V^S)$	.00796	-.0956	.00021	.00017
D. <u>Total Private Welfare</u>				
$\Delta E(\Pi)$	-.03675	.5953	-.0008	-.00029
$\Delta E(V)$	-.01359	.8228	.00002	.00051
E. <u>Government</u>				
G.	.03200	.0030	.00086	.00062
F. <u>Total Welfare</u>				
$\Delta E(\Pi^T)$	-.00475	.5983	.00006	.00033
$\Delta E(V^T)$	.01841	.8258	.00088	.00113

$\mu = 10$

	Parameter Sets 1	3	4	6	10
A. <u>Producers</u>					
$\Delta E(\Pi^P)$	-.4394	3.0580	.0456	.00047	.10437
$\Delta[\text{var } \Pi^P(1)]$	11.8337	4.2244	4.839	.02921	106.634
$\Delta E(V^P)$	-1.0310	2.8468	.0214	.00033	.05106
B. <u>Consumers</u>					
$\Delta E(CS)$	.8025	3.8211	-.0028	-.00044	-.00068
C. <u>Speculators</u>					
$\Delta E(\Pi^S)$	1.5683	1.1817	.0105	.00057	.01537
$\Delta[\text{var } \Pi^S(1)]$	13.60997	9.8989	.0488	.03005	1.06698
$\Delta E(V^S)$	.8878	.6868	.0080	.00042	.01003
D. <u>Total Private Welfare</u>					
$\Delta E(\Pi)$	1.9315	8.0608	.0533	.0006	.1191
$\Delta E(V)$	.6593	7.3547	.0266	.00031	.0604
E. <u>Government</u>					
G.	-2.7401	-5.86574	-.0699	-.00140	-.13716
F. <u>Total Welfare</u>					
$\Delta E(\Pi^T)$	-.8086	2.1951	-.0166	-.0008	-.0181
$\Delta E(V^T)$	-2.0808	1.4890	-.0433	-.0011	-.0768

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FOOTNOTES

1/ A survey of much of this literature is contained in Turnovsky (1978a). For a more recent discussion of more recent developments see Newbery and Stiglitz (1981).

2/ See, e.g., Turnovsky (1978b), Newbery and Stiglitz (1979).

3/ See, for example, Working (1985), Stein (1961), Peck (1976), Turnovsky (1979 and 1983), and Kawai (1983).

4/ See also Kawai (1983) for a similar framework.

5/ The futures contracts we consider are for single period which coincides with the production period.

6/ The assumption that costs depend upon planned rather than actual output can be justified if costs are incurred on non-stochastically determined inputs, chosen at the time the production decision is made. The random fluctuations in output appearing in revenue are due to stochastic disturbances in production conditions, which occur after the inputs have been purchased.

7/ The additive form of the disturbance term appearing in (2) corresponds to a production function $f(m_t)$ of the form $y = f(m_t) + v_t'$, where m_t is a fector of inputs. The result we shall obtain below, that production and futures trading decisions may be "separated," depends on this assumption and would not hold for a more general specification of the disturbance term $y_t = f(m_t, v_t')$; see Danthine (1978).

8/ Since the function (3) can be derived from formal expected utility maximization under at best only highly restrictive conditions, we prefer to simply postulate (3) on the grounds of being a plausible and familiar ad hoc ojective, which is analytically tractable, rather than attempting to justify it in any very rigorous way. The function is consistent with expected utility maximization if (i) the utility function has constant absolute risk aversion and (ii) the spot price is normally distributed and output is non-stochastic, in which case π_t^p is normally distributed. It is not consistent with expected utility maximization if the firm's output is stochastic, since then π_t^p will be non-normal. In this case the constant absolute risk averse utility function can be handled using the method of Bray (1981). The implied optimal decision rules for firms using this more general procedure turn out to differ from those derived below by terms which are of $0(1/n_f)$. Provided the number of firms is sufficiently large, as we are assuming, our approach provides an adequate approximation to this more general procedure.

9/ See Feder, Just and Schmitz (1980), although they assume $\sigma_v^2 = 0$.

10/ For simplicity, the model assumes a zero interest rate. Since the introduction of such a rate would be exogenous to the market, its exclusion involves no essential loss of generality.

11/ The fact that $i_{j,t-1}$ may be negative is of no particular concern, since our specification here refers to only the speculative component of inventory holdings. To take care of the nonnegativity of total inventory holdings we simply assume that a sufficiently large fixed stock of inventories are held for non-speculative purposes to more than offset any short speculative position taken.

More explicitly, one can introduce a based level of inventories $\bar{i}$ say and treat the speculative decision as involving the choice of $(i_t - \bar{i})$. This leads to the addition of the base level $\bar{i}$ in the inventory demand function and provided $\bar{i}$ is taken to be sufficiently large, the problem of the nonnegativity of inventory demand disappears.

12/ In the case of speculators, the demand functions (8a), (8b) would no longer obtain with a multiperiod objective function if costs were in terms of changes in inventory holdings, say.

13/ Rules 1 and 3 assume that the stabilization authorities know the equilibrium level of spot and futures prices, respectively. An analysis of intervention when the authorities do not know the target price exactly has been carried out in a simple model (with only a spot market) by Turnovsky (1978b).

14/ Rules such as (22a) are prevalent in the recent literature analyzing exchange market intervention.

15/ Note that Rule 1 requires the authorities to hold inventories of the commodity. No account is taken of the carrying costs of these inventories in assessing the costs and benefits of stabilization. Note further that the stocks of inventories held by the authorities at time t is given by

$$I_t^g = I_o^g + \sum_{i=1}^{t} G_t$$

where I_o is the initial level. Asymptotically, the expected value of this converges to the initial level, I_o.

16/ While these rules are arbitrarily specified, they are plausible. Both Rules 1 and 3 have well established histories in both theoretical and applied work. One can of course specify an optimality criterion for the government and derive a corresponding optimal intervention policy. For example, if the stabilization authority chooses to maximize its utility of revenue from intervening in both the spot and futures market, then one can derive G and H functions of the form (8a), (8b), respectively. In this case, the effects of intervention can be analyzed through changes in the parameters, d, k.

17/ Note further, that as the variance of prices tends to zero, the futures price converges to the equilibrium spot price.

18/ The solution procedure essentially follows that of Turnovsky (1983).

19/ The detailed derivations of the solutions under the various intervention regimes are available from the authors.

20/ Note that we evaluate the welfare effects of the stabilization rules in terms of their asymptotic properties. While this seems reasonable, and is standard procedure, it may tell us little about the short run effects, especially when prices start from some initial point distant from their long run equilibrium.

21/ The derivations of the welfare expressions (30) and (32) are available from the authors on request.

22/ The reason for looking at the asymptotic one-period variance $E[\Pi_t - E_{t-1}(\Pi_t)]$ is that this is the asymptotic expectation of the variance measure appearing in the welfare function. This is not to be confused with asymptotic variance of profit, $E[\Pi_t - E(\Pi_t)]$, which measures long-run variability.

23/ The aggregation of the government's profit form intervention with the sum of private utilities is not entirely satisfactory. However, given that these utility measures are assumed to reflect dollar evaluations, the aggregate provides a measure for the potential for achieving Pareto optimality through redistribution.

24/ The equations were solved using a nonlinear solution subroutine contained in the Australian National University Program library and is based on a method discussed in Brent (1973).

25/ The following features of the benchmark solutions given in Table 3 merit attention: (i) the mean spot price $\bar{P}$ is lower under supply than under demand disturbances; (ii) the mean futures price $\bar{P}^f$ is higher under supply than under demand disturbances; (iii) $\bar{P}^f < \bar{P}$ under demand disturbances; (iv) $\bar{P}^f > \bar{P}$ under supply disturbances; (v) the variances of the spot and futures prices respectively are both lower under demand disturbances than under supply disturbances; (vi) the variance of the futures price is less than the variance of the spot price.

26/ The analogous tables for values 1, 100 are available form the authors.

27/ See McCafferty and Driskill (1980).

28/ In the limiting case when $\lambda = \infty$ and the spot price is stabilized exactly, gains to the stabilization authority are reduced to zero.