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A COMPARISON OF TOURNAMENTS AND CONTRACTS

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ABSTRACT

Tournaments, reward structures based on rank order, are compared with individual contracts in a model with one risk-neutral principal and many risk-averse agents. Each agents's output is a stochastic function of his effort level plus an additive shock term that is common to all the agents. The principal observes only the output levels of the agents. It is shown that in the absence of a common shock, using optimal independent contracts dominates using the optimal tournament. Conversely, if the distribution of the common shock is sufficiently diffuse, using the optimal tournament dominates using optimal independent contracts. Finally, it is shown that for a sufficiently large number of agents, a principal who cannot observe the common shock but uses the optimal tournament, does as well as one who can observe the shock and uses independent contracts.

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1. Introduction

At the Olympics prizes are awarded not on the basis of absolute performance, but rather on the basis of relative performance. Similarly, in most organizations one of the most important rewards is promotion. If the hierarchical structure of the organization is fixed, employees at any one echelon are competing for a fixed, smaller number of positions at the next higher echelon. The goal for these employees is not just to do well, but to do better than their peers.

The existing literature on incentives and contract design has been concerned primarily with the case where a principal employs only one agent, and rewards him on the basis of absolute performance. Exceptions include two recent papers about tournaments--reward structures based on rank order, and one that considers more general compensation schemes for multi-agent settings.

The first is by Lazear and Rosen [1981], who examine the problem of a risk-neutral firm with two employees. The output of each agent depends stochastically on his own effort and on an additive shock that is common to both. The agents do not know the value of the common shock at the time they choose their effort levels; they do know its distribution. It is shown that if the agents are risk-neutral, an optimal two-person tournament is equivalent, for all three parties, to offering the optimal incentive contract to each agent independently. In either case, because the agents are risk-neutral, the moral hazard problem can be avoided costlessly by shifting all risk onto the agents. Lazear and Rosen also compare linear piece rates and tournaments for the case of risk-averse agents and a normally distributed shock. They show that if the variance of the shock term is sufficiently large, the optimal tournament yields higher expected utilities.

Stiglitz [1981] compares tournaments and independent contracts using a somewhat different specification of the production technology. Here the shock is multiplicative, so that it affects the marginal product of labor. A wide variety of cases and examples are studied.

Finally, Holmstrom [1981] examines arbitrary relative performance schemes for risk-averse agents. He shows that for either additive or multiplicative shocks, because the mean output level is a sufficient statistic for all of the information about the common shock, optimal contracts can be designed in which each agent's reward depends only on his own output level and on the mean output level.

In all of these models, compensation schemes that base an agent's reward on the performance of his peers as well as his own, take advantage of the fact that the vector of output levels for the whole group is a source of information about the common shock--which by assumption the principal cannot observe directly. Optimal compensation schemes for groups of agents can, in general, have arbitrary and complicated functional forms--depending on assumptions about tastes, technology, and distributions for the error terms. In practise, on the other hand, rather simple schemes are often used. Consequently, it is useful to study the properties of simple schemes--to understand when they perform "almost" as well as "optimal" contracts. That is the viewpoint adopted here, where we compare the efficiency of independent contracts and tournaments. Under the former each agent's reward depends only on his own output level, while under the latter it depends only on his rank order. These represent the extreme cases of reward structures based on absolute and relative performance.

We consider a situation in which one risk-neutral principal employs a group of identical risk-averse agents. As in the Lazear-Rosen model, each agent's output is assumed to depend stochastically on his own effort and a common additive shock term. The common shock might represent economic conditions which affect all of the agents. We allow agents to observe private signals, correlated with this common shock, before they choose their effort levels. The realized output of each agent then is a stochastic function of his effort and the value of the common shock. The principal observes only the output levels of the agents.

We assume throughout that the principal is constrained to offer a fixed minimum level of expected utility to each agent, so that we can judge the relative performance of contracts and tournaments by examining the expected payoff of the principal. The principal's objective function is the sum of the outputs of all the agents minus the sum of the rewards paid to all of them.

We show that for any finite number of agents, in the absence of a common error term, using the optimal tournament is dominated by using optimal independent contracts. In the absence of a common shock, the output levels of the rest of the group convey no information about the effort level of an agent. Using a tournament in this case only introduces extraneous noise into the payoff function that agent faces. Since the agents are risk-averse, this is costly for the principal.

Conversely, given any group of at least two agents, if the distribution of the common error term is sufficiently diffuse, then the optimal tournament dominates using optimal independent contracts. In this case using tournaments eliminates a major source of noise, while adding a relatively minor one.

Finally, given any fixed distribution for the common error term, for a sufficiently large number of agents, using the optimal tournament dominates using optimal independent contracts. In fact, if the number of agents is sufficiently large, a principal who cannot observe the value of the common shock and uses an optimal tournament can do as well as a principal who can observe the value of the shock and uses general, interdependent contracts. For a large enough group of agents, an agent's rank order is an extremely accurate signal about his output level net of the common additive shock.

The rest of the paper is organized as follows. In Section 2 tastes, technology, distributions, and feasible sets of tournaments and contracts are described; in Section 3 tournaments and independent contracts are compared; and in Section 4 the conclusions are discussed.

2. The Model

We consider the problem faced by a principal who employs a fixed group of agents, $i = 1,..,n$. The agents are all identical *ex ante*. The preferences of each agent i over his income, m_i, and his effort, x_i, are represented by the von Neumann-Morgenstern utility function

$$U^i(m_i,x_i) = u(m_i) - x_i \ , \quad m_i > 0, \ x_i > 0, \ i = 1,...,n; \tag{1}$$

where $u: R_+ \rightarrow [0,B]$ is strictly increasing and strictly concave.[1]

The output of agent i, y_i, depends stochastically on his effort level, x_i. In particular,

$$y_i = z_i + \eta, \tag{2}$$

where $\eta \in R$ is a random variable affecting all of the agents, and z_i is a random variable whose distribution depends on x_i. Let $F(\ ;x_i)$ denote the conditional distribution function for z_i given x_i; since the agents are identical *ex ante*, F does not depend on i. Assume that for any effort level x > 0, the distribution function for output, $F(\ ;x)$ has a continuous density function f(;x) which is positive everywhere and continuously differentiable in x.

The agents observe private signals about η before choosing their effort levels; let $\sigma_i \in R$ denote the signal observed by agent i, and let G denote the distribution function for (η,σ). Note that this formulation includes situations where all agents observe the same signal, independent signals, signals that reveal η completely, and signals that are uncorrelated with η. Assume that z_i and (η,σ) are independent, and assume that η has zero mean.

$$\int \eta dG(\eta,\sigma) = 0 \tag{3}$$

(Except where otherwise indicated, integration is over the entire range.)

The principal's problem is to design a reward structure for the n agents. The principal is risk-neutral and seeks to maximize the expected sum of the outputs net of total payments to the agents.

$$E[\sum_{i=1}^{n} (y_i - m_i)] \tag{4}$$

By assumption the principal observes only the output levels of the agents, $y = (y_1, y_2, \ldots, y_n)$; he cannot directly observe either the agents' effort levels or the realization of any random variable. Under independent contracts agent i's reward depends only on his own output level, y_i, while under a tournament it depends only on the rank order of y_i in y.

Given any reward structure, the problem facing each agent is to choose a level of effort. First consider the situation under independent contracts. Since the agents are identical, we can consider the problem facing a representative agent i. It is convenient to think of the principal as constructing the reward function in terms of utility. For any reward function $R(y)$, let $v(y)$ be defined as its representation in utility terms, $v(y) \equiv u(R(y))$. The cost to the principal of providing this level of utility is then given by $\gamma(v(y))$, where $\gamma \equiv u^{-1}$. Since u is strictly increasing and strictly concave, γ is strictly increasing and strictly convex. Agent i observes σ_i, and then chooses the level of effort that maximizes his expected utility. Since the optimal level of effort will depend on the value of σ_i, the optimal decision rule for the agent is a function $X(\sigma_i)$.

The principal's problem is to choose (v,X) to maximize (4) subject to the two constraints that X be an optimal decision rule for the agent given v, and that the expected utility of the agent be at least u^o. Given G, define $S_{ci}(G)$ to be the set of contracts that are feasible for the i^{th} agent:

$$S_{ci}(G) \equiv \{(v,X) |\ v:R_+ \to [0,B],\ X:R \to R_+;$$

$$X(\sigma_i) \in \underset{x}{\text{argmax}} \int v(y) \int f(y-\eta;x)dG(\eta,\sigma_{\sim i}|\sigma_i)dy - x,\ \forall\ \sigma_i \qquad (5a)$$

$$\iint (v(y) - X(\sigma_i))f(y-\eta;X(\sigma_i))dG(\eta,\sigma)dy \geq u^o\}; \qquad (5b)$$

and define $P_{ci}(v,X,G)$ to be the expected payoff of the principal from the contract (v,X):

$$P_{ci}(v,X,G) \equiv \iint (y - \gamma(v(y))f(y-\eta;X(\sigma_i))dydG(\eta,\sigma).$$

The feasible set is always non-empty, since it always contains the "no-incentive" contract: $(v \equiv u^o, X \equiv 0) \in S_{ci}(G)$, for all G,i. Note that the expected payoff to the principal under this contract, call it P^o, is independent of G.

$$P^o \equiv P_{ci}(v^o, X^o, G) = \iint \left(z + \eta - \gamma(u^o)\right) f(z;0)\,dz\,dG(\eta,\sigma)$$

$$= \int z f(z;0)\,dz - \gamma(u^o)$$

Next consider the situation under a tournament. As above it is convenient to express rewards in terms of utility. In an n-person tournament with prizes $(W_1, W_2, \ldots, W_n)$, define $w = (w_1, w_2, \ldots, w_n)$ by $w_i \equiv u(W_i)$, $\forall i$. (We use the numbering conventional in the study of order statistics: "first place" is the lowest outcome, and w_1 is the prize received by the agent with the lowest output, etc.) We will consider symmetric Nash equilibria of the game in which each agent's strategy is his effort level.[2]

Since each agent's output is given by $y_i = z_i + \eta$:

$$y_i > y_j \iff z_i > z_j.$$

That is, the rank order of the outputs depends only on the z_i's and not on η. Therefore, the realization of (η, σ) does not affect the game played by the agents, and the equilibrium effort level will be independent of σ. Hence we can analyze the game in terms just of the z_i's. In an n-person tournament, agent i wins prize w_j if and only if z_i is the j^{th} order statistic of $(z_1, \ldots, z_n)$. Define:

$$\Phi_{jn}(z;x) \equiv \frac{n!}{(n-j)!(j-1)!} f(z;x) F^{j-1}(z;x)\left(1-F(z;x)\right)^{n-j}$$

That is, $\Phi_{jn}(z;x)$ is the density function for the jth order statistic in a sample of size n drawn from the distribution F(;x).

As above, the principal is constrained to offer the agents an expected utility of at least u^o. We are interested only in tournaments that have symmetric Nash equilibria. Given n and G, the set of feasible n-person tournaments, $\hat{S}_T(n,G)$, is defined by:

$$\hat{S}_T(n,G) \equiv \{(w,\bar{x}) | w \in [0,B]^n, \bar{x} \in R_+;$$

$$\bar{x} \in \underset{x}{\operatorname{argmax}} \frac{1}{n} \sum_{j=1}^{n} w_j \int \frac{f(z;x)}{f(z;\bar{x})} \phi_{jn}(z;\bar{x})dz - x; \tag{6a}$$

$$\frac{1}{n} \sum_{j=1}^{n} w_j - \bar{x} \geq u^o\} \quad , \tag{6b}$$

Note that $\hat{S}_T(n,G) = S_T(n)$, for all G satisfying (3).

Given n, G, and $(w,\bar{x}) \in S_T(n)$, let $\hat{P}_T(n,w,\bar{x},G)$ denote the principal's expected net payoff per agent, under the tournament $(w,\bar{x})$.

$$\hat{P}_T(w,w,\bar{x},G) \equiv \iint y \; f(y-\eta;\bar{x})dG(\eta,\sigma)dy - \frac{1}{n} \sum_{j=1}^{n} \gamma(w_j)$$

$$= \int zf(z;\bar{x})dz - \frac{1}{n} \sum_{j=1}^{n} \gamma(w_j)$$

Hence $\hat{P}_T(n,w,\bar{x},G) = P_T(n,w,\bar{x})$. We summarize these results in Lemma 1.

<u>Lemma 1:</u> The set of feasible tournaments, the expected payoff of the principal under any feasible tournament, and hence the optimal tournament, each depend on the number of players n, and on the distribution function F, but <u>not</u> on the distribution function G.

Lemma 1 is interesting in its own right, since it says that tournaments are robust against lack of information or lack of agreement about G. It will also be useful for our later results. Note that the set of feasible tournaments is always non-empty, since it always contains the "no-incentive" tournament: $((u^o,u^o,\ldots,u^o),0) \in S_T(n)$, for all n. The payoff per agent to the principal under this tournament is P^o.

3. Comparison of Tournaments and Contracts

First we will show that for any number of agents n and any function F satisfying (2), if there is no common error term, i.e. if:

$$\int_{\sigma \in R^n} dG(\eta,\sigma) \equiv \begin{cases} 0 & \text{for } \eta < 0 \\ 1 & \text{for } \eta \geq 0 \end{cases} \tag{7}$$

then for any feasible tournament there exists a feasible contract that dominates it.[3] Note that if G satisfies (7), then it also satisfies (3).

Proposition 1: For any F, G satisfying (7), and $n \geq 2$, given $(w,\bar{x}) \in S_T(n)$, there exists $(v,X) \in S_{ci}(G)$, $i = 1,\ldots,n$, such that:

$$P_{ci}(v,X,G) \geq P_T(n,w,\bar{x}), \qquad i = 1,\ldots,n.$$

The inequality is strict unless $(w,\bar{x}) = ((u^o,u^o,\ldots,u^o),0)$.

Proof: Let F, G, $n \geq 2$, and $(w,\bar{x}) \in S_T(n)$ be given. We will show that the contract (v,X) defined by:

$$v(y) \equiv \frac{1}{n} \sum_{j=1}^{n} w_j \, \phi_{jn}(y;\bar{x})/f(y;\bar{x}) \,, \qquad \text{for all } y;$$

$$X(\sigma_i) = \bar{x} \,, \qquad \text{for all } \sigma_i;$$

satisfies the required conditions.

First we will show that the proposed contract satisfies (5a) and (5b). Since G satisfies (7), any agent's optimal effort function under v is given by:

$$X(\sigma_i) = \underset{x}{\operatorname{argmax}} \int v(z)f(z;x)dz - x$$

$$= \underset{x}{\operatorname{argmax}} \int \frac{1}{n} \sum_{j=1}^{n} w_j \frac{\Phi_{jn}(z;\bar{x})}{f(z;\bar{x})} f(z;x)dz - x , \quad \text{for all } \sigma_i.$$

Since $(w,\bar{x})$ satisfies (6a), $X(\sigma_i) \equiv \bar{x}$ and (5a) is satisfied.

Moreover, since $(w,\bar{x})$ satisfies (6b), the expected utility of the agent under v is given by:

$$\int v(z)f(z;\bar{x})dz - \bar{x}$$

$$= \int \frac{1}{n} \sum_{j=1}^{n} \frac{\Phi_{jn}(z;\bar{x})}{f(z,\bar{x})} w_j f(z;\bar{x})dz - \bar{x} > u^o.$$

Hence (5b) is also satisfied.

The expected utility of the principal is higher under (v,X) than under $(w,\bar{x})$ since:

$$P_c(v,X,G) = \int \left[z - \gamma(v(z))\right] f(z;\bar{x})dz$$

$$= \int zf(z;\bar{x})dz - \int \gamma\left(\frac{1}{n} \sum_{j=1}^{n} \frac{\Phi_{jn}(z,\bar{x})}{f(z;\bar{x})} w_j\right)f(z;\bar{x})dz$$

$$> \int z\, f(z,\bar{x})dz - \frac{1}{n} \sum_{j=1}^{n} \gamma(w_j) \int \Phi_{jn}(z,\bar{x})dz$$

$$= P_T(n,w,\bar{x})$$

The inequality follows from Jensen's inequality and the fact that γ is strictly convex. If $w \neq (\bar{w},\ldots,\bar{w})$, the inequality is strict. If $w = (\bar{w},\ldots,\bar{w})$, then $\bar{x} = 0$, and unless $\bar{w} = u^o$, the contract $(v \equiv u^o,\ X \equiv 0)$ dominates $(w,\bar{x})$. Q.E.D.

An obvious corollary of Proposition 1 is that when there is no common error term, the optimal contract dominates the optimal tournament.

Corollary 1: Let F, G satisfying (7), and $n > 2$ be given. Then

$$\underset{(v,X)\varepsilon S_{ci}(G)}{\text{Max}} P_{ci}(v,X,G) \geq \underset{(w,\bar{x})\varepsilon S_T(n)}{\text{Max}} P_T(n,w,\bar{x}), \text{ for all } i.$$

The inequality is strict unless $(v \equiv u^o, X \equiv 0)$ maximizes the left-hand side.

Next, we will compare independent contracts and tournaments as the common error term becomes diffuse. We will consider sequences $\{G_k\}_{k=1}^{\infty}$ such that for all k:

$$\begin{aligned} &G_k \text{ satisfies (2);} \\ &G_k \text{ has a density function } g_k; \\ &\int g_k(\eta,\sigma_{\sim i}|\sigma_i)d\sigma_i \equiv g_{ki}(\eta|\sigma_i) < 1/k, \text{ for all } \eta,\ \sigma_i,\ i. \end{aligned} \tag{8}$$

In Proposition 2 we show that for any sequence $\{G_k\}$ satisfying (8), for all k sufficiently large, the optimal contract is the "no-incentive" contract

($v \equiv u^o$, $X \equiv 0$). Hence the principal's expected payoff under the optimal contract falls to P^o along the sequence. However, as shown above, the optimal tournament and the principal's expected payoff under it--which is at least P^o, will be unchanged along the sequence.

Proposition 2: Let F, $\{G_k\}_{k=1}^{\infty}$ satisying (8), and $n \geq 2$, be given. Assume that $f_x(z;x)$ is a function of bounded variation in z, for all $x > 0$, and that the bound, M, is uniform in x. Then there exists K such that for all $k > K$:

$$\underset{(w,\bar{x})\varepsilon S_T(n)}{\text{Max}} P_T(w,\bar{x},n) \geq \underset{(v,X)\varepsilon S_{ci}(G_k)}{\text{Max}} P_{ci}(v,X,G_k), \qquad i = 1,\ldots,n. \qquad (9)$$

The inequality is strict unless the lefthand side is equal to P^o.

Proof: Let $\{(v^*_{ki},X^*_{ki})\}$ be a sequence of optimal contracts for agent i. Note that:

$$X^*_{ki}(\sigma_i) > 0 \Rightarrow 1 = \int v^*_{ki}(y) \int f_x(y - \eta;X^*_{ki}(\sigma_i))g_{ki}(\eta|\sigma_i)d\eta dy. \quad (10)$$

However, since f_x is of bounded variation, (8) implies that:

$$\lim_{k\to\infty} |\int f_x(y-\eta;X^*_{ki})g_{ki}(\eta|\sigma_i)d\eta| \leq \lim_{k\to\infty} \frac{1}{k} \int |f_x(y - \eta;X^*_{ki})|d\eta \leq \lim_{k\to\infty} \frac{M}{k} = 0.$$

Since $v^*_{ki}(y) \varepsilon [0,B]$, for all y, (10) cannot hold. Hence for k sufficiently large ($v^*_{ki} \equiv u^o, X^*_{ki} \equiv 0$), and $P_c(v^*_{ki},X^*_{ki},G_k) = P^o$.

By Lemma 1, S_T and P_T are independent of G, so that the left hand side of (9) is at least P^o and is constant along the sequence $\{G_k\}$. Q.E.D.

Our final result concerns the relative efficiency of tournaments and contracts as the number of agents grows. We will consider sequences of distributions $\{G_n(\eta,\sigma_1,\ldots,\sigma_n)\}_{n=2}^{\infty}$ such that the marginal distribution function for η is unchanged throughout.

$$\int_{\sigma\varepsilon R^n} dG_n(\eta,\sigma) = \bar{G}(\eta) , \qquad \text{for all } n = 2,3,\ldots. \tag{11}$$

We will show that as $n \to \infty$, not only does the optimal tournament dominate the optimal contract, but in fact the optimal tournament approaches the full-information solution. That is, as $n \to \infty$ the principal does as well as if he could observe η directly. There are two steps in the proof.

In Lemma 2 we show that any contract for which the payoff function is piecewise continuous and the agent's optimal effort level is unique, can be approximated arbitrarily closely by a payoff function that is a step function. Then in Lemma 3 we show that each of these step function contracts can be approximated arbitrarily closely by tournament with a sufficiently large number of players.

Let G satisfying (7) be given. As noted above, when G satisfies (7) we can restrict attention to contracts (v,X) for which $X(\sigma_i) \equiv \bar{X}$ is a constant function. For these contracts $S_{ci} = S_c$ and $P_{ci} = P_c$, for all i. First we will show that given any feasible contract (v,X), we can construct a sequence of contracts $\{(v_k,\bar{X}_k)\}_{k=1}^{\infty}$ such that v_k is a step function with k steps, $\bar{X}_k$ is a constant function, and $v_k \to v$ in measure.

Let $(v(y),\bar{X}) \in S_c(G)$ be given, and let $I_{k1},\ldots,I_{kk}$, be intervals corresponding to quantiles of the distribution $F(;\bar{X})$:

$$I_{kj} \equiv \{z \mid (j-1)/k < F(z;\bar{X}) \leq j/k\}, \quad j = 1,\ldots,k; \; k = 1,2,3,\ldots. \tag{12}$$

Let $\bar{v}_{k1},\ldots,\bar{v}_{kk}$ be the expected payoff of the agent under $(v,\bar{X})$ on each of these intervals:

$$\bar{v}_{kj} \equiv \int_{I_{kj}} v(z)f(z;\bar{X})dz, \qquad j = 1,\ldots,k;\ k = 1,2,\ldots \tag{13}$$

Next, define the step function $\hat{v}_k(z)$ by:

$$z \in I_{kj} \quad ==> \quad \hat{v}_k(z) = \bar{v}_{kj}, \quad \text{for all } z,k; \tag{14}$$

as shown in Figure 1. Note that as $k \to \infty$, $\hat{v}_k(z) \to v(z)$ in measure. Finally, define:

$$\bar{X}_k \equiv \underset{x}{\text{argmax}} \int \hat{v}_k(z)f(z;x)dz\ , \qquad \text{for all } k; \tag{15}$$

$$c_k \equiv u^o + \bar{X}_k - \int \hat{v}_k(z)f(z;\bar{X}_k)dz\ , \qquad \text{for all } k; \tag{16}$$

$$v_k(z) = \hat{v}_k(z) + c_k\ , \qquad \text{for all } z,k. \tag{17}$$

Note that by construction, for G satisfying (7), $(v_k,\bar{X}_k) \in S_c(G)$, for all k.

__Lemma 2:__ Let F, G satisfying (7) and $(v,\bar{X}) \in S_c(G)$ be given. If v is piecewise continuous and if $\bar{X}$ is the unique solution of:

$$\underset{x}{\text{Max}} \int v(z)f(z;x)dz - x\ , \tag{18}$$

then for the sequence $\{(v_k,\bar{X}_k)\}_{k=1}^{\infty}$ defined by (12)-(17),

$$\lim_{k\to\infty} P_c(v_k,\bar{X}_k,G) = P_c(v,\bar{X},G).$$

and for k sufficiently large, $\bar{X}_k$ is the unique solution of:

$$\underset{x}{\text{Max}} \iint v_k(y)f(y - \eta;x)dG(\eta)dy - x .$$

Proof: Since f(;x) is continuous in x, $\hat{v}_k \to v$ in measure, and $\bar{X}$ is the unique maximizer of (18), from (15) it follows that $\bar{X}_k \to \bar{X}$. Hence $c_k \to 0$ and $v_k \to \hat{v}_k$. By construction then,

$$\lim_{k\to\infty} \int \gamma(v_k(z))f(z;\bar{X}_k)dz = \int \gamma(v(z))f(z;\bar{X})dz ,$$

so that the desired conclusion follows immediately. Q.E.D.

Next, we will show that for G satisfying (7), given any contract $(v,\bar{X}) \in S_c(G)$, where v is a step function, and $\bar{X}$ is a constant function, we can construct a sequence of tournaments that approximate it. Define y_{ni} by $F(y_{ni};\ \bar{X}) \equiv i/(n+1)$, as shown in Figure 2, and define $\hat{w}_{ni}$ by:

$$\hat{w}_{ni} \equiv v(y_{ni}). \tag{19}$$

Then where it exists define:

$$\bar{x}_n = \underset{x}{\text{argmax}} \int f(z;x) \frac{1}{n} \sum_{i=1}^{n} \hat{w}_{ni} \frac{\Phi_{in}(z;\bar{x}_n)}{f(z;\bar{x}_n)} dz - x ; \tag{20}$$

and let:

$$a_n \equiv u^o + \bar{X}_n - \frac{1}{n} \sum_{i=1}^{n} \hat{w}_{ni} , \quad (21)$$

$$w_{ni} \equiv \hat{w}_{ni} + a_n , \qquad i = 1,\ldots,n. \tag{22}$$

Lemma 3: Let F, G satisfying (7), and $(v,\bar{X}) \in S_c(G)$ be given, where v is a step function, $\bar{X}$ is a constant function, and $\bar{X}$ is the unique solution of (18). Let $\{(\hat{w}^n)\}_{n=1}^{\infty}$ and $\{(w_n,\bar{x}_n)\}_{n=1}^{\infty}$ be the sequences defined by (19)-(22). For n sufficiently large, $\bar{x}_n$ as defined by (20) exists and $(w_n,\bar{x}_n) \in S_T(n)$. Moreover,

$$\lim_{n\to\infty} P_T(n,w_n,\bar{x}_n) = P_c(v,\bar{X},G) , \tag{23}$$

Proof: Consider a representative player j in an n-person tournament with prizes $\hat{w}_n$. Suppose that the n-1 other players all adopt the effort level $\bar{X}$, and consider the conditional distribution of prizes for player j, given that his observed output is y_j. Unless y_j is a point of discontinuity in the step function v, as the number of players increases, by the law of large numbers the conditional probability that j has a rank order such that his prize is equal to $v(y_j)$ approaches one. That is:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \hat{w}_{ni}\Phi_{in}(z;\bar{X})/f(z;\bar{X}) = v(z;\bar{X}) \text{ , for almost all z.}$$

Since $\bar{X}$ is the unique solution of (18), for n sufficiently large, $\bar{x}_n$ as defined by (20) exists, and $\lim_{n\to\infty} \bar{x}_n = \bar{X}$. Thus (23) follows immediately.

Q.E.D.

As a reference point, we want to define the optimal contract and expected payoff for a principal who can obseve η. If the principal can observe η directly, the optimal contract (v^*,X^*) is independent of G, and has the form:

$$v(y,\eta) = v^*(y - \eta) \text{ , for all } y,\eta;$$

$$X(\eta) = X^*, \qquad \text{for all } \eta;$$

The principal's expected payoff in this case is given by:

$$P^* \equiv \int (z - v^*(z))f(z;X^*)dz$$

Moreover, note that for any G^o satisfying (7):

$$(v^*,X^*) = \underset{(v,X)\varepsilon S_c(G^o)}{\operatorname{argmax}} P_c(v,X,G^o) .$$

<u>**Proposition 3:**</u> Let F, and $\{G_n\}_{n=2}^{\infty}$ satisfying (3) and (11) be given, and let (v^*,X^*) and P^* be as defined above. Then,

$$\lim_{n\to\infty} \underset{(w,\bar{x})\varepsilon S_T(n)}{\operatorname{Max}} P_T(n,w,\bar{x}) = P^* \geq \underset{(v,X)\varepsilon S_c(G)}{\operatorname{Max}} P_c(v,X,G) . \tag{24}$$

The inequality is strict unless $\bar{G}$ defined in (11) satisfies (7) or $P^* = P^o$.

<u>**Proof**</u>: Let $\{(v_k,X_k)\}_{k=1}^{\infty}$ be the sequence of contracts defined for (v^*,X^*) by (12)-(17), and for each k = 1,2,..., let $\{(w_n^k,\bar{x}_n^k)\}$ be the sequence of tournaments defined by (19)-(22). By Lemma 2, for any G satisfying (7):

$$\lim_{k\to\infty} P_c(v_k,X_k,G) = P^*,$$

and by Lemma 3,

$$\lim_{n\to\infty} P_T(w_n^k,\bar{x}_n^k,n) = P_c(v_k,X_k,G).$$

Since by Lemma 1, $P_T(w_n^k, \bar{x}_n^k, n)$ is independent of G,

$$\lim_{n\to\infty} \max_{(w,\bar{x})\in S_T(n)} P_T(w,\bar{x},n) \geq \lim_{k\to\infty} \lim_{n\to\infty} P_T(w_n^k, \bar{x}_n^k, n) = P^*.$$

However, unless $\bar{G}(\eta)$ satisfies (7) or $P^* = P^o$, the righthand side of (24) is strictly less than P^* (Holmstrom [1979], Corollary 2). Q.E.D.

4. Conclusions

In the model above, all agents' output levels are subject to the same random shock. Thus, the output levels of the group provide the principal with information about the value of the common shock, and consequently about the portion of any particular agent's output that is attributable to effort. Relative performance schemes--of which tournaments are an extreme form--allow the principal to make use of this information.

Obviously tournaments employ available information in a rather inefficient and inflexible way. In the model above, tournaments tend to reduce the randomness of any agent's compensation by filtering out the common shock term. However, they also tend to increase the randomness in any agent's compensation by making his reward depend on the idiosyncratic shocks of his peers. Proposition 1 and 2 show that the relative advantage of tournaments vis a vis contracts depends on which effect dominates.

Despite the fact that a tournament makes inefficient use of information, Proposition 3 shows that this entails no loss if the number of agents is sufficiently large. In large groups, the rank order of an agent's observed output is a very accurate esimator of his output net of the common shock.

Tournaments are not, in general, "optimal" contracts. Why then are rankings so commonly used as an evaluation criterion? First, it may be substantially easier to determine agents' rankings than to measure their output levels. In addition, as shown in Lemma 1 neither the set of feasible tournaments nor the optimal tournament depends on the distribution function for the common shock and agents' signals. This is an obvious advantage if that distribution is unknown or imprecisely known--as it would be, for example, in nonstationary environments. Since in large group the inefficiency due to information loss is negligable, tournaments will perform very well in such settings.

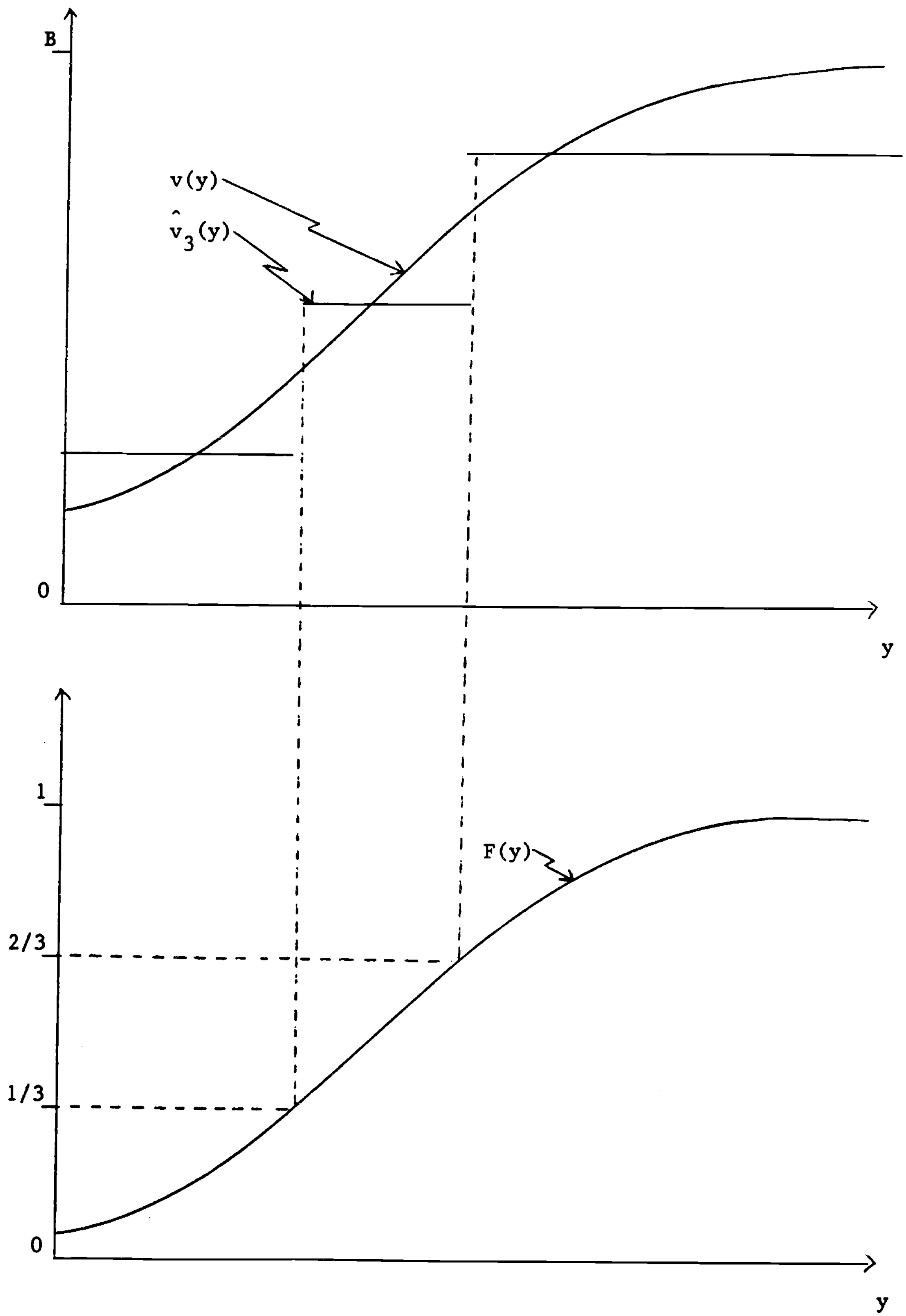

Figure 1

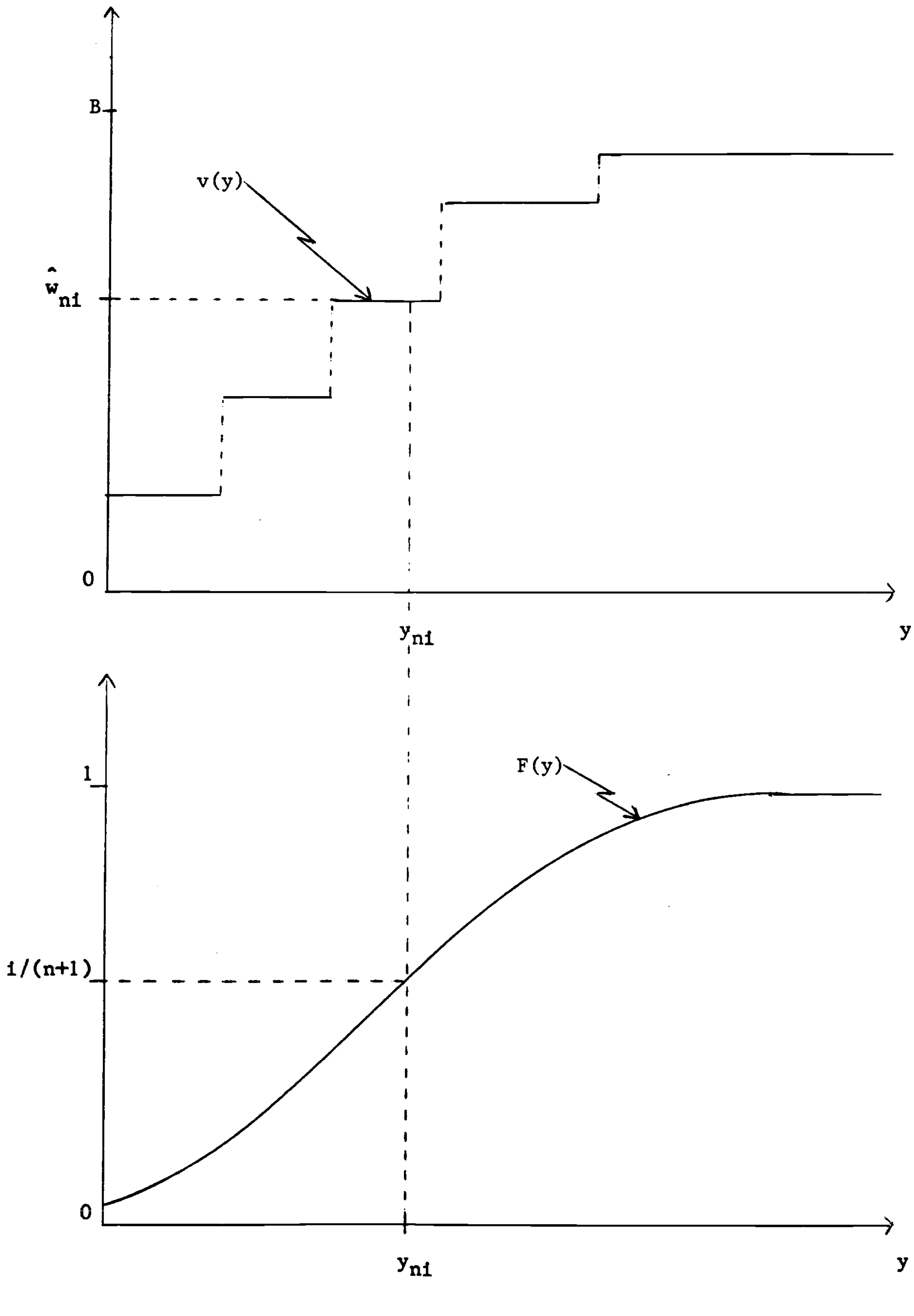

Figure 2

FOOTNOTES

[1]The fact that "effort enters linearly into each agent's utility function involves no loss of generality. Since "effort" is never aggregated across agents, units of effort are simply defined as whatever causes units of disutility to the agent. Under this units convention F reflects both the concavity of the agent's utility function in "hours" (or some other standard unit of measurement), as well as the concavity of the production function in "hours." If at least one of these functions is strictly concave, then (3) holds. The agent's utility of income is bounded to avoid problems of the type discussed by Mirrlees [1975].

[2]For arbitrary prize structures, there may be no Nash equilibrium--symmetric or otherwise. This is of no importance to us, since we are considering only tournaments that are designed so that they do have a symmetric Nash equilibrium. The restriction to tournaments with symmetric equilibria is purely on the grounds of tractability.

[3]This result is (almost) a special case of Theorem 10 in Holmstrom [1981]. (In Holmstrom's model agents do not have access to any information about the common shock.) Proposition 1 is included here for the sake of completeness.

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