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EXPECTATIONS AND THE VALUATION OF SHARES

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ABSTRACT

This is a study using a unique body of expectations data collected over the decade of the 1960s. After describing the data, this paper first looks at the extent of consensus among those financial institutions providing the forecasts and measures the accuracy of the forecasts. We then ask if the forecasts are consistent with the hypothesis that the expectations are "rational".

We then turn to the relationship of the forecasts to security valuation. We develop our own variant of the popular capital asset pricing model using a framework suggested by Ross for this arbitrage model. Alternative specifications are developed relating expected returns to risk variables and relating securities prices to expectations and risk variables. We find that the expectations data of the sort we have collected do appear to influence security prices in the manner suggested by the theory.

We also find that the expected security returns implied by the expectations data are related to "systematic" risk measures appropriately defined. Nevertheless, we find that, even when a variety of systematic influences are used, other risk measures, possibly related to their own variance of the securities, appear to play some role in security valuation.

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For years economists have emphasized the importance of expectations in a variety of problems.[1] The extent of agreement on the significance of expectations is almost matched, however, by the paucity of data that can be considered even reasonable proxies for these forecasts. One area in which expectations are highly important is the valuation of the common stock of a corporation. The price of a share and the anticipated future returns from stocks are determined primarily by investors' current expectations about the future values of variables that measure the relevant aspects of corporations' performance and profitability, particularly the anticipated growth rate of earnings and dividends per share.[2] Moreover, modern financial literature has emphasized the link between anticipated risk and return.[3] This theoretical emphasis is matched by efforts in the financial community where security analysts spend considerable effort in forecasting the future earnings and in judging the risk levels of the companies they study.

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[1] A number of studies of anticipations data have been collected in two National Bureau Volumes (1955) and (1960). Some more recent work on the assessment of expectations or forecasts has been done by Zarnowitz (1967), Mincer (1970), Pesando (1975), and Friedman (1978).

[2] The classic theoretical statement of the anticipations view of the determination of share valuation may be found in J. B. Williams (1938). This position is also adopted in the standard text in the field by Graham, Dodd, and Cottle (1962). The emphasis on the importance of earnings growth on the prices of common stocks can be found in Gordon (1962), Holt (1962) and Malkiel (1965).

[3] The capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966) has grown into a widely used and refined approach to security valuation.

This is a study using a unique body of expectations data collected over the decade of the 1960s. After describing the data, this paper first looks at the extent of consensus among those financial institutions providing the forecasts and measures the accuracy of the forecasts. We then ask if the forecasts are consistent with the hypothesis that the expectations are "rational." We then turn to the relationship of the forecasts to security valuation. We develop our own variant of the popular capital asset pricing model using a framework suggested by Ross (1977) for his arbitrage model. Alternative specifications are developed relating expected returns to risk variables and relating securities prices to expectations and risk variables. We find that the expectations data of the sort we have collected do appear to influence security prices in the manner suggested by the theory.

We also find that the expected security returns implied by the expectations data are related to "systematic" risk measures appropriately defined. Nevertheless, we find that, even when a variety of systematic influences are used, other risk measures, possibly related to the own variance of the securities, appear to play some role in security valuation.

1. The Data Employed

The data used in the study were collected from 17 major investment firms during the decade of the 1960s.[4] 178 individual

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[4]We are deeply grateful to the participating firms, who wish to remain anonymous.

companies were covered in our survey. The company sample was not selected as a random sample but rather on the basis of data availability. The companies included in the sample thus tended to be the large corporations in whose securities investment interest is centered.

It should be emphasized that each company was not predicted by each forecaster in each year. Indeed, there is unfortunately a substantial lack of overlap, which seriously hampers our ability to do time series and other analyses, as we shall see below. For a company to be included in the sample at least three securities firms had to furnish expectations estimates in each of two years. But no one forecaster ever made forecasts for all the 178 companies. Furthermore, some forecasters were available only for two or three years. Only one forecaster was available for all nine years of the sample period.

We would not argue that these estimates necessarily give an accurate picture of general market expectations. It would, however, seem reasonable to suggest that they are representative of opinions of some of the largest professional investment institutions and that they may not be wholly unrepresentative of more general expectations.[5] Also, insofar as other security analysts and investors follow the same sorts of procedures as those used by

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[5]That several of our participating firms found it worthwhile to publish these projections and provide them to their customers provides _prima facie_ evidence that a certain segment of the market placed some reliance on such information in forming its own expectation.

our sample security analysts in forming expectations, general investors' expectations would resemble those of the analysts.

Consequently, these predictions may well serve as acceptable proxies for market expectations and they surely seem worthy of detailed analysis. It should be noted that security analysts are not limited to published data in forming their expectations. They frequently visit the companies they study and discuss the corporations' prospects with their executives.

The major data collected concerned estimated growth rates for corporate earnings over the short term (explicitly, for the following year) and over the long term (for the following five years). The growth rates of the individual predictors were used separately in a study of forecast consensus and accuracy reported in the next sections and they were averaged for use in studies of security valuation reported subsequently. These forecasts are also of particular interest because one can observe divergence of opinion among different individuals dealing with the same quantities. Thus, as we shall see, it is possible to use a measure of the dispersion of these forecasts as an expectational measure.

In addition to the expected growth figures, we collected analysts' estimates of so-called "normalized earnings" for each company each year. These normalized earnings are estimated to be the earnings that would obtain at a normal level of economic activity if the company were experiencing normal operations--that is, operations not affected by such nonrecurring items as strikes, natural disasters, and so forth. Also, one firm provided an "instability index" of earnings which represented a measure of the past variability of earnings (around trend) adjusted by the security analyst to indicate potential future variability.

2. The Extent of Consensus Among the Predictors

The question of agreement among predictors is important in the development of the capital-asset pricing model. It has been traditional in that literature to assume that all market predictors have identical expectations concerning all security returns. Our data reveal that this assumption is clearly invalid. Simple correlations and Spearman rank correlation coefficients were calculated between each pair of long and short-term predictors. In addition, Kendall's coefficient of concordance was calculated where a substantial overlap occurred in the companies covered.

Although this analysis indicated that there was considerable consensus among predictors, the lack of agreement the correlations reveal is substantial. Even among the long-term predictions, where agreement was greatest, the average correlation between pairs of predictors was only about 3/4. This indicates that little more than half the variance in one individual's predictions can be "explained" by agreement with another analyst. Interpreted differently, at most 75 percent of both predictors) variances could be accounted for by variation common to each. Among the short-term predictors, agreement was substantially less. On average the correlation coefficients between pairs of predictors was not much above 1/2. What clearly emerges here is a finding of considerable non-homogeneity of expectations. This finding in turn has significant implications for the study of capital-asset pricing.

The extent of agreement among the predictors can also be evaluated by comparisons of the predicted growth rates with earlier predictions and with historical growth rates of earnings. We found that the correlations of long-term growth predictions with historical growth rates are generally high and not much different from those found in comparing the predictions with each other. These comparisons suggest that the apparent agreement among the predictors may reflect little more than use by all of them of the historic figures. In investigating this possibility, the partial correlations among the predictions of long-term growth, holding constant past growth rates, were calculated. These correlations were considerably smaller than the simple correlations, though almost all were significant beyond the .05 level. Thus, while a substantial part of the agreement among predictors appears to result from their use of historic growth figures, there is also evidence that security analysts tend to make similar adjustments to the past growth rates.

Finally, we examined the extent of agreement among predictors by comparing their forecasts with the price-earnings ratios of the corresponding securities. By utilizing a normative valuation model (see e.g., those cited in note 2) it is possible to calculate an implicit growth rate from the market-determined earnings multiple of a security. This model ignores differences in risk among securities which complicate the calculation. Nevertheless, comparisons of the predictions with price-earnings ratios may be interpreted as examinations of the relationship between the forecasts and market-expected growth rates. The correlations between the predicted growth rates and these implicit growth rates are about the same as those obtained when the forecasts were compared with each other. Since price-earnings multiples are affected by several variables other than expected growth rates, this exercise underscores the extent of disagreement among the forecasters.

3. The Accuracy of the Predictions

Two basic methods were employed to evaluate the accuracy of the predictors: The first method of evaluation employed simple parametric and nonparametric correlations between predictions and realizations. In order to remove the scale effect from per share figures, all per share predictions and realizations were converted into percentage changes from the values as of the date the predictions were made.

The second method of evaluation was to calculate an inequality coefficient similar to that developed by Theil (1966). This coefficient is given by

$$(3.1) \quad T = \frac{\sum_{i=1}^{N} (P_i - R_i)^2}{\sum_{i=1}^{N} R_i^2}$$

where P_i is the predicted and R_i the realized growth rate for the i^{th} company. The inequality coefficient gives a comparison between perfect prediction ($T = 0$) and a naive prediction of zero growth for all corporations ($T = 1$). Thus, the higher the inequality coefficient the worse the predictions. It is even possible, of course, for the inequality coefficient to be greater than unity. In such a case, the predictors would have done worse than a naive prediction of zero growth for all companies.

Using this framework, we also investigated the extent to which errors in predictions were related to 1) errors in predicting the average overall earnings growth of the sample firms; 2) errors in predicting the average growth rate of particular industries; and 3) errors in predicting the growth rates of firms within industries. To accomplish this, we decomposed the numerator of (i) into three parts. The first comes from the average prediction for all companies not being equal to the average realization. The second part arises from differences among the average industry predictions not being equal to the corresponding differences among industry realizations. The third arises from the differences in predictions for the corporations within an industry not being the same as the differences in realizations.

Letting P_{kj} and R_{kj} be the predicted and realized growth rates for the k^{th} company ($k = 1,\ldots,N_j$) in the j^{th} industry ($j = 1,\ldots,J$), we can write the numerator of (3.1) as:

$$(3.2)\quad \sum_{j=1}^{J} \sum_{k=1}^{N_j} (P_{kj} - R_{kj})^2 = [\sum_{j=1}^{J} N_j(\bar{P} - \bar{R})^2] +$$

$$[\sum_{j=1}^{J} N_j[(\bar{P}_j - \bar{P}) - (\bar{R}_j - \bar{R})]^2] +$$

$$[\sum_{j=1}^{J} \sum_{i=1}^{N_j} [(P_{kj} - \bar{P}_j) - (R_{kj} - \bar{R}_j)]^2] \quad ,$$

when $\bar{P}_j$, $\bar{R}_j$ are the averages for the j^{th} industry and $\bar{P}$ and $\bar{R}$ are the overall means. The three terms in square represent the decomposition referred to above. The proportions of T arising from these three sources will be called T^M, T^{BI}, and T^{WI} respectively for mean errors, between-industry errors, and within-industry errors.

Statistics summarizing the forecasting ability of the short-term predictors from 1961 through 1968 are presented in Table 3-1. Here we present the correlation coefficients (r), the number of observations (N), Spearman's rank correlation coefficients (ρ) and inequality coefficients (T) comparing the predictions of short-term (one-year) earnings growth for the 11 firms (out of 17) that provided forecasts with the realized growth rates. By and large, the correlations of predicted and realized growth rates are fairly low, although most of them are significantly greater than zero, and the inequality coefficients are large. Indeed, in many cases, the inequality coefficients are larger than 1. The only exception to this seems to be the superior short-term forecasts of Predictor E. However, this apparent superiority is largely illusory: Predictor E tended to concentrate on large,

Table 3-1

Comparison of Earnings Growth Forecasts of Security Analysts with Realized One-Year Earnings Growth for the Year Following the Forecast

ictor Term (one- Forecasts	1961 Predictions				1962 Predictions				1963 Predictions				1964 Predictions				1965 Predictic			
	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T
	.47	109	.22	1.11	.18	112	.21	1.01	.21	112	.21	.73	.34	111	.36	.59	.33	112	.42	1.0
	.40	63	.20	.80	.41	64	.25	.80	.52	64	.27	.46	.51	62	.37	.44	.58	66	.39	.4
	.20	106	.15	1.10	.03	126	.11	1.00	.45	126	.49	.51	.52	128	.34	.55	.43	129	.40	1.2
	.29	186	.21	1.41	.03	192	.06	1.07	.04	184	.02	.82	.59	115	.33	.49	.15	117	.24	.7
	.87	30	.68	.16	.63	32	.39	.36	.44	31	.57	.37	.40	35	.49	.47	.55	36	.57	.5
	--	--	--	--	--	--	--	--	.42	74	.27	.60	.44	74	.28	.49	.57	76	.47	.4
	.49	143	.24	.70	.01	146	.09	1.24	.31	166	.13	.66	.33	161	.14	.75	.15	163	.16	.8
	.25	42	.30	1.61	.44	48	.38	.79	.19	55	.28	.72	.63	54	.35	.36	.39	56	.45	.6
	.56	176	.20	.63	.34	187	.16	.84	.20	191	.18	.74	.34	191	.24	.75	.27	194	.35	1.1
	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	.44	52	.28	.7
	.36	87	.36	.81	.29	88	.25	.73	.34	88	.15	.56	.35	87	.36	.57	.54	88	.43	.5

	1966 Predictions				1967 Predictions				1968 Predictions			
	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T
	--	--	--	--	-	--	--	--	.10	108	-.09	.97
	.14	165	-.15	1.09	.10	155	.05	.97	.04	142	-.08	.95
	.23	154	.18	1.37	.28	151	.21	.91	.30	150	.23	1.21

r = correlation coefficient

N = no. of observations

ρ = Spearman's Rho

T = Theil statistic

relatively stable companies; and, we suspect, predictions were made only when there was *a priori* reason to believe that the forecasts would be reliable. The validity of this conjecture is suggested by the finding that all the other forecasters did better for the set of companies for which Predictor E make forecasts than for the larger set.

In Table 3-2 we present a comparison of the 5-year earnings growth forecasts of nine of our forecasters who provided long-term predictions. The correlation coefficients in Table 3-2 tend to be somewhat higher and the inequality coefficients lower than those found in assessing the value of the short-term predictions. For these long-term predictions, a naive prediction of average long-run GNP growth tended to be slightly inferior to our predictors. In the case of the short-term predictors, the naive forecast produced lower mean squared errors. As we shall see below, there was a tendency in the early years for the forecasters to underestimate the realized growth. In later years, however, we did not find any systematic evidence of underestimation of change.

Turning to the industry breakdowns of the forecasts, we found that failure to forecast industry means accounted for only a very small proportion of the inequality coefficient. The main sources of inequality were the within-industry errors.

Looking at the correlations of predictions with future growth rates within industries permits us to assess which industries were most difficult to forecast in an *ex post* sense. For the long-term predictions, we found that the correlation coefficients

Table 5-2

Comparison of Five-Year Earnings Growth Forecasts of Security Analysts with Realizations

ic-	1961 Predictions vs. Growth 1961-66				1962 Predictions vs. Growth 1962-67				1963 Predictions vs. Growth 1963-68				1964 Predictions vs. Growth 1964-69				1965 Predictions vs. Growth 1965-70			
	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T
	.50	119	.09	.38	.24	175	.03	.53	.21	173	-.02	.56	.27	173	.18	.74	.40	168	.33	[illegible]
	.33	116	.16	.39	.32	173	.16	.49	.25	171	.11	.53	.35	167	.33	.70	.34	165	.29	[illegible]
	.80	42	.41	.28	--	--	--	--	.48	122	.31	.43	.53	67	.46	.33	.33	72	.15	[illegible]
	--	--	--	--	.75	57	.39	.22	.75	59	.46	.21	.50	123	.49	.53	.40	121	.35	[illegible]
	--	--	--	--	.49	172	.34	.35	.42	172	.29	.39	.68	103	.28	.29	.37	145	.32	[illegible]
	.55	37	.21	.32	.62	62	.49	.39	.69	37	.40	.43	.43	163	.30	.67	.31	158	.33	1.[illegible]
	--	--	--	--	.57	61	.37	.34	.51	60	.32	.33	.38	173	.27	.69	.42	163	.36	1.[illegible]
	.36	60	.21	.39	--	--	--	--	--	--	--	--	.66	54	.57	.40	.64	55	.59	[illegible]
	--	--	--	--	--	--	--	--	--	--	--	--	.40	39	.22	.37	.53	45	.40	[illegible]

ic-	1966 Predictions vs.				1967 Predictions vs.				1968 Predictions vs.				1969 Predictions vs.			
	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T	r	N	ρ	T
	.38	112	.40	1.17	.10	108	.23	.90	.11	107	.16	.64	.05	118	.09	.63
	.09	160	.29	1.36	.09	148	.23	.96	.14	137	.14	1.29	.18	129	-.12	.85
	.15	162	.16	1.40	.36	153	.33	.88	.21	156	.32	.69	--	--	--	--

r = correlation coefficient

N = no. of observations

ρ = Spearman's Rho

T = Theil statistic

between predictions and realizations tended to be highest in the oil, food and stores, and a number of "cyclical" industries. For the short-term there were really no industries that were particularly easy to predict compared with the others. It is interesting to observe that the "stable" electric utility industry turned out to be one of the more difficult industries for which to make long-term forecasts.

In general, we had little success in associating forecasting success with any industry or company characteristics. The differences between industries in forecasting success were only moderately related either to the average growth rates to be realized or to the variances of the realized growth rates.

The picture that emerges thus far is one of rather mediocre performance by our sample of forecasters. Short-term forecasting performance may fairly be described as poor, while long-term forecasting success was only slightly better. A variety of supplementary tests will help us to buttress these conclusions and enable us to appraise more fully security analysts' ability to forecast.

The simple correlation coefficients reported in Tables 3-1 and 3-2 probably give more favorable an impression of predicting ability than is justified. The fairly high correlations of long-term forecasts with realizations may create an impression of quite satisfactory performance. In our study, however, two or three companies turned out to be rapid growers and easy to predict. Were these companies omitted, the correlations (especially the simple ones) would decline substantially. For example, if IBM, Polaroid, and Xerox are omitted from the sample, the correlation coefficients decline quite sharply. The importance of a few outliers in boosting the reported correlation coefficients weakens the evidence in favor of superior forecasting ability.

The record of the forecasters raises the question, "Does any naive forecasting device based on historical data yield as good forecasts as the painstaking efforts of the security analysts?" Several alternative historical growth rates were compared with the predictors' forecasts in order to assess their forecasting ability better. Our conclusions can be stated very briefly. Mechanically calculated growth rates and those based on historical rates of return are not very effective predictors. They tend to be inferior to the analyst. This finding is similar to that of I.M.D. Little (1962) for British corporations and Glauber and Lintner for U.S. companies. On the other hand there is a naive forecasting method that is as good as the analyst. As we shall describe below, the market determined price-earnings multiple was generally as good as the forecasts made by the analysts.

The question of the usefulness of security analysts forecasts is further examined in Table 3-3. The first row of the table demonstrates the potential benefits from accurate earnings forecasting during four of the nine years covered by our sample. It shows that if one could predict in advance what group of companies will actually realize above-average earnings growth and confine one's purchase to that group, an investor would have received well-above-average returns. In the columns of Table 3-3 we simply

Table 3-3

Mean Rate of Return Over Subsequent Year
For Alternative Groupings of Companies

	1962 Companies for Which				1963 Companies for Which			
	$q \geq \bar{q}$	N	$q < \bar{q}$	N	$q \geq \bar{q}$	N	$q < \bar{q}$	N
Short-term Realized Growth	0.352	92	0.126	166	0.257	98	0.117	159
Short-term Predicted Growth								
Predictor A	0.235	42	0.239	70	0.152	41	0.199	71
B	0.212	18	0.179	46	0.085	19	0.149	45
C	0.179	53	0.206	72	0.169	50	0.148	75
D	0.192	79	0.223	112	0.131	72	0.173	111
E	0.314	11	0.224	21	0.113	12	0.221	19
F	--	0	--	0	0.177	28	0.184	46
G	0.186	54	0.193	89	0.132	58	0.179	105
H	0.276	28	0.247	28	0.151	24	0.207	31
I	0.212	74	0.241	110	0.129	74	0.192	114
J	--	0	--	0	--	0	--	0
K	0.195	27	0.209	60	0.133	37	0.193	50
Long-term Predicted Growth								
Predictor 1	0.148	64	0.202	111	0.128	89	0.186	85
2	0.166	67	0.191	106	0.119	61	0.178	111
3	--	0	--	0	0.103	48	0.184	75
4	0.233	23	0.147	34	0.090	20	0.133	39
5	0.180	59	0.185	113	0.118	51	0.173	121
6	0.195	21	0.209	41	0.096	13	0.094	24
7	0.229	23	0.183	38	0.109	22	0.143	39
8	--	0	--	0	--	0	--	0
9	--	0	--	0	--	0	--	0

Table 3-3 (continued)

Mean Rate of Return Over Subsequent Year
For Alternative Groupings of Companies

	1964				1965			
	Companies for Which				Companies for Which			
	$g \geq \bar{g}$	N	$g < \bar{g}$	N	$g \geq \bar{g}$	N	$g < \bar{g}$	N
Short-term Realized Growth	0.459	95	0.076	162	0.034	107	-0.142	148
Short-term Predicted Growth								
Predictor A	0.350	47	0.153	64	-0.025	50	-0.110	62
B	0.349	30	0.091	32	-0.002	22	-0.107	43
C	0.232	69	0.053	57	-0.074	65	-0.095	61
D	0.282	48	0.119	67	0.008	42	-0.087	74
E	0.388	12	0.248	23	0.048	13	-0.143	22
F	0.231	29	0.159	45	-0.059	30	-0.099	45
G	0.323	60	0.198	98	-0.050	65	-0.082	94
H	0.213	21	0.153	33	-0.056	23	-0.111	32
I	0.301	95	0.233	93	-0.004	80	-0.120	109
J	--	0	--	0	-0.131	17	-0.109	34
K	0.188	50	0.081	36	-0.081	34	-0.101	52
Long-term Predicted Growth								
Predictor 1	0.204	87	0.169	87	-0.038	67	-0.083	102
2	0.228	74	0.144	94	-0.049	75	-0.082	92
3	0.437	23	0.112	44	-0.001	22	-0.055	50
4	0.259	57	0.128	67	-0.031	55	-0.102	69
5	0.331	28	0.119	76	-0.023	64	-0.099	83
6	0.250	68	0.143	96	-0.031	68	-0.098	93
7	0.238	75	0.147	99	-0.005	63	-0.102	104
8	0.342	20	0.151	35	0.024	21	-0.107	34
9	0.105	16	0.147	23	-0.102	18	-0.100	27

divided the companies into two groups by cross-sectional average. Column 1 shows the group for which the short-term realized (or predicted) earnings growth was greater than average. Column 2 shows the results for the companies for which the realized (or predicted) earnings growth was less than average. In each of the four years shown, investors could have earned higher rates of return by investing in companies for which realized earnings growth was greater than the average earnings growth in the sample. (Similar results were found for other years.)

The rest of Table 3-3 runs the same experiment for the predictors A through K who made short-term predictions, and 1 through 9 who forecast long-run earnings growth. The numbers in the table are average rates of return including dividends and capital gains or losses for the group of companies for which above- and below-average growth was forecast. This test allows us to see if there is any useful information in the forecasts that would permit an investor to realize superior returns. The answer seems to be that the forecasts offer no consistent help. In 1963, investors would have done worse in buying the companies for which the analysts forecast larger growth. In 1964 and 1965 they would have done better. 1962 is mixed. Similar kinds of results were found for other years. This is precisely the sort of finding one would expect in a random process.

In summary, we find the value of analysts predictions of future earnings to be open to serious question. I.M.D. Little and Glauber and Lintner have convincingly shown that earnings growth in past periods is not a useful predictor of future earnings growth. The remarkable conclusion of our analysis is that the careful estimates of the security analysts participating in our survey, the bases of which are not limited to public information, perform little better than these past growth rates. Moreover, as we shall see below, the market price-earnings ratios themselves were as good as the analysts' predictions and the best forecasts based on historical data. This suggests that whatever little information content there is in security analysts' forecasts is quickly impounded into market prices.

4. Are the expectations "rational"?

In this section we report on several tests of the "rationality" of our forecasters. At the outset we should stress that for a variety of reasons these tests are at best only slightly indicative of whether the rational expectations hypothesis is appropriate, though the analyses may be of interest in any case to indicate features of our data and the quantities being predicted.

The difficulty with using our data to investigate the rational-expectations hypothesis is that they are primarily cross-sectional. Unfortunately, we do not have consistent time series of forecasts, despite having nine years of data. Even nine observations would be of rather limited use for time-series analysis, and we do not have data for our forecasters in all years. Some forecasters were in the sample for only two or three years. Moreover, even when a forecaster was available throughout the forecast period, that forecasting firm did not cover each of the companies in our sample in all years. Thus, many obvious time-series tests of forecast rationality have not been performed because of lack of available data.

To make an investigation we shall have to treat the forecasts made for different companies as giving a random sample of forecasts. Such an assumption would, of course, also have been implicit in the comparisons made in section 3, had they been used to draw formal inferences. The problem with this assumption may be two-fold. First, realizations of earning growth may be highly correlated with a few major influences, particularly the over-all growth of the economy. Insofar as differences among forecasts reflect implicitly a forecast for the economy and the forecaster's preceptions of the sensitivity of different firms to the level of the economy, differences in realization from forecasts may be positively or negatively correlated with predictions to reflect this common feature. Secondly, differences between realizations and forecasts may be correlated across firms, affecting the significance levels and powers of tests. Unfortunately, there is no obvious _a priori_ structure to impose on such correlations, while we lack observations sufficient to estimate them. Hence, we shall have to make the standard cross-sectional assumption of independent residuals.

A critical requirement for considering a forecast to be "rational" in the sense used by Muth (1961) is that the mathematical expectation of the conditional realization on the forecast should be the forecast, i.e., we can express the realized growth over period t as

(4.1) $G_{R,t} = {}_tg_{p,t-1} + u_t$,

where ${}_tg_{p,t-1}$ is the predicted growth rate for period t, made at period t-1, and u_t is a random variable with mean zero uncorrelated with ${}_tg_{p,t-1}$. Following Theil (1966) and Friedman (1978), who recently tested the "rationality" of interest-rate forecasts, we regress the forecasters' predictions (${}_tg_{p,t-1}$) on the corresponding realizations ($G_{R,t}$) according to

(4.2) $G_{R,t} = a + b\,{}_tg_{p,t-1} + u_t$,

and investigate the null hypothesis of unbiasedness defined by H_o: $(a,b) = (0,1)$.

Table 4-1 presents the results of our experiments for the long-term forecasters for whom comparatively long runs of data are available. The findings are typical: in general we find that the b coefficients are not significantly different from unity. Some constant terms are significantly positive, but the majority do not differ from zero significantly. Testing the joint hypothesis H_o, we find that in the vast majority of cases we cannot reject a finding of unbiasedness. Unfortunately, it does not seem to be appropriate to aggregate these results over time. The five-year nature of the forecasts and of the realizations means that successive values of u_t may very well be correlated. For this reason also, one would not expect in this case a lack of serial correlation between the u_t to be an implication of rational expectations.

The pattern found for the short-term forecasts is rather different from the longer-term ones. Table 4-2 performs much the same experiments with five short-term forecasters.[1] Here we find

[1] Unfortunately, the predictors who forecast for five ... years did not make the forecasts later than 1965.

Table 4-1

Long-Term Forecasts

Estimates of Equation 4.2

Prediction Period	Predictor 2 Intercept (std.error)	Predictor 2 $\hat{b}$ (std.error)	Predictor 3 Intercept (std.error)	Predictor 3 $\hat{b}$ (std.error)	Predictor 6 Intercept (std.error)	Predictor 6 $\hat{b}$ (std.error)
66/61	.057 (.014)	.803 (.217)			-.036 (.054)	1.57 (.398)
67/62	.042 (.012)	.924 (.211)			-.054 (.040)	1.84 (.375)
68/63	.057 (.011)	.570 (.203)	.015 (.013)	1.18 (.167)	-.035 (.028)	1.43 (.254)
69/64	-.008 (.017)	1.25 (.232)	-.016 (.021)	1.31 (.259)	-.035 (.019)	1.43 (.258)
70/65	-.047 (.018)	1.15 (.273)	-0.11 (.020)	.692 (.249)	-.029 (.015)	.808 (.193)
71/66	-.055 (.016)	1.06 (.240)	-0.007 (.017)	.260 (.237)	-.032 (.019)	.550 (.238)
72/67	.023 (.018)	.281 (.258)	.029 (.019)	.205 (.230)	-.025 (.016)	.819 (.173)
73/68	.043 (.015)	.474 (.211)	.055 (.011)	.112 (.079)	.024 (.015)	.459 (.168)
74/69	.069 (.031)	.236 (.370)	.113 (.020)	-.533 (.264)		

that the null hypothesis; a=0, b=1, can generally be rejected except for predictor E. Furthermore, we may now look at whether the residuals are serially correlated--that is whether any information contained in the extent to which the previous forecast erred seems to be fully incorporated in the forecast. These correlations are also reported in Table 4-2--using the residual from the fitted versions of (4.2) rather than those from (4.1). We find the correlation coefficient of the residuals to be significant at the .05 level in the vast majority of cases. Such correlation of forecast errors is inconsistent with expectations rationality.

To shed further light on this issue, Table 4-3 presents the average errors of our short-term and long-term forecasters taken as a group. The average errors are measured by the differences between the realization and the average prediction of those forecasters making predictions for the companies in question. In the early period, both the short and long-term predictors tended to make forecasts that were too low. Realizations tended to exceed forecasts during most of this early period. This may simply indicate a failure to anticipate the continuation of the economic expansion experienced through the period. It may also reflect the underestimation of change frequently found in investigating forecasts and reported by Theil (1966). In the later period, however, a similar underestimation of change did not characterize the forecasts. In some years these forecasts were too high, but during other years they were too low. The only consistent pattern found was that in years when the forecasts were too high (low), all the forecasters

Table 4-2

Short-Term Forecasts

Estimates of Equation 4.2

Production Period	Predictor A: Intercept (std. error)	Predictor A: $\hat{b}$ (std. error)	Predictor A: Correlation of residuals	Predictor B: Intercept (std. error)	Predictor B: $\hat{b}$ (std. error)	Predictor B: Correlation of residuals
62/61	.066 (.025)	.369 (.068)		.012 (.026)	.669 (.141)	
			.08			-.01
63/62	.108 (.026)	.217 (.143)		.055 (.020)	.419 (.122)	
			.28*			.29*
64/63	.184 (.024)	.309 (.130)		.111 (.025)	.539 (.136)	
			.44*			.56*
65/64	.118 (.022)	.533 (.141)		.060 (.021)	.747 (.159)	
			.38*			.23*
66/65	.067 (.018)	.332 (.092)		.004 (.023)	.823 (.150)	

Production Period	Predictor C: Intercept (std. error)	Predictor C: $\hat{b}$ (std. error)	Predictor C: Correlation of residuals	Predictor D: Intercept (std. error)	Predictor D: $\hat{b}$ (std. error)	Predictor D: Correlation of residuals
62/61	.054 (.022)	.254 (.120)		.056 (.019)	.256 (.063)	
			-.11*			.03
63/62	.085 (.012)	.131 (.130)		.104 (.016)	.143 (.123)	
			.24*			.34*
64/63	.100 (.017)	.585 (.103)		.140 (.018)	.318 (.114)	
			.44*			.28*
65/64	.086 (.014)	.602 (.089)		.077 (.016)	.751 (.139)	
			-.38*			.23*
66/65	.210 (.034)	-.131 (.249)		.077 (.021)	.292 (.165)	

*Significant at the .05 level.

Table 4-2 Continued

Production Period	Predictor E: Intercept (std.error)	Predictor E: $\hat{b}$ (std.error)	Correlation of residuals
62/61	.013 (.024)	.892 (.101)	
			-.35*
63/62	.037 (.037)	.870 (.195)	
			.08
64/63	.129 (.042)	.534 (.213)	
			.51*
65/64	.096 (.038)	.538 (.211)	
			.26*
66/65	.029 (.031)	.684 (.180)	

*Significant at the .05 level.

Table 4-3

Average Forecast Errors Across Sample

Long-Term Forecasts

Forecast Period	Average Error
1966/61	0.0450
1967/62	0.0521
1968/63	0.0300
1969/64	0.0014
1970/65	-0.0336
1971/66	-0.0667
1972/67	-0.0308
1973/68	-0.0246
1974/69	0.0126

Short-Term Forecasts

1962/61	-0.0328
1963/62	0.0586
1964/63	0.1062
1965/64	0.0345
1966/65	0.0315
1967/66	-0.0841
1968/67	0.0441
1969/68	-0.0288

tended to be too high (low). Moreover, insofar as the continuation of economic expansion lies behind the results, it does not seem to lie behind the values of $\hat{b}$ that were found, since we do not find accentuation of forecast differences as we would expect.

Another important property of rational expectations is that they efficiently incorporate all available information including the information contained in previously realized outcomes. Thus, the change in a forecast made for a given future time should not be related to errors made prior to the current forecast. Any useful information contained in the forecast error during period t-2 should have affected the t-1 forecast and should not affect the change in the forecast from period (t-1) to t.

Unfortunately, this notion cannot be applied easily to our data, because successive forecasts are for different times. We may, however, consider the hypothesis that the short-term growth rate can be considered a parameter following a random walk measured with error and so investigate the usual error-learning hypothesis on a cross-sectional basis. Explicitly, this hypothesis states that the <u>change</u> in forecast can be considered a linear function only of the difference of the most recent realization from its forecast value and not of previously observed differences. Specifically, we can investigate

$$(4.3) \quad {}_{t+1}g_{p,t} - {}_{t}g_{p,t-1} \; A + B\,(G_{R,t} - {}_{t}g_{p,t-1}) + C(G_{R,t-1} - {}_{t}g_{p,t-2}) + u_t \,,$$

and then consider the null hypothesis to be that C=0. In considering this hypothesis, we assume that B is constant across companies. This is possibly not the case and a different value of B for each company might seem appropriate. This would suggest a random coefficients model--leading to heteroskedasticity. However, it also seems unlikely that the u_t can themselves be treated as homoskedastic but we have not basis for specification of the structure of the heteroskedasticity.

Table 4-4 shows the results for the short-term predictors. The estimate of the coefficient C shows the effect of the error made in the period prior to the previous forecast on the current forecast change. The table shows that the estimated coefficient, $\hat{C}$, generally is not significantly different from zero. Nevertheless, there are many instances in the table where one-period prior forecast errors did appear to influence forecast revisions, a finding inconsistent with the simple error-learning model.[2]

As Table 4-4 also shows, the values of R^2 are generally quite low. Although part of the variables not accounted for may come from the coefficient B varying among companies, the general impression is that the error-learning hypothesis is not strong. Whether this is because earnings do not follow an appropriate model for such expectations formation to be reasonable remains an open question.

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[2] It should also be noted that in some cases the changes in the forecasts from year to year were correlated.

Table 4-4

Estimates of $\hat{C}$ from Equation (4.3)

Dependent Variable:	Forecast Change, 63/62		Forecast Change, 64/63		Forecast Change, 65/64	
Predictor	$\hat{C}$ and its Standard Error	R^2 for Regression	$\hat{C}$ and its Standard Error	R^2 for Regression	$\hat{C}$ and its Standard Error	R^2 for Regression
A	.032 (.065)	.02	.093* (.370)	.23	-.151* (.759)	.06
B	.090 (.100)	.02	.319* (.118)	.23	.005 (.082)	.18
C	.103 (.089)	.08	-.086 (.099)	.19	-.124* (.063)	.15
D	.011 (.050)	.00	.040 (.092)	.01	-.087 (.059)	.04
E	-.227 (.131)	.10	.100 (.156)	.02	.357* (.096)	.52
G	.136* (.036)	.18	.059 (.047)	.04	.214* (.052)	.19
H	.111 (.125)	.15	.115 (.072)	.05	.133* (.046)	.19
I	.075* (.037)	.02	.046 (.047)	.03	.176* (.035)	.17
K	.085 (.555)	.03	.077 (.077)	.03	-.018 (.080)	.10

*Indicates that coefficient is significantly different from zero at .05 level.

A broader test of expectations rationality is whether the forecasters effectively incorporate all historical information available. In these experiments, we asked if there was some combination of historical information and analysts' forecasts that together might be better than using any individual piece of information. We asked, ex post, how an analyst could have made the best linear one-year prediction for earnings in, say, 1962, given that he had available to him the information available to us. To deal simply with the problems of missing observations, however, we concentrate on the average of the available predictions rather than on each individually or with some best linear combinations of them. Needless to say, in each year there is a combination of anticipated short-term growth, long term growth, 8-10 years historical growth, etc., which would be more highly correlated with realization than any one alone. This fact is demonstrated in comparing the seventh column of Table 4-5 with the earlier columns. In the earlier columns $\bar{g}_p$ is the average predicted growth rate and g_{h1} through g_{h3} are the best historical growth rates. The increase over the average of the predictors is not great. More dramatic, is the result when the coefficients of the best linear combination for one year are used with the data for the next. The superiority of the linear combination disappears and it is decidedly inferior to the average of the predictors. These correlations are shown in Column 8 of Table 4-5. Stated differently, there was no consistent combination of analysts' forecasts and historical information that could be used to make better predictions.[3]

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[3]For a similar analysis in the field of interest rates, see Friedman (1978), pp. 8, 9.

Table 4-5

Correlations of Various
Predictions with Realizations
1961/1965

(1) Forecast Period	(2) Forecast Date	(3) $\bar{g}_p$	(4) g_{h1}	(5) g_{h2}	(6) g_{h3}	(7) Best Linear Combination of Forecast and Historical Information: Present Year	(8) Prior Year	(9) P/E
A. Long-term Forecasts:								
66/61	61	0.5585	0.0900	0.3863	0.2436	0.6463	--	0.7129
67/62	62	0.5280	0.3305	0.3990	0.3511	0.5980	0.1409	0.5155
68/63	63	0.6172	0.3802	0.5162	0.4871	0.6533	0.5243	0.5488
69/64	64	0.5814	0.1104	0.5026	0.2469	0.6253	0.1042	0.4829
70/65	65	0.4760	-0.1062	0.4063	0.1677	0.5842	0.4746	0.4234
B. Short-term Forecasts:								
62/61	61	0.3583	0.1886	0.2338	0.1642	0.4694	--	0.5546
63/62	62	0.3228	0.1279	0.2646	0.2533	0.4379	0.2584	0.2964
64/63	63	0.5830	0.3137	0.4227	0.4269	0.6425	0.2526	0.4782
65/64	64	0.4652	0.2607	0.3796	0.2379	0.5437	0.3520	0.3344
66/65	65	0.4852	0.2186	0.2549	0.1394	0.5340	0.4726	0.3331

This result suggests that there is no systematic relationship between historical and realized growth that is not directly incorporated into the forecasts. This analysis was done only for five years because of lack of data for later years for g_{h2} and g_{h3}, which were past growth rates as "perceived" by two of our forecasters.

The last column of Table 4-5 shows the correlation of the simple price-earnings multiple with the realized earnings growth. The price-earnings multiple was as good as and often better than the average of the analysts' predictions. Since P/E multiples are influenced by more than forecasted growth (as will be shown below), this is a surprising result. It suggests that whatever information there is in the forecasts gets quickly assimilated. Thus, while the forecasts we have collected may not be wholly "rational", it appears that any useful information in the forecasts gets rationally included in share prices. To put the point a different way, there does not seem to be any more information in the predictions themselves than is already contained in market prices.

5. Growth Forecasts and the Valuation of Shares

In this section, we examine in some detail the theoretical relationship which may exist between growth-rate forecasts and the price of common stocks. For this purpose the obvious starting point is the Sharpe-Lintner-Mossin capital-asset pricing model (CAPM). In its simple version this model predicts that the price of security j, p_j, can be expressed as

$$(5.1) \qquad p_j = \frac{\mu_j}{(1+\bar{\rho})} + \frac{(\rho_M-\bar{\rho})}{(1+\bar{\rho})} B_{jM} .$$

In this equation μ_j is the mathematical expectation of the (one-period) gross return per security (that is, the price of the security in the future plus intervening dividend payments), B_{jM} is the regression coefficient of this return with total return to all securities, ρ_M is the expected rate of return to the "market" portfolio in which the holding of each security is in the same proportion to the total outstanding and $\bar{\rho}$ is the risk-free rate of interest. More commonly this relationship is expressed as

$$(5.2) \qquad \rho_j = \bar{\rho} + (\rho_M-\bar{\rho})\beta_{jM} ,$$

with ρ_j being the expected rate of return of security j ($\rho_j=\mu_j/p_j$) and β_{jM} being the regression coefficient of this rate of return on the rate of return of the market portfolio.

It may be objected that a risk-free security does not exist, especially if investors are concerned about real returns, and that not all relevant income streams stem from marketable securities. Within the mean-variance framework of the CAPM, these concerns may be accomodated by altering the valuation model so that equation (5.1) becomes

$$P_j = \phi_1 \mu_j + \phi_2 B_{jM} + \phi_3 B_{jN} \quad . \tag{5.3}$$

Here B_{jN} is the regression coefficient of the j^{th} return on total non-marketable return. Corresponding to (5.2) we can also express the rate of return on security j as

$$\rho_j = \bar{\rho} + (\rho_M - \bar{\rho}) \beta_{jM} + \psi \delta_{jN} \quad . \tag{5.4}$$

$\bar{\rho}$ now is the rate of return on a portfolio uncorrelated with marketable and unmarketable returns and δ_{jN} is the partial regression of the j^{th} rate of return on non-marketable return holding marketable returns constant. ψ is a coefficient (which could be expressed in terms of the expected return on a portfolio of securities having zero correlation with the "market" and maximum correlation with non-marketable return and of the regression coefficient of this portfolio's return on the non-marketable return).

These expressions are derived under the hypothesis of homogeneous expectations, but continue to hold when different investors hold different expectations provided that appropriate assumptions are made about how the expectations differ. Unfortunately, they do not also remain valid in the face of other changes in assumptions. In particular, market imperfections that have the effect of limiting the range of securities in which it will be feasible to take a position, either by recognizing the costs of short-selling, prohibitions on negative holdings or the costs associated with the number of different securities held destroy the model. Since those taking positions in one security are not necessarily those taking positions in others, the covariance with the market ceases to be clearly the relevant risk measure. In addition, concerns of investors that extend beyond the one-period return on their portfolios, so that the associations of returns on securities with other quantities become relevant or receive different weights. Finally the tax laws by creating differences among investors in the relative returns from various securities also renders the simple CAPM results less plausible.

Added to the theoretical difficulties are the ones emphasized by Roll (1977) concerning empirical implementation. Roll points out that the CAPM theory may be regarded simply as stating that the prices of securities adjust in such a way that the market portfolio is made mean-variance efficient. The theory does not predict that the valuation equation will hold for commonly used indexes that do not include the whole market with appropriate weights, but the theory will "hold" directly for any index which includes the security if the index is selected to represent a mean-variance efficient portfolio, whatever may be the process of valuation. Measurement of the market portfolio is possibly impossible and certainly infeasible, especially as recognition of non-marketable returns requires their measurement as well. In consequence, the usefulness of the CAPM for our purposes is less than apparent.

Fortunately, an alternative valuation approach can be developed using the return generation process suggested by Ross (1976) and (1977), and the common observation that many portfolios containing the securities of major corporations can be considered widely diversified. This yields the same valuation formula as Ross' arbitrage model, though on possibly more robust grounds. We call it the diversification model.

Suppose that the investor regards the covariances of the various returns as stemming from their linear dependence on a limited number, K, of random factors, f_k. Such a model has found empirical support, for example in King (1967) and in Roll and Ross (1979). Furthermore, in explaining the likely fortunes of particular securities and the contingencies they face, our impression is that analysts discuss them in terms of association with a comparatively small number of important variables -- or developments in the economy and parts of it -- rather than in terms of associations with all other companies. A list of such variables might include interest rates, national income, returns to the stock market as a whole, and so on. For purposes of our argument it is important that investors think in terms of a few general factors rather than that this necessarily be a full and correct specification of returns. Specifically, the supposition is that the return to a security, r_j, is given by

$$r_j = \mu_j + \sum_{k=1}^{K} \gamma_{kj} f_k + e_j \quad . \tag{5.5}$$

e_j is a random variable with mean zero which is independent of the f_k and of other e_j. The f_k are also assumed to have mean zero, so that μ_j remains the expected return.

Suppose next that the portfolio selection of a typical investor, i, can be described as maximizing the expected utility of end-of-period return, $\sum_{j=1}^{J} r_j v_{ji}$, by choosing security holdings v_{ji} subject to the budget constraint, $\sum_{j=1}^{J} p_j v_{ji} = \sum_{j=1}^{J} p_j \bar{v}_{ji}$, where the prices p_j are considered to be given and $\bar{v}_{ji}$ are his initial holdings. His utility function, $U_i(\sum_{j=1}^{J} r_j v_{ji})$, is assumed to be monotonically increasing and concave. First-order conditions then give

$$E(r_j U_i') - \lambda_i p_j = \tag{5.6}$$

$$\mu_j E(U_i') + \sum_{k=1}^{K} \gamma_{jk} E(f_k U_i') + E(e_j U_i') - \lambda_i p_j = 0 \quad .$$

Hence

$$p_j = \mu_j a_{oi} + \sum_{k=1}^{K} \gamma_{jk} a_{ki} + E(e_j U_i')/\lambda_i \quad . \tag{5.7}$$

Here λ_i is the usual Lagrange multiplier from the constrained maximization problem and U_i' is the first derivative of U_i with respect to end of period return.

The last term in (5.7) has sign opposite to v_{ji} by Jensen's inequality and the assumptions of concavity of U_i and zero expectation for e_j. If v_{ji} is zero, the last term also is zero (and is continuous in v_{ji}). By contrast, the coefficients of γ_{jk}, a_{ki}, depend on the total holding of f_k, $\sum_{j=1}^{J} \gamma_{jk} v_{ji}$, rather than only on the holding of security j.

In at least a broad sense the last two terms in (5.7) may be considered to be the contribution of the security to the risk of the portfolio since they arise only from the concavity of the utility function and the stochastic nature of returns. Sharpe's (1964) insight that with diversification only "systematic" risk matters can be interpreted as stating that the term $\sum_{k=1}^{K} \gamma_{jk} a_{ki}$ is large relative to the last term which gives the marginal effect of the security's own variation. If this is the case, and recalling that the last term in (5.6) does go to zero as v_{ji} does, we may approximate the equation by

$$(5.7) \qquad p_j = \mu_j a_{oi} + \sum_{k=1}^{K} \gamma_{jk} a_{ki} \quad .$$

That is, there is an approximately linear relationship between the price of the security, the expected return for the security and its coefficients for the various factors.

Suppose next that the number of securities that are held by investor i for which (5.7) yields an adequate approximation exceeds K. Then a constraint is placed on prices in the sense that for any set of prices and coefficients (5.7) would not normally hold and so prices must have adjusted to bring (5.7) about. Furthermore, the coefficients a_{oi} and a_{ki} must be the same for all investors who hold more than K of the same securities in well diversified portfolios, that is ones with holdings of individual securities small enough that the own variation term of (5.6) may be ignored. More generally then, if investors tend to diversify widely and if a group of securities is held within these portfolios, the prices of these securities obey approximately the equations

$$p_j = \mu_j a_o + \sum_{k=1}^{K} \gamma_{jk} a_k \quad . \tag{5.8}$$

In rate of return terms we can express this equation as

$$\rho_j = b_o + \sum_{k=1}^{K} \alpha_{jk} b_k \tag{5.9}$$

where $\alpha_{jk} = \gamma_{jk}/p_j$. Which version is more immediately appropriate depends in part on whether it is better to regard the total return or the rate of return of security j as following a factor-analysis model such as (5.5). We would stress (5.8) since it indicates more directly that the adjustment that occurs in the market to produce the linear relationship takes place in current prices. The equation is the same as the one derived by Ross (1976), using a somewhat different argument.

The diversification argument about security pricing is fairly robust to variations in the details of the institutional and behavioral setting to which the CAPM is sensitive. The assumption that utility is defined over end-of-period return is no longer a major part of the theory. The relationship of a security's return to other matters of concern to the investor is presumably captured through correlation with the common or systematic factors, f_k, and so depends on the coefficient γ_{jk}. The interpretation and value of the coefficients a_{ki} may then change, but the linear relationship still exists. Secondly, the model is robust to non-negativity constraints, extra costs of going short, or small costs that depend on the number of securities held. The argument is that if diversification proceeds so far that investors hold more securities than there are factors and that the last term of (5.7) becomes trivial, then utility maximization still can be expressed approximately by (5.6) with a neglibible final term. The problem that returns vary among investors because of the different tax treatment of dividends and capital gains may also be overcome within the diversification theory. While these tax differences affect many individuals, they are not relevant to many of the largest institutional investors such as pension funds who dominate the market for many securities, particularly the common stock of large well-known corporation of the sort for which the expected growth-rate data were prepared. If these investors hold well diversified portfolios of these securities, then the structure of prices derived from the diversification model will apply using the before-tax expected returns. One would not expect investors for whom the tax differences matter to hold these securities -- at least not in small quantities.

Diversity of expectations such as we found in our sample of institutional investors produces a more delicate problem for the diversification model than these other divergences from the assumed conditions, though surprisingly one can still argue -- at least under certain circumstances -- that the suggested price structure will continue to prevail. Suppose that investors' perceptions of the process generating returns correspond to (5.3) but that they do not use the same parameters so that investor i perceives returns to be generated by

$$r_j = \mu_{ji} + \Sigma\gamma_{jki} f_k + e_j \quad . \tag{5.10}$$

Suppose also that diversity of expectations is captured by

$$\mu_{ji} = \mu_j + \varepsilon_{ji} \quad ; \tag{5.11}$$

$$\gamma_{jki} = \gamma_{jk} + \eta_{jki}$$

where the additional terms can be treated as random and independent over the investors and as having mean zero. The individual choice equations then give rise to

$$p_j = (\alpha_j + \varepsilon_{ji}) a_{oi} + \sum_{k=1}^{K} (\gamma_{jk} + \eta_{jki}) a_{ki} + E(e_j U_i')/\lambda_i \quad . \tag{5.12}$$

If the portfolio is well-diversified so that again the last term is negligible, then the prices of the securities held obey approximately a linear relationship in the parameters the investor perceives. Suppose another investor, say m, also holds more than (K+1) of these securities in a well-diversified portfolio. Then approximately for more than (K+1) securities

$$(5.13)\qquad (\mu_j + \varepsilon_{ji})a_{oi} + \sum_{k=1}^{K} (\gamma_{jk} + \eta_{jki})a_{ki}$$
$$= (\mu_j + \varepsilon_{jm})a_{om} + \sum_{k=1}^{K} (\gamma_{jk} + \eta_{jkm})a_{km} \;.$$

Equation (5.13) maintains that a linear relationship must exist approximately among the perceptions of the two investors, apparently in part contradicting the diversity of expectations that was supposed. In effect, as long as the diversification motive leads to substantial overlap in the holdings of securities, it is possible for the price structure suggested by the model to hold. While some investors may take substantial positions in particular securities, one might well expect others to hold only small quantities. Equation (5.13) would hold approximately, for example, if the random terms were negligible, as with smooth symmetric distributions might well be true of typical perceptions. In that case, the values of a_{ki} , k=0,...,K, must be the same for all such investors and the security prices will obey (5.7) .

To make this argument more explicitly, rewrite (5.12) as

$$(5.14)\qquad E(e_j U_i')/\lambda_i = p_j - (\mu_j+\varepsilon_{ji})a_{oi} - \sum_{k=1}^{K} (\gamma_{jk}+\eta_{jki})a_{ki} \;.$$

If typically diversification is expected in the sense that the expected value (taken across investors) of the left hand of (5.14) is zero, then

(5.15)
$$p_j = E(\mu_j+\varepsilon_{ji})a_{oi} + \sum_{k=1}^{K} E(\gamma_{jk}+\eta_{jki})a_{ki}$$
$$= \mu_j a_{oi} + \sum_{k=1}^{K} \gamma_{jk} a_{ki}$$

for all i. (5.15) is the same as (5.7) which produced the valuation formula of the diversification model.

Unfortunately, the argument which we have been making does not stand up well to a prohibition on negative holdings (or substantial costs to short-selling). This is especially true if it is the case that most portfolios, even well-diversified ones do not contain most securities. This feature implies, given the other assumptions, that most investors perceive most securities to be over-valued. While it remains true that (5.15) must hold for any pair of investors holding over-lapping, well-diversified portfolios, there is no implication that the coefficients a_{ki} will tend to be the same for different investors. With the restriction on negative holdings, investors hold only a scattering of securities which they perceive to be overvalued. For given coefficients a_{ki}, this amounts to saying that the value of the linear combination $(a_{oi}\mu_{ji} + \sum_{k=1}^{K} \gamma_{jki}a_{ki})$ for securities actually held tends to fall in the upper tail of the distribution of these sums. Since in that case, with smooth probability density functions, the density decreases as this quantity increases, a small holding is more probable than any particular larger holding. While we may properly conclude that investors who hold small quantities of security j have the same value of this sum, this in turn does not directly tell us about expressions in terms of more objective -- or agreed upon parameters.

How devastating this argument actually is to the diversification theory is questionable. To be sure, it is reasonable to presume that the costs of short-selling are substantial (if not strictly speaking prohibitive) and some major market participants are directly prohibited from short-selling. Certainly most securities tend not to be held in any particular portfolio. However, this may arise from other sources. There are costs associated with achieving an informed opinion. These may be the reasons for investors not holding most securities, and one can safely infer only that they do not perceive securities which they do not hold to offer unusually favorable opportunities, not that such securities are considered to be poor investments. These considerations make it more plausible that an investor's opinions about securities he holds in small quantities represent average or typical opinion rather than an outlying one. Given that an investor may hold a fairly large fraction of securities about which he has formed an opinion, it is reasonable to suppose that many of these opinions will be near the means of their distributions for securities for which his opinions are unusually favorable, an investor will of course have a large holding while he will hold none of securities he judges a poor investment. This behavior does not upset the basic pricing of the diversification model, though the valuation does rely on an investor's small holdings tending to represent an average opinion. That this condition may hold is rendered more plausible by remembering that if the diversification model holds in the expected values (across investors) of the parameters, and if part of an investor's beliefs and portfolio holdings correspond to these values (so that his values of a_{ki} correspond to the market coefficients), then this investor may plausibly assume that any security that he has not investigated in detail will be found on investigation to offer only an expected return and coefficients for the factors that would not make it a clearly compelling candidate for inclusion or exclusion in a portfolio. Such an expectation is exactly what we need to preserve the plausibility of the diversification model. The crucial question is why typically an investor does not hold a particular security. If the reason for the omission is that the investor does not really know much about it or that he thinks its value is about right but it offers only opportunities roughly equivalent to ones available from securities already included in the portfolio, no problems may arise for the diversification model. If, however, the omission can be explained only by an expectation that the security offers a poor return relative to those included in the portfolio, then the earlier conclusion that diversity of expectations and restrictions on short-selling destroy the basis of the diversification model would be compelling.

The conclusion of these arguments is simple: the diversification model suggests a theory for security prices which rests on arguments that are not as vulnerable as those of other formulations to relaxing many standard counter-factual assumptions. It remains, however, to relate this theory to the expectations data we have gathered.

The basic quantity which we have been considering is the total return to a security, presumably consisting of the dividend plus the price in the next period. Letting t be the time subscript and d_{jt} be the dividend about to be paid immediately after the price determination, we can express the expected return as

$$\mu_{jt} = E(d_{jt}) + E(p_{jt+1}) \quad . \tag{5.16}$$

Clearly the value of this expression depends on the price expected in the next period. It does not take a very perceptive investor to realize that other investors' attitudes towards the securities in the next period are going to affect the prices in the next period. On this basis there are nevertheless a number of ways in which we might proceed. The simplest is to assume that the investor expects the rate of return to be the same in every period, say ρ_j , in the sense that for all future t,

$$E(p_{jt}) = [E(d_{jt}) + E(p_{jt+1})]/(1+\rho_j) \quad . \tag{5.17}$$

Repeated substitution of this expression in (5.16) yields

(5.18) $$\mu_{jo} = \sum_{t=0}^{T-1} E(d_{jt})/(1+\rho_j)^t + E(p_{jT})/(1+\rho_j)^T \quad .$$

If we presume that the first term is convergent and that investors do not expect some other valuation aberation as $T\to\infty$, and since $\mu_{jo}/p_{jo} = (1+\rho_j)$, this development then produces the standard valuation expression

(5.19) $$p_{jo} = \sum_{t=0}^{\infty} E(d_{jt})/(1+\rho_j)^t \quad .$$

Determination of ρ_j now depends on the future path of dividends. The easiest assumption to make is that they are expected to grow indefinitely at a constant rate g_j . Expression (5.19) then becomes

(5.20) $$p_{jo} = d_{jo}\sum_{t=0}^{\infty} (1+g_j)^t/(1+\rho_j)^{t+1}$$

$$= d_{jo}/(\rho_j-g_j)$$

Solving (5.20) for ρ_j yields

(5.21) $$\rho_j = g_j + d_{jo}/p_{jo} \quad ,$$

so that if the simple CAPM is valid

(5.22) $$g_j + d_{jo}/p_{jo} = \bar{\rho} + (\rho_M-\bar{\rho})\beta_{jM} \quad ,$$

while using the diversification model would suggest

$$(5.23) \qquad g_j + d_{jo}/p_{jo} = b_o + \sum_{k=1}^{K} a_{jk} b_k \quad .$$

A similar argument, based on the price equation, rather than on the expected rate of return, yields the pricing equation

$$(5.24) \qquad p_{jo} = d_{jo}/(\bar{\rho}-g_j) + \sum_{k=1}^{K} b_k \gamma_{jk}/(\bar{\rho}-g_j)$$

or

$$(5.25) \qquad p_{jo}/e_{jo} = (d_{jo}/e_{jo})/(\bar{\rho}-g_j) + \sum_{k=1}^{K} b_k (\gamma_{jk}/e_{jo})/(\bar{\rho}-g_j) \quad .$$

The last formulation brings out the relationship between this valuation model and more traditional approaches. There the risk term is ignored so that the valuation expression is given by only the first term of (5.23). It shares with such approaches difficulties which have led a number of writers to formulate finite-horizon models for share prices. Similarly, more complicated relationships between dividends, earnings and growth than suggested in (5.23) may seem plausible. It is quite easy to generate a large variety of models using variants of quite plausible simple assumptions. They share the property of producing quite complicated, non-linear explicit specifications which might all be expressed in terms of some expected rate of return function with the relevant arguments:

$$(5.26) \qquad \rho_j = \rho(g_j \, , \, d_{jo}/e_{jo} \, , \, d_{jo}/p_{jo}) \quad ;$$

or as a price-earnings function:

$$p_{jo}/e_{jo} = f(d_{jo}/e_{jo}, g_j, V_j) \tag{5.27}$$

with V_j being an appropriate vector of risk variables.

One may now proceed to use a linear approximation (from an expansion in Taylor's series) to produce a linear model for p_{jo}/e_{jo} and this is in essence the approach we adopt here. We have found, for example, that a linear approximation fits the finite horizon model suggested by Malkiel (1963) with considerable accuracy. We also fit models for the expected rate of return measure derived from (5.21) as well as from more complicated formulations.

These arguments relate the valuation models to the expectations data we have available. To complete the argument we need only assume that the growth rate figures may be taken as estimates of g_j so that their average may be considered a variable measuring this parameter, albeit with error. However, this still leaves unmeasured the risk parameters relevant to the valuation models. The only risk measure we were able to obtain was the Instability Index provided by one of the participating firms. It is far from clear that this can be taken as a measure of the relevant quantity, since it claims to indicate total instability rather than a regression coefficient, whether with the "market" or with some "factor". We are therefore forced to look elsewhere for our data.

As we noted, a major weakness of the CAPM is that it requires β coefficients for the true market portfolio which itself cannot be measured. Luckily, the diversification model has less problems of this sort.

One approach to the problem would be to fit the factor analysis model (5.5) to data for returns to the companies in our sample over some period of time. This would parallel the empirical work done with considerable success by King (1967) and, in conjunction with testing the main prediction of the arbitrage or diversification model for ex-post data, by Roll and Ross (1979). This approach would encounter severe computational problems arising from the number of companies in our sample. Even the approach of fitting models to smaller number of companies, adopted by Roll and Ross (1979), runs into problems from normalization of factors. For the present we shall instead adopt an approach based on regression coefficients which has the advantage of being somewhat more straightforward.

For the moment, assume that there is only one factor so that (5.5) becomes

(5.28) $$r_j = \mu_j + \gamma_j f + e_j \quad .$$

Suppose that we have a variable, z, which is generally correlated with the return of companies. Presumably such correlations arise from association with f so that we can also express z as

(5.29) $$z = \mu_z + \gamma_z f + e_z$$

with e_z uncorrelated with f. It is also presumably uncorrelated with e_j also, since otherwise the factor model is not adequate for r_j.

A regression of r_j on z may be regarded as estimating $(\gamma_j/\gamma_z)/[1+E(e_z^2)/E(\gamma_z^2 f^2)]$, that is a coefficient which is proportional to γ_j, with the same factor of proportionality for all companies. This is, of course, simply the multivariate-regression analogue of the standard result when there is only one variable measured with error. With one factor, and with one general proxy for it, the regression coefficient can then serve as a proxy for the coefficient of the factor.

Matters become more complicated if there is more than one factor. With the same number of measured variables on which to base regression coefficients, there will be a linear relationship between the regression coefficients for the returns on these variables and the coefficients of the factors in the factor model, but there is no simple correspondence, even if a clear normalization rule for the factors themselves is adopted, allowing clear interpretations of the results. Furthermore, should one have more variables than factors, all variables would have non-zero coefficients in the theoretical regression and might have significant coefficients in the empirical regressions, even though the factor model is correct. As a result, a great deal of caution is warranted in interpreting the coefficients.

These problems are made worse by the fact that the estimated coefficients in turn measure the population coefficients only with error.

Hence, when we regress our rate of return measures on various, previously calculated regression coefficients, all the problems of errors-in-variables plague the analysis. In particular, there will be no necessarily direct correspondence between the coefficients being estimated and their population values in a regression using these variables measured without error. Furthermore, variables which actually have zero coefficients in the regressions using variables measured without error will have non-zero error.

These considerations might seem to preclude useful empirical work which does not explicitly allow for errors-in-variables, itself a formidable undertaking. While this claim has some merit, the picture is not entirely black for simpler approaches. First, finding significant associations can be interpreted as indicating that the variables are at least related to the relevant ones. Second, when one set of variables produces a closer fit than another set, it is fair to interpret the second as providing a more accurate measurement than the other of the relevant underlying quantities. Finally, the associations found may well be of interest in their own right as descriptive statistics worth examining before a more complicated model is attempted. We therefore proceed in the next section to report some of the regression equations suggested by the approaches described in this section.

6. Growth Forecasts and the Valuation of Shares

Our final task is to investigate the relationships between the earnings-growth expectations data and the market values of the corresponding shares based on the formulations developed in section 5. This endeavor may be regarded in one of two ways. Given that the growth-rate expectations are a major input into assessment of the expected return to the security, our investigation tests the validity of the valuation model. Conversely, if we maintain the validity of the valuation model, we may be regarded as testing the hypothesis that earnings-growth expectations play a major role, together with the other specified variables, in investors' evaluations of expected security returns.

We begin by specifying alternative measures of risk. The first set of variables employed are measures of so-called "market risk" derived from the regressions of the experienced rates of return on various market-wide variables. We have experimented with several market indexes including the S&P 500 Stock Index, the Dow Jones Industrial Average (of 30 stocks), and the weighted and unweighted indexes made available from the University of Chicago's Center for Research in Security Prices (CRSP). Our results turn out to be insensitive to alternative market indexes so we report here only the results for the CRSP weighted index.

Correlation with other types of variables may also yield needed measures whether the extended CAPM or the diversification model is assumed. The variables used along with the rate of return of the market index are the rate of change in national income, the short-term interest rate measured as the ninety-day treasury bill rate and the rate of increase of the Consumer Price Index. These may be considered typical measures of some risks to which investors are subject, stemming from variation in other (nonmarket) sources of income, from the changes in interest rates, and from inflation.

The period over which these coefficients should be calculated is not clear *a priori*. It is not even clear that only past values should be used. The theory involves the covariance of returns with various quantities in the future. They could safely be estimated from part data if they did not change or if investors did not perceive change. This is unlikely, however. Changes in the nature and type of activities that corporations pursue and alterations in the structure of the economy make it likely that the appropriate regression coefficients change through time. Insofar as investors can perceive and even anticipate these changes, it might be sensible to use values estimated with data following the time at which the valuation took place.

The approach we adopted after some experimentation is a compromise. The regression coefficients are calculated using quarterly observations over ten-year periods. The periods used covered the three years prior to the valuation date and the seven years following it. The nature of the results is not very sensitive to variations in the details of this procedure, but use of data entirely from past periods was found to be less satisfactory. We found that extending the estimation period into future periods improved the values of R^2 and was particularly important for obtaining some precision in evaluating the effect of inflation.

One other measure is available to us which may be considered to indicate risk in some broad sense. This is the variance of the predicted growth rates of each company made by the different forecasters. The lack of agreement with respect to expected growth may be considered to stem from uncertainty about the future of the company and so to indicate risk. This measure can be taken to represent the own variance of the security rather than an additional measure of systematic risk. Theoretically then, its use is dubious unless one is prepared to assume that the variability itself stems from systematic risk. More plausibly, this quantity serves as a variable whose significance, if demonstrated, would raise queries about the validity of the underlying theory.

We investigate first whether there is association between the most straightforward measure of expected return to be derived from our data and various risk measures. The rate of return variable is the one suggested in equation (5.21) denoted here as ρ, which is defined as $\bar{\rho}_{jt} = \bar{g}_{jt} + d_{jt+1}/p_{jt}$. The variable $\bar{g}_{jt}$ is the average of the available long-term growth rate predictions, d_{jt+1} are dividends expected to be paid per share in the course of the next year (as estimated by the predictor who furnished data in all years) while p_{jt} is the end-of-year closing price.

Simple regressions of this expected return measure on the various risk variables are summarized in Table 6.1 where we report the t-values of the regression coefficients. (The coefficient of determination, r^2 , is a monotonic transformation of this quantity.) The first risk measure is the regression coefficient of the (excess) rate of return of each security on the (excess) rate of return to the CRSP value weighted market index. It is denoted β_M and obtained by estimating the equation

$$(6.1) \quad r_{jt} - \rho_t = \beta_{jM}(r_{Mt} - \rho_t) .$$

Here ρ_t is the short-term (90-day) treasury bill rate taken to represent the risk free rate and r_{Mt} is the rate of return of the CRSP index. We then proceed to estimate the equation

$$(6.2) \quad \bar{\rho}_{jt} = \alpha_0 + \alpha_1 \hat{\beta}_{jM} ,$$

and the resulting t-values for α_1 appear in Table 6.1 .

In estimating (6.2), the signs of the t-values are positive and usually significant, as suggested by the theory. The strength of the association is not great, however; the r^2 corresponding to the highest t-value is only .16 . As noted above, the results did not vary if alternative indexes were used in place of the CRSP weighted index. Moreover, the results were not very sensitive to varying the period over which the β_{jM} coefficients were estimated, as long as the period used contained at least some observations from quarters after the forecast period.

Although the regression coefficient with the CRSP Index gives significant results, strong t-values (and coefficients of

Table 6-1

Risk Measures and Naive Expected Return

Independent Variables YEAR	β_M	β_Y	s_g^2
	t-values from simple regressions of ρ_{jt} on various variables as in equation (6.2)		
1961	3.65	3.98	2.57
1962	3.32	3.84	6.30
1963	0.55	2.70	3.74
1964	1.65	2.87	5.95
1965	3.79	3.42	6.97
1966	5.35	3.27	2.79
1967	3.49	3.85	2.54
1968	3.35	3.91	12.86

β_M - Coefficient of CRSP weighted market index.

β_Y - Coefficient of rate of change of National Incomes.

s_g^2 - Variance of predicted growth.

determination) are also obtained from using the regression coefficients of the securities' returns on the rate of change of National Income, indicated by β_{jY} in Table 6.1, in place of β_{jM} in estimating equation (6.2). In most years, these are stronger than those for the coefficient for the CRSP Index. Hence, there is as much of a prima facie case for covariance with income being the relevant quantity as for covariance with a market index, if only one quantity is assumed to be relevant.

Before congratulating ourselves on the strength of these results, the column of Table 6.1 headed s_g^2 stands as a warning. s_g^2 is the variance of the predictions of long-term growth for each company and may possibly be interpreted as a measure of own variance. It also has a positive coefficient with our expected return measure and for the years 1962 through 1965, when our sample was widest, s_g^2 gives stronger results than either of the regression measures.

Table 6-2 presents the results of including all three risk variables (β_{jM}, β_{jY}, and s_g^2) simultaneously in equation (6.2). It will be noted that while all three variables generally continue to have their proper signs, it is s_g^2 that appears to hold up best when the three risk measures are used together. Whether this result should be attributed to the importance of own variance is explaining the structure of expected returns or whether s_g^2 is simply serving as a proxy for systematic risk remains an open question.

We can shed light on these matters by estimating the wider specification where additional regression coefficients for the association of each security's return with other variables are used.

Table 6-2

Expected Return Regressions Using Our Risk Measures

(t-values in parentheses)

Year	Constant	β_M	β_Y	s_q^2	R^2	F (degrees of freedom)
1961	0.06 (7.75)	0.15 (1.93)	0.126 (0.91)	0.637 (2.72)	.19	7.37 (3,93)
1962	0.08 (14.08)	0.005 (1.01)	0.032 (1.06)	0.536 (5.23)	.24	16.49 (3,156)
1963	0.07 (9.67)	0.011 (1.57)	0.002 (0.03)	0.373 (3.04)	.10	6.14 (3,157)
1964	0.06 (8.27)	0.020 (2.80)	-0.161 (2.49)	0.540 (5.66)	.23	15.92 (3,156)
1965	0.08 (14.13)	0.005 (0.83)	0.034 (0.44)	0.522 (6.01)	.23	15.72 (3,156)
1966	0.09 (20.92)	0.013 (2.90)	0.142 (1.83)	0.010 (0.62)	.12	6.65 (3,147)
1967	0.09 (16.82)	0.016 (2.80)	0.275 (3.51)	0.024 (2.18)	.20	11.65 (3,144)
1968	0.09 (14.70)	0.020 (3.23)	0.152 (1.41)	0.054 (10.96)	.59	62.86 (3,133)

The coefficients were obtained from the multiple regression of the rate of return of each security on the CRSP (value weighted) index and on National Incomes, as used above, and also on the treasury bill rate, and on the rate of inflation, denoted as δ_M, δ_Y, δ_I and δ_P respectively. The specification of (6.2) is then expanded to give

$$(6.3) \quad \rho_{jt} = \alpha_0 + \alpha_1 \delta_{jM} + \alpha_2 \delta_{jY} + \alpha_3 \delta_{jI} + \alpha_4 \delta_{jP} .$$

Estimates of this equation are given in Table 6-3.

A number of things are worth noting about this table. Of most importance, each type of coefficient is significant in at least some years. Second, in the first part of the period only the market coefficient is significant with regularity. However, toward the end of the period other coefficients tend to be important, especially those measuring the systematic relationship with inflation and treasury bill rates. Taking these results at face value, two explanations come to mind. First, in the more stable early part of the period, estimates of the δ coefficients may be sufficiently imprecise that in the subsequent estimation of (6.3) the relatively greater errors of measurement lead to lack of significance. Second, investors may have become more concerned about the other sources of risk such as inflation as the decade proceeded. Overall, the results suggest strongly that all influences play a role, though, again, it is an open question whether this is because they act as proxies for other variables.

The signs of the coefficients tend to be the same across the different equations. Though with errors-in-variables we must be cautious in attaching much importance to th sign of the coefficients, they do usually conform to intuition. Positive association with

Table 6-3

Estimation of the Extended Equation (6.3) for the Expected Rate of Return

(t-values in parentheses)

Year	α_0	δ_M	δ_Y(x100)	δ_I(x100)	δ_P(x100)	R^2	F (degrees of freedom)
1961	0.070 (10.74)	0.016 (2.49)	0.233 (2.04)	-0.025 (0.80)	-0.034 (0.62)	.15	5.49 (4,126)
1962	0.078 (13.11)	0.012 (2.11)	0.201 (2.50)	0.021 (0.70)	-0.018 (0.37)	.10	4.47 (4,155)
1963	0.069 (9.54)	0.016 (2.25)	0.050 (0.66)	-0.030 (0.79)	-0.009 (0.18)	.07	2.89 (4,156)
1964	0.060 (7.92)	0.026 (3.46)	-0.058 (0.88)	-0.031 (2.19)	-0.061 (1.18)	.12	5.27 (4,155)
1965	0.083 (13.54)	0.008 (1.30)	0.110 (1.55)	-0.075 (2.35)	-0.095 (2.10)	.07	2.98 (4,155)
1966	0.099 (22.00)	0.009 (2.12)	0.168 (2.57)	-0.090 (3.74)	-0.019 (4.61)	.19	8.70 (4,152)
1967	0.098 (17.88)	0.013 (2.30)	0.251 (3.32)	-0.015 (6.03)	-0.030 (5.74)	.26	12.93 (4,147)
1968	0.088 (9.79)	0.040 (4.61)	0.421 (2.75)	-0.025 (5.93)	-0.052 (5.60)	.28	13.29 (4,134)

either the market return or with income raises the expected rate of return as theory suggests. Correspondingly positive partial correlation with the rate of inflation, so that the stock tends to act as a hedge against inflation, lowers the expected rate of return. Finally, the coefficient for the treasury bill rate usually has a negative sign. This might be taken to indicate that the relevant risk really arises from association with realized rates of return on fixed income securities, which tends to vary inversely with the short-term rate. It is worth noting, however, that there is a good deal of correlation (roughly about .6) between the coefficients for the treasury bill rate and for the rate of inflation so that one may partly be serving as an additional proxy for the other. Otherwise, multicollinearity problems are small, making it less plausible that all the different measures serve as proxies for some single variable.

Inclusion of all these different regression coefficients does not, however, account for the odd strength we found early for the variance of the predictions. When that variable was included along with the others, it usually was highly significant with a positive coefficient. Other coefficients tended to retain their same signs, though with lessened significance. The reason for this finding may be that it stems from errors-in-variables problems or misspecification; but it may also indicate inadequacy of the theory. However, it is important to note that the values of R^2 are sufficiently high and so very highly significant that there is no question about whether there is some underlying systematic association among the variables. Equally clearly, our results do not give entirely straightforward support to the joint hypothesis that our growth rates allow measurement of expected return and that the valuation model is appropriate.

Further information on the connection between predictions of growth and valuation may be obtained from examining the alternative form of the valuation equation which was suggested in Section 5. This yields a more traditional formulation where the price earnings ratio is the dependent variable, and earnings growth, the payout ratio and our three risk measures were treated as independent variables. We can now ask whether our growth-rate expectations are associated with market earnings multiples and again examine which risk measures appear strongest. In carrying out this investigation, both price and dividends were divided by normalized earnings to give the equation

(6.4) $P/\overline{NE} = a_0 + a_1\bar{g}_p + a_2 D/\overline{NE} + a_3 RISK$.

In considering this equation, we first treated each of the risk measures as being alternatives, as we had with (6.1). In these regressions, both the average expected growth rate and dividend payout ratio almost always had positive and significant coefficients throughout the sample period, and it is fair to say that these results tend to indicate that measures such as our expectations of growth appear to enter valuation. The pattern for the risk measures is more complicated. Table 6.3 corresponds to Table 6.1. The t-values, being transforms of the partial-correlation coefficients, still indicate the relative strength of the alternative measures. A negative sign in these regressions should be expected for the risk measures based only on theoretical considerations and ignoring data

Table 6-3

P/$\overline{NE}$ Regressions Treating Risk Measures as Alternatives

Value t-ratios

Year	β_M	β_Y	s_q^2
1961	-.28	1.72	-.81
1962	-4.29	-1.21	-6.70
1963	-.73	-.85	-2.49
1964	-2.02	-3.23	-7.28
1965	.63	1.79	-2.77
1966	-1.82	-1.17	-.43
1967	1.57	.82	-3.24
1968	2.73	.12	-2.12

difficulties. Although both β measures more often than not have the correct negative values, the t-values indicate that they are only occasionally significant. The variance of the predictions always has the appropriate negative sign, though its significance varies considerably, reflecting variation in the magnitude of the coefficient. Table 6-4 shows the full estimates of equation (6.4), using s_g^2 as the risk variable.

As did the results for the naive expected rate of return, these results suggest that there are promising and meaningful associations among our data. Growth projections of the sort we have been investigating do appear to play an important role in valuation and earnings multiples are related to some of our risk measures. Again, however, we find that stronger associations are obtained with the risk proxy not directly related to traditional asset-pricing theory.

What is the source of the unexpected findings is not clear. We suspect that a major part of the difficulty stems from error-in-variables problems. These have yet to be dealt with, though our data do afford certain opportunities for handling errors-in-variables that are not usually present. Furthermore, the list of proxies may well be expanded fruitfully. But to do so meaningfully requires solving the errors-in-variables problem.

7. Concluding Comments

The basic results of our empirical investigation are notable and indicative of the pattern of empirical results we have been finding in this research. The set of variables given by our expected

Table 6-4

P/$\overline{NE}$ Regressions: Estimates of Equation (6.2)

(t-values in parentheses)

Year	Constant	$\bar{g}_p$	D/NE	s_g^2	R^2	F (degrees of freedom)
1961	1.88 (0.49)	3.91 (18.08)	1.22 (0.24)	-0.57 (0.81)	.81	137.70 (3,100)
1962	3.30 (2.12)	2.23 (22.11)	8.41 (4.09)	-1.17 (6.70)	.75	171.10 (3,167)
1963	2.85 (1.53)	2.70 (22.67)	6.71 (2.69)	-0.59 (2.49)	.77	182.80 (3,167)
1964	2.53 (1.93)	2.15 (23.45)	13.16 (7.02)	-1.09 (7.28)	.77	185.00 (3,166)
1965	1.76 (0.73)	2.82 (16.86)	4.73 (1.41)	-0.66 (2.77)	.67	112.40 (3,166)
1966	0.22 (0.11)	1.74 (13.57)	7.42 (3.00)	-0.01 (0.43)	.57	68.41 (3,156)
1967	1.88 (0.79)	2.35 (15.10)	-1.05 (0.37)	-0.09 (3.24)	.69	112.00 (3,150)
1968	2.18 (0.70)	1.78 (9.43)	5.13 (1.38)	-0.04 (2.12)	.52	49.47 (3,138)

growth rates and risk measured by the variance of the growth rate, give a closer account of the valuation of securities than do alternatives. Since closer fitting equations are the result one would expect with smaller errors of measurement, one can presume that they are more similar to the expectations being valued in the market than are measures based on ex post realized growth or regression coefficients. Earlier we saw that there is a great deal of diversity of expectation among forecasters, an aspect of reality with which valuation models do not usually cope. We also found that, while hardly being strong predictors, the expectations data appear to yield forecasts at least as accurate as naive forecasts based on ex post realizations. Furthermore, we found we could not find a linear contribution where superior forecasting performance continued over time.

Efficient market hypotheses suggest that valuation should reflect information available. Insofar as analysts' forecasts are more precise than other types, we should therefore expect them to be reflected in the market. It therefore is noteworthy that our regression results do support this hypothesis, even though the valuation model as a whole remains unclear.

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