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HOW ELASTIC IS THE DEMAND FOR LABOR?

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SUMMARY

This paper investigates the magnitude of the elasticity of demand for labor in time series data using more general and complete models of demand than have been previously employed. It argues that previous analyses have imposed two invalid constraints in calculations, which bias downward estimated elasticities. The first invalid constraint is the assumption that real capital prices have an equal opposite effect to real wages in the demand equation. We show on measurement error grounds that this constraint should not be imposed in econometric work even when longrun homogeneity of prices correctly characterizes the market. The constraint is rejected in the data. The second invalid constraint is that all explanatory variables have the same lag distribution. We argue that this constraint is invalid when decisions are made under uncertainty and find that it is also rejected by the data. The principal positive empirical finding is that with the constraints relaxed, the elasticity of demand with respect to real wages is much larger than the estimates in the literature, indicating much greater price responsiveness on the demand side of the labor market than has previously been thought.

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Time series studies of the determinants of employment have tended to find relatively low elasticities of response to factor prices. One body of literature has ignored relative price effects and focused exclusively on the adjustment of employment to output.[1] Other studies have entered relative factor prices into adjustment models, to find negligible elasticities. The recent study by Nadiri and Rosen, which represents the most comprehensive work to date, obtained virtually zero elasticities of demand in aggregate and two-digit manufacturing data. Reviewing the literature, Hamermesh concluded that a concensus value of the fixed output response of employment to wages in the long run was a bare -.15 (Hamermesh, table 1).

The econometric evidence of low elasticities of demand may be explained in several ways: it could result from the particular type of model specified, with alternative models yielding different results; it could reflect peculiar variation or lack of variation in the factor price variables, creating poor empirical "experiments"; it could result from inadequate measurement of variables; it could reflect correlation between the wage and error terms due to simultaneity; or it could be, in fact, that demand for labor is highly inelastic.

This study develops a new and more general model of the demand for labor under cost minimization than those used in previous work and uses the model to analyze the time series evidence on the magnitude of the elasticity of demand for labor in United States manufacturing. The principal finding is that, contrary to much of the literature, the fixed output elasticity of demand for labor in manufacturing is fairly sizeable. The common finding of negligible factor price effects appears due to the imposition of the constraint, which is strongly rejected by the data, that real capital prices have as large a positive effect on demand as the negative effect of real wages.

We begin by examining the patterns of change in the principal variables that enter demand analysis: wages, prices, employment and output. The second section examines alternative models of the demand for labor, with specific focus on the appropriate empirical specification of relative prices. Section three presents the basic empirical results. The paper concludes with a brief summary of the findings.

I Patterns of Wage and Employment Change

The patterns of change in the principal demand variables in manufacturing are presented in Table 1. The left hand side of the table records standard deviations in the log changes of quantity variables and of ratios of quantity variables, by quarter, from first quarter 1950 to third quarter 1976. The right hand side of the table presents comparable standard deviations in the log changes of the relevant prices and earnings from quarter to quarter. With one exception, the basic data are obtained directly from the Bureau of Labor Statistics establishment survey and from the U.S. Department of Commerce, as described in the table source. The exception is the price of capital, which we have measured using the concept of user cost developed in several articles by Jorgenson, and various associates (Jorgenson, Hall and Jorgensen). Our measure takes into account the differential tax treatment of equipment and structures and the presence of both equity and debt finance, and adjusts the cost of capital for depreciation and expected capital gains. The formulas and sources of data for the cost of capital are given in the Appendix.

The table reveals two striking aspects about postwar changes in the prices and quantities of labor, goods, and capital which condition econometric analysis of demand relations.

First, there is a markedly different pattern of variation between quantities and prices in the labor market, the product market, and the capital market. In the labor and product markets, the standard deviations show much greater variation over time in quantity variables than in price variables. In the labor market variation in production worker employment is more than twice as large as the variation in nominal wages and 1.8 times that in real wages. The standard deviation for non-production worker employment is smaller but still sizeable. In the product market, the pattern is similar, with the standard deviation in real shipments far exceeding the standard deviation in prices. The pattern is reversed in the capital market. The nominal price of capital shows the greatest variation of any price-side measure and is more than twice as variable as the stock of capital.

Second, although the variation in nominal and deflated (real) earnings is smaller than the variation in employment, the standard deviations of the two earnings variables are still sufficiently sizeable as to provide a reasonable "experiment" from which to estimate demand schedules. Indeed, the basic data suggest that demand for labor may be quite responsive to changes in real wages. If, as a crude first approximation, one were to take the ratio of employment to shipments as indicative of movements along a fixed demand schedule and changes in real wages as the relevant price-side variable, the figures show greater variation along the quantity than price axis. Given a high negative correlation between the variables, this implies an elasticity of demand above unity.[2] In fact, relative to trend, the ratio of employment to shipments is substantially negatively related to the real wage variable: in 86 of 107 quarters, the variables move in the

Table 1

Patterns of Change in Major Demand Variables, 1950:1 to 1976:3

Group	Quantity Variables: category	Quantity Variables: standard deviation of log change, by quarter	Price Variables: category	Price Variables: standard deviation of log change, by quarter
labor market	Employment of production workers	7.9	Average hourly earnings of production workers, nominal units	2.8
	Employment of nonproduction workers	5.2		
product market	Shipments	11.0	W.P.I. in manufacturing	5.4
capital market	Capital stock	2.0	User cost of capital nominal units	11.7
input/output ratios	Employment of production workers/shipments	6.4	Average hourly earnings of production workers, deflated by W.P.I. in manufacturing	4.5
	Employment of nonproduction workers/shipments	11.1	User cost of capital, deflated by W.P.I. in manufacturing	10.3
labor/capital ratios	Employment of production workers/capital	8.5	Average hourly earnings/user cost of capital	11.3
	Employment of nonproduction workers/capital	5.3		

Variable Definitions and Sources

Shipments: value of shipments in manufacturing, Bureau of Census M.3 Report (published in Manufacturers' Shipments, Inventories and Orders) deflated by the WPI for manufactured goods. We used the 1958 benchmark series, which is available over the entire period. The more recent benchmark of the shipments series does not extend back beyond 1958.

Producation and nonproduction worker employment: number of production and nonproduction workers in manufacturing--Bureau of Labor Statistics, establishment survey (published in Employment and Earnings)

Wages: average hourly earnings of production workers--Bureau of Labor Statistics, establishment survey (published in Employment and Earnings)

Output Price: WPI for manufactured goods—Bureau of Labor Statistics

User Cost of Capital: See Appendix

Capital: computed from series on real gross investment in plant and equipment using the perpetual inventory method. The depreciation rate was an average based on the depreciation rates for plant and equipment (.01775 and .03375) with weights being the average share of equipment (plant) in total investment over the period 1950-1976. The benchmark for 1950 was taken from J. Fawcett et al., Capital Stocks Study.

appropriate direction; their simple correlation is -0.84. While these patterns are hardly to be viewed as providing evidence of the magnitude of elasticities, they highlight the fact that the data evince demand-type patterns of change.

Much of the variation in real wages occurs, it should be stressed, in the 1969-75 period when real wages dropped sharply; decomposition of the overall variance of quantity changes into the variance from 1950 to 1969 and the variance from 1969 to 1976 shows that 56% of the total variation occurred in the latter period.[3] Whereas in the "average" postwar NBER reference cycle, real wages rose by 2.45% from trough to peak and by 1.34% from peak to trough, a very different pattern is found in the 1970-75 cycle: from 1970:4 to 1973:4 real wages rose only 0.5%; from 1973:4 to 1975:1 they fell by 7.9%. This highlights the extent to which the decline in real wages in the 1970s provides a distinct "experiment."[4]

Third, the standard deviations in the log quarterly changes of the ratio of employment to capital and of the ratio of average hourly earnings to the price of capital reveal a very different pattern of variation in the relative quantity of inputs than in the relative price of inputs. Not unexpectedly, the relative variation in the labor/capital ratio tends to be dominated by movements in the numerator. More important in terms of ensuing analysis, however, is the fact that <u>variation in the price of labor relative to the price of capital is dominated by changes in the cost of capital</u>: the variance in wages/cost of capital exceeds the variance in nominal wages and is approximately the same as the variance in the cost of capital. This implies that models which relate demand for labor to relative factor prices are essentially relating demand for labor to the price of capital rather than to the price of labor.

II Alternative Models of Demand

The standard, static theory of cost minimization subject to a production constraint can be used to derive an expression for long run labor demand as a function of planned output and relative prices. For simplicity assume that the production process in the representative firm can be approximated by a Cobb-Douglas function of the form:

$$Q_t = a + \beta K_t + \gamma L_t \tag{1}$$

where K and L represent the log of capital and labor input respectively, Q is the log of output and a is a constant. When the firm takes all prices as given, and varies all inputs, cost minimization leads to the following labor demand function:

$$L_t^* = \frac{b}{\theta} + \frac{1}{\theta} Q_t^p - \frac{\beta}{\theta}(W-C)_t \tag{2}$$

where $\theta = \gamma + \beta$ and measures returns to scale, W and C are the log of the prices of labor and capital, Q_T^P is the log of planned output, and b is a constant.

Equation (2) depicts the long run demand for labor. If the firm faces costs of adjustment, discrepancies between current labor input and long run demand may arise as the firm adjusts with a lag to changes in output or relative prices. A common formulation of the adjustment process makes use of some variant of the flexible accelerator. A simple version is given by

$$\Delta L_t = \psi(L_t^* - L_{t-1}) \tag{3}$$

Substituting for L_t^* yields an equation for observed labor input:

$$L_t = \frac{\psi b}{\theta} + \frac{\psi}{\theta} Q_t^p - \frac{\psi}{\theta}\beta(W-C)_t + (1-\psi)L_{t-1} \tag{4}$$

Equation (4) places two constraints on the demand equation. First, it imposes the constraint that the price of labor and the price of capital have equal, opposite signed, coefficients. This restriction derives from the homogeneity of the production process and is assumed to hold in each period.

The second constraint embodied in (4) is that the explanatory variables W, C, and Q have the same lag structure. The equation imposes this constraint by specifying that changes in these variables affect labor demand through changes in the latent variable L_t^*. This restriction assumes that the cost of moving to the new target is the only source of lagged adjustment in the system and that expectations are static or are formed identically for all variables.[5]

We argue that neither of these constraints should necessarily be imposed on the data. The homogeneity constraint, though theoretically valid in models of the demand for labor by a firm, should not be imposed on the data when capital prices are subject to sizeable measurement error. The constraint that all explanatory variables have the same lagged effect on employment is not appropriate when decisions are made under uncertainty.

Measurement error and the relative price constraint

Assume that in the long run W_t and C_t have equal opposite signs but that the price of capital is measured with considerable error. In this case, one can get a better estimate of the long run elasticity of demand for labor by letting wages and capital enter the equations separately than by imposing the constraint the they enter with equal opposite signed coefficients.

To see the implications of measurement error in C_t assume that the true model is (conditional on other variables)

$$L_t = a_0 - \alpha(W_t - C_t) + e_t \tag{5}$$

where W and C represent logs of wages and cost of capital measured in real terms. The price of capital is measured with error, so that

$$\tilde{C}_t = C_t + v_t \tag{6}$$

where $\tilde{C}_t$ is the observed price and v_t is the error. Equations (5) and (6) yield the following observable model

$$L_t = a_0 - \alpha(W_t - \tilde{C}_t - v_t) + e_t \tag{7}$$

We assume that measurement error in capital and the error in the equation are independent of the other variables and are independent of each other: that is $E(Wv) = E(We) = E(Ce) = E(Cv) = E(ev) = 0$.

Least squares regression of L on $W-\tilde{C}$ will yield a $\hat{\beta}$ that differs on average from the true value by

$$\frac{\sigma_v^2 \beta}{\sigma_v^2 + \sigma_\epsilon^2 + \sigma_w^2 - 2\sigma_{cw}} = \frac{\phi\sigma_{\tilde{c}}^2 \beta}{\sigma_{\tilde{c}}^2 + \sigma_w^2 - \sigma_{\tilde{c}w}} \tag{8}$$

where ϕ is the proportion of variation in $\tilde{C}$ due to measurement error, $(\phi = \sigma_v^2/\sigma_{\tilde{c}}^2)$ and $\sigma_{\tilde{c}w}$ $(=\sigma_{cw})$ is the covariance between $\tilde{C}$ and W in the data.

When W and $\tilde{C}$ are entered separately, on the other hand, we obtain the following regression

$$L_t = a_0 - \hat{\beta}_w(W_t) + \hat{\beta}_c(\tilde{C}_t) + e - \beta_c v \tag{9}$$

where $\hat{\beta}_w$ and $\hat{\beta}_c$ are the coefficients on W and $\tilde{C}$ respectively. Because of the measurement error in $\tilde{C}$, $\hat{\beta}_w$ is biased downward relative to the true response coefficient β. The magnitude of the bias on $\hat{\beta}_w$ can be evaluated from (Griliches Ringstad, p. 197):

$$\text{plim } (\hat{\beta}_w - \beta) = \left[\frac{\phi r_{\tilde{c}w} \left(\frac{\sigma_{\tilde{c}}}{\sigma_w}\right)}{1-r_{\tilde{c}w}^2}\right]\beta \tag{10}$$

Comparing (10) and (8) we see that $\hat{\beta}_w$ will be a less biased estimate of β than $\hat{\beta}$ when

$$\frac{r_{\tilde{c}w} \left(\frac{\sigma_{\tilde{c}}}{\sigma_w}\right)}{1 - r_{\tilde{c}w}^2} < \frac{\sigma_{\tilde{c}}^2}{\sigma_{\tilde{c}}^2 + \sigma_w^2 - 2\sigma_{\tilde{c}w}} \tag{11}$$

which simplifies to[6]

$$\sigma_{\tilde{c}w} < \sigma^2_{\tilde{c}}$$

That is, the bias in $\hat{\beta}_w$ is less than the bias in $\hat{\beta}$ when the covariance between the capital price and the wage is less than the variance in the price of capital (all variances partialled on the other relevant variables).

Using the data described in table 1 and measuring $\tilde{C}$ and W in deflated units, one obtains, conditional on real shipments, lagged employment and time, a value for $\sigma^2_{\tilde{c}}$ of 7.84 and a value of $\sigma_{\tilde{c}w}$ of 0.60. These values imply that the β estimated in the constrained model is subject to a much larger downward bias than the $\hat{\beta}_w$ obtained from the unconstrained model. Indeed, the estimated variance in W conditional on real shipments, lagged employment and time is 0.50, which together with the estimates of $\sigma^2_{\tilde{c}}$ and σ^2_{cw} gives a bias for the constrained model of -1.1ϕ and a bias for the unconstrained model of -0.19ϕ. With these values, the unconstrained model is to be preferred.

Decision making under uncertainty and the lag structure

The implications of uncertainty in factor prices for the lag structure used in labor demand analysis can be illustrated in the context of a slightly modified version of equation (2). Consider a risk neutral firm faced with the problem of choosing an optimal input mix given a planned level of output. We retain the basic framework specified in (1) and (2), but assume that W and C are stochastic. The firm's objective, is to minimize expected cost, subject to the production constraint given by (1):

$$\min_{\ell, k} H = E(w_t \ell_t + c_t k_t) - \lambda(q_t - Ak_t^{\beta} \ell_t^{\gamma}) \qquad (13)$$

where lower case letters have been used to indicate variables in their natural units, and E is the expected value operator. The first order conditions for a minimum in addition to the production function are

$$\frac{\partial H}{\partial \ell} = E(w_t) - \lambda\gamma\frac{q_t}{\ell_t} = 0 \quad ; \qquad \frac{\partial H}{\partial k} = E(c_t) - \lambda\beta\frac{q_t}{k_t} = 0 \tag{14}$$

which may be expressed as

$$E(w_t/\lambda) = \gamma\frac{q_t}{\ell_t} \qquad E(c_t/\lambda) = \beta\frac{q_t}{k_t} \tag{15}$$

Solving (15) and the production function for ℓ_t and taking logs yields

$$\ell_t = \frac{a_o}{\theta} + \frac{1}{\theta} Q_t^p - \frac{\beta}{\theta} \log \left[\frac{E(w)}{E(c)}\right]t \tag{16}$$

Equation (16) differs from equation (4) in the specification of relative factor prices. In (16) demand depends on the ratio of the expected value of w to the expected value of c, not on the ratio of factor prices or the expectation of the ratio. When c and w do not follow the same stochastic process, this implies that their lag structures should differ. To illustrate, consider the situation in which the "true" cost of capital in the current period is related to previous values by $c_t = c_{t-1} + u_t$, where u_t is a zero mean, white noise input. Assume further that measurement of the cost of capital is subject to error, so that $\tilde{c}_t = c_t + v_t$. where $\tilde{c}$ is the observed or measured cost, and v_t is a zero mean, white noise input, independent of u_t. The process of determining an expected cost of capital can be treated as a problem of choosing an optimal predictor of $\tilde{c}$. For simplicity we restrict the predictor to be a linear combination of past observations of c. Assuming that the firm's objective is to minimize the mean square error of the forecast, the optimal predictor of c can be expressed as

$$c = (1-\delta) \sum_{j=0} \delta^j \tilde{c}_{t-j} \tag{17}$$

which is the standard form for adaptive expectations (see Nerlove). The parameter δ determines the shape of the distributed lag, and is a decreasing function of $\frac{\sigma_u^2}{\sigma_v^2}$, the relative size of the true component in the total variation of measured capital costs. In effect, the more "noisy" the observed cost of capital, (i.e. the lower $\frac{\sigma_u^2}{\sigma_v^2}$) the less rapid will the firm adjust to changes in $\tilde{c}$.

The distributed lag given by (17) is based on a highly simplified stochastic process; a more realistic characterization would yield a much more complicated lag structure. The important point here, however, is that the nature of the distributed lag depends on the characteristics of the process generating observed values of w and c. Since wages and capital costs are likely to follow different stochastic processes, the lag structure associated with each will be different. As long as past observations of w and c are used in forming expectations and the stochastic processes generating w and c differ, $\frac{E(w)}{E(c)} \neq E(\frac{w}{c})$. Hence, the firm will not look at past values of $(\frac{w}{c})$ in determining likely means of the ratio of expected wages to expected costs of capital. Even though variations in wage rates and capital costs may influence labor demand through their effect on the relative price term, the firm must examine the past history of each variable separately in forming expectations.

The empirical implication of uncertainty in factor prices is that W and C should be allowed to have different lag distributions in labor demand analysis and thus that past values of these variables should enter the equation separately. This is true even if we impose the long run homogeneity constraint. While the ultimate long run effect of W and C may be identical, the speed of adjustment and other aspects of the time pattern of response may be quite different.

Taken together, the presence of uncertainty and errors of measurement imply that the impact of wage rates and the cost of capital should be examined separately. Ignoring adjustment costs for the moment, the basic two-factor model can be written as

$$L_t = \frac{a_0}{\theta} + \frac{1}{\theta} Q_t^p - \frac{\beta_w}{\phi} \ln E(w_t) + \frac{\beta_c}{\phi} \ln E(c_t) \qquad (18)$$

If, as noted earlier, we assume that planned output is based on a forecast of shipments, then Q_t^p can be replaced by expected real shipments $E(S_t)$. Depending on the specification of expectation formation, L_t will depend on current and lagged values of the variables, and lags in the system will be solely attributed to problems of information and uncertainty. Adjustment costs can be introduced through an equation like (3), so that (18) becomes

$$L_t = \psi[\frac{a_0}{\theta} + \frac{1}{\theta}\ln E(s_t) - \frac{\beta_w}{\phi}\ln E(w_t) + \frac{\beta_c}{\phi}\ln E(c_t)] + (1-\psi)L_{t-1} \quad (19)$$

Clearly, (19) may be rewritten to eliminate L_{t-1} so that L_t depends only on current and past values of the independent variables. Lags due to adjustment costs would be confounded with lags due to expectations, and without further restrictions the separate influence of each could not be identified. However, sorting out the separate influence of expectations and adjustment costs and determining the specific time pattern of response is of secondary importance in the current analysis. In terms of evaluating the elasticity of demand, the precise pattern of adjustment and the source of lags in the system are of interest only insofar as they affect estimates of the elasticity parameter or raise questions about the appropriate model of demand. With respect to estimated elasticities, empirical evidence to be presented in Section III shows that different adjustment processes affect the timing but not the magnitude of the estimated demand response. Accordingly we shall focus on the question of whether all independent variables have the same lag distribution. If a common distributed lag cannot be rejected, it could be argued that adjustment costs are the principal source of lags and that the lagged effects of the independent variables work through the latent variable L* as posited in the standard flexible accelerator model. Significant differences in the lag structure will reject the standard adjustment model and provide support for the specification based on uncertainty and expectations presented earlier.[7]

III Empirical Results

This section presents estimates of the models of labor demand developed in section II using data for U.S. manufacturing from first quarter 1950 to third quarter 1976. We focus first on the size of elasticity estimates obtained with models of the firm that have different specifications of relative prices and processes of adjustment. Subsequent analysis examines the effect of changes in several assumptions on the estimated elasticities. The principal finding is that, as long as wages and the price of capital are allowed to have different impacts on demand for labor, the estimated elasticities are quite sizeable, from 2 1/2 to 3 times the values obtained when the variables are constrained to have the same impact.

Estimates of the 2-factor model

The basic empirical framework used in this section is provided by a fairly general form:

$$L_t = \frac{1}{\theta}\left[a + \sum_{i=0}^{n_1} b_i S_{t-i} - \beta_w \sum_{i=0}^{n_2} d_i W_{t-i} + \beta_c \sum_{i=0}^{n_3} m_i C_i - \delta_t\right] + \varepsilon \quad (20)$$

where ε is the error term and all variables are in logs. The data used cover the manufacturing sector as described in table 1. L is the log of production worker employment, S is the log of shipments, W is the log average hourly earnings, and C is log of the cost of capital, all expressed in real terms by deflating by the WPI for manufacturing.

The distributed lags in (20) will be examined along three lines. To develop a benchmark we will impose the constraint that the lag distributions follow a geometric pattern, with $b = d_i = m_i$ for all i, so

(20) may be estimated with a Koyck lag. This constraint will then be relaxed, and the lag weights estimated using Almon polynomials. Finally, unconstrained lagged values of the independent variables will be entered into the equations. The length of the lag appropriate for each variable will be chosen empirically, based on minimizing the error sum of squares.

Our principal focus in the analysis which follows is the magnitude of β_w and β_c. Given the methods of estimation outlined above, estimates of β_w and β_c (and θ) can be obtained only under the assumption that $\Sigma b_i = \Sigma d_i = \Sigma m_i = 1$, since no attempt will be made to estimate the long run parameters and the lag weights separately. The assumption holds exactly for the Koyck lag, and seems reasonable in the other situations as well. It should be noted that the most likely alternative is that the sum of the weights is less than one. Thus if the unity constraint on the sum of the weights is not true, it is likely that the estimated long run elasticities will be downward biased.

Table 2 presents estimates of equation (20) under several specifications of the lag structure and relative prices. In columns 1 and 2 a Koyck lag has been imposed; column 3 presents results with an unconstrained lag, while column 4 contains results when the coefficients follow a second degree polynomial. The relative prices are entered separately in each column except the first, where the long run homogeneity constraint is imposed. The estimated elasticity of demand for labor in column 1, which imposes the same lag distribution on the two variables, is consistent with other results in the literature: the long run elasticity of -.19 is of the same magnitude as Hamermesh's summary value -.15 (Hamermesh, 1976). We argued earlier that imposition of the constraint of $\beta_w + \beta_c = 0$ may not be appropriate in the presence of measurement error and uncertainty. Entering the prices separately finds empirical support in column 2, where C and W obtain very different coefficients. The evidence suggests a wage elasticity of -.253 in the short run, and a value of -.463 over the longer term. The cost of capital, in contrast, is estimated to have a short run impact of .062 and a long run effect of .114. Thus, both wage rates and the cost of capital are found to influence the demand for labor, with wages estimated to have an impact about four times as large. These results imply that models which specify identical, opposite signed effects of factor prices are not consistent with the data.[8] It appears that one reason for the small elasticities found in previous work is imposition of a model ($\beta_w + \beta_c = 0$) which is rejected by the data.

The results in columns 1 and 2 are based on a model of the adjustment process in which changes in wages and the cost of capital influence labor demand through their effect on desired employment. Costs of adjustment yield a lagged response which has the same time pattern in all variables. The assumption of a common geometric lag structure is dropped in columns 3-4 and more general lag distributions estimated. Changing the lag structure has little impact on the magnitude of long run elasticities of demand for labor. The long run wage parameter lies between -.472 and -.489, while the long run output elasticity ranges from 1.058 to 1.069. Only in the case of the cost of capital did the lag specification have any impact on the magnitude of the elasticity. In experiments not reported here, shortening the lag from 8 to 3 quarters reduces the effect by almost 30 percent. The magnitude of the capital cost elasticity was sensitive to the length of the lag up until about 8 quarters out. Beyond that point the value of the coefficient remained relatively close to 0.100.

The actual lag structures in the variables, given in figure 1, suggests that the lagged response to changes in the cost of capital is

Table 2: Estimates of the Elasticity of Demand for Labor With Alternative Factor Price Constraints and Alternative Adjustment Processes[a]

Equation	Factor Price Constraint	Lag Structure	Long run Elasticity S	Long run Elasticity W	Long run Elasticity C	Regression Coefficients and Standard Error	$\bar{R}^2$	SEE	ρ
(1)	yes	geometric	.891	-.192	.192	$\underset{(.103)}{.134} - \underset{(.0002)}{.003T} + \underset{(.041)}{.422L_{t-1}} + \underset{(.032)}{.515S_t}$ $- \underset{(.018)}{.099W_t} + \underset{(.018)}{.099C_t}$	.984	.00794	.720 (.077)
(2)	no	geometric	.980	-.463	.114	$\underset{(.080)}{.001} - \underset{(.0002)}{.003T} + \underset{(.034)}{.443L_{t-1}} + \underset{(.031)}{.546S_t}$ $- \underset{(.035)}{.253W_t} + \underset{(.015)}{.062C_t}$	.986	.00745	.525 (.091)
(3)	no	unconstrained distributed lag	1.058	-.472	.105	$\underset{(.199)}{-.209} - \underset{(.0004)}{.005T} + \underset{(.040)}{.432S_t} + \underset{(.045)}{.226S_{t-1}}$ $+ \underset{(.048)}{.097S_{t-2}} + \underset{(.038)}{.057S_{t-3}} + \underset{(.035)}{.046S_{t-4}} - \underset{(.110)}{.213W_t}$ $- \underset{(.124)}{.347W_{t-1}} + \underset{(.120)}{.061W_{t-2}} + \underset{(.033)}{.062C_t} - \underset{(.031)}{.034C_{t-1}}$ $+ \underset{(.030)}{.053C_{t-2}} - \underset{(.031)}{.036C_{t-3}} + \underset{(.030)}{.054C_{t-4}}$ $- \underset{(.031)}{.039C_{t-5}} + \underset{(.031)}{.027C_{t-6}} + \underset{(.030)}{.011C_{t-7}} + \underset{(.031)}{.013C_{t-8}}$	.988	.00693	.858 (.059)
(4)	no	Almon lag with 2nd degree polynomial	1.069	-.484	.097	$\underset{(.188)}{-.249} - \underset{(.0004)}{.005T} + \underset{(.031)}{.597S_t} + \underset{(.016)}{.283S_{t-1}}$ $+ \underset{(.021)}{.092S_{t-2}} + \underset{(.016)}{.022S_{t-3}} + \underset{(.028)}{.075S_{t-4}} - \underset{(.092)}{.144W_t}$ $- \underset{(.102)}{.411W_{t-1}} + \underset{(.101)}{.038W_{t-2}} + \underset{(.023)}{.024C_t} + \underset{(.014)}{.016C_{t-1}}$ $+ \underset{(.010)}{.011C_{t-2}} + \underset{(.011)}{.007C_{t-3}} + \underset{(.012)}{.005C_{t-4}} + \underset{(.011)}{.005C_{t-5}}$ $+ \underset{(.009)}{.007C_{t-6}} + \underset{(.012)}{.011C_{t-7}} + \underset{(.022)}{.017C_{t-8}}$	.988	.00706	.840 (.059)

[a] Dependent variable is the log of production worker employment.

somewhat slower than the response to shifts in sales, and quite a bit slower than the response to changes in the wage. The strikingly different pattern of response shown in figure 1 is consistent with an uncertainty/expectations rationale for lagged adjustment, in which the speed and pattern of adjustment depends on the stochastic structure generating observed values of the independent variables. The longest response time is in the variable with the highest noise-to-signal ratio, the cost of capital. In contrast, firms appear to respond relatively rapidly to changes in wage rates which are more easily predicted. The differences in lag response provide further empirical justification for treating wages and the cost of capital separately, and for relaxing the assumption of a common lag distribution.

Given the preceding adjustments to the basic 2-factor model, the evidence suggests that the wage elasticity of demand for production workers in manufacturing lies between -.15 and -.20 in the short run (one quarter), with the full effect of approximately -.48 completed after 2-3 quarters.

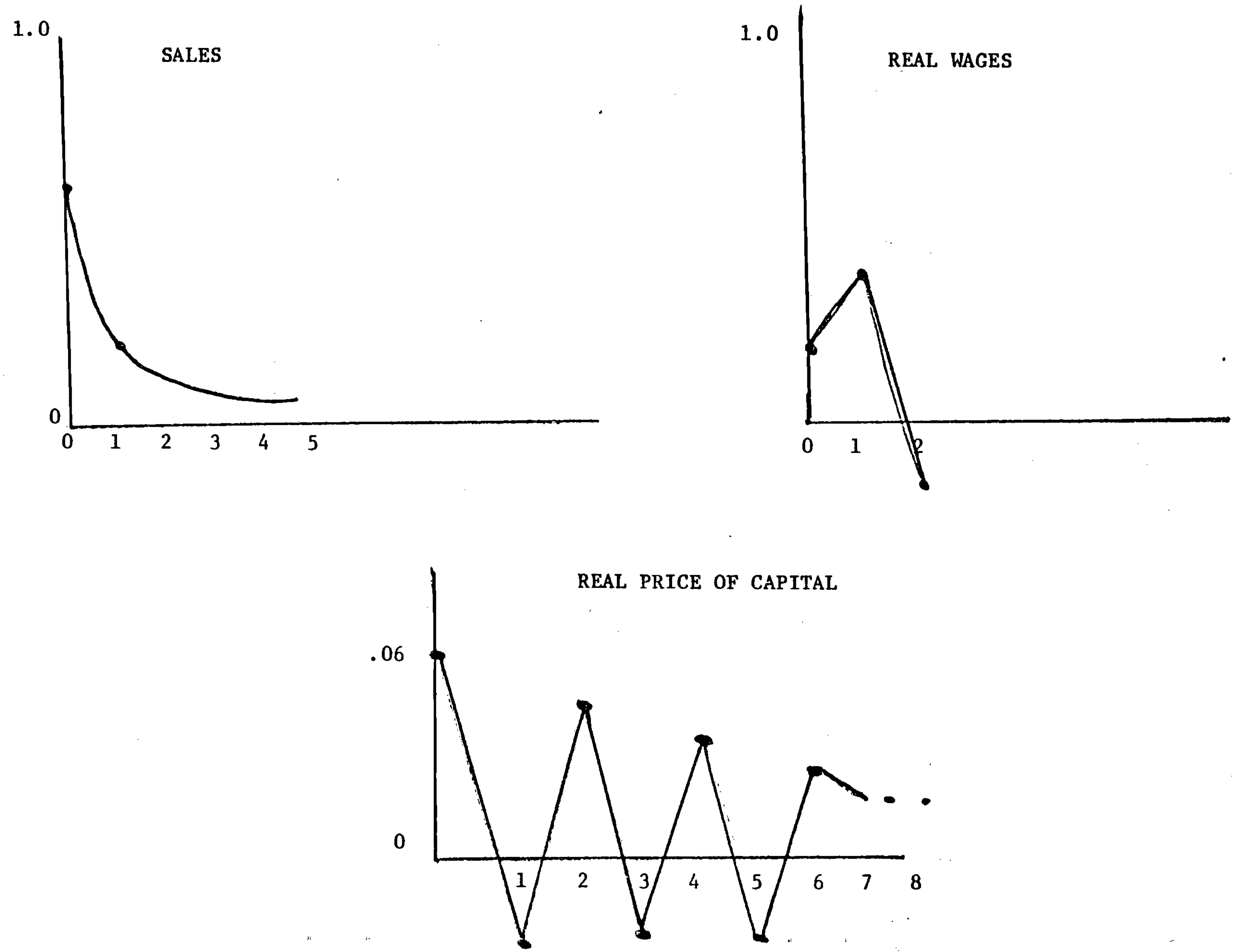
Analysis of assumptions

The elasticity estimates in table 2 are based on a 2-factor model of the production process, in which the firm treats prices as parameters and input decisions are made conditional on an output plan derived from a sales forecast. The 2-factor framework ignores the influence of variation in other factors of production by implicitly assuming that they move in proportion to changes in the number of production workers or that they are fixed. The model further assumes that sales can be treated as an exogenous term, thereby ignoring questions of simultaneity in the determination of output, employment, and factor prices. Finally, the adjustment process underlying the basic results does not allow for feedback between the factors of production of the type stressed by Nadiri and Rosen, among others.

Table 3 presents estimates of the labor demand equation which alters

Figure 1

Estimated Time Patterns of Response From Table 2



several of these postulates. For comparison, line 1 reproduces the coefficients from the basic model (column 2, table 2). The potential impact of simultaneity is examined in lines 2-4. In line 2 we treat shipments as endogenous, instrumenting it on the exogenous measures of aggregate demand specified in the table note. In line 3, we instrument factor prices as well as shipments on the aggregate demand measures. Use of the demand side instruments in lines 2 and 3 has very little effect on the estimated coefficients. As expected, the long run elasticity increases somewhat, but the change is marginal. In line 4 we shift from demand to supply factors, instrumenting the real wage on measures of shifts in supply. With the supply instruments, the short run elasticity increases by 20 percent, while the estimated long run impact is -.53. This suggests the possibility that the "demand relation" between employment and wages in the data is contaminated by simultaneity on the supply side leading to underestimates of the elasticity of demand.

Expansion of the 2-factor model to include additional inputs (non-production workers, inventories) requires more complete data on factor prices. As Nadiri and Rosen have shown, appropriate measures would include wage rates, fringe benefits, search and training costs, and inventory carrying costs. The availability of such data would permit a distinction between stock and flow demands, and would allow estimation of the elasticity of demand for production workers holding constant the prices of additional relevant inputs. The full range of factor price data is not available on a time series basis,and the usual procedure is to assume that all facets of the user cost of labor and the price of inventories vary proportionally with average hourly earnings, or the cost of capital. Without additional data, the implications of this assumption cannot be examined. However, some feel for the influence of additional inputs may be obtained by using available data on hours per worker, the number of non-production workers and the stock of capital to modify our estimating equations further.

TABLE 3: Estimates of the Elasticity of Demand under Alternative Assumptions[1]

	S	W	C	L_{-1}	long run wage elasticity	R^2	SEE	ρ
Specific Variant								
1) Result from column 2 table 2	0.55 (0.03)	-0.25 (0.04)	0.06 (0.02)	0.44 (0.03)	-0.45	0.986	7.45	0.53 (0.09)
Corrections for Simultaneity								
2) Correction for simultaneity in shipments[2]	0.55 (0.04)	-0.26 (0.03)	0.05 (0.01)	0.50 (0.03)	-0.48	0.954	5.27	0.51 (0.07)
3) Correction for simultaneity in shipments, real wages, price of capital[3]	0.53 (0.04)	-0.25 (0.03)	0.05 (0.02)	0.47 (0.03)	-0.47	0.951	5.04	0.49 (0.06)
4) Correction for simultaneity in real wages, using supply instruments[4]	0.57 (0.03)	-0.30 (0.04)	0.04 (0.02)	0.45 (0.03)	-0.55	0.986	7.27	0.52 (0.09)
Alternative Dependent Variables								
5) Using Manhours as dependent variable	0.77 (0.04)	-0.35 (0.04)	0.07 (0.01)	0.32 (0.04)	-0.51	0.981	9.62	0.47 (0.09)
6) Using total employment[5] as dependent variable	0.42 (0.02)	-0.16 (0.03)	0.02 (0.01)	0.52 (0.03)	-0.33	0.995	5.64	0.56 (0.09)
Addition of Other Control Variables								
7) Addition of nonproduction workers as independent variable [6]	0.54 (0.03)	-0.22 (0.04)	0.04 (0.02)	0.48 (0.04)	-0.42	0.986	7.41	0.51 (0.09)
8) Addition of capital as independent variable[7]	0.55 (0.03)	-0.29 (0.05)	—	0.41 (0.05)	-0.49	0.984	7.90	0.71 (0.08)
Interrelated Adjustment								
9) Lagged values of hours, inventories, capital, and non-production workers added[8]	0.54 (.04)	-0.24 (.05)	0.05 (.02)	0.47 (.06)	-0.49	0.985	7.56	0.49 (0.10)

[1] Dependent variable is the log of production worker employment except in lines 5 and 6, where the dependent variable is changed. All regressions include time and a constant.

[2] Instruments (in addition to current and lagged values of included exogenous variables): real government expenditures on durable and nondurable goods and services (both federal and state and local) real exports of goods and services, and total population 18-64.

[3] Instruments were the same as in (2) except that current and lagged values of C were excluded.

[4] Instruments (in addition to the current and lagged values of included exogenous variables) were: labor force less manufacturing production workers; compensation per manhour in private nonfarm sector deflated by the CPI.

[5] W is total compensation per manhour deflated by the WPI for manufacturing.

[6] Non production labor entered the equation with a coefficient (and standard error) of -0.06 (0.04).

[7] Capital entered the equation with a coefficient (and standard error) of 0.02 (0.03).

[8] The other coefficients (standard errors) were as follows: constant -.02 (.61); TIME -.003 (.0002); inventories -.03 (.05); capital .03 (.03); hours per worker -.02 (.17); non-production workers -.02 (.05).

In lines 5 and 6 we have altered the dependent variable to reflect manhours and total employment rather than employment of production workers. With manhours as the dependent variable, the coefficient on real wages rises while that on the lagged adjustment term falls. Since hours are likely to respond more quickly than employment, this is a reasonable result. The long run elasticity is somewhat larger than that obtained in the production employment calculations. Changing the dependent variable to total employment by inclusion of nonproduction workers in line 6 has the opposite effect: the short run elasticity with respect to wages falls; the estimated adjustment coefficient declines, as does the long run elasticity. This reflects the addition of a relatively quasi-fixed factor (nonproduction workers) to the dependent variable.

A different type of amendation is made in lines 7 and 8, where two other control variables are added to the regressions: employment of non-production workers (line 7) and the stock of capital (line 8). Addition of these variables has a slight impact on estimated elasticities. In line 7, the long run response falls to 0.42, while controlling for the capital stock in line 8 raises the estimate to -.49. In the absence of a complete system of demand equations, however, the long run elasticities implied by the adjustment process are to be viewed as no more than illustrative of the impact of specification changes on estimates. In the long run, one clearly does not want to hold these other inputs fixed.[9]

An alternative more desirable way of treating additional inputs is to analyze interrelations in terms of the effect of discrepancies between actual and desired levels of other inputs. One way of doing this is to generalize the flexible accelerator as in:

$$\Delta X_{it} = \sum_i \psi_i (X^*_{it} - X_{it-1}) \qquad (21)$$

where X's are inputs (see Nadiri and Rosen). Assuming X^*_{it} is determined by expected sales and relative prices, solution of (21) for X_{it} yields an equation similar to the basic Koyck lag model, with the addition of lagged values of all other inputs. In the current setting, interrelated adjustment is examined by relating production worker employment to the usual variables, but including lagged values of hours-per-worker, inventories, non-production employment, and the capital stock (all variables in logs). The results are presented in line 9 and footnote 8 of table 3. Allowing for interrelated adjustment has only a marginal effect on the short run elasticity, which falls to -.24. To calculate a long run elasticity it is necessary to estimate comparable equations for all inputs and solve the resultant difference equation system. For production workers, this procedure yields a long run elasticity of -.49. Together with the evidence in table 2, these results underscore the point that alternative lag structures have little impact on the estimated elasticity.

IV Conclusion

The examination of demand for labor in this paper has yielded substantive empirical results regarding the magnitude of the elasticity of demand and more technical findings regarding alternative model structures.

The principal finding is that under a variety of specifications, the elasticity of demand for production labor is quite sizeable, far from the very small estimates traditionally found in the literature. The low elasticities found in previous work appear to be due to the imposition of the long run homogeneity constraint in the presence of measurement error. We have found that while our basic results are invariant to changes in the lag structure, the data reject both the imposition of identical lag structures on the explanatory variables, and the restriction of equal opposite signed factor price coefficients. The rejection of a common lag structure on all explanatory variables suggests the need for new models of the adjustment process in the labor market, with emphasis on the role of uncertainty and expectations.

The evidence that demand for labor is more responsive to changes in real wages than had been previously thought suggests that greater attention be given to policies, like employment tax credits, which seek to stimulate employment by inducing movements along demand schedules.

Footnotes

[1] See, for example, the studies of Brechling and O'Brien, Fair, Soligo, and Sims.

[2] Simply applying the definition of a least squares estimator we have $b_{(W/Q)(W/P)} = \frac{r_{(W/Q)(W/P)} \quad (E/Q)}{\sigma_{w/p}}$; if (E/Q) and (W/P) were perfectly negatively correlated, the elasticity would be -1.44.

[3] The sum of squared deviations from the mean for the 1950-1969 period is 930.8, and for the latter period 1186.5.

[4] The NBER reference cycles were taken from Business Conditions Digest, published monthly by the Department of Commerce. The trough for the 1973-1975 recession has not been officially determined and our use of 1975:1 is only an approximation.

[5] Since the model in equation (6) is based on adjustment lags, the assumption is that expectations are static. A similar geometric distribution could, however, be generated assuming expectations for all variables were formed according to adaptive expectations.

[6] Since $r_{\tilde{c}w} = \sigma_{\tilde{c}w}/\sigma_w\sigma_{\tilde{c}}$, we rewrite (11) as

$$(\sigma^2_{\tilde{c}} + \sigma^2_w - 2\sigma_{\tilde{c}w})(\sigma_{\tilde{c}w}/\sigma_w\sigma_{\tilde{c}})(\sigma_{\tilde{c}}/\sigma_w) < \sigma^2_{\tilde{c}}(1 - \sigma^2_{\tilde{c}w}/\sigma^2_w\sigma^2_{\tilde{c}})$$

which simplifies to

$$(\sigma^2_{\tilde{c}} + \sigma^2_w - 2\sigma_{\tilde{c}w})(\sigma_{\tilde{c}w}/\sigma^2_w) < \sigma^2_{\tilde{c}} - \sigma^2_{\tilde{c}w}/\sigma^2_w$$

Thus:

$$\left(\frac{\sigma^2_{\tilde{c}}}{\sigma^2_w} + 1\right)\sigma_{\tilde{c}w} - \sigma^2_{\tilde{c}w}/\sigma^2_w < \sigma^2_{\tilde{c}}$$

Rearranging terms yields

$$\sigma^2_{\tilde{c}}(1 - \sigma_{\tilde{c}w}/\sigma^2_w) > \sigma_{\tilde{c}w}(1 - \sigma_{cw}/\sigma^2_w)$$

which gives (12) in the text.

[7] The analysis in the text follows existing literature by analyzing demand for labor by an individual cost-minimizing firms and thus on the fixed output demand elasticity. Because the data relate to industry level aggregates, however, questions arise about whether models of the firm offer the appropriate tool of analysis. The principal difference between the fixed-output demand schedule of a firm and the industry-level demand schedule relates to the exogeneity of the scale of output. Whereas the cost-minimizing firm takes output as exogenous and the profit-making firm takes the price of output as exogenous, the level of output and price are endogenous variables to the industry. As is well known, with price and output endogenous, the elasticity of demand depends not only on substitution but also on "scale effects." An increase in the wage reduces demand for labor by causing substitution against labor and also by reducing industry output through increased costs and prices. An increase in the price of capital, by contrast, raises demand for labor by causing substitution toward labor but also reduces demand for labor by reducing industry output through increased costs and prices. The differential scale and substitution effects induced by capital price changes compared to those induced by wage changes provides another reason for relaxing the assumption that W and C enter the demand equation with equal opposite signs.

Estimates of industry demand curves with output endogenous require, however, detailed investigation of the factors that shift the demand for output, which goes beyond the scope of the current analysis (see Houthakker and Taylor for empirical estimates of demand for industry ouput). Hence, no effort is made in this study to estimate the indusry level demand curves. Since we focus solely on fixed-output demand relations, the "full" elasticity of demand for labor exceeds the estimates on our models.

[8]The important finding that the relative factor price model if rejected by the data can be examined in another way. Associated with the optimal level of employment is a factor price frontier relating price of output to the price of inputs. With a Cobb-Douglas constant returns production function the factor price frontiers can be written as

$$(1) \quad P = \alpha W + (1-\alpha)C + aT$$

where P = log of output price; W = log of wage rate; C = log of cost of capital; and T = time, used to index of technical change. If the quarterly data lie on the long run frontier regressions of P on W, C and T should yield positive coefficients on the two input prices with coefficients that sum, at least approximately, to unity. After considerable effort to obtain estimates consistent with the model, the following "best" results were obtained:

$$(2) \quad P = \underset{(0.095)}{-0.622} + \underset{(0.112)}{0.311}\, W - \underset{(0.024)}{0.075}\, C + 0.003T \qquad \bar{R}^2 = 0.999, \quad \sigma_u x10^3 = 7.81$$

$$(3) \quad P = \underset{(0.122)}{-0.482} + \underset{(0.140)}{0.409}\, W + \underset{(0.024)}{0.068}\, C + \underset{(0.098)}{0.693}\, P_{-1} \underset{(0.001)}{-0.003} \qquad \bar{R}^2 = 0.998, \quad \sigma_u x10^3 = 7.83$$

where the calculations include a correction for second-order autocorrelation. In (2) the coefficients sum to .23; in (3), where a simple geometric distribution is applied, they sum to 1.55 due to the high correlation between P and P_{-1}. Both sums of the coefficients diverge significantly from unity, providing further evidence against the standard relative factor price model. The model simply does <u>not fit</u> the data.

[9]The proper way to deal with several inputs is to imbed the labor demand function into a consistent system of factor demand relations. As a further check on our empirical findings we estimated elasticities of demand from a three input translog cost function, using annual data on production workers, nonproduction workers, and capital. While most attention has been given to the translog production function in analysis, we believe that the translog cost function offers a more fruitful means of estimating the elasticity of demand. The factor demand equations obtained from the cost function relates shares to factor prices, consistent with the single equation models,

while the production function relates shares to factor quantities. In analyzing a single sector such as manufacturing the first specification seems more plausible.

Given factor prices and input levels, we calculated cost as the sum of factor outlays and then derived the cost shares by division. We treated technological change as an additional input, estimated as the rate of growth of total cost deflated by the price of output less a weighted average of input growth weights, with input shares in cost as weights.

The parameters of the constrained translog cost system were jointly estimated using an iterative version of Zellner's minimum distance estimator. To correct for the possible endogenous determination of factor prices we combined the iterative Zellner procedure with two stage least squares using the same instruments as in the single equation analysis (see table 3, note 2). This iterative version of three stage least squares is asymptotically equivalent to full information maximum likelihood (see Berndt, Hall, Hall and Hausman).

With the full translog constraint only two of the share equations are needed. Assuming neutral technical change the blue collar and white collar share equations are:

$$\alpha_B = \underset{(.004)}{.55} - \underset{(.14)}{.08} \log (p_B/c) - \underset{(.13)}{.01} \log(p_W/c) \qquad \bar{R}^2 = .57$$

$$D.W. = 1.15$$

$$\sigma_u \times 10^2 = 1.74$$

$$\alpha_W = \underset{(.002)}{.31} - \underset{(.13)}{.01} \log (p_B/c) + \underset{(.13)}{.143} \log(p_W/c) \qquad \bar{R}^2 = .93$$

$$D.W. = 1.70$$

$$\sigma_u \times 10^2 = 0.85$$

where α_B = share of blue collar labor, α_w = share of white collar labor, p_B = price of blue collar labor, p_w = price of white collar labor, and c = cost of capital.

Solving the system for the elasticity of demand for labor (see Binswanger) for the algebraic methodology) yielded an estimated elasticity of demand for production worker labor of -.58 with a standard error of .25. This estimate is consistent with the single equation estimates in the text. For further analysis of the translog estimates see Clark and Freeman.

References

Berndt, E., and L. Christensen. "The Translog Function and the Substitution of Equipment, Structures and Labor in U.S. Manufacturing, 1929-1968." Journal of Econometrics (1973), pp. 81-114.

Berndt, E., B. Hall, R. Hall, and J. Hausman, "Estimation and Inference in Non-linear Structural Models," Annals of Economic and Social Measurement (1974).

Binswanger, H. "A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution," American Journal of Agricultural Economics (May, 1974), pp. 377-386.

Brechling, F. and P. O'Brien, "Short-Run Employment Functions in Manufacturing Industries," Review of Economics and Statistics (1967), pp. 277-287.

Clark, K. and R. Freeman, "Time Series Models of the Demand for Labor in Manufacturing," Harvard Institute of Economic Research, Discussion Paper 575, 1977.

Fair, R., The Short-Run Demand for Workers and Hours (Amsterdam: North Holland, 1969).

Griliches, Z., "Capital-Skill Complementarity," Review of Economics and Statistics (November, 1969), pp. 465-468.

Griliches, Z., and V. Ringstad, Economies of Scale and the Form of the Production Function (Amsterdam: North Holland, 1974).

Hamermesh, D. "Econometric Studies of Labor Demand and Their Application to Policy Analysis," Journal of Human Resources (Fall, 1976), pp. 507-525.

Hall, R. and D. Jorgensen, "Tax Policy and Investment Behavior," American Economic Review (June, 1967), pp. 391-414.

Houthakker, H. and L. Taylor, Consumer Demand in the United States: Analysis and Projects (Harvard Economic Studies v. 126), 1970.

Jorgensen, D., "Capital Theory and Investment Behavior," American Economic Review (May, 1963), pp. 247-259.

Mundlak, Y., "Elasticities of Substitution and the Theory of Derived Demand," Review of Economic Studies (1968), pp. 225-236.

Nadiri, I. and S. Rosen, A Disequilibrium Model of the Demand for Factors of Production (NBER: New York, 1973).

Nerlove, M. "Distributed Lags and Unobserved Components in Economic Time Series" in M. Fellner, et al. Ten Economic Studies in the Tradition of Irving Fisher, 1967, p. 127.

Soligo, R. "The Short-Run Relationship between Employment and Output," Yale Economic Essays (Spring, 1966), pp. 161-125.

Solow, R. "The Short-Run Productivity Puzzle," (Cambridge: MIT Press, 1970), processed.

Appendix

This appendix describes the definitions and sources of the variables used to construct the user cost of capital.

user cost of capital: computed as a weighted of the user cost of equipment (C_{eq}) and structure (C_{st}), with the share of equipment (structures) in total investment taken from the National Income and Product Accounts, hereafter NIPA.

C_{eq} = user cost of producers durable equipment

$$= Q_e[r + \delta - \pi_E][\frac{1 - k - uz + Dkuz}{1 - u}]$$

C_{st} = user cost of nonresidential structures $= Q_S[r + \lambda - \pi_S][\frac{1 - ux}{1 - u}]$

Q_e = implicit deflator for investment in nonresidential durable equipment -- NIPA

r = $.45(1/PE) + .55[RGBS(1 - u)]$

PE = four quarter moving average of ratio of Standard and Poors Index of stock prices and average after tax earnings per share for 500 large companies

RGBS = yield on long term government bonds

u = statutory tax rate on corporate profits

δ = quarterly depreciation rate for equipment = 0.03375 (from Office of Business Economics (Capital Stocks Study)

π_E = expected rate of inflation in the price of new equipment estimated using the following procedure: beginning with the third quarter of 1976, we estimated a first order autogregressive, first order moving average process using the history of inflation up to that point; we then forecast the rate of inflation for each quarter up to t + 40 (10 years); the expected rate of inflation for period t was then calculated as the average of the forecasted values; we then deleted one quarter of data from the sample and repeated the process, generating a new set of coefficients and a new forecast; the procedure continued back to the second quarter of 1953, when estimates of the model became unstable. For the period 1950:1 to 1953:2, we did not re-estimate the model, but used forecasts from a model estimated over the period 1947 to 1953:3, together with the fitted values of the model for those periods in which no forecast was available.

k = effective rate of investment tax credit

z = present value of depreciation for equipment

formula for z: $(1 - g)[(\frac{1}{r}T)(1 - e^{-rT})] + g[(\frac{2}{r}T)(1 - [\frac{1 - e^{-rt}}{rt}])].$

g = proportion of firms using accelerated depreciation (sum of the years digits)--calculated using a learning function (see Wales)

T = lifetime of equipment for tax purposes

D = dummy for years in which Long Amendment was in effect

Q_s = implicit deflator for investment in non-residential structures -- NIPA

λ = quarterly depreciation rate for structures = 0.01775

π_S = expected rate of inflation in the price of new structures (estimated as a first order autogregressive, first order moving average process -- see above for details)

x = present value of depreciation for structures (formula same as for equipment except that T is replaced by lifetime of structures for tax purposes)