

NBER WORKING PAPER SERIES

IRREVERSIBILITY, UNCERTAINTY,
AND CYCLICAL INVESTMENT

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Working Paper No. 502

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

July 1980

The research reported here is part of the NBER's research program in Economic Flucations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

The optimal timing of real investment is studied under the assumptions that investment is irreversible and that new information about returns is arriving over time. Investment should be undertaken in this case only when the costs of deferring the project exceed the expected value of information gained by waiting. Uncertainty, because it increases the value of waiting for new information, retards the current rate of investment. The nature of investor's optimal reactions to events whose implications are resolved over time is a possible explanation of the instability of aggregate investment over the business cycle.

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But I suggest that the essential character of the trade cycle...is mainly due to the way in which the marginal efficiency of capital fluctuates... (T)he marginal efficiency of capital depends, not only on the existing abundance or scarcity of capital-goods and the current cost of production of capital-goods, but also on current expectations as to the future yield of capital-goods.... But, as we have seen, the basis for such expectations is very precarious. Being based on shifting and unreliable evidence, they are subject to sudden and violent changes.

Keynes, in Chapter 22 of The General Theory

1. Introduction

The fluctuations of the marginal efficiency of capital, and the attendant high variation in the aggregate demand for investment goods, are still an important feature of business cycles. Our understanding of these fluctuations, however, has not substantially advanced since Keynes' day. The earlier accelerator models¹ of investment exhibited the right cyclical properties, but their decision-theoretic bases were weak. More recent literature on investment demand has stressed long-run considerations: Investment is predicted to occur when expected returns over the life of the project exceed its costs.² This later work may explain average investment over the longer term, but it does not do much to help us understand the sometimes radical swings in investment spending that can occur in a relatively short period.

This paper presents a theory of investment in which optimizing behavior and short-run investment fluctuations are compatible. Central to the theory is an analysis of how newly-arriving information - Keynes' "shifting and unreliable evidence" - affects the investment decision. We will show that, when new information arrives continually over time, knowledge of long-run expected return is not very helpful for determining current optimal investment levels. Of much greater importance is the

short-run consideration of whether deferring projects in order to receive additional information is likely to be worthwhile.

Two assumptions are crucial to our theory. First, we assume that real investments are strictly irreversible. This is a reasonable description of most investments. Once a machine tool is made, for instance, it cannot be transformed into anything very unlike a machine tool without a loss of economic value that we can take to be prohibitive. This assumption does rule out some interesting possibilities of partial reversibility, e.g., the conversion of oil-fired plants to coal; however, the model can be extended (profitably, we think) to handle such cases. The existence of second-hand markets is not an important problem for this assumption; on this point, see Part 6 (especially Note 8).

Our second assumption is that new information relevant to judging investment returns arrives over time. This appears innocuous, but it is a departure from the usual specification. Most theories distinguish two classes of information -- that which the investor already possesses, and that which he will not receive until after the investment has been made. (Ignorance of the latter is the source of "risk".) We add here the class of information which the investor does not have but can obtain at the cost of waiting. The distinction between reducible and irreducible ignorance ("uncertainty" and "risk") was first made by Frank Knight and A. G. Hart³ but is not often seen currently.

Under these two assumptions, investment in a given project becomes a problem in stochastic dynamic optimization. In each period the potential investor must decide whether to commit himself immediately or to defer commitment. The costs of deferral are lost output, or increased construction costs. The gain is additional information, which may reveal that the proposed investment should not be undertaken. The more disparate

are the probable information-outcomes, the more likely it is that the investor will defer commitment. "Uncertainty" is seen to retard investment, independently of considerations of risk or expected return. Introduction of uncertainty can be associated with slack investment; resolution of uncertainty with an investment boom.

The rest of this paper is in six parts. Part 2 sets up the model for the case of an indivisible project. A sequential analysis similar to that used in the theories of optimal search and optimal sampling is employed. Part 3 introduces a measure of uncertainty and shows its relation to investment. Part 4 extends the model to divisible investments and gives an example based on the assumption of Dirichlet priors. In Part 5 we develop an example in which an investment cycle is generated by a single event with uncertain implications. Part 6 discusses the role of irreversible investment in Keynesian and equilibrium macromodels. Part 7 concludes. Proofs of the propositions are given in the Appendix.

2. The Case of a Single Project

This section presents a simple model that illustrates the relation between irreversible investment and newly-arriving information. Assume that there is a single, indivisible investment project available, and a single investor. At time t available information allows the investor to form an estimate of the excess of long-run returns to the project over the returns to an alternative, liquid asset. The problem is sequential. In the discrete period of time t the investor has two options: He may commit himself (irreversibly) to the project; or, he can wait a period, deferring his decision.⁴ Each action has a cost. Commitment to the project risks the possibility that information arriving later will show him that the investment was a mistake, one that he cannot undo. Waiting, on the other hand, is assumed to increase the cost of the project (should

it ever be undertaken); this reflects, for example, output foregone in the short-run or the higher costs of speedier construction. We would like to know what strategy the investor should pick to go about maximizing his expected return.

In order to answer this question, we introduce some notation. It is simplest to work mainly with vectors, designated by capital letters. Vector elements are notated by a superscript. The problem is made finite by the assumption that there is a period T after which the project is no longer available, or is prohibitively costly. We let

$$I_{t'} = \text{the set of possible information states in period } t' \\ (t' = t, t + 1, \dots, T)$$

$I_{t'}$ is of dimension $n_{t'} \times 1$, where $n_{t'}$ is the number of possible information-states in t' .

The information-state vectors are assumed to have a known joint probability distribution, so that conditional probability matrices of the form

$$P(I_{t'} | I_{t''}) \quad t' > t''$$

are well-defined. The matrix $P(I_{t'} | I_{t''})$ is dimensioned $n_{t'} \times n_{t''}$.

Define the $n_T \times 1$ vector

$$R = R(I_T)$$

to be the expected long-run excess return of the project, given all information available in the last period. R may be measured in dollars (for the risk-neutral) or in expected utiles (for other risk preferences). Note that R is a random variable with respect to information-state vectors $I_{t'}$, $t' < T$.

We let $c(t')$ be the (scalar) cost of building the project in period t' ; we will take $c(t')$ to be nonstochastic. $c(t')$ is nondecreasing in t' . For convenience, define

$$C(t') = c(t') \times i$$

where i is an $n_{t'} \times 1$ column-vector of ones.

Applying familiar dynamic programming methods, we can show that the irreversible project should be undertaken if the expected excess return of the project less building cost exceeds a non-negative reservation level:

Proposition 1. A necessary and sufficient condition for the investment in this problem to be optimal, given information set $(I_t)^j$ in period t , is given by

$$[P(I_T | I_t) \times R]^j - c(t) \geq [Q(t, I_t)]^j$$

where the $n_t \times 1$ vector $Q(t, I_t)$ is defined recursively by

$$Q(T, I_T) = (0, 0, \dots, 0)$$

$$Q(t', I_{t'}) = P(I_{t'+1} | I_{t'}) \times \max (P(I_T | I_{t'+1}) \times R - C(t'+1), \\ Q(t'+1, I_{t'+1}))$$

where the operator $\overline{\max}(A,B)$ forms a new vector whose j -th element is $\max(A^j, B^j)$.

A proof sketch of this (and of all the later propositions) is given in the Appendix.

The reservation-level-of-return vector, $Q(t, I_t)$, is the value function for each of the n_t possible information-states in period t ; it may be thought of as "the expected value of waiting". Since the return to doing nothing has been normalized to zero, Q will always be non-negative; typically, Q will have some strictly positive elements. Hence, it may happen that the project's expected excess return less building costs is greater than zero, yet it is not economic for the investment to be made. Irreversibility may command a "negative premium".⁵

We can also motivate Proposition 1 by analogy to the theory of finance. The right to build the irreversible project may be thought of as an option ("an American call", with an increasing exercise price) held by the investor. The option will be exercised when its expected exercise value (the left side of Prop. 1) exceeds the option's (typically positive) expected holding value.

An alternative expression of Proposition 1 is now given:

Proposition 2. An equivalent condition to Proposition 1 is

$$c(t+1) - c(t) \geq -[P(A(I_{t+1})|I_t) \times (P(I_T|A(I_{t+1}))) \times R - C(t+1) - Q(t+1, A(I_{t+1})))]^j$$

where $C(t+1)$ is understood to have been made conformable, and where $A(I_{t+1})$ is the subvector composed of elements of I_{t+1} that satisfy

$$[P(I_T|I_{t+1}) \times R]^j - c(t+1) < [Q(t+1, I_{t+1})]^j;$$

i.e., $A(I_{t+1})$ is the subset of possible information-states in $t+1$ in which the investor would choose not to commit to the project.

The interpretation of this condition is as follows: It will pay to invest in an irreversible project when the cost of waiting (the incremental building cost $c(t+1)-c(t)$) exceeds the expected gains from waiting. The expected gain from waiting is the probability that information arriving in $t+1$ will make the investor regret his decision to invest ($P(A(I_{t+1})|I_t)$), times what the investor would be willing to pay in that circumstance to regain his lost option ($-P(I_T|A(I_{t+1})) \times R - C(t+1) - Q(t+1, A(I_{t+1}))$). Heuristically we write

Rule: Invest in an irreversible project

if and only if

Costs of delay \geq Probability that a current commitment will be revealed to be a mistake in $t+1$ \times Expected magnitude of the mistake, given that a mistake is revealed in $t+1$

The motive for waiting is seen to be essentially caution-concern over the possible arrival of unfavorable news. This is made a bit more formal in the next section.

3. Uncertainty and Investment

We define a new vector quantity:

$$Z(t') = P(I_T|I_t) \times R - C(t') - Q(t', I_t)$$

$Z(t')$ is an $n_t \times 1$ vector that gives the excess of net return over the value of waiting for each information-state in t' . Those information-

states in t' corresponding to negative components of $Z(t')$ make up the set $A(I_t)$, the information-states in t' in which no commitment would be made (if none had been made already).

Returning to our decision problem in period t , let us define the random variable $z(t+1, I_t^j)$. $z(t+1, I_t^j)$ takes on the value of the k -th element of $Z(t+1)$ with probability $p(I_{t+1}^k | I_t^j)$. $z(t+1, I_t^j)$ is the random outcome of investing in t , as revealed in $t+1$.

Approximating the discrete probability density with a continuous one for esthetic reasons, we draw a possible distribution for z in Figure 1. The probability mass to the right of zero corresponds to information-states in $t+1$ in which the investment would be made, if still available; probability to the left of zero corresponds to states in $t+1$ in which investment would be deferred.

Figure 1 is not sufficient to tell us whether it pays to commit or to wait in period t , since, although it describes the gain from waiting (equal to

$$(3.2) \quad - \int_{z < 0} f(z) z = -E(z | z < 0) \text{pr}(z < 0)$$

it does not give waiting's cost $(c(t+1) - c(t))$. For a given cost of waiting, however, we can describe transformations of $f(z)$ that make investment in period t less and less attractive.

Consider two arbitrary p.d.f.-c.d.f. pairs, (f, F) and (g, G) .

We shall say that g is a negative spread of f if

- i) $G(x) \geq F(x)$, all $x \leq 0$
- ii) There is some $x < 0$ such that $G(x) > F(x)$

Figures 2, 3, and 4 are pictures of negatively spread densities. The dotted lines represent the original densities f . Note that what happens to the right of zero is irrelevant to determining if one density negatively spreads another.

Proposition 3. Suppose that the costs of delay and the density f of $z(t+1, I_t^j)$ are such that the investor is indifferent in period t between waiting and committing himself to the project. Then, for the same waiting costs but a new density g such that g negatively spreads f , the investor will necessarily choose to defer investment in t .

Thus any of the transformations of f given in Figures 2-4 would reduce the likelihood of investment in period t . Figure 2 pictures what is usually meant by "an increase in investor uncertainty"; both very good and very bad news are more likely, so that waiting to get information becomes a more valuable option. As the value of information increases, the propensity to make a commitment in the current period declines. This is similar to well-known results in sampling theory and the theory of optimal search; increased dispersion of possible outcomes makes extra sampling (or an extra period of search) more attractive. Results of this sort do not, of course, have any essential connection to risk preferences; Proposition 3 is true for risk-lovers as well as risk-aversers and -neutrals.

Figures 3 and 4 bring home the asymmetry of our result, however. The spread in Figure 3 makes the project look less attractive in expected value terms, while in Figure 4 the spread makes the project look more attractive. Nevertheless, the difference between current expected net return and the reservation return unambiguously declines under both transformations. For period $t < T$, only the left side of the outcome distribution is relevant; this formalizes the idea that waiting is done principally

for reasons of caution.

In this connection we also note that the net expected present value of a project is an unreliable criterion for irreversible commitment. The present value of the project is irrelevant unless it is negative, in which case the project should be deferred; or, if the investor is in the last decision period, so that deferral is not an option. For other cases, it is easy to construct examples where an investment with a small positive present value should be undertaken, while an investment with an arbitrarily large present value should simultaneously be deferred.

4. Extension to Divisible Investments, With an Example

The analysis can be applied to the case of continuously divisible projects, with similar results. Suppose that, in period t and with information-state I_t^j , the potential investor is not required to make an all-or-nothing decision about the project. Instead, he may choose to invest in any positive quantity K of the project (or of the type of capital under consideration). In subsequent periods he may increase, but not decrease, his holdings. The other parts of the problem are modified appropriately. We let

$R(K, I_T)$ = the marginal expected long-run return on the project, given final holding K , for each information-state in I_T . Assume, $R_K < 0$.

$c(t')$ = the cost per unit of constructing capital at time t' . $c(t')$ is increasing in t' .
As before, $C(t') = C(t') \times i$.

With these conditions maximal long-run return is achieved by investing in each period to the point where marginal return equals a given reservation level.

Proposition 1'. The optimal capital holding for the investor in this model, given information set I_t^j at t , is the maximum of the inherited holding and K^* , where K^* satisfies

$$[P(I_T | I_t) \times R(K^*, I_T)]^j - c(t) = [Q(K^*, t, I_t)]^j$$

where the function $Q(K, t, I_t)$ is defined by

$$\begin{aligned} Q(K, T, I_T) &= 0 \text{ for all } K \geq 0 \text{ and all } I_T^j \in I_T \\ Q(K, t', I_{t'}) &= P(I_{t'+1}) \times \overline{\max} (P(I_T | I_{t'+1}) \times R(K, I_T) - C(t'+1), \\ &\quad Q(K, t'+1, I_{t'+1})) \text{ for all } K \geq 0, I_{t'}^j \in I_{t'} \end{aligned}$$

Existence of K^* is discussed in the Appendix.

Again, the net excess return required to expand holdings of an irreversible investment is always non-negative and is typically greater than zero.

The optimality condition can be written in a form analogous to Proposition 2. Define $A(K, I_{t+1})$ to be the set of components of I_{t+1} that satisfy

$$[P(I_T | I_{t+1}) \times R(K, I_T)]^j - c(t+1) < (Q(K, t+1, I_{t+1}))^j,$$

i.e., $A(K, I_{t+1})$ is the set of information-states in $t+1$ in which an investor with irreversible holdings K would not increase his investment.

Then we have

Proposition 2'. An equivalent definition of K^* (the optimal unrestricted holding in t given I_t^j) is given by

$$c(t+1) - c(t) = -[P(A(K^*, I_{t+1}) | I_t) \times (P(I_T | A(K^*, I_{t+1})) \times R(K^*, I_T) - C(t+1) - Q(K^*, t+1, A(K^*, I_{t+1})))]$$

This condition says that the holding of K should be expanded until the cost of waiting (which is independent of K) is equal to the gains from waiting (which are increasing in K). The gain from waiting is equal to the probability that the irreversible holding chosen is revealed in $t+1$ to be too high, times the premium the investor would be willing to pay at the margin in that circumstance to "undo" his investment.

Increased uncertainty reduces the propensity to make divisible irreversible investments, just as it does indivisible ones. This can be shown using the idea of negative spreads. Rather than pursue an absolutely parallel discussion to the previous sections, however, we shall illustrate this point with an example that we found instructive.

A Dirichlet example. We give a specific structure to the information sets and the evolution of the agent's priors. Suppose that in each period it is possible to observe one of only two outcomes, $s = 1$ or $s = 2$. (An information-state in t' is a history of outcomes from $t = 1$ to t' .) These outcomes are generated independently each period by a binomial distribution with parameters p and $1-p$, corresponding to $s=1$ and $s=2$. The true parameter p is unknown to the investor; he only has his subjective estimate \hat{p}_t , at time t' . Long run marginal returns are given by

$$pR_1(K) + (1-p) R_2(K)$$

and expected long-run returns by

$$R(K, I_T) = \hat{p}_T R_1(K) + (1 - \hat{p}_T) R_2(K).$$

We take $R_1(K) > R_2(K)$, all K , so that observations of $s=1$ are favorable and of $s=2$ relatively unfavorable. The investor has an incentive to improve his knowledge of p ; and the higher his \hat{p} , the happier he is with his prospective investment (at the margin).

The investor's priors on the parameter p we shall take to be in the form of a Dirichlet distribution.⁶ The Dirichlet has a number of useful properties, one of which is that it implies a particularly simple belief-updating rule. For the binomial case, this rule can be illustrated by imagining that the investor holds an urn containing known numbers of white and black marbles. His estimate of p at time t , \hat{p}_t , can be represented by the fraction of marbles in his urn that are white. Now period $t+1$ begins, and a new outcome is observed. The investor adds one white marble to the urn if he observed $s=1$, one black marble if he observed $s=2$. His updated estimate of p , \hat{p}_{t+1} , corresponds to the new ratio of white marbles to the total.

Algebraically, let the investor's priors at t be described by (\hat{p}_t, n_t) . This is updated by

$$\hat{p}_{t+1} = \begin{cases} \frac{\hat{p}_t n_t + 1}{n_t + 1} & \text{if } s_{t+1} = 1 \\ \frac{\hat{p}_t n_t}{n_t + 1} & \text{if } s_{t+1} = 2 \end{cases}$$

$$n_{t+1} = n_t + 1$$

Note that n_t , the "number of marbles in the urn," is a natural measure of how certain the investor is about the true long-run returns. With a high n_t , new observations have very little impact on priors; when n_t is low, a new observation contains a relatively large amount of information. We would expect that when the investor is uncertain (his n_t is low), he will be less willing to make irreversible investments. This is indeed the case.

Proposition 4. Define $K^*(t, \hat{p}_t, n_t)$ to be the optimal holding of the irreversible asset at t when the investor's priors are given by (\hat{p}_t, n_t) . Then $\partial K^*/\partial \hat{p} > 0$, $\partial K^*/\partial n > 0$.

This result illustrates, in a specific case, the dependence of desired capital stock on both a measure of long-run return (\hat{p}) and a measure of subjective certainty (n).

5. Irreversibility, Uncertainty, and Cyclical Investment: An Example

We have developed a model in which the current demand for investment depends not only on long-run returns but on the presence of reducible uncertainty. It is difficult to see how the former factor could cause the short-run "bunching" of investments observed in the business cycle. The latter factor, however, is likely to cause bunching (if the uncertainty is pervasive in the system); and, it is intrinsically short-run in its effects. Indeed, uncertainty which is not likely to be shortly reduced does not affect the investment decision.

If we attribute investment swings in the short-run at least in part to bouts of uncertainty, we may be accused of substituting one deus ex

machina for another. But it is not necessary to imagine uncertainty expanding and receding in three-to-five year periods in order to explain the observed phenomena. Rather, exogeneous uncertainty which arrives in the form of a single unexpected event, or sequence of events, will create a cyclical response in the economy as the implications of the event are resolved over time. We illustrate this principle with another example.

Example: Investment and an Energy Cartel of Uncertain Duration

We introduce a simple model of investment and output in an energy-importing economy after the unanticipated formation of an energy-exporters' cartel. It will be shown that uncertainty can make investment collapse, even though capital goods seem to dominate the alternative asset in each period.

The identical agents in our model energy-importing economy are assumed to be risk-neutral. There are three perfectly durable assets with which they can form their portfolios:

- 1) Energy-using capital (K^e)
- 2) Energy-saving capital (K^s)
- 3) Investible resources (W)

The two forms of capital can be used to produce a homogeneous consumption good according to a relation to be specified shortly. Investible resources⁷ have no direct productivity and pay no return; they are valuable, however, because they can be converted costlessly and on a one-for-one basis into units of K^e or K^s . Conversion of investible resources into a specific form of capital is an irreversible process. For simplicity, we shall assume that investible resources are rained like manna from the sky at a constant rate of ΔW per period; all that is required for our results, however, is that the supply of W in each period not be perfectly elastic.

The state of nature in each period depends on the status of the energy-exporters' cartel. We define the state of nature s_t by

$$s_t = \begin{cases} 1, & \text{if the cartel exists in period } t \\ 0, & \text{otherwise} \end{cases}$$

The cartel is assumed to have formed in period t_0 . Agents' beliefs about its continued existence are given by

$$\Pr (s_t=1 | s_{t-1}=1) = p_t$$

$$\Pr (s_t=1 | s_{t-1}=0) = 0$$

where $dp_t/dt > 0$, $\lim_{t \rightarrow \infty} p_t = 1$

Thus if the cartel fails, everyone assumes it will be gone forever. The longer the cartel lasts, the greater is the common subjective probability of its survival through the next period. If the cartel survives long enough, it is assumed to be permanent.

The net production of the consumption good y in each period by each unit of energy-using capital depends on the contemporaneous state of nature. If the cartel is in existence ($s_t=1$), each unit of K^e has a (small) net output in period t of $R^{e,1}$. With no cartel ($s_t=0$), each unit of K^e produces (large) net output $R^{e,0}$. Energy-saving capital produces an intermediate quantity of R^s per period per unit, independent of the state of nature. Since machines can be shut down, net production is assumed to be greater than or equal to zero. In summary:

$$Y_t = \begin{cases} R^{e,1} K_t^e + R^S K_t^S, & \text{if } s_t = 1 \\ R^{e,0} K_t^e + R^S K_t^S, & \text{if } s_t = 0 \end{cases}$$

where $0 \leq R^{e,1} < R^S < R^{e,0}$

The agents' goal is to maximize their discounted consumption stream, $E_t \sum_{t'=t}^{\infty} \beta^{t'-t} Y_{t'}$, over the set of possible portfolio strategies. Here we

will chart their optimal investment path under the assumption that the cartel stubbornly refuses to disappear. ("Investment" in this context refers to the rate of conversion of investible resources into K^e or K^S .)

A naive approach to this problem is to note that since 1) investors are risk-neutral, 2) investment in either K^e or K^S is guaranteed a positive return, 3) uninvested resources pay no return, and 4) investment is free up to the exogenous resource constraint, capital appears to dominate the alternative in a traditional risk-return sense. Thus it might appear that the best strategy is to convert all investible resources into some form of capital as soon as they become available.

This conclusion is false. It is possible in this model to have an investment 'pause', during which even risk-neutral investors are content to cumulate barren liquid resources in order to wait for new information. This is shown by

Proposition 5. Jointly sufficient conditions for agents to make no investments in period t are given by

$$1) \quad R^S < a_1 R^{e,0} \quad a_1 = \frac{\beta(1-p_t)}{1 - \beta + \beta(1-p_t)}$$

and

$$2) \quad R^{e,1} < a_2 R^S \quad a_2 = \frac{\beta}{1 - \beta + \beta p_t}$$

Since p_t is monotonic for $s=1$, if either of these conditions is true it will be true over a continuous interval of time. The continuous interval in which these conditions intersect is the investment 'pause'. It is numerically plausible that this intersection will exist. Suppose $\beta = .9$, $p_t = .5$. Then no investment takes place if $R^{e,1} < .82R^s$ and $R^s < .82R^{e,0}$.

A picture of output, investment, and capital stock over time in this economy is given in Figure 5. The pause runs from t_1 to t_2 . (We show $t_1 > t_0$, the period of the cartel's formation; an alternative possibility is $t_1 = t_0$). The history of the economy is as follows. From t_0 to t_1 investors give insufficient credence to the cartel to desist from energy-intensive investment. By t_1 the future has become sufficiently ambiguous that investors prefer to remain liquid and wait for new information. Finally, at t_2 , the continued existence of the cartel seems sufficiently likely that investors commit themselves with a bang to energy-saving capital. There is an investment spurt as cumulated investible resources are transformed to a stock of K^s . Output varies over time as capital composition changes; this minor effect is swamped, of course, if we expand our definition of output to include the processings of investible resources into capital. Then a pronounced cyclical effect is observable.

The value of information in this model depends on three factors:

- 1) β , the discount factor. The less one discounts the future, the more likely one is to sacrifice current consumption to get more information.
 - 2) p_t , the probability of cartel survival. Values of p_t far from zero or one are more likely to satisfy the conditions for a pause. This is the only factor that changes over time, and hence it is the source of the dynamics of the result.
 - 3) The disparateness of $R^{e,0}$, R^s , and $R^{e,1}$.
- The farther apart these profitabilities are, the more valuable is new

information, the more likely it is that a pause will occur, and the longer it will be if it does occur. An example: Say that we have at t , a set of profitabilities $(R^{e,0}, R^s, R^{e,1})$ such that agents invest all available resources. Now suppose that $R^{e,0}$ were to be multiplied by a thousand, R^s by a hundred, and $R^{e,1}$ by ten. This huge increase in the value of the capital stock will likely drive current investment to zero! This is because the increased return from waiting for more information more than offsets the improvement in current returns.

We remind the reader that the joint conditions of Proposition 5 are sufficient, not necessary. They are emphasized here because of their simplicity and ready interpretation. Finding the necessary condition is not difficult and is left as an exercise. As might be suspected, the necessary condition depends on dp/dt , the rate at which uncertainty is being resolved.

6. Irreversible Investment In the Macro-Model

The example illustrates how events whose implications for the economy are resolved over time can cause fluctuations in the rate of aggregate investment. To conclude that this is a source of business cycles as well, we must go one step further and specify how these elements could enter into a complete model of the macroeconomy. This task is straightforward; we will briefly consider the potential role of our theory in both Keynesian and non-Keynesian macro-models.

Our analysis of investment ties in neatly with the standard Keynesian approach. In terms of the usual IS-LM diagram, investment fluctuations caused by uncertainty can be viewed as autonomous shifts of the IS curve. Thus the investment "pause" of our example moves the IS curve down, driving

the economy into a period of reduced output and low interest rates. The end of the pause brings a resurgence. Multiplier effects cause the changes in output to exceed the magnitude of the shock to investment.

The notion that the demand for investment, made volatile by the random arrival of new information, is a source of the business cycle would have been a congenial one for Keynes; he takes a very similar viewpoint in Chapter 22 of the General Theory (from which our opening quotation is taken). That the sources of cycles are essentially exogenous shocks to the efficiency of investment was also espoused by Schumpeter.

The irreversible investment theory also fits easily into the principal non-Keynesian alternative, the models of the cycle proposed by modern "equilibrium theorists." The occurrence of an event with uncertain implications for the appropriate level and mix of investment is a real shock of the type alluded to by Lucas (1977). During the period in which the uncertainty is resolved, it is desirable that investible resources be saved for future use and that workers in investment goods industries take more leisure. The resulting variability of output and employment, unlike that of the Keynesian case, is completely efficient. As Black (1978) argues, in this line of reasoning there is no presumption that the government must try to smooth aggregate production over time.

It would not be difficult to recast our example of the energy-importing economy in an equilibrium business cycle mold. As given, the economy of that example is best thought of as being run by a central planner. However, it can be verified that the introduction of competitive markets for output, investible resources, and used capital into that setup does not affect the path of any real variable. The investment pause still occurs, motivated by speculation in investible resource stocks and the realization by investors

that choosing the "wrong" kind of capital will be penalized by low output and low second-hand values in the future. In particular, allowing firms to sell old capital does not change any result gotten by assuming irreversibility.⁸

The result of this exercise could be called an equilibrium business cycle model in the following sense: The model economy would exhibit serially correlated deviations of investment and output away from the average growth path. These deviations would be consistent with full market clearing at each moment and, as in the centrally-planned version, would represent a completely efficient response to the uncertain events.

7. Conclusion

This paper has argued that when investment is irreversible, it will sometimes pay agents to defer commitment of scarce investible resources in order to await new information. Uncertainty which is potentially resolvable over time thus exists a depressing effect on current investment. This may help explain the short-run investment fluctuations associated with the business cycle.

There are many potentially instructive generalizations of this model. These include

- 1) the incorporation of information flows that are not purely exogenous. For example, the possibility of "learning-by-doing" induced by the investment process may create a positive incentive for investment in some uncertain situations.

- 2) the removal of the "zero-one" character of irreversibility in our model. If we allow for partial convertibility of capital stock, we can analyze the decision to commit to, say, flexible (but higher cost) technologies versus more restrictive options.

This work also has microeconomic applications. An example is the problem of choosing a technique in a field where the technology is changing rapidly. Should a firm buy the current-generation computer system or speculate by waiting for a system that is better and cheaper? The decision to wait in a given period depends not only on expected system improvement and delay costs, but also on how much one can expect to learn in the short run about long-run technical possibilities.

APPENDIX

This appendix sketches the proof of the Propositions stated in the text. For the sake of brevity some details are excluded.

Proposition 1. This is a standard application of backward induction. $Q(t', I_t)$ is the expected value of deferring the project at t' . Investment takes place if the expected value of current investment exceeds that of waiting. Finiteness is assumed to avoid technical problems.

Proposition 2. Subtract $[P(I_T | I_t) \times R]^J - c(t+1)$ from both sides of the condition in Prop. 1, noting $[P(I_T | I_t) \times R]^J = [P(A(I_{t+1}) | I_t) \times P(I_T | A(I_{t+1})) \times R]^J + [P(A'(I_{t+1}) | I_t) \times P(I_T | A'(I_{t+1})) \times R]^J$ and that $[Q(t, I_t)]^J = [P(A(I_{t+1}) | I_t) \times Q(t+1, A(I_{t+1}))]^J + [P(A'(I_{t+1}) | I_t) \times (P(I_T | A'(I_{t+1})) \times R - C(t+1))]^J$, where $A'(I_{t+1})$ is the complement of $A(I_{t+1})$ in I_{t+1} .

Proposition 3. The spread unambiguously increases $-\sum_{z < 0} f(z)z$, which is the right-hand side of the expression in Prop. 2.

Proposition 1' and 2'. Again we apply backward induction. $Q(K, t', I_t)$ is the value of an option to build an extra unit of capital, given committed holdings of K and information I_t . The value of an option is the maximum of its exercise value and its holding value. Prop. 1' says the investor should expand K in t until he is indifferent between exercising and holding his marginal option. This maximizes as long as $\frac{\partial}{\partial K} [P(A(K, I_{t+1}) | I_t) \times ((P(I_T | A(K, I_{t+1})) \times R(K, I_T)) - C(t+1)) - Q(K, t+1, A(K, I_{t+1}))]^J < 0$, which can be shown by induction. Prop. 2' is derived from Prop. 1' in a manner analogous to the derivation of Prop. 2.

Since the condition in Prop. 1' and Prop. 2' will have a finite number of discontinuities, there is a "measure-zero possibility" that there will

be no K^* exactly fulfilling the condition. In this case we designate K^* to be such that moving from $K^*+\varepsilon$ to $K^*-\varepsilon$ reverses the inequality.

Proposition 4. We show $\partial K^*/\partial n > 0$. By Prop. 1', we need $\frac{\partial}{\partial n_t}$

$Q(K^*(t, \hat{p}_t, n_t), t, \hat{p}_t, n_t) < 0$. Will show by induction that $\partial Q/\partial n < 0$ for all K such that $K^*(t, \hat{p}_t, n_t) \leq K \leq K_{\max}(t, \hat{p}_t, n_t)$, where $K_{\max}(t, \hat{p}_t, n_t) \equiv K^*(T, (\hat{p}_t, n_t + (T-t))/(n_t + (T-t)), n_t + (T-t))$ is the highest possible desired holding in T , given information up to t . For $T-1$: $\partial Q/\partial n = \partial/\partial n [\hat{p}_{t-1}$
 $\frac{\hat{p}_{t-1} n_{t-1} + 1}{n_{t-1} + 1} R_1(K) + \frac{(1-\hat{p}_{t-1}) n_{t-1}}{n_{t-1} + 1} R_2(K) + (1-\hat{p}_{t-1}) 0] < 0$ for $K^* \leq K \leq K_{\max}$.

For t : $\partial Q/\partial n = \frac{\partial}{\partial n} [\hat{p}_t (\max(\frac{\hat{p}_t n_t + 1}{n_t + 1} R_1(K) + \frac{(1-\hat{p}_t) n_t}{n_t + 1} R_2(K), Q(K, t+1, \frac{\hat{p}_t n_t + 1}{n_t + 1}, n_t + 1))$
 $+ (1-\hat{p}_t) Q(K, t+1, \frac{\hat{p}_t n_t}{n_t + 1}, n_t + 1)] < 0$, for $K^* \leq K \leq K_{\max}$, using the inductive hypothesis and the fact that $Q(K, t+1, \frac{\hat{p}_t n_t + 1}{n_t + 1}, n_t + 1) > \frac{\hat{p}_t n_t + 1}{n_t + 1} R_1(K) + \frac{(1-\hat{p}_t) n_t}{n_t + 1} R_2(K)$
 only for K s.t. $K > K^*(t+1, \frac{\hat{p}_t n_t + 1}{n_t + 1}, n_t + 1)$.

Proposition 5. Sufficient conditions for waiting to be optimal according to Prop. 2' are

- 1) $R^s < (1-p_t) \frac{\beta}{(1-\beta)} (R^{e,0} - R^s)$ and
- 2) $R^{e,1} < p_t \frac{\beta}{(1-\beta)} (R^s - R^{e,1})$, which are equivalent to Prop. 5.

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FOOTNOTES

1. See, for example, Samuelson (1939).
2. The "fundamental" approach (which looks at long-run returns) is the basis of standard works like Hall-Jorgenson (1967); Eisner (1967); and Tobin-Brainard (1977).
3. Knight (1933) differentiated uncertainty and risk; but see Hart (1942).
4. With zero decision-making costs, there is never any point in ruling a project out completely.
5. This is noted in another context by Arrow and Fisher (1974).
6. For a derivation of the Dirichlet's properties, the reader should consult DeGroot (1970) or Murphy (1965). For an application of this family in the theory of search, see Rothschild (1974).
7. The interpretation of "investible resources" varies with the context. For a small firm, the resources are the available line of credit; for an industry (e.g., electric power), potential plant sites or markets; for a national economy, real resources like labor, land, and raw materials.
8. In our example, in which agents are identical, the existence of a used capital market is irrelevant; prices in that market would always be such as to make agents indifferent between selling and holding their capital. In the more general case, the existence of second-hand markets makes the problem look different to the individual agent but does not materially affect general equilibrium results obtained by assuming irreversibility. The economy as a whole must still hold all irreversible investment, a fact that is reflected in second-hand prices. Second-hand markets are important only if beliefs or preferences are so heterogeneous that there is no agreement on what constitutes good or bad news for a particular investment.

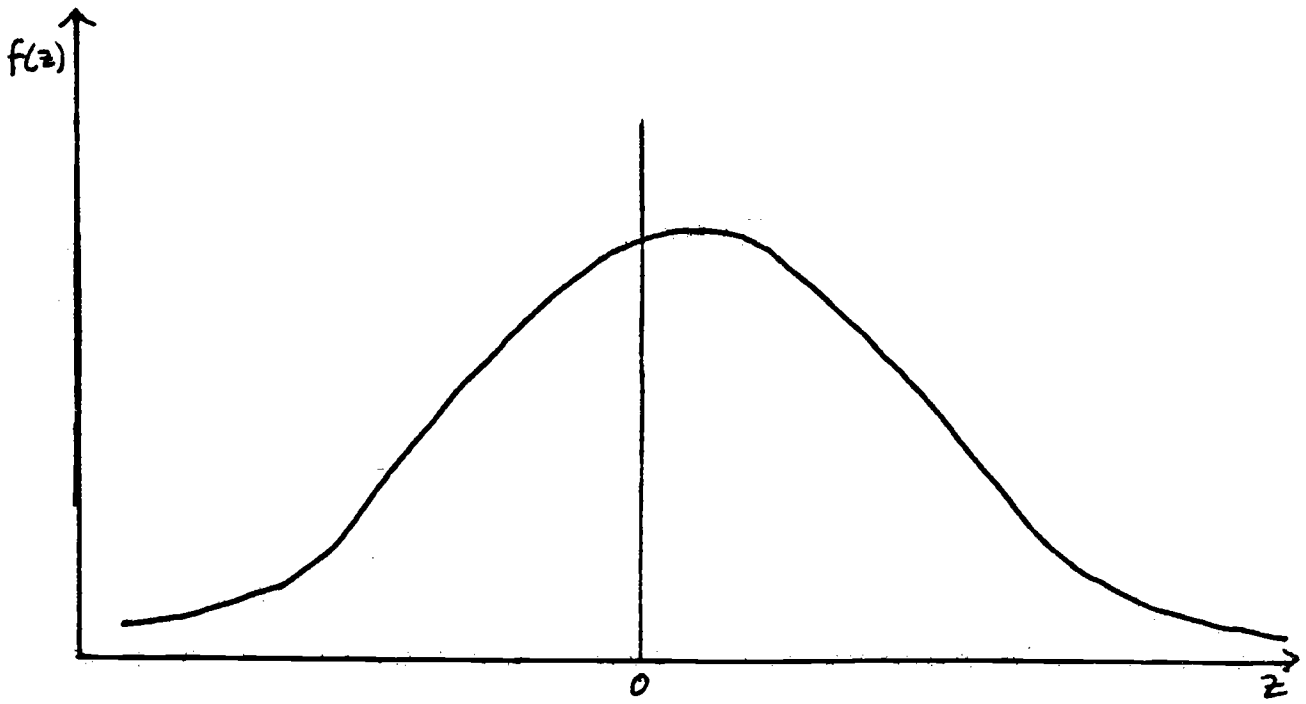


Figure 1

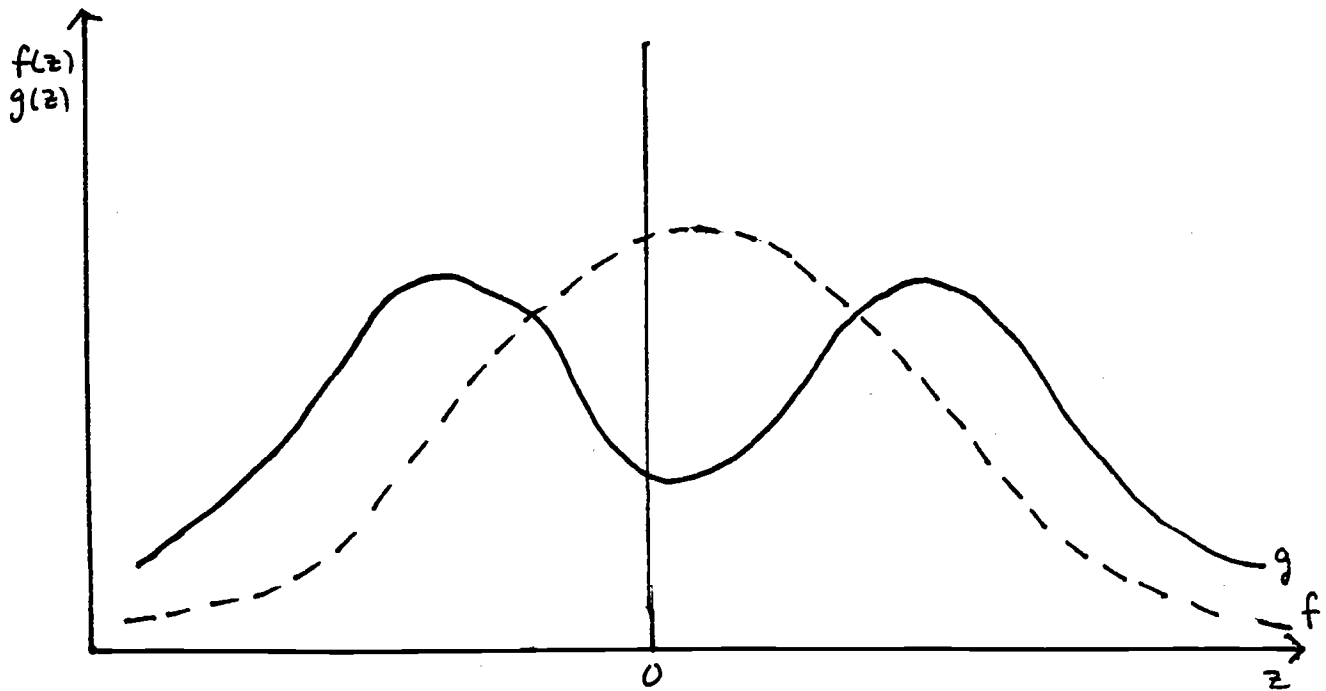


Figure 2

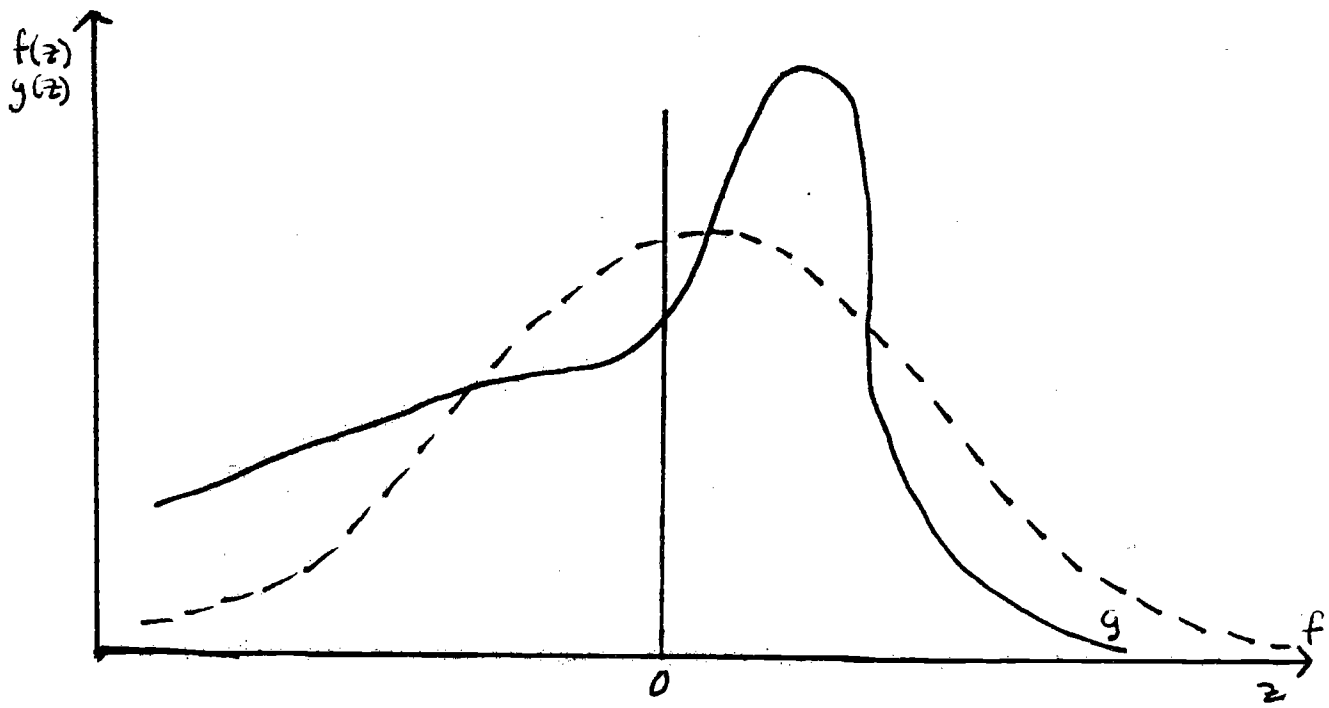


Figure 3

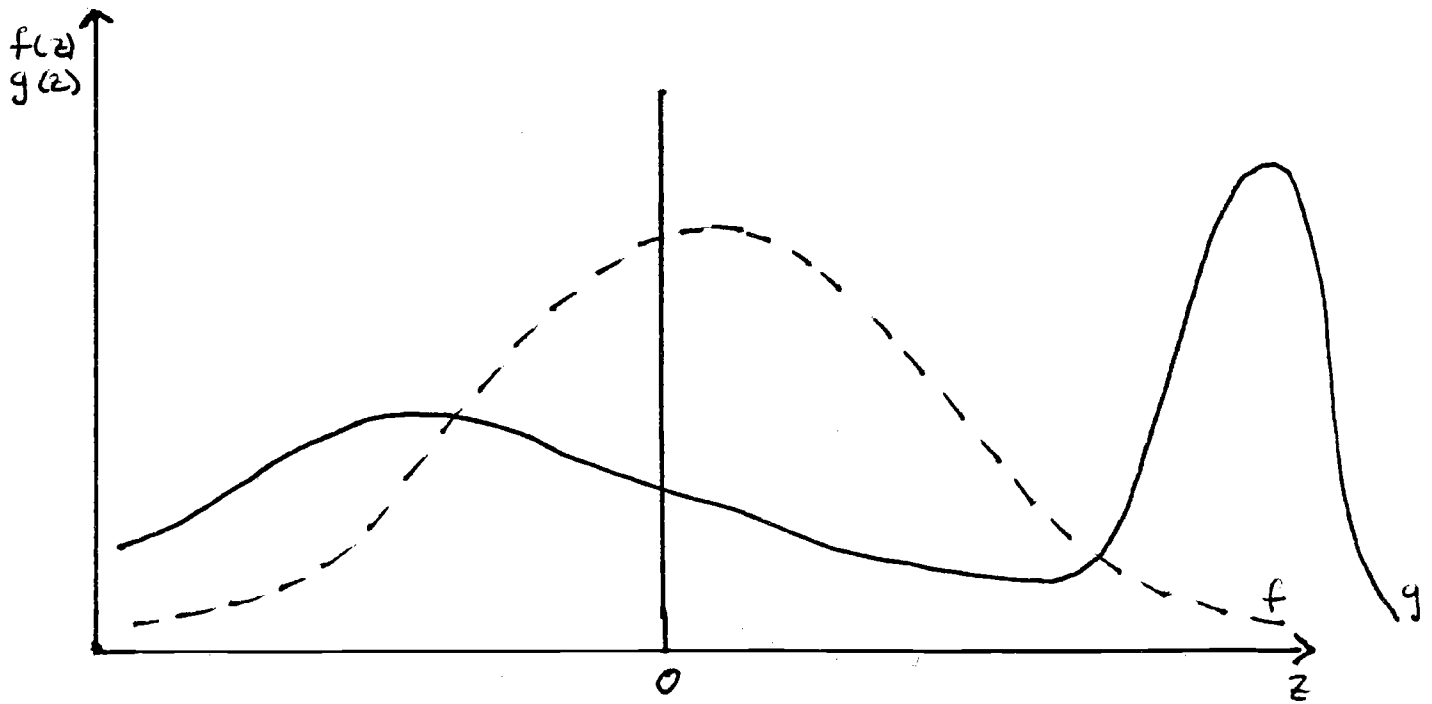


Figure 4

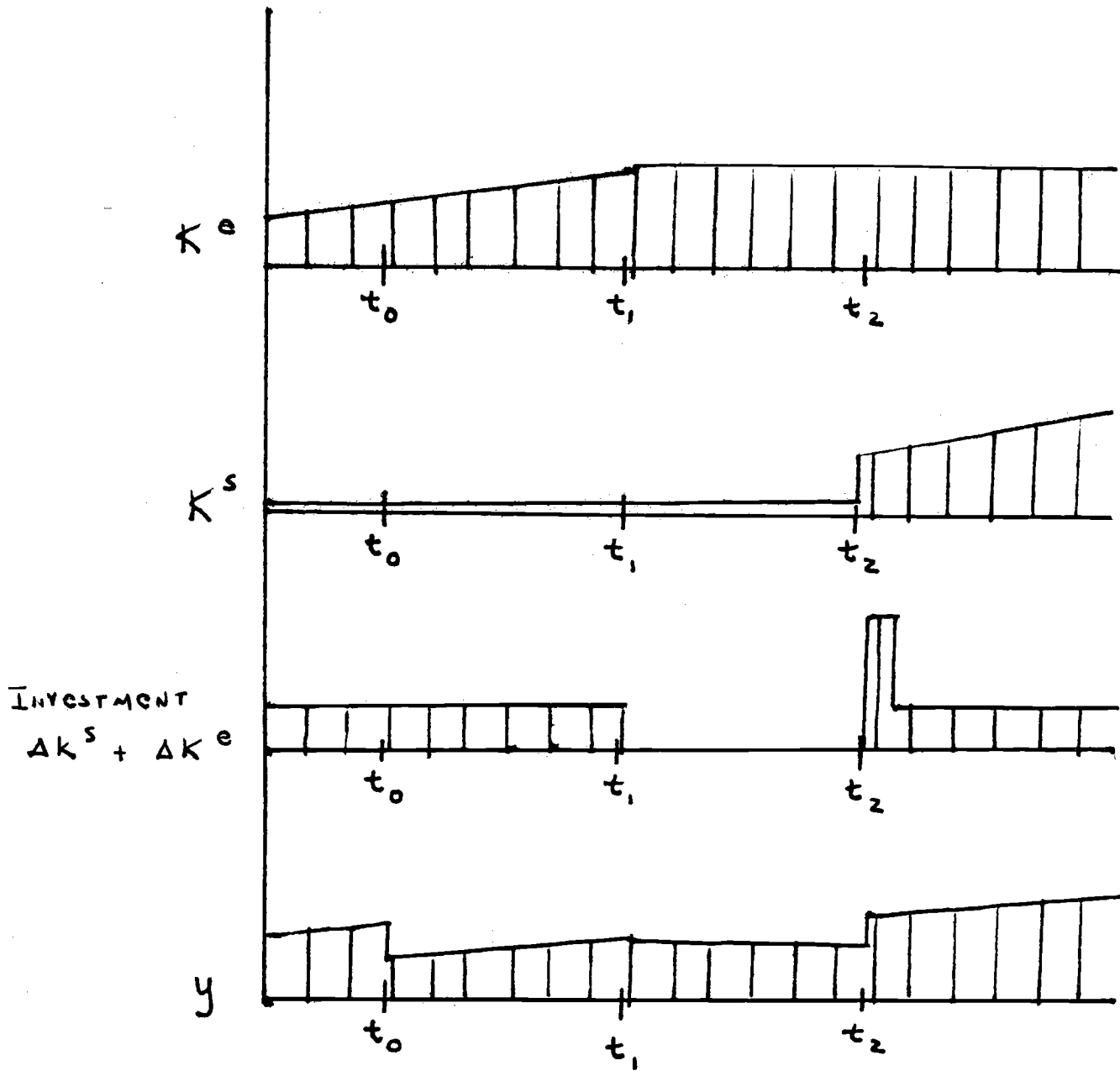


Figure 5