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THE LOCATION AND ALLOCATION OF ASSETS IN PENSION AND CONVENTIONAL SAVINGS ACCOUNTS

John B. Shoven

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ABSTRACT

This paper addresses two important parts of the problem of saving for retirement. They are (1) if assets are to be held in both conventional (and hence taxable) accounts and pension accounts, which assets should be held in each? and, (2) if the investor is substantially risk averse, what is the optimal mix of stocks and bonds for retirement saving? It is shown that the conventional wisdom of first placing heavily taxed corporate bonds in the pension account (and holding equity mutual funds outside the account) is the wrong asset location strategy for most people and most circumstances. It is also shown that even very risk averse retirement savers should allocate more than half of their portfolio to stocks if asset returns have the same means, variances, and covariances as have been observed over the past seventy years.

John B. Shoven
Department of Economics
Stanford University
Stanford, California 94305
and NBER
shoven@leland.stanford.edu

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I. Introduction

One of the biggest financial challenges facing households is saving for retirement. There are many aspects to the overall problem --- how much to save, what to invest in, whether to rebalance the portfolio through time, which investments to hold in pension accounts and which in conventional accounts, etc. This paper deals with two important aspects of the problem. First, due to limitations on how much individuals can contribute to tax qualified (i.e. tax deferred) pension accounts, households may want to or be forced to accumulate funds both inside a pension account and outside such an account (that is in a taxable environment). If this is the case, then one important question is which assets should be held inside the pension environment and which outside. For the most part, this paper will deal with the question of where to locate two portfolios -- one a diversified portfolio of stocks and the other a long-term bond portfolio -- either in pension or conventional taxable accounts. I will also examine the location problem for different stock portfolios. That is, if someone wanted to hold two different stock mutual funds (for instance, a small-cap stock fund and a large-cap fund), which should be given preference for placement in the pension account.[1] The second major aspect of the overall retirement saving problem addressed in this paper is the question of the optimal overall allocation between stocks and bonds for risk

[1] The topic of which assets should be held in pension funds was briefly addressed in Shoven and Wise (1998).

averse individuals (risk neutral investors would clearly choose the asset with the higher mean return -- stocks).

The analysis is presented in the next three sections. Section Two begins the examination of which assets should be held in pension accounts and which in conventional accounts. The analysis is simplified -- perhaps overly so -- in that it focuses on the impact of taxes, the length of the accumulation period and rates of return on the answer, but it ignores uncertainty. Among the tax aspects that are considered are the preferential treatment of long-term capital gains, the tax-free nature of interest on municipal bonds, and the resetting of the basis on conventionally held assets that pass through estates. The importance of municipal bonds -- and their relatively low implicit tax rate -- is explored. The third section remedies the certainty weakness of Section Two by presenting extensive Monte Carlo simulations of different allocation strategies, using realistic parameters for the means, variances and covariances for the alternative asset classes. The interesting aspect of the results is that in many cases one strategy of locating assets in pensions and conventional accounts usually outperforms the alternative strategy. Further, the difference between adopting the optimal strategy and its opposite can be very large in terms of the spendable wealth available in retirement.

Section Four examines the overall allocation of assets between stocks and bonds in providing retirement wealth, again through Monte Carlo simulations. In general, the results indicate that even very risk averse individuals should have a substantial fraction of their portfolio in stocks. Section Five summarizes the results of the paper.

II Allocating Assets Between Pensions and Conventional Accounts -- Certainty Analysis

This section and the next examine the preferred domain for investments; that is, if retirement accumulation assets must be held in conventional taxable accounts as well as in tax-sheltered pension accounts, which of them should be first allocated to the pension environment? This is an important problem because of asymmetries in the way taxes apply to the returns of different assets in the two accounts. All asset returns (dividends, interest, rents, realized and unrealized capital gains, etc.) are treated identically inside a pension account. However, these various forms of capital income are taxed very differently from one another if they are held outside of a qualified pension plan. The treatment of capital gains and the taxation of interest (on corporate, and federal and municipal bonds) are particularly important. Long-term capital gains have been taxed more lightly than ordinary income, at least for high income taxpayers, for a long time. Currently the top federal marginal rate of taxation on realized capital gains on assets held for more than 12 months is 20 percent. The tax on accrued or unrealized capital gains is deferred until realization, while the federal marginal tax rate faced on gains of assets held less than 12 months can be as high as 41 percent (factoring in the phaseout of itemized deductions). One special feature of capital gains taxation is that the cost basis of assets passing through an estate is reset to the current value, thereby opening up the possibility that capital gains may completely escape income taxation. This resetting is only relevant for assets held outside of pension plans. Corporate bond interest faces full ordinary income taxation; interest on

federal government debt is exempt from state income taxes, and the interest on state and local obligations can be exempt from both state and federal income taxation.[2]

The advantage of one location allocation choice for assets over an alternative is close to a pure efficiency gain. Exactly the same assets are acquired, involving the same gross returns and, in the next two sections, the same risk. "Efficiency gain" loosely means "something for nothing," or in this case extra retirement income for simply placing a given set of investments in their most advantageous environments. The qualifier "close to a pure efficiency gain" is necessary because rearranging the location of investments does trigger tax differences and subsequent differences in total wealth and risk. These effects may be of second order importance compared to the potential retirement wealth improvements from wisely determining the location of various types of investments. In simulations that are reported in the next section of the paper, I present cases where the portfolio is rebalanced every year in terms of the stock-bond split of total assets as well as cases where such rebalancing is not performed. Particularly with rebalancing (since the relative amount of various asset types is periodically reset so as to remain similar under the different asset location strategies), any wealth improvement from one asset location strategy over another is very close to a pure efficiency gain.

I begin by looking at a one-time saving decision of how to allocate investments between pension and conventional saving. The simplest case of all involves two pools of money, one a pension account and one a conventional nonpension brokerage account. Take the example where there is precisely the same amount of money in each account, say $5,000

[2] I am not going to consider the Alternative Minimum Tax in this analysis. The interest on some municipal bonds is exempt from the AMT calculation, while the interest on other bonds must be considered for AMT purposes.

in each. Assume that these amounts are taken as given; that is, it is not possible to enlarge the pension account and reduce the conventional one through simple transfers. The household is considering investing in three different types of assets in these accounts: an equity mutual fund; long-term corporate bonds (or a corporate bond fund); and long-term municipal bonds (or a municipal bond fund). Consider the case where the "what to invest in" question has already been answered to the extent that the decision has been made to initially invest $5,000 in a particular stock fund and $5,000 in a long-term bond fund. Once the initial investment allocation has been made no changes will be considered until retirement in T years. The municipal bonds (or fund) are debt issues of governments within the state of the household's residence, so their interest is completely free of both federal and state income taxation. Because of this tax free nature of municipal bond interest, the interest rate on these securities is below the rate on equally safe corporate bonds. Assume that the implicit tax rate on the municipal bond's interest, defined as one minus the ratio of the municipal bond's interest rate to the corporate bond's interest rate, is less than the household's marginal tax rate on ordinary income (and hence, on corporate interest receipts). With this assumption, municipal bonds dominate corporate bonds when held outside of a pension plan. Given the way this problem has now been set up, there really are only two location alternatives: either hold the stock fund in the pension and municipal bonds outside or hold corporate bonds in the pension and the stock fund outside. Which is optimal? Does it make a lot of difference?

The equations for this particular problem are very simple. The $5,000 in pension assets will grow to $W_P(T)$ in T years, where

(1) $W_P(T) = (1 - t_R)\ 5{,}000\ (1 + r)^T$.

The expression $W_P(T)$ is the consumable wealth at retirement. If the money is taken out of the pension at the time of retirement, then it is subject to taxation at rate t_R (the combined marginal federal and state income tax rate of the household at time T). The variable r in equation (1) is the gross rate of return of the asset over the T accumulation years. If corporate bonds are to be held in the pension, then r would be their interest rate, i_c. If a stock fund is held inside the pension, then r would be the gross rate of return of the fund including dividends, realized capital gains and unrealized appreciation. Studying this simple equation leads to one conclusion, namely that the consumable wealth from a pension account grows at the full gross rate of return earned by the assets in the account. To a first order approximation, pension wealth is subject to a wealth tax, but not to an income tax on the returns.[3] The first order approximation statement is necessary because it is possible that differences in gross rates of return would be so significant as to change the household's tax rate upon retirement, t_R. Without that exception, however, the amount of consumption that can be funded by a pension contribution is equivalent to what could be accumulated with one minus t_R times the contribution with the money growing at the full gross rates of return. Pension saving is thus treated on a consumption tax basis.

If the conventional savings account were invested in municipal bonds, the consumable conventional wealth after T years would be

[3] This "wealth tax" could be considered a deferred income tax on the money originally contributed to the pension. The point here, however, is that consumable retirement wealth grows directly with the full compounded gross rate of return earned on the assets in the pension.

(2A) $W_C(T) = \$5{,}000\ (1 + i_m)^T$

where i_m is the interest rate on long-term (20-30 year) municipal bonds. On the other hand, if the conventional (i.e. taxable) saving account is invested in a stock fund, the expression is more complicated. In that case, the composition of the stock fund's return matters since dividends, realized capital gains and accrued capital gains are all taxed differently. Let the total rate of return r of the stock fund be composed of three parts: r_d, the dividend (and short-run realized capital gains) rate of return, r_c, the long-term realized capital gains rate of return, and, r_a, the accrued or unrealized capital gain rate of return. The amount in the conventional account invested in equities will compound at the rate

$$r^S = r_d\,(1 - t_d) + r_c(1 - t_c) + r_a$$

assuming that the after-tax dividend and realized long-term capital gains distributions are reinvested. The final consumable wealth at retirement in this case is

(2B) $W_C(T) = \$5{,}000\ (1 + r^S)^T - t_c(\$5{,}000(1 + r^S)^T - B_T)$,

where B_T is the cost` basis of the mutual fund investment after T years. The expression for B_T is derived in Appendix 1; the formula is

$$B_T = \$5{,}000\left[1 + (r_d\,(1 - t_d) + r_c\,(1 - t_c))\,\frac{(1 + r^S)^T - 1}{r^S}\right].$$

Total consumable wealth at retirement is $W_P(T)$ plus $W_C(T)$ and therefore is either equation (1) with r set as the gross rate of return on the stock portfolio plus equation (2A) or

equation (1) with r set as the corporate bond rate, $i_{c,}$ plus equation (2B). And, ... the winner is?

Despite the fact that I have made a myriad of simplifying assumptions, the only way to answer the question is to specify particular interest rates, stock fund returns and tax rates. So, to complete this example, I assume the structure of interest rates that prevailed in September 1997 when the 30-year AAA corporate bond interest rate was 7.15 percent and the 30-year municipal bond interest rate was 5.36 percent. The implicit tax rate (combined federal and state income tax) on municipal bond interest was 25 percent, meaning that municipals dominated corporate bonds for all taxpayers with federal marginal tax rates of 28 percent or more.[4] First consider someone who has a very high income both while working and in retirement. They are assumed to be in the top current federal marginal tax bracket (39.6%) and that rate is assumed to apply to them in T = 30 years when they retire. They also face a state marginal income tax rate of 9.3% (the top rate in California). The answer to the question of whether to put the stock fund in the pension and hold municipal bonds outside or to hold corporate bonds in the pension and hold stocks outside depends on the composition of the return of the stock fund (as well as on the gross return and T). Table 2.1 shows the results for five hypothetical stock mutual funds, all with nominal gross rates of return of 12 percent. Case 1 may be the most realistic depiction of typical equity mutual funds. Notice that in this case most of the nominal return is distributed to shareholders triggering current taxation. Before concluding that the dividend yield is unrealistically high for this case, recall that for tax purposes gains on assets held for less

[4] See Appendix 2 for data regarding interest rates and the implicit tax rate on municipal bonds from 1950-97.

Table 2.1 Nominal Wealth Accumulated After 30 Years by a Top Tax Rate Household from $5,000 in Pensions and $5,000 in Conventional Saving

Stock Fund Characteristics					Net Wealth	
Case Number	Total Return	Dividend Yield	Long-term Capital Gains	Accrued Capital Gains	Stocks in Pension; Munis outside	Bonds in Pension; Stocks outside
1	12%	4%	6%	2%	**$104,241**	**$75,612**
2	12%	3%	4%	5%	**$104,241**	**$87,297**
3	12%	2%	3%	7%	**$104,241**	**$98,072**
4	12%	2%	2%	8%	**$104,241**	**$101.921**
5	12%	1%	1%	10%	**$104,241**	**$115,395**

than one year are subject to full ordinary income tax rates and thus taxed like dividends. Even this hypothetical fund is probably more tax efficient than most of those actually available. The Case 1 result is that the strategy of holding the municipal bonds outside and stocks inside the pension plan generates 38 percent more wealth than the alternative of holding corporate bonds in the pension plan and the stock fund outside. As efficiency gains go, this is enormous, although the advantage cannot be characterized as a pure efficiency gain -- more on that later. Case 3 depicts a very tax efficient equity mutual fund. Nonetheless, the optimal plan for someone who wants to invest half their money in the Case 3 stock fund and half in bonds is to hold the equity fund inside the pension and the bonds outside (in the form of municipals). Case 5 depicts an extremely tax efficient mutual fund -- one that is more efficient than almost any in actual existence. If the investor had such a fund available and chose to invest in it, then the optimal allocation strategy would be reversed with corporate bonds held in the pension account and the stock fund held outside.

Already, some answers to our question are offered. High tax bracket households who want to invest some of their money in stocks and some of their money in bonds, should first place their equity mutual funds in their pension accounts and hold long-term municipal

bonds outside (as long as the composition of their stock returns is similar to Cases 1-4 of Table 2.1). The reason why this is so can be easily understood. In all of the cases in the table, the strategy of holding the stocks inside the pension fund and municipal bonds outside results in the two pools of money compounding at 12 and 5.36 percent. In Case 1, the opposite strategy results in stocks compounding at r^S = 8.5 percent (with some final capital gains taxation payable at the end of the thirty years) and bonds compounding at 7.15 percent. With equal initial weighting, the combination of 12 and 5.36 percent beats the 8.5 and 7.15 percent pair for all horizons. One additional interpretation of this is that the pension environment is tax-preferred for all types of assets. By placing the asset with the higher gross rate of return in the pension plan, this preferred environment grows more rapidly to the advantage of the retirement saver.

The existence of municipal bonds and their relatively low imputed tax rate certainly affects the results of Table 2.1. However, it is not the entire story. Consider the same example, but without municipal bonds. Now, the choice, of course, is even simpler -- the only alternatives are corporate bonds in the pension and stocks outside or *vice versa*. The interest on corporate bonds is taxed at full ordinary income tax rates (assumed to be a combined state and federal marginal rate of 46.41 percent). Certainly for this high income/high tax household, corporate bond interest is taxed more heavily than the returns on any of the hypothetical stock mutual funds we just introduced in Table 2.1. Nonetheless, the answer to the asset location question is not as simple as "put the otherwise relatively heavily taxed bonds in the pension and hold the relatively lightly taxed stocks outside." In fact, the best allocation is the opposite of that for Cases 1 and 2 as shown in Table 2.2.

Table 2.2 Wealth Accumulated After 30 Years from $5K in Pensions and $5K in Conventional Saving -- The Analysis Without Municipal Bonds

Stock Fund Characteristics					Net Wealth	
Case Number	Total Return	Dividend Yield	Long-term Capital Gains	Accrued Capital Gains	Stocks in Pension; Bonds outside	Bonds in Pension; Stocks outside
1	12%	4%	6%	2%	**$95,725**	**$75,612**
2	12%	3%	4%	5%	**$95,725**	**$87,297**
3	12%	2%	3%	7%	**$95,725**	**$98,072**
4	12%	2%	2%	8%	**$95,725**	**$101.921**
5	12%	1%	1%	10%	**$95,725**	**$115,395**

For the first two cases, which are the most accurate depiction of typical actively managed equity mutual funds, putting the stock fund investment in the pension plan and holding the heavily taxed corporate bonds outside is still the better strategy than the opposite. Indeed, it still can make a huge difference in retirement wealth. Note that the advantage of strategy 1 (stocks in; bonds out) over strategy 2 (bonds in; stocks out) in Case 1 is 26.6 percent. The combination of 12 percent and 3.83 percent rates of return dominates the alternative of 7.15 percent and 8.5 percent for all horizons.

Clearly, the fact that the gross return on the stock funds has been taken as 12 percent whereas the gross return on bonds is only 7.15 percent is partly driving the results given that pensions allow accumulation at the full gross rate of return. The assumed rate of return premium of 4.85 percent per year of stocks over corporate bonds is actually rather conservative compared to the historical difference. However, to check on the sensitivity of the results to this assumption, I examined the impact of lowering the assumed gross nominal rate of return on stocks to 10 percent. The results are shown in Table 2.3.

Table 2.3 Nominal Wealth Accumulated After 30 Years by a Top Tax Rate Household from $5,000 in Pensions and $5,000 in Conventional Saving -- Gross Nominal Return on Stocks Set to 10%.

Stock Fund Characteristics					Net Wealth	
Case Number	Total Return	Dividend Yield	Long-term Capital Gains	Accrued Capital Gains	Stocks in Pension; Munis outside	Bonds in Pension; Stocks outside
1	10%	4%	6%	0%	**$70,719**	**$54,318**
2	10%	3%	4%	3%	**$70,719**	**$60,918**
3	10%	2%	3%	5%	**$70,719**	**$67,048**
4	10%	2%	2%	6%	**$70,719**	**$69,167**
5	10%	1%	1%	8%	**$70,719**	**$76,849**

The optimal location rule is once again to hold stocks inside the pension and municipal bonds outside for Cases 1-4. The gain in Case 1 from this strategy relative to the opposite is still a substantial 30 percent (compared with 38 percent in Table 2.1). The tax efficiency of the mutual fund continues to affect the calculation dramatically, but for the most realistic representations of actively managed funds, equities should be held in the pension.

So far we haven't simulated one advantage of holding stocks outside of pensions -- the fact that if stocks are held until they pass through an estate, then their cost basis is reset to their value as of date of death of the owner and capital gains accrued but unrealized at that time completely escape income taxation. In order to see whether this effect is potentially large enough to reverse the optimal allocation rule in the example we have been examining, I calculate how much the initial $10,000 would have grown to if it was part of an estate after 30 years. The results are shown in Table 2.4.

Table 2.4 Nominal Wealth Accumulated After 30 Years by a Top Tax Rate Household from $5,000 in Pensions and $5,000 in Conventional Saving -- If the Assets Are Part of An Estate

Stock Fund Characteristics					Net Wealth	
Case Number	Total Return	Dividend Yield	Long-term Capital Gains	Accrued Capital Gains	Stocks in Pension; Munis outside	Bonds in Pension; Stocks outside
1	12%	4%	6%	2%	**$104,241**	**$79,019**
2	12%	3%	4%	5%	**$104,241**	**$97,585**
3	12%	2%	3%	7%	**$104,241**	**$114,633**
4	12%	2%	2%	8%	**$104,241**	**$121,861**
5	12%	1%	1%	10%	**$104,241**	**$144,106**

Clearly, if the assets are held until death, then the step-up of cost basis makes the strategy of holding stocks outside of a pension more attractive. It also increases the advantage of tilting the composition of returns towards unrealized capital gains. In Case 1, where only 1/6th of the returns are deferred, it still is best to hold the stock fund inside the pension. At the other extreme, Case 5, where 5/6ths of returns are deferred, there is a large potential advantage to holding the stocks outside. This advantage is realized only in the event of the death of the owner of the assets.

We can examine two additional sensitivity analysis cases for the example of this section. Return now to the base situation of Table 2.1, but consider a household with much lower income and therefore lower tax rates. In particular, Table 2.5 looks at the same assets and location choices as Table 2.1, except that the household's federal tax rate, both now and in the future, is taken as 28 percent (with an 8 percent marginal rate of state income tax).

Table 2.5 Nominal Wealth Accumulated After 30 Years by a Household in the 28% Federal Tax Bracket from $5,000 in Pensions and $5,000 in Conventional Saving

Stock Fund Characteristics					Net Wealth	
Case Number	Total Return	Dividend Yield	Long-term Capital Gains	Accrued Capital Gains	Stocks in Pension; Munis outside	Bonds in Pension; Stocks outside
1	12%	4%	6%	2%	**$123,191**	**$91,543**
2	12%	3%	4%	5%	**$123,191**	**$102,597**
3	12%	2%	3%	7%	**$123,191**	**$111,839**
4	12%	2%	2%	8%	**$123,191**	**$115,827**
5	12%	1%	1%	10%	**$123,191**	**$126,885**

Such rates would apply to married couples filing jointly with taxable income in the range of $40,000 to $100,000. Interestingly, municipal bonds still dominate corporate bonds in a taxable environment for such households since they carry an implicit tax rate of 25 percent vs. an explicit combined state and federal tax rate on corporate bonds of 33.76 percent. Table 2.5 indicates that this middle-income household should prefer the stocks in the pension/municipal bonds outside allocation for the first four hypothetical equity mutual funds. In fact, for all cases except for number 1, the "stocks inside the pension fund" strategy is relatively more advantageous for this middle income household than it was for the high income household of the previous tables. Part of the reason is that the decrease in the tax faced on pension money withdrawn in retirement, t_R , is greater than the reduction in the rate of taxation of equity returns for stock portfolios held outside pensions. Recall that the federal tax rate on long-term realized capital gains is the same for these middle-income households (at 20 percent) as it is for the highest income taxpayers.

The allocation choice is obviously dependent on time horizon. Table 2.6 shows the results for the same case of investing $5,000 inside a pension account and $5,000 held

conventionally, but this time the holding period is only 15 years. The stakes in choosing the best allocation strategy are clearly much smaller over 15 years than over 30.

Table 2.6 Nominal Wealth Accumulated After 15 Years by a Top Tax Rate Household from $5,000 in Pensions and $5,000 in Conventional Saving

Stock Fund Characteristics					Net Wealth	
Case Number	Total Return	Dividend Yield	Long-term Capital Gains	Accrued Capital Gains	Stocks in Pension; Munis outside	Bonds in Pension; Stocks outside
1	12%	4%	6%	2%	**$25,613**	**$23,767**
2	12%	3%	4%	5%	**$25,613**	**$24.986**
3	12%	2%	3%	7%	**$25,613**	**$26,043**
4	12%	2%	2%	8%	**$25,613**	**$26,340**
5	12%	1%	1%	10%	**$25,613**	**$27,512**

However, the pattern is pretty much the same with the stocks inside/munis outside policy besting the corporate bonds inside/stocks outside choice for Cases 1 and 2. In Case 1 the advantage of the preferred location allocation is still a significant 7.8 percent after 15 years.

III. Allocating Assets Between Pensions and Conventional Accounts -- Monte Carlo Analysis

A legitimate complaint about the previous section is that there is no explanation of why stocks are assumed to earn an annual nominal return of 12 percent (or 10 percent for Table 2.3), whereas taxable bonds earn only 7.15 percent. In this section, I report on a series of Monte Carlo simulations of alternative strategies of locating stock and bond funds,

taking into account the variances and co-variances in asset returns. The historical long-run inflation adjusted returns are reported in Table 3.1 and taken from Ibbotson(1997). In forming the base parameters for the Monte Carlo simulations

Table 3.1 Inflation-Adjusted Rates of Return, 1926-96

Asset Category	Arithmetic Mean Return	Standard Deviation
Large Company Stocks	9.4%	20.4%
Small Company Stocks	14.1%	33.5%
Long-Term Corp Bonds	2.9%	10%

I have superimposed a rate of inflation of 2.6%. Further, in most of the cases I have used a contemporary long-term high-grade corporate bond yield (7.15%) as the mean return on bonds; this rate, of course, appears to incorporate a higher real return than corporate bonds have realized on average over the past 70 years. I report on some calculations in the next section with lower real bond yields. The Monte Carlo simulations assume that annual returns are serially uncorrelated (which accords with the historical evidence). The actual (nominal) parameters that were used for the base case Monte Carlo simulations are:

	Mean Return	Standard Deviation
Diversified Common Stock Portfolios	12.0%	20%
Corporate Bond Portfolio	7.15%	10%
Municipal Bond Portfolio	5.3625%	8%

with the following covariances and cross correlations:

	Co-Variance	Cross-Correlation
Stocks - Corporate Bonds	.0050	.25
Stocks - Municipal Bonds	.0024	.15
Corp Bonds - Municipal Bonds	.0076	.95

Each of the return relatives (one plus the rate of return) are assumed to be generated by a lognormal distribution constructed in such a manner to reflect these means, variances and covariances. The precise way in which the Monte Carlo simulations were generated to match these moments is described in Appendix 3. The outcomes of each allocation strategy are determined for 1,000 randomly generated sequences of returns. For instance, if a portfolio is to be held for 30 years, it is evaluated for 1,000 sequences of 30 returns. Each alternative portfolio strategy is tested against the same set of 1,000 sequences.

As in the previous section, different equity mutual funds, with varying dividend yields and realization policies are examined. The dividend yields are taken as fixed; all of the randomness in stock returns is reflected in realized and unrealized capital gains. The assumptions of the five cases are summarized at the bottom of Table 3.2, which, like Table 2.1, is for one-shot 30 year investments with two $5,000 pools of money (one a pension account and one a conventional savings account) and a high income investor. It is assumed that the individual is going to invest $5,000 in stocks and $5,000 in bonds and not rebalance. Strategy 1 involves investing the pension pool in stocks and the conventional money in municipal bonds; under strategy 2 the investor holds corporate bonds inside the pension and stocks outside. As in Section 2, Case 1 is probably the most realistic of a typical actively managed equity mutual fund, whereas Case 5 is probably unrealistically tax efficient.

Table 3.2 Nominal Wealth Accumulated After 30 Years By A Top Tax Rate Household -- Results from Monte Carlo Simulations

Case Number	Mean Strategy 1	Mean Strategy 2	Median Strategy 1	Median Strategy 2	25th %tile Strategy 1	25th %tile Strategy 2	Probability of Strat 1 > Strat 2

1	107,848	74,767	77,687	60,077	51,330	42,388	.988
2	107,848	87,946	77,687	67,560	51,330	45,252	.972
3	107,848	99,770	77,687	75,113	51,330	48,620	.866
4	107,848	104,266	77,687	77,171	51,330	49,426	.760
5	107,848	118,902	77,687	85,138	51,330	53,491	.127

Assumptions: E(rs) = .12, E(rb) = .0715, E(rm) = .0536, Var(rs) = .04, Var(rb) = .01, Var(rm) = .0064, Cov(rs,rb) = .005, Cov(rs,rm) = .0024, Cov(rb,rm).0076

1. Dividend Yield = 4%, 75% of appreciation is realized as long-term capital gain
2. Dividend Yield = 3%, 4/9ths of appreciation is realized as long term capital gain
3. Dividend Yield = 2%, 30% of appreciation is realized as long term capital gain
4. Dividend Yield = 2%, 20% of appreciation is realized as long-term capital gain
5. Dividend Yield = 1%, 1/11th of appreciation is realized as long-term capital gain

The results are very much the same as those in Table 2.1. Strategy 1 (holding the stock fund in the pension) has a higher mean, median, and 25th percentile outcome for Cases 1-4. For Case 1, the improvement in the mean outcome is 44.2 percent, whereas the median outcome improves by 29.3 percent and the 25th percentile outcome by 21.1 percent. Out of 1,000 simulated sequences of 30 years of asset returns, location strategy 1 bested strategy 2 by a score of 988-12.

The results of Table 3.2 provide another conclusion. Neither of the two asset location strategies should be considered safer than the other or more appropriate for risk-averse investors. Once the decision has been made to initially invest half of the total portfolio ($10,000 in the example) in one of these stock funds and the other half in long-term bonds, the preferred location choice strategy does not depend on risk preference. The same policy (strategy 1 in cases 1-4) which has the better mean outcome also has the better median and 25th percentile outcome. Although the results aren't reported in the table, an extremely risk averse person who focused on the 5th percentile outcome (or even the 1st percentile result) would choose the same location policy as less risk averse savers.

The results of Table 3.3 confirm those of the earlier Table 2.5 -- namely, that the superiority of strategy 1 is not only applicable for the rich, but also for the upper-middle class who face a federal marginal income tax rate of 28 percent. As with Table 3.2, we see that strategy 1 results in more wealth when one compares mean, median, and 25th percentile outcomes for cases 1-4. Strategy 1 beats strategy 2 in at least 75 percent of the

Table 3.3 Nominal Wealth Accumulated After 30 Years By A Middle-Income Household -- Results from Monte Carlo Simulations

Case Number	Mean Strategy 1	Mean Strategy 2	Median Strategy 1	Median Strategy 2	25th %tile Strategy 1	25th %tile Strategy 2	Probability of Strat 1 > Strat 2
1	125,340	88,913	82,805	67,814	52,422	46,750	.934
2	125,340	100,944	82,805	74,528	52,422	48,981	.889
3	125,340	111,253	82,805	79,567	52,422	50,901	.808
4	125,340	115,958	82,805	81,591	52,422	51,462	.757
5	125,340	128,076	82,805	87,627	52,422	53,915	.240

simulated thirty year sequences of returns. The margin of victory is often quite high.

The next issue to examine is the impact of horizon on the preferred location decision in this Monte Carlo uncertainty framework. It is assumed that the investor has chosen the same 50-50 stocks-bonds initial investment allocation regardless of horizon. At least for the moment, we are not asking whether such a choice is optimal. The only question we are addressing is how the investment horizon affects the optimal location decision. The results are shown in Table 3.4 for a high income/highly taxed investor whose equity portfolio is represented by Case 1 of Table 3.2. What can be seen is that strategy 1 performs better at all horizons and at all points in the distribution of outcomes.

Table 3.4 Nominal Wealth Accumulated From Alternative Locational Strategies As A Function of Time Horizon -- Monte Carlo Simulations for A Top Tax Rate Household Investing in 50% Bonds and 50% Case 1 Stocks

Time Horizon	Mean Strategy 1	Mean Strategy 2	25th %tile Strategy 1	25th %tile Strategy 2	5th %tile Strategy 1	5th %tile Strategy 2	Probability of Strat 1 > Strat 2
5	11,344	11,214	9,652	9,307	8,074	7,606	.584
10	17,134	16,448	13,253	12,618	9,726	9,172	.803
15	26,017	23,751	17,881	16,484	12,638	10,994	.956
20	41,442	34,988	24,467	22,172	16,661	14,688	.991
25	65,945	50,799	33,610	29,062	21,742	18,863	991
30	108,229	74,535	50,230	40,664	29,067	24,296	989
40	310,403	161,356	103,077	75,740	54,538	45,247	.988

Still, with randomness, strategy 1 has only a 58.4 percent chance of being *ex-post* superior over a five year horizon. Its chances of besting strategy 2 rise to 80.3 percent after 10 years and 95.6 percent after 15 years. In general, these results indicate that there is no reason to shift the location of stocks and bonds as the retirement horizon shortens. That is not to say that the overall stock-bond balance should not be adjusted.

A systematic program of lifetime saving for retirement can be thought of as a sequence of one-shot investments (which we have just examined). Nonetheless, it is useful to calculate the outcome of 30 years of annual saving. Table 3.5 reports the wealth accumulated by a high income/high tax household that began saving 30 years before retirement. In the first year of their retirement saving program, they saved a total of $10,000, half in pensions and half in conventional accounts. Just as in the analyses summarized in the previous tables, half of the saving is invested in one of five different stock funds and the other half is invested in long-term bonds.

Table 3.5 Nominal Wealth Accumulated With 30 Years of Annual Contributions By A Top Tax Rate Household -- Monte Carlo Simulations

Case Number	Mean Strategy 1	Mean Strategy 2	Median Strategy 1	Median Strategy 2	25th %tile Strategy 1	25th %tile Strategy 2	Probability of Strat 1 > Strat 2
1	1,587,059	1,361,462	1,372,628	1,229,341	1,060,824	957,901	.983
2	1,587,059	1,469,231	1,372,628	1,295,145	1,060,824	1,000,280	.936
3	1,587,059	1,564,164	1,372,628	1,363,271	1,060,824	1,034,237	.605
4	1,587,059	1,596,171	1,372,628	1,381,504	1,060,824	1,044,212	.435
5	1,587,059	1,707,971	1,372,628	1,446,725	1,060,824	1,088,068	.189

The amount saved increases by four percent each year, always split 50-50 between a pension account and conventional (taxable) accounts. The new investments also continue to be split 50-50 between stocks and bonds. In this case, there is no rebalancing meaning that stocks are likely to grow to substantially more than 50 percent of the total portfolio over time. Table 3.5 shows that strategy 1 has a higher mean, median, and 25th percentile outcome for Cases 1-3. For Case 1, the mean outcome is 16.6 percent greater with strategy 1 than strategy 2. The difference between the 25th percentile outcomes is 10.7 percent. Perhaps most impressively, in 983 of the 1,000 Monte Carlo sequences of 30 annual stock and bond returns, asset location strategy 1 results in more wealth than location strategy 2.

I have calculated the ratio of the wealth with strategy 1 to the wealth with strategy 2 for each of the 1,000 sequences of 30 sets of returns and for each of the hypothetical funds.

Figure 3.1
Relative Performance of Strategy 1 vs. Strategy 2:
Annual Contributions, Case 1

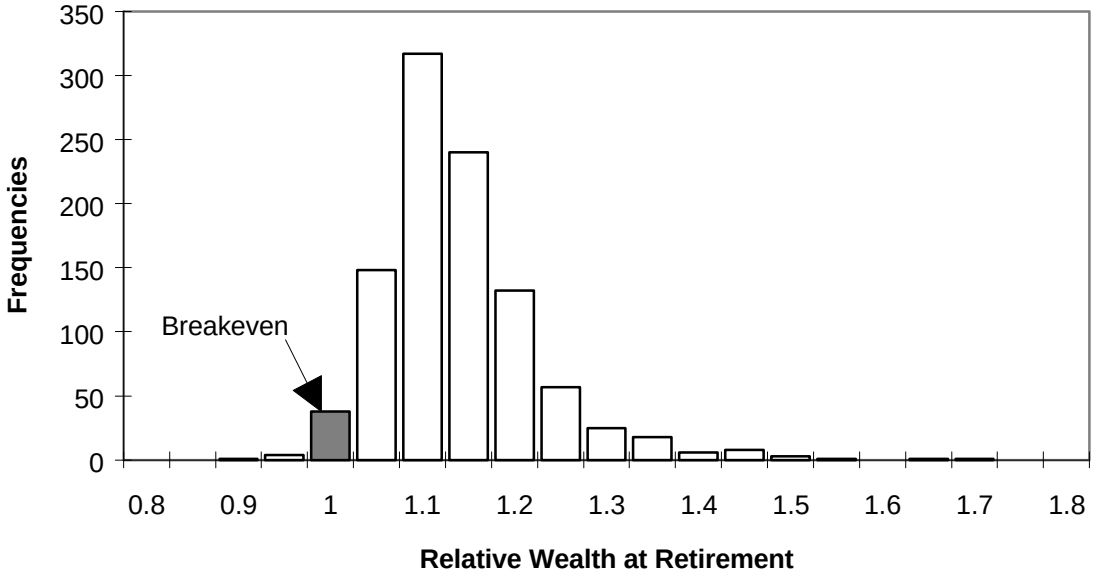


Figure 3.1 graphs the distribution of the relative performance of the two strategies when the initial investments are the Case 1 mutual fund and municipal bonds. The minimum relative wealth in this case is .919, meaning that the worst scenario out of one thousand has strategy 1 losing by only 8.1 percent relative to strategy 2. The median relative wealth is 1.124 and the mean is 1.139. I find the fact that strategy 1 outperforms strategy 2 by 13.9 percent on average impressive. The average relative wealth for Cases 2, 3, and 4 is 1.070, 1.017, and 1.002. For Case 5, the average relative wealth is .950.

The next simulation looks at systematic saving over thirty years with annual rebalancing to restore the asset split to half bonds and half stocks. The rebalancing may require selling assets (usually stocks), thus triggering capital gains taxation. In general, rebalancing leads to lower exposure to stocks in the long run and to lower average wealth in retirement. The results for the same pattern of saving as in Table 3.5 are shown in Table

3.6. Strategy 1 is superior to strategy 2 for the first three cases and it is a not-so-obvious close call for stock funds 4 and 5. In the first three cases, strategy 1 has substantially higher mean, 25th percentile, and 5th percentile outcomes. While rebalancing leads to lower average wealth in retirement ($1.458 million vs. $1.583 million), the outcomes

Table 3.6 Nominal Wealth Accumulated With 30 Years of Annual Contributions By A Top Tax Rate Household -- Results from Monte Carlo Simulations with Annual Rebalancing to a 50-50 Stock-Bond Split

Case Number	Mean Strategy 1	Mean Strategy 2	25th %tile Strategy 1	25th %tile Strategy 2	5th %tile Strategy 1	5th %tile Strategy 2	Probability of Strat 1 > Strat 2
1	1,458,451	1,377,274	1,091,064	1,030,497	792,649	748,347	.843
2	1,458,245	1,416,566	1,091,064	1,035,690	791,354	738,149	.700
3	1,458,068	1,454,554	1,090,483	1,052,452	790,441	736,717	.518
4	1,458,037	1,461,046	1,090,200	1,051,302	790,236	732,919	.494
5	1,457,838	1,503,589	1,089,889	1,067,231	788,796	736,175	.375

with and without rebalancing are virtually the same at the twenty-fifth percentile ($1.09 million vs. $1.06 million). The fifth percentile outcomes under strategy 1 are noticeably better with rebalancing than without ($792,649 vs. $768,323). As one might expect, rebalancing makes sense for strongly risk averse savers.

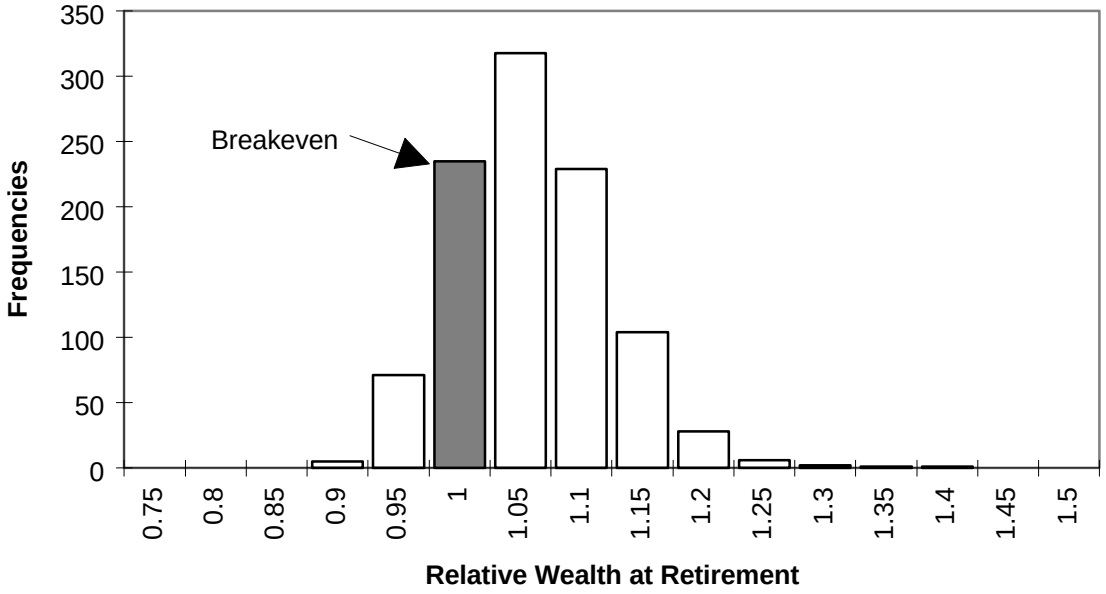
The relative performance figures for the two strategies are summarized in Table 3.7. The mean and median relative wealth outcomes favor strategy 1 for the first three cases. Figure 3.2 plots the frequency distribution of the ratio of wealth with the two strategies for the high income investor (for Case 1 with rebalancing). It clearly shows that there is a considerable chance of gaining between five and fifteen percent by choosing strategy 1 over strategy 2 and a very small chance of losing more than ten percent.

Table 3.7 The Distribution of the Ratio of Wealth Generated by Strategy 1 to Wealth Generated by Strategy 2 --Annual Contributions and Rebalancing

Case Number	Mean of Relative Wealth (Median)	Chance of Losing More Than 5% By Choosing Strategy 1	Chance of Losing More Than 10% By Choosing Strategy 1
1	1.059 (1.053)	2.4%	0.1%
2	1.037 (1.029)	7.1%	0.5%
3	1.015 (1.003)	15.2%	2.3%
4	1.013 (0.999)	18.5%	3.1%
5	0.988 (0.974)	35.9%	10.6%

Given these odds, it would seem irrational not to choose to hold the Case 1 stock fund in the pension fund and municipal bonds outside.

Figure 3.2
Relative Performance of Strategy 1 vs. Strategy 2:
Annual Contributions and Rebalancing, Case 1



In order to get a rough idea of the impact of the decrease in the taxation of long-term capital gains in the Taxpayer Relief Act of 1997 on the choice between strategy 1 and strategy 2, I ran the annual contribution/annual rebalancing simulations with the long-run

capital gains tax rate set to 28 percent (rather than 20 percent). The results were that strategy 1 was even more advantageous before the recent tax change. So, the conclusion is that the tax law change reduced the advantage of strategy 1 over strategy 2. Nonetheless, in most circumstances, strategy 1 still frequently generates substantially more retirement wealth than strategy 2, particularly for the typical, not-so-tax-efficient, equity mutual fund.

All of the analysis so far has dealt with locating stocks and bonds between pension and conventional savings accounts, given that the investor has already decided to invest half of the total money in stocks and half in bonds. The same type of analysis, however, can be undertaken for allocating different stock portfolios across the two environments. If two stock portfolios have the same expected return, it almost certainly makes sense to place the less tax efficient one in the pension plan first and hold the more tax efficient one outside. However, the more interesting case involves two stock portfolios with different expected returns, risk characteristics, and dividend policies. Consider a high income/high tax investor who has decided to equally divide all of her retirement saving between two stock funds, one a small-cap fund with an expected nominal annual return of 16.5 percent, a dividend (and realized short-run capital gains) return of 2.0 percent, and a standard deviation in the total rate of return of 33.5 percent. The other fund is a large-cap fund with an expected rate of return of 11.8 percent, a 4.0 percent dividend yield, and a total return standard deviation of 20.4 percent. The covariance in the returns of the two stock funds is 0.0555, which corresponds to a cross-correlation in returns of 80 percent. Both funds realize 75 percent of their experienced appreciation as long-term capital gains. The small-cap fund is clearly the more tax efficient due primarily to the lower dividend yield. The question is which stock fund should be given preference in placement inside the pension. As we have seen

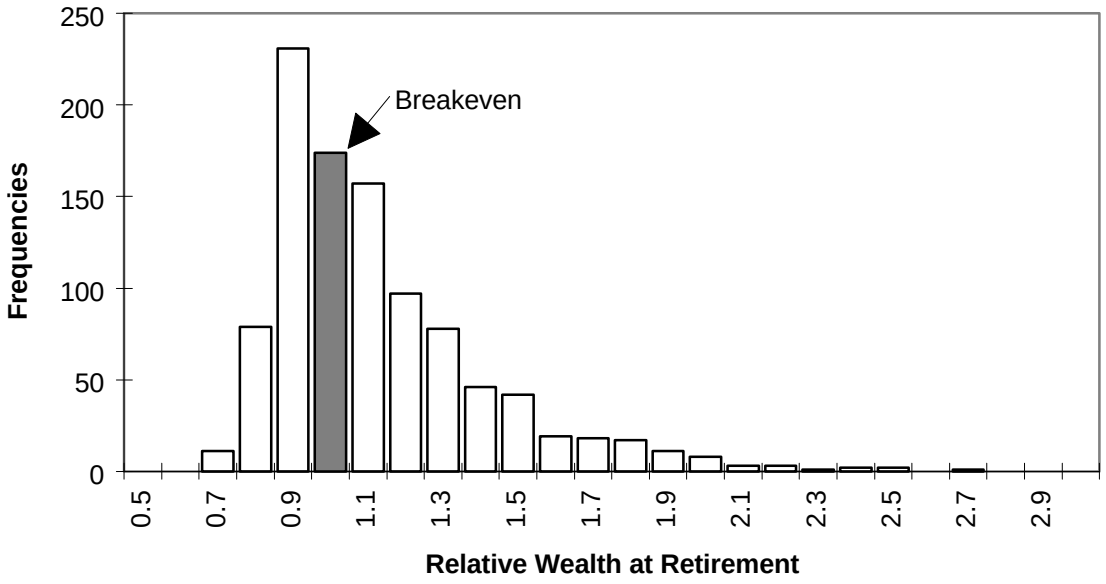
with the stock-bond analysis, there are many particular circumstances that could be evaluated; here we are only going to look at one. Consider a one-shot location decision (without rebalancing) and a thirty year horizon. Label the policy of placing the small-cap fund in the pension and the large-cap fund outside strategy 1. Strategy 2 is obviously the opposite locational choice. The results are shown in Table 3.8.

Table 3.8 Nominal Wealth Accumulated After 30 Years By A Top Tax Rate Household -- Results from Monte Carlo Simulations of A Large-Cap/Small-Cap Initial Investment

Mean Strategy 1	Mean Strategy 2	25th %tile Strategy 1	25th %tile Strategy 2	5th %tile Strategy 1	5th %tile Strategy 2	Probability Strat 1> Strat 2
370,658	249,985	55,888	58,320	17,976	20,177	.579

The payoff from this all-stocks strategy is illustrated by comparing these results with those of Table 3.2, where strategy 1 returned an average of $107,848 rather than the $370,658 shown in Table 3.8 for the two stock funds portfolio. Of course, the riskiness of this all-stocks strategy is also apparent. The 25th percentile outcome, while less than one-sixth of the mean outcome, is still better than the corresponding position for the stock-bond investor (where the better strategy returned $51,330), but the 5th percentile outcome is substantially worse (the 5th percentile outcome for the stock-bond saver using strategy 1 was $30,366). Which of the two location strategies is better for the small-cap/large-cap investor is also less clear, although strategy 1 (putting the high-yield/high-risk small-cap fund in the pension) appears preferable. It outperforms strategy 2 fifty-eight percent of the time and results in a much higher average wealth. The frequency distribution of relative performance of the two strategies in this case is shown in Figure 3.3.

Figure 3.3
Relative Performance of Strategy 1 vs. Strategy 2:
30 Year Horizon, One-Shot Investment, Small- & Large-Cap Stocks



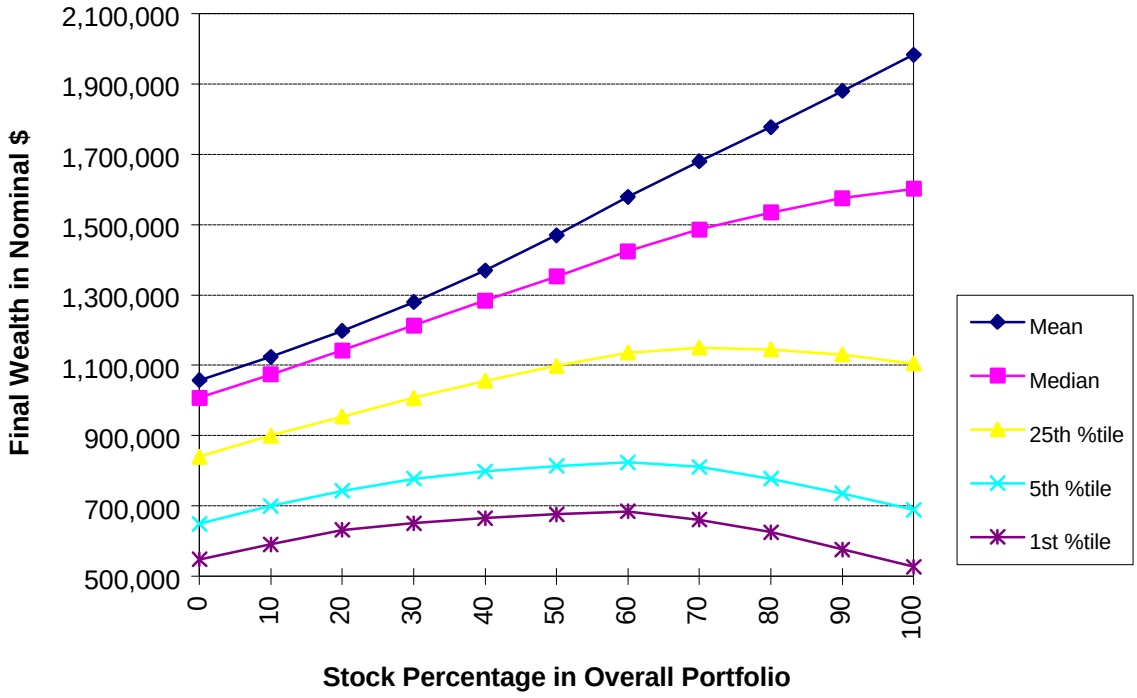
IV. The Overall Allocation Between Stocks and Bonds in Saving for Retirement

So far, with the exception of the final example of the previous section, we have taken as given that the saver was going to allocate half of his or her money to stocks and half to bonds. In some cases we simulated annual rebalancing to maintain the 50-50 stock-bond split. This section looks at the overall allocation of assets between stocks and bonds on the distribution of retirement resources. Given that stocks are assumed to have (and observed to have) higher average rates of return, it shouldn't come as any surprise that the higher the fraction of stocks in the retirement portfolio, the greater will be the mean retirement wealth outcome. Of more interest, of course, is how different stock-bond investment ratios affect the more unlucky outcomes in the distribution (say, for instance, the fifth percentile outcome). Rather than deal directly with the structure of investors' preferences *per se*, we will attempt to capture attitudes towards risk by what part of the

distribution of Monte Carlo outcomes different investors maximize. Risk neutral investors would choose the policy that maximizes the mean outcome. Mildly risk averse households should pay attention to the median outcomes under alternative investment allocation choices or perhaps those for the 25th percentile. Strongly risk averse savers would choose the policy with the best outcome at the fifth or even the first percentile.

Using this approach, consider a high income/high tax rate investor who is forced to invest half of his or her retirement saving in conventional accounts and half in pension accounts, just like in the previous two sections of the paper. This investor saves for thirty years with annual installments growing at four percent per year. The stock fund which will be used is the same as Case 1 of the previous section with an expected total nominal return of 12 percent per year, a standard deviation of returns of 20 percent, a 4 percent dividend and realized short term capital gains return, and 75 percent of the remaining return realized as long-term capital gains. Under strategy 1, whatever stocks are acquired, they will first be placed inside the pension fund. If the investor wants to save more than half of the new money in stocks, then stocks will be held outside the pension as well as inside. Under strategy 2, corporate bonds (yielding 7.15 percent) go first into the pension saving. If necessary, municipal bonds (with an interest rate of 5.36 percent) are held outside. I have run 10,000 Monte Carlo simulations of the consequences of choosing each of eleven different ratios of stocks to bonds. The ratios examined range from zero percent stock to one hundred percent stocks. Once a ratio is chosen, it is maintained in the portfolio through annual rebalancing. The results of adopting strategy 1 (where the stock fund is given priority for placement in the pension account) are pictured in Figure 4.1.

Figure 4.1
Stock-Bond Proportions and Accumulated Wealth:
Strategy 1 With 30 Annual Contributions and Rebalancing



It is no surprise that the mean and median outcomes are maximized with a policy of 100 percent stock investment. Similarly, it is not unexpected that the optimal policy for someone who concentrates on the 25th percentile results is to choose an allocation of 70 percent stocks and 30 percent bonds. What I find surprising in the results is that even the most risk averse individuals imaginable (referring to those who maximize either the fifth or first percentile of possible outcomes) should systematically invest 60 percent of their retirement saving in stocks.[5] While clearly this result depends on the particular parameters of the Monte Carlo simulations, I would argue that the underlying assumptions chosen here

[5] While it is not shown in the figure, the result for someone who is infinitely risk averse in the sense that they simply try to maximize the worst outcome from the 10,000 simulated sets of returns would choose to invest in 40 percent stocks. Of course, there is an extremely small chance of experiencing outcomes worse than any generated in the 10,000 Monte Carlo simulations.

are conservative. Certainly the difference between the expected return on bonds and the expected return on stocks (modeled here as 4.85 percent) is significantly lower than the historical difference. Even the fact that wealth is being calculated at the end of thirty years strikes me as quite conservative. The average pension saver probably begins saving at roughly age thirty and the average withdrawal in retirement may take place at approximately age 75. Here we are calculating the results of saving for thirty years, perhaps between the ages of 32 and 62.

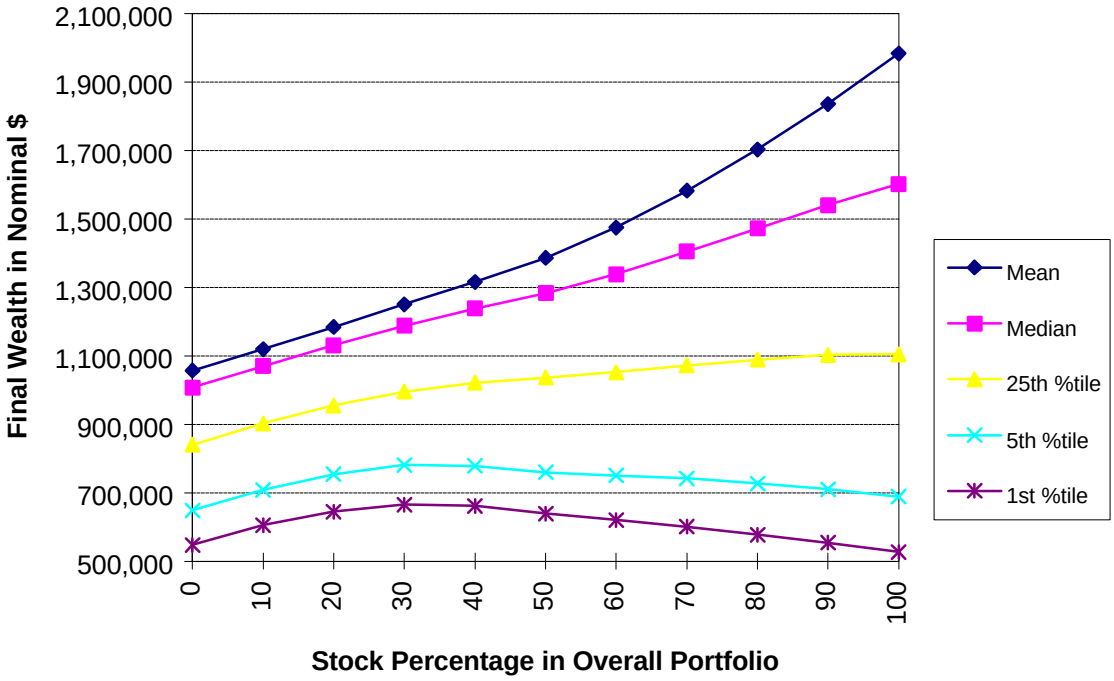
Not only do the results indicate that risk averse savers should allocate 60-70 percent of their investments to stocks, but Figure 4.1 displays major losses for those who use all bonds (and considerable losses for risk averse households using all stocks). Consider the very risk averse household that concentrates on the fifth percentile results -- the fifth percentile outcome with 60 percent stocks is 27 percent greater than with all bonds. For the less risk averse person who studies the twenty-fifth percentile outcomes, the result with 70 percent stocks exceeds the all-bonds outcome by 37 percent (and the all-stocks outcome by 4.1 percent).

Still referring to Figure 4.1, one can think about adding bonds to what was initially a 100 percent stock portfolio. For concreteness, say a retirement saver is contemplating placing 40 percent of the overall portfolio in bonds rather than holding all stocks. Adding these bonds is analogous to buying insurance. The (expected) cost of the insurance is the difference between the mean return with 100 percent stocks and the mean return with 60 percent stocks. In this case, the cost is $404,512. The payoff of adding the bonds (i.e. buying the insurance) is that the lower portion of the retirement wealth distribution is better. The 25th percentile outcome is $31,279 better, the 5th percentile outcome is $135,222 better, and the 1st percentile outcome is $158,213 better. Obviously, whether adding the bonds makes sense depends on the investor's level of risk aversion. Personally, losing more

than $400,000 in the mean retirement wealth strikes me as an expensive price for the improvements shown in the outcomes below the 25th percentile.

Figure 4.2 displays the results of the same calculations as Figure 4.1 except that the results are now for strategy 2. It is not surprising that the optimal fraction of stocks, at least for those who are very risk averse, is lower if strategy 2 is used -- after all, with this

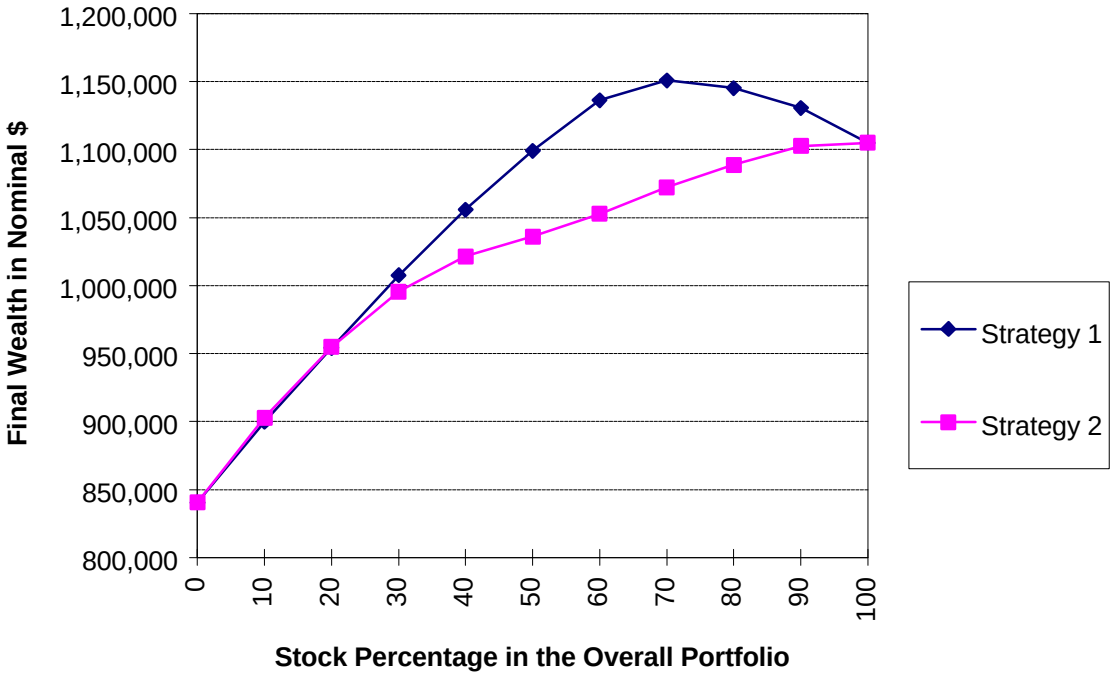
Figure 4.2
Stock-Bond Proportions and Accumulated Wealth:
Strategy 2 With 30 Annual Contributions and Rebalancing



strategy bonds get the first priority for inclusion in the pension plan. Nonetheless, even the very risk averse should choose to put at least 30 percent of their saving in stocks. Those whose risk aversion is such that they maximize their wealth at the 25th percentile of the distribution or higher should invest in one hundred percent stocks.

The previous two sections found that strategy 1, giving stocks priority in placement within the pension fund, consistently outperformed strategy 2, particularly for the realistic equity mutual fund (Case 1) we have been modeling here. The magnitude of the benefit from using strategy 1 is vividly demonstrated in Figure 4.3, which plots the 25th percentile outcomes for the two alternative asset location strategies. Obviously, if one

Figure 4.3
The Two Strategies At The 25th Percentile

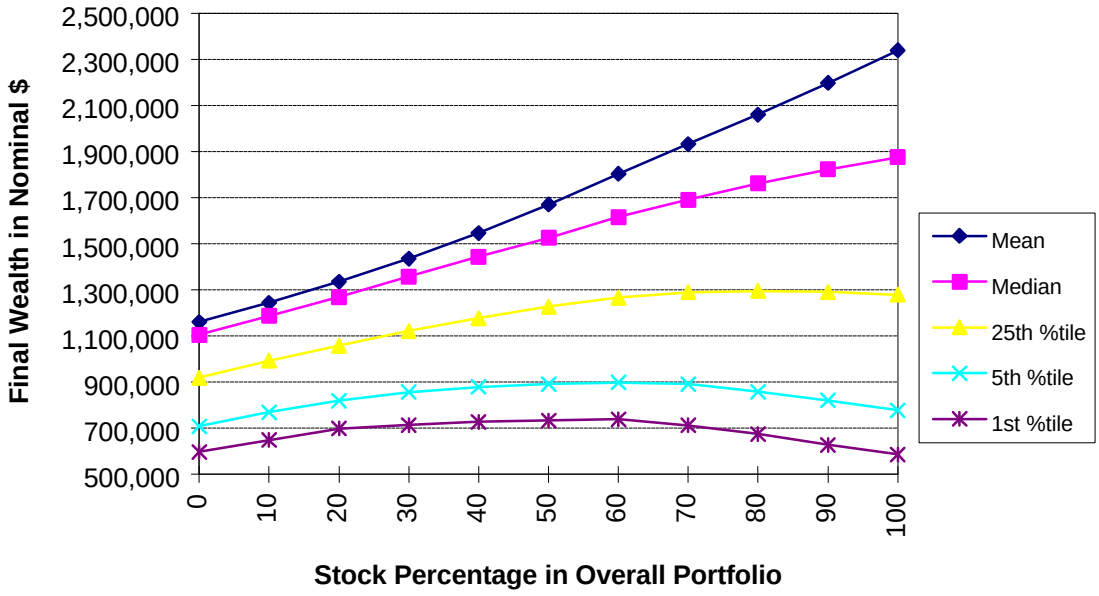


is investing in 100 percent stocks or bonds there is no difference between strategies 1 and 2 -- there is only one type of asset to locate. Similarly, location decisions are not very important if the annually rebalanced portfolio is going to be either 10-90 or 90-10. However, notice that the optimal choice for this risk averse saver is strategy 1with 70 percent stocks. With that overall allocation, the advantage of strategy 1 over strategy 2 is

$78,555 or approximately seven percent at the 25th percentile of the distribution of outcomes. Once again, this gain is nearly a pure efficiency gain and, as such, is enormous. One way to stress this is that if efficiency gain means "something for nothing" then $78,555 for nothing is a big deal.

In order to check the robustness of the results of Figures 4.1, I have rerun the analysis for a middle-income investor in the 28 percent federal income tax bracket.

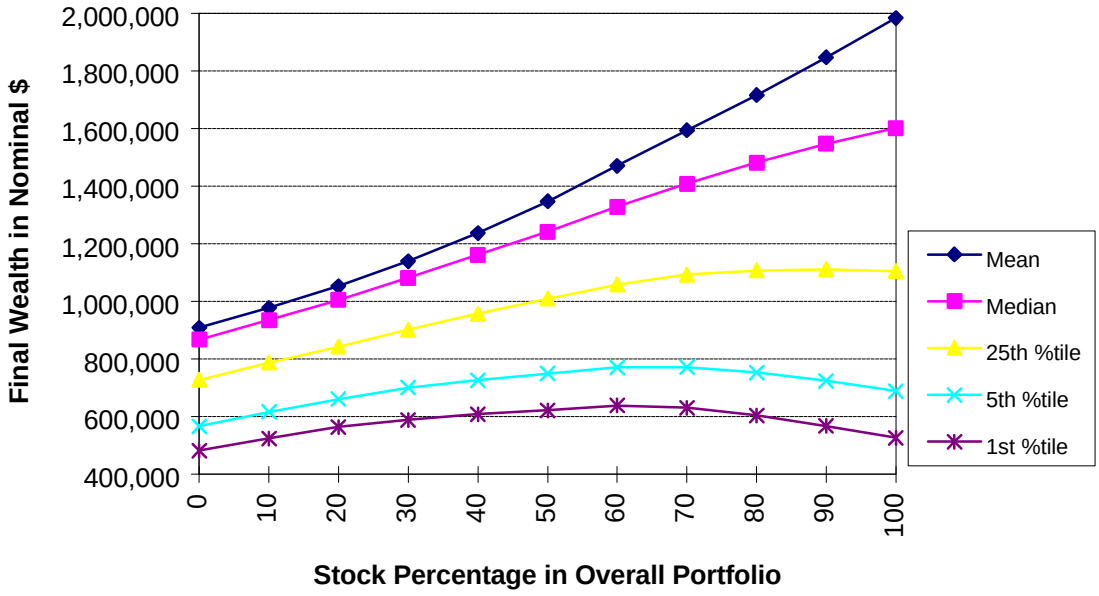
Figure 4.4
Stock-Bond Proportions and Accumulated Wealth:
Strategy 1 With Annual Rebalancing; 28% Tax Rate



The results are much the same as can be seen in Figure 4.4. The 100 percent stock choice maximizes the mean and median outcome with strategy 1, the 80 percent stock choice has the best 25th percentile outcome, and the 60 percent stock choice is best for the fifth and first percentile outcomes. Strategy 1 outperforms strategy 2 by at least as much as for the higher income household.

I also ran the base analysis with a lower assumed rate of return on bonds (6.0 percent instead of 7.15 percent). The reason I looked at this case is that the implied six percent premium in the expected return on stocks over bonds correspond closer to historical experience. The outcomes for this case are displayed in Figure 4.5. As might be expected, the results favor having slightly more stocks in the overall portfolio. Now, the best choice at the 25th percentile of outcomes is 90 percent stocks and the

Figure 4.5
Stock-Bond Proportions and Accumulated Wealth:
Strategy 1 With Annual Rebalancing; 6% Bonds



fifth and first percentile outcomes are best with 70 and 60 percent stocks, respectively. The basic shape of the curves is unaffected. My conclusion is that the results are quite robust and they can be summarized as follows: someone who is risk neutral should invest in all stocks for their retirement saving. Those who are mildly risk averse should consider holding 70 to 90 percent stocks. Finally, even those who are extremely risk averse should choose a portfolio with between 60 and 70 percent stocks over their accumulation period.

V. Summary and Conclusions

This paper addresses two important parts of the problem of saving for retirement. They are: (1) if assets are to be held in both conventional (and hence taxable) accounts and pension accounts, which assets should be held in each? and, (2) if the investor is substantially risk averse, what is the optimal mix of stocks and bonds for retirement saving?

Obviously, the results of the paper are only as good as the parameters selected for the examples and simulations. The future means, variances and covariances of asset returns could certainly differ from those chosen or those observed in the past. Similarly, future tax rates may be quite different from those modeled or now in place. Despite these disclaimers, my feeling is that the results appear robust and unambiguous. They are applicable to households with middle-level incomes and higher (those whose federal tax bracket is 28 percent or more). I list the results as follows:

1. If retirement assets consist of stock funds and bond funds (or stocks and bonds) and if some of the assets must be held outside of pensions, then the stocks or stock funds should be given preference for inclusion in the pension accounts. The only exceptions to this rule involve stock funds that are extraordinarily (probably unrealistically) tax efficient. If the retirement saver chooses to invest in 50 percent stocks and 50 percent bonds, the retirement resources advantage of giving stocks rather than bonds preference for placement within pensions can be eight percent or more.

2. The reduction in taxation for realized long-run capital gains enacted in the Taxpayer Relief Act of 1997 reduced, but far from eliminated, the advantage of strategy 1 (giving stocks preference for location within pension accounts).

3. Long-term bonds held outside of pensions should be municipal bonds, not obligations of the federal government or private corporations.

4. If bonds are held in a pension account, they should be corporate bonds.

5. Risk-neutral retirement savers should allocate all of their investments to stocks. Mildly risk averse investors should invest in 70 to 90 percent stocks (and possibly rebalance to maintain this allocation). Very risk averse savers should allocate roughly 60 percent of their retirement assets to a diversified portfolio of common stocks. The 60 percent allocation to stocks not only results in a higher average outcome than an all-bonds allocation, it also has a better first percentile outcome as well.

Certainly there is plenty of additional work that should be done on this topic. A comprehensive study of what people actually do is called for. Two recent studies that make substantial progress on this are TIAA-CREF (1997) and Bodie and Crane (1997). A broader analysis of asset allocation, including housing and human capital, would be valuable. A more detailed examination of the tax efficiency of actual mutual funds and how they respond to the new tax law would be profitable. However, despite the need for more research, the results reached here are sufficiently striking to consider acting upon. The two action items to take away from the paper are (1) strongly consider giving stocks preference for placement inside the pension account, with bonds held outside, if necessary, in the form of municipals, and (2) consider investing at least sixty percent of retirement assets in a diversified portfolio of common stocks even if you are cautious and risk averse. Hopefully, these lessons were worth the price of reading the paper.

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APPENDIX 1

The Calculation of the Cost Basis of an Equity Fund Held T Years

Anyone who has sold a mutual fund after a long holding period knows that calculating the tax basis is a nontrivial undertaking. The same is true in terms of the examples in this paper. The assumptions are that every year the fund experiences a total return of r divided into three components: r_d, the dividend rate of return, r_c, the realized long-term capital gains rate of return, and r_a,, the unrealized or accrued rate of return. All of these rates of return are relative to the total net asset value at the beginning of each year. The paper assumes that the distributed returns are reinvested after the payment of the relevant tax (i.e. the amount $r_d(1 - t_d) + r_c(1 - t_c)$ is reinvested). The paper defines the rate at which the investment compounds as $r^S = r_d(1 - t_d) + r_c(1 - t_c) + r_a$. The cost basis of the investment equals the initial amount invested B_0 plus all of the reinvested amounts.

The cost basis follows a recursive relationship through time. Namely, the cost basis at time T is related to the cost basis at time T-1 by the following equation:

$$(1)\ B_T = B_{T-1} + B_0(1 + r^S)^{T-1}\{r_d(1 - t_d) + r_c(1 - t_c)\}$$

By substituting the expression for B_{T-1} into equation (1) one gets

$$(2)\ B_T = B_{T-2} + B_0(1 + r^S)^{T-2}\{r_d(1-t_d) + r_c(1 - t_c)\} + B_0(1 + r^S)^{T-1}\{r_d(1 - t_d) + r_c(1 - t_c)\}$$

Continuing the substitutions one gets

$$(3)\ B_T = B_0 + B_0\{1 + (1 + r^S) + (1 + r^S)^2 + \ldots + (1 + r^S)^{T-1}\}\{r_d(1 - t_d) + r_c(1 - t_c)\}$$

This can be simplified to

$$(4)\ B_T = B_0[1 + \{1 - (1 + r^S)^T)/(1 - (1 + r^S))\}\{r_d(1 - t_d) + r_c(1 - t_c)\}]$$

or, finally, the equation (2D) in the text,

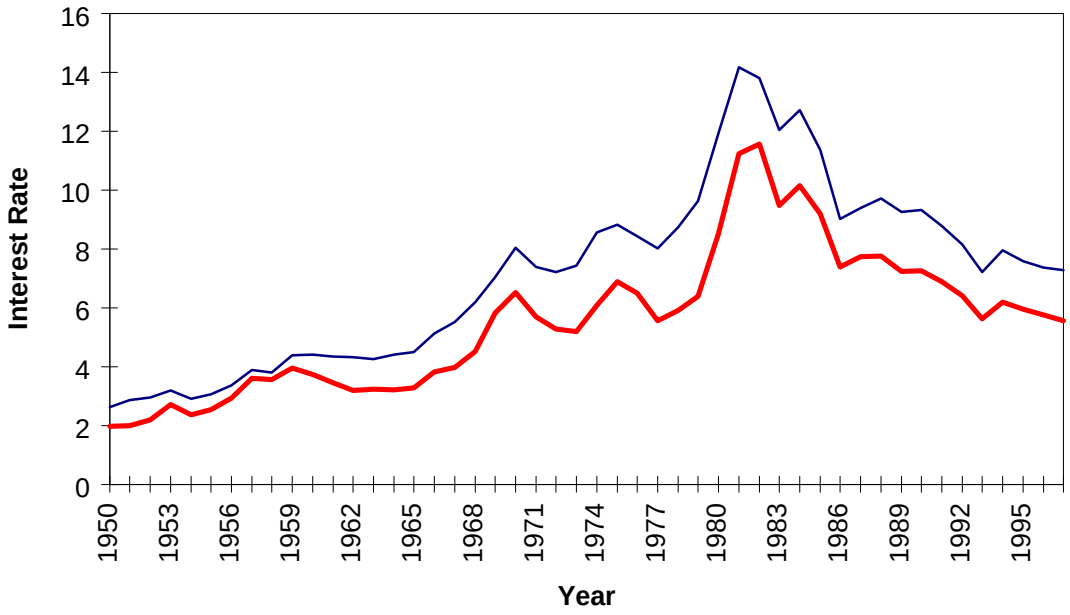
$$(5)\ B_T = B_0[1 + \{r_d(1 - t_d) + r_c(1 - t_c)\}\{((1 + r^S)^T - 1)/r^S\}]$$

APPENDIX 2

Corporate and Municipal Bond Yields

The interest rates on long-term high-grade corporate and municipal bonds track one another as is shown in Figure A1. Given that the interest on municipal bonds is not subject to either federal or state personal income taxes (with the possible exception of the AMT) whereas corporate bond interest is subject to both levies, it is interesting how small the discount is in municipal bond yields. Figure A1 plots the interest rate on Moody's Aaa corporate bonds and Standard & Poors High Grade Municipal bonds for the period 1950-97.

Figure A1
Aaa Corporate Bond and High-Grade Municipal Bond Interest Rates



If we define the implicit tax rate on municipal bonds to be the tax rate which gives munis and corporates the same after tax yield, then we have

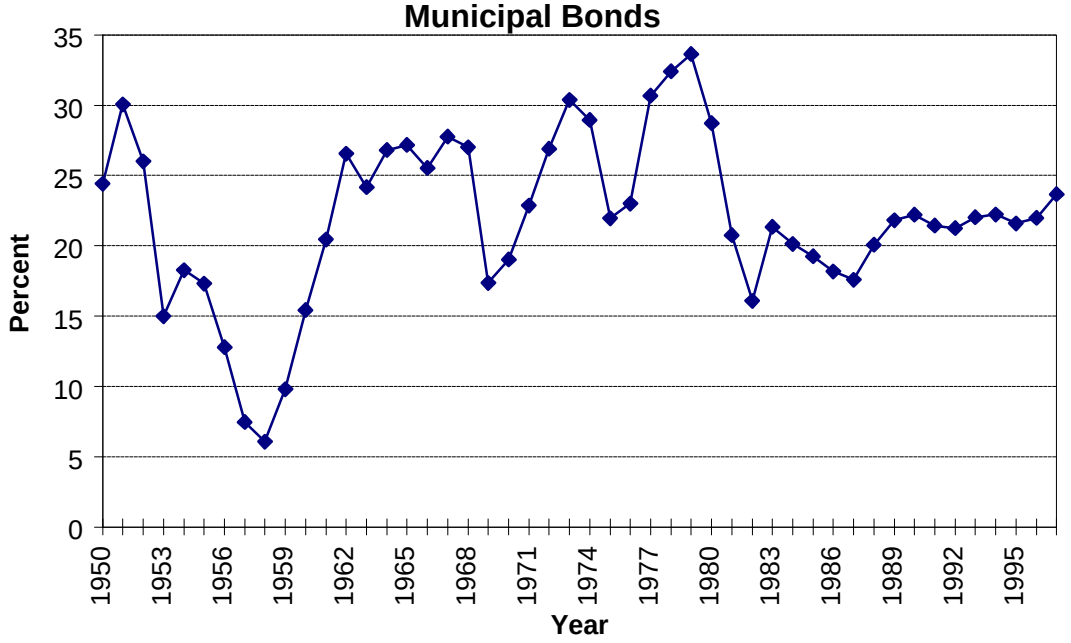
$$i_m = i_c(1 - t)$$

so, the implicit tax rate t can be determined as

$$t = 1 - i_m/i_c \ .$$

Figure A2 plots the implicit tax rate from 1950-97. It can be seen that the 25 percent rate assumed in the paper is well above the average for the period and also above all of the rates experienced since 1980.

Figure A2
The Combined Federal and State Implicit Tax Rates on Municipal Bonds



The fact that the implicit tax rate on long-term municipal bonds is so far below the maximum statutory personal income tax rates can be considered an anomaly and has been the subject of considerable research. For this paper, the fact that the implicit tax rate is relatively low is important; the explanation for why that is so is not necessary for our purposes.

Appendix 3

Explanation of the Monte-Carlo-Simulations

1. Basic Case (One-Shot-Investment)

Simulation of the Return Series

The random return series for each of the 1,000 simulations are generated as follows. The simple return relatives (i.e., one plus the simple return) of stocks, corporate bonds, and municipal bonds are assumed to be distributed log-normally. The log-normal distribution is a better description of simple returns than the normal distribution. It is skewed to the right and ensures that simple returns can not fall below negative 100% and thus that asset prices are non-negative. This model implies that the logarithms of asset return relatives are normally distributed. The simple return is abbreviated with *R* and the logarithm of the simple return relative with *r*. The moments of *r* (denoted with Greek letters) can be expressed as functions of the moments of *R* (denoted with Latin letters). *m* and μ denote the means, *s* and σ the standard deviations, and s_{ij} and σ_{ij} the covariances.

(1) $$\mu_i = \frac{1+m_i}{\sqrt{1+\left(\frac{s_i}{1+m_i}\right)^2}}$$

(2) $$\sigma_i^2 = \log\left(1+\frac{s_i^2}{\left(1+m_i\right)^2}\right)$$

(3) $$\sigma_{ij} = \log\left(1+\frac{s_{ij}}{\left(1+m_i\right)\left(1+m_j\right)}\right)$$

In a first step three independent standard-normal random variables *z* are generated for each simulation using Matlab's random number generator. Those three random numbers are combined such that the returns *r* have the desired variances and covariances.

(4) $$r_s = \mu_s + \sigma_s z_1$$

(5) $$r_b = \mu_b + \frac{\sigma_{sb}}{\sigma_s} z_1 + \left(\sigma_b^2 - \left(\frac{\sigma_{sb}}{\sigma_s}\right)^2\right)^{0.5} z_2$$

(6) $$r_m = \mu_m + a z_1 + b z_2 + c z_3$$

$$a = \frac{s_{sm}}{s_s},\ b = \frac{s_{bm} - \frac{s_{sm} s_{sb}}{s_s^2}}{\left(s_b^2 - \left(\frac{s_{sb}}{s_s} \right)^2 \right)^{0.5}},\ c = \sqrt{s_m^2 - a^2 - b^2}$$

The simple returns *R* are determined using the following transformation:

(7) $$R_i = \exp(r_i) - 1$$

Appreciation
All returns of the corporate bonds and the municipal bonds are assumed to be coupon payments or short term capital gains. Short term capital gains and coupon payments are treated like dividend payments. Appreciation of stocks in each time period is defined as the return minus the dividend payments. A fixed proportion of long-run capital gains of the stocks are distributed to the shareholders and are subject to capital-gains taxation. Capital losses are not distributed. Capital gains are only distributed when there are no accumulated past capital losses.

Wealth Levels of the two Allocation Strategies
Under strategy 1 the pension fund invests only in stocks and the conventional savings consist only of municipal bonds. The wealth at retirement at time *T* of conventional savings of municipal bonds and of pension savings of stocks amounts to:

(8) $$wr_m^v = i^v \prod_{i=1}^{T} (1 + r_m)$$

(9) $$wr_s^p = i^p (1 - t_R) \prod_{i=1}^{T} (1 + r_s)$$

wr_m^v denotes the wealth at retirement in the conventional savings account which is invested in municipal bonds, and i^v denotes the initial investment in the savings account. wr_s^p denotes the wealth at retirement in the pension fund which is invested in stocks. i^p denotes the initial investment in the pension fund, and t_R denotes the income tax rate during retirement. The total wealth at retirement generated by strategy 1 is equal to the sum of (8) and (9).

(10) $$wr^1 = wr_s^p + wr_m^v$$

Under strategy 2 the pension fund invests only in corporate bonds and the conventional savings consist only of stocks. The wealth at retirement at time *T* of conventional savings of stocks and of pension savings of corporate bonds amounts to:

$$(11) \quad wr_s^v = i^v \prod_{i=1}^{T}\left(1+r^*_s\right) - t_C\left(i^v \prod_{i=1}^{T}\left(1+r^*_s\right) - b_T\right)$$

$$r^*_s = \left(1-t_D\right)d_s + \left(1-t_C\right)cgd_s + \left(r_s - d_s - cgd_s\right)$$

$$r'_s = \left(1-t_D\right)d_s + \left(1-t_C\right)cgd_s$$

$$b_T = i^v\left(1+\sum_{i=1}^{T} r'_s \prod_{j=1}^{i-1}\left(1+r^*_s\right)\right)$$

$$(12) \quad wr_b^p = i^p\left(1-t_R\right)\prod_{i=1}^{T}\left(1+r_b\right)$$

t_D and t_C denote the tax rates of dividends (and short-run capital gains) and long-term capital gains, respectively. *d* denotes the dividend yield (including short-run capital gains), *b* the cost basis for the capital gains taxation, and *cad* the distributed capital gains. The total wealth at retirement generated by strategy 2 is equal to the sum of (11) and (12).

$$(13) \quad wr^2 = wr_b^p + wr_s^v$$

2. Annual Contributions and Allocation between Stocks, Corporate Bonds and Municipal Bonds

Under strategy 1 the pension fund invests only in stocks and the conventional savings consist only of municipal bonds. The wealth at retirement at time *T* of conventional savings of municipal bonds and of pension savings of stocks amounts to:

$$(14) \quad wr_m^v = \sum_{j=1}^{T}\left(1+g^v\right)^j i^v \prod_{i=j}^{T}\left(1+r_m\right)$$

$$(15) \quad wr_s^p = \sum_{j+1}^{T}\left(1+g^p\right)^j i^p\left(1-t_R\right)\prod_{i=j}^{T}\left(1+r_s\right)$$

g^v and g^p denote the growth rate of the annual contributions to the conventional savings account and the pension fund, respectively. The total wealth at retirement generated by strategy 1 is equal to the sum of (14) and (15).

$$(16) \quad wr^1 = wr_s^p + wr_m^v$$

Under strategy 2 the pension fund invests only in corporate bonds and the conventional savings consist only of stocks. The wealth at retirement at time *T* of conventional savings of stocks and of pension savings of corporate bonds amounts to:

(17) $$wr_s^v = \sum_{j=1}^{T}\left(1+g^v\right)^j i^v \prod_{i=j}^{T}\left(1+r^*_s\right) - t_C\left(\sum_{j=1}^{T}\left(1+g^v\right)^j i^v \prod_{i=j}^{T}\left(1+r^*_s\right) - b_T\right)$$

$$r^*_s = \left(1-t_D\right)d_s + \left(1-t_C\right)cgd_s + \left(r_s - d_s - cgd_s\right)$$

$$r'_s = \left(1-t_D\right)d_s + \left(1-t_C\right)cgd_s$$

$$b_T = \sum_{k=1}^{T} i^v\left(1+g^v\right)^k\left(1+\sum_{i=1}^{T} r'_s \prod_{j=1}^{i-1}\left(1+r^*_s\right)\right)$$

(18) $$wr_b^p = \sum_{j+1}^{T}\left(1+g^p\right)^j i^p\left(1-t_R\right)\prod_{i=j}^{T}\left(1+r_b\right)$$

The total wealth at retirement generated by strategy 2 is equal to the sum of (17) and (18).

(19) $$wr^2 = wr_b^p + wr_s^v$$

3. Annual Contributions and Allocation between Stocks, Corporate Bonds and Municipal Bonds with Annual Rebalancing

Rebalancing sets the stock-proportion of the portfolio after taxes equal to a predetermined level at the end of each time period. If capital gains result from rebalancing then these capital gains are taxed as long-run capital gains. If capital losses result from rebalancing then the total capital gains taxes decrease correspondingly.