NBER WORKING PAPER SERIES

AN INVESTIGATION OF RISK AND RETURN IN FORWARD FOREIGN EXCHANGE

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Working Paper No. 1180

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1983

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ABSTRACT

This paper examines the determination of risk premiums in foreign exchange markets. The statistical model is based on a theoretical model of asset pricing, which leads to severe cross-equation constraints. Statistical tests lead to a rejection of these constraints. We examine the robustness of these tests to time variation in parameters and to the presence of heteroskedasticity. We find that there is evidence for heteroskedasticity and that the conditional expectation of the risk premium is a nonlinear function of the forward premium. Accounting for this nonlinearity, the specification appears to be time invariant. Out of sample portfolio speculation is profitable but risky.

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Since the advent of generally flexible exchange rates for the major currencies of the world in 1973, there has been considerable interest among policy makers, commercial firms, and research economists into questions related to the efficiency of the forward foreign exchange market.[1] Policy makers and their advisors are concerned that the volatility of spot exchange rates reflects an incorrect amount of speculation in the forward market, and evidence on the predictive ability of forward exchange rates in forecasting future spot exchange rates is used in arguments for or against intervention by central banks in the exchange markets.[2] Commercial firms are concerned with obtaining accurate information on the price that they pay to hedge exchange risk where the price of hedging is the deviation between the forward exchange rate and the firm's expected future spot rate. In response to this demand, a large number of advisory services now sell forecasts of future spot rates.[3]

Academic and other research economists have contributed a substantial literature on the question of the efficiency of the forward market. Early empirical studies by Frenkel (1977) and Levich (1979a) were interpreted by the profession as providing considerable support for the proposition that the forward rate was an unbiased predictor of the future spot rate which was taken as an indication of the efficiency of the market. Indeed, the unbiasedness hypothesis continues to command a substantial following within the profession. On the other hand, a burgeoning empirical literature suggests that this hypothesis can be rejected at all but the smallest of marginal levels of significance for a variety of currencies and sample periods. Of course, this does not imply that the market operates inefficiently. A recognized alternative hypothesis is that a risk premium exists, although inefficiency certainly is an alternative hypothesis.

Within the profession there are now several well-defined positions on these issues. Many of those who continue to defend the unbiasedness hypothesis take refuge in the fact that the empirical studies which reject the hypothesis are often based on asymptotic distribution theory and hence may be subject to small sample bias. A particularly common criticism is that the studies may be subject to the "peso problem."[4] Such a criticism is not totally unwarranted, although longer sample periods and Monte Carlo studies may serve to resolve the issue. Presumably, the prevalence of the assumption of uncovered interest rate parity in most of the current theoretical models of international macroeconomics must be predicated on such an assumption.

A second position which provides another reason why the profession continues to ignore the rejection of the unbiasedness hypothesis has been articulated by Frankel (1982). He argues that most of the rejections of the unbiasedness hypothesis fail to provide evidence into the nature of a risk premium separating the forward rate from the expected future spot rate. They merely demonstrate that some information available to investors at the time the forward rate is set is potentially useful in predicting the forward rate forecast error. In Frankel's theory of the risk premium, which is the popular portfolio balance model of macroeconomics, the outstanding quantities of nominal government bonds are important fundamental determinants of the deviation of the forward rate from the expected future spot rate. Since he was unable to reject unbiasedness using the outstanding stocks of government bonds, Frankel (p. 263) concluded that "These results carry some weight against those who argue that the case for a risk premium has been firmly established." Given this finding, many researchers using portfolio balance models may feel justified in assuming that nominal government assets denominated in different currencies are perfect substitutes which is equivalent to the uncovered interest parity assumption, although the model of the risk premium discussed in this paper is inconsistent with such a proposition.

The third distinct position within the profession on the efficiency of the forward market has been articulated by Bilson (1981). In his investigation of the "speculative efficiency" hypothesis which is the unbiasedness proposition, Bilson (p. 449) found that information in the forward premium could be used to develop a trading strategy which has the property that "the profit/risk ratio appears to be too large to be accounted for in terms of risk aversity."

A somewhat related position has been taken by Dooley and Shafer (1982) who demonstrate the out-of-sample profitability of certain filter rules without discussing the riskiness of the strategies. The filter rules borrow depreciating currencies and lend appreciating currencies. After investigating various rules, Dooley and Shafer (p. 24) stated that "many currencies either were not efficient in their use of price information or real interest differentials were large and variable during the sample period."

The final position within the profession is occupied by those who explicitly recognize the possibility that time-varying risk premia can separate the forward rate from the expected future spot rate.[5] The theoretical models of Hodrick (1981) and Stulz (1981) demonstrated this possibility, but these models did not lend themselves to easy empirical implementation. These theoretical models also demonstrate that the expected real interest rate differential between nominal riskless assets denominated in two different currencies is exactly the same as the risk premium separating forward rates from expected future spot rates.

In order to construct statistical tests about the nature of the divergence of forward exchange rates from expected future spot rates, Hansen and Hodrick (1983) relied on the first order conditions of a rational representative investor. Since they placed auxiliary assumptions on the endogenous variables, they were unable to claim that their statistical tests were direct tests of an equilibrium model. Nevertheless, since they were unable to reject one of their restricted statistical models, they concluded (p. 33): "using a single beta latent variable model to measure risk, we found risk premiums to be important in at least two of the five currencies studied."

The purpose of this paper is to reexamine the conclusions of Hansen and Hodrick (1983) and to investigate further the potential role of risk premiums in explaining the rejection of the unbiasedness hypothesis. In Section II, we discuss the nature of the risk premium in a complete dynamic general equilibrium model of interest rate and exchange rate determination developed by Lucas (1982). We show that the model discussed by Hansen and Hodrick (1983) (henceforth abbreviated to HH) is consistent with that of Lucas, and in Section III, we test the restrictions of the HH model with nonoverlapping monthly data that includes twenty-one months of additional data.

In Section IV, we examine the unconstrained HH model for heteroskedasticity, and test for the time-invariance of the specification. In Section V, we examine the risk-return trade-off from following the trading strategy proposed by Bilson (1981) for our data set. Conclusions from our study are presented in Section VI.

II. The Lucas Model

In this section we describe some implications of the model developed in Lucas (1982) for the relationship between forward exchange rates and expected future spot rates. The Lucas model is a complete, dynamic, two country, general equilibrium model which provides some useful insights into the possible nature of risk premiums in the forward foreign exchange market. Given the highly stylized nature of the model and the generality of its stochastic structure, direct empirical tests of the model are impossible without additional restrictions. We do not pursue that strategy here; rather, we use the implications of the Lucas model to motivate a reexamination of the empirical analysis of Hansen and Hodrick and the trading strategy of Bilson.

In the Lucas model, the world consists of two countries whose agents have identical preferences but different stochastic endowments of the two consumption goods. In period t, citizens of country 0 are endowed with ξ_t units of commodity x, and nothing of y, and citizens of country 1 are endowed with η_t units of commodity y, and nothing of x. Each agent of country i wishes to maximize

$$(1) \qquad E_t\{ \sum_{t=0}^{\infty} \beta^t U(x_{it}, y_{it})\}, \quad 0 < \beta < 1$$

where x_{it} and y_{it} are the representative agent's consumptions in country i in period t of good x and y, respectively. The function U is assumed to be bounded, continuously differentiable, increasing in both arguments, and strictly concave, and β is a constant discount factor. The current real state of the system is given by $s_t = (\xi_t, \eta_t)$ which is assumed to be a realization of a known Markov process with transition function $F(s_{t+1}, s_t)$ where s_{t+1} represents next periods real state. In the equilibrium, agents pool risk perfectly, and each representative agent consumes half of the endowment of each country. In such an equilibrium, the relative price of y in terms of x, $p_y(s_t)$, depends only on the real state of the system and is given by the ratio of the marginal utility of y to the marginal utility of x:

$$(2) \qquad p_y(s_t) = \frac{U_y(\frac{1}{2}\xi_t, \frac{1}{2}\eta_t)}{U_x(\frac{1}{2}\xi_t, \frac{1}{2}\eta_t)} .$$

In the flexible exchange rate version of the model, agents are required to purchase the endowment of a country only with the money of that country. The timing of trade is such that all uncertainty about the state of the economy is realized prior to trade in securities and goods. Given this, the finance constraint is binding for all agents, and the nominal prices of goods x and y are simply

$$(3) \qquad p_x(s_t, M_t) = M_t/\xi_t$$

and

$$(4) \qquad p_y(s_t, N_t) = N_t/\eta_t$$

where M and N are "dollars" and "pounds," the monies of country 0 and country 1, respectively.

There is also nominal uncertainty in the world. In each period t there is a lump sum dollar transfer, $w_{0t}M_{t-1}$, to agents of country 0 and a lump sum pound transfer, $w_{1t}N_{t-1}$, to agents of country 1. The transition function for the two monies is also characterized by a known, exogenous Markov process, $K(w_{t+1},w_t,s_{t+1},s_t)$, where $w_t = (w_{0t},w_{1t})$ is the vector of stochastic growth rates for the two monies between periods t-1 and t.

Given the relative price of the two goods in (2) and the dollar and pound prices of x and y in (3) and (4), the equilibrium exchange rate is given by the arbitrage equation:

$$e(s_t,M_t,N_t) = \frac{p_x(s_t,M_t)}{P_y(s_t,N_t)}\, p_y(s_t) = \frac{M_t n_t}{N_t \xi_t}\, p_y(s_t). \tag{5}$$

Asset pricing in this world is similar to the intertemporal asset pricing models of Rubinstein (1976), Lucas (1978), Brock (1980), and others.[6] The equilibrium price of an asset is such that the marginal utility foregone by purchasing the asset is equal to the conditional expectation of the marginal utility of the return from holding the asset. The conditional expectation is taken with respect to the distribution functions F and K in this case.

Consider, for example, the derivation of the dollar price in period t of a claim to one dollar with certainty in period t+1. Such a claim is equivalent to $1/p_x(s_{t+1},M_{t+1}) = M_t^{-1}(1+w_{0t+1})^{-1}\xi_{t+1} \equiv \Pi^M_{t+1}$ units of x in period t+1 which is an uncertain amount that depends on the purchasing power of the dollar, Π^M_{t+1}. The Π^M_{t+1} units of x will be valued by agents in period t+1 at the marginal utility of x, $U_x(s_{t+1})$, which must be discounted back to period t by multiplication by the discount factor β. The x-unit price of the claim to one dollar is therefore $E_t[\beta U_x(s_{t+1})\Pi^M_{t+1}U_x(s_t)^{-1}]$ which is obtained by taking the conditional expectation of the marginal value of the payoff on the asset and dividing by the marginal utility of x in period t since the opportunity cost of the investment is its x-unit price times the marginal utility of x in period t. The dollar price of the investment is then obtained by multiplication of the x-unit price by $\rho_x(s_t,M_t)$ or division by Π^M_t. Therefore, the period t dollar price of a discount bill paying one dollar in period t+1 is

$$(6)\qquad b_x(s_t,w_t) = E_t\left[\frac{\beta U_x(s_{t+1})\Pi^M_{t+1}}{U_x(s_t)\Pi^M_t}\right].$$

Similarly, by replacing x with y in the above argument, the period t pound price of a claim to one pound in the next period is found to be

$$(7)\qquad b_y(s_t,w_t) = E_t\left[\frac{\beta U_y(s_{t+1})\Pi^N_{t+1}}{U_y(s_t)\Pi^N_t}\right],$$

where $U_y(s_t)$ is the marginal utility of y in period t and Π^N_t is the purchasing power of the pound in terms of y.

The discount bill prices in (6) and (7) are conditional expectations of the intertemporal marginal rates of substitution of dollars and pounds, respectively. Since these random variables are central to the discussion of risk in a monetary economy, we define them as

$$(8) \qquad Q_{t+1}^{M} = \frac{\beta U_x(s_{t+1})\Pi_{t+1}^{M}}{U_x(s_t)\Pi_t^{M}} \quad \text{and} \quad Q_{t+1}^{N} = \frac{\beta U_y(s_{t+1})\Pi_{t+1}^{N}}{U_y(s_t)\Pi_t^{M}} .$$

The intertemporal marginal rate of substitution of money is an index that weights the change in the purchasing power of the money by the intertemporal marginal rate of substitution of goods between the two periods. Since the exchange rate is the relative price of two monies, each of the rates of substitution is important in determining the risk premium in the forward foreign exchange market.

In order to determine the nature of the risk premium in the forward foreign exchange market, we must derive the forward price of foreign exchange, that is, the contract price set in period t at which one can buy and sell foreign exchange in period t+1.[7] If there is no default risk on either nominal investment discussed above or on the forward contracts, investors must be indifferent between investing in the sure dollar denominated asset in which case the return is $1/b_x(s_t,w_t)$ per dollar invested and the alternative covered interest arbitrage strategy of converting dollars into pounds, investing in sure pound denominated assets, and selling the proceeds in today's forward market at price $f(s_t,w_t,M_t,N_t)$ of dollars per pound. The covered pound investment strategy yields $[1/e_t(s_t,M_t,N_t)][1/b_y(s_t,w_t)][f_t(s_t,w_t,M_t,N_t)]$ per dollar invested. Equating the two investment strategies and substituting from above gives

$$(9) \qquad f_t(s_t, w_t, M_t, N_t) = e(s_t, M_t, N_t) \frac{b_y(s_t, w_t)}{b_x(s_t, w_t)} .$$

Taking the conditional expectation of next period's exchange rate from (5) and subtracting (9) gives an expression for the risk premium in this model:

$$(10) \quad E_t(e_{t+1}) - f_t = E_t\left[\frac{M_t(1+w_{0t+1})\eta_{t+1}}{N_t(1+w_{1t+1})\xi_{t+1}} p_y(s_{t+1})\right] - \frac{M_t\eta_t}{N_t\xi_t} p_y(s_t) \frac{E_t(Q^N_{t+1})}{E_t(Q^M_{t+1})}$$

Dividing both sides of (10) by e_t normalizes the scale of the expression and provides an insight into the nature of the risk premium which depends upon the two currencies intertemporal marginal rates of substitution:

$$(11) \qquad \frac{E_t(e_{t+1}) - f_t}{e_t} = E_t\left[\frac{Q^N_{t+1}}{Q^M_{t+1}}\right] - \frac{E_t(Q^N_{t+1})}{E_t(Q^M_{t+1})} .$$ [8]

Both real and monetary uncertainty enter the determination of the risk premium as well as the preferences of agents which act as weights in determining the importance of the fundamental sources of uncertainty represented by the real and monetary shocks to the two economies.

In order to develop an empirically testable hypothesis regarding the possibly time-varying risk premium in (11), Hansen and Hodrick (1983) exploited the fact that covered and uncovered investments in the pound-denominated riskless nominal return yield two dollar denominated returns that must satisfy the representation of the risk-return trade-off given by a conditional capital asset pricing model based on the conditional mean-variance frontier. That is, in equilibrium any dollar-denominated return, R_{t+1}, must satisfy

$$(12) \qquad E_t(R_{t+1}-R^z_{t+1}) = \beta_t E_t(R^b_{t+1}-R^z_{t+1})$$

where $\beta_t = C_t(R_{t+1};R^b_{t+1})/V_t(R^b_{t+1})$, R^b_{t+1} is the dollar-denominated return on an appropriately chosen benchmark return on the conditional mean variance frontier, and R^z_{t+1} is the return on an asset that is conditonally uncorrelated with the return on the benchmark asset.[9] When there exists a riskless nominal investment such as the one period bill described above, R^z_{t+1} can be chosen to be the nominal return R^f_{t+1} = $1/b_x(s_t,w_t)$.

Hansen, Richard and Singleton (1982) establish that any return on the conditional mean variance frontier is an appropriate benchmark return, and each of these returns satisfies:

$$(13) \qquad R^b_{t+1} = \omega_t R^c_{t+1} + (1-\omega_t)R^f{}_{t+1}$$

where R^c_{t+1} is the minimum second moment return conditioned on the informaton set of agents, and ω_t is a possibly random weight that may depend on the conditioning set and which has the property that the probability of the event $\{\omega_t=0\}$ is zero. When markets are complete as in the Lucas model, one can think of agents having the opportunity to trade an asset with nominal return

$$(14) \qquad R^M_{t+1} = Q^M_{t+1}/E_t(Q^M_{t+1})^2,$$

and it is easily verified that this return is the minimum second moment return.[10]

Now consider the dollar denominated returns mentioned above, covered and uncovered investments in pounds. Each return must satisfy (12), and taking the difference of the two returns gives

$$(15) \qquad E_t[(e_{t+1}-f_t)/e_t] = \beta_t^e E_t(R_{t+1}^b - R_{t+1}^f)$$

where $\beta_t^e = C_t[(e_{t+1}-f_t)/e_t; R_{t+1}^b]/V_t(R_{t+1}^b)$. This alternative represenation of the risk premium is perfectly consistent with the representation in (11), and it is representation (15) that Hansen and Hodrick (1983) exploited in their empirical tests.

The next section of the paper discusses extensions of the empirical model of HH to longer sample periods.

III. The Hansen-Hodrick Model

In (15), the beta is conditional on the information set of agents. Without a more detailed specification of the stochastic properties of the exogenous processes of a model such as the Lucas model, an assumption that the beta is constant is strictly an empirical hypothesis that allows one to proceed empirically. Consequently, while we use the discussion of the previous section to motivate a representation of the risk-return trade-off in the foreign exchange market, one must remember that the tests reported here, as in the case of HH, are not tests of an explicit equilibrium model.[11]

The empirical specification of the HH model begins with an assumption that the betas on several forward foreign exchange contracts that satisfy (15) are constant. The expected return on the benchmark portfolio in excess of the nominal riskless return is assumed to vary through time and is treated as an unobserved variable. This allows the empirical model to be written as

$$(16) \qquad y_{t+1} = \beta^* x_t + u_{t+1}$$

where y_{t+1} is a vector of actual normalized forecast errors, $(e^i_{t+1}-f^i_t)/e^i_t$ for several currencies, $x_t = E_t(R^b_{t+1}-R^f_{t+1})$, β^* is a vector of the β^e's in (15), and u_{t+1} is a vector of conditional expectation forecast errors with typical element, $u^i_{t+1} = y^i_{t+1} - E_t y^i_{t+1}$. The vector stochastic process u_t satisfies the condition $E(u_t u'_{t-j}) = 0$, $j \geq 1$, and $E(u_t h_t) = 0$ for all h_t in the conditioning set, but we do not specify how $E_t(u_{t+1}u'_{t+1})$ depends on elements in the time t information set.[12]

Since x_t is assumed to be unobservable by the econometrician, the empirical test is constructed by substituting into (16) the best linear predictor of x_t based on a subset of the information in agents' conditioning set. That is, let

$$(17) \qquad x_t = \alpha_0^* + \alpha_1^{*\prime} z_t + \varepsilon_t$$

where z_t is a vector of instrumental variables and ε_t is the prediction error which has mean zero and is orthogonal to z_t. Substituting (17) into (16) gives the complete model:

(18) $$y_{t+1} = \beta^*\alpha_0^* + \beta^*\alpha_1^{*\prime}z_t + \nu_{t+1}$$

where $\nu_{t+1} = u_{t+1} + \beta^*\varepsilon_t$ implying that ν_{t+1} is also orthogonal to z_t.

The original sample period for the model estimated in HH was February 1976 to December 1980. The data were spot and one month forward exchange rates of U.S. dollars for the French franc, the Japanese yen, the Swiss franc, the U.K. pound, and the Deutsche mark. The data set consisted of a semi-weekly sample in which Tuesday forward rates predicted Thursday spot rates thirty days in the future and Friday forward rates predicted the corresponding Monday spot rates.[13] There were 512 overlapping observations in the data set. For purposes of comparison across the various models of this paper, the original data set was sampled to form 57 nonoverlapping observations, and 21 additional nonoverlapping observations were added from 1981 and 1982.

In HH, the instrumental variables were chosen to be the five currently occurring forward rate forecast errors, i.e., $z_t = y_t$. In reestimating the model with the new longer data set, we compared the power of the original instruments against the power of using the five current forward premiums as instruments. Since the forward premiums provided a more powerful test, only the results with these instruments are reported here, i.e., z_t is a vector with typical element $z_t^i = (f_t^i - e_t^i)/e_t^i$.

The first result to examine is the reestimation of the model with the forward premiums as instruments for the sample period that coincides with the initial estimation period of HH. Estimation of the parameters of (18) requires a system estimation technique, and as in HH, we applied Hansen's (1982) Generalized Method of Moments (GMM) procedure.[14] The results are presented in Table 1. The first beta is normalized to one reflecting the identification problem that arises when one treats the expected return on the benchmark portfolio as an unobserved variable. The overall test of the model's restrictions is a test statistic which is asymptotically chi-square distributed with twenty degrees of freedom reflecting the difference between the number of orthogonality conditions exploited in the estimation, thirty, and the number of estimated parameters, ten. The value of the test statistic, 24.239, indicates that the restrictions are not rejected at standard levels of significance. These results are similar to the test statistic reported in Table 5 of HH in which, with the forecast errors as instruments, the test statistic had a value of 18.834.[15]

One noticeable difference between the results of the estimation of the model with the forecast errors as instruments versus the results with the forward premiums as instruments is in the joint tests of the significance of the reduced form coefficients, which are defined for each currency as $\hat{\theta}_i = \hat{\beta}_i \hat{\alpha}_1$. In the original HH specification with the forecast errors as instruments, the tests of the significance of the reduced form coefficients had low marginal levels of significance for the Japanese yen and the Swiss franc. In Table 1, only the test of the Swiss franc reduced form coefficients has a low marginal level of significance. This partly reflects the fact that in the unconstrained specification the forward premiums are not particularly powerful explanatory variables during this sample period.

On the basis of their inability to reject the restrictions of the model and having found significant explanatory power in at least two currencies, HH concluded that the latent variable model provided a convenient vehicle for the interpretation of the rejection of the unbiasedness hypothesis. If investment in the forward market is risky in the sense described in the previous section, investors will have to be compensated for bearing risk. At a point in time, the expected returns on the various forward contracts will be proportional to each other, but the expected return on the benchmark portfolio may vary. The conclusion of their study was that this proportionality remained sufficiently stable through time that its assumed constancy could not be rejected by the data.

In Table 2 we investigate the model for the sample period February 1976 to September 1982 adding twenty-one nonoverlapping observations to those used in Table 1. The results are very different from those in Table 1. The explanatory power of the constrained model is somewhat improved. In the tests of the significance of the reduced form coefficients, the marginal levels of significance for the Japanese yen and the Swiss franc are now very small. In contrast to Table 1, though, the chi-square test of the constrained model has a value of 34.497, which indicates that the restrictions of the model are rejected at all marginal levels of significance greater than .023. If the source of the rejection of the unbiasedness hypothesis is a time-varying risk premium, it appears that the assumptions of the HH model are too strong. Either the betas in (15) are not constant, or some other model of risk and return is necessary to describe the risk premium.

It is interesting to compare the results of Table 2 with the estimation of the unconstrained model presented in Table 3. These are ordinary least squares (OLS) estimates of the unconstrained reduced form coefficients.[16] Note the differences between the two sets of estimates.

In the OLS regressions the coefficients of the instrumental variables that have weak explanatory power do not always have the same algebraic sign across currencies. This is true in the case of the constant terms and the coefficients of the forward premiums of the French franc and the U.K. pound although in none of the cases is the set of parameters particularly precisely estimated. Also, in the case of the coefficients which do have strong explanatory power in the unconstrained model, that is the coefficients of the Swiss franc and Deutsche mark forward premiums, the rank ordering across currencies is striking, but the proportionality is not of the same order of magnitude in each case. Finally, the imposition of the constraints causes a relatively severe loss in explanatory power as measured by the R^2 for the French franc, the U.K. pound, and the Deutsche mark.

Given the rejection of the model of risk and return postulated in this section, it is important to reiterate that the model was a statistical hypothesis and not a precisely stated theory. Ideally, we would like to test a representation of dynamic equilibrium such as that set forth by Lucas and discussed in the previous section. At this point in time, the demands on the data to test such a model make it an exceedingly difficult task. For now, we set that task aside in order to investigate the stability of the reduced form coefficients presented in Table 3. This is done in the next section of the paper.

IV. Parameter Stability

In this section, we investigate whether the rejection of the constraints in the HH model, documented in the previous section, is due to time-varying parameters and if so, why this might arise.

The theoretical analysis of Section II only postulates the existence of a trade-off between risk and return at a point in time, as in (12) and (15). It does not impose the restriction that the conditional covariance between the return on an asset and the return on the benchmark portfolio is constant or that the conditional variance of the benchmark portfolio is constant.

We shall work with the unconstrained model, estimated in Table 3. The reason is that even though the latent variable model is rejected relative to the unconstrained model, the cross-equation constraints may still be valid at any point in time as argued above, but the coefficients of the unconstrained model may not be constant through time. Alternatively, the coefficients of the unconstrained model could be constant and the restrictions of the HH model not hold under alternative hypotheses regarding the nature of risk and return in the forward foreign exchange market. This motivates our investigation of the stability of the coefficients of the unconstrained model.

There are several reasons why the coefficients of the unconstrained model may not be constant. For example, if we interpret the reduced-form equation

$$(19) \qquad (e^i_{t+1}-f^i_t)/e^i_t = a_i + \sum_{j=1}^{5} b_{ij}(f^j_t-e^j_t)/e^j_t + u^i_{t+1}$$

as a conditional expectation, then we are imposing the assumption that this conditional expectation is a linear function of the variables of the information set. The true conditional expectation may be a nonlinear function of the forward premiums. This could arise, for instance, even if the conditional expectation of the forecast error is a linear function of variables in the complete information set of agents. Equation (19) may always be interpreted as a linear least squares projection; however, testing for the stability of this projection requires assumptions on the error term which make the projection a conditional expectation.

Cumby and Obstfeld (1983) argue that the error terms in equations such as (19) are characterized by the presence of conditional heteroskedasticity. One scenario under which conditional heteroskedasticity might arise is the following. In the theoretical model of Section II, the risk premium depends on the intertemporal marginal rates of substitution of the two currencies. Therefore, changes in the actual variances of monetary growth rates can lead to time variation in the risk premium as well as the presence of conditional heteroskedasticity. Ignoring this potential problem in estimation and hypothesis testing could lead one to conclude that the coefficients were not constant when in fact they actually were. In the tests in Section III, the covariance matrices of the parameters allowed for conditional heteroskedasticity. Here, we will demonstrate that there is strong evidence against conditional homoskedasticity, and we will perform a stability test that does not impose such an assumption.

The presence of conditional heteroskedasticity can be detected by using a test analogous to one used by Cumby and Obstfeld (1983). Under the null hypothesis of conditional homoskedasticity, the conditional variance of the residuals is a constant and consequently, uncorrelated with information in the conditioning set. The test consists of regressing the squared residuals from the estimation in Table 3 on instrumental variables from the information set, and testing whether the coefficients of these variables are significantly different from zero. The instrumental variables we use in the test are the forward premiums and the squared forward premiums as in the following regression:

$$(20)\quad (u^i_{t+1})^2 = a_i + \sum_{j=1}^{5} b_{ij}(f^j_t - e^j_t)/e^j_t + \sum_{j=1}^{5} c_{ij}[(f^i_t - e^j_t)/e^j_t]^2 + \varepsilon^i_{t+1}.$$

The results of the test are presented in Table 4. The chi-square statistics indicate strong evidence against the null hypothesis of conditional homoskedasticity in the case of the French franc, the Swiss franc and the Deutsche mark. Remember that since the test of the latent variable model in Section III is based on a covariance matrix that allows for conditional heteroskedasticity, the finding against conditional homoskedasticity here does not invalidate our previous test; but it does indicate that traditional tests of structural change in coefficients such as the Chow test and the Brown, Durbin and Evans test are not appropriate.

An appropriate test for stability of coefficients in the presence of conditional heteroskedasticity can be derived from the asymptotic covariance matrices of the coefficients estimated over two sample periods. Hansen (1982) and Cumby, Huizinga, and Obstfeld (1983) describe procedures for estimation which do not require the traditional assumptions of strict exogeneity of the regressors and conditional homoskedasticity of the error term. These procedures were followed in the estimation of the constrained and unconstrained models of Section

III and are described briefly in the appendix. Our test is appropriate given their regularity conditions.

In the unconstrained model, the GMM estimator for a sample of size T_i is strongly consistent and asymptotically normally distributed. That is,

$$(21) \qquad \sqrt{T_i}(\hat{\beta}_i-\beta^*) \sim N(0,\Omega_i)$$

where $\hat{\beta}_i$ is the GMM estimate of β^*, which reduces to the OLS estimate in this case, and $\Omega_i = \Sigma_i^{-1}S_i\Sigma_i^{-1}$, where the covariance matrix, Ω_i, is constructed from the following sample moments:

$$\Sigma_i = \frac{1}{T_i}\sum_{t=1}^{T_i} z_t^i z_t^{i\prime}$$

(22) and

$$S_i = \frac{1}{T_i}\sum_{t=1}^{T_i} z_t^i z_t^{i\prime}(u_{t+1}^i)^2$$

where z_t^i is the vector of instruments and u_{t+1}^i is the corresponding error term for the equation. Under the maintained hypothesis of no serial correlation in the error process, and under the null hypothesis that $\beta_1 = \beta_2$, the test statistic

$$(23) \qquad (\hat{\beta}_1-\hat{\beta}_2)'\Omega^{-1}(\hat{\beta}_1-\hat{\beta}_2)$$

has an asymptotic chi-square distribution with m degrees of freedom, where m is the dimension of β^*, $\hat{\beta}_1$ and $\hat{\beta}_2$ are the estimates of β^* over the two subsamples, and $\Omega = (\Omega_1/T_1 + \Omega_2/T_2)$.[17] The results are presented in Table 5.

We performed three sets of tests. The first test examines the HH conjecture that the observations from the transitional years of the flexible exchange rate period from July 1973 when our data series begin until the formal ratification of the Rambouillet agreement in January 1976 should be omitted. The ratification amended the Articles of Agreement of the International Monetary Fund to allow countries to adopt a flexible exchange rate as their de jure system. We performed the tests between the periods July 1973 to January 1976 and February 1976 to December 1980. The latter is the sample period employed by HH. The tests indicate strong evidence against the null hypothesis of constant coefficients for the case of the Japanese yen, the Swiss franc, and the British pound.

The second test examines the hypothesis that the coefficients of the unconstrained model did not differ significantly when the twenty-one additional observations were added to the HH sample. The results from these tests provide some evidence against the null hypothesis for the British pound and the Deutsche mark. It is interesting to note that there is no strong evidence against the null hypothesis for the two currencies for which we obtained the most explanatory power in the constrained model, namely the Japanese yen and the Swiss franc.

The third test compared the estimated coefficients before and after the Carter intervention in October 1979 and the resulting change in Federal Reserve Board operating procedures. The two samples were February 1976 to October 1979 and November 1979 to September 1982. This appeared to be a natural point at which to perform the test given the change in U.S. policy. Somewhat surprisingly, we found evidence against the null hypothesis only in the case of the French franc and the British pound. The yen and the Swiss franc tests again demonstrate no evidence against structural change.

We turn next to the interpretation of these tests. There is some evidence against the hypothesis of no structural change. Given only this evidence, though, one might conclude that the linear model with constant coefficients was a good approximation of the true conditional expectation. However, there is additional strong evidence that the conditional expectations of the forecast errors are nonlinear functions of the forward premiums. In particular, if we also include squared forward premiums as right-hand side variables in equation (19), we find that the coefficients on these additional terms are highly significant. The results of the estimation are presented in Table 6.

Nonlinearity of the conditional expectation could be responsible for the evidence against time-invariant parameters and also for the evidence against conditional homoskedasticity. Some evidence for this interpretation is provided by the fact that when the squared forward premiums are added as additional instruments to the specification in (19), we cannot reject time-invariance of the coefficients. The results of this last test are given in Table 7.[18]

The nonlinearity of the conditional expectation of the forecast error in the forward premiums is inconsistent with the assumed constancy of the betas in the HH model which is the likely reason for its rejection. Since the tests of this section provide strong evidence only against the unbiasedness hypothesis without providing a truly convincing model of the risk premium, we turn in the next section to an examination of speculative trading strategies based on equations like (19).

V. Speculative Profits

In this section, we examine Bilson's (1981) contention that the risk-return trade-off from speculating in forward currency markets is too favorable to be consistent with risk averse behavior.

His strategy is to forecast spot exchange rates with a model analogous to that represented by equation (19). Using the covariance matrix of the error terms in the equations, he forms a portfolio of positions in the forward market to minimize the variance of the portfolio subject to an expected profit constraint. Denoting by θ_t the estimated covariance matrix of the error terms in the equations, the portfolio weights in period t, q_t, are chosen as follows:

$$(24) \qquad \min_{q_t} q_t' \theta_t q_t \quad \text{subject to } q_t' r_t = \pi^*$$

where r_t is the vector of expected forecast errors and π^* is the desired profit. The solution to the problem in (24) is

$$(25) \qquad q_t^* = \theta_t^{-1} r_t (r_t' \theta_t^{-1} r_t)^{-1} \pi^*$$

where the variance of the portfolio is given by

$$(26) \qquad \sigma_t^2 = q_t^{*\prime} \theta_t q_t^*.$$

Note that this model implies a linear portfolio efficient frontier in each period and presumes that the investor cares only about the first two moments of his forward market portfolio, and not about its covariation with other asset returns or his consumption stream, as would be implied by the Lucas model of Section II.

The basis of Bilson's position is an examination of standardized expected profits (SRE) which are defined to be expected profits divided by the standard deviation of the portfolio and standardized actual profits (SRA) which are analogously defined using actual profits. In his research, an equation like (19) was estimated using a basket of nine currencies for the sample period July 1974 to January 1980.[19] The estimated parameters were used to form expected profits which were combined with the estimated covariance matrix to construct portfolios as in (25). The out-of-sample profitability of following this strategy for one year was computed. His estimates yielded an average SRE of 0.929, and an average SRA of 0.857. Applying a two-standard deviation rule, this implies that expected profits are one and the two-sigma band runs from -1.153 to 3.153, which forms the basis for his contention.

The result that average SRE is approximately one is indeed striking and prima facie evidence against efficiency of the market. In order to examine the risk-return trade-off from following this strategy for our sample, we conducted two experiments. These are described next, followed by a discussion of the results.

<u>Experiment 1</u>: The first experiment consists of sequentially estimating and simulating the trading strategy in the following model:

$$(27) \qquad (e^i_{t+1}-f^i_t)/e^i_t = \beta_{i0} + \sum_{j=1}^{5} \beta_{ij}(f^j_t-e^j_t)/e^j_t + u^i_{t+1} \quad i = 1,\ldots,5.$$

We used the first twenty-five observations to compute the first estimate of θ_t and the coefficient vector, β_t. Combining β_t with the values of the next set of forward premiums yielded r_t, the vector of expected values of the five forecast errors, and the first set of portfolio weights. The matrix θ_t, which is the covariance of the residuals in (27), was estimated by the maximum likelihood estimator:

$$(28) \qquad \theta_t = (U'_{t-1}U_{t-1})/(t-1)$$

where U_{t-1} is the (t-1 by 5) matrix of residuals up to time t-1. This procedure of OLS estimation and formation of portfolios was then repeated until the end of the sample by adding an observation at each date.

<u>Experiment 2</u>: The second experiment allows for stochastic parameter variation through time. It assumes that the coefficients follow the first-order stochastic process,

$$(29) \qquad \beta_t = A\beta_{t-1} + \varepsilon_t.$$

The updated coefficients are then given by the Kalman filtering formula:

$$(30) \qquad \beta_t = A\beta_{t-1} + (AP_tx'_t)(x_tP_tx'_t+\theta_t)(y_t-x_tA\beta_{t-1}),$$

where $E(\varepsilon_t) = 0$, and x_t is the vector of right-hand side variables in (27).[20] The covariance matrix of β_t is

$$(31) \qquad P_t = AP_{t-1}A' + Q_t,$$

where Q_t is the covariance matrix of ε_t. In order to run the experiment, A, β_0, P_0, Q_t and θ_t have to be specified. The prior on the coefficients, β_0, was specified to be the OLS estimate of β based on the first twenty-four periods. P_0 was specified in the same way. Since we did not have a prior on the matrix Q_t, we assumed that $Q_t = P_{t-1} - AP_{t-1}A'$ which implies that the covariance of the coefficients is constant over time and equal to P_0.

We measured θ_t as in (28), and the matrix A was specified as follows:

$$(32) \qquad A = \{a_{ij}\} = \begin{matrix} 0 & \text{if } i \neq j \\ .75 & \text{if } i = j. \end{matrix}$$

In both experiments π^* was set equal to 1. This completes the descriptions of the two experiments which were run for 83 nonoverlapping monthly observations, and we turn now to the interpretation of the results.

The results of the first experiment show that over the sample, average SRE was 0.871 and average SRA was 0.211. The values of SRE ranged between 0.255 and 2.516.

It is possible to test if profits at time t are drawn from a normal distribution with mean π^* and variance σ_t^2. Let $\hat{\sigma}_t^2$ denote the estimated portfolio variance at t. Assuming that the distribution of profits is normal, standardized unexpected profit, $(\pi_t-\pi^*)/\hat{\sigma}_t$, has a t-distribution with (t-6) degress of freedom. Hence, by Liapunov's Central Limit Theorem, the statistic

$$(33) \qquad \frac{\sum_{t=t_0}^{T} (\pi_t-\pi^*)/\hat{\sigma}_t}{\sum_{t=t_0}^{T} (t-6)/(t-8)},$$

has an asymptotic standard normal distribution where $t_0 = 26$ and $T = 108$ which are the 83 observations for the experiments.[21] We tested whether π_t was significantly different from one and zero. For the null hypothesis $\pi^* = 0$, the test statistic was 1.883 which corresponds to a marginal level of significance of .060. For the null hypothesis $\pi^* = 1$, the test statistic was -5.889 which corresponds to a marginal level of significance smaller than .001. Both the null hypotheses are rejected by the data, the latter more strongly than the former.

The results of the second experiment show that over the sample, average SRE was 0.660 and average SRA was 0.620. The values of SRE ranged between 0.061 and 4.573. Once again, we tested whether π_t was significantly different from one and zero. For the null hypothesis $\pi^* =$ 0, the test statistic was 5.536 which corresponds to a marginal level of significance smaller than .001. For the null hypothesis $\pi^* = 1$, the test statistic was -0.358 which corresponds to a marginal level of significance of 0.72. In this case, we cannot reject the hypothesis that $\pi^* = 1$, while the hypothesis that $\pi^* = 0$ is rejected.

In experiment 2, since we cannot reject $\pi^* = 1$, it makes sense to examine the implied risk-return trade-off as measured by the mean and standard deviation of profits. Applying the two standard deviation rule, expected profits are one, and the two-sigma band runs from -2.030 to 4.030. This is a less favorable risk-return trade-off than that found by Bilson. Once again, this trade-off is based on the average SRE. When SRE was equal to 0.061, the implied two-sigma band was -31.787 to 33.787, and when SRE was equal to 4.573, the band ran from 0.563 to 1.437. The latter trade-off is extremely favorable, while the former is highly unfavorable. It is not obvious how seriously one should take these extreme values since they depend on the estimated values of the parameters. Nevertheless, it would appear from the volatility of the risk-return trade-offs at different points in time that a speculator in foreign exchange must be willing to bear a considerable amount of risk, even if risk is measured in the way described above, ignoring consumption risk, etc.

VI. Conclusions

The analysis conducted in the paper was motivated by an attempt to explain the now common rejection of the unbiasedness hypothesis. As was discussed in the introduction, various explanations have been offered for this finding. One explanation is based on the existence of a risk premium, and the analysis in this paper addressed the problem from this perspective.

There are strong empirical and theoretical reasons for believing a priori in the existence of a risk premium. For instance, Ibbotson and Sinquefield (1976) have documented the existence of large differences in the average holding period returns on a variety of assets. Most financial economists view these differences as reflecting risk premiums, and one would therefore expect to find a risk premium in the forward foreign exchange market especially given the modern approach to exchange rate determination, which argues that foreign exchange rates are determined in asset markets. In intertemporal asset pricing theory, the covariation between intertemporal marginal rates of substitution on monies and the nominal returns on assets induces a risk premium on an asset. In the Lucas model of Section II, the risk premium on a forward contract depends on the same covariation, since forward contracts are risky nominal assets.

Hansen and Hodrick (1983) point out the difficulties of testing the equilibrium model of Section II. As in that paper, we have not attempted to measure the intertemporal marginal rates of substitutions of currencies directly, nor have we attempted to specify an explicit equilibrium econometric study. Our goals have been more modest, yet we believe that the results presented here provide some insights into the workings of forward exchange markets.

We found that one reason for the rejection of the Hansen-Hodrick model is the assumed constancy of the betas which is inconsistent with the observed nonlinearity of the conditional expectations of the forecast errors in the forward premiums. We observed that this could also be responsible for the presence of heteroskedasticity reported by Cumby and Obstfeld (1983).

In the introduction, we noted that since much of our work is of necessity based on asymptotic distribution theory, proponents of the unbiasedness hypothesis will probably remain skeptical about the rejection of the unbiasedness hypothesis which appears throughout this paper. Such a position is tenable, but as sample sizes have grown, the numerous rejections of the hypothesis which are now commonplace form a substantial body of evidence which is increasingly difficult to ignore.

With regard to the second position within the profession, which argues that the rejection of the unbiasedness hypothesis ought to be related to the outstanding stocks of government bonds, we note that the discussion of the Lucas model in Section II was independent of the existence of such assets. Nominal government bonds may be important determinants of the purchasing power of a currency, in which case we would expect them to have a role in the determination of a risk premium. However, the lack of significant explanatory power of such assets in an equation like (19) does not constitute evidence against the existence of a risk premium.

The last section of our paper investigates the claim that particular trading strategies in the forward foreign exchange market yield a risk-return trade-off which is too favorable to be accounted for by risk aversion. Upon conducting experiments based on the trading strategy of Bilson (1981), we found that the strategy was profitable, but it also required willingness on the part of the speculator to absorb a substantial variance of profits. The experiments were run over an eight-year period and produced statistically significant out-of-sample profits. This profitability is consistent either with the existence of a risk premium or with market inefficiency. In any case, it provides further evidence against the unbiasedness hypothesis. The volatility of actual profits and the magnitude of average standardized profit suggest to us that a risk premium is the likely explanation.

Appendix

In this appendix, we describe the estimation of the parameters and their asymptotic covariance matrices for the two models discussed in the text. Estimation in both models is an example of the procedure referred to by Hansen (1982) as the Generalized Method of Moments (GMM). The estimation procedure is also described in Hansen and Singleton (1982) and in Hansen and Hodrick (1983).

The HH model is a system of five equations,

$$\text{(A1)} \qquad y_{t+1} = \beta^{*}\alpha_0^{*} + \beta^{*}\alpha_1^{*\prime}z_t + \nu_{t+1}$$

in ten parameters, $\delta' = (\beta^{*\prime}, \alpha_0^{*}, \alpha_1^{*\prime})$. We assume that the stochastic process z_t is stationary and ergodic. The orthogonality conditions are

$$\text{(A2)} \qquad E_t(z_t \quad \nu_{t+1}) = 0,$$

which is a thirty element vector formed from the unobservable error term. Estimation proceeds by defining two functions of the observable data and the parameters to be estimated:

$$\text{(A3)} \qquad \begin{aligned} f(y_{t+1}, z_t, \delta) &\equiv (y_{t+1} - \beta^{*}\alpha_0^{*} - \beta^{*}\alpha_1^{*\prime}z_t) \quad z_t \\ &\equiv h(y_{t+1}, z_t, \delta) \quad z_t \end{aligned}$$

and by forming the moment estimator of the function, $f(y_{t+1}, z_t, \delta)$, for a sample of size T:

$$\text{(A4)} \qquad g_T(\delta) = \frac{1}{T}\sum_{t=1}^{T} f(y_{t+1}, z_t, \delta).$$

For large values of T, $g_T(\delta)$ ought to be "close" to zero if the model is true. Estimation of the parameters requires the choice of a weighting matrix W_T, and the parameters are chosen to minimize the criterion function,

$$\text{(A5)} \qquad J_T(\delta) = g_T(\delta)' W_T g_T(\delta).$$

Hansen (1982) describes the optimal choice of W_T. It is optimal in the sense of minimizing the asymptotic covariance matrix of the parameters for the class of estimators that exploit the same orthogonality conditions.

The covariance matrix of the parameters is

$$\text{(A6)} \qquad \Omega(\delta) = (D_T' W_T D_T)^{-1}$$

where

$$\text{(A7)} \qquad D_T = \frac{1}{T}\sum_{t=1}^{T} \frac{\partial h}{\partial \delta}(y_{t+1}, z_t, \delta) \quad z_t$$

and

$$\text{(A8)} \qquad W_T = \{\frac{1}{T}\sum_{t=1}^{T} f(y_{t+1}, z_t, \delta) f(y_{t+1}, z_t, \delta)'\}^{-1}$$

in this case since we assume ν_{t+1} to be mean zero and serially uncorrelated. We followed the suggestion in Hansen and Singleton (1982) of removing the sample means $g_T(\delta)g_T(\delta)'$ from the cross-products $f(y_{t+1}, z_t, \delta)f(y_{t+1}, z_t, \delta)'$ in computing W_T. They note that this adjustment has no effect on the asymptotic properties of the estimates or the test statistics, yet under alternative hypotheses $g_T(\delta)$ may not be zero. The adjustment improves the power of the test.

Hansen demonstrates that T times the value of the criterion function at its minimum is asymptotically chi-square distributed with degrees of freedom equal to the number of orthogonality conditions minus the number of estimated parameters. This is the test statistic for the model.

The estimation of the unconstrained model is a GMM procedure which reduces to equation by equation ordinary least squares. As in the above discussion, the derivation of the covariance matrix does not impose conditional homoskedasticity. The covariance matrix of the parameters for a particular equation has the same form as (A6), but D_T reduces to $(1/T)Z'Z$ where Z is the (Txk) matrix of instruments, and W_T reduces to

$$\text{(A9)} \qquad W_t = \frac{1}{T} \sum_{t=1}^{T} z_t z_t' u_{t+1}^2 .$$

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Footnotes

* We thank Lars Peter Hansen and Katherine Schipper for useful discussions and Ken Singleton for providing us with his computer program.

1. A large literature now exists on this topic. Major empirical contributions to the area have been made by Dooley and Shafer (1976, 1982), Frenkel (1977, 1981), Stockman (1978), Levich (1978, 1979a), Geweke and Feige (1979), Frankel (1980, 1983), Hansen and Hodrick (1980, 1983), Bilson (1981), Cumby and Obstfeld (1981, 1983), Hakkio (1981a, 1981b), Longworth (1981), and Hsieh (1982).
2. McKinnon (1979, p. 156) has argued "that the supply of private capital for taking net positions in either the forward or spot markets is currently inadequate. Exchange rates can move sharply in response to random variations in the day-to-day demands by merchants or from monetary disturbances. Once a rate starts to move because of some temporary perturbation, no prospective speculator is willing to hold an open position for a significant time interval in order to bet on a reversal--whence the large daily and monthly movements in the foreign exchanges and sometimes high bid-ask spreads. Bandwagon psychologies result from the general unwillingness of participants to take net positions against near-term market movements that are necessarily accentuated by the behavior of nonspeculative merchants."
3. See Levich (1979b) for an analysis of commercial forecasting services.

4. Michael Mussa made this criticism at the NBER Conference on Exchange Rates and International Macroeconomics held in Cambridge, Massachusetts in November 1981. Krasker (1980) argued that the existence of a particular event such as a discrete devaluation could bias the sampling distribution of the test statistics such that they are poorly approximated by their asymptotic distribution under the null hypothesis. This particular problem is not unique to studies of the foreign exchange market because it plagues much of modern time series analysis.

5. This point is generally acknowledged by those who reject the unbiasedness hypothesis. The theoretical models of Grauer, Litzenberger, and Stehle (1976), Kouri (1977), Stockman (1978) Fama and Farber (1979), Frankel (1979), and Roll and Solnik (1979) provided reasons for the existence of a risk premium without necessarily demonstrating how or why it would vary through time.

6. See Breeden (1979) and Grossman and Shiller (1983) for a discussion of the conversion of these models into "consumption beta" models. Stulz (1981) generalized the Breeden approach to consider pricing of international assets. Hansen and Singleton (1983) conduct econometric analysis of the intertemporal models using aggregate consumption data.

7. A discussion of the determination of the forward foreign exchange rate and the risk premium that separates it from the expected future spot rate was included in early drafts but excluded from the published version of Lucas (1982).

8. An alternative representation of the right-hand side of (11) is obtained by taking the conditional expectation of the second order Taylor series expansion of (Q^N_{t+1}/Q^M_{t+1}) around $E_t(Q^N_{t+1})$ and $E_t(Q^M_{t+1})$. The resulting expression is $[1/E_t(Q^M_{t+1})]^2\{[E_t(Q^N_{t+1}/E_t(Q^M_{t+1})]V_t(Q^M_{t+1}) - C_t(Q^M_{t+1};\ Q^N_{t+1})\}$ where $V_t(\cdot)$ and $C_t(\cdot;\cdot)$ are the conditional variance and the conditional covariance, respectively.

9. As demonstrated by Roll (1977) and extended to conditional environments by Hansen, Richard, and Singleton (1982), the content of the restriction embodied in (12) is that the benchmark return is on the conditional mean-variance frontier. The static capital asset pricing model is often given empirical content through the assumption by the econometrician that measurements on an aggregate wealth portfolio are mean-variance efficient. As in HH, no such assumption is made here.

10. Equation (14) follows immediately once one recognizes that all nominal dollar denominated returns satisfy $E_t(R_{t+1}Q^M_{t+1}) = 1$.

11. Singleton (1983) is relatively optimistic about the ability of econometricians to estimate directly form equations such as (11) by using observations on aggregate consumption series and price indexes. We are suspect of what one may gain from such an approach given the severe measurement error problems that are encountered in using macro time series although see Hansen and Singleton (1983).

12. Cumby and Obstfeld (1983) argue that forward rate forecast errors are characterized by conditional heteroskedasticity. Hence, we do not assume homoskedastic disturbances. This issue is investigated in Section 4.

13. Hakkio (1983) argues correctly that the forward rates are not matched precisely with the appropriate value date one month in the future. Riehl and Rodriguez (1977) discuss the rules which regulate the determination of the exact delivery day when the contract is to be executed. Hsieh (1982) and Cumby and Obstfeld (1983) match the data precisely taking account of holidays, etc., with no difference in inference regarding evidence against the unbiasedness hypothesis.
14. See the Appendix for a discussion of the estimation procedure.
15. Using the forecast errors as instruments, sampling the HH data to form a data set of 57 observations, and reestimating the HH model gives a $\chi^2(20) = 18.459$. Hence, the sampling procedure does not appear to have reduced the power of the test significantly.
16. Note that we allow for some forms of conditional heteroskedasticity in the construction of the covariance matrix of the parameters. See the Appendix for details. Cumby and Obstfeld (1983) and Hsieh (1982) argue for this approach, which was proposed by White (1980).
17. Note that this is an asymptotic test, and in theory, requires two infinitely large, disjoint samples.
18. Note that estimation of the HH model with eleven instruments and five currencies would impose fifty-five orthogonality conditions in the estimation. At this point, this is computationally impractical which is why we did not attempt to reestimate the HH model with this specification.

19. Bilson's specification is

$$(e^i_{t+1}-f^i_t)/e^i_t = \alpha_1[(f^i_t-e^i_t)/e^i_t]^S + \alpha_2[(f^i_t-e^i_t)/e^i_t]^L + u^i_{t+1}$$

where superscript S and L refer to "small" and "large." Small forward premia are those less than 10 percent in absolute value. He uses the five currencies of this study plus the Canadian dollar, the Belgian franc, the Italian lira, and the Dutch guilder. The estimation imposes the constraint that the coefficients are identical across currencies.

20. See Schweppe (1973).

21. See Dhrymes (1974).

Table 1: Hansen-Hodrick Latent Variable Model

$$y_{t+1} = \beta^* \alpha_0^* + \beta^* \alpha_1^{*\prime} z_t + \nu_{t+1}$$

Sample Period: February 1976 to December 1980; Number of Observations: 57

Currency	$\hat{\beta}_i$ (Std.Err.)	Reduced Form Coefficients $\hat{\theta}_{i0}$ (Std.Err.) Confidence	$\hat{\theta}_{i1}$ (Std.Err.) Confidence	$\hat{\theta}_{i2}$ (Std.Err.) Confidence	$\hat{\theta}_{i3}$ (Std.Err.) Confidence	$\hat{\theta}_{i4}$ (Std.Err.) Confidence	$\hat{\theta}_{i5}$ (Std.Err.) Confidence	$\chi^2(5)$ All $\hat{\theta}_{ij}$'s $j \geq 1 = 0$ Confidence	R^2
1. French franc	1.0	4.916 (3.754) 0.810	-0.809 (0.493) 0.899	-0.022 (0.429) 0.040	-1.441 (1.012) 0.845	0.837 (0.502) 0.905	2.244 (1.425) 0.885	4.529 0.524	0.053
2. Japanese yen	1.592 (0.711)	7.828 (6.872) 0.745	-1.289 (0.638) 0.957	-0.035 (0.679) 0.041	-2.294 (1.797) 0.798	1.333 (0.719) 0.936	3.574 (2.241) 0.889	5.270 0.616	0.047
3. Swiss franc	3.152 (0.664)	15.498 (9.957) 0.880	-2.551 (1.070) 0.983	-0.068 (1.352) 0.040	-4.541 (2.519) 0.929	2.639 (1.060) 0.987	7.076 (3.075) 0.978	27.503 0.999	0.154
4. U.K. pound	1.683 (0.664)	8.275 (5.184) 0.889	-1.362 (0.835) 0.897	-0.037 (0.727) 0.040	-2.425 (1.377) 0.922	1.409 (0.804) 0.921	3.778 (2.035) 0.937	8.296 0.859	0.000
5. Deutsche mark	0.721 (0.424)	3.544 (3.993) 0.625	-0.583 (0.579) 0.686	-0.016 (0.310) 0.040	-1.038 (1.074) 0.666	0.604 (0.623) 0.667	1.618 (1.611) 0.685	1.253 0.060	0.030

Test of the Constrained Model: $\chi^2(20) = 24.239$; Confidence = 0.768.

Note: The dependent variables are the forward rate forecast errors $y^i_{t+1} = (e^i_{t+1} - f^i_t)/e^i_t$, U.S. dollars for the French franc, the Japanese yen, the Swiss Franc, the U.K. pound, and the Deutsche mark. The instrumental variables are the forward premia for the same five currencies. Confidence is 1 minus the marginal level of significance. Values of the confidence term close to 1 indicate evidence against the null hypothesis that one or a set of coefficients equals zero. All forecast errors and forward premia are expressed in percent at an annual rate.

Table 2: Hansen-Hodrick Latent Variable Model

$$y_{t+1} = \beta^* \alpha_0^* + \beta^* \alpha_1^{*\prime} z_t + \nu_{t+1}$$

Sample Period: February 1976 to September 1982; Number of Observations: 78

Currency	β_i (Std.Err.)	Reduced Form Parameters: $\hat{\theta}_{i0}$ (Std.Err.) Confidence	$\hat{\theta}_{i1}$ (Std.Err.) Confidence	$\hat{\theta}_{i2}$ (Std.Err.) Confidence	$\hat{\theta}_{i3}$ (Std.Err.) Confidence	$\hat{\theta}_{i4}$ (Std.Err.) Confidence	$\hat{\theta}_{i5}$ (Std.Err.) Confidence	$\chi^2(5)$ All $\hat{\theta}_{ij}$'s $j \geq 1 = 0$ Confidence	R^2
1. French franc	1.0	13.198 (6.164) 0.968	-0.080 (0.206) 0.302	-1.244 (0.741) 0.907	-4.467 (1.945) 0.978	0.450 (0.469) 0.663	6.080 (2.719) 0.975	5.972 0.691	0.034
2. Japanese yen	2.059 (0.800)	27.181 (9.052) 0.997	-0.164 (0.419) 0.304	-2.562 (1.113) 0.979	-9.199 (2.614) 0.999	0.927 (0.870) 0.713	12.521 (3.567) 0.999	19.998 0.999	0.153
3. Swiss franc	2.607 (0.698)	34.411 (11.473) 0.997	-0.208 (0.524) 0.308	-3.243 (1.409) 0.979	-11.646 (3.066) 0.999	1.173 (1.114) 0.707	15.852 (4.138) 0.999	22.910 0.999	0.179
4. U.K. pound	1.227 (0.373)	16.188 (7.753) 0.963	-0.098 (0.245) 0.310	-1.526 (0.860) 0.924	-5.478 (2.205) 0.987	0.552 (0.579) 0.659	7.457 (2.954) 0.988	7.258 0.798	0.026
5. Deutsche mark	1.184 (0.222)	15.622 (7.582) 0.961	-0.094 (0.243) 0.302	-1.472 (0.868) 0.910	-5.287 (2.268) 0.980	0.533 (0.562) 0.657	7.196 (3.130) 0.979	6.222 0.715	0.024

Test of the Constrained Model: $\chi^2(20) = 34.497$; Confidence = 0.977

Note: See Table 1.

Table 3: $(e^i_{t+1} - f^i_t)/e^i_t = a_i + \sum_{j=1}^{5} b_{ij}(f^j_t - e^j_t)/e^j_t + u^i_{t+1}$

Sample Period: February 1976 to September 1982; Number of Observations: 78

Currency	$\hat{a}_i$ (Std. Err.) Confidence	$\hat{b}_{i1}$ (Std. Err.) Confidence	$\hat{b}_{i2}$ (Std. Err.) Confidence	$\hat{b}_{i3}$ (Std. Err.) Confidence	$\hat{b}_{i4}$ (Std. Err.) Confidence	$\hat{b}_{i5}$ (Std. Err.) Confidence	$\chi^2(6)$ All Coeffs. = 0 Confidence	$\chi^2(5)$ All $\hat{b}_{ij}$'s $j \geq 1 = 0$ Confidence	R^2	Resid. Var.
1. French franc	-8.436 (19.880) 0.329	-1.115 (2.085) 0.407	-1.535 (1.716) 0.629	-4.993 (3.053) 0.898	-1.482 (1.057) 0.839	10.230 (4.470) 0.978	8.391 0.789	8.329 0.861	0.079	1284.68
2. Japanese yen	29.008 (13.238) 0.972	-0.195 (0.956) 0.161	-6.131 (1.588) 0.999	-6.953 (3.184) 0.971	1.319 (1.294) 0.692	12.363 (4.437) 0.995	36.199 0.999	36.155 0.999	0.222	1325.61
3. Swiss franc	27.568 (17.401) 0.886	-0.971 (1.489) 0.485	-2.659 (2.253) 0.762	-13.024 (3.676) 0.999	0.238 (1.438) 0.131	18.146 (5.775) 0.998	19.365 0.996	18.783 0.998	0.188	1755.74
4. U.K. pound	-2.302 (11.391) 0.160	0.713 (0.584) 0.778	-1.692 (1.420) 0.767	-4.933 (2.647) 0.938	-2.793 (1.243) 0.975	10.161 (3.458) 0.997	25.833 0.999	25.715 0.999	0.163	1106.15
5. Deutsche mark	4.857 (18.013) 0.213	-1.638 (1.702) 0.664	-2.474 (2.141) 0.752	-2.554 (3.844) 0.494	0.043 (1.470) 0.023	5.402 (6.113) 0.623	10.006 0.876	7.928 0.840	0.080	1630.13

Note: See Table 1.

Table 4

Test for Conditional Homoskedasticity: Eqs(20)		
Sample: February 1976 to September 1982; Number of Observations: 78		
Currency	Test Statistic $\chi^2(10) b_{ij} = c_{ij} = 0 \;\forall\; j$	Confidence
1. French franc	32.559	0.999
2. Japanese yen	14.485	0.848
3. Swiss franc	96.655	0.999
4. U.K. pound	10.557	0.607
5. Deutsche mark	26.038	0.996

Table 5

Tests for Constant Coefficients		
Currency	Test Statistic	Confidence
July 1973 to January 1976 and February 1976 to December 1980		
1. French franc	3.725	0.286
2. Japanese yen	32.633	0.999
3. Swiss franc	14.175	0.972
4. U.K. pound	22.877	0.999
5. Deutsche mark	9.220	0.838
February 1976 to December 1980 and January 1981 to September 1982		
1. French franc	7.101	0.688
2. Japanese yen	4.900	0.443
3. Swiss franc	5.501	0.518
4. U.K. pound	10.996	0.911
5. Deutsche mark	10.942	0.909
February 1976 to October 1979 and November 1979 to September 1982		
1. French franc	17.246	0.991
2. Japanese yen	4.423	0.380
3. Swiss franc	6.859	0.665
4. U.K. pound	10.670	0.900
5. Deutsche mark	6.448	0.625

Table 6

$$(e^i_{t+1} - f^i_t)/e^i_t = a_i + \sum_{j=1}^{5} b_{ij}(f^j_t - e^j_t)/e^j_t + \sum_{j=1}^{5} c_{ij}[(f^j_t - e^j_t)/e^j_t]^2 + u^i_{t+1}$$

Sample: February 1976 to September 1982; Number of Observations: 78

Currency	$\chi^2(5)$ $b_{ij}=0$ $j=1,5$ Confidence	$\chi^2(5)$ $c_{ij}=0$ $j=1,5$ Confidence	$\chi^2(10)$ $b_{ij}=c_{ij}=0$ $j=1,5$ Confidence
1. French franc	19.171 0.998	21.741 0.999	26.951 0.997
2. Japanese yen	9.029 0.892	9.753 0.917	44.527 0.999
3. Swiss franc	22.782 0.999	23.163 0.999	59.059 0.999
4. U.K. pound	8.578 0.873	10.900 0.947	68.693 0.999
5. Deutsche mark	13.119 0.978	13.292 0.979	23.633 0.991

Table 7

$$(e^i_{t+1}-f^i_t)/e^i_t = a_i + \sum_{j=1}^{5} b_{ij}(f^j_t-e^j_t)/e^j_t + \sum_{j=1}^{5} c_{ij}[(f^j_t-e^j_t)/e^j_t]^2 + u^i_{t+1}$$

Period: February 1976 to October 1979
and November 1979 to September 1982

Currency	Test Statistic $\chi^2(11)$	Confidence
1. French franc	12.807	0.694
2. Japanese yen	8.839	0.363
3. Swiss franc	13.018	0.708
4. U.K. pound	14.220	0.779
5. Deutsche mark	13.392	0.732