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MONETARY INSTRUMENTS AND POLICY RULES IN A RATIONAL EXPECTATIONS ENVIRONMENT

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ABSTRACT

This paper explores the implications of rational expectations and the aggregate supply theory advanced by Lucas (1973) for analysis of optimal monetary policy under uncertainty along the lines of Poole (1970), returning to a topic initially treated by Sargent and Wallace (1975). Not surprisingly, these two "classical" concepts alter both the menu of feasible policy choice and the desirability of certain policy actions. In our setup, unlike that of Sargent and Wallace (1975), the systematic component of monetary policy is a relevant determinant of the magnitude of "business fluctuations" that arise from shocks to the system. Central bank behavior--both the selection of monetary instruments and the framing of overall policy response to economic conditions--can work to diminish or increase the magnitude of business fluctuations. However, the "activist" policies stressed by the present discussion bear little (if any) relationship to the policy options rationalized by the conventional analysis of monetary policy under uncertainty. In particular, in contrast to Poole's analysis, money supply responses to the nominal interest rate are not important determinants of real economic activity. Rather, the central bank should focus on policies that make movements in the general price level readily identifiable by economic agents.

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I. Introduction[1]

How should the central bank design its monetary policies, so as to minimize the magnitude of business fluctuations? The modern answer to this question builds on Poole's (1970) analysis of the implications of three alternative instrument selections for output variability, within a simple stochastic Keynesian model: (i) an interest rate peg at a predetermined level, in which the monetary authority allows the money supply to adjust so as to accommodate the quantity of money demanded; (ii) a simple money supply rule, in which money follows a predetermined path; and (iii) a "combination policy," which permits the money supply to respond to interest rate departures from a predetermined "target" level. Within this setup, the contemporaneous information about economic activity contained in the nominal interest rate can best be exploited by a combination policy, with the optimal magnitude of money supply response to nominal interest rates depending on the relative variability of real ("IS") and nominal ("LM") shocks.[2]

The present discussion augments Poole's model by introducing an aggregate commodity supply schedule that depends on relative prices as perceived by agents at a specific point in time. If expectations about the current and future price level are adaptive, then Poole's results obtain. This is no longer the case, for two basic reasons, if expectations are rational and are based on contemporaneous information such as prices and the nominal interest rate.

First, agents optimally exploit the information contained in the nominal interest rate. Second, changes in the money supply have real effects only to the extent that the resulting price level movements are perceived as changes in real opportunities. Consequently, all combination policies imply the same distribution of real activity as a simple money stock rule, since agents can identify the money supply response to interest rate movements. Further, since private agents find the interest rate to be a useful indicator of aggregate conditions, a policy of pegging the interest rate is undesirable. That is, if the interest rate is removed as a source of information, the extent of economic fluctuations is increased (although aggregate output is insulated from money demand disturbances).

Under the rational expectations assumption, we also examine two types of monetary policies that may improve on a simple monetary rule. First, the nominal interest rate may be made into a better indicator of movements in the general price level--aiding suppliers in their efforts to sort out relative and general price movements--by an announced policy of regularly responding to changes in economic conditions, along the lines described previously by King (1982) and Weiss (1980). Second, structural reforms of the money supply process may alter the covariance between money and the aggregate shocks that give rise to business fluctuations. Some analogous considerations have been discussed by McCallum and Whitaker (1979), in their analysis of "automatic stabilizers" in a rational expectations environment.

The organization of the remainder of the paper is as follows: The macroeconomic model and alternate monetary policy rules are described in Section II, with Poole's results replicated for an adaptive expectations version of the model in Section III. After outlining the rational expectations solution of the model (Section IV), the implications of alternative policy rules are discussed in Section V. In Section VI, we review related literature and provide some concluding comments.

II. The Macroeconomic Model

This section describes the economy that will be extensively analyzed under alternative expectational schemes in Sections III and V below.

The Private Economy

Two basic components of the linear macroeconomic model are a conventional aggregate demand or "IS" function (1) and a conventional money demand function (2).

$$(1) \qquad y_t^d = k^d - ar_t + u_t$$

$$(2) \qquad M_t^d = P_t + dy_t - cR_t + v_t$$

Equation (1) indicates that the logarithm of aggregate demand depends negatively on the real interest rate (r_t) and positively on a shock u_t, which is independently normally distributed with mean zero and variance σ_u^2. Equation (2) states that the logarithm of nominal money demand is positively related to the log of the price level (P_t), the log of output (y_t), and a stochastic shock as well as being negatively related to the nominal interest rate (R_t). The money demand shock is also independently normally distributed with mean zero and variance σ_v^2. With a fixed price level and the corresponding Keynesian assumption that production is demand-determined as well as the requirement of monetary equilibrium ($M_t^s = M_t^d$), equations (1) and (2) are Poole's model.

Lucas's formulation (1973, pp. 327-328) of the aggregate supply schedule derives from initially considering one of a large number of localities or industries, which are indexed by $z = 1, 2, \ldots Z$. In that location, supply depends positively on the perceived relative price of the product, written as the difference between the log of the local price $P_t(z)$ and the supplier's estimate of the log of the general price level, $E_z P_t$.

$$(3a) \qquad y_t^s(z) = k^s + b(P_t(z) - E_z P_t)$$

The economy-wide average or aggregate[3] supply ($y_t^s \equiv Z^{-1}\sum_z y^s(z)$) depends on the difference between the (log of the) price level ($P_t \equiv Z^{-1}\sum_z P_t(z)$) and the economy-wide average expectation about this variable ($\overline{E_z P_t} \equiv Z^{-1}\sum_z E_z P_t$).[4]

$$\text{(3b)} \qquad y_t^s = k^s + b(P_t - \overline{E_z P_t})$$

A key aspect of this supply schedule is that movements in the general level of prices alter output only to the extent that a representative supplier does not perceive them, i.e., if $\overline{E_z P_t}$ does not adjust one to one with P_t.

Taken together with a definition of the real rate of interest as $r_t = P_t + R_t - \overline{E_z P_{t+1}}$ and equilibrium conditions that specify $M_t^s = M_t^d$ and $y_t^d = y_t^s$, equations (1), (2) and (3b) represent a fairly standard macroeconomic model, closely related to that analyzed by Sargent-Wallace (1975). The model can be solved for P_t, y_t and R_t as functions of the money stock, the shocks u_t and v_t and the expectational terms $\overline{E_z P_t}$ and $\overline{E_z P_{t+1}}$, with comparative statics results that are wholly conventional.

However, to study the model's behavior under rational expectations, it is necessary to specify further elements of the "micro structure" of the model. Specifically, demand in a particular location z differs from the economy-wide average or aggregate value (see footnote 3) by a random shock,

$$\text{(4)} \qquad y_t^d(z) = y_t^d + \varepsilon_t(z),$$

where $\varepsilon_t(z)$ is an independent normal variate with mean zero and variance σ_ε^2 that is the same for all locations.[5] Finally, the number of markets is large enough so that average value of the $\varepsilon_t(z)$ shocks is zero in each period.

Monetary Policy

In our analysis of monetary policy under uncertainty, we consider five alternative specifications (see Table 1 below). A common feature of these specifications is a fixed long-run money growth path ($\tilde{M}_t$ with $\tilde{M}_t - \tilde{M}_{t-1} = n$), so that the policies differ solely in the conditions under which temporary movements in the money stock occur. The policies are as follows:

The Simple Money Stock Rule specifies that no responses to economic conditions occur. However, departures of the money supply (M_t^s) from the long-run path may arise as a result of control errors, which also arise under other specifications considered below. Such money shocks (x_t) are assumed independently normally distributed with mean zero and variance σ_x^2--since all money stock variables are in logarithms, these errors correspond to percentage deviations.

The Interest Rate Peg implies that the money stock is demand-determined during the period. With this specification, we seek to depict a policy regime in which the nominal interest rate is an "instrument" for control of an intermediate target such as the money stock or a target such as output. That is, the central bank controls the interest rate over a short period (a single unit of our discrete time setup) but, in principle, the level of the "peg" might be adjusted from period to period.[6]

The Combination Policy is the third policy option considered by Poole (1970). This policy involves money supply response to departures from an interest rate target, i.e., the interest rate is an information variable which alerts the monetary authority to changes in the conditions of the private economy.[7]

A Feedback Policy involves reaction to past real or nominal aggregates or, more basically, to the shocks that underly these aggregates. The simple form considered here involves response to only date t-1 shocks.

Structural Reforms of monetary institutions could alter the pattern of the money stock's response to the underlying shocks to the system. Due to the aggregate character of our model, we do not discuss the nature of the institutional changes involved but simply specify that a component of money (h_t) responds to aggregate shocks.[8]

The monetary authority's criterion is based on the gap between actual and full information output ($y_t(z) - y_t^*(z)$), where full information output ($y_t^*(z)$) is the level of output that would occur if agents know all current prices and the values of all contemporaneous shocks. Barro (1976) has argued persuasively that monetary policy should employ such a criterion, as opposed to an aggregate output gap measure such as $y_t - y_t^*$, so as to capture the impacts of aggregate policies on resource allocation. The policy goal in our model is to minimize the variance of local output about its full information value, denoted $V(y_t(z) - y_t^*(z))$.[9]

Table I: Alternative Monetary Policy Rules

Money Stock Rule*	$M^s = \tilde{M}_t + x_t$
Interest Rate Peg	$M_t^s = M_t^d = P_t + dy_t - cR_t^{peg} + v_t$
Combination Policy**	$M_t^s = \tilde{M}_t + g_R(R_t - R_t^T) + x_t$
Feedback Policy	$M_t^s = \tilde{M}_t + f_{t-1} + x_t;\quad f_{t-1} = f_x x_{t-1} + f_v v_{t-1} + f_u u_{t-1}$
Structural Reform	$M_t^s = \tilde{M}_t + h_t + x_t;\quad h_t = h_x x_t + h_v v_t + h_u u_t$

*$\tilde{M}_t$ is a long-run growth path of money

**R_t^T is a target interest rate, taken to be ${}_{t-1}E R_t$ below.

III. Optimal Policy Under Adaptive Expectations

In this section, the optimal policy under uncertainty is analyzed given the assumptions that (i) the monetary authority knows the structure of the model, but not the values of current shocks, and (ii) the private sector forms its expectations according to a simple adaptive mechanism. The analysis essentially replicates Poole's conclusions about the implications of money supply control policies, pegs, and combination policies. That is, for the money supply rule (5), which incorporates feedback (f_{t-1}) and contemporaneous response to departures from an interest rate target ($g_R(R_t - R_t^T)$), we find that an optimal policy typically involves a non-zero value of g_R--using the new information in the interest rate--and adjustment of either f_{t-1} or R_t^T to past information about the state of the economy.

(5) $\quad M_t^s = \tilde{M}_t + f_{t-1} + g_R(R_t - R_t^T) + x_t$

For the remainder of this section, we consider only policy specification (5).

The specific form of the adaptive mechanism employed is $E_z P_t = \lambda P_t(z) + (1-\lambda)(P_{t-1} + n)$.[10] This mechanism implies that agents are not fooled by sustained inflation at the rate n, but do not fully recognize irregular movements in the general price level as they are taking place. That is, $\overline{E_z P_t} = \lambda P_t + (1-\lambda)(P_{t-1} + n)$ so that $P_t - \overline{E_z P_t} = (1-\lambda)(P_t - P_{t-1} - n)$. Analogously, it is assumed that agents believe that the future price level will be the current price level plus the normal rate of inflation, i.e., $\overline{E_z P_{t+1}} = \overline{E_z P_t} + n$.

The solution for the price level, nominal interest rate and real output--given the specified adaptive mechanisms--is obtained by requiring that portfolio balance and commodity market equilibrium occur and solving for the values of these endogenous variables. (Appendix A reports the results of this procedure.) In this model, variations in aggregate and local output stem, respectively, from movements in $P_t - \overline{E_z P_t}$ and $P_t(z) - E_z P_t$, so that the discussion focuses on these two terms. The price level solution implies that the gap between the price level and its economy-wide average expected value has the following form:

$$
\begin{aligned}
(6) \qquad P_t - \overline{E_z P_t} &= (1-\lambda)(P_t - P_{t-1} - n) \\
&= \frac{(1-\lambda)a}{\Delta}\{f_{t-1} - g_R(R_t^T - \hat{r} - n) \\
&\qquad + \tilde{M}_t - d\hat{y} + c(\hat{r} + n) - (P_{t-1} + n)\} \\
&\quad + \frac{1-\lambda}{\Delta}\{(g_R + c)u_t - a(v_t - x_t)\},
\end{aligned}
$$

where $\Delta = (g_R + c)(a + b)(1-\lambda) + a(1 + db(1-\lambda)) > 0$ and $\hat{y} \equiv k^s$ and $\hat{r} \equiv (k^d - k^s)/a$.[11]

This expression displays two basic aspects of policy choice in the adaptive expectations version of the model. First, if last period's price level differed from the value $(M_t - n + dy + c(\hat{r} + n))$ as a result of shocks, etc., it is desirable to use temporary adjustments of money growth to eliminate this source of output departures from $\hat{y}$. In this case, direct adjustment of money via feedback (f_{t-1}) or adjustments to the interest rate target (R_t^T) are perfect substitutes. Put alternatively, in a manner corresponding to Poole (1970), the "instrument problem" is unimportant under conditions that are equivalent to certainty. Second, the money supply response to interest rate departures from R_t^T is an important determinant of output variability.

Next, we analyze the movement in expected relative prices, $P_t(z) - E_zP_t$, under the conditions that f_{t-1} is set to eliminate the effects of departures in last period's price level from its full information expected value and that $R_t^T = \hat{r} + n$. It is found that

$$(7) \qquad P_t(z) - E_zP_t = (1-\lambda)\ (P_t(z) - P_{t-1} - n)$$
$$= \frac{1}{b}\,\varepsilon_t(z) + \frac{(1-\lambda)}{\Delta}[(g_R + c)u_t - a(v_t - x_t)].$$

The supply schedule, $y_t^s(z) = k^s + b(P_t(z) - E_zP_t)$, then provides local output. The "full information" value of output is that which would prevail if agents know the aggregate shocks affecting price level ($P_t = E_zP_t$), so that $y_t^s(z) = k^s + \varepsilon_t(z)$. Therefore, the variance of local output around its full information value may be shown to be

$$(8) \qquad V(y_t(z) - y_t^*(z)) = b^2(1-\lambda)^2[\frac{(g_R - c)^2}{\Delta^2}\,\sigma_u^2 + (\frac{a}{\Delta})^2(\sigma_v^2 + \sigma_x^2)].$$

The value of g_R which minimizes this variance is

$$(9) \qquad g_R = \frac{a(a+b)(1-\lambda)(\sigma_x^2 + \sigma_v^2)}{(b(1-\lambda)d+1)\sigma_u^2} - c.$$

Thus, the optimal pattern of contemporaneous policy response depends positively on the variance of nominal disturbances (x, v), negatively on the variance of real disturbances (u), and negatively on the interest rate responsiveness of the demand for real cash balances.

In summary, Poole's general conclusions (1970, p. 208) are maintained in the adaptive expectations version of the model. Typically, the monetary authority should choose a strategy of allowing the money supply to respond to nominal interest rate movements, exploiting the contemporaneous information provided by the interest rate. To investigate how the policy choice problem is sensitive to efficient private sector use of information (rational expectations), we next solve the model of Section II under that alternative assumption.

IV. Rational Expectations Solution

Our evaluation of alternative monetary policy rules under rational expectations is based on solutions of the macroeconomic model (see Appendix B), using the "method of undetermined coefficients" previously employed by Lucas and Barro in similar contexts. The basic economic requirements of the model derive from the local condition(s) of commodity equilibrium and from aggregate portfolio balance. Equilibrium in the local commodity market, ($y_t^d(z) = y_t^s(z)$), yields

$$(10) \qquad P_t(z) = E_zP_t + (1/b)(k^d - k^s) - (a/b)(P_t + R_t - \overline{E_zP_{t+1}}) + (1/b)u_t + (1/b)\varepsilon_t(z).$$

This solution for $P_t(z)$ is incomplete since it contains the endogenous variable R_t as well as expectations about the current and future aggregate price level, E_zP_t and $\overline{E_zP_{t+1}}$.

Equating money supply and money demand provides a comparable equation for the nominal interest rate.

$$(11a) \qquad (g_R + c)R_t = P_t + d\hat{y} + db(P_t - \overline{E_zP_t}) - \tilde{M}_t - g_RR_t^T - f_uu_{t-1} - f_xx_{t-1} - f_vv_t - h_uu_t - h_vv_t - h_xx_t + (v_t - x_t)$$

In formulating (11a), a money supply incorporating an interest rate target, one-period feedback, and a structural reform was employed. In the case of an interest rate peg, an analogous equation for money would be obtained.

$$(11b) \qquad M_t = P_t + d\hat{y} + db(P_t - E_zP_t) - cR_t^{peg} + v_t$$

Again, (11a) and (11b) are not complete solutions since they involve the endogenous price level and expectations of the current price level.

Under the method of undetermined coefficients, we hypothesize a linear solution for local prices and the global interest rate (or money supply in the case of a peg) that involves all the predetermined and stochastic variables of the model.

$$(12) \qquad P_t(z) = \pi_0 + \pi_1\tilde{M}_t + \pi_2(v_t - x_t) + \pi_3u_t + \pi_4\varepsilon_t(z) + \pi_5(v_{t-1} - x_{t-1}) + \pi_6u_{t-1}$$

$$(13) \qquad R_t = \psi_0 + \psi_1\tilde{M}_t + \psi_2(v_t - x_t) + \psi_3u_t + \psi_5(v_{t-1} - x_{t-1}) + \psi_6u_{t-1}$$

These hypothesized solutions employ a composite excess money demand shock $(v_t - x_t)$, as is appropriate so long as v and x exert symmetric effects on the system (which requires that $f_x = -f_v$).

Rational expectations implies that terms such as E_zP_t are mathematical expectations, conditional on the information available to agents. Throughout the analysis, agents are assumed to have full information concerning all lagged variables in the model, which is summarized by the information set I^*_{t-1}. In addition, following Lucas (1973), we restrict agents to contemporaneous price information, which in our model is the local price $P_t(z)$ and the nominal interest rate R_t. Specifically, the current shocks (x_t, v_t and so forth) are not observable by either the monetary authority or the private sector. However, at some points in the discussion below, we discuss the implications of permitting the money supply to be observable.

In view of (12) and (13), the requirement that the price level expectations E_zP_t and E_zP_{t+1} are truly conditional expectations implies that these have the form of the following "regressions," as a result of the linear normal nature of the model.

$$E_zP_t = EP_t|(I^*_{t-1}, P_t(z), R_t)$$
$$= EP_t|I^*_{t-1} + \beta_1(P_t(z) - EP_t|I^*_{t-1}) + \beta_2(R_t - ER_t|I^*_{t-1})$$

and

$$E_zP_{t+1} = EP_{t+1}|(I^*_{t-1}, P_t(z), R_t)$$
$$= EP_{t+1}|I^*_{t-1} + \gamma_1(P_t(z) - EP_t|I^*_{t-1}) + \gamma_2(R - ER_t|I^*_{t-1})$$

The "regression coefficients" β and γ are specified functions of the variance-covariance matrix of the variable being predicted (e.g., $P_t - EP_t|I^*_{t-1}$) and the information variables employed, $(P_t(z) - EP_t|I^*_{t-1})$ and $(R_t - ER_t|I^*_{t-1})$). Consequently, as Lucas has stressed, the sensitivity of expectations to movements in information variables depends on characteristics of the economic environment facing individuals, including the form of the money supply rule. As an extreme example, if the interest rate is pegged, individuals lose a source of information, so that E_zP_t is altered. Thus, evaluation of alternative monetary policies under rational expectations must consider the effects that these policies have on the quality and type of information transmitted to individuals. Below, we use β_1, γ_1, etc. to refer to the generic expectational parameter and denote the value under a particular policy rule by a superscript, i.e., β_1^{MS} for the case of a money stock rule.

A rational expectations solution involves finding values of the unknown coefficients π, ψ, β and γ that satisfy (10, 11), (12, 13) and the restrictions that link β and γ to elements of the variance-covariance matrices discussed above (which are functions of the π and ψ parameters). In this class of models, the π and ψ coefficients can readily be expressed in terms of the behavioral parameters (a, b, etc.) and regression coefficients (β, γ). Although it is not possible to explicitly solve for the regression coefficients, the nonlinear restrictions that implicitly determine solution values can be used to demonstrate important general properties of the rational expectations model (see Appendix B and the discussion below).

Before proceeding to the implications of more general environments, it is useful to consider the restricted model where there are no nominal shifts (x_t and v_t are absent) and no contemporaneous or feedback responses. (The monetary policy specification is just $M_t^s = M_t = \tilde{M}_0 + nt$). The general problem that a supplier in location z faces is to isolate the relative price $P_t(z) - P_t = \pi_4 \varepsilon_t(z)$. Since the agent is assumed to observe $P_t(z)$ but not $\varepsilon_t(z)$, he will typically be uncertain as to whether a particular movement in $P_t(z)$ derives from aggregate conditions (P_t) or relative conditions ($\pi_4 \varepsilon_t(z)$). However, if there is only one aggregate shock, then movements in R_t actually convey movements in the aggregate price level (that is, R_t is perfectly correlated with P_t). This characteristic will carry over to the discussion below, where policies that make the interest rate a better indicator of the general price level are desirable.

V. Monetary Policies Under Rational Expectations

This section compares four types of activist monetary policies (interest rate pegs and targets, feedback policies, and structural reforms) to a simple money stock rule, under the assumption that agents form their expectations rationally.

The variance of output about its full information value has the simple form $V(y_t(z) - y_t^*(z)) = [\beta_1/(1-\beta_1)]\sigma_\varepsilon^2$ under all policy structures. Thus, monetary policies which induce individuals to put a smaller weight on local prices in forming expectations of the price level lead to a lower value of this variance.

A Money Stock Policy

Under a money stock policy, the model displays many of the standard features associated with the equilibrium approach to business cycles. (For a recent survey, see Barro (1981)). In particular, nominal disturbances have real consequences only to the extent that suppliers cannot disentangle relative from absolute movements in prices. Local prices take the form

$$(14) \qquad P_t(z) = c(\hat{r}+n) - d\hat{y} + \tilde{M}_t + \pi_2(v_t - x_t) + \pi_3 u_t + \pi_4 \varepsilon_t(z).$$

The π coefficients are functions of the structural parameters and the expectational coefficients, with the exact nature of this dependence spelled out in Appendix B. For example, π_4 is equal to $1/(b(1-\beta_1^{MS}))$, which reflects the fact that suppliers partially attribute a shift in relative demand to movements in the general price level. As a result, prices will be more responsive to such shocks than under full current information, where $\pi_4^* = 1/b$ (the asterisk denotes full current information). Although the solution for β_1^{MS} is only implicitly determined, it can be shown that $0 < \beta_1^{MS} < \sigma_u^2/(\sigma_u^2 + \sigma_\varepsilon^2) < 1$. Therefore, a change in local prices causes a less than one-to-one change in agents' expectations of the general price level.

The behavior of local output is derived from the local supply schedule, $y_t^s(z) = k^s + b(P_t(z) - E_zP_t)$. This can be rewritten as

$$y_t^s(z) = \hat{y} + b(\pi_2 - \beta_1^{MS}\pi_2 - \beta_2^{MS}\psi_2)(v_t - x_t) + b(\pi_3 - \beta_1^{IS}\pi_3 - \beta_2^{MS}\psi_3)u_t \tag{15}$$
$$+ b(\pi_4 - \beta_1^{MS}\pi_4)\varepsilon_t(z) \equiv \hat{y} + y_m(v_t - x_t) + y_u u_t + y_\varepsilon \varepsilon_t(z).$$

It is shown in Appendix B that $y_m < 0$, $y_u > 0$, $y_\varepsilon = 1$.[12] That is, the magnitude of response to the money excess demand shocks (which have a negative impact on output) and aggregate demand shocks (which have a positive impact on output) is greater than under full information.

An Interest Rate Peg

In a rational expectations environment, there are two basic constraints on a monetary authority which seeks to peg the interest rate. First, the level of the peg must be consistent with the expected real interest rate and a level of expected inflation implied by money growth. Second, it must also specify a long-run path of money ($\tilde{M}_t$ in this model), so as to insure that price level movements are not arbitrarily accommodated. This latter condition avoids price level determinacy problems discussed by Sargent and Wallace (1975) and McCallum (1981). Such a "monetary anchor" might derive ultimately from the requirements of fixed exchange rates or a legal requirement that forces the central bank to hit some monetary target. In the context of this model, the condition imposed is that $EM_{t+1}|I_t^* = \tilde{M}_{t+1}$, which pins down the conditional mean of the price level at date t+1. This condition can be met in one of two ways. The monetary authority can make the peg temporary, promising to return to some form of money stock rule next period, or to "settle up" at the end of each period so as to hit the monetary target $\tilde{M}_{t+1}$ on average.

Another method for achieving determinacy, discussed by McCallum, is to set the interest rate peg so as to hit a monetary target. Essentially all of these policies provide some anchor for expectations and, therefore, allow the price level to be determined.

Under the peg, the local price is the only piece of contemporaneous information. Consequently, agents place greater weight on this information in expectation formation. (Appendix B formally demonstrates that $\beta_1^P > \beta_1^{MS}$).

The stochastic behavior of output and prices is qualitatively different under the peg. The solution for local prices no longer involves money demand disturbances, as these are absorbed by changes in the money supply.

$$(16) \quad P_t(z) = c(\hat{r}+n) - d\hat{y} + \tilde{M}_t + [1/(a+b(1-\beta_1^P))]u_t + [1/b(1-\beta_1^P)]\varepsilon_t(z)$$

Local output is also insulated from velocity shocks, although aggregate demand shocks produce departures from full information output.

$$(17) \quad y_t(z) = k^s + \frac{b(1-\beta_1^P)}{a+b(1-\beta_1^P)} u_t + \varepsilon_t(z)$$

Importantly, even though output is insulated from monetary disturbances, the peg is inferior to a policy which allows interest rates to vary. The intuitive reason for this is that the peg destroys the information concerning aggregate conditions that is normally conveyed by the nominal interest rate. Mathematically, this translates into agents placing greater weight on local prices in forming expectations about the aggregate price level, making $\beta_1^P > \beta_1^{MS}$, and a higher output variance under the peg.

Our assumption that the money supply is not observable is central to this result. If agents could accurately observe $M_t - \tilde{M}_t$ under the peg, then they could gain the same information yielded by the interest rate under the money stock rule. We conjecture, but have not shown, that if money stock figures were 'noisy' signals, then the peg would also destroy information and raise output variability.

An Interest Rate Target

In the adaptive expectations model of Section II, it was shown that a policy which systematically responded to interest rates altered the distribution of output and that such a "combination policy" dominated both a simple money stock rule and an interest rate peg. However, under rational expectations, such a policy is neutral with respect to real economic activity, relative prices, and the real rate of interest.

The reason for this neutrality is straightforward, as it results from two basic characteristics of this type of model. First, perceived changes in the money stock have an equal impact on local prices and the perceived general price level. Since relative prices do not respond to perceived movements in money, suppliers do not supply more output. Second, as both the nominal interest rate and the form of the money supply rule are part of the individual's information set, changes in the level of money resulting from responses to nominal interest rates are fully perceived.

By contrast, the nominal interest rate will be sensitive to the magnitude of the monetary authority's response to interest rate departures from target, because this brings about changes in expected inflation. However, this alteration of the stochastic behavior of the nominal interest rate does not alter information that it conveys about current and future prices. Interest rate targeting has no effect on real activity in our model. Consequently, we turn to discussion of policies that may improve on simple money supply growth rules.

Feedback Policies

When agents are heterogeneously informed and have access to common opportunities for exchange of goods and assets, King (1982) and Weiss (1980) have demonstrated that feedback policies can alter the "information content of prices" and, hence, the distribution of real activity. The present setup involves agents in each location z knowing $P_t(z)$ but not the complete vector of commodity prices prevailing at a point in time, so that agents are heterogeneously informed. Further, all agents trade in the economy-wide bond market. Consequently, in contrast to Sargent and Wallace (1975), feedback policies are relevant.

In our model, the nominal interest rate involves two elements: (i) direct influences of aggregate shocks (u_t, v_t, and x_t); and (ii) influences of economy-wide averages of expectations of the current and future price level ($\overline{E_zP_t}$ and $\overline{E_zP_{t+1}}$).

$$(18)\quad (R_t - ER_t|I^*_{t-1}) = \{(\frac{1+db}{a+b} u_t + v_t - x_t) + \frac{b(1-ad)}{a+b}(\overline{E_zP_t} - EP_t|I^*_{t-1})$$

$$+ \frac{a(1+db)}{a+b}(\overline{E_zP_{t+1}} - EP_{t+1}|I^*_{t-1})\}/[g_r + c + \frac{a(1+db)}{a+b}]$$

If expectations were homogeneous, then the nominal interest rate would effectively convey only the direct influences, as given by the first term in square brackets. Feedback policy would be irrelevant to agent expectations about the general price level, although it could influence the expected future price level and, hence, the nominal interest rate.[13]

However, in the present setup, agents find it valuable to use local information, $P_t(z) - EP_t|I^*_{t-1}$, in the formation of expectations about the current and future price level. Consequently, the nominal interest rate conveys the following information.[14]

$$(19) \quad [\frac{1+ab}{b+a} u_t + (v_t - x_t)] + [\frac{b(1-ad)}{b+a} \beta_1 + \frac{a(1+db)}{b+a} \gamma_1](P_t - EP_t|I^*_{t-1})$$

Policy may affect the information content of the interest rate by altering the weights that agents attach to local prices in forming E_zP_t and E_zP_{t+1}.

It should be stressed that <u>current</u> feedback responses $(f_x x_{t-1} + f_v v_{t-1} + f_u u_{t-1})$ are neutral as in Sargent and Wallace (1975) and in contrast to the adaptive expectations model of Section III. Rather, it is <u>prospective</u> feedback actions that are important for the information content of prices. That is, future policy responses to shocks that are only imperfectly known by agents at the current point in time $(f_x x_t + f_v v_t + f_u u_t)$ affect expectations of next period's price level and therefore affect current prices. Ideally, feedback policy would make the nominal interest rate a perfect "indicator" of the general price level, so that (as in the one-shock case of Section IV above) agents might correctly identify relative opportunities. However, the present setup does not permit the feedback rule to attain this ideal position, unlike some earlier examples provided by Weiss (1980) and King (1981a). Indeed, it has so far not been possible to analytically solve for an ideal feedback structure, although it is possible to show that feedback policies exist which dominate the simple money stock rule.[15]

In our model, as in other recent macroeconomic analyses, prospective feedback can alter the information content of prices in ways that are desirable. However, as discussed by King (1982), much detailed knowledge about the workings of the economy must be accumulated before it is possible to systematically evaluate alternative monetary regimes and to determine those that have low output variance.

Structural Reforms

A key aspect of the interest rate peg described above is that unobservable fluctuations in the quantity of money work to offset shocks to velocity (both disturbances are assumed not directly observable by private agents). However, this correlation between money and other aggregate shocks comes about at a significant cost, namely that the interest rate is no longer a signal about aggregate economic conditions. If this correlation could be obtained without imposing such a cost, it is reasonable to conjecture that output fluctuations could be reduced. Structural reforms which link the quantity of money to aggregate shocks are a possibility in this regard. Specifically, we consider a monetary policy of the form $M_t^s = \ldots + h_u u_t + h_x x_t + h_v v_t + x_t$, where $h_u u_t + h_x x_t + h_v v_t$ embody the structural reforms.

The reason we view the above linear combination of current shocks as a structural reform is that we wish to stick to the requirement that the central bank has no greater information than the private sector. In particular, no consideration is given to the possibility that the central bank knows the contemporaneous value of the aggregate shocks, since it is presumably the case that a policy of releasing the information dominates a direct response.[16] Rather, we are concerned with variations in central bank rules which alter individuals' optimal responses to shocks. Previous authors who have considered this type of idea include Friedman (1957), who indicates how a commodity currency standard can lessen the informational requirements of central bank policy, and McCallum and Whitaker (1979), who discuss how specified tax schedules may stabilize income over the course of a business cycle.

To be concrete, suppose that the money stock obeyed the specification $M_t = \tilde{M}_t + \phi(\hat{y}-y_t) + x_t$,[17] so that money responds positively to declines in output as in Friedman's (1948) proposal for economic stability. The portfolio balance condition now becomes

$$(20) \qquad (g_R+c)R_t = P_t + d\hat{y} + (d+\phi)b(P_t-E_zP_t) - \tilde{M}_t + (v_t-x_t).$$

That is, shifts in ϕ are identical to changes in the income elasticity of money demand. Furthermore, increases in ϕ reduce the variance of output around its full information value (see Appendix B) by lowering β_1. Therefore, a policy which introduced a negative correlation between money and output fluctuations would reduce the magnitude of these fluctuations.

A fully efficient policy would have the money supply responses that eliminated autonomous events ($h_x = -1$) and accomodated shifts in money demand ($h_v = 1$), so that the interest rate would depend only on the aggregate real demand factor u_t. As discussed above, this would yield the full information value of output in every period.

Our sense is that the interesting and realistic proposals along this line involve restructuring the regulations of the banking sector, a decentralized provider of money, so as to achieve a desired aggregate money response to shocks that are not directly observable by the central bank. However, the example again serves to indicate that a very different type of "activist policy" is desirable in the present rational expectations environment.

VI. Related Literature

A valuable reference point for our discussion is the analysis of Sargent and Wallace (1975), which also emphasizes rational expectations and Lucas's aggregate supply hypothesis (1975, p. 254), but that differs from our discussion in two key respects. First, although S-W contrasted money supply rules with and without feedback to past economic conditions, they did not investigate money supply responses to contemporaneous observable indicators such as nominal interest rates. Second, while appealing to Lucas's work as the foundation for their aggregate supply specifications, S-W formulated the aggregate supply schedule as $y_t^s = k^s + \alpha(P_t - EP_t|I_{t-1}^*)$, in the notation of the present paper.

As McCallum (1980) and others have noted, the Lucas supply hypothesis leads to the S-W supply schedule only if local prices are the sole contemporaneous source of information. This is clearly indicated in our model by replacing $\overline{E_z P_t}$ in the aggregate supply schedule with its rational expectation, obtaining an expression of the form $y_t^s = k^s + \beta_1 b(P_t - EP_t|I_{t-1}^*) + \beta_2 b(R_t - ER_t|I_{t-1}^*)$. If R_t were not observable, the last term would drop out, leaving an expression similar to that of S-W. However, in our setup, the variables β_1 and β_2 are not arbitrary, but rather are functions of the parameters of the economic environment including those of the money supply rule.

Fischer (1977) provides an alternative rationalization for an aggregate supply function $y_t^s = k^s + \alpha(P_t - EP_t|I_{t-1}^*)$, stressing labor contracts. In particular, contracts made by factor suppliers and demanders contain rational expectations of the price level conditioned on the information available at the time the contract is signed. (A contract signed at the end of the previous period would involve terms such as $EP_t|I_{t-1}^*$ and would exclude the use of period t information.) On the other hand, Barro (1977) has made a forceful argument that suboptimal contracts of this form should not exist, demonstrating that this type of contract is not optimal to either party. Efficient contracts would presumably not make employment/production contingent on the price level, but rather on perceived relative prices.

Woglom (1979) analyzes contemporaneous policy response to nominal interest rates within a variant of the S-W model that incorporates a dependence of supply on $P_t - EP_t | I^*_{t-1}$, without explaining its origins. In that analysis, the Poole-type conclusions about the optimal magnitude of monetary response go through. Indeed, the results of our analysis of the "adaptive expectations" model of Section III and Woglom's setup are nearly identical.[18] From our perspective, this emphasizes that Woglom's results basically derive from the fact that agents do not efficiently utilize contemporaneous information.

Canzoneri, Henderson and Rogoff (1981) provide a more complete development along these lines. They stress that a role for active monetary response to interest rates may arise if suppliers do not utilize interest rate information efficiently (as in our adaptive expectations model of III above) or if there are preexisting nominal contracts of the Fischer-Phelps-Taylor form. They contrast the real effects of combination policies in such environments with the irrelevance that occurs if all agents rationally employ the nominal interest rate in forming expectations. Furthermore, these authors stress that although nominal contrasts might similarly incorporate a dependence on nominal interest rates, an activist policy can save the private sector the costs of such "indexation."

Finally, Weiss (1981) provides an example of an environment in which a policy of pegging the nominal interest rate (perversely) leads to an increase in the information content of a vector of prices observed by private agents. In Weiss's setup, agents are heterogeneously informed about future aggregate shocks to money demand, which contaminate the current price and interest rate. A policy of pegging the future nominal interest rate implies that such shocks will have no impact on the future price level. Hence, such future nominal disturbances are no longer relevant to the current price, which implies that it serves as a better guide to current real opportunities. The loss of the nominal interest rate as a signal about current circumstances in unimportant, in Weiss's analysis, because the price level and nominal interest rate convey identical information under the "flexible rate" scheme. It seems to us that Weiss's result is principally a curiosity, although it does serve to warn that information-based arguments against pegging nominal interest rates may not always be as strong as in the model developed in Section V.

In summary, the present paper and other recent analyses lend support to the idea that activist monetary policies may be beneficial in a model that incorporates Lucas's supply hypothesis and rational expectations. However, in contrast to Poole-type analyses, contemporaneous response to nominal interest rates is not central. Rather, feedback policies and structural reforms which make price level movements more discernible by private agents are rationalized by the present analysis.

Footnotes

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[2]B. Friedman (1975) provides an overview of further developments along these lines, which generalize Poole's results without changing the basic message.

[3]Throughout the discussion, the term "aggregate" is used to describe variables which are (more precisely) economy-wide averages. This permits a ready movement between a representative (or average) market and a specific market z, without normalizing constants.

[4]Our discussion abstracts from aggregate supply shocks, for the sake of simplicity. We are confident that our central results would go through with a modification to incorporate an aggregate supply shock in equation (3). However, such a modification implies that full information output is stochastic and, consequently, complicates the solution of the model.

[5]It is difficult to tell a story consistent with optimal individual use of information that produces this demand specification. However, we wish to keep the model as close in spirit to the models in the standard literature, e.g., Poole (1970). In any event, the major results of this paper can be replicated in a model where local demand is sensitive to the local price level.

[6]Proceeding along the lines of McCallum (1981), it should be possible to study an "adjustment peg" in a situation where the real rate of interest changes over time. Our nondynamic analysis, by contrast, focuses on the informational implications of an interest rate peg at a level that must be constant over time. However, the main informational implications would appear to carry over to a model (such as King, 1981b) that incorporates capital accumulation and dynamic behavior of the real interest rate.

[7]The particular formal structure employed emphasizes departures of the interest rate from a target level R_t^T, i.e., $M_t^s = \tilde{M}_t + g_R(R_t - R_t^T) + x_t$. (As $g_R \to \infty$, the combination policy approaches a peg and as $g_R \to 0$, the combination policy approaches a simple money stock rule.)

[8]Santomero and Siegel (1981) provide a detailed discussion of bank regulatory policies' implications for the price level, within a model that involves no effect of price level changes on output.

[9] However, no result of this paper would be altered if the aggregate criterion were employed ($V(y_t - y_t^*)$). This is because the local shocks $\varepsilon_t(z)$ always have an impact on local production that is equivalent to the full information impact, which derives from the assumption that the local demand curve is vertical.

[10] We realize that the mechanism chosen (primarily for its mathematical tractability) is not the usual way of expressing adaptive expectations. What is important is that agents make systematic mistakes and do not take advantage of all easily available information. The reader may wish to verify that a standard adaptive mechanism produces the same qualitative results.

[11] $\hat{y}$ can be thought of as the "long run" value of aggregate output while $\hat{r}$ is the "long run" value of real interest rate.

[12] In some other papers relative shocks have a smaller impact on output under incomplete information than under full information (see Lucas (1973) and Barro (1976)). Hence, the vertical local demand curve leads to an identical effect.

[13] In the case where all expectations are homogeneous (i.e., there is no local information), aggregate output will be affected by $P_t - EP_t|\tilde{I}_t$. Writing $P_t = \pi_0 + \pi_1 \tilde{M}_t + \pi_2 m_t + \pi_3 u_t + \pi_5 m_{t-1} + \pi_6 u_{t-1}$ and $R_t = \psi_0 + \psi_1 M_t + \psi_2 m_t + \psi_3 u_t + \psi_5 m_{t-1} + \psi_6 u_{t-1}$, one can solve for the undetermined coefficients in an analogous way to the method used in Appendix B. In this case $P_t - EP_t|I_t = \pi_2(m_t - Em_t|I_t) + \pi_3(u_t - Eu_t|I_t)$, where the conditional expectations in this expression are derived from observing the linear combination $\psi_2 m_t + \psi_3 u_t$ embedded in R_t. Therefore $Em_t|I_t = \frac{\theta}{\psi_2}(\psi_2 m_t + \psi_3 u_t)$ and $Eu_t|I_t = \frac{1-\theta}{\psi_3}(\psi_2 m_t + \psi_3 u_t)$, where $\theta \equiv \psi_2^2\sigma_m^2/(\psi_2^2\sigma_m^2 + \psi_3^2\sigma_u^2)$. A complete solution yields the result that $P_t - EP_t|I_t = -\frac{1-\theta}{a+b} m_t + \frac{\theta}{a+b} u_t$, which is independent of all feedback parameters. It can also be shown that this solution is equivalent to the solution which obtains under heterogeneous expectations when $\sigma_\varepsilon^2 \to \infty$. [This can be seen in Appendix B, equation B-5a, where as $\sigma_\varepsilon^2 \to \infty$, $\beta_1 \to 0$ and $\beta_2 \to (\pi_2 \psi_2 \sigma_m^2 + \pi_3 \psi_3 \sigma_u^2)/(\psi_2^2 \sigma_m^2 + \psi_3 \sigma_u^2)$. In that case $E_z P_t = EP_t|I_{t-1}^* + \beta_2(\psi_2 m_t + \psi_3 u_t)$, which is the same as the solution for $EP_t|I_t$ when expectations are homogenous.] This is intuitively plausible since a value of $\sigma_\varepsilon^2 = \infty$ implies that $P_t(z)$ is useless in conveying information about the aggregate price level. Also it is important to observe that when expectations are heterogeneous, the solution for γ_1 (and γ_2) in equation B-7c (and B-7d) involve the terms π_5 and π_6 which contain the feedback parameters f_m and f_u. It is through these regression coefficients that f_m and f_u directly enter into the solution for output. If there were no money supply response next period to this periods shocks there would be no role for feedback, since $E_z P_{t+1}$ would be unaffected. Consequently, we describe the feedback effects as being "prospective" in nature.

[14] This may be obtained by substituting the expectations rules into (18), averaging over markets, and deleting the influences of interest rate surprises on expectations.

[15] Consequently, Appendix B provides a simple example of a feedback policy that reduces the variance of output about its full information value. Specifically, the variance is $[\beta_1^{FB}/(1-\beta_1^{FB})]\sigma_\varepsilon^2$ and it is shown that $\beta_1^{FB} < \beta_1$ so that the variance under feedback is smaller. (This result can be generated by a linear combination of f_m and f_u in a feedback specification of the form of $f_m(v_{t-1} - x_{t-1}) + f_u u_{t-1}$.) We do not attach a great deal of importance to the particular example chosen, as it was selected for analytical convenience.

[16] See Barro (1976) for an argument along these lines. For a counter argument based on the private cost of information processing see Howitt (1981).

[17] As an example of how this specification could be rationalized in the Section II setup, consider an economy which had a local government office in each location z. These offices would make a fixed level of real purchases, $\bar{g}$, in their respective locations and collect nominal taxes equal to $\tau + y_t(z) + P_t(z)$. These offices then either subtract from (or add to) the local stock of money to the extent that $P_t(z) + \tilde{g} < \tau + y_t(z) + P_t(z)$. Consequently, assume $M_t(z) = \tilde{M}_t + x_t + \phi(\hat{y} - y_t(z))$ and that (correspondingly) $M_t = \tilde{M}_t + x_t + \phi(\hat{y} - y_t)$.

[18] Within our setup, this is easiest to see if expectations don't adjust at all to the current price ($\lambda = 0$), so that the perceived relative price term $P_t - \overline{E_z P_t}$ can be written as $P_t - \hat{P}_t$, where P_t is a predetermined expectation. This perceived relative price may be decomposed into two parts $(P_t - EP_t|I^*_{t-1}) + (EP_t|I^*_{t-1} - \hat{P}_t)$, where the former is a pure expectation error and the latter represents the departure of the expectations scheme from rationality. In our Section III analysis, it is assumed that the feedback component of monetary policy is chosen to eliminate $(EP_t|I^*_{t-1} - P_t)$. Hence, the supply of output depends positively on $P_t - EP_t|I^*_{t-1} = \{(g_R + c)u_t - a(v_t - x_t))\}/\Delta$. Since our model and Woglom's model are similar in other respects, we find that a simple adaptive framework produces the nearly same results as his model with rational, but predetermined expectations.

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Appendix A

Adaptive Expectations Solution

The adaptive expectations version of the model specifies that $E_zP_t = \lambda P_t(z) + (1-\lambda)(P_{t-1} + n)$ and that $E_zP_{t+1} = E_zP_t + n$. Solving the aggregate system comprised of (1), (2), (3b), (5) and economy-wide average expectations yields

$$\text{(A1)}\quad P_t = (P_{t-1} + n) + \frac{a}{\Delta}\,(f_{t-1} + \tilde{M}_t - g_R(R_t^T - \hat{r} - n) - d\hat{y} + c(\hat{r}+n) - (P_{t-1}+n)) + \frac{1}{\Delta}\,((g_R+c)\,u_t - a(v_t - x_t))$$

$$\text{(A2)}\quad R_t = (\hat{r}+n) - \frac{(a+b)(1-\lambda)}{\Delta}(f_{t-1} + \tilde{M}_t - g_R(R_t^T - \hat{r} - n) - d\hat{y} + c(\hat{r}+n) - (P_{t-1}+n)) - \frac{(a+b)(1-\lambda)}{a\Delta}((g_R+c)u_t - a(y_t - x_t)) + 1/a\ u_t$$

$$\text{(A3)}\quad y_t = \hat{y} + \frac{b(1-\lambda)a}{\Delta}(f_{t-1} + \hat{M}_t - g_R(R_t^T - \hat{r} - n) - d\hat{y} + c(\hat{r}+n) - (P_{t-1} + n)) + \frac{b(1-\lambda)}{\Delta}((g_R+c)u_t - a(v_t - x_t))$$

where $\Delta = a(1+db(1-\lambda)) + (g_R+c)(a+b)(1-\lambda) > 0$. As discussed in the text, adjustment of either f_{t-1} or R_t^T can make the second term in the output expression zero. We assume that f_{t-1} has been so adjusted and that $R_t^T = \hat{r}+n$. Then $y_t = k^s + \frac{b(1-\lambda)}{\Delta}\,(a(x_t - v_t) + (g_R + c)u_t)$.

Local output is given by $y_t(z) = y_t + \varepsilon_t(z)$ and $y_t^*(z) = \hat{y} + \varepsilon_t(z)$ so that

$$\text{(A4)}\quad V(y_t(z) - y_t^*(z)) = E\left[\frac{b(1-\lambda)}{\Delta}(a(x_t - v_t) + (g_R + c)u_t)\right]^2 = \frac{b^2(1-\lambda)^2}{\Delta^2}\left[(g_R + c)^2\sigma_u^2 + a^2(\sigma_x^2 + \sigma_v^2)\right]$$

Differentiating this variance with respect to g_R, setting the resulting derivative to zero, and solving for a minimizing value of g_R leads to the expression reported in the text (equation (9)).

Appendix B

Construction of Rational Expectations Solutions

Throughout this appendix,* one of the two reduced form relationships is given by text equation (11), the local commodity market clearing value of the product price,

$$\text{(B-1)}\qquad P_t(z) = \frac{a}{b}\hat{r} - \frac{a}{b}(P_t + R_t - \overline{E_zP_{t+1}}) + E_zP_t + \frac{1}{b}(u_t + \varepsilon_t(z)),$$

where $\hat{r} = (k^d - k^s)/a$. By contrast, the implications of the monetary equilibrium condition must be developed under the alternative policy specifications.

1. The Flexible Rate Cases

The solutions under the three basic flexible interest rate cases can be developed by considering the composite money supply specification, $M_t^S = \tilde{M}_t + g_R(R_t - R_t^T) + x_t + f_{t-1}$, which implies that the monetary equilibrium condition may be written as

$$\text{(B-2)}\qquad (g_R + c)R_t = (1+db)P_t + d\hat{y} - db\overline{E_zP_t} - \tilde{M}_t + g_R(R_t^T) + m_t - f_m m_{t-1} - f_u u_{t-1},$$

where we have defined the excess money demand shock $m_t = v_t - x_t$. Further, we assume throughout this appendix that $R_t^T = ER_t|I^*_{t-1}$.

The undetermined coefficients representations of local prices and the economy-wide nominal interest rate then take the following forms:

$$\text{(B-3a)}\qquad P_t(z) = \pi_0 + \pi_1\tilde{M}_t + \pi_2 m_t + \pi_3 u_t + \pi_4\varepsilon_t(z) + \pi_5 m_{t-1} + \pi_6 u_{t-1}$$

$$\text{(B-3b)}\qquad R_t = \psi_0 + \psi_1\tilde{M}_t + \psi_2 m_t + \psi_3 u_t + \psi_5 m_{t-1} + \psi_6 u_{t-1}$$

(Note that since the average value of $\varepsilon_t(z)$ is zero across markets, it follows that the price level is $P_t = \pi_0 + \pi_1\tilde{M}_t + \pi_2 m_t + \pi_3 u_t + \pi_5 m_{t-1} + \pi_6 u_{t-1}$). Before solving the model, the nature of the expectational terms E_zP_t, etc. must be examined. Individuals in market z are receiving two signals at date t, the local price and nominal

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*For a more detailed discussion, see the appendices to our Federal Reserve Bank of New York Working Paper (number 8114).

interest rate. Due to the linear multivariate normal nature of the model, the expectactions E_zP_t and E_zP_{t+1} take the following forms:

$$\text{(B-4a)} \qquad E_zP_t = {}_{t-1}E_1P_t + \beta_1(P_t(z) - {}_{t-1}E_1P_t) + \beta_2(R_t - {}_{t-1}E_1R_t)$$

$$\text{(B-4b)} \qquad E_zP_{t+1} = {}_{t-1}E_1P_{t+1} + \gamma_1(P_t(z) - {}_{t-1}E_1P_t) + \gamma_2(R_t - {}_{t-1}E_1R_t),$$

where the notation ${}_{t-1}E_1$ is a short-hand for an expectation conditional on I^*_{t-1}, which will be employed throughout this appendix. From (B-3a, b), it follows that ${}_{t-1}E_1P_t = \pi_0 + \pi_1\tilde{M}_t + \pi_5m_{t-1} + \pi_6u_{t-1}$ and that ${}_{t-1}E_1R_t = \psi_0 + \psi_1\tilde{M}_t + \psi_5m_{t-1} + \psi_6u_{t-1}$.

The expressions (B-3a, b) and (B-4a, b) imply the following equations for the expected price level in market z and its economy-wide counterpart.

$$\text{(B-5a)} \qquad E_zP_t = {}_{t-1}E_1P_t + \beta_1(\pi_2m_t + \pi_3u_t + \pi_4\varepsilon_t(z)) + \beta_2(\psi_2m_t + \psi_3u_t)$$

$$\text{(B-5b)} \qquad \overline{E_zP_t} = {}_{t-1}E_1P_t + \beta_1(\pi_2m_t + \pi_3u_t) + \beta_2(\psi_2m_t + \psi_3u_t),$$

since the economy-wide average value of $\varepsilon_t(z)$ is zero. The analogous expressions for the expected future price level are

$$\text{(B-6a)} \qquad E_zP_{t+1} = {}_{t-1}E_1P_{t+1} + \gamma_1(\pi_2m_t + \pi_3u_t + \pi_4\varepsilon_t(z)) + \gamma_2(\psi_2m_t + \psi_3u_t)$$

$$\text{(B-6b)} \qquad \overline{E_zP_{t+1}} = {}_{t-1}E_1P_{t+1} + \gamma_1(\pi_2m_t + \pi_3u_t) + \gamma_2(\psi_2m_t + \psi_3u_t).$$

where ${}_{t-1}E_1P_{t+1} = \pi_o + \pi_1\tilde{M}_{t+1}$.

Now, use (B-5) and (B-6) to eliminate expectational terms in (B-1) and (B-2). Then, the requirement that the resulting expressions hold identically with (B-3a, b) leads to a set of restrictions on the π and ψ cofficients that may be solved to yield: $\pi_0 = c(\hat{r}+n) - d\hat{y}$, $\pi_1 = 1$, $\pi_2 = \{b\beta_2 - a(1-\gamma_2)\}/A$, $\pi_3 = \{g_R + c + db\beta_2\}/A$, $\pi_4 = 1/\{b(1-\beta_1)\}$, $\pi_5 = f_m/\{1 + c\}$, $\pi_6 = f_u/\{1 + c\}$ and $\psi_0 = \hat{r}+n$, $\psi_1 = 0$, $\psi_2 = \{b(1-\beta_1) + a(1-\gamma_1)\}/A$, $\psi_3 = \{1+db(1-\beta_1)\}/A$, $\psi_5 = -f_m/\{1 + c\}$, $\psi_6 = -f_u/\{1 + c\}$, where $A = b(g_R + c + db\beta_2)[(1-\beta_1) + \frac{a}{b}(1-\gamma_1)] + b(1 + db(1-\beta_1))[-\beta_2 + \frac{a}{b}(1-\gamma_2)]$.

Conditional Expectations

The requirement that expectations are rational restricts the solution values of the β and γ coefficients. See King (1981a, appendix) for a detailed discussion in a parallel context.

$$\text{(B-7a)} \qquad \beta_1 = \frac{1}{\delta}[\pi_2\psi_3 - \pi_3\psi_2]^2\sigma_u^2\sigma_m^2$$

$$\text{(B-7b)} \qquad \beta_2 = \frac{1}{\delta}[\pi_2\psi_2\sigma_m^2 + \pi_3\psi_3\sigma_u^2]\pi_4^2\sigma_\varepsilon^2,$$

$$\text{(B-7c)} \qquad \gamma_1 = \frac{1}{\delta}\ \{(\pi_5\psi_3 - \pi_6\psi_2)(\pi_2\psi_3 - \pi_3\psi_2)\sigma_u^2\sigma_m^2\}$$

$$\text{(B-7d)} \qquad \gamma_2 = \frac{1}{\delta}\ (\psi_2\pi_5\sigma_m^2 + \psi_3\pi_6\sigma_u^2)(\pi_4^2\sigma_\varepsilon^2) + (\pi_2\pi_6 - \pi_3\pi_5)(\pi_2\psi_3 - \pi_3\psi_2)\sigma_u^2\sigma_m^2,$$

where $\delta = (\pi_2\psi_3 - \pi_3\psi_2)^2\sigma_u^2\sigma_m^2 + [\psi_2^2\sigma_m^2 + \psi_3^2\sigma_u^2]\pi_4^2\sigma_\varepsilon^2$. Noting that $\pi_2\psi_3 - \pi_3\psi_2 = -1/A$, it can be shown that

$$\text{(B-8)} \qquad \beta_1 = \frac{\sigma_u^2\sigma_m^2}{\sigma_u^2\sigma_m^2 + \{[a(1-\gamma_1) + b(1-\beta_1)]^2\sigma_m^2 + [1+db(1-\beta_1)]^2\sigma_u^2\}\pi_4^2\sigma_\varepsilon^2}.$$

Further, employing the fact $\gamma_1 = \dfrac{\pi_5\psi_3 - \pi_6\psi_2}{\pi_2\psi_3 - \psi_2\pi_3}\beta_1$, it follows that

$$\text{(B-9)} \qquad \gamma_1 = \frac{-f_m[1+db(1-\beta_1)] + f_u b(1-\beta_1) + f_u a}{1 + c + af_u\beta_1}\beta_1$$

Comparable simplifications for β_2 and γ_2 may also be obtained. However, these expressions will not be relevant to the discussion below.

Output Response and Variance

Working with the local supply schedule and expression (B5a) for E_zP_t, it follows that $y_t(z) \equiv y_m m_t + y_u u_t + y_\varepsilon \varepsilon_t(z)$ is equal to

$$\text{(B-10)} \qquad y_t(z) = b(\pi_2 - \beta_1\pi_2 - \beta_2\psi_2)m_t + b(\pi_3 - \beta_1\pi_3 - \beta_2\psi_3)u_t + b\pi_4(1-\beta_1)\varepsilon_t(z).$$

Using the above conditional expectations formulae, it may be shown that $y_m = b\beta_1\psi_3\pi_4^2\sigma_\varepsilon^2/\{(\pi_2\psi_3 - \pi_3\psi_2)\}\sigma_m^2$ and using the π and ψ coefficients, this simplifies to $y_m = -\beta_1(1+db(1-\beta_1))\sigma_\varepsilon^2/\{b(1-\beta_1)^2\sigma_m^2\}$, which is negative since the form of (B-8) restricts $0 < \beta_1 < 1$. Analogously, it can be shown that $y_u = \beta_1(b(1-\beta_1) + a(1-\gamma_1))\sigma_\varepsilon^2/\{b(1-\beta_1)^2\sigma_u^2\}$, which is positive for values of γ_1 that are in the interval $0 \leq \gamma_1 \leq 1$. Further, since $\pi_4 = 1/\{b(1-\beta_1)\}$, it follows that $y_\varepsilon = 1$.

The variance of output about its full information value, $y_t^*(z) = k^s + \varepsilon_t(z)$, thus has the form $V(y_t(z) - y_t^*(z)) = y_m^2\sigma_m^2 + y_u^2\sigma_u^2$. Working with the first-stage simplifications of y_m and y_u, this may be shown to equal $b^2\beta_1(1-\beta_1)\pi_4^2\sigma_\varepsilon^2$. Using the value of π_4, it follows that $V(y_t(z) - y_t^*(z)) = \beta_1\sigma_\varepsilon^2/(1-\beta_1)$.

The following special cases correspond to the policy rules discussed in the text:

Money Stock Rule ($g_R = 0$, $f_m = f_u = 0$). In this case and the next, all disturbances are strictly temporary, so that current prices and interest rates do not respond to past shocks ($\pi_5 = \pi_6 = \psi_5 = \psi_6 = 0$). Consequently, agents' expectations of the future price level do not respond to either current prices or interest rates ($\gamma_1 = \gamma_2 = 0$). We denote the value of β_1 that solves (B-8) as β_1^{MS}.

Combination Policy ($g_R \neq 0$, $f_m = f_u = 0$). Although the value of g_R is relevant for the response of nominal interest rates and the price level to m_t and u_t shocks (entering in the coefficients π_2, π_3, ψ_2, and ψ_3), it is not relevant for the response of output to any shock. For example, continuing the manipulation discussed in the context of output variance above, it may be shown that $b(\pi_2 - \beta_1\pi_2 - \beta_2\psi_2) = -[1+d(1-\beta_1)]\sigma_\varepsilon^2/\{b(1-\beta_1)^2\sigma_m^2\}$. Since β_1 does not depend on the value of g_R in this case (see (B-8) above with $\gamma_1 = 0$), it follows that the response of output to m_t shocks is independent of g_R. A similar demonstration can be made for y_u and y_ε.

Feedback Policy ($g_R = 0$, $f_m \neq 0$, $f_u \neq 0$). The value of β_1 is affected by the selection of f_u, f_m, as may be seen by substituting (B-9) into (B-8). Unfortunately, the complicated form of the resulting restriction has precluded determination of a minimum variance (using values of f_u and f_m as controls). Nevertheless, the variance can be lowered below that of a pure money stock policy. For example, if γ_1^{FB} is set equal to $-\frac{b}{a}\beta_1^{FB}$, then it follows that β_1 coefficient satisfies the following (simpler) restriction (derived from B-8).

$$\bar{\beta}_1^{FB} = \frac{\sigma_u^2\sigma_m^2}{\sigma_u^2\sigma_m^2 + \{(a+b)^2\sigma_m^2 + [1+db(1-\bar{\beta}_1^{FB})]^2\sigma_u^2\}\pi_4^2\sigma_\varepsilon^2}$$

where an over-bar ($\bar{\beta}_1^{FB}$) is used to distinguish this special case from the more general solution. The selection of $f_m = 0$ and $f_u = -\frac{b(1+c)}{a(a+b)}$ yields the desired value of $\bar{\gamma}_1^{FB} = -\frac{b}{a}\bar{\beta}_1^{FB}$.

Structural Reforms. To analyze the impact of the structural reforms outlined in the text, we replace d with $d+\phi$ in restriction (B-8), under the assumption that $\gamma_1 = \gamma_2 = 0$ as in the money supply rule case. The resulting value is denoted β_1^{SR}.

2. The Pegged Interest Rate

Under the assumption that the interest rate is pegged at the level R^T, the money supply is demand determined

$$\text{(B-11)} \qquad M_t^s = M_t^d = P_t + dy_t - cR^T + v_t$$

Consequently, we hypothesize that the money stock is governed by the following undetermined coefficients representation,

$$\text{(B-12)} \qquad M_t = \mu_0 + \mu_1\tilde{M}_t + \mu_2 v_t + \mu_3 u_t.$$

As discussed by Sargent-Wallace (1975) and McCallum (1980), there is a price level determinancy problem if the condition $M_t^s = M_t^d$ is the only aspect of monetary policy.

By initially considering the system under full information, it is easy to exposit this indeterminacy and the additional constraint that we impose on monetary policy to make the price level determinate. Under full current information, agents are assumed to know the shocks u_t, v_t and $\varepsilon_t(z)$. Consequently, the π and μ coefficients are determined by the requirement that the local price equations (B-1) and (B-3a) and the money stock equations (B-11) and (B-12) hold identically, when $E_zP_{t+1} = \pi_0 + \pi_1(\tilde{M}_t+n)$ and $E_zP_t' = P_t = \pi_0 + \pi_1\tilde{M}_t + \pi_2v_t + \pi_3u_t$ have been substituted in. The restrictions on the constant terms and the coefficients on $\tilde{M}_t$ are $\pi_0 = \frac{a}{b}(\hat{r}+\pi_1n - R^T) + \pi_0$, $\pi_1 = \pi_1$, $\mu_0 = \pi_0 + d\hat{y} - cR^T$, and $\mu_1 = \pi_1$. Now, if the nominal interest rate target is not equal to $\hat{r}+\pi_1n$, then it follows that there is no value of π_0 that solves $\pi_0 = \frac{a}{b}(\hat{r}+\pi_1n-R^T) + \pi_0$. If R^T is equal to $\hat{r}+\pi_1n$, then there is no meaningful restriction on π_0.

In order to have a system of interest rate pegging that uniquely determines the price level and money stock, one must provide some kind of nominal anchor. This can be done in a number of ways, all of which are essentially related. In this appendix, we impose the conditions that the peg be consistent ($R^T = \hat{r}+n$) and that the money stock obeys $EM_{t+1}|I_t^* = \tilde{M}_{t+1}$, with the latter condition insuring that $\mu_0 = 0$ and that $\mu_1 = 1$. Consequently, it follows that $\pi_0 = d\hat{y} + cR^T$ and that $\pi_1 = 1$. We think of this condition as requiring that the monetary authority "settle up" at the end of each period, returning to a preannounced path for the money stock ($\tilde{M}_t$).

Under incomplete information, the peg case is actually simpler because agents have only a single signal about current conditions, the local price $P_t(z)$. Consequently, the expectations about the price level are $E_zP_t = {}_{t-1}E_1P_t + \beta_1^P(P_t(z) - {}_{t-1}E_1P_t)$ $= \pi_0 + \pi_1\tilde{M}_t + \beta_1^P(\pi_2v_t + \pi_3u_t + \pi_4\varepsilon_t(z))$, where $\beta_1^P = (\pi_2^2\sigma_v^2 + \pi_3^2\sigma_u^2)/(\pi_2^2\sigma_v^2 + \pi_3^2\sigma_u^2 + \pi_4^2\sigma_\varepsilon^2)$ is the single "regression coefficient."

Working through analogous restrictions to those in the previous case yields the following solution values for the π and μ coefficients, in terms of the structural parameters and the regression coefficient β_1^P: $\pi_0 = -d\hat{y} + c(\hat{r}+n)$, $\pi_1 = 1$, $\pi_2 = 0$, $\pi_3 = 1/(a+b(1-\beta_1^P))$, $\pi_4 = 1/b(1-\beta_1^P)$, $\mu_0 = 0$, $\mu_1 = 1$, $\mu_2 = 1$ and $\mu_3 = \pi_3$. As discussed in the text, the fact that $\pi_2 = 0$ and $\mu_2 = 1$ involves the full accommodation of money demand shock under an interest rate peg. These solutions imply that the restriction on the regression coefficient simplifies to $\beta_1^P = \sigma_u^2/\{\sigma_u^2 + [a+b(1-\beta_1^P)]^2\pi_4^2\sigma_\varepsilon^2\}$.

3. Comparison of Output Variances

Our comparison of the money stock policy (or combination policy), the peg, the feedback policy and structural reform is straightforward since in all cases the variance is equivalent to $\beta_1\sigma_\varepsilon^2/(1-\beta_1)$. Consequently, to show the variance relationship discussed in the text ($V_{peg} > V_{MS} > V_{FB}$ and $V_{MS} > V_{SR}$ for $\phi > 0$), it is simply necessary to show that $\beta_1^P > \beta_1^{MS} > \bar{\beta}_1^{FB}$ and $\beta_1^{MS} > \beta_1^{SR}$ for $\phi > 0$. The following relationships implicitly determine the alternative β_1 coefficients.

$$\beta_1^P = \sigma_u^2/\{\sigma_u^2 + [a+b(1-\beta_1^P)]^2\pi_2^2\sigma_\varepsilon^2\}$$

$$\beta_1^{MS} = \sigma_u^2/\{\sigma_u^2 + [a+b(1-\beta_1^{MS})]^2\pi_4^2\sigma_\varepsilon^2 + [1+db(1-\beta_1^{MS})]\pi_4^2 \frac{\sigma_\varepsilon^2\sigma_u^2}{\sigma_m^2}\}$$

$$\bar{\beta}_1^{FB} = \sigma_u^2/\{\sigma_u^2 + [a(1+\tfrac{a}{b}\bar{\beta}_1^{FB}) + b(1-\bar{\beta}_1^{FB})]\pi_4^2\sigma_m^2 + [1+db(1-\bar{\beta}_1^{FB})]\pi_4^2 \frac{\sigma_\varepsilon^2\sigma_u^2}{\sigma_m^2}\},$$

where $\pi_4 = 1/b(1-\beta_1)$, with the appropriate β_1. Recall that β_1^{SR} can be obtained by setting $d = d+\phi$ in the expression for β_1^{MS}. Notice that all the β's are between zero and one and that as $\sigma_m^2 \to \infty$, $\beta_1^{MS} \to \beta_1^P$.

We have found it easiest to use a graphical approach, after taking the transformation of variables ($\omega_P = 1-\beta_1^P$, $\omega_{MS} = 1-\beta_1^{MS}$ and $\omega_{FB} = 1-\beta_1^{FB}$). Then, the restrictions can be written as

$$G(\omega_p) \equiv \left(\frac{a+b\omega_p}{b\omega_p}\right)^2 \frac{\sigma_\varepsilon^2}{\sigma_u^2} = \frac{\omega_p}{1-\omega_p} \equiv F(\omega_p)$$

$$H(\omega_p) \equiv \left(\frac{a+b\omega_{MS}}{b\omega_{MS}}\right)^2 \frac{\sigma_\varepsilon^2}{\sigma_u^2} + \left(\frac{1+db\omega_{MS}}{b\omega_{MS}}\right)^2 \frac{\sigma_\varepsilon^2}{\sigma_m^2} = F(\omega_{MS})$$

$$I(\omega_p) \equiv \left[\frac{a(1+\frac{a}{b}(1-\omega_{FB})) + b\omega_{FB}}{b\omega_{FB}}\right]^2 \frac{\sigma_\varepsilon^2}{\sigma_u^2} + \left[\frac{1+db\omega_{FB}}{b\omega_{FB}}\right]^2 \frac{\sigma_\varepsilon^2}{\sigma_m^2} = F(\omega_{FB})$$

For any ω in the interval $0 < \omega < 1$, note that $G(\omega) < H(\omega) < I(\omega)$. A graph of these functions (Figure B-1) shows the uniqueness of the solutions for ω and the fact that $\omega_p < \omega_{MS} < \omega_{FB}$, which implies that $\beta_1^{FB} > \beta_1^{MS} > \beta_1^{FB}$. Finally, it is direct that an increase in d (say, to $d+\phi$) shifts the H function up to the right, so that $\omega_{SR} > \omega_{MS}$, for $\phi > 0$. In turn, this implies that $\beta_1^{SR} < \beta_1^{MS}$.

Figure B-1