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ABSTRACT

We investigate the choice of exchange-rate regime – fixed or floating – in a dynamic, intertemporal general equilibrium framework. Our framework extends Devereux and Engel (1998) by investigating the implications of internationalized production. We examine the role of price-setting -- whether prices are set in the currency of producers or the currency of consumers – in determining the optimality of exchange-rate regimes in an environment of uncertainty created by monetary shocks. We find that when prices are set in producers' currencies, floating exchange rates are preferred when the country is large enough, or not too risk averse. On the other hand, floating exchange rates are always preferred when prices are set in consumers' currencies because floating exchange rates allow domestic consumption to be insulated from foreign monetary shocks. The gains from floating exchange rates are greater when there is internationalized production in this case.

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Introduction

The modern literature on the choice of fixed versus flexible exchange rate regimes traces back to Friedman (1953). Friedman's contribution emphasized the enhanced flexibility of relative prices afforded by floating exchange rates in a world where nominal goods prices adjust slowly. A country can be insulated to some degree from foreign demand shocks under floating exchange rates because relative price movements absorb some of the changes in demand that would have to be met by changes in quantities produced under fixed exchange rates.

Friedman's early work was supplemented and refined in the 1960s by Mundell (1960, 1961a, 1961b, 1963) who explored the role of capital mobility in the choice of exchange-rate regimes. Whether fixed or floating exchange rates are better depends in Mundell's work on whether the source of shocks was monetary or real; on the degree of capital and other factor mobility; and, on the relative size of countries.

A large literature in the 1970s and 1980s, in turn, extended Mundell by incorporating a role for expectations. Contributors include Turnovsky (1976, 1983), Fischer (1976), Hamada and Sakurai (1978), Flood (1979), Flood and Marion (1983), Weber (1981), Kimbrough (1983), Aizenman and Frenkel (1985), and Glick and Wihlborg (1990). These papers evaluated the optimality of exchange-rate regimes on ad hoc criteria, generally involving variance of output and inflation.

We investigate the optimality of exchange rate regimes from a welfare maximization standpoint. Others have studied the welfare properties of alternative exchange-rate systems, including Lapan and Enders (1980), Helpman (1981), Helpman and Razin (1982), Eaton (1985), Aizenman (1994), Chinn and Miller (1998), and Neumeyer (1998). These papers, however, do not assume any sort of nominal price stickiness, and therefore do not follow directly in the tradition of Friedman and Mundell.

The model we use is a two-country, infinite-horizon model of optimization under uncertainty. Consumers get utility from consumption, leisure and real balances. We assume that there is perfect capital mobility, because we allow a complete market of state-contingent nominal bonds. The sources of uncertainty are random monetary shocks at home and abroad. We assume that monopolistic firms must set nominal prices prior to the realization of monetary shocks. Prices adjust fully after one period.

Two important features of the economy that we examine are motivated in part by the empirical research of Robert Lipsey. First is the nature of how price setting affects the optimal choice of exchange-rate regime. Friedman and Mundell assumed that producers set prices in their own currency, and that those prices do not adjust when exchange rates change. Indeed, in their models, the law of one price holds for all goods.

Lipsey has demonstrated how poor the law of one price assumption is. The seminal paper in this regard is Kravis and Lipsey (1978).[1] That paper demonstrates that prices show substantial deviations from purchasing power parity. Moreover, relative prices move with exchange rates: countries with weaker currencies have lower prices. These movements appear to be attributable to a large extent to failures of the law of one price: "There are reasons for believing that there may be substantial deviations from the law of one price even for traded goods (p.227)." Furthermore, it appears from microeconomic data that firms set different prices for consumers in different countries. They conclude, "A given seller may charge different prices for a given product to different destinations (p. 234)," and, "Price discrimination … is quite common in international trade (p. 234)."

[1] See also Kravis and Lipsey (1987).

More recent empirical research confirms those findings.[2] Swedenborg and Lipsey (1998) conclude that even for highly tradable food items, a significant temporary factor affecting relative prices across countries is exchange rate movements. We conclude that a better description of price setting is that nominal prices are sticky in the currency of the consumer. We refer to this type of price stickiness as “pricing to market”.

We have argued that the type of price stickiness may be of critical importance in the analysis of fixed versus floating exchange rates (Devereux and Engel (1998)). We find that under floating exchange rates and pricing to market, foreign monetary shocks do not affect domestic consumption.[3] In contrast, when prices are set in the producers’ currencies, the prices paid by home residents for foreign goods changes as the exchange rate changes. This introduces a channel through which the foreign monetary shocks can affect domestic consumption. The larger the share of foreign goods in consumption, the more vulnerable will consumption be to foreign money shocks. The type of pricing behavior also influences the average level of prices, consumption and leisure under floating exchange rates.

We extend our earlier paper by examining the role of the price-setting behavior of firms when there is internationalized production. Lipsey’s empirical work has documented the large and increasing role for multinationals in U.S. and world production. Lipsey (1998) finds that internationalized production grew from four and a half percent of world output in 1970 to seven percent of world output in 1995. In manufacturing, the share rose from twelve percent in 1970 to sixteen percent in 1990 (and probably much higher by 1995.) Blomstrom, Fors and Lipsey (1997) find that U.S. firms allocated more labor-intensive operations to foreign affiliates. Kravis and

2 See especially Engel (1993, 1998), Rogers and Jenkins (1996), and Engel and Rogers (1996).

3 However, we shall see that foreign monetary variance may influence the expected level of home consumption.

Lipsey (1982, 1992), and Lipsey (1988, 1993) document the increasing role for multinational production for U.S. firms.

For purposes of comparison with standard models that assume no internationalized production, we make the extreme assumption that all production for export occurs in foreign affiliates. Kravis and Lipsey (1992) document that by the late 1980s, U.S. multinationals export more from their foreign affiliates abroad than from the U.S.

The model in this paper of floating exchange rates with prices set in producers' currency is based on Obstfeld and Rogoff (1998). That model, in turn, is influenced by the non-stochastic models of Corsetti and Pesenti (1998) and Obstfeld and Rogoff (1995). These models are examples of recent international models with optimizing agents and prices which are sticky in producers' currencies which include Svensson and van Wijnbergen (1989), Kollman (1996) and Hau (1998).

Our pricing to market model shares some of the characteristics of Bacchetta and van Wincoop's (1998) examination of how exchange rate regime affects the volume of trade and capital flows in a stochastic two-period model of pricing to market. Our model of pricing to market is also related to the work of Betts and Devereux (1996, 1998a, 1998b). Other recent general equilibrium models in which prices are sticky in consumers' currencies include Chari, Kehoe and McGrattan (1997), Tille (1998a, 1998b) and Engel (1996).

None of the aforementioned models, however, allow for any internationalized production.

The general models are laid out in section 1. In section 2, we investigate the welfare comparisons of fixed versus floating exchange rates under the two models of price-setting behavior. Section 3 compares the results with those of Devereux and Engel (1998), so that we can ascertain the role of internationalized production. The concluding section points to some potential weaknesses of our analysis and directions for future research.

1. The Model

Here we lay out the main features of the models we examine. The model for consumers follows exactly that of Devereux and Engel (1998), which in turn closely follows the model of consumption in Obstfeld and Rogoff (1998). Consumers maximize expected lifetime utility, taking prices and wages as given. Home-country agents own home-country firms (and similarly for foreign country agents). Firm managers make production and pricing decisions. Each consumer receives a share of profits from every firm in which he owns shares. Monetary authorities in each country let the money supply fluctuate randomly in the floating exchange rate models, and this is the only source of uncertainty in the model. In the fixed exchange rate model, the foreign monetary authority determines the foreign money supply (randomly), while the domestic central bank controls the domestic money supply in order to keep the exchange rate fixed.

There are sticky goods prices in our models. Producers must set prices prior to the realization of monetary shocks. One could justify this constraint with an appropriate menu-cost model, though we do not model this and simply view the stickiness of prices as an institutional constraint. Prices adjust fully to monetary shocks after one period. There is no persistence to the price-adjustment process. We consider two separate types of price setting. In the first type, producers must set prices in their own currency. For example, the home currency price of home goods is preset, and unresponsive to monetary shocks. This implies that the price for home goods paid by foreign consumers varies when the exchange rate changes in response to monetary shocks. In the other type of model, producers preset prices in consumers’ currencies. Home firms set one price for home-country consumers in the home currency and another price for foreign-country consumers in the foreign currency. In the fixed exchange rate model, the currency in which prices are set is irrelevant since the exchange rate is assumed to be fixed permanently. In each of the models, the objective of the firm managers is to maximize the expected utility of the representative owner of the firm.

We shall contrast the exchange-rate regime choice when all production by home firms is accomplished with domestic factors of production (as in Devereux and Engel (1998)), with a production configuration in which the domestically-owned firm produces goods for sale to foreigners using foreign factors of production. (Similarly, goods sold by the foreign-owned firm to home residents are produced by home-country factors.)

Consumers

The representative consumer in the home country maximizes:

$$U_t = E_t\left(\sum_{s=t}^{\infty} b^{s-t} u_s\right), \quad 0 < b < 1$$

where

$$u_s = \frac{1}{1-r} C_s^{1-r} + \ln\left(\frac{M_s}{P_s}\right) - hL_s \qquad\qquad r > 0.$$

C is a consumption index over home and foreign consumption:

$$C = \frac{C_h^n C_f^{1-n}}{n^n (1-n)^{1-n}}.$$

There are n identical individuals in the home country, $0 < n < 1$, and 1-n in the foreign country. C_h and C_f, in turn, are indexes over consumption of goods produced at home and in the foreign country, respectively:

$$C_h = \left[n^{-1/l} \int_0^n C_h(i)^{l-1/l} di\right]^{l/l-1}; \qquad C_f = \left[(1-n)^{-1/l} \int_n^1 C_f(i)^{l-1/l} di\right]^{l/l-1}$$

The elasticity of substitution between any two goods produced within a country is l , which we assume to be greater than 1. There is a unit elasticity of substitution between the home goods and foreign goods bundles. M/P represents domestic real balances, and L is the labor supply of the representative home agent.

The utility function we consider here is one case investigated by Devereux and Engel (1998). They examine a more general welfare function in which real balances enter as a power function, and labor enters quadratically. We choose this simpler representation for utility here for several reasons. The model can be solved analytically in closed form for this welfare function, while the more general welfare function requires us to use second-order Taylor series approximations. We found in Devereux and Engel (1998) that the assumption of real balances entering logarithmically makes little difference for the conclusions on the optimality of fixed vs. floating exchange rates.

We justify the linearity of labor on two grounds. First, some literature has found that when individuals face a discrete choice of working a fixed number of hours or not working at all, the appropriate aggregate representation for leisure in the utility function is linear.[4] Second, we do not believe there is a strong tradition in economics of considering the effects of risk-averse behavior toward uncertainty about consumption of leisure. Risk-neutrality toward leisure is probably a plausible assumption.[5]

P, the price index, is defined by

(1) $P = P_h^n P_f^{1-n}$,

where

$$P_h = \left[\frac{1}{n} \int_0^n P_h(i)^{1-l}\, di \right]^{1/(1-l)}, \qquad P_f = \left[\frac{1}{1-n} \int_n^1 P_f(i)^{1-l}\, di \right]^{1/(1-l)}.$$

[4] See Hansen (1985) and Rogerson (1988).

The preferences of foreign consumers are similar to home country residents' preferences. The terms in the utility function involving consumption are identical in the two countries. Note that this implies that if the law of one price should hold for both goods, then purchasing power parity obtains: $P_t = S_t P_t^*$, where S_t is the home-currency price of foreign currency. The functional form for real balances and labor are also the same, but, for foreign residents, they are functions of foreign real balances and foreign labor supply.

The optimal intratemporal consumption choices take on simple forms:

$$C_h(i) = \frac{1}{n}\left[\frac{P_h(i)}{P_h}\right]^{-l} C_h, \qquad C_f(i) = \frac{1}{1-n}\left[\frac{P_f(i)}{P_f}\right]^{-l} C_f;$$

$$P_h C_h = nPC, \qquad P_f C_f = (1-n)PC;$$

and,

$$\int_0^n P_h(i) C_h(i) di = P_h C_h, \qquad \int_n^1 P_f(i) C_f(i) di = P_f C_f.$$

We assume that there are complete asset markets -- residents of each country can purchase state-contingent nominal bonds. As Obstfeld and Rogoff (1998) emphasize, the structure of the utility functions ensures that there is complete consumption risk sharing when the law of one price holds, irrespective of what assets are traded.[6] But the law of one price does not hold in the pricing-to-market model. We consider the assumption of complete asset markets to be convenient, and an approximation to the assumption of perfect capital mobility.

Given the intratemporal consumption choices above, the budget constraint of the representative home agent is:

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[5] The existence of overtime pay, however, suggests that there is some desire to smooth leisure.

$$P_t C_t + M_t + \sum_{z^{t+1}} q(z^{t+1}, z^t) B(z^{t+1}) = W_t L_t + p_t + M_{t-1} + B_t + T_t \,.$$

$B(z^{t+1})$ are contingent home-currency denominated nominal bonds whose prices at time *t* are $q(z^{t+1}, z^t)$, where z^t represents the state at time t. p_t is the representative agent's share of profits from all home firms. T_t represents monetary transfers from the government. W_t is the wage rate.[7]

In addition to the consumption demand equations, we can derive the money demand equation for the representative home-country resident:

$$(2)\quad \frac{M_t}{P_t} = \frac{c C_t^r}{1 - d_t},$$

where d_t is the inverse of the gross nominal interest rate, equal to:

$$d_t = E_t\left(b \, \frac{C_{t+1}^{-r} P_t}{C_t^{-r} P_{t+1}} \right).$$

The trade-off between consumption and leisure is given by:

$$(3)\quad \frac{W_t}{P_t C_t^r} = h \,.$$

Before turning to the role of government, and the production and pricing decision of firms, it is convenient to express the equilibrium condition that arises from the complete market in nominal bonds:

$$(4)\quad \frac{S_t P_t^*}{P_t} = \left(\frac{C_t}{C_t^*} \right)^r .$$

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6 See Cole and Obstfeld (1991) for an analysis of how the terms of trade changes can serve as a substitute for capital mobility.

7 To avoid excess notation, we will not continue to use this state-contingent notation from here on. The important point to remember is that complete asset markets are necessary to sustain the full risk-sharing condition in equation (1.10) below.

There are immediately two important things that arise out of condition (4). First, when the law of one price holds (as it does in the PCP and FER models defined below), then home consumption always equals foreign consumption: $C_t = C_t^*$.[8]

Second, the factor price equalization holds. This arises immediately out of equations (3) and (4), and does not require that the law of one price hold for either good. We have:

$$W_t = S_t W_t^* .$$

Government

Government alters the money supply with direct transfers. The government budget constraint (in per capita terms) is

$$M_t = M_{t-1} + T_t .$$

Firms

Here we shall discuss the production and pricing decisions for firms with internationalized production. Ultimately we will contrast our welfare results in this model with those in Devereux and Engel (1998). That paper assumes all production by home firms takes place domestically, and all production by foreign firms takes place in the foreign country.

Firms are assumed to be monopolistic competitors. The production function by the home-country firm *i* for sale to domestic residents is given by:

$$X_{ht}(i) = L_t(i) .$$

So, total output sold to domestic residents by firm *i*, $X_{ht}(i)$, is produced using only labor. Similarly, the production function for sales to foreign consumers by firm *i* is given by:

[8] It is not necessary that markets be complete to sustain this condition in the PCP model.

$$X^*_{ht}(i) = L^*_t(i).$$

The objective of the domestic firms is to set prices to maximize the expected utility of the owners, who are the domestic residents. Firms must set prices before information about the random domestic and foreign money supplies is known. We will consider three models:

PCP: PCP refers to producer-currency pricing. Producers set the price in their own currency. The price that foreigners pay for home goods, and the price that home residents pay for foreign goods fluctuates when the exchange rate changes. This model is examined by Obstfeld and Rogoff (1995, 1998).

PTM: PTM refers to pricing to market. Producers set the price in the consumers' currency. Prices consumers face do not respond to exchange rate changes. A PTM model was introduced by Devereux and Engel (1998).

FER: FER refers to permanently fixed exchange rates. Prices are set ahead of time, but the choice of currency is irrelevant since there is essentially a single currency.

No state-contingent pricing is allowed in the three models.

The firm owners maximize the expected present value of profits using the market nominal discount factor for the owners of the firm. This is equivalent to maximizing expected utility of the owners. In the PCP model, the firms choose a single price for both markets. Given there is no intertemporal aspect to the firms' optimization problems (see, Obstfeld and Rogoff (1998)), this reduces to maximizing:

$$E_{t-1}\Big(d_{t-1}\big((P_{ht}(i) - W_t)X_{ht}(i) + (P_{ht}(i) - S_t W^*_t)X^*_{ht}(i)\big)\Big).$$

Given that factor price equalization holds in all of our models, this objective function can be simplified to:

$$E_{t-1}\Big(d_{t-1}\big((P_{ht}(i) - W_t)(X_{ht}(i) + X^*_{ht}(i))\big)\Big)$$

Then, using the fact that home and foreign consumption are equal in the PCP model, the optimal price set by the home firm is:

$$P_{ht}(i) = P_{ht} = \frac{l}{l-1}\frac{E_{t-1}(C_t^{1-r}W_t)}{E_{t-1}(C_t^{1-r})}.$$

From equation (3), this price-setting rule can be rewritten as:

$$(5)\ P_{ht} = \left(\frac{l h}{l-1}\right)\frac{E_{t-1}(P_t C_t)}{E_{t-1}(C_t^{1-r})}.$$

In a world of certainty, the price would be a mark-up over unit labor costs. Here, there is a risk premium incorporated in the goods price (as Bacchetta and van Wincoop (1998) and Devereux and Engel (1998) discuss) arising from the covariance of the firm's profits with the marginal utility of consumption.

The law of one price holds for the price charged to foreigners by the home firm:

$$(6)\ P_{ht}^* = P_{ht}/S_t.$$

Analogous relationships hold for the prices set by the foreign firms:

$$(7)\ P_{ft}^* = \left(\frac{l h}{l-1}\right)\frac{E_{t-1}(P_t^* C_t^*)}{E_{t-1}(C_t^{*1-r})},$$

$$(8)\ P_{ft} = S_t P_{ft}^*.$$

In the PTM model, the firm chooses two different prices – one for residents of its own country, and for residents of the other country. The typical home firm maximizes:

$$E_{t-1}\left[d_{t-1}\left(P_{ht}(i)X_{ht}(i) + S_t P_{ht}^*(i)X_{ht}^*(i) - W_t(X_{ht}(i) + X_{ht}^*(i))\right)\right],$$

where we have used factor-price equalization. The price charged by the home firm to the home residents is the same as in the PCP model, and is given in equation (5). But, making use of the risk-sharing condition (4), we find the price charged to foreign residents is given by:

(9) $P^*_{ht} = \left(\frac{l}{l-1}\right)\frac{E_{t-1}W^*_t C^{*1-r}_t}{E_{t-1}(C^{*1-r}_t)} = \left(\frac{lh}{l-1}\right)\frac{E_{t-1}P^*_t C^*_t}{E_{t-1}(C^{*1-r}_t)}$.

Likewise, the price charged by foreign firms to its own residents is the same as in the PCP model, and is given by equation (7), but the price charged to home-country consumers in the PTM model is:

$$(10) \qquad P_{ft} = \left(\frac{l}{l-1}\right)\frac{E_{t-1}(C^{1-r}_t W_t)}{E_{t-1}(C^{1-r}_t)} = \left(\frac{lh}{l-1}\right)\frac{E_{t-1}(P_t C_t)}{E_{t-1}(C^{1-r}_t)}.$$

Comparing equation (10) to (5), we see that in the PTM model, home residents pay exactly the same price for the home and the foreign good. Likewise, equations (9) and (7) show that the foreign resident pays the same price for the two goods (although the prices do not necessarily equal the price paid by home residents.)

In the FER model, the exchange rate is fixed at all times, so the pricing rule is the same whether the firms state prices in their own currency or the foreign currency.

Equilibrium in the home market for labor requires that labor employed equal to demand for home goods and foreign goods by home residents:

$$(11) \qquad L_t = n\frac{P_t C_t}{P_{ht}} + (1-n)\frac{P_t C_t}{P_{ft}}.$$

In both of the models with floating exchange rates, some simplifications of equation (11) are readily derived. In the PTM model, we note that since $P_{ht} = P_{ft} = P_t$, we arrive at

$$(12) \qquad L_t = C_t.$$

The logic of that relationship is straightforward. Since the relative price of home goods to foreign goods is always one, and both goods are produced by domestic labor, then the amount of labor demanded will equal the amount of the good consumed (since the production function is $Y_t = L_t$.)

The expectation of equation (11) can be simplified in the PCP model. From equation (5):

(13) $$E_{t-1}\left(\frac{P_t C_t}{P_{ht}}\right) = \frac{l-1}{l h} E_{t-1}\left(C_t^{1-r}\right).$$

If we use the facts that $\frac{P_t}{P_{ft}} = \frac{P_t^*}{P_{ft}^*}$, and $C_t = C_t^*$, then we get, using equation (7):

(14) $$E_{t-1}\left(\frac{P_t C_t}{P_{ft}}\right) = E_{t-1}\left(\frac{P_t^* C_t^*}{P_{ft}^*}\right) = \frac{l-1}{l h} E_{t-1}(C_t^{*1-r}) = \frac{l-1}{l h} E_{t-1}(C_t^{1-r}).$$

Equations (11), (13) and (14) then tell us in the PCP model:

(15) $$E_{t-1}(L_t) = \frac{l-1}{l h} E_{t-1}(C_t^{1-r}).$$

These equations suffice to solve the three models. In the next section, we will analyze welfare under fixed exchange rates, and under the two models of floating exchange rates.

2. Welfare Comparisons

We shall assume the money supply follows a random walk:

(16) $$E_t({}^{M_t}\!/\!_{M_{t+1}}) = m.$$

Under these assumptions, it is easy to verify from the money demand equation, (2.2), that consumption is a function only of the real money supply:

(17) $$C_t^r = \left(\frac{1-mb}{c}\right)\frac{M_t}{P_t}.$$

An analogous equation holds for the foreign country.

This equation implies that the home and foreign nominal interest rates must be constant. The gross nominal interest rate is given by the inverse of d_t, which is defined is section 1. The ex post real interest rate is determined by the rate of change of consumption: higher consumption growth is associated with higher real interest rates.

Consider a monetary expansion in the home country under flexible exchange rates. In each of our models, a monetary expansion lowers the real interest rate. Since money is neutral in the long-run (i.e., after one period), current consumption must grow. A monetary expansion also leads to expected inflation. The future domestic price level increases more than the current price level in each of our models. When money enters the utility function logarithmically, the increase in expected inflation exactly offsets the decline in the real interest rate, leaving the nominal interest rate unchanged.

It follows immediately that in both the PCP and PTM models, the exchange rate must follow a random walk. In our model, uncovered interest parity may not hold exactly, because there may be a foreign exchange risk premium. However, under assumptions that will be introduced shortly (specifically, that the variance of monetary shocks is constant over time), the risk premium is constant. Since domestic and foreign interest rates are also constant, the expected change in the (log of) the nominal exchange rate must be zero. Because money is neutral in the long run, the current change in the exchange rate is proportional to changes in domestic money (with a positive sign) and foreign money (with a negative sign.) In fact, from equations (4) and (17):

$$(18) \qquad S_t = \frac{M_t(1-mb)}{M_t^*(1-m^*b)}.$$

We can now derive one of the chief results of this section: in the PTM model, foreign monetary shocks have no effect on domestic consumption, but they do affect home consumption in the PCP model. Recall that $P = P_h^n P_f^{1-n}$. In the PTM model, P is predetermined, so it is not affected by foreign (or domestic) money shocks. It follows from equation (17) that domestic consumption is determined in the short-run entirely by the domestic money supply. Changes in the foreign money supply have no effect on domestic consumption. In contrast, in the PCP model, P_f increases when the price of the foreign currency increases, since that price is fixed in foreign currency terms. So, a one-percent jump up in the foreign money supply leads to a one-percent decrease in S, a one-percent decrease in P_f, and a $1-n$ percent decline in P. So, a one-percent increase in the foreign money supply induces a $\frac{1-n}{r}$ percent increase in C.

We have compared the two floating exchange rate models (PCP and PTM.) How do these models compare to the fixed-exchange rate (FER) model? To keep exchange rates fixed, the domestic money supply must move in proportion to the foreign money supply (see equation (18).) It follows that in response to a one-percent shock to the foreign money supply, the domestic money supply must change one percent. Since goods prices do not change, equation (17) tells us there must be a $\frac{1}{r}$ percent change in domestic consumption.

If our only concern were how the variance of foreign money shocks affected the variance of domestic consumption, it is clear that floating rates dominate fixed exchange rates. Under fixed rates, a one percent shock to foreign money leads to a $\frac{1}{r}$ percent changes in domestic consumption, compared to a change in domestic consumption of only $\frac{1-n}{r}$ percent in the PCP model; and zero percent in the PTM model. Table 1 shows how the variance of (the log of) consumption is related to the variance of (the log of) the foreign money supply in each model.[9]

[9] These results are derived formally in Appendix 1 in equations (A.9), (A.16) and (A.23)

Table 1

Variance of Domestic Consumption
(holding domestic money supply constant)

	s_c^2
PCP	$s_{m^*}^2 (1-n)^2 / r^2$
PTM	0
FER	$s_{m^*}^2 / r^2$

But, the variance of consumption this is not the only welfare consideration. Even ignoring the effect on welfare coming from money in the utility function, the variance of foreign monetary shocks has further effects on utility. Leisure enters utility linearly, so greater variance of output (and hence leisure) does not directly influence welfare. But, the variance of foreign monetary shocks has further effects on welfare because it influences the means of both consumption and leisure. This channel arises from the effects that monetary variances have on goods prices.

We assume shocks to money supplies are log-normally distributed. Using equation (16):

$$(19) \qquad m_{t+1} - m_t = -\ln(m) + \frac{1}{2} s_m^2 + u_{t+1},$$

where m_t is the log of M_t (we will follow the convention that lower-case letters are logs of upper-case letters), u_{t+1} is the white-noise shock to domestic money, and s_m^2 is the variance of u_{t+1}. An analogous equation holds for the foreign money supply process.

We can write equations (17) and (18) as

$$(20) \qquad r c_t = m_t - p_t + \ln(1 - mb / c),$$

$$(21) \qquad s_t = m_t - m_t^* + \ln(1 - mb) - \ln(1 - m^* b).$$

In log terms, equation (3) can be written as:

(22) $w_t = p_t + rc_t + \ln(h)$,

and analogously for the foreign country.

The solutions for goods prices, consumption and leisure depend on the particular model of price-setting behavior. The Appendix derives solutions in terms of the exogenous variables – the domestic and foreign money supplies – for each of our models.

The average levels of consumption and leisure can be different depending on the exchange-rate regime. This may seem surprising, since it implies average long-run consumption depends on a monetary choice –fixed or floating exchange rates. The intuition for this outcome can be seen from equation (3). The level of consumption is related to the mark-up of prices over wages: when the mark-up is smaller, the level of consumption is higher. The level of prices is affected by the exchange-rate regime because the risk premium incorporated in prices differs between regimes. In turn, since labor is used to produce consumption goods, the expected level of leisure is also different under the different exchange-rate regimes.

We compare the influence of the variance of foreign money shocks on the expected level of consumption by setting the variance of domestic money equal to zero in equations (A.10), (A.18) and (A.24). These values are reported in Table 2.

Table 2

Expected Level of Domestic Consumption
(holding domestic money supply constant)

	$E(C)$
PCP	$\left(\frac{l-1}{lh}\right)^{1/r} \exp\left[-\left(\frac{(1-n)^2 + r(1-n)(1-2(1-n))}{2r^2}\right)s^2_{m^*}\right]$
PTM	$\left(\frac{l-1}{lh}\right)^{1/r}$
FER	$\left(\frac{l-1}{lh}\right)^{1/r} \exp\left[-\left(\frac{1-r}{2r^2}\right)s^2_{m^*}\right]$

From Table 2, the expected level of consumption is higher in the FER model compared to the PCP model when $r > \frac{2-n}{3-2n}$. A sufficient condition is $r > 1$. The condition of $r > 1$ is necessary and sufficient for expected consumption to be higher in the FER model than in the PTM model. Although there is little agreement empirically about the correct value for r, virtually all studies agree that $r > 1$. So, while the variance of consumption is higher for fixed exchange rates than in either floating exchange rate model, there is a higher average level of consumption under fixed exchange rates.

This implies a trade-off in utility. Examining for the moment the consumption term alone, using the fact that consumption is log-normal, we can write:

$$(23) \qquad \frac{1}{1-r}E(C^{1-r}) = \frac{1}{1-r}(E(C))^{1-r} \cdot \exp\left(\frac{-r(1-r)}{2}s^2_c\right).$$

Welfare is positively related to the expected level of consumption, but falls with increases in the variance of consumption.

Utility also depends on expected leisure. The greater the average level of employment, the lower is welfare. Table 3 reports the expected levels of employment in each model (taken from equations (A.12), (A.18) and (A.26).

Table 3

Expected Level of Domestic Employment
(holding domestic money supply constant)

	$E(L)$
PCP	$\left(\frac{l-1}{lh}\right)^{1/r} \exp\left[-\left(\frac{(1-n)(1-r)(1-n(1-r))}{2r^2}\right)s_{m^*}^2\right]$
PTM	$\left(\frac{l-1}{lh}\right)^{1/r}$
FER	$\left(\frac{l-1}{lh}\right)^{1/r} \exp\left[-\left(\frac{1-r}{2r^2}\right)s_{m^*}^2\right]$

Employment is higher under fixed exchange rates, as compared to floating under PCP, when $1 < r < \frac{2-n}{1-n}$. When *n* is large (that is, close to one), it is likely that expected employment is higher and expected leisure lower under fixed exchange rates. But, for a small country (*n* close to zero), average employment under fixed exchange rates will be smaller unless r falls in the narrow range between one and two. So, in comparing leisure effects between FER and PCP, the size of the country is quite important. On the other hand, expected employment is higher and expected leisure lower under FER compared to PCP when $r > 1$.

In comparing welfare under fixed and floating exchange rates, **w**e look at the effects of the variance of foreign money shocks on domestic welfare. A fixed exchange rate system eliminates the possibility of domestic money shocks, but even under floating rates central banks can choose a zero variance. So, we set $\sigma_m^2 = 0$ in our welfare analysis.

Real money balances enter the utility function. It may not be wise to evaluate exchange rate systems in terms of how they affect the real balance part of the utility function. Money in the utility function is a convenient way to create demand for an asset, money, that would otherwise be dominated by other assets. But there are other ways to model demand for money that may be more realistic (and more complicated) that do not involve welfare being directly influenced by holdings of real balances. In making welfare comparisons, we will assume real balances are not important in welfare ($\chi \to 0$).

Utility, then, can be expressed in terms of expected consumption, the variance of consumption, and expected leisure. Tables 1, 2 and 3 compare the models for each of these components of utility. The comparisons seem ambiguous: FER is the worst in terms of the variance of consumption, the best in terms of expected consumption, and there is some ambiguity about the ranking in terms of expected leisure. We can further clarify the matter by looking at all three effects together.

PCP vs. FER models

In the PCP and FER models we can use equation (15) to write the welfare expression simply as a function of the mean and variance of consumption:

(24) $$E(u) = \frac{1+\rho(\lambda-1)}{\lambda(1-\rho)} E(C^{1-\rho}) = \frac{1+\rho(\lambda-1)}{\lambda(1-\rho)} (E(C))^{1-\rho} \cdot \exp\left(\frac{-\rho(1-\rho)}{2}\sigma_c^2\right).$$

There is a trade-off in choosing between fixed and floating exchange rates. Fixed exchange rates have a higher expected level of consumption (when $\rho > \frac{2-n}{3-2n}$), but under floating rates the variance of consumption is lower.

We find that welfare is higher under floating rates when $r \leq \frac{2-n}{1-n}$. When $r \leq 2$, floating rates are always better. As *n* approaches 1, so that the home country is getting very large relative to the small country, then floating rates dominate. This accords with the intuition that smaller countries may find it desirable to fix their exchange rate to the currency of a larger country, but large countries are better off with their own independent currency.

As r increases, the fixed exchange rate system's advantage in terms of expected consumption increases. Its disadvantage in terms of variance also falls as the square of r increases, although that is offset by the fact that the importance of the variance in welfare rises with the square of r (see equation (24).) On net, for large values of r, fixed exchange rates become more desirable.

PTM vs. FER Models

We have seen from Table 1 that foreign monetary variance does not affect either the mean or variance of consumption in the PTM model. Compared to the FER model, the variance of consumption is lower, but the mean of consumption is also lower when $r > 1$. The PTM model has the advantage in terms of expected leisure when $r > 1$.

As is the case with the PCP and FER models (equation (15)), we have $E(L) = \frac{l - 1}{l h} E(C^{1-r})$.

In the PTM model, this follows from equations (12) and (A.14). So, conveniently, the analysis simplifies to a comparison of the models in terms of expected consumption and the variance of consumption. Equation (24) holds in the PTM model as well as the other two models.

Utility in the PTM model is not influenced by foreign monetary policy shocks. We can derive:

$$E(u_{PTM}) = \left(\frac{l-1}{hl}\right)^{1-r/r}\left(\frac{1+r(l-1)}{l(1-r)}\right).$$

This can be compared directly to welfare in the FER model:

$$E(u_{FER}) = \left(\frac{l-1}{hl}\right)^{1-r/r}\left(\frac{1+r(l-1)}{l(1-r)}\right)\exp\left(\frac{r-1}{2r^2}s^2_{m^*}\right).$$

It is clear from examination of these equations that any variance in foreign money shocks lowers welfare in the FER model, so

$$E(u_{PTM}) > E(u_{FER})$$

for all admissible parameter values. Even though, when $r > 1$, there may be less expected consumption under floating exchange rates and PTM, the fact that home consumption is completely insulated from foreign money shocks is enough to insure that floating rates always dominate fixed exchange rates in this model.

3. How Internationalized Production Matters

How do the comparisons between levels of welfare under fixed and floating exchange rates in the models of this paper, with internationalized production, relate to the comparisons in Devereux and Engel (1998) where there is no internationalized production?

There are surprising similarities in the welfare comparisons. A major reason why the welfare comparisons are not very different whether or not we assume internationalized production is that there is factor price equalization in our model. Since home-country wages equal foreign-country wages, the prices set by firms are the same whether or not domestic labor or foreign labor produces output. That is, there is a risk premium incorporated in nominal goods prices arising from the covariance of labor costs with revenues. But, that risk is the same whether domestic or foreign labor is employed because the two types of labor are paid the same wage.

Why is there factor-price equalization? From equation (3), the marginal utility of taking one more dollar's worth of leisure, $\frac{h}{W_t}$, is equal to the marginal utility from one more dollar's worth of consumption, $\frac{C_t^{-r}}{P_t}$. That is, each agent only works so much that the last dollar earned creates as much utility from consumption as it does disutility from working. But, our assumption of perfect capital mobility and complete nominal asset markets means that the marginal utility of a dollar for the home-country resident is worth the same for the foreigh agent: $\frac{C_t^{-r}}{P_t} = \frac{C_t^{*-r}}{S_t P_t^*}$. Together, these imply that the dollar value of wages must be the same in the two countries: $W_t = S_t W_t^*$. In short, the complete asset markets in nominal bonds assures that the marginal value of a dollar spend on leisure is the same in the two countries: $\frac{h}{W_t} = \frac{h}{S_t W_t^*}$.

Since wage equalization implies that the nominal price set by firms is not affected by the location of production, then neither exchange rates nor consumption levels (from equation (17)) are affected by the location of production. So, any welfare comparison involving consumption is not influenced by whether or not there is internationalized production.

Welfare is also influenced by leisure. One would expect that term to be different depending on where production is located. In Devereux and Engel (1998), domestic workers produce goods consumed by foreigners, so movements in foreign demand for domestic goods change employment. But, in the model of this paper, domestic workers produce only for domestic consumption: they produce both the home and foreign goods for home consumers.

Surprisingly, in the PCP model, even the leisure term is not affected by the location of production. In the PCP model, as Obstfeld and Rogoff (1998) emphasize, the current account is always balanced. Even though there is perfect capital mobility, the unitary elasticity of demand between foreign and domestic consumption combined with the law of one price assumption ensures that the value of goods purchased from abroad by home residents continuously equals the value of goods sold to foreign residents by home firms. But, the prices of home goods and foreign goods are expected to be the same (prior to the realization of monetary shocks.) Since the value of exports and imports are equal, and the expected prices are the same, the expected quantity of goods home firms produce for foreign residents always equals the expected quantity of goods foreign firms produce for home residents. So, it does not matter where the goods are produced – expected employment will not be affected by the location of production.

Welfare in the PCP model is not affected by the presence of internationalized production. The same is true for the FER model, since given the structure of the model, the FER model is identical to the PCP model with $n = 1$. The welfare comparisons for this paper between the PCP and FER worlds are exactly the same as in Devereux and Engel (1998) for these two models.

Expected leisure in the PTM model does depend on where output is produced. In the model of this paper, where domestic labor only produces for home consumers, there is no influence of foreign money shocks on expected employment. In the model of Devereux and Engel (1998), foreign money shocks change domestic employment because they affect foreign demand for home goods (which, in that model, are produced with domestic labor.) In that model, when $r > 1$, a higher foreign monetary variance raises expected demand for home goods and lowers expected home leisure. So, expected domestic leisure is higher when there is internationalized production. Still, whether or not there is internationalized production, floating rates in the PTM model always yield higher expected utility than fixed exchange rates.

4. Conclusions

We find that the hypothesis of how prices are set has implications for the optimal choice of exchange-rate regime. If we follow the traditional literature in assuming that prices are set in producers' currencies, and that the law of one price holds, there is no clear-cut answer on which regime is preferable. When a country is small, or very risk averse, it would prefer fixed exchange rates; otherwise, floating exchange rates are preferred.

However, the empirical evidence supports the model of pricing in which producers set different prices in different markets. Prices are set in consumers' currencies and adjust slowly to demand shocks. In that case, we find that floating exchange rates are unambiguously preferred to fixed exchange rates.

That conclusion is not altered by incorporating internationalized production in the model. Indeed, we find under pricing to market that the economy is even more sheltered from foreign shocks in a floating exchange rate system then in a world where all production of the domestic firm takes place in the home country. The reason is straightforward: one potential avenue for foreign shocks to hit the home economy is through employment changes in response to changes in foreign demand for the home good. If a monetary shock abroad changes foreign demand for the home good, that may alter domestic employment. But, to the extent that the foreign demand is met by production by foreign subsidiaries of the home country, domestic employment is sheltered from the foreign shock. So, one avenue through which foreign shocks could affect domestic employment and welfare under a floating exchange rate system with pricing to market is closed when the home good is produced internationally.

The model considered here is quite simple. We only look at a special case examined by Devereux and Engel (1998), in which real money balances enter the utility function logarithmically and leisure enters linearly. This specification simplifies the analysis considerably. For example, nominal interest rates are constant in equilibrium. But, it also loses some of the interesting and more realistic dynamic behavior of the more complex models. Still, in Devereux and Engel (1998), the flavor of the welfare analysis of exchange-rate regimes was not appreciably altered by consideration of the more complex model.

One important extension to this model would allow for price-adjustment that lasts longer than a single period. Indeed, it is possible that the speed of price adjustment could differ between exchange-rate regimes, and that could alter the welfare analysis. We would also like to consider investment questions. In this study, internationalized production is undertaken using only foreign labor as an input. More interesting would be an analysis that allowed for domestically-owned capital to be combined with foreign labor in producing output for sale abroad.

We are not certain which direction these extensions would tilt the welfare analysis. But, this study has highlighted some differences in the exchange-rate systems, and the roles of price-setting and internationalized production, that would be important in any extended model.

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Appendix

Recall that the pricing formulas for P_{ht} and P^*_{ft} are the same in all three models. Using the fact that prices and consumption will be log-normally distributed in equilibrium, we can write equation (5) as:

(A.1) $p_{ht} = m_{t-1} - \ln(m) + s^2_m + (1-r)s_{mc} + \ln(lh(1-mb) \big/ c(l-1)) ,$

where s_{mc} is the covariance of m_t and c_t conditional on $t-1$ information. (In general, we will use the notation s_{xz} to denote $Cov_{t-1}(x_t, z_t)$, and s^2_z to denote $Var_{t-1}(z_t)$.) Similar derivations yield from equation (7):

(A.2) $p^*_{ft} = m^*_{t-1} - \ln(m^*) + s^2_{m^*} + (1-r)s_{m^*c^*} + \ln(lh(1-m^*b) \big/ c(l-1)) .$

Equations (A.1) and (A.2) hold in all three models. We will also use the relationship

(A.3) $p_t = np_{ht} + (1-n)p_{ft} ,$

and the analogous expression for the foreign price level. These equations must be augmented by equations for p_{ft} and p^*_{ht}, as well as equations for output, in order to complete derivations in each model.

A.1 The PCP Model

The law of one price holds in this model, so

(A.4) $p_{ft} = s_t + p^*_{ft} ,$

(A.5) $p^*_{ht} = p_{ht} - s_t .$

Recall, also, the perfect risk-sharing property of this model, so that

$$c_t = c^*_t .$$

Then, using (21), (A.2), (A.3) and (A.4), we can derive for the domestic price level

$$\text{(A.6)}\ p_t = m_t - n(u_t - \frac{1}{2}s_m^2 - (1-r)s_{mc}) - (1-n)(u_t^* - \frac{1}{2}s_{m^*}^2 - (1-r)s_{m^*c}) + \ln(\frac{lh(1-mb)}{c(l-1)}).$$

From equations (21) and (A.6) we get

$$\text{(A.7)}\quad rc_t = n(u_t - \frac{1}{2}s_m^2 - (1-r)s_{mc}) + (1-n)(u_t^* - \frac{1}{2}s_{m^*}^2 - (1-r)s_{m^*c}) - \ln(\frac{lh}{l-1}).$$

We can derive expressions for s_{mc} and s_{m^*c} directly from equation (A.7):

$$s_{mc} = \frac{n}{r}s_m^2,$$

$$s_{m^*c} = \frac{1-n}{r}s_{m^*}^2.$$

Plugging these into (A.7), we arrive at our expression for domestic consumption:

$$\text{(A.8)}\quad c_t = \frac{n}{r}u_t + \frac{1-n}{r}u_t^* - (\frac{nr + 2n^2(1-r)}{2r^2})s_m^2 - (\frac{(1-n)r + 2(1-n)^2(1-r)}{2r^2})s_{m^*}^2 - \frac{1}{r}\ln(\frac{lh}{l-1}).$$

The variance of domestic and foreign money shocks affects utility also because they affect the variance of consumption. From equation (A.7),

$$\text{(A.9)}\qquad s_c^2 = \frac{n^2}{r^2}s_m^2 + \frac{(1-n)^2}{r^2}s_{m^*}^2.$$

The expected level of consumption is given by:

(A.10)

$$E(C) = \exp(Ec + {s_c^2}/{2}) = \left(\frac{l-1}{lh}\right)^{1/r} \exp\left(-\left(\frac{n^2 + rn(1-2n)}{2r^2}\right)s_m^2 - \left(\frac{(1-n)^2 + r(1-n)(1-2(1-n))}{2r^2}\right)s_{m^*}^2\right)$$

Expected utility depends on both the expected level and the variance of consumption:

(A.11)

$$\frac{1}{1-r}E(C^{1-r}) = \frac{1}{1-r}E(\exp((1-r)c)) = \frac{1}{1-r}(\exp((1-r)Ec + \frac{(1-r)^2}{2}s_c^2)) =$$

$$\left(\frac{1}{1-r}\right)\left(\frac{l-1}{hl}\right)^{1-r/r}\left(\exp\left(-\frac{n(1-r)(r+n(1-r))}{2r^2}s_m^2 - \frac{(1-n)(1-r)(1-n(1-r))}{2r^2}s_{m^*}^2\right)\right)$$

From equation (15), we can derive the expected level of output easily as:

(A.12)

$$E(L) = \frac{l-1}{hl}E(C^{1-r}) = \left(\frac{l-1}{hl}\right)^{1/r}\left(\exp\left(-\frac{n(1-r)(r+n(1-r))}{2r^2}s_m^2 - \frac{(1-n)(1-r)(1-n(1-r))}{2r^2}s_{m^*}^2\right)\right)$$

A.2 The PTM Model

From equations (5) and (10), we see that P_{ft} and P_{ht} are equal. Using this fact, and the definition of the price index in equation (1), we have:

(A.13) $P_{ft} = P_{ht} = P_t = \dfrac{hl}{l-1}\dfrac{E_{t-1}(P_t C_t)}{E_{t-1}(C_t^{1-r})}$.

But, since P_t is in the time *t*-1 information set, equation (A.13) gives us that

(A.14) $E_{t-1}(C_t^{1-r}) = \dfrac{hl}{l-1}E_{t-1}(C_t)$.

Writing this equation in logs gives us:

$$(1-r)E_{t-1}(c_t) + \frac{(1-r)^2}{2}s_c^2 = \ln\left(\frac{lh}{l-1}\right) + E_{t-1}(c_t) + \frac{1}{2}s_c^2.$$

Solving, we find

(A.15) $E_{t-1}(c_t) = \dfrac{-1}{r}\ln\left(\dfrac{lh}{l-1}\right) + \dfrac{r-2}{2}s_c^2$.

Since P_t is in the time *t*-1 information set, we have from equation (20)

(A.16) $$s_c^2 = \frac{1}{r^2} s_m^2 .$$

As compared to the PCP model (equation (A.9)), domestic monetary variance has a greater effect on the variance of domestic consumption. Substituting into equation (A.15), we get

(A.17) $$E_{t-1}(c_t) = \frac{-1}{r}\ln\left(\frac{lh}{l-1}\right) + \frac{r-2}{2r^2} s_m^2 .$$

The expected level of consumption is given by

(A.18) $$E(C) = \exp(Ec + s_c^2/2) = \left(\frac{l-1}{lh}\right)^{1/r} \exp\left(\frac{r-1}{2r^2} s_m^2\right).$$

The expected utility term involving consumption is derived directly from equation (A.14) using equation (A.18):

(A.19) $$\frac{1}{1-r} E(C^{1-r}) = \left(\frac{1}{1-r}\right)\left(\frac{l-1}{hl}\right)^{1-r/r} \left(\exp\left(\frac{r-1}{2r^2} s_m^2\right)\right).$$

From equation (12), $L_t = C_t$, so equation (A.18) gives us the expression for $E(L)$.

A.3 FER Model

We will assume the exchange rate is fixed at 1, so $s_t = 0$ for all *t*. From equation (21) it follows that

(A.20) $$m_t + \ln(1 - mb) = m_t^* + \ln(1 - m^* b) .$$

The domestic money supply moves in reaction to foreign money supply shocks in order to keep the exchange rate fixed.

Because exchange rates are fixed, $P_{ft} = P_{ft}^*$ and $P_{ht} = P_{ht}^*$. We can use equations (A.1) and (A.2) to derive expressions for prices. Noting that all prices are preset, the money demand equation (20) (combined with relation (A.20)) tells us

$$s_{mc} = s_{m^*c^*} = \frac{1}{r} s^2_{m^*}.$$

We can then derive:

(A.21) $$p_{ht} = p_{ft} = p_t = m^*_{t-1} + \frac{1}{r} s^2_{m^*} + \ln\left(\frac{l h (1 - m^* b)}{m^* c (l - 1)}\right).$$

Then, using equation (20) we get the expression for consumption under fixed exchange rates:

(A.22) $$c_t = \frac{1}{r} u^*_t + \frac{r - 2}{2r^2} s^2_{m^*} - \frac{1}{r} \ln\left(\frac{l h}{l - 1}\right).$$

The variance of consumption in the FER model is given by

(A.23) $$s^2_c = \frac{1}{r^2} s^2_{m^*}.$$

The expected level of consumption is given by:

(A.24) $$E(C) = \exp(Ec + s^2_c / 2) = \left(\frac{l - 1}{l h}\right)^{1/r} \exp\left(-\left(\frac{1 - r}{2r^2}\right) s^2_{m^*}\right).$$

The expected utility term involving consumption is

(A.25) $$\frac{1}{1 - r} E(C^{1-r}) = \frac{1}{1 - r} E(\exp((1 - r)c)) = \frac{1}{1 - r} (\exp((1 - r)Ec + \frac{(1 - r)^2}{2} s^2_c)) =$$
$$\left(\frac{1}{1 - r}\right)\left(\frac{l - 1}{h l}\right)^{1-r/r}\left(\exp\left(\frac{-(1 - r)}{2r^2} s^2_{m^*}\right)\right)$$

We can derive the expected level of output easily as:

(A.26) $$E(L) = \frac{l - 1}{h l} E(C^{1-r}) = \left(\frac{l - 1}{h l}\right)^{1/r}\left(\exp\left(-\frac{(1 - r)}{2r^2} s^2_{m^*}\right)\right).$$

Given the structure of the model, the fixed exchange rate model simply reduces to the PCP model with $n = 0$.