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CHAPTER 7

The Stability of Relative Income Status

So FAR attention has been restricted to factors that might explain why different professional men receive different incomes in the same year, why one man earns $10,000, another only $1,000. We have tried to account for these differences in terms of differences in attributes of individuals: the professions they practise, where they practise, the type and organization of their practices, the number of years they have practised, and their training and ability. Except for number of years in practice, these attributes are all relatively permanent. A man's ability and training remain the same except for changes connected with experience. He usually practises in the same community year after year and rarely changes the type or organization of his practice. Consequently, if factors like these were alone responsible for income differences, men in practice the same number of years would have the same positions in an array of practitioners by size of income—the same relative income status—year after year. True, their incomes in dollars would rise and fall with ups and downs in the fortunes of the economy and the profession as a whole, but these changes would affect all alike and leave their relative positions unaltered.

Of course, individuals do not have the same relative income status year after year. Some who rank high in one year rank low in another, and conversely. As just noted, these differences in relative income status can be ascribed only in small part to changes in the attributes considered in preceding chapters. They must be ascribed largely to accidental occurrences that impinge on the individual in one year but not in another: a stroke of good fortune may suddenly improve his status, a stroke of bad fortune may blight his career. This chapter is concerned with the effect of such accidental occurrences on the relative income status of the same individual in different years.

1 THE INCOMES OF THE SAME INDIVIDUALS IN DIFFERENT YEARS

The changes in relative income status that actually occur are illustrated by Table 55, which shows the incomes of 1,020 dentists in 1932 and 1934. Since the average income of the whole group is almost identical in the two years and variability of income does not differ widely,[1] the same actual income would imply approximately the same relative income status. Consequently, if there had been no changes in relative income status, all observations would fall in the diagonal cells printed in bold face in the table. In fact, only about a third of the observations are in the diagonal cells. And even some of the dentists whose incomes fall in the diagonal cells may have experienced a change in relative income status too small to move them from one income class to another. The two-thirds whose incomes are not in the diagonal cells clearly experienced a change in relative income status. For example, of the 138 dentists who had 1932 incomes between $1,000 and $1,500, one had a loss in 1934, another, an income between $4,000 and $5,000.

A simple summary measure of the degree of stability of relative income status shown by Table 55 is given by the coefficient of correlation between incomes in the two years.[2] The stronger the tendency for individuals to maintain their income status,

[1] The average income is $2,738 in 1932 and $2,726 in 1934; the standard deviation, $2,363 and $2,096 respectively.

Table 55 gives the total number of dentists as 1,020.5, and the number in one cell as 11.5. The explanation of the fraction is that, for the 1935 and 1937 samples, a return reporting income for only part of the year was counted as the fraction of the year it covered. See the explanatory note at the beginning of Appendix B.

[2] Since we are interested in shifts in relative position, the rank difference correlation might be preferable. Two sets of data might yield identical rank difference correlations but very different product moment correlations if both relationships were not linear. However, this difficulty is unimportant for our data (see below). Moreover, the rank difference correlation would be more arduous to compute with the large number of observations available.

TABLE 55

Relation between 1932 and 1934 Incomes

Dentists, 1935 Sample

INCOME IN 1932 (thousands of dollars) *	NUMBER OF DENTISTS WHOSE INCOME (THOUSANDS OF DOLLARS) IN 1934 WAS																					ALL CLASSES	
	−1 to 0	0	.001 to .5	.5 to 1	1 to 1.5	1.5 to 2	2 to 2.5	2.5 to 3	3 to 3.5	3.5 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	10 to 12	12 to 14 *	16 to 18	18 to 20 *	25 to 30 *	No.	Arith. mean income[1] in 1934 (dollars)
−2 to −1	1		1																			2	−125
−1 to 0	2		1																			3	−250
0		8		1		1																10	250
.001– .5	1	2	10	11.5	7	2																33.5	683
.5– 1	1	1	6	39	24	11	5	2														89	1,082
1 – 1.5	1		1	20	61	33	9	10	2		1											138	1,504
1.5– 2			1	5	31	49	44	13	8	1												152	1,928
2 – 2.5			2		6	37	55	31	16	11	1											159	2,387
2.5– 3					2	9	24	34	27	17	7	1	1									122	2,957
3 – 3.5		1	1		1	2	12	15	23	14	11		1									81	3,167
3.5– 4					1	1	10	8	13	9	10	1										53	3,283
4 – 5					1	1	6	8	11	17	24	9	2			1						80	3,975
5 – 6									4	3	7	11	6	1	1							33	5,189
6 – 7										3	5	10	1	1	1	1	1					23	5,728
7 – 8									2	1	1	3	7	3	1							18	5,986
8 – 9								1				1	2	1	2							7	6,538
9 –10														1	1	1	1					4	9,125
10 –12																2	1					3	10,000
12 –14																1	2	2				5	11,500
14 –16																	1					1	11,000
16 –18																			1			1	17,000
18 –20																			1			1	17,000
20 –25																							
25 –30																				1		1	19,000
30 –40																					1	1	27,500
All classes	6	12	23	76.5	134	146	165	122	106	76	67	36	20	7	6	6	6	2	2	1	1	1020.5	
Arith. mean income[1] in 1932 (dollars)	−42	375	685	861	1,349	1,770	2,335	2,650	3,177	3,586	4,216	5,653	6,200	7,500	7,667	9,250	11,333	13,000	18,000	22,500	35,000		

[1] Averages computed from frequency distributions in this table, not from reported incomes.

* The 1934 income classes '–2,000 to –1,000', '14,000–16,000', '20,000–25,000', and '30,000–40,000' have been omitted from this table for lack of space. No dentists reported income in 1934 in these classes.

i.e., the less shifting, the larger will be the correlation coefficient, and conversely. The correlation coefficient is not affected by changes in income level from one year to the next, since it depends on the differences between individuals' incomes and the average income in the profession.

The correlation coefficient for Table 55 is .907. Similar correlation coefficients for all pairs of years for which we have prepared correlation tables like Table 55 [3] are given in Table 56.

Except for consulting engineers, the correlation coefficients in Table 56 are all high, indicating a marked degree of stability in the positions that individuals occupy in an array of professional men by size of income. The coefficients are not sufficiently consistent from group to group or sample to sample to make possible an unambiguous ranking of the professions by variability of income status. The major differences are, however, clear. The correlation coefficients are largest for physicians and dentists and smallest for engineers. There is perhaps some tendency for the coefficients to be larger for physicians than for dentists, and for accountants than for lawyers, but the differences are slight and irregular and deserve little confidence. The temporal differences are even less consistent than the differences among professions. For example, the 1932–33 correlation is the lowest consecutive-year correlation for physicians and accountants, but the highest for dentists. Moreover, the temporal differences among the correlation coefficients are in general much smaller than the differences among the professions.[4]

[3] See the general note to Table 56 for the reasons why we excluded some pairs of years for some professions and restricted the analysis for certified public accountants, lawyers, and consulting engineers to individual practitioners.

[4] The statistical significance of the difference between the correlation coefficients can, of course, be tested in the usual fashion by transforming the correlation coefficients into $z = \frac{1}{2}\log_e \frac{1+r}{1-r}$, and making use of the fact that the standard deviation of z is $1/\sqrt{n-3}$. However, this test assumes that the basic data are normally distributed, an assumption to which our data patently do not conform. For this reason, we preferred to base our conclusions on the con-

One possible explanation of the greater stability of relative income status in the curative professions, medicine and dentistry, than in the business professions, is that it reflects a difference in the strength of competitive forces. In professions like medicine and dentistry that serve the ultimate consumer, custom may well play a more important role than in professions that serve primarily business enterprises. A consumer usually has 'his' physician or dentist from whom he is not easily won away. He is unlikely to 'shop around' extensively, and ordinarily has little competence to judge the technical quality of the services he purchases. 'Reputation' and 'renown' play a large part in his choice, and, in the main, reputations are not made or lost overnight. The enterprises that purchase services of the business professions are more addicted to rational calculation than the ultimate consumer. They are in a better position to judge the quality of services; they experiment more, seeking the best services at the lowest prices. They often purchase professional services in fairly large quantities and it is worth their while to devote a good deal of attention to getting the best 'buy'. Individuals who gain a reputation not justified by their professional competence are more likely to lose it than in the curative professions. Individuals who have been undervalued are more likely to gain the reputation they deserve.

The contrast just drawn is of course not sharp and definite. In business as in private life, custom guides day to day affairs. Firms have 'their' accountants just as individuals have 'their'

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sistency of the differences among the correlation coefficients rather than on their statistical significance as determined by the above test.

If the test of significance is applied, it will be found that the relatively large size of the samples on which the coefficients are based leads to significant differences between almost all pairs of coefficients in Table 56. For example, the difference between the (transformed values of the) 1933–34 correlation coefficients for physicians and dentists is more than twice its standard error; between the 1932–33 correlation coefficients, more than three times the standard error. Similarly, the difference between the correlation coefficients for physicians for 1929–30 and 1932–33 is more than twice its standard error; and the difference between the coefficients for physicians for 1932–33 and 1933–34, more than five times its standard error.

TABLE 56

Correlation Coefficients between Incomes in Different Years, and Number of Returns Correlated

YEARS CORRELATED	PHYSICIANS	DENTISTS	LAWYERS[2]	CERTIFIED PUBLIC ACCOUNTANTS	CONSULTING ENGINEERS
				(individual practitioners)	
		Correlation Coefficient			
Consecutive years					
1929 & 1930[1]	.932	.887		.901	.672
1932 & 1933	.920	.939	.834[3]	.848[4]	
1933 & 1934	.947	.936	.854[3]	.858[4]	
1934 & 1935				.906	
1935 & 1936				.868	
Nonconsecutive years, 1 year intervening					
1929 & 1931[1]	.939	.843		.786	.626
1932 & 1934	.889	.907	.795[3]	.731[4]	
1934 & 1936				.826	
Nonconsecutive years, 2 years intervening					
1929 & 1932[1]	.877	.761		.821	.521
		Number of Returns Correlated			
Consecutive years					
1929 & 1930	2,120	1,219		368	257
1932 & 1933	1,379	1,020	685[5]	783[5]	
1933 & 1934	1,446	1,057	730[5]	831[5]	
1934 & 1935				491	
1935 & 1936				505	
Nonconsecutive years, 1 year intervening					
1929 & 1931	2,110	1,220		365	251
1932 & 1934	1,381	1,020	674[5]	776[5]	
1934 & 1936				472	
Nonconsecutive years, 2 years intervening					
1929 & 1932	2,094	1,216		362	246

Several features of the table require explanation. (1) Not all the correlation coefficients that could have been computed from the 1933 samples are presented. Our initial analysis of these samples was later judged erroneous. It was possible, however, to use the results to approximate the correlation coefficients between 1929 and each of the other years by the method described in footnote 1. In view of the inevitable interdependence among the correlation coefficients computed

TABLE 56, NOTES (cont.)

from the same sample for different pairs of years, we decided that the large amount of labor needed to compute the correlation coefficients for the other pairs of years was not justified. To test the accuracy of the approximate method used to compute the correlation coefficient for the 1933 samples, we applied the same procedure to the 1935 accountancy sample with the accompanying results (the extreme case noted in footnote 4 is excluded throughout).

	Correlation Coefficient		
		APPROXIMATE METHOD, USING REGRESSION OF	
YEARS CORRELATED	USUAL PROCEDURE	LATER ON EARLIER YEAR	EARLIER ON LATER YEAR
1932 & 1933	.848	.836	.882
1933 & 1934	.858	.853	.862
1932 & 1934	.731	.752	.782

The agreement is, on the whole, good.

(2) No correlation coefficients are presented for the 1937 medical and legal samples because they are nonrandom among states. To have weighted the states before combining them as we did in computing the other measures presented for these samples, would have required inordinate labor and yielded results not easily susceptible to the kind of statistical analysis presented in the Appendix to this chapter. On the other hand, considerable ambiguity would have attached to correlation coefficients computed without correcting for the bias.

(3) This general rule of making no computations involving correction for biases also accounts for the restriction of our analysis of the legal and accountancy samples to individual practitioners. The data for firm members are subject to a twofold bias: (a) the sampling bias that led to overrepresentation of large firms; (b) the bias in measures of variability arising because the income of the firm as a whole, not of each member, was reported. The first bias but not the second could have been corrected by weighting.

(4) Likewise, correlation coefficients were computed only for consulting engineers who were individual practitioners although the data for engineers are subject to the second bias alone. For the 1933 accountancy and engineering samples, we have some evidence on the effect of excluding firm members since correlation coefficients were computed for both individual and all practitioners. In computing coefficients for all practitioners, the accountancy sample was not corrected for the firm member bias. The exclusion of firm members has little

	Correlation Coefficient			
	ACCOUNTANTS		ENGINEERS	
YEARS CORRELATED	Individual practitioners	All	Individual practitioners	All
1929 & 1930	.901	.873	.672	.680
1929 & 1931	.786	.697	.628	.509
1929 & 1932	.821	.665	.521	.367

effect on the correlation coefficients for consecutive pairs of years but alters markedly the other correlation coefficients.

TABLE 56, NOTES (concl.)

(5) We departed from our general rule of not analyzing samples that require weighting of parts to correct for bias only in analyzing the 1935 legal sample. This sample, it will be recalled, is subject to a size of community bias. In its analysis for the present purpose, we used the original data and did not correct for the size of community bias. This departure is explained by a desire to include some results for lawyers and by our inability to discover any respect in which the correlation coefficient would be seriously affected by not correcting for the size of community bias. It seems reasonable, however, that it is affected in some fashion. Consequently, less confidence should be attached to the coefficients for lawyers than for the other professions.

(6) Both dental samples are restricted to American Dental Association members. Since the resulting bias has not been adjusted for, any conclusions about dentists must be interpreted as referring to members alone.

[1] Correlation coefficients computed indirectly. Practitioners were arrayed by size of 1929 net income. For each decile of this array, average net income for each year 1929–32 was computed. Straight line regression equations were computed between average incomes for each year 1930–32 and average 1929 incomes. The slope of each equation was multiplied by the ratio of the standard deviation of income in 1929 to the standard deviation of income in the year considered, the standard deviations being based on the entire frequency distribution of income. The product is the figure entered in this table. For accountants and engineers, the deciles were computed from an array of all practitioners including firm members. The number of individual practitioners thus varied from decile to decile. The actual numbers in the deciles were used as weights in computing the regression equations.

[2] Correlation coefficients computed without correcting for size of community bias.

[3] Excludes one extreme return reporting an income of $8,357 in 1932; $50,471 in 1933; $3,165 in 1934. Correlation coefficients including this return are: 1932 and 1933, .759; 1933 and 1934, .757; 1932 and 1934, .794.

[4] Excludes one extreme return reporting an income of $41,000 in 1932; —$19,000 in 1933; $13,000 in 1934. Correlation coefficients including this return are: 1932 and 1933, .596; 1933 and 1934, .793; 1932 and 1934, .696.

[5] Excludes return omitted in computing correlation coefficient.

physicians. And 'nepotism' or 'favoritism' involving the purchase of an inferior service in preference to a superior one are by no means absent. Nevertheless, it is probably true that custom reigns more widely in private life than in business affairs, and this may be the reason why professional status, though relatively stable in all professions, is more stable in the curative than in the business professions.

The correlation coefficients in Table 56 are classified into three groups: for consecutive years, nonconsecutive years with one year intervening, and nonconsecutive years with two years intervening. As is to be expected, the correlation coefficients between nonconsecutive years are in general smaller than between consecutive years: shifts in income status are likely to be greater and to affect more people the longer the period considered. A more important question is whether the shifts over long periods are a random cumulation of the year to year shifts. If they were, the correlation coefficients between nonconsecutive years would be the product of the correlation coefficients for the intervening pairs of consecutive years. For example, the correlation coefficient between physicians' incomes in 1932 and 1933 is .920, and between their incomes in 1933 and 1934, .947. If the shifts in successive pairs of years were independent, the correlation coefficient between incomes in 1932 and 1934 would be .920 x .947 or .871; in fact, it is .889, indicating that the income status of physicians over the three years is more stable than the degree of shifting from one year to the next would suggest.

Stated in somewhat different terms, the question posed in the preceding paragraph is: among a group of individuals with the same incomes in, say, 1933, which are most likely to experience a rise in income in 1934? Those for whom the 1933 income represents an increase from 1932 or those for whom it represents a decrease? The answer is given by the partial correlation coefficient between incomes in one year and the second succeeding year, income in the intervening year being the variable eliminated.

	PARTIAL CORRELATION COEFFICIENT BETWEEN INCOMES IN	
	1932 AND 1934, INCOME IN 1933 BEING THE VARIABLE ELIMINATED	1934 AND 1936, INCOME IN 1935 BEING THE VARIABLE ELIMINATED
Medicine	.141	
Dentistry	.232	
Law	.288	
Accountancy	.0125	.188

All the correlation coefficients are positive. The conclusion suggested for physicians by the example worked out above is therefore confirmed for the other professions: there is a tendency for individuals whose income status changes to regain their initial positions. This tendency is, however, slight. The partial correlation coefficients are all below 0.3, the simple correlation coefficients are all above 0.7.

The smallness of the partial correlation coefficients probably reflects two forces working in opposite directions. On the one hand, the natural life cycle of earnings makes for a negative correlation. Young men just beginning practice tend to have steadily rising incomes; older men, in some professions at least, steadily falling ones.[5] On the other hand, individuals in practice the same length of time tend to return to the same position after reverses or successes. The observed correlation coefficients are positive, suggesting that the second tendency is stronger, at least for periods as brief as that covered by our analysis.

The last column and the last row of Table 55 give a more detailed summary of the interrelations between incomes in 1932 and 1934. The last column gives the average 1934 income of the dentists in each 1932 income class. For example, dentists whose 1932 incomes were between $500 and $1,000 had an average income of $1,082 in 1934; dentists whose 1932 incomes were between $7,000 and $8,000 had an average income of $5,986 in 1934. Similarly, the last row gives the average 1932 income of each 1934 income class. Averages like those in the last column and row of Table 55 are plotted in Chart 25 for other professions and pairs of years.

The chart contains 42 diagrams, each showing the average income in one year of ten classes obtained by grouping individuals by their incomes in another year. To illustrate, the first diagram depicts the relation between incomes of physicians in 1929 and 1932. The grouping is by 1929 income. The or-

5 This makes for a negative partial correlation because, of a group of persons whose incomes are the same in one year, those persons will tend to have higher incomes in the next year who had lower incomes in the preceding year, and conversely; i.e., those who have risen to this level will continue to rise and those who have fallen to this level will continue to fall.

CHART 25

Relation between Incomes of Same Individuals in Different Years

(Scales represent income in thousands of dollars in designated years)

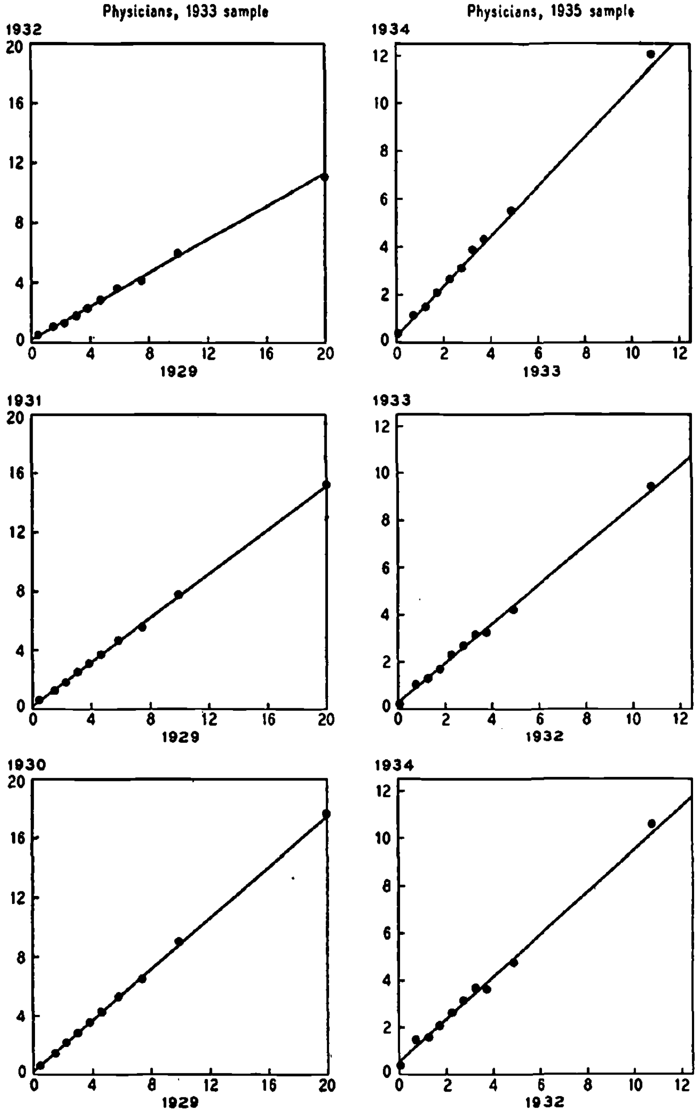


CHART 25 (CONT.)

(Scales represent income in thousands of dollars in designated years)

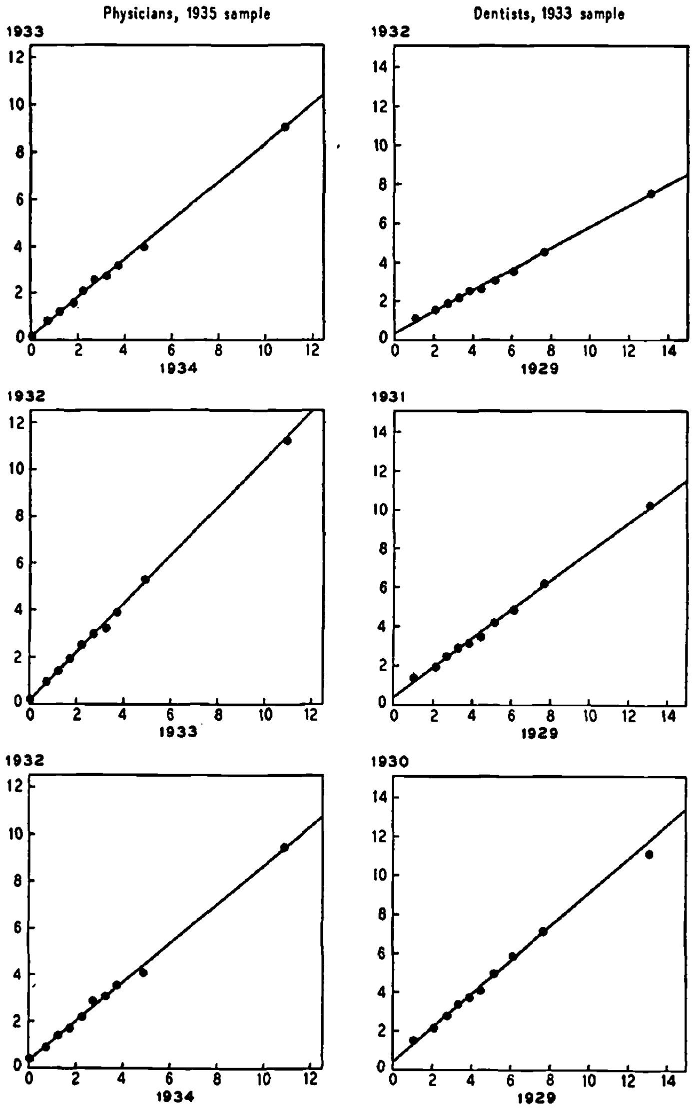


CHART 25 (CONT.)

(Scales represent income in thousands of dollars in designated years)

Dentists, 1935 sample

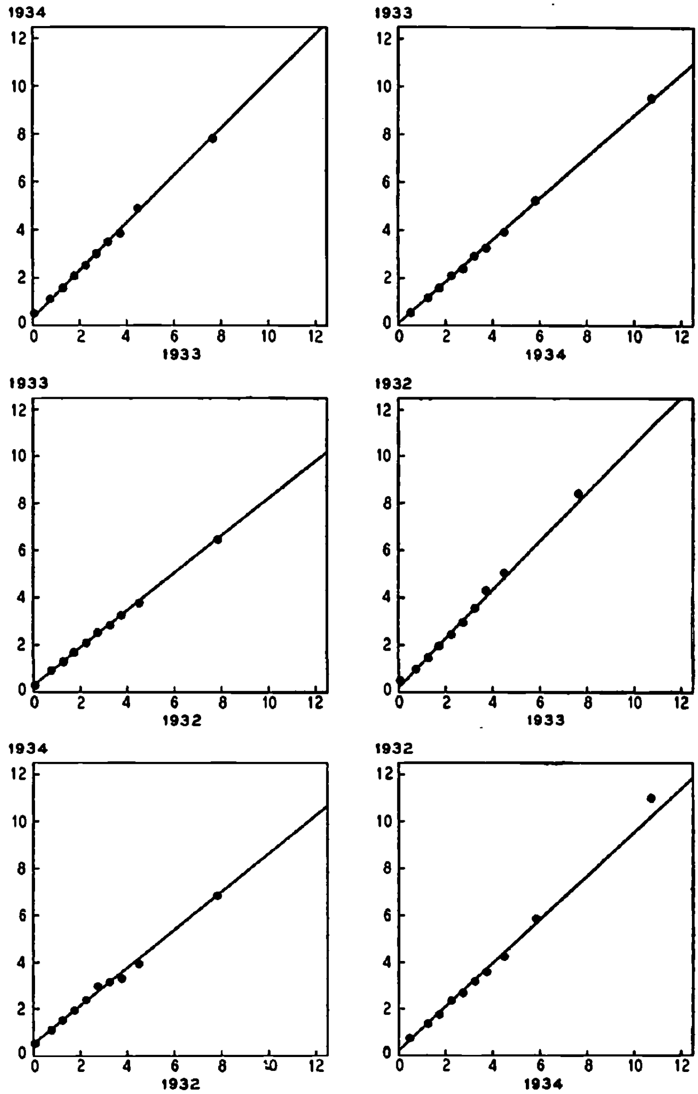


CHART 25 (CONT.)

(Scales represent income in thousands of dollars in designated years)

Lawyers, 1935 sample

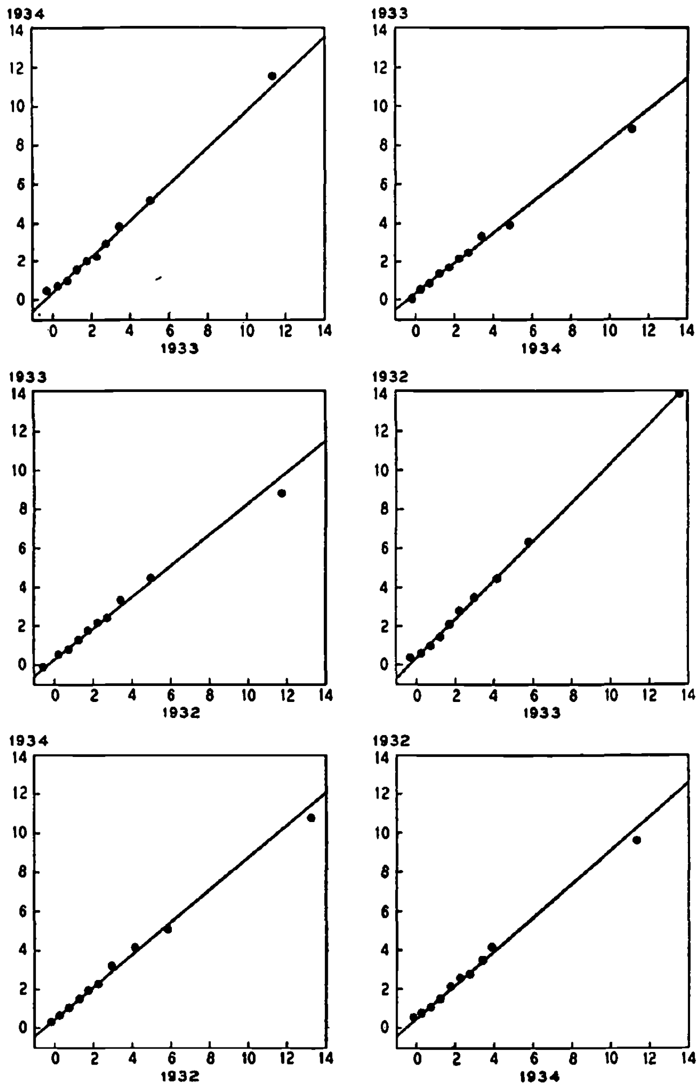


CHART 25 (CONT.)

(Scales represent income in thousands of dollars in designated years)

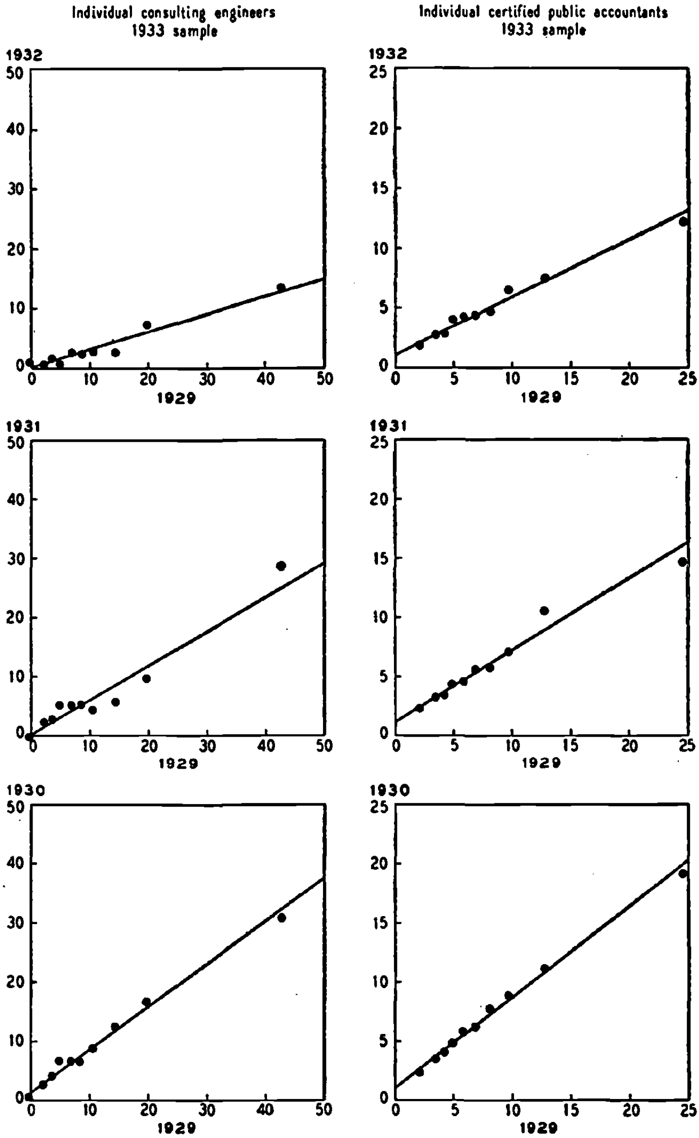


CHART 25 (CONT.)

(Scales represent income in thousands of dollars in designated years)

Individual certified public accountants
1935 sample

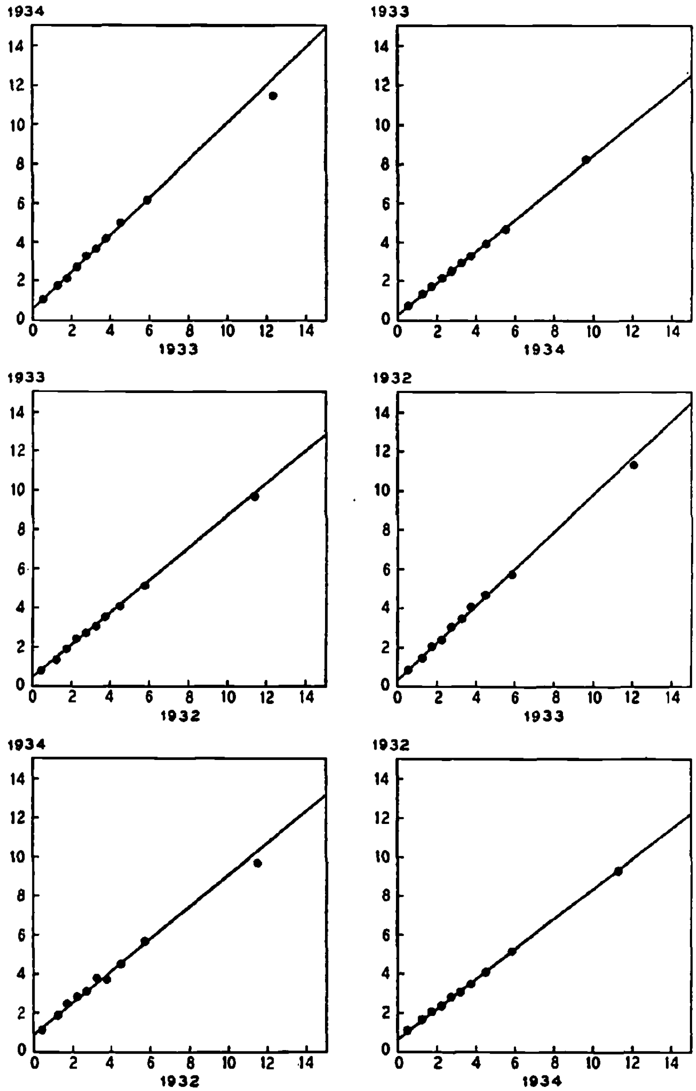
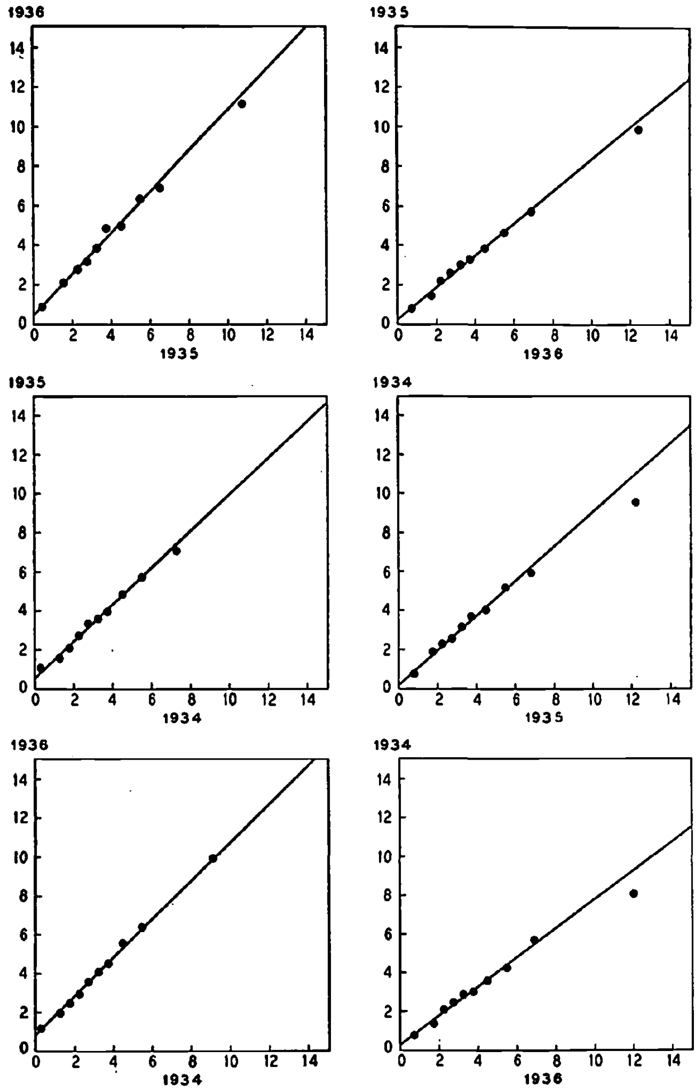


CHART 25 (CONCL.)

(Scales represent income in thousands of dollars in designated years)

Individual certified public accountants
1937 sample



dinate of any point (the vertical distance between the point and the horizontal axis) is the average income in 1932 of the 1929 income class whose average income in 1929 is given by the corresponding abscissa (the horizontal distance between the point and the vertical axis). In all diagrams, income in the year that serves as the basis for grouping the practitioners (in the diagram just cited, 1929) is measured along the horizontal axis. In order to limit the disturbing influence of sampling fluctuations, the income classes for which points are plotted are sometimes broader than those used in the correlation tables. In the main, this is true only of the lowest, highest, and less frequently, next to the highest income classes.[6] In addition to the actual averages, regression lines computed from the correlation tables are plotted on the chart. For the 1933 samples, the lines are based on the plotted points; for the other samples, on data for the finer class intervals. (The constants of the regression equations are given in Table 60, Section 1a of the Appendix to this chapter.[7])

[6] In obtaining the classes plotted, we subdivided none of the finer class intervals except in the analysis of the 1933 samples. For these samples, each class contains almost exactly the same number of practitioners. There is more variation among the classes for the other samples but, with few exceptions, the largest class contains fewer than three times as many practitioners as the smallest. It should be noted that if any set of observations falls on a straight line, their means will fall on the same straight line.

[7] For the 1933 samples the equations of the straight lines were computed from the ten pairs of means by the usual least squares procedure. For the other samples, however, this procedure was not followed. As we show below, the standard deviation of income in year 2 is not the same for all year 1 income classes: at first it decreases with income in year 1, then increases very rapidly. Consequently, the different income classes were weighted inversely to their estimated variances in computing the equations of the straight lines. See Section 1 of the Appendix to this chapter for a more detailed explanation of the method used in obtaining the weights.

The use of weights means that the regression equations plotted in Chart 25 for the 1935 and 1937 samples are not entirely consistent with the correlation coefficients in Table 56, since the correlation coefficients were computed in the usual fashion. The reason for this difference in procedure is that we have been unable to find a logically sound method for taking the unequal variances of the different income classes into account in estimating the correlation coefficient. We cannot see that the difference in procedure introduces any bias into our results. It means simply that a less accurate method of estimation has

The broad conclusion suggested by the diagrams is that the straight lines fit the points remarkably well. Of course, few of the points fall exactly on the straight lines, but the deviations are small and, of more importance, appear randomly distributed about the lines. The one tolerably clear exception to this generalization is in the diagrams for accountants; the last point is almost uniformly below the fitted line, and the first two points are very frequently below, i.e., there is some indication that the regression is concave to the horizontal axis. Though the deviations are small, the last point is below the line in 14, the first in 9, and the second in 10 of the 15 accountancy diagrams. The only other profession displaying anything like the same degree of consistent departure from the fitted lines is law, for which the first point is above the line in all six diagrams. However, this is the only point showing consistent departure; and since all six diagrams are based on the same sample, little importance should be attached to this phenomenon. Accountancy is the one profession for which data based on all three samples are presented. The analysis for medicine and dentistry is restricted to two samples, and for law and engineering, to one.

been used to obtain the correlation coefficients than to obtain the regression equations.

The geometric mean of the two weighted regression coefficients gives one possible alternative estimate of the correlation coefficient. This method seems reasonable by analogy to the relations when all observations are given equal weight. However, since we have been unable to find any other justification of its validity, we have not used the method. It yields results very similar to those obtained by the usual procedure, as the accompanying comparison indicates (r = correlation coefficient by usual procedure, r' = geometric mean of weighted regression coefficients).

	PHYSICIANS		DENTISTS		LAWYERS		ACCOUNTANTS	
YEARS COMPARED	r	r'	r	r'	r	r'	r	r'
1932 and 1933	.920	.931	.939	.909	.834	.893	.848	.880
1933 and 1934	.947	.934	.936	.920	.854	.869	.858	.889
1932 and 1934	.889	.874	.907	.874	.795	.849	.731	.793
1934 and 1935							.906	.906
1935 and 1936							.868	.921
1934 and 1936							.826	.863

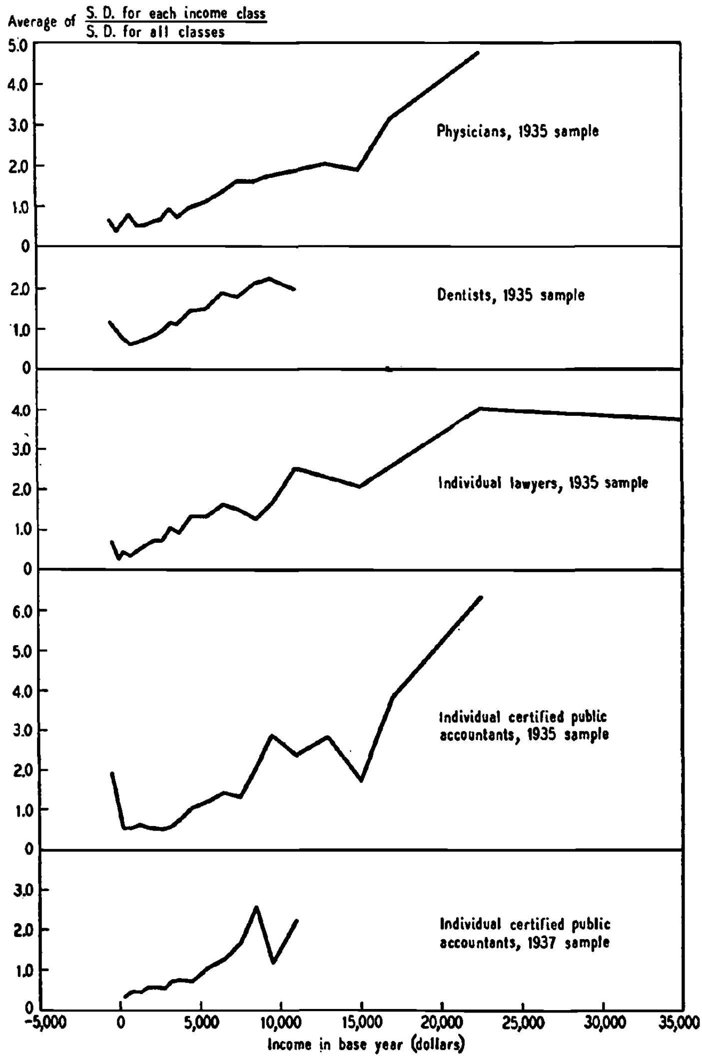
For reasons explained in detail in Section 1 of the Appendix to this chapter, it is difficult to make an exact statistical test of the hypothesis that the regressions are linear. Since the results of the tests there made are seriously affected by personal judgments, the basis for which cannot be fully presented, we have preferred to rest our case on the diagrams in Chart 25 and on the lack of consistency among the deviations of the points from the lines. Moreover, whether or not the deviations are greater than can reasonably be attributed to chance, they are so small that they could scarcely be considered 'significant' in any other than the statistical sense of that term.

The diagrams in Chart 25 summarize the relation between the average incomes in two years of individuals classified by their incomes in one of those years. To complete the description of the correlation tables, we must investigate the variability about these averages, the extent to which individuals in the same income class in one year are dispersed in another year. We use the standard deviation as a measure of absolute variability, the coefficient of variation as a measure of relative variability.

Chart 26 summarizes the relation between the standard deviation of income in one year and the size of income in another. It contains five sections—four based on the 1935 medical, dental, legal, and accountancy samples, and one based on the 1937 accountancy sample. Each section was prepared as follows: (1) Standard deviations of income in each year were computed for groups of practitioners classified by their incomes in each of the other years. For example, the standard deviation of 1934 income was computed for each 1932 and each 1933 income class; the standard deviation of 1933 income, for each 1932 and each 1934 income class; and the standard deviation of 1932 income, for each 1933 and each 1934 income class. This yielded six sets of standard deviations—two sets for each of the three years. In each case the fine class intervals, rather than the broad ones plotted in Chart 25, were used. (2) The standard deviations were expressed as ratios to a standard deviation obtained

CHART 26

Relation between Absolute Variability and Size of Income in Base Year



by combining all income classes.[8] This was done to eliminate differences in the levels of the six sets of standard deviations. The levels differed, first, because of differences from year to year in the absolute variability of the income distribution as a whole, second, because four sets of standard deviations were for incomes in a year immediately following or preceding the 'base' year while two sets were for incomes in a year two years before or after the 'base' year. The latter naturally tended to be greater than the former. (3) The six ratios for the same dollar income class were then averaged; e.g., the six ratios for the income class $1,000–$1,500. Fewer than six ratios were available for those income classes which in some years included either no practitioners or only one. No averages were struck for such classes. The use of the same dollar income classes for the different years is somewhat questionable because of changes in average professional income and hence in the position of each income class relative to the average. However, the changes in average income are small relatively to the range of income classes plotted. The possible refinement in the results hardly justified the arbitrariness of any alternative procedure. (4) These average ratios are plotted in the chart against the income classes to which they refer. The average ratios are measured along the vertical axis, the income classes along the horizontal. The income classes in the base year are not all the same width: the higher classes tend to be wider than the lower. Consequently, there is a bias toward greater variability in the higher income classes.

The lines in Chart 26 tell much the same story. Absolute variability at first falls for a short distance, then rises at an increasing rate throughout the rest of the range. The falling portion covers primarily the negative and extremely low posi-

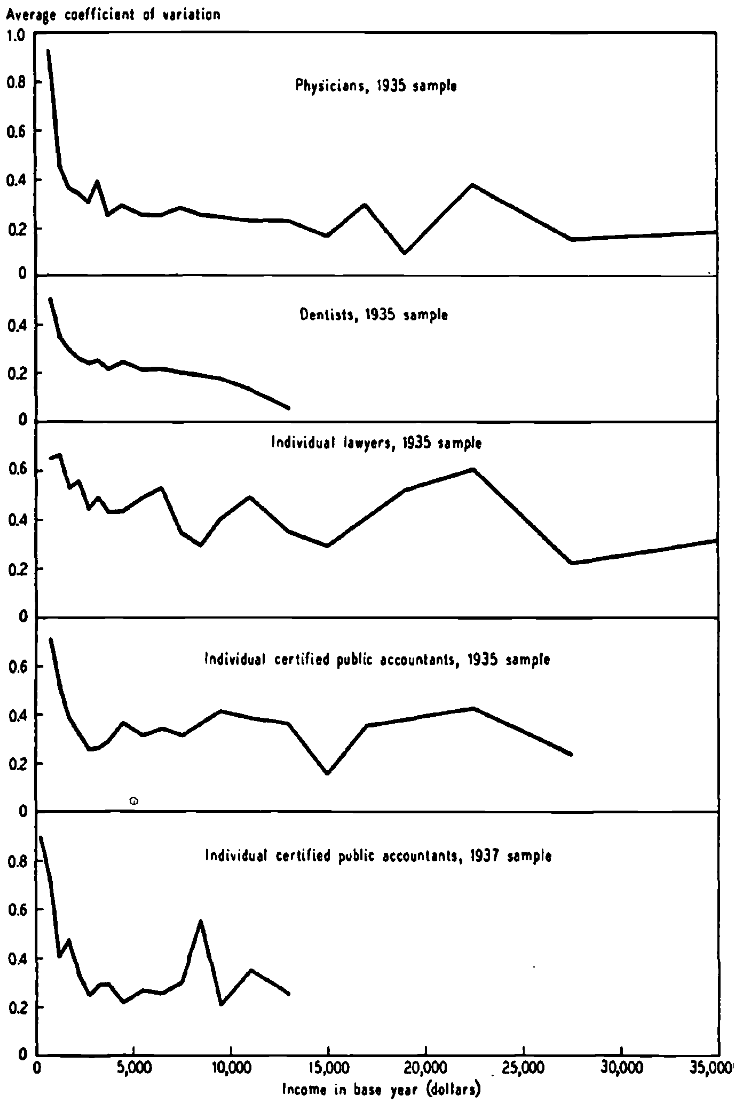
8 This average standard deviation was not a simple arithmetic average. It was the root mean square of the deviations of the observations about the class means. In other words, the sum of the squares of the observations about the class mean was computed for each class; these sums were added for all classes and the total divided by the total number of degrees of freedom (the number of observations minus the number of income classes). The square root of the quotient gave the standard deviation used as the base of the ratios.

tive income classes. This is, of course, reasonable. If individuals were to remain in these classes for any length of time they would almost certainly be forced out of professional practice entirely. The fact that they remain in practice means that years in which they lose money or receive negligible incomes are the exception rather than the rule. Nor is the large increase in dispersion as income increases surprising in view of the results of earlier chapters. We have almost uniformly found a positive relation between absolute variability and income level: the higher the income level, the greater the absolute variability. True, in Chart 26 the standard deviation of income is plotted against average income in a different year. But we found earlier in this chapter that there is a fairly close positive relation between incomes in the two years.

Chart 27, based on coefficients of variation instead of standard deviations, differs from Chart 26 in two other respects: (1) The figures plotted are the averages of the actual coefficients of variation rather than of ratios of the coefficients of variation to an average coefficient for all income classes. (2) Each figure plotted is the average of four instead of six coefficients of variation, the four used being those that refer to incomes in a year immediately following or preceding the 'base' year. Both changes were made for the same reason. Dividing the standard deviations by the average income eliminated the larger part of the differences in the level of the standard deviations. The four sets of coefficients of variation used were consequently at almost the same level. The other two sets—those for years two years before or after the 'base' year—were, however, on a higher level. Instead of correcting for differences in level so that all six sets could be combined, we analyzed only the four directly comparable sets. The two excluded sets yield curves similar to those in Chart 27. The coefficients of variation on which Chart 27 is based were obtained by dividing the standard deviation by the mean income for the same year; e.g., the coefficients of variation of 1932 income for 1933 income classes were obtained by dividing the standard deviation of 1932 income for each 1933 income class

CHART 27

Relation between Relative Variability and Size of Income in Base Year



by the average 1932 income of the same 1933 income class.

The initial decrease and then approximate constancy in the standard deviation as the mean income increases naturally reflects itself in an even more rapid decrease in the coefficients of variation. At an income level of about $2,000, however, the rate of decline in the coefficient of variation is drastically curtailed. Among physicians and dentists the decline continues thereafter but at a very much slower pace. Among accountants it apparently ceases entirely, giving way to irregular fluctuations about a stable level. Correction for the upward bias in variability noted above might lead to a continuance of the decline for accountants as well as for physicians and dentists. After the $2,000 point, the coefficients of variation for accountants are about the same as or somewhat greater than those for physicians, and these, in turn, are consistently larger than the coefficients of variation for dentists. Yet in Chapter 3 we concluded that the relative variability of the income distribution as a whole was greater for physicians than for either accountants or dentists, but almost the same for the two latter professions. The reason for the change in order is, of course, the higher correlation coefficients for physicians and dentists than for accountants. Considering only the variation within income groups eliminates a greater proportion of the variability for physicians and dentists than for accountants.

The findings of this section may be summarized very briefly. Relative income status is fairly stable in all professions other than engineering, and somewhat more stable in medicine and dentistry than in accountancy and law: while a group of individuals who are in the same income class one year will not be in the same class another year, they are not likely to be widely dispersed. The average income of this income class in the latter year is linearly related to its income in the base year, i.e., it tends to equal a constant multiple of the base year income plus or minus a constant amount. The absolute variability of income about this average is large for very small or negative incomes, small for intermediate incomes, and large for large incomes; relative variability decreases sharply for base year incomes below $2,000, and thereafter either remains constant or declines more slowly.

2 AN INTERPRETATION OF THE LINEAR REGRESSIONS

The results of the preceding section have an important bearing on whether the degree of stability of relative income status that characterizes a profession also characterizes separate income classes; stated differently, whether the uncertainties and accidental occurrences that accompany economic change affect with equal strength those who rank high in a profession and those who rank low. One significant interpretation of 'equal effect' can be shown to imply linear regression between incomes in different years, and conversely. We have already seen that the observed regressions are very nearly linear.

A man's relative income status in any two years will be determined in part by factors that are common to the two years: personal attributes such as training, ability, personality; attributes of the man's practice such as its location, type, organization; and accidental influences whose effects are present in both years. Superimposed on these factors are transitory influences that affect his income in only one of the two years; influences that are likely to be interpreted by the man affected as 'accidental' or 'chance' occurrences, though in reality they may be the result of definite causal factors at work, and may even reappear at intervals associated, for example, with cyclical fluctuations in general business activity. Let us call the part of a man's income determined by the first set of factors the 'permanent' component, and the part determined by the second set, the 'transitory' component. The magnitude of the two components will depend on the period covered. Factors that are 'permanent' for a particular pair of years may not be for a longer period, or a different pair of years; factors that are 'transitory' change correspondingly; lengthening the period considered will in general increase the range of factors considered 'transitory'. The separation could be fixed and constant only for a man's whole career treated as a unit.

When we ask whether the stability of relative income status is the same for all income classes, or whether economic change affects all income classes alike, it seems clear that implicitly we are thinking not of classes determined by actual income but of classes determined by what we have called the permanent component of income. Surely, there is little point in asking whether men who have very high incomes this year because they happen to have been the beneficiaries of strokes of good fortune are affected more by the occurrences that accompany economic change than the men who have low incomes this year because they have had bad 'luck'. The relevant question would seem to be whether the men whose personal and professional attributes would tend to make their relative income status high are more affected by these occurrences than other professional men.

There is of course no way of isolating the permanent and transitory components of the income of a particular man. We can measure only his actual income, and we can classify men only by their actual incomes. The difference between the average income of men in the same actual income class and the average income in the profession as a whole will consist of two parts: (1) the difference between the average permanent components for these men and for the profession as a whole, and (2) the average transitory component. (The average transitory component for the profession as a whole can, without loss of generality, be defined as zero since we are interested in relative income status.) If the permanent and transitory components of a man's income are uncorrelated[9] then both parts of the difference between the average income of an income class and the average income of the profession will tend to have

[9] Zero correlation between permanent and transitory components does not necessarily imply that the absolute value of the transitory component is the same no matter what the size of the permanent component. For example, suppose the transitory component was equally likely to be either + 10 per cent of the permanent component or — 10 per cent. The correlation between the transitory and permanent components would be zero, since the average transitory component would be the same (i.e., zero) for all values of the permanent component. The absolute value of the transitory component would be a fixed proportion of the permanent component.

the same sign; e.g., an income class above the average for the profession will tend to have an average permanent component above the average permanent component for the profession and a positive average transitory component.[10] In other words, an income class above the average in any year has on the whole been favorably affected by transitory factors, and conversely.

While we cannot isolate the transitory component of the income of a particular man, we can isolate the average transitory component of the average income of an income class. Suppose we have data on the incomes of the same men in two years (say 1932 and 1933) in which the average income in the profession as a whole is the same. (This assumption about average income will be removed presently; it is made here merely to simplify exposition.) For any 1932 income class,

average income in 1932 = average permanent component in 1932 + average transitory component in 1932.

If, as assumed above, the transitory and permanent components are uncorrelated, the average transitory component for this 1932 income class will be zero in 1933. It is not zero in 1932 because 1932 transitory components helped to determine the 1932 income class into which the individuals were grouped. But such a grouping is entirely random with respect to 1933 transitory components, since by definition these are not present in 1932. Consequently, for the same 1932 income class,

average income in 1933 = average permanent component in 1933.

10 Let x = income of an individual,
p = permanent component of his income,
t = transitory component of his income,
$\bar{x}$ = average income for profession as a whole,
$\bar{p} = \bar{x}$ = average permanent component for profession as a whole,
$\bar{t} = 0$ = average transitory component for profession as a whole,
r = correlation coefficient between variables indicated by subscripts.

Then

$$x = p + t.$$

If $r_{pt} = 0$, then r_{xp} and r_{xt} are greater than zero, from which it follows that the average value of p and t corresponding to an x greater than $\bar{x}$ will be greater than $\bar{p}$ and $\bar{t}$ respectively.

In two years in which average income in the profession is the same, it seems not unreasonable to assume that the permanent component is the same for each man separately. If we make this assumption, then

average income in 1933 = average permanent component in 1932.

It follows that the difference between the average incomes in 1932 and 1933 of a 1932 income class measures the average transitory component in 1932. This average transitory component divided by the difference between the average 1932 income of the 1932 income class and the average 1932 income in the profession measures the proportion of the latter difference that can be attributed to transitory factors. If this proportion is the same for all income classes we can say that they are affected equally by the occurrences that accompany economic change. The above reasoning applies equally well in reverse: the difference between the average incomes in 1933 and 1932 of a 1933 income class measures the average transitory component in 1933. It should be recalled, however, that the result obtained in this way will not be the same as the average transitory component in 1933 computed from data for, say, 1934. The former measures the effect of forces present in 1933 but not in 1932; the latter, the effect of forces present in 1933 but not in 1934.

We can remove the assumption that the average income in the profession is the same in the two years compared by broadening the assumption made above about the relation between permanent components in different years. Instead of assuming them equal we can assume that the permanent component changes in the same proportion as average income in the profession.[11] We can then correct the average 1932 income of a

11 This assumption implies that the relative variability of the permanent component among all members of the profession is constant in different years. Any change in the relative variability of actual incomes is attributed to a change in the strength of transitory forces. There is no reason why the relative variability of the permanent component must remain constant. It is perfectly possible that what we have called the permanent forces might not have effects

1932 income class for the change in the average income in the profession. For example, if average income in the profession is 10 per cent lower in 1933 than in 1932, the average 1932 income of each 1932 income class can be reduced by 10 per

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that were constant or that changed in proportion to the change in the average income in the profession. In that case, the variability of the permanent component might change from year to year. The likelihood of such changes taking place is increased if the data used are for a group that includes individuals in practice different numbers of years. 'Experience' is a permanent factor that obviously does not have constant effects. It would be preferable, therefore, to analyze each years-in-practice group separately. Unfortunately, we cannot do so with our data.

An alternative assumption that could be made is that the relative variability of the permanent component is proportional to the relative variability of actual income. If relative variability is measured by the coefficient of variation, this is equivalent to assuming that the standard deviation of the permanent component is proportional to the standard deviation of actual income, since the average permanent component for the profession is equal to average income. If permanent and transitory components are uncorrelated, this would imply that the standard deviation of the permanent component is also proportional to the standard deviation of the transitory component. This does not seem reasonable. It seems better to use an assumption that permits the transitory components to contribute a greater proportion of the variability of actual incomes in some years than in others.

The formula for the proportion contributed by the transitory factors under the alternative assumption (hereafter called the variability assumption) can be derived as follows:

Let x_{11}, p_{11}, t_{11} be the average income, permanent component, and transitory component, respectively, in year 1 of a year 1 income class;
x_{21}, p_{21}, t_{21} be the average income, permanent component, and transitory component, respectively, in year 2 of a year 1 income class.

Let $\bar{x}_1$, $\bar{x}_2$ be the average income of the profession in years 1 and 2, respectively (these will also be the average permanent components in the profession);
s_1, s_2 the standard deviation of income in years 1 and 2, respectively;
K'_1 the proportion contributed by the transitory factor in year 1 as computed under the variability assumption.

Then

$$p_{11} - \bar{x}_1 = (p_{21} - \bar{x}_2)\frac{s_1}{s_2} = (x_{21} - \bar{x}_2)\frac{s_1}{s_2},$$

$$(1) \qquad t_{11} = x_{11} - p_{11} = x_{11} - \bar{x}_1 - (x_{21} - \bar{x}_2)\frac{s_1}{s_2},$$

$$K'_1 = \frac{t_{11}}{x_{11} - \bar{x}_1} = 1 - \frac{x_{21} - \bar{x}_2}{x_{11} - \bar{x}_1}\cdot\frac{s_1}{s_2}.$$

The corresponding formula under the assumption stated in the text (hereafter called the mean assumption) can be derived in similar fashion. Let K_1

cent and then compared directly with the 1933 average income of the same class.[12]

The computation of the average transitory component is illustrated in Table 57. Column 7 gives the percentage contributed by the transitory component to the deviation of each 1932 income class from the 1932 average income of all dentists. Though computed with the aid of 1933 data, these percentages refer to 1932: e.g., in 1932, 13 per cent of the difference between the average income of dentists who received between \$500 and \$1,000 and the average income of all dentists is accounted for by transitory factors not present in 1933. The very high figure for the \$2,500–\$3,000 class (182) should be attributed little importance; it is for an income class whose average income was nearly the same as the average income of all dentists; consequently, both the permanent and transitory components are small and sampling errors might lead to wide variation in the percentage attributed to either component. The rest of the percentages are fairly similar, ranging from 7

represent the proportion contributed by the transitory factor in year 1 as computed under the mean assumption.

Then

$$p_{11} - \bar{x}_1 = (p_{21} - \bar{x}_2)\frac{\bar{x}_1}{\bar{x}_2} = (x_{21} - \bar{x}_2)\frac{\bar{x}_1}{\bar{x}_2},$$

$$(2) \qquad t_{11} = x_{11} - p_{11} = x_{11} - \bar{x}_1 - (x_{21} - \bar{x}_2)\frac{\bar{x}_1}{\bar{x}_2},$$

$$K_1 = \frac{t_{11}}{x_{11} - \bar{x}_1} = 1 - \frac{x_{21} - \bar{x}_2}{x_{11} - \bar{x}_1}\cdot\frac{\bar{x}_1}{\bar{x}_2}.$$

It is clear from (1) and (2) that if K is the same for all income classes, so is K', since s_1, s_2, and $\bar{x}_1$, and $\bar{x}_2$ do not vary from income class to income class. Hence, the two assumptions lead to the identical criterion of 'equal effect', although they do not give the same value for the magnitude of the contribution of transitory factors unless the coefficient of variation is identical in the two years compared.

12 In the symbols of the preceding footnote the formula described in the text is

$$K_1 = \frac{x_{11}\dfrac{\bar{x}_2}{\bar{x}_1} - x_{21}}{x_{11}\dfrac{\bar{x}_2}{\bar{x}_1} - \bar{x}_2}.$$

This formula can be derived from formula (2) by clearing fractions and then dividing numerator and denominator by $\bar{x}_1$.

TABLE 57

Computation of Transitory Component in Deviation of Average 1932 Income for 1932 Income Classes from Average Income for Profession

Dentists, 1935 Sample

1932 INCOME CLASSES	AVG. INCOME IN 1932	AVG. INCOME IN 1933	AVG. INCOME IN 1932 corrected for change in avg. for profession	AVG. TRANSITORY COMPONENT IN 1932	DEVIATION OF 1932 CORRECTED AVG. FROM AVG. FOR PROFESSION	TRANSITORY COMPONENT AS % OF TOTAL DEVIATION
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(dollars)				
−2,000 to 500	83	244	74	−170	−2,422	7.0
500 to 1,000	750	901	670	−231	−1,826	12.7
1,000 to 1,500	1,250	1,268	1,117	−151	−1,379	10.9
1,500 to 2,000	1,750	1,663	1,563	−100	−933	10.7
2,000 to 2,500	2,250	2,078	2,011	−67	−485	13.8
2,500 to 3,000	2,750	2,527	2,458	−69	−38	181.6
3,000 to 3,500	3,250	2,818	2,904	+86	+408	21.1
3,500 to 4,000	3,750	3,226	3,351	+125	+855	14.6
4,000 to 5,000	4,500	3,794	4,021	+227	+1,525	14.9
5,000 to 40,000	7,843	6,475	7,009	+534	+4,513	11.8
All classes	2,793	2,496	2,496			

Col. 4 = col. 2 × 2,496/2,793
Col. 5 = col. 4 − col. 3
Col. 6 = col. 4 − 2,496
Col. 7 = 100 × (col. 5/col. 6)

to 21, with all except two between 11 and 15, and varying erratically from class to class. On this showing we should be inclined to conclude that all income classes were affected roughly in the same degree.[18]

[18] A common fallacy is to regard all income classes as having been affected 'equally' only if the transitory component is zero. Individuals are classified by the size of their income in the base year, say 1932, and the average income of these classes computed for future years. Divergent movements in these averages are interpreted as reflecting a differential effect of the change in total income. For example, a larger decline in the average income of the upper 1932 income classes from 1932 to 1933 than in the average income of the lower 1932 income classes, would be interpreted as meaning that the upper income classes 'suffered' most from the decline in total income. According to this test, all income classes are affected equally if the average income of each income class changes in the same proportion as the average income of all classes, i.e., if the transitory component is zero.

As the discussion above indicates, this test would be valid only if individuals were classified by the permanent component of their income. It is completely fallacious as actually applied. (See Harold Hotelling, review of Howard Secrist, 'The Triumph of Mediocrity in Business', *Journal of the American*

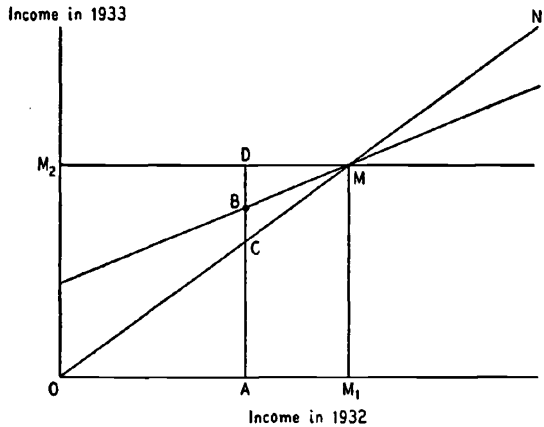
If the percentages in column 7 were equal, the averages in column 3 plotted against those in column 2 would fall on a straight line. Chart 28 serves to illustrate this point. Let us measure income in one year, say 1932, along the horizontal axis and income in 1933, along the vertical axis. Any point in the chart describes the income position of an individual or an income class in the two years. Suppose OM_1 is the average income of the profession in 1932 and OM_2 the average income in 1933, so that the point M represents the average incomes in the two years. Let point B represent the average 1932 and 1933 incomes of a particular income class, so that OA is the average income of this class in 1932, AB, its average income in 1933. Then AC will be the corrected average 1932 income (comparable to the figures in column 4 of Table 57). The transitory component in 1932 will be measured by BC, the permanent component by AB (or expressed as a deviation from the average permanent component for the profession, by BD), the total deviation from the average for the profession by CD, and

Statistical Association, Dec. 1933, pp. 463–5, and the subsequent discussion, *ibid.*, June 1934, pp. 196–200.) An alternative statement of the argument of the text indicates the essential fallacy. Suppose the actual figures on incomes are replaced by ranks. Let us give a rank of 1 to the lowest 1932 income, a rank of 2 to the next lowest income, and so on; and repeat the process for future years. Consider the nine individuals with the lowest 1932 incomes. Their average 1932 rank is obviously 5. Their average rank in 1933 can clearly be no less than 5; it will be 5 only if these same individuals have the lowest incomes in 1933; if there is any shifting in relative income status their average rank must of necessity increase. Similarly, if the average rank of the group of individuals with the highest incomes in 1932 changes at all from 1932 to 1933, it must of necessity decrease.

Nothing essential is altered by using actual incomes rather than ranks. The fact that individuals 'wander', that their relative income status shifts from one year to the next, means that the 'extreme' 1932 income classes will be less extreme in 1933 than in 1932; that the 1932 average incomes of 1932 income classes will tend to be more divergent than their 1933 average incomes. Strictly speaking, this is necessarily true only if the coefficient of variation does not increase from 1932 to 1933. The precise statement is that the 1933 average incomes of 1932 income classes necessarily diverge less than the 1933 average incomes of 1933 income classes. If the divergence among the latter is considerably greater than the divergence among the 1932 average incomes of 1932 income classes, it is possible for the 1933 average incomes of 1932 income classes to diverge more than the 1932 average incomes of the 1932 income classes.

CHART 28

Illustration of Transitory Component



the transitory component as a proportion of the total deviation by BC/CD. If this proportion were constant as A moved along the horizontal axis it is clear that the point B would generate a straight line.[14]

[14] Let x_{11} and x_{21} represent the abscissa and the ordinate, respectively, of the point B, and $\bar{x}_1$ and $\bar{x}_2$ the mean incomes in years 1 and 2, i.e., $\bar{x}_1 = OM_1$, and $\bar{x}_2 = OM_2$. (These are the symbols defined in footnote 11 above.) Then

$$AC = \frac{\bar{x}_2}{\bar{x}_1} x_{11},$$

$$BC = AB - AC = x_{21} - \frac{\bar{x}_2}{\bar{x}_1} x_{11},$$

$$CD = AD - AC = \bar{x}_2 - \frac{\bar{x}_2}{\bar{x}_1} x_{11},$$

$$\frac{BC}{CD} = \frac{x_{21} - \frac{\bar{x}_2}{\bar{x}_1} x_{11}}{\bar{x}_2 - \frac{\bar{x}_2}{\bar{x}_1} x_{11}}.$$

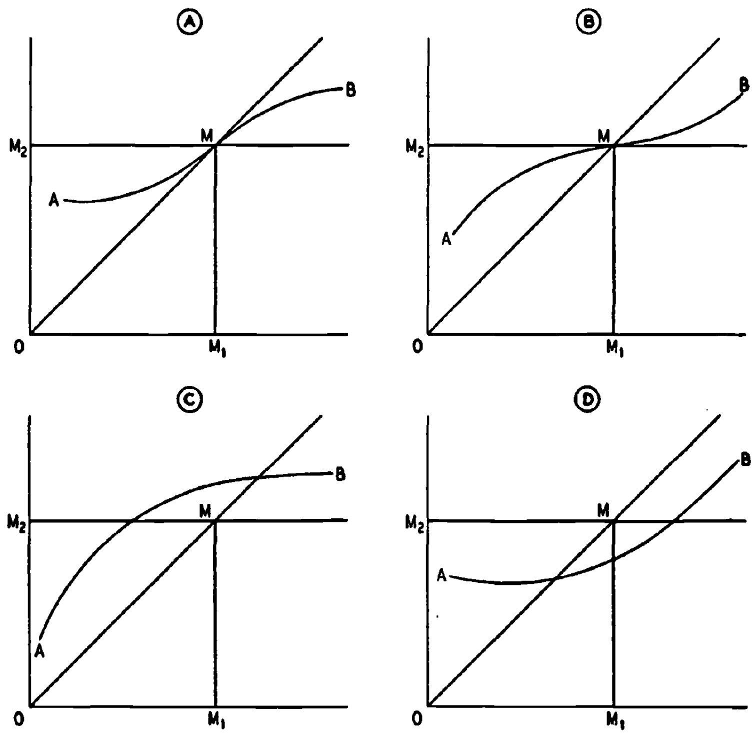
Setting the right hand member of this expression equal to K_1, multiplying both sides of the equation by the denominator of the fraction, and rearranging, gives:

$$x_{21} = K_1 \bar{x}_2 + \frac{\bar{x}_2}{\bar{x}_1} (1 - K_1) x_{11}.$$

It follows that if K_1 is a constant, x_{21} is linearly related to x_{11}. It is obvious that the converse is also true, namely, that if the regression is a straight line, then

CHART 29

Illustration of Different Types of Regression



If the regression is not linear, the strength of the transitory factors differs from income class to income class. Several possible types of regression are illustrated in Chart 29. The regression *AB* in panel A depicts a case in which transitory factors

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the proportion attributed to the transitory component is the same for all income classes. Further, since K'_1 is the same for all income classes when K_1 is (see footnote 11), the variability assumption also implies a straight line when the proportion attributed to the transitory factors is the same for all income classes. The only difference is that the mean assumption measures the proportion contributed by the transitory factors from the line, $y = \frac{\bar{x}_2}{\bar{x}_1} x_{1i}$; the variability assumption from the line, $y = \bar{x}_2 + \frac{s_2}{s_1}(x_{1i} - \bar{x}_1)$.

are relatively strongest in the extreme income classes; regression *AB* in panel B, one in which the transitory factors are relatively strongest in the intermediate income classes; the regression *AB* in panel C, one in which the transitory factors are relatively weakest in the low income classes, relatively strongest in the high income classes; the regression *AB* in panel D, one in which the transitory factors are relatively strongest in the low income classes, relatively weakest in the high income classes. It is perfectly possible for the regression of 1933 income on 1932 income to be linear but the regression of 1932 income on 1933 income, nonlinear. This would in no way be contradictory. It would merely imply, if our line of reasoning is valid, that transitory factors were equally important in all classes in 1932 but not in 1933.

We found in Section 1 that the regressions between incomes in different years are approximately linear. Accountants constitute the only exception. For them, there is some indication that transitory forces were relatively weaker in the low income classes and relatively stronger in the high income classes than in the intermediate classes. For the remaining professions, we can conclude that transitory forces affect all income classes equally. It follows that a single figure will suffice to indicate for all income classes the percentage of the difference between the average income of an income class and the average income in the profession that can be attributed to the transitory component. This figure can be computed from the average incomes in the profession in the two years and the slope of the regression line.[15] For the illustrative example in Table 57 it is

[15] Let b_{21} represent the slope of the regression line of year 2 on year 1. Then, from the equation in the preceding footnote containing K_1,

$$b_{21} = \frac{\bar{x}_2}{\bar{x}_1}(1 - K_1), \text{ or}$$

$$(1) \quad K_1 = 1 - b_{21}\frac{\bar{x}_1}{\bar{x}_2},$$

where K_1 is the proportion attributed to the transitory component. The equation in the preceding footnote also suggests an alternative method of computing K_1. If a is the intercept of the regression line, then

11.5 per cent. Similar figures for other years and professions are given in Table 58.

Table 58 brings out clearly the effect of the choice of the years compared on the size of the transitory component. For example, about 4 per cent of the deviation of the average income of each income class from the average income of all physicians is accounted for in 1933 by transitory factors, if these are defined as factors present in 1933 but not in 1932; about 9 per cent is accounted for by transitory factors, if these are defined as factors present in 1933 but not in 1934. As would be expected, the transitory component is uniformly larger if the comparison year is one year removed from the base year than if it immediately precedes or follows the base year.

$$a = K_1 \bar{x}_2, \text{ or}$$

$$(2) \quad K_1 = \frac{a}{\bar{x}_2}.$$

The first formula for K_1 can be derived from this one by substituting for a its equivalent,

$$\bar{x}_2 - b_{21} \bar{x}_1.$$

From equation (1) of footnote 11 it can be shown that under the variability assumption

$$(3) \quad K'_1 = 1 - b_{21} \frac{s_1}{s_2}.$$

If the regression is fitted by least squares

$$(4) \quad b_{21} = r \frac{s_2}{s_1},$$

where r is the correlation coefficient. Hence,

$$(5) \quad K'_1 = 1 - r.$$

It follows that if K'_2 is the proportionate contribution of the transitory component in year 2 computed from data for years 1 and 2 under the variability assumption,

$$K'_1 = K'_2.$$

I.e., if we use the variability assumption, transitory factors must be attributed the same strength in the two years. This was pointed out in somewhat different form in footnote 11.

As was also implied in that footnote, under the mean assumption $K_1 \gtreqless K_2$ if $V_1 \gtreqless V_2$, where V is the coefficient of variation for the profession in the year designated by the subscript. This can be shown by substituting (4) into (1), and the formula for b_{12} corresponding to (4) into the formula for K_2 corresponding to (1).

TABLE 58

Percentage Contribution of Transitory Component to Deviation of Average Income for Each Income Class from Average Income for Profession

BASE YEAR	COMPARISON YEAR	PERCENTAGE CONTRIBUTION OF TRANSITORY COMPONENT Physicians	Dentists	Lawyers	Certified public accountants
1932	1933	10.0	11.5	9.7	14.0
1933	1932	3.7	6.70	11.7	10.1
1933	1934	9.0	12.4	15.5	13.0
1934	1933	4.2	3.4	10.7	9.1
1932	1934	15.7	18.0	14.5	22.4
1934	1932	9.5	6.71	15.8	19.0
1934	1935				14.7
1935	1934				3.8
1935	1936				9.6
1936	1935				6.2
1934	1936				19.4
1936	1934				7.5

Percentage contributions computed from formula (1) of footnote 15, using weighted *b*'s in Table 60 and actual means.

There is nothing in the arithmetic of the computation that makes such a result inevitable; indeed, the arithmetic does not preclude negative percentage contributions, though no sensible meaning could be attributed to them. Our failure to get any illogical results is evidence in support of the assumptions on which the analysis is based.[16]

Our earlier finding that transitory factors are more important for lawyers and accountants than for physicians and dentists is reflected in the percentage contributions: in four of the six possible comparisons, the percentage contribution is less

16 Had we used the original *b*'s instead of the weighted *b*'s in computing the percentage contributions, we would have gotten two negative contributions, both for dentists, base year 1934. (See footnote 7 for the reason we computed weighted *b*'s.) The use of the weighted *b*'s also explains why there are no entries in Table 58 for years prior to 1932.

Negative contributions are impossible if the variability assumption is used in place of the mean assumption. This is obvious from equation (5) in footnote 15.

for physicians and dentists than for either lawyers or accountants; in the other two, the percentage contributions for physicians and dentists are between those for lawyers and accountants. In three of the four professions, the factors present in 1932 but not in 1933 were more important than those present in 1933 but not in 1932; in all four, the latter were less important than those present in 1933 but not in 1934, and these in turn were more important than those present in 1934 but not in 1933.[17] Apparently there were some factors associated with the downswing that carried over into 1932 but not through 1933; some did carry over into 1933, but were apparently eliminated in the course of the upswing that followed. This interpretation is confirmed by the percentage contributions for accountants for the later years, which are on the whole lower than those for the earlier years.

APPENDIX TO CHAPTER 7

1 STATISTICAL TESTS OF THE LINEARITY OF REGRESSION BETWEEN INCOME IN TWO YEARS

Linearity of regression is ordinarily tested by an analysis of variance that is equivalent to testing the significance of the difference between the squared correlation ratio and the squared correlation coefficient. The observations are grouped by the value of the independent variable (here, income in the base

[17] Despite the statement in the last paragraph of footnote 15 these changes do not parallel the changes in the coefficient of variation. The reason is that the percentage contribution is based on the weighted *b*'s, which are not given by formula (4) of footnote 15 if the correlation coefficient and standard deviations are computed from the original data. It is interesting that the changes described in the text are slightly more consistent than those in the coefficients of variation.

year), and the mean value of the dependent variable (income in the other year) is computed for each class. Two estimates of variance are then computed. (1) The difference between each class mean and the corresponding ordinate of a regression equation fitted by the ordinary least squares procedure is squared, multiplied by the number of observations on which the mean is based, and summed for all base year income classes. The resulting sum of squares divided by the proper number of degrees of freedom (two fewer than the number of classes) gives an estimate of variance based on the deviations of the means about the regression. (2) The difference between each observation and the mean of the class in which it falls is squared and summed for all observations. The resulting sum of squares divided by the proper number of degrees of freedom (the number of observations minus the number of classes) gives an estimate of variance based on the deviations of the observations about the class means.[1]

If a linear regression adequately describes the relation between the two variables, the deviations of the class means from the regression reflect chance fluctuation alone; consequently, the variance of the means about the regression should be equal to the variance of the observations about the means. Of course, the two are rarely exactly equal. Whether the linear regression is adequate thus turns on whether the first estimate of variance exceeds the second by more than can reasonably be attributed to chance alone. The ratio of the first variance to the second, called the analysis of variance ratio and denoted by F, fluctuates in repeated random samples from a normal universe in a known manner; published tables give the values of F that would be exceeded by chance in 20, 5, 1, and 0.1 per cent of random samples for many pairs of degrees of freedom.[2]

Values of the analysis of variance ratio computed in the manner described are given for the 1935 medical, dental, and legal samples, and for the 1935 and 1937 accountancy samples in the columns of Table 59 headed 'original'. At the bottom of the table are the approximate values of F that would be exceeded by chance in 5,

1 In practice, of course, short-cut methods are used to compute the two sums of squares; see R. A. Fisher, *Statistical Methods for Research Workers* (London: Oliver and Boyd, 1925), Sec. 44, 45, 46.

2 R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural, and Medical Research* (London: Oliver and Boyd, 1938), Table V.

TABLE 59

Tests of Linearity of Regression between Incomes in Different Years
Physicians, Dentists, Lawyers, and Certified Public Accountants

ANALYSIS OF VARIANCE RATIO (F)

REGRESSION	PHYSICIANS Original	PHYSICIANS Weighted	DENTISTS Original	DENTISTS Weighted	LAWYERS [1] Original	LAWYERS [1] Weighted	CERTIFIED PUBLIC ACCOUNTANTS [2] Original	CERTIFIED PUBLIC ACCOUNTANTS [2] Weighted
1933 on 1932	4.32	1.23	5.13	.84	7.37	2.60	5.61	1.11
1932 on 1933	31.69	1.88	5.54	1.47	6.52	1.41	3.03	.72
1934 on 1933	9.55	1.02	4.22	.89	15.80	1.20	7.30	.63
1933 on 1934	8.54	1.87	7.92	1.49	9.93	1.06	5.24	1.64
1934 on 1932	7.22	.68	2.81	1.17	10.68	1.23	5.16	1.26
1932 on 1934	5.78	1.13	5.51	1.20	7.02	.92	1.05	.63
1935 on 1934							9.99	10.29
1934 on 1935							4.66	.94
1936 on 1935							5.54	1.05
1935 on 1936							1.18	1.09
1936 on 1934							6.14	8.08
1934 on 1936							3.45	.78

PROPORTION OF RANDOM SAMPLES IN WHICH VALUE OF F WOULD BE EXCEEDED	APPROXIMATE VALUE OF F THAT WOULD BE EXCEEDED BY CHANCE [3]
1 in 20	1.55
1 in 100	1.85
1 in 1,000	2.20

[1] Individual practitioners only. Excludes in both original and weighted analysis for 1932-33 and 1933-34 one return reporting an income of $8,357 in 1932; $50,471 in 1933; $3,165 in 1934.

[2] Individual practitioners only. Excludes in both original and weighted analysis for the 1935 sample one return reporting an income of $41,000 in 1932; —$19,000 in 1933; $13,000 in 1934.

[3] Precise values of F vary because of differences in the number of degrees of freedom. They are slightly lower for physicians and slightly higher for the other professions. However, in no case would the probability attached to a variance ratio be altered by using the exact significance values. The number of degrees of freedom for means about the regression varies from 25 to 28 for physicians, 19 to 22 for dentists, 21 to 24 for lawyers, and 20 to 22 for accountants (both samples). The number of degrees of freedom for observations about the means varies from 1,350 to 1,416 for physicians, 996 to 1,036 for dentists, 650 to 705 for lawyers, 752 to 809 for the 1935 accountancy sample, and 450 to 483 for the 1937 accountancy sample.

1, and 0.1 per cent of random samples.[3] All except two of the thirty variance ratios are larger than the value of F that would be exceeded by chance in one in a thousand random samples, and most

[3] The values are 'approximate' because the number of degrees of freedom for the means about the regression and for the observations about the means are not the same for all professions and regressions. The variation in the number of degrees of freedom is too slight to justify giving separate significance values of F for each regression or profession. The use of exact significance values would not alter the probability attached to any variance ratio.

of them are very much larger. If the variance ratios were taken at face value we should be forced to conclude that the regression between incomes in two years is very definitely not linear.

The variance ratios cannot, however, be taken at face value. The significance values of F at the bottom of Table 59 that were used as the basis of comparison are computed on the assumption that the variance of the observations about the means is the same for all classes and that the observations are drawn from a normal universe. Consequently, in comparing the computed F's with these significance values we are in effect testing the composite hypothesis that (1) the regression is linear, (2) the variance of the observations about the means is the same for all classes, and (3) the observations in each class are a random sample from a normal universe. Chart 26 demonstrates conclusively that the variance of the observations about the means is not the same for all income classes and Chart 1 and Table 12 as well as the frequency distributions in Appendix B, suggest strongly that the distribution of the observations is far from normal.[4] Consequently, the high values of F may reflect the failure of these two conditions to be satisfied rather than nonlinearity of regression.

a Effect of unequal variances

The description of how the two estimates of variance are computed makes it clear that the contribution of each class to the sum of the squares of the observations about class means is proportional to the number of observations in the class minus one, but that all classes contribute equally to the sum of the squares of the means about the regression.[5] Chart 26 demonstrates that after a brief

[4] The evidence on nonnormality cited is not conclusive since it pertains to the distribution as a whole, whereas the relevant question in the present connection is the distribution of the observations within each base year income class. Inspection of these distributions tends to confirm the indication given by the distribution for each year as a whole: they too are definitely not normal but tend to be skewed.

[5] There is no inconsistency between this statement and the fact that the square of the difference between the class mean and the corresponding ordinate of the regression equation is multiplied by the number of observations on which the mean is based. The square of the difference between the class mean and the regression equation gives an estimate of the variance of a mean; the variance of a mean is the variance of an individual observation divided by the number of observations on which the mean is based; multiplication by the number of ob-

initial decline, the variance of the observations about the class mean increases consistently with the base year income. Moreover, the number of observations in an income class at first increases and then decreases with income so that the low and intermediate income classes include the most observations. Consequently, the high income classes, which have few observations, receive a much greater weight in the variance of the means about the regression than in the variance of the observations about the means. Since the high income classes have large variances, the former variance would consequently tend to exceed the latter, and the analysis of variance ratio would tend to exceed unity, even though the means differed from the regressions solely because of chance fluctuations.

If we knew the relations among the variances of the different income classes, we could correct for this bias in the *F*'s and test the hypothesis of linearity without assuming equal variance by weighting both the sums of the squares within classes and the squared differences between the class means and the regression by numbers proportional to the reciprocals of the class variances.[6] Since we do not know the relations among the variances of the different income classes, we estimated them from the data. Forced on us by our lack of knowledge, this procedure is not entirely accurate since estimating the relations among the variances uses up an unknown number of degrees of freedom, for which no allowance was made.

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servations merely converts an estimate of the variance of a mean into an estimate of the variance of an individual observation. The essential point in the present connection is rather that each additional class increases by one the number of degrees of freedom of means about the regression, but increases the number of degrees of freedom of observations about means by the number of observations in the class minus one.

6 Let $\sigma_i^2 = w_i \sigma_0^2$, where σ_i^2 is the 'true' variance of the *i*-th income class, and σ_0^2, the 'true' variance of an income class selected as the base. The sum of squared deviations of the observations in the *i*-th class from their mean divided by $n_i - 1$, where n_i is the number of observations in the *i*-th class, is an estimate of σ_i^2 based on $n_i - 1$ degrees of freedom. The squared deviation of the class mean from the regression multiplied by n_i is also an estimate of σ_i^2 if the 'true' regression is linear. If divided by w_i, these estimates are converted into estimates of σ_0^2, which, by definition and unlike σ_i^2, is constant for all income classes. Consequently, the weighted sum of squared deviations about the class means divided by the total number of degrees of freedom within classes and the weighted sum of squared deviations of means about the regression divided by the corresponding number of degrees of freedom both give estimates of the same thing, namely σ_0^2, if $1/w_i$ is used as the weight for the *i*-th class.

The method used to estimate the relations among the variances was to construct a chart for each sample similar to the sections of Chart 26 but based on class variances rather than standard deviations, and giving the points for each year rather than their average. The variance of each income class was expressed as a ratio to the mean variance for all classes, i.e., as a ratio to the denominator of the corresponding analysis of variance ratio in the columns of Table 59 headed 'original', and plotted against the midpoint of the proper base year income class. A free hand curve was drawn through the resulting scatter and extended to cover very high or very low income classes containing a single observation and therefore giving no estimates of the class variance though contributing to the sum of the squares of the class means about the regression.[7] The ordinates of the free hand curve corresponding to the successive income classes were read from the graphs and their reciprocals used as weights. The use of a single curve for all regressions from the same sample minimizes the number of degrees of freedom used in obtaining the weights. At the same time, the scatter about the final free hand curve was usually very large and the curve often had to be extrapolated a considerable distance in order to get weights for all income classes. In consequence, a large element of personal judgment entered into the determination of the weights and seriously limits the confidence that can be attached to the results.

The inequality of the class variances not only imparts an upward bias to the analysis of variance ratios but also renders the regressions computed by the ordinary least squares method inappropriate. A new set of regression equations was therefore computed by weighting the observations by the reciprocals of the ordinates of the free hand curve described in the preceding paragraph, i.e., by weighting the observations inversely to their estimated variances. These are the regressions plotted in Chart 25. The constants of both the original and weighted regressions are given in Table 60.

The weights from the free hand curves fitted to the variances and the weighted regressions make it possible to compute analysis of variance ratios that are independent of the assumption that the class variance is the same for all income classes. These analysis of

[7] We resorted to the use of free hand curves only after considerable experimentation with fitting mathematical functions to the class variances. No relatively simple function gave anything approaching an adequate fit.

TABLE 60

Constants of Original and Weighted Regression Equations

REGRESSION	CONSTANTS[1] Original a	Original b	Weighted a	Weighted b
		Physicians		
1930 on 1929	169	.876		
1931 on 1929	237	.749		
1932 on 1929	243	.546		
1933 on 1932	217	.866	285	.841
1932 on 1933	278	.976	136	1.031
1934 on 1933	154	1.108	259	1.057
1933 on 1934	174	.809	124	.825
1934 on 1932	309	.984	522	.912
1932 on 1934	413	.804	312	.837
		Dentists		
1930 on 1929	389	.875		
1931 on 1929	353	.749		
1932 on 1929	355	.541		
1933 on 1932	235	.810	286	.791
1932 on 1933	74	1.090	160	1.044
1934 on 1933	395	.963	329	.979
1933 on 1934	—45	.910	81	.864
1934 on 1932	535	.804	503	.816
1932 on 1934	—49	1.022	167	.937
		Lawyers[2]		
1933 on 1932	440	.726	258	.795
1932 on 1933	451	.957	289	1.004
1934 on 1933	274	1.000	366	.940
1933 on 1934	464	.730	283	.804
1934 on 1932	445	.818	425	.833
1932 on 1934	729	.770	473	.865
		Certified Public Accountants[3]		
1930 on 1929	1,048	.769		
1931 on 1929	1,197	.605		
1932 on 1929	1,152	.480		
1933 on 1932	497	.818	489	.824
1932 on 1933	562	.879	379	.939
1934 on 1933	681	.909	524	.969
1933 on 1934	330	.808	291	.816
1934 on 1932	1,085	.754	889	.823
1932 on 1934	883	.708	666	.764
1935 on 1934	731	.890	561	.933
1934 on 1935	629	.755	228	.880

TABLE 60 (cont.)

RECREGRESSION	CONSTANTS[1] Original a	b	Weighted a	b
		Certified Public Accountants[3] (cont.)		
1936 on 1935	831	.931	461	1.038
1935 on 1936	277	.808	262	.817
1936 on 1934	978	.963	818	.998
1934 on 1936	439	.708	304	.747
		Consulting Engineers[4]		
1930 on 1929	1,436	.724		
1931 on 1929	165	.585		
1932 on 1929	195	.302		

[1] General equation: $y = a + bx$, where $x =$ independent variable (income in base year); $y =$ dependent variable (income in other year); $a =$ intercept; $b =$ slope.

[2] Individual practitioners only. In computing both original and weighted constants for the 1932–33 and 1933–34 regressions, we excluded one return reporting an income of $8,357 in 1932; $50,471 in 1933; $3,165 in 1934.

[3] Individual practitioners only. In computing both original and weighted constants for the 1935 sample, we excluded one return reporting an income of $41,000 in 1932; — $19,000 in 1933; $13,000 in 1934.

[4] Individual practitioners only.

variance ratios are given in the columns of Table 59 headed 'weighted'. As already implied, these *F*'s were obtained by the same procedure as the original *F*'s except that the class means were expressed as deviations from the weighted regressions and that both the sum of the squares of the observations about the means and the sum of the squares of means about the regressions are weighted sums. All except two of the weighted *F*'s are smaller than the original *F*'s and most of the former are a small fraction of the latter. Whereas 28 of the 30 original *F*'s are larger than the value of *F* that would be exceeded by chance in less than one in a thousand random samples, only three of the weighted *F*'s are so large; two of the other 27 weighted *F*'s would be exceeded by chance in less than one in a hundred random samples, and one, in less than one in twenty. The remaining 24 values would be exceeded by chance more than once in twenty times and hence are entirely consistent with the hypothesis that the regressions are linear. Moreover, the two largest weighted *F*'s—those for the 1935 on 1934 and 1936 on

1934 accountancy regressions—reflect not a consistent departure from linearity but the wide deviations from the regressions of two of the almost 500 observations on which they are based. If these two observations are eliminated, the *F*'s are reduced from 10.29 and 8.09 to 1.19 and .97 respectively.[8]

The weighted analysis strongly suggests that linearity of regression is the rule and that the high values of the original *F*'s reflect the inequality of class variances rather than nonlinearity of the regressions. However, for the time being at least, this conclusion too must be accepted with the greatest reservation. For just as the extremely high values of the original *F*'s reflect the inequality of the variances, so the low values of the weighted *F*'s may conceivably reflect the nonnormality of the income distributions—the third element of our original three-pronged hypothesis.

b *Effect of nonnormality*

Several studies suggest that the effect of nonnormality on the analysis of variance is not serious.[9] However, since they related to effects of moderate departures from normality, whereas our data are markedly nonnormal, they do not justify the assumption that our results are equally little affected.

Unfortunately, we know no way of correcting adequately for the influence of nonnormality without more exact knowledge of the distribution of income for base year income classes than we possess, i.e., without knowing the equation of a mathematical formula that adequately describes these distributions. Some indirect indication of the possible effect of nonnormality is given by the efficiency of the ordinary estimate of variance for data whose logarithms are normally distributed. In the discussion in footnote 2 to Chapter 3 it was noted that though the logarithmic normal

[8] The two observations eliminated have the values: (1) 1934, $0; 1935, $7,856; 1936, $8,331; (2) 1934, —$5,903; 1935, $8,744; 1936, $5,926. The results of eliminating each observation separately are as follows:

	F FOR	
	1935 on 1934	1936 on 1934
Eliminating return (1)	2.70	1.60
Eliminating return (2)	9.26	7.86

[9] See E. S. Pearson, 'The Analysis of Variance in Cases of Non-normal Variation', *Biometrika*, 1931; and T. Eden and F. Yates, 'On the Validity of Fisher's *z* Test When Applied to an Actual Example of Non-normal Data', *Journal of Agricultural Science*, Jan. 1933.

curve does not describe income distributions adequately; it is probably better than any of the other curves suggested.

Let x represent the original observations and y their natural logarithms (i.e., logarithms to the base e); let μ_y and σ_y be the 'true' mean and standard deviation respectively of the logarithms of the observations, i.e., of the normal curve according to which the logarithms are distributed; and μ_x and σ_x the 'true' mean and standard deviation respectively of the observations themselves. The best estimates[10] of μ_y and σ_y from a sample of observations are given by the usual formulae:

$$(1)\ \text{Estimate of } \mu_y = \frac{\Sigma y}{n} = \frac{\Sigma \log x}{n} = \bar{y} = \overline{\log x};$$

$$(2)\ \text{Estimate of } \sigma_y = \sqrt{\frac{\Sigma(y - \bar{y}^2)}{n-1}} = \sqrt{\frac{\Sigma(\log x - \overline{\log x})^2}{n-1}} = s_y,$$

where n is the number of observations in the sample. The best estimates of μ_x and σ_x are not, however, given by the usual formulae but by

$$(3)\ \text{Estimate of } \mu_x = e^{\bar{y} + \frac{s_y^2}{2}} = \bar{x};$$

$$(4)\ \text{Estimate of } \sigma_x = \left(e^{\bar{y} + \frac{s_y^2}{2}}\right)\sqrt{e^{s_y^2} - 1} = s_x.$$

That is, the logarithm to the base e of the best estimate of the mean of x is the mean of the logarithms plus one-half the variance of the logarithms; to get the best estimate of the standard deviation of x we must find the value whose logarithm to the base e is the variance of the logarithms, subtract unity, take the square root, and multiply by the best estimate of the mean of x obtained as just described.[11]

10 In what follows the 'best estimates' will be interpreted as the maximum likelihood estimates.

11 If common instead of natural logarithms are used, i.e., if the logarithms are taken to the base 10 instead of to the base e, the formulae for the best estimates of μ_x and σ_x become:

$$\text{Estimate of } \mu_x = 10^{\bar{y} + 1\cdot151293\, s_y^2}$$

$$\text{Estimate of } \sigma_x = 10^{\bar{y} + 1\cdot151293\, s_y^2}\sqrt{10^{2\cdot30259\, s_y^2} - 1},$$

where $\bar{y}$ and s_y^2 are computed by the ordinary formulae from the common logarithms of the observations.

The efficiency of an estimate of a parameter is defined as the ratio of the variance of the best estimate of the parameter to the variance of the estimate under consideration. Let $\bar{x}'$ and s'_x be estimates of the mean and standard deviations of the observations computed by the usual formulae (i.e., formulae like (1) and (2) but applied to the original observations). It can be shown that the efficiency of $\bar{x}'$ is given by

$$(5)\qquad \frac{\sigma_{\bar{x}}^2}{\sigma_{\bar{x}'}^2} = \frac{\sigma_y^2 + \frac{\sigma_y^4}{2}}{e^{\sigma_y^2} - 1};$$

the efficiency of $(s'_x)^2$, i.e., of the estimate of the variance of the original observations computed by the usual formula, by

$$(6)\qquad \frac{\sigma^2_{s_x^2}}{\sigma^2_{(s'_x)^2}} = \frac{2\,\sigma_y^2\left[2(e^{\sigma_y^2} - 1)^2 + (2e^{\sigma_y^2} - 1)^2\sigma_y^2\right]}{e^{6\,\sigma_y^2} - 4\,e^{3\,\sigma_y^2} - e^{2\,\sigma_y^2} + 8\,e^{\sigma_y^2} - 4}.$$

As equations (5) and (6) indicate, the efficiency of the ordinary estimates of the mean and variance of observations from a logarithmically normal population depends on the variance of that population. It is, consequently, impossible to give any general indication of their efficiency. However, in the course of experimenting with alternative methods of testing the linearity of the regressions we computed the variance of the logarithms of the observations about their class means for the 1933 on 1934 accountancy regression. For this special case [12] the efficiency of the ordinary estimate is approximately 99.5 per cent for the mean and 80 per cent for the variance. The loss of information through using the ordinary estimate is negligible for the mean, but sizable for the variance.[13]

The possible inefficiency of the ordinary estimate of variance, if the observations are drawn from a logarithmically normal population, strongly suggests, though it does not conclusively demonstrate, that an analysis of variance of the type used above is an

12 The mean intraclass variance of the logarithms to the base e is .14135 for income classes above $1,500. This figure is used in the text in estimating the efficiency of the mean and variance.

13 If the mean variance of the logarithms is estimated for income classes above $500, instead of above $1,500, as in the preceding footnote, it becomes .26738. The efficiency of the mean is then 99 per cent, but of the variance only 59 per cent.

inefficient test of linearity. But whether or not this deduction can validly be made, it is clear that marked departures from normality can affect greatly the validity of statistical tests based on variances; and that in consequence, little confidence can be placed in the results of tests assuming normality when, as in the present instance, the data to which they are applied are known to deviate widely from normality.

c An alternative test of linearity[14]

A test of linearity that is independent of the assumptions of equal class variance and normal distributions can be based on the diagrams of Chart 25. Each diagram of Chart 25 shows ten points and a regression equation fitted to them. We can ask whether there is any consistency in the deviations of the points from the different regressions for the same profession.

An example designed to answer this question is Table 61, based on the nine regressions for physicians. Each column corresponds to one point on the diagrams of Chart 25—column 1 to the first point, column 2 to the second, and so on. Each row of the table is for a particular regression. The pluses and minuses indicate whether the point is above or below the relevant regression; a zero indicates that the point falls exactly on the regression line.[15] At the bottom of the table are entered the number of pluses and minuses in each row, and the total number of pluses and minuses in the table.

If the regressions are not really linear we should expect the points to differ systematically from the straight line. For example, if the true regression were convex to the horizontal axis, the first few points and the last few points should be consistently above the fitted straight line, and the intermediate points consistently below. This would be reflected in a preponderance of pluses in the first few columns of Table 61, of minuses in the intermediate columns, and of pluses in the last few columns. On the other hand, if regressions deviate little from linearity, the pluses and minuses should be randomly distributed; the proportion of pluses in each column should be approximately the same as in the whole table. To test

[14] This test was suggested by a procedure applied to an analogous problem by G. H. Moore and W. A. Wallis.

[15] At least, to the number of significant figures to which we have carried our computations.

whether the pluses and minuses are randomly distributed among the columns we can treat the two rows giving the number of pluses and minuses in each column as a two-by-ten contingency table, compute χ^2 for the table, and see how frequently the observed value of χ^2 would be exceeded by chance.[16] For Table 61, χ^2 is 14.6 and would be exceeded by chance in about one-tenth of random samples. The table gives no evidence of consistent departure from linearity.

TABLE 61

Test of Consistency of Deviation of Points from Regressions

Physicians

REGRESSION	SIGN OF DEVIATION FROM REGRESSION OF POINT NUMBER										TOTAL
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	
1930 on 1929	+	−	o	+	+	−	+	−	+	o	
1931 on 1929	+	−	−	+	+	+	+	−	+	+	
1932 on 1929	+	−	−	−	+	+	+	−	+	−	
1933 on 1932	−	+	−	−	+	+	+	−	−	+	
1932 on 1933	+	+	−	−	+	+	−	−	+	−	
1934 on 1933	+	+	−	−	−	−	+	+	+	+	
1933 on 1934	+	+	+	+	+	+	−	−	−	−	
1934 on 1932	−	+	−	−	+	+	+	−	−	+	
1932 on 1934	+	−	+	−	−	+	−	+	−	+	
Number of +'s	7	5	2.5	3	7	7	6	2	5	5.5	50
Number of −'s	2	4	6.5	6	2	2	3	7	4	3.5	40
Total	9	9	9	9	9	9	9	9	9	9	

Weighted regressions for 1935 sample; regressions fitted to ten plotted points for 1933 sample.

Before presenting the results for the other professions, we note one limitation of this test. While it makes no assumption about normality or equal variance it does assume that the items in each column are independent. Our data do not satisfy this assumption. The different regressions for any one sample are based on data for essentially the same individuals. Even more serious is the interdependence among the regressions for which the base year is the same; for these, each base year income class includes essentially the same individuals. For example, both the 1933 on 1932 and 1934 on 1932 regressions summarize the average incomes of individuals grouped by their 1932 income. If the individuals in a particular

[16] See Fisher, *Statistical Methods for Research Workers*, Sec. 21.

1932 income class have an abnormally high average income in 1933 there is some reason to suppose that they will have an abnormally high income also in 1934. This interdependence among the items in the same class tends to make for a concentration of pluses or minuses and hence imparts an upward bias to χ^2.

The importance of this interdependence is revealed by pairing the regressions from the same sample in two ways: first, into pairs with the same base year, e.g., the regressions 1933 on 1932, and 1934 on 1932, the regressions 1932 on 1933, and 1934 on 1933, and so on; second, into pairs with different base years but the same second year, e.g., the regressions 1932 on 1933, and 1932 on 1934, the regressions 1933 on 1932, and 1933 on 1934, and so on. For each pair we can count the points for which the signs entered in tables like Table 61 agree. For example, the 1933 on 1932, and 1934 on 1932 regressions for physicians agree in sign for all ten points. If this is done for all professions and all pairs of regressions from the 1935 and 1937 samples, the number of agreements in sign is 101 out of 150 for pairs with the same base year but only 76 out of 150 for pairs with different base years.[17] The effect of the interdependence is far from negligible.

The values of χ^2 computed from tables similar to Table 61, and the probability that each would be exceeded by chance, follow.[18] In view of the bias toward large values of χ^2 arising from the

PROFESSION	χ^2	PROBABILITY OF BEING EXCEEDED BY CHANCE
Physicians	14.6	.10
Dentists	18.2	.03
Lawyers	18.2	.03
Certified public accountants	24.5	.004

interdependence of regressions from the same sample, only the χ^2 for accountants, for whom this bias is least since data are available from three samples, seems sufficiently large to establish even a presumption in favor of nonlinearity. The χ^2 for both dentists and lawyers are indeed larger than the values we should ordinarily be willing to attribute to chance, but both are very close to the ordinary borderline of significance—a probability of .05—and, in

17 The difference between 101 and 76 is about three times its standard error.

18 No χ^2 is given for engineers since only three regressions are available.

addition, the χ^2 for lawyers is based on a single sample and hence is most subject to bias.

We are thus led to the conclusion enunciated in the text of this chapter; namely, that accountancy is the one profession for which there is a tolerably clear indication that the regression is not linear.

2 THE ANALYSIS OF CHANGES IN RELATIVE INCOME STATUS FOR PERIODS LONGER THAN TWO YEARS

Section 2 of this chapter outlines a procedure for interpreting changes in relative income status between any two years. The present section sketches a generalization of this procedure applicable to a period of any length. This generalized procedure is embodied in an appendix rather than in the text; first, because it is tentative and intended to be suggestive rather than definitive, second, because our data are not entirely suited to its application.

The dichotomy between permanent and transitory components of a man's income, as was noted above, necessarily does violence to the facts. An accurate description of the factors determining a man's income must substitute a continuum for the dichotomy. This continuum is bounded at one extreme by 'truly' permanent factors—those that affect a man's income throughout his career—and at the other by the 'truly' transitory—those that affect his income only during a single time unit, where a time unit is the shortest period during which it seems desirable to measure income, in our case, a year, but conceivably a month, a week, a day, an hour, or even a minute. Between these extremes fall what may be called 'quasi-permanent' factors, factors whose effects neither disappear at once nor last throughout a man's career. The quasi-permanent factors can be ordered according to the length of time during which their effects persist: two-year factors, three-year factors, etc.

A man's income can then be conceived as the sum of the parts attributable to each set of factors:

$$\begin{aligned}\text{Income in any year} &= \text{permanent component}\\ &+ \text{quasi-permanent component}\\ &+ \text{transitory component.}\end{aligned}$$

The quasi-permanent component in any year can in turn be expressed as:

Quasi-permanent component = (two-year component ending this year
+ three-year component ending this year
+ four-year component ending this year
+ etc.)
+ (two-year component ending next year
+ three-year component ending next year
+ four-year component ending next year
+ etc.)
+ (three-year component ending year after next
+ four-year component ending year after next
+ etc.)
+ etc.

This framework classifies factors affecting income according to the length of time during which their effects are present and the year in which their effects come to an end. How far the classification is carried depends on the period covered by the analysis. For example, if a three-year period is considered, the classification would stop with two-year components. A three-year component ending in the third year of the period would be permanent for these three years, a three-year component ending in the second year of the period would be equivalent to a two-year component ending in that year, and so on.

We can symbolize these relations for a three-year period (years 1, 2, and 3) as follows:

$$(1)\quad \begin{aligned} x'_1 &= p'_1 + t'_1 + q'_{11} + q'_{12}; \\ x'_2 &= p'_2 + t'_2 \qquad\quad + q'_{22} + q'_{23}; \\ x'_3 &= p'_3 + t'_3 \qquad\qquad\qquad + q'_{33} + q'_{34}, \end{aligned}$$

where x' is income; p', the component that is permanent for the three years considered; t', the transitory component; q', the quasi-permanent component, in this case, a two-year component; the subscripts to x', p', and t' denote the year in which the income, permanent component, and transitory component are received; the first subscript to q' denotes the year in which it is received, and the second, the year in which the component ends (i.e., the last year in which income is affected by the factors responsible for this component).

As already noted, the components that summarize the effects of factors present in two of the three years, for example, q'_{12} and q'_{22}, may include the final (or for q'_{23} and q'_{33}, the initial) effects of factors present for more than two years. From the point of view of the three years, however, these partial effects are indistinguishable from two-year effects proper. In the same way, data for this three-year period alone would never permit the segregation of those effects in year 1 that are a carry-over from preceding years but do not continue into the future (q'_{11}) from the purely transitory effects (t'_1); q'_{11} and t'_1, and in similar fashion q'_{34} and t'_3, will necessarily be merged in the data. We split the component present in year 1 (or year 3) but not in the other two years into two parts in order to get comparable components for different years. What will appear from the data to be transitory for year 1 ($q'_{11} + t'_1$) or for year 3 ($q'_{34} + t'_3$) is not comparable with what will appear from the data to be transitory for year 2 (t'_2).

It follows from the considerations just advanced that lengthening the period to four years, which would involve introducing three-year components, will leave the content of t'_1, t'_2, and t'_3 unchanged, but will reduce the content of the p' and q' terms, i.e., of the permanent and quasi-permanent components. It follows also that while we might hope from data for three years to segregate the permanent, quasi-permanent, and transitory components of year 2 incomes, we cannot do so for year 1 or year 3 unless we assume each component equally important in all years; otherwise, the quasi-permanent and transitory components will be merged.

Our analysis has so far precisely paralleled the two-year analysis in Section 2 of this chapter. At this point we part company with that analysis for a time. Instead of attempting to segregate the different components for a particular income class, we shall outline a procedure for estimating the average importance of each component for all income classes combined. This will lead to an alternative interpretation of the percentage contributions presented in Table 58. We shall then return to the problem of segregating the components for each income class separately. As in the analysis in the text of the chapter, however, some assumption about the relation between permanent (and quasi-permanent) components in different years must be made. We shall consider two alternative assumptions: the one used in the text of the chapter, designated the 'mean assumption'; and the one used in footnotes 11, 14, and 15 of the chapter, designated the 'variability assumption'.

a The mean assumption

For a group of men in practice the same number of years it seems reasonable to assume that the permanent and quasi-permanent components change in proportion to the change in the arithmetic mean income of the group as a whole; i.e., that factors common to several years would, if they alone were present, lead to a constant ratio between a man's income and average income in the profession. This ratio could then serve as a measure of relative income status. The same assumption would not seem reasonable for a group of men in practice varying numbers of years. Permanent forces whose effects depend on 'experience' would then lead to changes in income ratios: the ratios would tend to rise over time for the younger men, and to fall for older men past their peak earnings. Even for men in practice the same number of years the assumption is doubtless not entirely valid. The 'earnings life cycle' probably depends in a consistent fashion on other permanent factors; for example, it is probably different for physicians who specialize than for physicians who engage in general practice. However, eliminating the influence of differences in the number of years in practice probably eliminates most of the changes in the permanent and quasi-permanent components not related to changes in general economic conditions in the profession.

i *The procedure.*[19] In symbols, the mean assumption can be written:

$$\begin{aligned} &\frac{p'_1}{\bar{x}'_1} = \frac{p'_2}{\bar{x}'_2} = \frac{p'_3}{\bar{x}'_3} = p; \\ (2)\quad &\frac{q'_{12}}{\bar{x}'_1} = \frac{q'_{22}}{\bar{x}'_2} = q_2; \\ &\frac{q'_{23}}{\bar{x}'_2} = \frac{q'_{33}}{\bar{x}'_3} = q_3, \end{aligned}$$

19 The reader not interested in the derivation of the procedure should pass directly to Section 2a ii below.

where $\bar{x}'$ is the arithmetic mean income in the year indicated by the subscript. Dividing the equations (1) by the corresponding mean incomes, we have

$$
(3)\quad
\begin{aligned}
x_1 &= p + t_1 + q_1 + q_2;\\
x_2 &= p + t_2 \qquad + q_2 + q_3;\\
x_3 &= p + t_3 \qquad\qquad + q_3 + q_4,
\end{aligned}
$$

where $x = x'/\bar{x}'$; $t = t'/\bar{x}'$, the subscripts indicating the year; and $q_1 = q'_{11}/\bar{x}'_1$; $q_4 = q'_{34}/\bar{x}'_3$.

If the different components of the income of any year are uncorrelated (i.e., if $r_{pt} = r_{pq} = r_{tq} = r_{q_1 q_2} = r_{q_2 q_3} = r_{q_3 q_4} = 0$, where r is the correlation coefficient between the variables indicated by the subscripts), then

$$(4)\quad s^2_{x_1} = s^2_p + s^2_{t_1} + s^2_{q_1} + s^2_{q_2};$$

$$(5)\quad s^2_{x_2} = s^2_p + s^2_{t_2} + s^2_{q_2} + s^2_{q_3};$$

$$(6)\quad s^2_{x_3} = s^2_p + s^2_{t_3} + s^2_{q_3} + s^2_{q_4},$$

where s stands for the standard deviation of the variable indicated by the subscript. The standard deviation of the income ratio (x) is of course equal to the coefficient of variation of actual income (x').

If further, the transitory components in different years are uncorrelated (i.e., $r_{t_1 t_2} = r_{t_1 t_3} = r_{t_2 t_3} = 0$), then

$$(7)\quad r_{x_1 x_2} s_{x_1} s_{x_2} = s^2_p + s^2_{q_2};$$

$$(8)\quad r_{x_2 x_3} s_{x_2} s_{x_3} = s^2_p + s^2_{q_3};$$

$$(9)\quad r_{x_1 x_3} s_{x_1} s_{x_3} = s^2_p.$$

From equations (4) through (9),

$$(10)\quad s^2_p = r_{x_1 x_3} s_{x_1} s_{x_3} = (9);$$

$$(11)\quad s^2_{q_2} = r_{x_1 x_2} s_{x_1} s_{x_2} - r_{x_1 x_3} s_{x_1} s_{x_3} = (7) - (9);$$

$$(12)\quad s^2_{q_3} = r_{x_2 x_3} s_{x_2} s_{x_3} - r_{x_1 x_3} s_{x_1} s_{x_3} = (8) - (9);$$

$$(13)\quad s^2_{t_2} = s^2_{x_2} - s^2_p - s^2_{q_2} - s^2_{q_3} = (5) - (7) - (12);$$

$$(14)\quad s^2_{t_1} + s^2_{q_1} = s^2_{x_1} - s^2_p - s^2_{q_2} = (4) - (10) - (11);$$

$$(15)\quad s^2_{t_3} + s^2_{q_4} = s^2_{x_3} - s^2_p - s^2_{q_3} = (6) - (10) - (12).$$

If now we designate the proportionate contribution of the permanent, quasi-permanent, and transitory components to the total variance of income in any year as P, Q, and T, indicating the year by subscripts, we have

$$(16) \quad P_1 = \frac{s_p^2}{s_{x_1}^2} = \frac{(10)}{(4)};$$

$$(17) \quad P_2 = \frac{s_p^2}{s_{x_2}^2} = \frac{(10)}{(5)};$$

$$(18) \quad P_3 = \frac{s_p^2}{s_{x_3}^2} = \frac{(10)}{(6)};$$

$$(19) \quad T_2 = \frac{s_{t_2}^2}{s_{x_2}^2} = \frac{(13)}{(5)};$$

$$(20) \quad Q_2 = \frac{s_{q_2}^2 + s_{q_3}^2}{s_{x_2}^2} = 1 - P_2 - T_2 = 1 - (17) - (19).$$

As was implied in the discussion above, we cannot segregate the transitory from the quasi-permanent component for years 1 and 3. But we can derive

$$(21) \quad T_1 + Q_1 = 1 - P_1 = 1 - (16);$$
$$(22) \quad T_3 + Q_3 = 1 - P_3 = 1 - (18).$$

These proportionate contributions can be derived from statistical data on the incomes in three successive years of a group of professional men in practice the same number of years, since s_x^2 is given by the square of the coefficient of variation of actual income, and the correlation coefficient between income ratios in two years is numerically identical with the correlation coefficient between actual incomes.[20]

[20] Instead of using income ratios, we could have used logarithms and assumed that the deviations of the logarithms of the permanent components from their mean were identical in different years, and similarly for the quasi-permanent components. This set of assumptions would lead to the same equations that we obtained, except that the correlation coefficients between dollar incomes would be replaced by the correlation coefficients between the logarithms of income and the coefficients of variation by the standard deviations of the logarithms. The use of logarithms has some advantages, since it may well be more reasonable to assume that logarithmic components are additive and uncorrelated than to assume that dollar components are.

The relation between these results and those presented in Section 2 of the text of this chapter can be seen most easily if we express P_1 in terms of the regression coefficient. It can easily be shown that

$$P_1 = b_{31}\frac{\bar{x}'_1}{\bar{x}'_3},$$

from which,

$$Q_1 + T_1 = 1 - b_{31}\frac{\bar{x}'_1}{\bar{x}'_3}.$$

This is the formula used in computing the percentage contribution of the component considered transitory for years 1 and 3 (see footnote 15 of the text). It follows that the percentages attributed to the transitory and permanent components in Table 58, there interpreted as contributions to the deviation of the mean income of each income class from the mean income in the profession, can also be interpreted as the percentages of the total variance of income attributable to the two components.[21]

Equations (16) through (22) enable us to compute the percentage contribution of the different components to the variance of the income distribution as a whole. We cannot get an equally detailed breakdown for separate income classes. Let a bar over each symbol designate an arithmetic mean, and a subscript to the left of the symbol, the year that serves as a basis for grouping the individuals. For example, ${}_1\bar{x}_2$ will stand for the average income ratio in year 2 of a year 1 income class. If the transitory and quasi-permanent components are defined so that their average for all individuals is zero, then not only the average income ratio but also the average permanent component expressed as a ratio to the average income (p) will be unity for the income distribution as a whole. We can then write the following sets of equations:

$$(23)\quad {}_1\bar{x}_1 = {}_1\bar{p} + {}_1\bar{t}_1 + {}_1\bar{q}_1 + {}_1\bar{q}_2;$$
$$(24)\quad {}_1\bar{x}_2 = {}_1\bar{p} + {}_1\bar{q}_2;$$
$$(25)\quad {}_1\bar{x}_3 = {}_1\bar{p};$$
$$(26)\quad {}_2\bar{x}_1 = {}_2\bar{p} + {}_2\bar{q}_2;$$
$$(27)\quad {}_2\bar{x}_2 = {}_2\bar{p} + {}_2\bar{t}_2 + {}_2\bar{q}_2 + {}_2\bar{q}_3;$$

21 Formulae (17) to (22) can also be expressed in terms of the regression coefficients.

(28) ${}_2\bar{x}_3 = {}_2\bar{p} + {}_2\bar{q}_3;$
(29) ${}_3\bar{x}_1 = {}_3\bar{p};$
(30) ${}_3\bar{x}_2 = {}_3\bar{p} + {}_3\bar{q}_3;$
(31) ${}_3\bar{x}_3 = {}_3\bar{p} + {}_3\bar{t}_3 + {}_3\bar{q}_3 + {}_3\bar{q}_4.$

It follows that for year 1 and year 3 income classes we can isolate the permanent component and one part of the quasi-permanent component but not the transitory component; for year 2 income classes we cannot isolate the permanent component alone, but can only get it in combination with part of the quasi-permanent component. To compute percentage contributions the isolated parts should be expressed as deviations from their mean for the distribution as a whole (unity for p, zero for t and q) and divided by the deviation of the average income ratio in the base year from the average income ratio for the distribution as a whole (unity). In actual computation it would be more convenient to use dollar incomes and the procedure outlined in Table 57. By analogy with the analysis of Section 2 of the text of this chapter, if the percentage contributions of the components that can be isolated are the same for all classes, the regression equations will be linear. In that case, formulae (16) through (22) will measure the percentage contributions for separate income classes as well as for the distribution as a whole.

ii *Illustrative computations of percentage contributions.* The logic of the preceding analysis calls for its application to data for individuals in practice the same number of years. Unfortunately, our data do not satisfy this requirement. Number of years in practice was reported only on the 1937 medical sample, and we have not prepared correlation tables for this sample because of the adjustments that would have been necessary. Despite this defect, we have thought it desirable to use our data to illustrate the kinds of results that can be obtained by the technique outlined above.

The blank spaces in Table 62, which summarizes the numerical results, are for components whose contribution cannot be isolated from data for three years. One 'nonsense' result was obtained, for the 1934 incomes of dentists. The percentage of the total variance attributable to the permanent component turns out to be over 100 per cent, an absurd but not arithmetically impossible result.

TABLE 62

Percentage of Total Variance Attributable to Permanent, Quasi-permanent, and Transitory Components

Computed under Mean Assumption

PROFESSION & YEAR	% OF TOTAL VARIANCE ATTRIBUTABLE TO: PERMANENT COMPONENT	QUASI-PERMANENT COMPONENT: Carried over from preceding year	QUASI-PERMANENT COMPONENT: Carrying over into following year	TRANSITORY COMPONENT
Physicians				
1932	91		1	
1933	87	1	7	5
1934	87	7		
Dentists				
1932	80		10	
1933	85	11	1	3
1934	100.5	1		
Lawyers *				
1932	82		1	
1933	83	1	6	10
1934	75	6		
Certified public accountants * 1935 sample				
1932	71		14	
1933	66	13	16	5
1934	73	17		
1937 sample				
1934	77		13	
1935	77	13	5	5
1936	87	5		

* Individual practitioners only.

This result may reflect errors in our data, the grouping of individuals in practice varying numbers of years, or the falsity of the assumptions on which the theoretical framework rests. The absence of any other 'nonsense' results gives some grounds for accepting the first interpretation.[22]

The transitory component is more important than the quasi-permanent component in law, less important in medicine, dentistry, and accountancy. Moreover, the transitory component contributes about the same percentage to the total variance in the three last professions—about 5 per cent. Differences among these professions in the stability of relative income status apparently reflect primarily differences in the strength of quasi-permanent forces. The lesser stability of relative income status in law than in medicine or dentistry, on the other hand, reflects primarily the greater strength of transitory forces. Apparently, the incomes of lawyers were affected by sizable random influences in 1933.

This analysis would be much more illuminating if we could name the specific factors that are included in the purely formal categories of permanent, quasi-permanent, and transitory factors, and isolate the contribution of each. It is probably not possible to do so for the transitory factors; these are likely to be largely 'chance' influences that have no counterpart in objectively determinable attributes of the individual or his practice, although this need not always be the case. There is more likelihood of naming the factors considered 'permanent' or 'quasi-permanent'. Indeed, the earlier chapters of this book, as already noted, are largely concerned with measuring the influence of specific permanent factors. Using our results for physicians, we can exemplify the

22 However, another 'nonsense' result would have been obtained if we had used an alternative value of the coefficient of variation for accountants. Because our basic correlation analysis was for pairs of years, we had two coefficients of variation for each year. For example, for 1933 we had one coefficient of variation computed from data for individuals who reported their 1932 and 1933 incomes, another computed from data for the slightly different group who reported their 1933 and 1934 incomes. We used the square of the larger of the two coefficients of variation as the base of the percentages. Had we used the smaller coefficients of variation for the 1933 incomes of accountants, the percentage attributable to the transitory component would have been — 0.6. An accurate analysis would call for the use of data only for individuals who reported their incomes in all three years.

kind of breakdown of the total variability of income that would be desired:

Percentage of total variance of 1933 incomes of physicians attributable to:		
Size of community, number of years in practice, and type of practice[23]	22	
Other factors present in all three years, 1932–34	65	
All 'permanent' factors		87
Factors present in		
1932 and 1933 but not 1934	1	
1933 and 1934 but not 1932	7	
All 'quasi-permanent' factors		8
Transitory factors present in 1933 but not 1932 or 1934		5
All factors		100

The 65 per cent attributable to 'other factors present in all three years' may in part reflect chance forces whose influence was present in all three years; in the main, however, it is a measure of the extent to which we have failed in our attempt to isolate permanent factors. It presumably reflects primarily the influence of factors that we have either neglected to consider or whose contribution we have not been able to measure (e.g., training and ability). Ideally, the 22 per cent attributable to size of community, years in practice, and type of practice should be segregated into the parts attributable to each.[24]

[23] We have arbitrarily assumed that the percentage contributed by these factors is the same for 1933 as for 1936, the only year for which we analyzed number of years in practice. The standard deviation of the 1936 incomes of physicians, computed from the 1937 medical sample, is $3,631. If the variation associated with size of community, number of years in practice, and type of practice is eliminated, the standard deviation is reduced to $3,214; i.e., this is the average standard deviation of incomes for physicians in the same size of community, in practice the same number of years, and engaged in the same type of practice. The percentage cited above, 22, was computed from these standard deviations by squaring them, subtracting the smaller square from the larger, and dividing the difference by the larger square.

[24] The variance of each factor that would be computed in an analysis of variance would not be the appropriate variance for such a breakdown for two reasons: first, the variance among, say, size of community classes includes the variance of the other factors as well; second, and more important, even eliminating the variance of the other factors, the part that is left is an estimate of the variance among the size of community income components of not all individuals in the

b The variability assumption

The variability assumption is that both the standard deviation of the permanent component and the standard deviation of the quasi-permanent component are proportional to the standard deviation of actual income.

i *The procedure.* In the symbols used above, except that s here denotes the standard deviation of actual income in the year designated by the subscript, the variability assumption is:

$$(32)\quad \frac{p'_1 - \bar{x}'_1}{s_1} = \frac{p'_2 - \bar{x}'_2}{s_2} = \frac{p'_3 - \bar{x}'_3}{s_3};$$

$$(33)\quad \frac{q'_{12}}{s_1} = \frac{q'_{22}}{s_2};$$

$$(34)\quad \frac{q'_{23}}{s_2} = \frac{q'_{33}}{s_3}.$$

It follows from this assumption that the proportionate contribution of the permanent components, though not necessarily the breakdown of the remainder between quasi-permanent and transitory components, will be equal in all years. By a method similar to that used in the last section these proportionate contributions, if the different components are uncorrelated, can be shown to be:

$$(35)\quad P^* = r_{13};$$

$$(36)\quad Q^*_2 = r_{12} + r_{23} - 2r_{13};$$

$$(37)\quad T^*_2 = 1 - r_{12} - r_{23} + r_{13};$$

$$(38)\quad T^*_1 + Q^*_1 = T^*_3 + Q^*_3 = 1 - r_{13}.$$

The starred symbols have the same meaning as the unstarred symbols previously used, except that they are computed under the variability assumption; r is the correlation coefficient between actual incomes in the years indicated by the subscripts. The breakdown of Q^*_2 into the proportionate contributions of q'_{22} and q'_{23} is given by:

$$(39)\quad Q^*_{22} = r_{12} - r_{13};$$

$$(40)\quad Q^*_{23} = r_{23} - r_{13}.$$

---

profession but a hypothetical sample containing one individual in each size of community class. For both reasons, the variance among size of community classes will tend to be too large.

We have preferred the 'mean assumption' because it does not seem reasonable to assume that the percentage contribution of the permanent factors is the same in all years.

ii *Illustrative computations of percentage contributions.* The results obtained by using the variability assumption do not differ greatly from those obtained by using the mean assumption (compare Table 63 with Table 62). The transitory component is again

TABLE 63

Percentage of Total Variance Attributable to Permanent, Quasi-permanent, and Transitory Components

Computed under Variability Assumption

PROFESSION & YEAR	% OF TOTAL VARIANCE ATTRIBUTABLE TO			
		QUASI-PERMANENT COMPONENT		
	PERMANENT COMPONENT [1]	Carried over from preceding year	Carrying over into following year	TRANSITORY COMPONENT
1933				
Physicians	89	3	6	2
Dentists	91	3	3	3
Lawyers [2]	79	4	6	11
Certified public accountants [2]	73	12	13	2
1935				
Certified public accountants [2]	83	8	4	5

[1] Percentage attributable to permanent component same in 1932 and 1934 as in 1933 and same in 1934 and 1936 as in 1935.
[2] Individual practitioners only.

smaller than the quasi-permanent component in medicine, dentistry, and accountancy, and larger than the latter in law. The transitory component is more important in law than in any other profession and about equally important in medicine, dentistry, and accountancy. No 'nonsense' results were obtained, though it is arithmetically possible to get negative results for all except the permanent component. In one respect the results under the variability assumption seem more reasonable than the results under the mean assumption: the percentages attributed to the two parts of the quasi-permanent component are less widely divergent.