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Appendix A. Effects of Different Methods of Computing Finance Charges on Maturity Patterns of Finance Charges

FINANCING AGENCIES now use add-on, discount, and per cent per month methods to compute finance charges on individual instalment contracts. The first part of this appendix deals with the effects of each of these methods on the pattern of finance charges as maturities vary. In the second part, finance charges are assumed to be directly determined or computed in terms of effective rates, and the effects of different effective rate formulas on maturity patterns of finance charges are examined. This second situation exists in residential mortgage financing and is instructive for comparative purposes.

In both parts of the appendix we are interested in the maturity patterns of finance charges for varying periods of time rather than the absolute level of the finance charges at any one time. The latter is largely a function of the level of the rate used to compute the finance charge.

The analysis is carried on in both monthly and yearly time periods for several reasons. Some existing computational rates are monthly, some are yearly, most instalment contracts require monthly payments, and the year is a common period for financial analysis, financial reporting, and tax reporting.

Add-On, Discount, and Per Cent Per Month Rates

The maturity pattern of finance charges for each computational method is the same as the effective rate pattern for that method. Given this similarity, we can demonstrate the maturity pattern of finance charges for each method separately by determining, first, the equivalence of computational and effective rates under identical contract terms, and, second, the effect of different maturities on effective rates (the term structure). For any given computational rate, each of the several general computational methods results in a different effective rate level and an alternative term structure. The general order of these relationships differs for each computational method and is largely independent of computational rate levels.

TABLE A-1

Selected Computational Rates and Equivalent Effective Monthly Finance Rates for Varying Maturities (per cent)

Method of Computing Finance Charges	Computational Rate	Equivalent Effective Monthly Rates for Indicated Maturities in Months[a]					
		6	12	18	24	30	36
Annual add-on[b]	6	0.851	0.908	0.923	0.927	0.926	0.923
	8	1.133	1.205	1.221	1.223	1.219	1.212
	10	1.412	1.498	1.514	1.513	1.505	1.493
	12	1.691	1.788	1.804	1.793	1.785	1.767
	14	1.968	2.076	2.089	2.078	2.058	2.034
Annual discount[b]	6	.877	.965	1.012	1.049	1.082	1.115
	8	1.179	1.307	1.382	1.445	1.504	1.566
	10	1.486	1.659	1.770	1.867	1.966	2.071
	12	1.797	2.024	2.179	2.324	2.476	2.646
	14	2.113	2.400	2.609	2.813	3.040	3.302
Per cent per month[c]	1	1.000	1.000	1.000	1.000	1.000	1.000
	2	2.000	2.000	2.000	2.000	2.000	2.000
	3	3.000	3.000	3.000	3.000	3.000	3.000

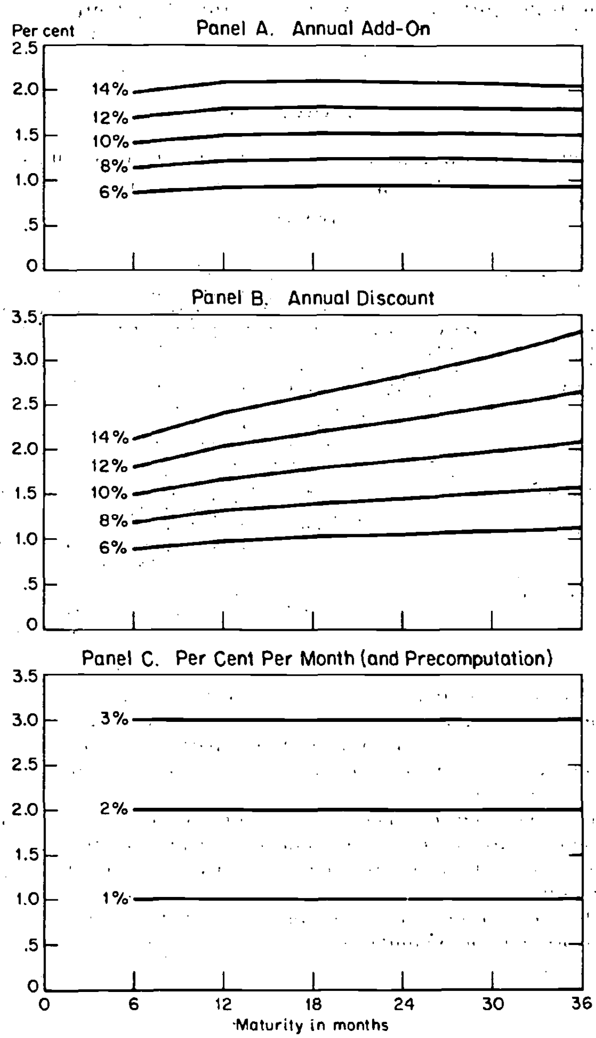
a The equivalent effective monthly rates are based on a 360-day year and a 30-day month. There are in practice two ways of measuring a year, 360 or 365 days, and four ways of measuring a fractional part of a year. For a description of these four ways, see M. R. Neifeld, *Neifeld's Guide to Instalment Computations,* Easton, 1951, pp. 48–51.

b The equivalent effective monthly rates for the annual add-on and annual discount methods are based on the annuity principle, i.e., that each instalment payment is applied first to interest and any remaining amount is applied to reducing the principal.

c The effective monthly rates here apply to both per cent per month on declining balance and precomputation.

CHART A-1

Effective Monthly Finance Rates Equivalent to Selected Annual Add-On, Annual Discount, and Per Cent Per Month Rates on Monthly Payment Contracts, by Maturity



Source: Table A-1.

RELATIONSHIP OF ANNUAL ADD-ON, DISCOUNT, PER CENT PER MONTH, AND PRECOMPUTATION TO EFFECTIVE MONTHLY AND ANNUAL FINANCE RATES

Table A-1 and Chart A-1 give for annual add-on, annual discount, per cent per month on declining balance, and precomputation the equivalence of selected computational rates and effective monthly rates. They show that, as maturities lengthen, effective monthly rates behave in the following way:

1. They stay the same under the per cent per month on declining balance and precomputation methods.

2. They follow an inverted saucer pattern with the annual add-on method. In general, the effective rate rises noticeably up to twelve-month maturities and then rises more gradually for a time thereafter as maturities lengthen. The maturity at which the effective monthly rate levels off becomes shorter as the add-on rate level increases. The decline in effective rates starts after twenty-four months with an 8 per cent annual add-on rate and after eighteen months with a 14 per cent annual add-on rate. Monthly add-on rates give an effective rate pattern similar to annual add-on rates, except on revolving credit where they give a horizontal effective rate pattern.

3. They rise quite markedly and continuously with discount. Using a 6 per cent (annual) discount rate and a twelve-month contract for comparison, the effective monthly rate rises 8 per cent on a twenty-four-month contract, 11 per cent on a thirty-month contract, and 16 per cent on a thirty-six-month contract. The corresponding percentages increase as maturities lengthen and become successively higher with higher discount rates.

A number of state laws providing ceiling rates on cash lending and retail instalment financing contain maturity limits. In such laws which designate discount or discount plus, the usual maturity limits range from twenty-five to sixty-one months. Thus a single discount computational rate ceiling results in progressively higher effective rate ceilings up to the maturity limit.

Assuming no change in maturity patterns during a year and no change in the amount of loanable funds, a financing agency's total annual finance charges are twelve times its monthly finance charges. While there is no conflict about the relation between annual and monthly finance charges, there are two conflicting views on how to establish the equivalent level of effective rates for a given computational rate. This conflict concerns level rather than pattern. Both views give effective annual rates which vary with maturity in patterns similar to those shown in Table A-1 for effective monthly rates. While we cannot end the conflict about level, we can indicate the assumption underlying each view.

According to one view, effective annual rates are twelve times the corresponding effective monthly rates. For convenience, we call these annually based effective annual rates, or y_a for short.[1] According to the second view, effective annual finance rates are obtained by compounding effective monthly rates according to the following formula:

$$y_m = (1 + k)^{12},$$

where y_m is the effective annual rate and k is the effective monthly rate, as shown in Table A-1.[2] For convenience, we call these monthly based effective annual rates, or y_m for short.

Table A-2 gives y_a and y_m for several selected computational rates. The y_a rates are based on the assumption that a year is the time limit for compounding (i.e., the compounding interval) and the y_m rates are based on a compounding interval of one month. They are two methods of measuring effective annual finance rates, much as yards and meters are two methods of measuring length.

To illustrate, the effective monthly rate for a twelve-month contract at a 6 per cent add-on rate is .9083 per cent (rounded off to .908 per cent in Table A-1). If a year is used as the compounding interval, the equivalent effective annual rate is $12 \times .9083$ per cent, or 10.90 per cent.

If a month is used as the compounding interval, the effective annual rate is equal to $(1 + .9083\%)^{12} - 1$, or 11.46 per cent, i.e., the effective monthly rate times a compound interest factor. The reasoning here is that, since a month is the compounding interval, pay-

[1] See Ralph A. Young and associates, *Personal Finance Companies and Their Credit Practices,* New York, NBER, 1940, p. 127, and M. R. Neifeld, "Analysis of Methods of Computing Interest on Amortized Loans," *Bankers Monthly,* March 1938, p. 135.

[2] See Milan V. Ayres, *Instalment Mathematics Handbook,* New York, 1946, pp. 233–235.

ment of 91 cents a month for twelve months represents a higher yearly rate than payment of twelve times 91 cents or $10.92 in one payment at the end of the year. This is based on the assumption that the debtor, in paying 91 cents a month, is deprived of earning the interest that he could have earned on that money if he had invested the 91 cents at interest each month and made one payment at the end of the year.

TABLE A-2

Selected Computational Rates and Equivalent Effective Annual Finance Rates for Varying Maturities (per cent)

Method of Computing Finance Charges	Computational Rate	Symbol[a]	Equivalent Effective Annual Rates for Indicated Maturities in Months[b]					
			6	12	18	24	30	36
Annual add-on	6	y_a	10.21	10.90	11.08	11.12	11.11	11.08
	6	y_m	10.72	11.46	11.67	11.72	11.71	11.66
Discount	6	y_a	10.52	11.58	12.14	12.59	12.98	13.38
	6	y_m	11.06	12.15	12.80	13.37	13.74	14.23
Per cent per month[c]	3	y_a	36.00	36.00	36.00	36.00	36.00	36.00
	3	y_m	42.58	42.58	42.58	42.58	42.58	42.58

[a] The symbol y_a means yearly based effective annual rates. These rates are twelve times corresponding monthly effective rates. The figures shown were obtained by using effective monthly rates to four decimal places and thus vary slightly from twelve times the two-demical rates shown in Table A-1.

The symbol y_m means monthly based effective annual rates. These rates were determined by the formula $y = (1 + k)^{12} - 1$, where y_m is the effective annual rate and k is the effective monthly rate. To illustrate, the monthly based effective annual rate on a twelve-month instalment contract at an annual add-on rate of 6 per cent is: $(1 + .9083\%)^{12} - 1 = 11.46\%$. The .9083% is shown rounded off to .91% in Table A-1.

[b] The effective annual rates are based on a 360-day year and a 30-day month.

[c] The effective annual rates here apply to both per cent per month on declining balance and precomputation.

The conflict is essentially over the compounding interval.[3] It does not directly concern financing agencies and sellers under present conditions because their monthly and yearly finance charges are not

[3] See Le Baron R. Foster, "Instalment Credit Costs and the Consumer," *Journal of Business,* January 1935, pp. 31–34. There are, of course, other possible compounding intervals besides a year and a month, e.g., a quarter, a half-year, a week, a day, and infinitely small intervals of time.

affected by which yardstick is used and they are not required to quote effective annual rates to consumers.

RELATIONSHIP OF ANNUAL ADD-ON PLUS AND DISCOUNT PLUS TO EFFECTIVE MONTHLY AND ANNUAL FINANCE RATES

The main question here is the effect of maturity on effective rate patterns when a service or investigation charge is added to an annual add-on or discount charge in determining the finance charge. The usual form of additional charge is a flat per cent of principal owed or borrowed, ranging from 1 to 8 per cent, with 1 to 4 per cent being most common. In some laws, the additional charge is subject to a maximum dollar limit, usually from $10 to $25. Table A-3 shows that lengthening maturities cause effective monthly rates to fall with annual add-on plus and to rise, fall, or behave in a U-shaped manner with annual discount plus. These patterns are not altered if the additional charge is subject to a flat dollar limit. The effect of such a limit is to lower effective monthly rates as loan size increases but to leave unchanged the maturity pattern of effective monthly rates for any given loan size. The loan-size effect is not a function of the method of computing finance charges but of the rate levels.

The patterns in Table A-3 result from the fact that the additional charge is a flat amount and does not, like the add-on or discount charge, increase proportionately as maturity lengthens in periodic intervals. This causes the "over-all annual charge" rate to fall as maturity lengthens. Thus, with an annual add-on rate of 6 per cent and an additional charge of 2 per cent, the over-all annual charge rate is 8 per cent on a twelve-month contract and 7 per cent on a twenty-four-month contract. The downward pull of the additional charge as maturity lengthens (1) meets the inverted saucer effect on annual add-on as shown in Table A-1 and causes the annual add-on-plus pattern to fall, and (2) meets the rising effect of annual discount as shown in Table A-1 and causes the varying discount-plus patterns shown in Table A-3; each pattern depends on the relative strength of the downward and upward pulls.

In a few discount-plus laws, the additional charge is determined on a sliding scale, e.g., 8 per cent on the first $600 of principal owed and 4 per cent on any excess. This arrangement does not alter the maturity patterns shown in Table A-3. It is simply another way of varying over-all annual charge rates by loan size rather than by maturity for any given loan size. In at least one law, the additional charge is computed as a function of the principal owed and time. The maturity patterns in this case are the same as those in Table A-1.

Effective Rates

The discussion in the first part of this appendix has been based on the assumption that finance charges are computed by using add-on, discount, and per cent per month rates and that effective rates are derived from finance charges so computed. We call this situation 1. This second part is based on the assumption that finance charges and finance charge ceilings are determined directly from effective rates. We call this situation 2.

TABLE A-3

Equivalent Effective Monthly Rates for Selected Annual Add-On Plus and Annual Discount-Plus Computational Rates for Varying Maturities (per cent)

Annual Add-On or Discount Rate[a]	Additional Charge in Per Cent of Principal Owed	Effective Monthly Rates for Indicated Maturities in Months		
		12	24	36
6	1	1.06	1.00	.97
6	2	1.21	1.08	1.04
6	4	1.50	1.22	1.12
6	1	1.13	1.14	1.20
6	2	1.31	1.24	1.26
6	4	1.49	1.45	1.42
10	1	1.84	1.98	2.17
10	2	2.02	2.09	2.25
10	4	2.40	2.32	2.45

[a] The first three are add-on rates; the others are discount rates.

As has already been shown, in situation 1 the choice of the formula used to determine effective rates does not have any influence on the gross revenue of financing agencies. In situation 2 the choice of the formula affects the gross revenue of financing agencies, for different formulas give different maturity patterns of finance charges.[4]

To illustrate situation 2, assume a $1,000 contract on which dollar finance charges are based on an effective annual rate of 12 per cent. The dollar finance charges for varying maturities under each of several formulas are as follows:

	Finance Charge in Dollars		
Formula	12-Month Contract	24-Month Contract	36-Month Contract
Monthly based annuity formula (y_m)[5]	62.67	122.76	185.03
Constant ratio formula (y_c)	65.00	125.00	185.00
Annually based annuity formula (y_a)[6]	66.19	129.76	195.72

The differences in dollar charges are as follows:

	Differences in Finance Charges in Dollars		
Item	12-Month Contract	24-Month Contract	36-Month Contract
Excess of y_c over y_m	2.33	2.24	−.03
Excess of y_a over y_c	1.19	4.76	10.72

As the above example shows, the y_m formula gives somewhat lower finance charges than the y_c formula on shorter maturities, with the differences narrowing progressively as maturities lengthen. Compared with the y_c formula, the y_a (or y_d) formula gives slightly higher finance charges on shorter maturities and progressively higher dollar charges as maturities lengthen. The y_m charges are lower than the y_c charges because, as Chart 1 shows, the compounding effect is stronger with the y_m formula and hence a lower dollar charge is needed to get a given effective rate (12 per cent in our sample). The difference in compounding effect also explains why the y_c dollar charge is higher than the y_a (or y_d) dollar charge.

If a state were to adopt any one formula in setting effective rate

[4] See Robert W. Johnson, *Methods of Stating Finance Charges,* Columbia University, New York, 1961, pp. 113–118.

[5] This formula is based on monthly compounding.

[6] This formula is based on yearly compounding. The y_d formula gives roughly the same figures as the y_a formula and is not shown separately.

ceilings, the formula adopted would, relative to other possible formulas, introduce a different maturity pattern of charges. This problem is not unique to situation 2, however, for, as has been shown earlier in this appendix, the same problem arises in situation 1, i.e., the add-on, discount, and per cent per moth methods of computing finance charges contain differences in maturity patterns of finance charges. The maturity pattern problem is thus independent of the existence of effective rates or, given their existence, of whether they are derivative or computational rates.