Eastern Economic Journal, Volume XVI, No. 3, July–September 1990

The Demand for Labor with Heterogeneous Hours

Bryan L. Boulier, Vincy Fon, and Robert Goldfarb*

The usual microeconomic model of the firm's demand for labor gives no insight into the choice between number of workers and hours per day, since all man-hours are assumed to be homogeneous. Models that allow for variation in worker productivity over the day permit analysis of this choice [Barzel, 1973; Ehrenberg, 1971(a), 1971(b); and Rosen, 1968, 1969, 1978]. Our previous work on heterogeneous hours [Fon, et al, 1985] develops this approach for one grade of labor in the presence of fixed costs of hiring and legal requirements for time-and-one-half overtime pay. This paper generalizes our earlier work by considering how hours for several grades of labor are determined at the firm level in a framework that takes explicit account of the imperfect substitutability between workers and hours within each labor category.

This extension generates a number of results which testify to the fruitfulness of the model. First, the model allows the derivation of plausible conditions under which a firm would choose to employ one class of labor full-time and another type of labor part-time or overtime. Second, the analytical framework provides a more fundamental theoretical explanation for ambiguous empirical results (Gramlich, 1976; McKee and West, 1984) about the effects of raising the minimum wage on relative levels of part-time versus full-time employment. Third, the general comparative static properties of the model produce a striking implication about the demand for labor. While the firm's demand for total labor hours is negatively related to the wage, the demand for the number of employees need not be. That is, the demand for the number of employees can be upward sloping. Besides its obvious theoretical interest, this finding has useful implications for interpreting empirical studies of the demand for labor.

THE MODEL

In this paper, we assume that the firm produces output using two grades of labor as inputs. For simplicity, we exclude capital and other inputs used in production. The production function is:

$$Q = f(L_1 g_1(H_1), \quad L_2 g_2(H_2)), \tag{1}$$

where L_i is the number of homogeneous workers of type i, H_i is the *total* number of hours per day per worker of type i, and $g_i(H_i)$ is a labor effectiveness function for labor of type i. The labor effectiveness function is assumed to be S-shaped and its derivative $g_i'(H_i)$, the "marginal effectiveness per extra hour" function, is an inverted U. The S-shape results from low marginal productivity at the start of the day due to set-up costs and low marginal productivity at the end of the day because of boredom and fatigue (Rosen, 1978, p. 150). The product of $g_i(H_i)$ and L_i is labor input of type i. The way equation (1) is written assumes that the production function is weakly separable in L_i and H_i and that the shape of $g_i(H_i)$ is independent of capital and of the number of employees of either labor type. We indicate below how some of our comparative static results are altered when the assumption of separability is relaxed.

Suppose the firm is faced with a legal requirement to pay time-and-one-half after 40 hours and must pay fixed costs of hiring workers, where F_i is the amortized daily fixed cost of hiring a worker of type i (Oi, 1962).[1] Given our assumption that the worker's effectiveness function $g_i(H_i)$ is identical for each day of

*George Washington University, Washington, D.C. 20052.
We would like to thank Michael Bradley, Daniel Hamermesh, Mark Montgomery, Susan Vroman, Anthony Yezer, and participants in seminars at George Washington University and Georgetown University for comments on an earlier version of this paper.

the week, the firm will want to hire each worker of type i for an equal number of hours per day. That is, ten hours per day for two days would produce more output than nine hours one day and eleven the next. Consequently, the assumption of a five day week turns a legal requirement to pay time-and-one-half after forty hours in a week into an implicit requirement to pay time-and-one-half after eight hours per day.

The profit-maximizing firm wants to pick hours and number of workers of each type e to maximize:

$$\Pi = pf(L_1g_1(H_1 + H_{1o}), L_2g_2(H_2 + H_{2o})) - \sum_i (w_iH_iL_i + 1.5w_iH_{io}L_i + F_iL_i) \qquad (2)$$

subject to

$$0 \le H_i \le 8$$
$$0 \le H_{io}$$
$$0 \le L_i \qquad (i = 1, 2)$$

where H_i is the number of straight-time hours per worker (less than or equal to 8), H_{io} represents overtime hours (hours in excess of 8) for labor of grade i, w_i is the hourly wage of grade i, and the terms containing 1.5 w_i represent the legal requirement to pay time-and-one-half for overtime. While each grade of labor can have a different wage rate, it is assumed that the supply of any one grade of labor to the firm is perfectly elastic *in both the number of individuals and hours.* Not only can the firm hire all the people it wants at the going wage, but it can hire each person to work whatever number of hours the firm prefers at that wage.[2]

To maximize the above equation subject to the relevant constraints, form the Lagrangian:

$$\mathcal{L} = pf(L_1g_1(H_1 + H_{1o}), L_2g_2(H_2 + H_{2o})) - \sum_i (w_iH_i + 1.5w_iH_{io} + F_i)L_i + \sum_i \lambda_i(8 - H_i). \qquad (3)$$

Equation (3) yields the following first order conditions:

$$\begin{aligned}
&\mathcal{L}_{L_i} = pf_ig_i(H_i + H_{io}) - w_iH_i - 1.5w_iH_{io} - F_i \le 0, && L_i[\mathcal{L}_{L_i}] = 0,\\
&\mathcal{L}_{H_i} = pf_ig_i'(H_i + H_{io})L_i - w_iL_i - \lambda_i \le 0, && H_i[\mathcal{L}_{H_i}] = 0,\\
&\mathcal{L}_{H_{io}} = pf_ig_i'(H_i + H_{io})L_i - 1.5w_iL_i \le 0, && H_{io}[\mathcal{L}_{H_{io}}] = 0,\\
&\mathcal{L}_{\lambda_i} = 8 - H_i \ge 0, \quad \lambda_i[8 - H_i] = 0, \quad \lambda_i > 0,
\end{aligned} \qquad (4)$$

for i = 1,2. Note that f_i is the partial derivative of f with respect to the ith argument and g_i' the derivative of g_i with respect to hours. We assume that the second order conditions hold. This requires, among other things, that g_i'' be negative at optimal H_i.

There are numerous possible solutions that arise from these conditions. For example, both grades of labor can work part-time; both grades of labor can work full-time (8 hours) or both can work overtime; one grade can work part-time and the other full-time; and so forth. When developing comparative static results, we intend to focus on three cases: (1) both grades work part-time; (2) one grade works eight hours, the other works overtime; and (3) one grade works eight hours and the other part-time.

First, however, it is useful to explore the content of the first order conditions by showing what they imply about equilibria in several different cases. As a first example, consider a case which has one or both grades of labor working part-time. For this case it can easily be shown from the first order conditions that equilibrium for the grade(s) of labor working part-time requires:

$$\frac{g_i'(H_i)}{w_i} = \frac{g_i(H_i)/H_i}{w_i + F_i/H_i}. \qquad (5)$$

The left-hand side (LHS) of equation (5) represents the marginal output per dollar cost of extending the worker's day by one hour. The RHS gives the extra hourly output per dollar of costs (wage plus an "hour's worth" of fixed cost) from hiring another worker. Equilibrium with positive L_i requires that these two quantities be equal, an intuitively plausible condition.[3]

Next, consider a case where one (or both) of the grades of labor works overtime. Equilibrium for the grades working overtime requires:

$$\frac{g_i'(8 + H_{io})}{1.5w_i} = \frac{g_i(8 + H_{io})/(8 + H_{io})}{(8w_i + 1.5w_iH_{io} + F_i)/(8 + H_{io})}, \qquad (6)$$

which has an interpretation analogous to equation (5) above.

Finally, consider the case where one or both of the grades of labor works exactly eight hours. Then equilibrium requires that the following inequalities be satisfied:

$$\frac{g_i'(8)}{1.5w_i} \le \frac{g_i(8)/8}{w_i + F_i/8} \le \frac{g_i'(8)}{w_i}, \qquad (7)$$

The LHS of the inequality represents the marginal output per dollar cost of lengthening the worker's day beyond eight hours; the 1.5 in the denominator indicates that hours in excess of eight must be paid at time-and-one-half. The LHS must be less than or equal to the middle term, which gives the extra hourly output per dollar of cost (wage plus an hour's worth of fixed costs) from hiring another worker. The middle term must in turn be less than or equal to the RHS, which represents the output per dollar of cost for the eighth hour. The LH inequality ensures that it does not pay to raise hours above eight. The RH inequality ensures that it does not pay to cut back hours below eight.

THE FIRM'S DEMAND FOR PART-TIME VS. FULL-TIME VS. OVERTIME HOURS OF WORK

The equilibrium conditions embodied in equations (5), (6), and (7) can be used to investigate the conditions under which a firm would choose to employ one class of labor part-time while simultaneously employing another class full-time and another overtime. Rearrangement of (5) yields the following result, where the subscript *a* is used to indicate labor of type a:

$$\frac{F_a}{w_a} = \frac{g_a(H_a)}{g_a'(H_a)} - H_a. \qquad (8)$$

This condition must hold if labor of type a is to be employed part-time. Rearrangement of (7) for full-time workers produces

$$\frac{g_b(8)}{g_b'(8)} - 8 \le \frac{F_b}{w_b} \le 1.5\left[\frac{g_b(8)}{g_b'(8)} - \frac{8}{1.5}\right], \qquad (9)$$

where the *b* indicates labor of type b. Finally, manipulation of equation (6) for workers hired for more than eight hours produces:

$$\frac{F_c}{w_c} = 1.5\left[\frac{g_c(8 + H_{co})}{g_c'(8 + H_{co})} - \frac{8}{1.5} - H_{co}\right], \qquad (10)$$

where the subscript *c* is used to indicate labor of type c.

Using (8), (9), and (10) and *assuming identical g functions for each grade of labor,*[4] then

$$\frac{F_a}{w_a} < \frac{F_b}{w_b} < \frac{F_c}{w_c}. \qquad (11)$$

That is, firm equilibrium requires that the ratio of fixed cost per day to the wage must be *lower* for part-time workers than for full-time workers, and lower for full-time than for overtime workers. Thus, the lower the ratio of fixed cost per day to the wage, the fewer the hours per worker the firm is likely to desire. In addition, for a firm to hire one class of labor part-time and another full-time or overtime, the part-time labor must have a lower ratio of fixed cost per day to its wage than does the full-time or overtime labor.

The result in (11) is derived as follows. Using equation (8) and the LH inequality of (9), $F_a/w_a < F_b/w_b$ if

$$\frac{g(H_a)}{g'(H_a)} - H_a < \frac{g(8)}{g'(8)} - 8 \tag{12}$$

(where $H_a < 8$). But this inequality must hold because the function $[g(H)/g'(H) - H]$ can be shown to have a positive first derivative with respect to H. A similar rearrangement of (9) and (10) can be used to show that $F_b/w_b < F_c/w_c$.

COMPARATIVE STATIC ANALYSIS

Further interesting results can be obtained from comparative static analysis. In particular this analysis leads to surprising findings about the shape of the "head count" demand for labor, and to results about increasing the minimum wage on hours of work relevant to the conflicting findings of Gramlich and McKee-West. We present comparative statics for three cases:

Case 1: $L_1,L_2 > 0$ and $0 < H_1,H_2 < 8$.
Case 2: $L_1,L_2 > 0$, $H_1 = 8$, $H_{1o} > 0$, $H_2 = 8$, $H_{2o} = 0$, where H_{io} represents overtime hours for labor of type i. This case assumes overtime hours for type 1 labor and full-time employment at eight hours for type 2 labor.
Case 3: $L_1,L_2 > 0$, $H_1 < 8$, $H_2 = 8$, $H_{2o} = 0$. This case assumes part-time employment for type 1 labor and full-time employment at eight hours for type 2 labor.

The comparative statics derivations of results in this section are available from the authors on request.

For case 1, where both labor types are employed part-time, comparative static results are shown in Table 1. As one might expect, a rise in w_1 lowers hours per worker (H_1) and labor input (H_1L_1). What is at first surprising is the ambiguous effect of a rise in w_1 on the number of workers hired, L_1. This result also holds when there is only one grade of labor.[5] The explanation is straightforward. An increase in w_1 has both substitution and output effects.[6] Holding output constant, an increase in w_1 leads to substitution of workers for hours. However, because the marginal cost of producing output increases with a rise in w_1, the profit maximizing level of output declines, resulting in a decrease in L_1. Whether L_1 rises or falls depends upon whether the substitution effect is larger or smaller than the output effect.

TABLE 1
Comparative Static Results When Both Types of Employees Work Part-Time ($H_1 < 8$, $H_2 < 8$)*

	dL_1	dH_1	$d(L_1H_1)$	dL_2, $d(L_2H_2)$	dH_2
dw_1	?	−	−	complement − substitute +	0
dF_1	−	+	?	complement − substitute +	0
dp	complement + substitute ?	0	complement + substitute ?	complement + substitute ?	0

*Labor inputs are defined as complements when $f_{12} > 0$ and substitutes when $f_{12} < 0$.

It can be shown that the elasticity of demand for labor with respect to the wage ($\eta_{L_iw_i}$) equals

$$\eta_{L_iw_i} = \frac{w_iH_i}{F_i}(\eta_{L_iF_i} - \eta_{H_iw_i}), \tag{14}$$

where the elasticity of demand for labor with respect to fixed costs ($\eta_{L_iF_i}$) and the elasticity of demand for hours per worker with respect to the wage ($\eta_{H_iw_i}$) are both less than zero.[7] If the demand for hours is sufficiently elastic that $|\eta_{H_iw_i}| > |\eta_{L_iF_i}|$, then L increases when the wage rises. In addition, the absolute value of $\eta_{L_iw_i}$ is positively related to the ratio of variable costs per worker (w_iH_i) to fixed costs per worker (F_i). Finally, the wage elasticity of the demand for total hours ($\eta(_{L_iH_i})_{w_i}$) equals the sum of the wage elasticities of demand for workers and for hours:

$$\eta(_{L_iH_i})_{w_i} = \eta_{L_iw_i} + \eta_{H_iw_i}. \tag{15}$$

While $\eta_{L_iw_i}$ can be positive, $\eta(_{L_iH_i})_{w_i}$ must be negative. Consequently, even when $\eta_{L_iw_i} < 0$, the demand for total hours is more elastic with respect to a change in the wage than is the demand for workers, since $\eta_{H_iw_i} < 0$.

As for cross wage effects, Table 1 indicates that a rise in w_1 has no effect on H_2. Note, however, that this result is generated by the assumption of separability. Because H_2 is unchanged, the effect of w_1 on L_2H_2 must be of the same direction as the effect on L_2 alone. The table indicates that the effect of w_1 depends on whether effective hours ($L_ig_i(H_i)$) of type 1 and type 2 labor are complements ($f_{12} > 0$) or substitutes ($f_{12} < 0$), an intuitively appealing result.

The effects of a rise in F_1 on L_1 and H_1 are also plausible; as F_1 rises, hours rise to economize on fixed costs per employee, and L_1 falls because employees are more expensive. These conflicting effects on L_1 versus H_1 produce the ambiguity of the effect of an increase of F_1 on L_1H_1. The cross effects of F_1 on H_2, L_2, and L_2H_2 are precisely analogous to the cross-wage effects.

The price effects are also of some interest. Price increases do not affect hours per worker, because they do not affect the relative cost of hiring one more worker versus raising hours. (This can also be seen by examining equation (5) and the associated discussion). That demand for L_1 and L_2 will rise with a price increase if the two grades of labor are complements agrees with intuition; the ambiguity if the two are substitutes represents the fact that an output increase may only cause one of the two inputs to rise.

The reader may wonder how dependent these effects—especially the ambiguous sign of the "head count" demand for labor—are on the assumption of weak separability between number of workers and hours per worker. If a more general production function without separability is assumed, the comparative static results become less determinate. In fact, the only remaining determinate comparative static effects are the negative effect of w_i on L_iH_i and the negative effect of F_i on L_i. Thus, the ambiguous sign on the "head count" demand for labor is not a result of the separability assumption.

In Case 2, type 1 labor works overtime ($H_{1o} > 0$) and type 2 labor works exactly 8 hours ($H_2 = 8$, $H_{2o} = 0$). Comparative static results are given in Table 2. Note that the format of Table 2 differs from that of Table 1. In particular, the dH_2 column and dp row of Table 1 are missing and a new row labelled "dw_2 or dF_2" is added. The dH_2 column is omitted because all dH_2 effects are zero by assumption.[8] The dp row is omitted because it is similar to that of Table 1. The new row is added because, unlike the situation in Table 1, there is no symmetry of effects between the two grades of labor.

Consider first the table entries obviously consistent with Table 1. The entries in the first two rows are identical to those in Table 1 and have the same explanations. Note that dL_1/dw_1 is still ambiguous but that total overtime hours $[L_1(8 + H_{1o})]$ declines in response to an increase in w_1.

The bottom entry in column one, representing cross-effects of w_2 or F_2 on L_1, are the "mirror images" of the cross-effects given by the upper two entries in the fourth column, so that it is not surprising that the same signs are displayed. The "zero" in row three, column two results from our separability assumption, as

TABLE 2
Comparative Statics When Type 1 Labor Works Overtime and Type 2 Labor Works Full-Time
($H_1 = 8, H_{1o} > 0, H_2 = 8$)*

	dL_1	dH_{1o}	$d[L_1(8 + H_{1o})]$	dL_2, $d(L_2H_2)$
dw_1	?	–	–	complement: – ; substitute: +
dF_1	–	+	?	complement: – ; substitute: +
dw_2 or dF_2	complement: – ; substitute: +	0	complement: – ; substitute: +	–

*Labor inputs are defined as complements when $f_{12} > 0$ and substitutes when $f_{12} < 0$.

explained in our comments on Table 1. The "complement/substitute" result in row three, column three follows directly from the first two entries in row three. After all, if the hours effect is zero, then row three, column three must be identical to row three, column one, since both cells embody only the identical effect on L_1.

The effect in row three, column four is quite striking in that the head count demand for labor of grade 2 is downward sloping, in contrast to our *ambiguous* result for part-time labor (Table 1) and overtime labor (Table 2, upper lefthand cell). The reason is straightforward. For labor "bunched" at eight hours, we *assume* (as explained in footnote 8) that exogenous comparative static changes do not change hours. Thus, the ambiguous effect of a wage increase on L_1 which can arise when hours fall is ruled out when hours are constant.

In Case 3, type 1 labor works part-time ($0 < H_1 < 8, H_{1o} = 0$) and type 2 labor works exactly eight hours. We have not provided a separate table for these results, because they are identical to Table 2 with column two relabelled dH_1 and column 3 relabelled $d(L_1H_1)$.

TWO APPLICATIONS OF THE COMPARATIVE STATICS

We believe that the ambiguous sign of the head count demand for labor in several of the cases is an interesting finding. Such an ambiguity *does not,* to the best of our knowledge, arise in other standard labor demand models. Thus, adding an intuitively plausible assumption about how labor effectiveness varies with hours produces a notable departure from standard results.

While this result is of interest for itself, it is also of interest in the context of the analysis of minimum wage effects. The result suggests that head count measures of the employment effects of minimum wages may considerably understate the true employment consequences in terms of manhours. The overwhelming majority of studies of the employment effects of minimum wages measure head counts, not manhours. Thus, the general empirical result of "small" employment effects may understate the true magnitude of actual decreases in hours worked. Two studies that do attempt to take account of the effects of the minimum wage on hours variation are consistent with this hypothesis. Zucker (1973, p. 270) reports that the elasticity of total hours of work with respect to the minimum wage, −.9, is larger (in absolute value) than the elasticity of the number of workers, −.7. Brown, Gilroy and Kohen (1983) try to deal with the hours issue by converting part-time teenage workers into full-time-equivalents. They do this by assuming the average part-timer works half the hours of the average full-time teenage employee. They then estimate the effect of the minimum wage on full-time-equivalent (FTE) employment. They argue that if raising the minimum wage results in a rise in the fraction of part-time workers, then the minimum wage in FTE equations should be larger than in simple head count equations. This result is precisely what they find, leading them to conclude that the minimum wage has a larger effect on hours-corrected employment than on simple counts of the number of employed.

A second application of the comparative statics concerns the Gramlich-McKee-West results about how a rise in the minimum wage would affect the relative quantities of full-time versus part-time work. Gramlich (1976) produced data indicating that a rise in the U.S. minimum wage increased the proportion of part-time work. McKee and West (1984) claim that contrasting outcomes are possible and use Canadian data to show a rise in the proportion of full-time employment. Our framework analyzes the question at the firm level. Clearly, several conflicting effects are possible. Consider Case 3 first, and suppose that the part-time labor L_1 is working at the minimum wage ($w_1 = w_m$) while full-time labor L_2 is working at $w_2 > w_m$. H_1 and L_1H_1 clearly fall when the minimum rises, but L_1 could even rise, and L_2 could either rise or fall. Clearly, this case could result in L_1/L_2 rising or falling; other things being equal, if effective hours ($L_ig(H_i)$) of type 1 and type 2 labor are substitutes, L_1/L_2 is more likely to fall. What about L_1H_1/L_2H_2? If effective hours of the two types of labor are substitutes, a rise in the minimum wage *lowers* this ratio; if they are complements, the result is uncertain. Thus, as this particular example shows, the effect of a change in the minimum wage is not always determinate even at the firm level. McKee and West rely on differential coverage of the minimum wage across firms to explain ambiguous effects of minimum wage increases on the ratio of part-time to full-time employment. Our analysis reveals that this indeterminacy has an even more fundamental cause and would exist even without differential coverage.

QUALIFICATIONS AND POTENTIAL EXTENSIONS

While we believe the analysis given above is useful and illuminating, there are a number of relevant aspects of the firm's hours choice that it does not incorporate. One important example is that it may be necessary for the firm to co-ordinate the hours of both types of labor because they must work in physical proximity. This kind of constraint is not incorporated in our analysis. Another example concerns a firm's decision to hire some workers in a single grade of labor part-time and others *in the same grade* full-time. Some of this behavior is undoubtedly a reaction to daily peak load demand (e.g., extra waiters at lunch). Peak-load demand is not included in the current model. Such practices may also arise when there is an upward sloping supply curve of hours among workers and there are differential fixed costs of hiring (Montgomery, 1988b). An extension of the current model to incorporate upward sloping supply curves of hours worked would be especially desirable.

There are other possible applications or extensions of the model. Equation (11) indicates that hours per worker depend on the ratio of F to w. This finding implies that hiring subsidies have different effects on the daily hours worked by subsidized workers than do wage subsidies. Moreover, the analysis of employment subsidies is complicated by the fact that the response of total hours (L_iH_i) to a decrease in fixed costs is of ambiguous sign because of the substitution of employees for hours per worker. This result suggests that hiring subsidies based on number of employees, job banks, and other programs that reduce fixed costs of employment have more complex effects than has been supposed. A quite different application involves analyzing the biases in head count based measures of elasticities of substitution. The labor input in our production function model $g(H_i)L_i$ is considerably more complex than simple head counts or even simple hours counts. Given that hours per worker will vary with the wage in our model, what biases would head count measures of substitution elasticities reflect?

NOTES

1. For example, if it costs Z dollars to hire a worker and his expected length of stay is T days, F is obtained as the solution to the equation $\Sigma_{t=1}^{T} [F/(1 + r)^t] = Z$, where r is the daily interest rate relevant to the firm in question. Depending upon institutional arrangements, contributions to social welfare schemes and unemployment and disability insurance may vary with the number of workers employed and not hours of work. The daily amortized value of these non-wage costs are also included in F_i. For a detailed treatment of non-wage labor costs, see Hart (1984).
2. Barzel (1973) considers the case where the market supply curve of labor is not perfectly elastic. Montgomery (1988a, 1988b) considers the firm's demand for full-time versus part-time employees when it faces an upward sloping wage-hours function. In Montgomery's model, hours of part-time and full-time labor are perfect substitutes.
3. If $F_i = 0$, hours per worker do not vary with the wage. When $F_i = 0$ in equation (5), the firm maximizes average effectiveness per hour of each worker, setting the marginal product of another hour equal to the average product. As w_i changes, labor services are adjusted by altering L_i; the firm continues using each worker for the number of hours that maximizes his average "input effectiveness" per hour.
4. Obviously, *different* g(H) functions can be assumed which will "explain" virtually any differences between full-time and part-time work across groups. We want to see how far we can get in explaining hours differences without assuming "convenient" variations in g(H) functions.
5. Our earlier paper (1985) incorrectly implied that $\partial L_i/\partial w_i$ was unambiguously negative.
6. In a model similar to the one used here, Hart (1984, pp. 76–78) derives the substitution effect of a change in the hourly wage on the number of workers employed, but does not consider output effects of wage changes.
7. Derivations of the elasticity formulae and the output and substitution effects of wage changes are in an appendix available from the authors on request.
8. The comparative static analysis assumes that $H_2 = 8$ and that H_2 does not vary with changes in exogenous parameters. It is, of course, possible that changes in exogenous parameters could be of sufficient magnitude that this corner solution would not be optimal.

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