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PROPERTIES OF SOME SHORT-RUN BUSINESS FORECASTS

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Empirical analyses of forecasts of demands, production, or work force requirements have been carried out by a number of authors. These studies have been concerned first with the accuracy of the forecasts, compared with that of "naive" or regression models. Whether forecasts incorporate information not otherwise available is relevant, for example, in deciding on the data to use for the analysis of current business conditions. The second major concern has been the power of the forecasts to explain changes in other economic variables, such as inventory investment or employment levels. This is the sole purpose of the implicit expectations hypothesis of Mills [8], and related models of Lovell [6], Pashigian [11], and Orr [10]. Neither concern is, of course, independent of expectations formation. All of these studies have examined expectations formation solely in terms of the past history of the series being predicted.

The main concern of this study is not the accuracy of the forecasts, but determining how alternative models of expectations formation compare with forecasts prepared by certain business firms. Hirsch and Lovell [5] had this as their main objective using quarterly data from the manufacturers' inventory and sales expectations survey of the Office of Business Economics.

In this study it is not necessary to examine expectations formation solely in terms of the past history of the series being predicted. Detailed information about several individual firms is available. The latest information known to the forecaster about realized production, new orders, deliveries, backlogs of unfilled orders, and inventories are available, as are indicators of industry, regional, and general economic activity.

The results of the analysis do not support the hypotheses of the naive, exponential, extrapolative, regressive, or rational models. Only the expectations revision model used by Meiselman [7] is consistently supported by the statistical results. There is some support for the errors-in-variables model, but several inconsistencies are noted. Data concerning new orders, deliveries, backlogs of unfilled orders, or inventories is seldom important in the formation of expectations. Exogenous information does not appear to be very important either, although some significant, plausible coefficients are found. These conclusions should be regarded as highly tentative and only suggestive, however, because of the small number of firms studied.

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I. Expectations Hypotheses

The following notation will be used in the paper. A representative element of a time series is denoted by A. The time subscript is ordinarily suppressed and lagged values have a subscript indicating the extent of the lag. The last forecast made of A is indicated by F, with earlier forecasts for the same time period indicated by F' and F''.[1] Forecasts of A_{-1} have the same subscript. The symbol A' is sometimes used to refer to preliminary information about A. (With the data used in this paper, it represents production during the first ten days of month t.) All forecasts referred to here are of levels, and not to either absolute or relative changes.

The simplest forecast is the *naive* forecast, which is taken to be

$$F = A_{-n} \tag{1}$$

where n is the lag in information at the time the forecast is prepared. This has been used in a number of models in mathematical economics, especially cobweb models and models of inventory fluctuations, and as a minimum-performance forecast to compare with those of business firms or econometric models. Of course, analysis of predicted changes, rather than levels, implicitly includes this criterion.

The *extrapolative* and *regressive* models satisfy the equation:

$$F = A_{-n} + a(A_{-n} - A_{-L}) \tag{2}$$

where L is a lag greater than n. The forecast is said to extrapolate changes if a is positive. It is regressive if a is negative. The regress condition has been found by Ferber [4] in the railroad shippers' forecast data.[2] When the forecast is regressive, it is a weighted average of A_{-n} and A_{-L}. This fact is of some importance in the errors-in-variables model described below.

Exponential forecast models are of the following form:

$$F = F_{-n} + b(A_{-n} - F_{-n}) \tag{3a}$$

$$= b\sum_{k=1}^{\infty} (1 - b)^k A_{-kn} \tag{3b}$$

The forecasts F and F_{-n} differ by an amount proportional to the last observed error. Many variants exist for predicting in the presence of trend and seasonal factors (see Brown [1] and Winters [12]). The solution to the difference equation (3a) is the moving-average expression of equation (3b). It has been used in a great number of econometric studies, although it is usually interpreted as a delayed action in response and not an expectations phenomenon.

A further mechanism, called expectations revision, satisfies equations of the following form:

$$F = F' + c(A_{-n} - F_{-n}) \tag{4}$$

1. The symbol E is often used in the literature for this purpose, but it is reserved here for the mathematical expectation (operation). The words "expectation" and "forecast," however, are used here interchangeably.

2. This formulation does some violence to Ferber's model because factors corresponding to seasonal adjustment have been omitted. The lag in his model corresponds to an annual lag in quarterly data.

With this model the forecast for a time period differs from an earlier one prepared for the same period by an amount proportional to the latest observed error. This has been used by Meiselman [7].

The forecast models, therefore, have the predictions about the regression coefficient of explanatory variables.

Rational forecasts are characterized by the property

$$F = M \tag{5a}$$

where $M = EA$. The actual equals the forecast plus an error term

$$A = F + e \tag{5b}$$

where e is a random variable with $Ee = 0$ and $EeF = 0$. Muth [9] argued that allowing the specific hypothesis about expectations to depend upon factors governing the realizations has some merit. The results of Hirsch and Lovell, as well as information reported below, show that the rational hypothesis is *not* in agreement with the facts about forecasts of demand and production.

Other forecasts may be rational in the sense used here. In particular, we show that rational forecasts, for any moving-average process, necessarily satisfy the condition of the expectations revision model. Let the series to be predicted be the moving-average process

$$A = \sum_{k=0}^{\infty} w_k e_{-k} + \mu , \tag{6a}$$

where e_{-k} are mutually independently distributed with zero means and unit variances. Rational forecasts made one and two periods ahead are given, respectively, by

$$F = \sum_{k=1}^{\infty} w_k e_{-k} + \mu \tag{6b}$$

$$F' = \sum_{k=2}^{\infty} w_k e_{-k} + \mu , \tag{6c}$$

The difference between F and F', from equations (6b) and (6c), is:

$$F - F' = w_1 e_{-1} \tag{7}$$

The forecast error in the previous period is

$$A_{-1} - F_{-1} = w_0 e_{-1} \tag{8}$$

Hence, equation (4) is satisfied for $n = 1$ if $c = w_1/w_0$. It must be noted that the converse is not true. Forecasts need not be rational in order to satisfy the expectations revision model, however.

A modification of the rational model is in better, but imperfect, agreement with the facts, and includes both the rational model and Mills's [8] implicit model as special cases. The hypothesis, which is identified here as the *errors-in-variables* model, has the form:

$$F = M + f \tag{9a}$$

while the actual has the form:

$$A = M + e \tag{9b}$$

where $M = EA$ and $Ef = EMf = Ee = EMe = 0$.

II. Actual and Predicted Production

The data used in this study have been collected for a number of years by the Bureau of Business Research, the University of Pittsburgh, for a survey of current business prospects in the Pittsburgh, Pennsylvania region.[3] Information for five firms was suitably complete for analysis. With the exception of one electric utility firm, the plants are moderately large facilities in basic iron and steel production or metal fabricating industries. The data used have been reported monthly for a period, with interruptions, of thirteen years from the beginning of 1957.

The data, reported monthly by the firms, give the following information: (1) Level of operations (production, in physical units) during the previous two months and the first 10 days of the current month. (2) Predictions of the level of operations, in the same units, for the next three. (The survey is made in the middle of each month). (3) Incoming new orders (last two months). (4) Deliveries (sales). (5) Backlog of unfilled orders. (6) Total inventories (materials, work in progress, finished goods). There is quite a bit of information about operations available each month, with minimal problems as to month-to-month comparability. (Firms do not often fail to report, although steel and other strikes lead some firms to interrupt their predictions of future production.)

These data have the advantages of providing a firm's history, monthly for several years, of being quite rich and of providing three successive predictions.

For purposes of economic analysis, however, these data present a number of problems.

1. Predictions are not forecasts of sales or of incoming orders, but of production. Since the production rate is a variable very largely within the control of the firm, the whole problem of forecasting gets compounded with budgets, quotas, and other goal-setting activities. As a result, we may have self-fulfilling (or self-defeating) expectations.

2. Units of measurement are not comparable. It is not, therefore, possible to reconstruct a set of production, deliveries, and inventory data in any consistent system of measurement. This problem is not a particularly devastating one, but it does mean that the values of most regression coefficients have no relevance to the questions involving expectations or short-term production adjustments. The tables of regression results present t-ratios (plus sign) only.

3. It is not always clear that the stated "predictions" are in fact taken seriously. Although there is some attempt to make sure that the questionnaires are prepared by responsible people in the organizations, it is not obvious that the reported figures are widely shared by the relevant people in the organizations.

4. Since the information is collected in the middle of the month, data for the current month (t) essentially is skipped. Actual production for months up to and including t are known, but the first forecast is for month $t + 2$. The time lags in the collection and summarization of the information are such that forecasts for month t are not very meaningful.

3. I am indebted to R.J.A. Pratt, Bureau of Business Research, University of Pittsburgh for making the forecast data available to me.

Summary information of the actual and predicted production rates for one firm is given in Table I.[4]

Table I. Summary Statistics: Forecasted and Actual Production for One Firm

	Means	Standard Deviation	Correlation	Error Standard Deviation
Actual	1.000	.254		
10 Days	1.014	.331		
Third	1.024	.278	.732	.196
Second	1.033	.282	.678	.217
First	1.039	.285	.640	.230

Additional variables were used in the regression equations as surrogates for activity in the relevant industry, the Pittsburgh region, and general business activity. (The variables are listed in Table II and summarized in Table III.) For each industry the FRB index of industry production, not seasonally adjusted (NSA), and the relative change in wholesale prices (except for the electric utility) were used. The change in wholesale prices was used for all firms except that firm as an indicator of information relevant to inventory speculation. The regional index of activity was that prepared by the Bureau of Business Research, the University of Pittsburgh on the basis of information supplied by the firms utilized in this study and about a dozen more. The indicators of general business activity include an index of real physical output (the FRB Index of Total Production, NSA), a monetary variable (Prime Commercial Paper Rate, New York, four-six months), indicators of defense spending (Military Prime Contract Awards and DOD Obligations Incurred),[5] and employment (BLS data on Total Percent Unemployment, Average Working Hours (SA)), and strikes (BLS series of Estimated Idle Time from Work Stoppages and a series, constructed from The *New York Times* Index of the fraction of days during the month that steel contract negotiations are in progress). It was assumed in all cases that all the information above is available to the forecaster with only a one-half month time lag. This is far from the case. These data are, however, assumed to be surrogates for other information about the economy which is available to the forecaster.

In addition to the variables listed above, seasonal dummy and linear trend variables were used. The dummy variables for the monthly seasonals represent the difference from the month in question from that of January. The intercept term is the intercept for January.

III. Standard Deviations of Expectations

The summary information for one firm contained in Table I shows that the standard deviation of the forecast is inconsistent with the rational expectations hypothesis. The last

4. Such data for the other firms are available from the author. The production data were adjusted for the number of working days in each month, when the original data were given for calendar months. New orders and deliveries were similarly adjusted. The new orders, deliveries, backlogs of unfilled orders, and inventories were deflated by BLS wholesale price index for that industry. Preliminary trials suggested, however, that these adjustments had little effect on the variables.

5. Both series deflated by the implicit GNP deflator of Federal Government Purchases of Goods and Services, are interpolated to convert the quarterly series to a monthly deflator. These series have sometimes been found to be important in explaining inventory investment. See Darling and Lovell [2].

forecast for four firms have standard deviations greater than the standard deviation of the actual. Since the rational forecast specifies $A = F + e$, where $EFe = 0$, the variance of A must clearly exceed that of F. The rational hypothesis similarly requires the variance of the third forecast to exceed that of the second, and the variance of the second to exceed that of the first. Three firms, including the sample one, report forecasts with the opposite property: the standard deviation of the forecast increases with the length of the forecast span. The errors-in-variables model admits any variance level as a possibility and is consistent with the data of Table I.

The standard deviation of the forecast errors, however, always increases as the forecast span increases. The mean values of the forecasts do not markedly differ from the actual for any firm, except possibly one. There is otherwise considerable diversity among the firms in the variability of the actual production and the standard deviation of the forecast errors.

IV. Tests Concerning the Formation of Expectations

This section is concerned with comparing alternative expectations hypotheses by using a forecast as a dependent variable in a regression model. Since the rational and errors-in-variables hypotheses cannot be tested with this method, analyses of these are described in the subsequent sections. In contrast to usual econometric practice, the statistical significance of variables is tested in a model general enough to include most of the expectations hypotheses. This approach, adopted by Hirsch and Lovell [5], is to compare the properties of regressions based on each hypothesis alone. The results are then judged on the basis of closeness of fit (R^2), serial correlation of the residuals (the Durbin-Watson statistic, d), and the plausibility of the signs and magnitudes of the coefficients. Statistical dependence among such comparisons is all but ignored. The alternative adopted here is often considered undesirable because of the collinearity of the independent variables. It is felt that the large number of observations is sufficient to control the standard errors of individual variables even in the presence of the high collinearity among the variables. Indeed, several coefficients of nearly collinear variables prove to be significant and the pattern is reasonably consistent, though hardly surprising, among the firms.

The t-ratios of the coefficients for the one representative firm are given in Table II. The variables are sometimes arranged according to three levels of information.

1. The minimum level of information consists of the forecasts, plus the intercept, seasonal dummies, and linear trend terms. It therefore includes no information beyond the forecast itself.

2. The second level includes recent information about the firm's operations, the previous forecasts, previous realizations, new orders, deliveries, unfilled orders, and inventories.

3. The most information (third level) includes the additional information about economic conditions for the industry, the region, and the general economy. Tables giving statistical analyses for the various firms arrange the variables in this order. Moreover, significance tests nest alternative hypotheses according to this order.

Column 1 contains the results for the first forecast for one firm, which is made approximately two and one-half months before the beginning of the month whose production is being predicted. Column 2 applies to the second forecast, prepared one month ahead, and column 3 to the third forecast, prepared one-half month ahead. The R^2 are quite high for the first forecast for about half the firms. The Durbin-Watson statistic appears well-behaved, except for the positive serial correlation of the residuals indicated for one of the

Table II. T-Ratios of Regressions for One Firm

	(1) First Forecast (F")	(2) Second Forecast (F')	(3) Third Forecast (F)	(4) First Forecast Error	(5) Second Forecast Error	(6) Third Forecast Error
	.086	.058	.053	.169	.163	.149
R^2	.8964	.9546	.9611	.5354	.4851	.4856
d^f	102	102	102	98	99	101
d	1.89	1.94	2.12	1.52	1.39	1.54
Information: Minimal						
Intercept	−0.27	−1.22	−0.71	−3.51	−2.27	−1.18
Seasonal Variable-Feb.	−0.43	−0.62	−0.86	−0.72	0.70	0.91
-Mar.	−1.56	1.89	0.39	−2.40	−0.43	−0.94
-Apr.	−0.83	−0.85	0.24	−0.71	−2.09	−0.91
-May	−0.17	0.52	−1.09	−0.01	−0.84	−2.24
-June	−1.75	−0.17	−1.93	0.47	0.20	0.77
-July	2.10	0.68	1.43	0.40	0.86	0.50
-Aug.	0.06	−1.13	−0.99	0.40	−0.21	0.21
-Sep.	−1.72	−1.22	−0.96	1.36	0.49	−0.32
-Oct.	−1.34	0.22	−0.61	1.01	0.57	0.03
-Nov.	−1.23	1.25	0.85	0.70	0.86	−0.27
-Dec.	−0.48	−1.07	0.00	0.80	0.81	−0.56
Trend (t)	−2.74	−1.11	−1.89	−0.37	0.26	0.96
Information: Second Level						
Second Forecast (for t)			8.45			−2.39
First Forecast (for t)		11.05	−0.31		−2.22	0.80
Third Forecast (for t-2)	4.86	−0.56	−0.88	−4.31	−0.85	−0.98
First Ten Days Actual (t-1)	1.32	0.96	−0.07	0.12	0.42	−0.48
Actual (t-2)	2.50	3.65		2.52	2.08	2.00
Actual (t-3)	2.50	3.65	3.47	−0.41	0.10	0.28
New Orders (t-2)	3.38	−1.86	−0.51	0.29	−0.01	−0.38
Deliveries (t-2)	−1.12	−0.70	−0.46	0.39	0.17	0.39
Backlog of Unfilled Orders (end of t-2)						
Inventories (end of t-2)	0.61	1.00	0.02	0.39	0.46	0.17
Information: Third Level						
FRB Index—Industry (NSA)	2.10	3.19	1.03	−0.07	0.11	0.27
Changes in Wholesale Prices—Industry	−0.17	1.70	0.77	1.42	0.86	0.04
Production Index—All Firms—Pittsburgh	1.28	0.22	−0.36	1.30	1.83	2.53
FRB Index—Total Production (NSA)	0.38	−1.27	0.47	0.38	−0.05	−0.67
Prime Commercial Paper Rate (N.Y., 4–6 mos.)	−2.59	−1.23	−0.27	−1.11	−1.36	−1.12
Mil. Prime Contract Awards (Defl.)	0.33	0.56	1.37	−0.22	−0.81	0.03
DOD Obligations Incurred (Defl.)	−0.13	−0.58	−1.00	−1.29	−1.44	−1.34
Total Percent Unemployment	1.64	1.70	1.34	1.88	0.39	−1.74
Working Hours (SA)	−1.01	0.66	−0.20	3.00	2.30	1.90
Estimated Idle Time From Work Stoppages	0.08	−0.02	0.47	−0.11	−0.33	−3.32

firms and negative serial correlation for another.[6] With only one exception the R^2's decline as the forecast span is increased.

Table II presents a fairly representative picture. Since A_{-3} is never significant, there is no support for the extrapolative or regressive model.[7] Because of the importance of previous forecasts, the naive model appears to be ruled out for all firms, except one. The third forecast for $t-2$ (i.e., F_{-2}) is significant for one firm, but it has the wrong sign to be consistent with the exponential forecast model. The expectations revision model seems to receive the best support from these data, because the previous forecast (where applicable) and either the First Ten Days Actual (A'_{-1}) or the Actual (A_{-2}) are nearly always significant. In two cases, the First Forecast (F'') is significant in addition to the Second Forecast (F').

The first forecast, column 1 for the sample firm, differs, of course, because there is no previous forecast to fall back on. This firm appears to behave in agreement with the exponential forecast, while the coefficient of F_{-2} for another firm still has the wrong sign. The other variables in the regression take on a significance which the variables of the simple predicting models do not share. New orders and inventories are significant for one firm, and the backlog of unfilled orders is significant for two others. Production indexes or employment indicators are significant for all. Several seasonal variables are significant for all firms but one. In addition, the trend is significant for half the firms (and the negative sign may be the correct one for these data).

For the second and third forecasts, the situation is somewhat different. Only one firm appears to make seasonal adjustments in the third forecasts (another has one coefficient greater than twice its standard error, but this is of little significance because of its small number of observations). Data on new orders, deliveries, unfilled orders, and inventories are all insignificant. None of the industry, region, or general economic indicators exhibit an importance consistently among firms. Production indexes, DOD Obligations Incurred, and Working Hours are sometimes significant. The coefficient for the third forecast have "wrong signs" as often as not. Estimated Idle Time Due to Work Stoppages and the Fraction of the Month with Steel Negotiations in Progress are not significant in the formation of expectations, although they frequently are in regressions with forecast errors as the dependent variables.

The conclusions about the forecasting models are contrary to those found in other studies, in particular those of Hirsch and Lovell. The only model showing up well is that of expectations revision. The errors-in-variables model and the rational expectations hypothesis cannot be tested with these methods because they do not specify dependence on any particular variables. Only statistical properties are specified. Additional data about the firm, industry, region, economy, as well as seasonal and trend factors are of distinctly minor importance for the second and third forecasts, for which previous forecasts are available. Many such variables are important, however, for the first forecast. The forecasting models depending on past levels of production do not appear to be applicable for the first forecast either.

6. Malfunction of program, compiler, or operating system led to overflows or inadmissible values indicated by dashes in the tables. That this is not solely truncation error is indicated by different values of the Durbin-Watson statistic on successive runs with no change in source program or data. Some clue might be provided by the fact that the program was written in Algol.

7. The regressive model here differs markedly from that used by Ferber [3] because the lag is only three months. To have been comparable, a twelve-month lag would have been required.

V. Tests Concerning Expectation Errors

The rational and errors-in-variables models both specify properties of the forecast errors and do not, in the absence of a complete econometric model, specify a dependency upon particular variables. The main requirement of the rational hypothesis, however, is that the forecast error (taken here as $A - F$) be independent of the independent variables, including the forecast itself. The errors-in-variables model allows the coefficient of the forecast to be significant, because of the presence of the "response" error f in both the dependent and one independent variable. Nevertheless, the errors-in-variables model requires the forecast error to be independent of the remaining variables.

The errors of the third forecast were analysed in more detail than those of the other forecasts. In all firms but one there is a significant reduction in the error due to using the third forecast, an intercept, seasonal dummies, and trend terms. The coefficient of the third forecast is significant for half the firms. This result, for the firms having the largest numbers of observations, is consistent with the errors-in-variables model, but *not* the rational expectations hypothesis in its original form. Several of the seasonal dummy variables, particularly for May, appear to be significant as well. Those of June, November, and December are sometimes significant.

Further reductions in the errors can be made for three firms by using the additional data concerning their operations. One firm appears to ignore information about the First Ten Days Actual. Another appears to place too much reliance on the First Forecast and Actual ($t-2$).

The information about industry and regional production and general business conditions leads to significant reductions in the variance for two firms, and possibly two others. The Total Percent Unemployment is important for two. Work stoppage variables (Estimated Idle Time from Work Stoppages or Number of Days Steel Negotiations in Progress) are significant for three. It is not clear that the signs of the coefficients, however, are correct.

It is difficult to escape the conclusion, that the forecast errors can be significantly reduced by using information represented by the variables in these regressions. The variables in the first two levels (requiring no additional information, and using information about the firm's operations) are available to the decision-maker at the time the forecasts are prepared. The remaining variables are not all available at this time, but are interpreted as surrogates for more informal sources. If such sources are not available, then the significance of these variables must be discounted accordingly.

Columns 4-5 of Table II give the corresponding regressions for the second and first forecasts. Seasonal variables have less significance, except for one firm. Unemployment and Working Hours variables appear more significant than for the third forecast (column 6).

As long as the variables used are correlated with the forecast, the presence of errors in the forecast variables can result in the others being significant. As long as the independent variables, excluding the forecasts themselves, are imperfect predictors of future production, and the forecasts are imperfect as well, then the model with errors in the two independent variables might explain the significance of the regression variables other than the forecasts themselves.

VI. Tests Concerning the Equality of Expected and Actual Structures

Some of the difficulties arising from errors in the variables may be handled by treating the forecasts and actual as the dependent variables in separate regression functions and

testing whether the two equations with coefficients are equal. This does not remove problems arising from incorrect specification of the independent variables, but a response error of the type postulated for the errors-in-variables model of expectations behaves as an error in the equation. Since it is desirable to allow the errors in the two equations to have unequal variances and to be correlated with each other, methods based on ordinary regression are not directly applicable. The model is a specialization of Zellner's [13; 14] unrelated regressions.

When the equations relating the forecasts and realizations are applied to the kinds of variables used in the regression analysis, the results obtained are rather similar to those of the regression analysis. There are some differences, however, and additional information is obtained.

Table III. Tests of Structural Equality for One Firm

Information Level	Variables	Third Forecast		Second Forecast		First Forecast	
		df	lr	df	lr	df	lr
	$\rho = 0$	1	1.64	1	1.17	1	7.09**
Minimal	Seasonal, Intercept & Trend	13	9.87	29	91.7**	28	86.6**
Second	Firm Variables	8	6.46 **				
Third	General Economic Variables	10	18.2				
	$\hat{\rho}$		−0.126		−0.107		−0.262
	N		133				

NOTES: ** Significant at 1% level
df Number of degrees of freedom
$\text{lr} = -2 \ln \lambda$, where λ is the likelihood ratio.

The results of testing the series of hypotheses discussed above are illustrated in Table III for all three forecasts for the sample firm. In addition, the test of the equality of the coefficients for the third forecast was run in a way similar to that of the regression analysis, for equations involving successive groups of variables. The nesting was based on not imposing equality restrictions on the successive groups of variables.

From Table III we see that the hypothesis $\rho = 0$ is accepted for the sample Firm; it is rejected for all the others, with the correlation being particularly high in one case. It appears that the relevant variables are especially incomplete for one of the firms. The seasonal, intercept, and trend variables are significantly different for the forecasts and realizations only for another firm. This conclusion differs from that before because the forecast itself is no longer an independent variable. Including the firm variables leads to significant differences between the regression functions for three firms. However, the addition of the industry, regional, and general economic variables does not appear to lead to additional discrepancies between the two lines.

Tests for the equality of all coefficients for the second and first forecasts lead to similar results, but the equations for one firm also appear to differ. At the microscopic level at which we are dealing, the conclusions which can be drawn are almost entirely negative. The regression functions appear to differ significantly from one another. However, the residuals of the two functions appear to be significantly correlated with each other for the variables used here.

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