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Money Balances in the Production Function: A Retrospective Look

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1. INTRODUCTION

Although a number of writers, such as Levhari and Patinkin (1968), Nadiri (1970, p. 1153, fn. 17) (1969, p. 175), and Bailey (1962, 1971), had developed theoretical models that incorporated real money balances as a factor of production, empirical testing of whether real balances are an input in the production function began with Sinai and Stokes (1972).[1] Since that time, there has been a large number of papers concerning the best way to specify a production function containing real balances. In this paper, we briefly survey the literature in this area, formally reply to some new papers, present some new results, and discuss possible new ways to model the effects of developments in the financial sector on real output.

A major motivation of our research concerning real money balances as a factor of production is to attempt to capture the effects of changes in financial institutions on real output. While our early work has focused on real balances, other work by Neuburger and Stokes (1974, 1975, 1976, 1978) has focused on the effect of changes in financial market efficiency on real output. These papers were concerned with testing the important theories of Alexander Gerschenkron (1962) regarding economic development and are related to the general question, How do we model the effect of changes in the financial market on real output?

Unlike the markets for labor and capital, which in theory do not contain constraints, the creation of the money supply is restricted by institutional and legal arrangements. The question becomes, How optimum is the money supply? Presumably, the more optimum the money supply, the greater the level of output because firms will optimally hold more real balances. We next have to define and measure what we mean by "the optimum quantity of money." Friedman (1969, p. 34) has argued for one approach. In his words, "our final rule for the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal rate of interest equal to zero." In Friedman's world, interest was paid to holders of money balances. In the real world, a case can be made for arguing that the lower the nominal interest rate, the more optimal the money supply, because the level of money demanded will be greater and technology of production will shift to incorporate technology involving a less restricted financial sector. The lower the interest rate, the less the impact of monetary restrictions on the real economy. The implications of this approach are tested below when we add the nominal interest rate to the production function as a shift parameter in models containing real balances as an input and time as a shift parameter.

In conclusion, we argue that the development of the financial system is an important determinant of economic growth. The empirical research to date has been concerned with attempting to measure some aspects of this financial development. A major task of future research will be to develop improved measures of the changes in the financial system over time.

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2. LITERATURE SURVEY

Sinai-Stokes (1972) was the first paper to add real balances, defined as m1, m2 and m3, to a production function for annual data for the United States from 1929 to 1967. The data on labor, capital, and output were obtained from Christensen-Jorgenson (1969, 1970) and the data on money was obtained from Friedman-Schwartz (1970). In this early work, Sinai-Stokes (1972) used second-order GLS to estimate a Cobb-Douglas production function containing real balances. Sinai-Stokes (1975, 1977) replied to a number of critics. These included Niccoli (1975), who argued for investment in the production function rather than real balances; Prais (1975a, 1975b), who suggested that the results obtained in the Sinai-Stokes (1972) paper were due to differencing the monetary variable, and Khan-Kouri (1975), who supported our findings with the estimation of a simultaneous equations model. Sinai-Stokes (1975) raised some questions concerning a number of problems in Khan-Kouri (1975) that include making capital and labor exogenous and the form of the money demand equation used. Ben-Zion and Ruttan (1975) provided a further comment on our work. They proposed an alternative specification of the production function that contained real balances as an input and the percent change in real balances as a shift parameter. Their finding was that the "rates of change in the real money supply seem to have stronger and more significant effects than the level of real balances. This is clearly consistent with the induced innovation approach, but not with the production approach." Sinai-Stokes (1975) had problems replicating their finding empirically and theoretical problems with their interpretation of the specification used.

Additional theoretical work in the early 70s included Pierson (1972), who argued for a more broad definition of the monetary aggregate used, and Moroney (1972) who argued that "it may seem justifiable to include real balances as an input of an aggregate production function" but commented, "the sources of the productivity of money are not clearly enough exposed." Fisher (1974) argued that there was a "well defined sense in which real balances may be said to be a factor of production" but that there were problems in estimating a production function containing real balances because real balances might not be "an adequate index of the resources used in transacting." In his view, this was "unlikely . . . if there is technical progress in transactions which is not explicitly modelled."

Further empirical evidence concerning the role of real balances in the production function included an important paper by Short (1979) using revised input data. This work used the more general translog production function to find evidence for real balances in the production function when the model had been corrected for any possible simultaneity bias. We view this work as more comprehensive than that of Khan-Kouri (1975). Additional simultaneous equations results were provided by Butterfield (1975), who found real balances were a significant input in a Diewert generalized Leontief production function. Later work on the original Sinai-Stokes (1972) data included Subrahmanyam (1980), who developed a translog production function model for the period 1947–1967 and found evidence for real balances. In related work, Simos (1981) studied the problem using further revisions of the data over the period 1929–1972 and the translog production function. His major finding was "rejection of the hypothesis that the hardware relation between capital and labor is independent of the level of real balances." Further findings were that "real money balances are substitutes for capital but complements with labor," and that "real money balances do contribute to the aggregate supply. Thus the theoretical and empirical foundations of existing models should be carefully re-examined." The above works support the assertion that real balances are a significant input in alternative production functions that have been corrected for possible simultaniety problems.

Boyes-Kavanaugh (1979) argued for the CES form of the production function in place of the Cobb-Douglas form used by Sinai-Stokes (1972). Sinai-Stokes (1981b) argued that Boyes-Kavanaugh (1979) mistakenly estimated their CES model conditionally and assumed unity of the constant. Sinai-Stokes (1981b) estimated a CES model, using nonlinear methods with and without time and with and without GLS corrections, and found real balances were a significant input in the production function. In addition, new data containing quarterly data on the nonfinancial corporate sector 1953:1 to 1977:3 was used to show that real balances significantly enter a Cobb-Douglas production function in which real balances were defined as m1, m2 and FA (real financial assets held by nonfinancial corporations), respectively. In Sinai-Stokes (1981a), Japanese data were used to estimate an aggregate production function containing labor, capital, and real balances for annual data in the period 1952–1968. Real balances were found to be a significant input in the production function and, in this paper, we extend the results of Nguyen (1986) who argued for subperiod effects using a new data series. No evidence was found for entering real balances as a shift parameter, as was suggested by Moroney (1972).

The above research supports the addition of real balances in an aggregate production function to capture the effect of the financial sector on real output. In related research, Neuburger-Stokes (1975, 1976, 1978) tested the important insights[2] of Gerschenkron (1962) on the effect of changes in the financial system on output. Gerschenkron's (1962, p. 46) hypothesis was that the backward countries that experience successful industrialization do so by making industrial substitutions that enable them to compensate for their initial deficiencies of productive inputs. Neuburger-Stokes (1974) chose to investigate the role of the Credit Banks (Kreditbaken) in Germany in the period 1883–1913. Over this period, the influence of the Credit Banks on certain industrial sectors was growing. This was typified by 1905 when the eight major Credit Banks' influence on industry had grown to 819 directors of industrial firms. The German financial system involved a system in which the Credit Banks made long-term loans at short-term rates to those industrial firms on which they had influence in the form of directors. The net effect of this institutional arrangement was to bias the capital market toward the favored firms by giving them long-term loans at short-term rates. A measure of this bias was current account credit extended by banks in this manner (CA) divided by total credit extended by banks for productive purposes (MB). Neuburger-Stokes (1974) chose to model this effect by estimating a production function containing labor (L) and capital (K) as inputs, and time (V_1) and various lags of (CA/MB) as $V_2, \ldots, V_n$ of the form

$$Q = Ae^{\delta}L^{\alpha}K^{\beta}e \tag{2.1}$$

where

$$\delta = \mu_1 V_1 + \mu_2 V_2 + \cdots + \mu_n V_n. \tag{2.2}$$

If there was a negative effect on output arising from the bias in the financial market (as measured by CA/MB), some of the values of $\mu_2, \cdots \mu_n$ would be negatively significant. Neuburger-Stokes (1974) found such effects for Germany. It should be noted that Gerschenkron argued for positive effects, and in his analysis neglected the dead weight loss to the economy of a discriminatory capital market. Neuburger-Stokes (1975) tested basically the same model for Japan in the period 1952–1968. Here the negative effects found for Germany were not observed. In the Japanese model, the level of imported technology was explicitly modelled as an additional shift parameter. Neuburger-Stokes (1975) argue that Japan allowed the banking system to obtain influence on certain industries, and rationed the importing of technology to counter the output loss associated with the bias in the capital market.

This brief literature survey is intended to outline some of the research into the effect on real output of changes in the financial sector. It is clear that an aggregate production function just containing labor and capital is misspecified. The above research supports the hypothesis that changes in the financial sector have a significant impact on real output. What is less clear is the best way to model these changes. In the next section, we extend the works of Nguyen (1986).[3]

3. EXTENSION OF WORK OF NGUYEN

Unlike many of the other writers who have used the original Sinai-Stokes (1972) dataset, Nguyen (1986) uses more recent data (1930–1978). His paper argues that in some subperiods, real balances were not significant in the production function and that the correct specification of the model should be

$$\ln Q = \ln A + \Phi + \Gamma(\delta m/m)t + \alpha \ln L + \beta \ln K + \tau \ln m + u. \qquad 3.1$$

where $(\delta m/m) = (m_t - m_{t-1})/m_t$. Specifically, while Nguyen finds real balances are significant in the period 1930–1967, in the period 1947–1967, he finds that "money, either as m1 or m2, is not statistically significant in regressions with or without the time trend." In the more recent period 1947–1978, m2 is only significant in models without the trend. Although it is possible that in the period 1929–1967, the financial system may have changed in ways not captured by m1 or m2 and time, the subperiods 1947–1967 and 1947–1978 contain only 21 and 32 observations, respectively, which may not be sufficient for models with six independent variables. We have obtained Nguyen's data[3] and have estimated models for the period 1930–1978 to investigate the complete period. These results are listed in Table 1. Equations 1 and 3 show models that do not contain time and that have significant values for m1 and m2 of 9.11 and 5.91, respectively. Equations 2 and 4 contain time and real balances and show significant values for m1 and m2 of 3.51 and 2.93, respectively. In fact, the levels of significance for the real balances variable in equations 1–4 are larger than in the original Sinai-Stokes (1972) dataset. Table 1 uses SAS PROC AUTOREG, which uses maximum likelihood estimation of a second-order GLS model in which the autoregressive parameters are estimated jointly with the coefficients.

Using recursive residual methods developed by Brown-Durbin-Evans (1975) and Dufour (1982) and available in B34S (Stokes [1989]), stability tests were performed on equations 1–4. These include the CUSUMSQ and Quandt likelihood ratio tests and plots and lists of the recursive coefficients and the recursive residual.[4] Although there is some indicated instability, there is no significant evidence that there was a structural shift in the complete period. However, the recursive OLS coefficients[5] for equations 1–4, which are listed in Tables 2–5, give an indication of possible changes in the structure of the economy.

Tables 2 and 4 show models that do not contain time, while Tables 3 and 5 show models that do contain time. What is remarkable is that in the models that do not contain time, the coefficient on real balances is relatively stable. For example, in Table 2 the coefficient for real balances is gradually increasing. From 1948 to 1978 it increased from .2509 to .3598. For Table 4, comparable numbers for m2 show the coefficient increasing from .2661 to .3899. Tables 3 and 5 show models for lnml and lnm2 with time. In Table 3, the coefficient for real balances in 1948 was .2819; by 1966, it had fallen to .1380; and by 1978, it had risen again to .2451. In Table 5, a similar pattern, although somewhat delayed, is seen for the coefficient for lnm2 as it moves from .2097 in 1948, to .0607 in 1975 and then rises to .1134 in 1978. In all tables, the coefficient for lnk was negative up until around 1949.

TABLE 1

Estimates of the Parameters of the Cobb-Douglas Production Function With Real Money Balances and With and Without Time 1930–1978.

$$\ln Q = \text{Ln } A + \Phi t + \alpha \ln L + \beta \ln K + \tau \ln m + u'$$

With	1 m1	2 m1, t	3 m2	4 m2, t
ln A	−3.29	−16.74	−3.20	−24.33
	(−10.98)	(−1.82)	(−8.55)	(−2.75)
Φ		.0076		.0120
		(1.47)		(2.39)
α	.8336	.8180	.9132	.8553
	(8.99)	(8.83)	(10.02)	(9.28)
β	.4929	.3289	.3063	.1040
	(11.36)	(2.66)	(4.33)	(1.11)
τ	.3436	.2545	.3936	.2577
	(9.11)	(3.51)	(5.91)	(2.93)
AR (1)	−.9367	−.9363	−1.141	−1.088
	(−6.26)	(−6.02)	(−7.27)	(−7.01)
AR (2)	.2697	.3283	.3210	.3764
	(1.82)	(2.19)	(2.14)	(2.46)
$\alpha + \beta + \tau$	1.670	1.399	1.613	1.217
R^2	.988	.999	.998	.984
R^2 (OLS)	.996	.997	.993	.996
SSE	.0230	.0228	.0242	.0233
SSE (OLS)	.0348	.0330	.0481	.0381
DW	1.89	1.91	1.91	1.93

t scores have been listed in parentheses. All calculations have been made with SAS (1984) ETS version 5.18, using PROC AUTOREG with METHOD = ML and second-order GLS. Data were obtained from Nguyen (1986). t = time trend, 1930 = 1930. AR(1) and AR(2) are the autoregressive coefficients. R^2 = the coefficient of determination for GLS2 equation. R^2 (OLS) = the coefficient of determination for OLS equation. SSE = standard error of estimate of the GLS equation. SSE (OLS) = SSE for OLS equation. DW = Durbin Watson for GLS2 equation.

It appears clear that there have been shifts in the structure of the economy that are not captured fully by the variables in the equation. The relationship between real balances, which is a proxy for the financial sector, and time, which is a proxy for technological change, is changing, although the change is not yet significant as measured by the CUSUMSQ test. This finding suggests that Nguyen's attempt to reformulate the model (equation 3.1) is a useful approach. We present an alternative approach, based on the optimum quantity of money literature below.

TABLE 2

Listing of Recursive Coefficients for Equation 1

Last Year	LNL	LNK	LNM1	CONSTANT
1934.	1.552	−0.8502	1.121	−4.058
1935.	1.263	−0.4258	0.1663	−0.4172
1936.	1.249	−0.4243	0.1236	−0.1581
1937.	1.212	−0.4208	0.8614E-01	0.1960
1938.	1.079	−0.1449	0.2764	−1.244
1939.	1.067	−0.1090	0.3024	−1.468
1940.	1.073	−0.1304	0.2890	−1.337
1941.	1.076	−0.1153	0.2966	−1.459
1942.	1.083	−0.6669E-01	0.3126	−1.814
1943.	1.084	−0.6932E-01	0.3113	−1.800
1944.	1.073	−0.5762E-01	0.3190	−1.831
1945.	1.104	−0.8157E-01	0.3037	−1.821
1946.	1.151	−0.1141	0.2649	−1.758
1947.	1.163	−0.2028	0.2496	−1.318
1948.	1.161	−0.1902	0.2509	−1.371
1949.	1.075	−0.1661E-01	0.2784	−1.865
1950.	1.018	0.1448	0.2958	−2.415
1951.	1.005	0.2453	0.2976	−2.847
1952.	0.9863	0.3026	0.3016	−3.043
1953.	0.9730	0.3403	0.3043	−3.166
1954.	0.9416	0.3806	0.3120	−3.220
1955.	0.9249	0.4136	0.3152	−3.302
1956.	0.9212	0.4264	0.3153	−3.345
1957.	0.9122	0.4415	0.3166	−3.373
1958.	0.8895	0.4623	0.3215	−3.368
1959.	0.8837	0.4746	0.3220	−3.397
1960.	0.8832	0.4833	0.3208	−3.433
1961.	0.8764	0.4941	0.3214	−3.450
1962.	0.8786	0.5049	0.3189	−3.505
1963.	0.8805	0.5135	0.3168	−3.549
1964.	0.8843	0.5214	0.3141	−3.600
1965.	0.8973	0.5254	0.3089	−3.671
1966.	0.9102	0.5265	0.3040	−3.730
1967.	0.9092	0.5263	0.3045	−3.724
1968.	0.9046	0.5253	0.3063	−3.701
1969.	0.8867	0.5249	0.3129	−3.625
1970.	0.8814	0.5189	0.3163	−3.579
1971.	0.8815	0.5141	0.3173	−3.561
1972.	0.8749	0.5126	0.3201	−3.527
1973.	0.8559	0.5144	0.3268	−3.456
1974.	0.8220	0.5160	0.3398	−3.326
1975.	0.8173	0.5111	0.3438	−3.291
1976.	0.8083	0.5119	0.3473	−3.259
1977.	0.7999	0.5135	0.3502	−3.232
1978.	0.7707	0.5206	0.3598	−3.141

Model estimated with OLS, adding one observation at a time. For data sources and variable definitions, see Table 1.

TABLE 3

Listing of Recursive Coefficients for Equation 2

Last Year	LNL	LNK	LNM1	YEAR	CONSTANT
1935.	0.9144	−1.488	0.4826	−0.5791E-01	117.4
1936.	1.073	−1.034	0.3745	−0.3312E-01	66.82
1937.	1.163	−0.7406	0.2587	−0.1715E-01	34.47
1938.	1.126	−0.8875E-01	0.1732	0.6189E-02	−13.30
1939.	1.120	−0.5618E-01	0.1859	0.6661E-02	−14.40
1940.	1.125	−0.6308E-01	0.1751	0.6948E-02	−14.90
1941.	1.128	−0.4930E-01	0.1766	0.7202E-02	−15.49
1942.	1.148	0.5192E-02	0.1564	0.9144E-02	−19.54
1943.	1.128	0.3481E-02	0.1994	0.7043E-02	−15.54
1944.	1.082	−0.1971E-01	0.2811	0.2772E-02	−7.266
1945.	1.097	−0.1622E-01	0.2619	0.3725E-02	−9.128
1946.	1.145	−0.4840E-01	0.2229	0.3742E-02	−9.099
1947.	1.169	−0.2718	0.3091	−0.5018E-02	8.434
1948.	1.158	−0.1976	0.2819	−0.2392E-02	3.169
1949.	1.084	−0.2746E-01	0.2589	0.1320E-02	−4.328
1950.	1.054	0.8690E-01	0.2315	0.4190E-02	−10.15
1951.	1.058	0.1420	0.2070	0.5877E-02	−13.61
1952.	1.047	0.1806	0.2024	0.6396E-02	−14.72
1953.	1.039	0.2071	0.1998	0.6712E-02	−15.40
1954.	1.014	0.2366	0.2018	0.7014E-02	−16.00
1955.	1.003	0.2575	0.1985	0.7411E-02	−16.79
1956.	1.002	0.2629	0.1959	0.7575E-02	−17.12
1957.	0.9979	0.2698	0.1937	0.7780E-02	−17.52
1958.	0.9828	0.2802	0.1931	0.8070E-02	−18.04
1959.	0.9828	0.2808	0.1881	0.8400E-02	−18.66
1960.	0.9856	0.2786	0.1835	0.8658E-02	−19.14
1961.	0.9857	0.2768	0.1786	0.8988E-02	−19.75
1962.	0.9943	0.2685	0.1675	0.9594E-02	−20.88
1963.	1.001	0.2627	0.1594	0.1003E-01	−21.70
1964.	1.008	0.2573	0.1511	0.1045E-01	−22.50
1965.	1.020	0.2515	0.1428	0.1078E-01	−23.12
1966.	1.029	0.2486	0.1380	0.1092E-01	−23.41
1967.	1.026	0.2485	0.1392	0.1090E-01	−23.37
1968.	1.023	0.2471	0.1404	0.1092E-01	−23.39
1969.	1.005	0.2467	0.1470	0.1092E-01	−23.31
1970.	1.002	0.2358	0.1466	0.1114E-01	−23.67
1971.	1.004	0.2286	0.1458	0.1126E-01	−23.87
1972.	0.9982	0.2259	0.1476	0.1131E-01	−23.92
1973.	0.9786	0.2283	0.1547	0.1129E-01	−23.82
1974.	0.9433	0.2319	0.1691	0.1121E-01	−23.54
1975.	0.9347	0.2343	0.1782	0.1090E-01	−22.95
1976.	0.9103	0.2517	0.1945	0.1028E-01	−21.76
1977.	0.8770	0.2749	0.2142	0.9569E-02	−20.40
1978.	0.8186	0.3121	0.2451	0.8563E-02	−18.43

Model estimated with OLS, adding one observation at a time. For data sources and variable definitions, see Table I.

TABLE 4
Listing of Recursive Coefficients for Equation 3

Last Year	LNL	LNK	LNM2	CONSTANT
1934.	1.381	−1.380	0.8548	0.2852
1935.	1.229	−0.5692	0.2435	0.3993E-01
1936.	1.211	−0.5174	0.1930	0.1278
1937.	1.164	−0.4705	0.1750	0.2469
1938.	1.046	−0.3958	0.3750	−0.4132
1939.	1.030	−0.3832	0.4151	−0.5802
1940.	1.032	−0.3861	0.4089	−0.5483
1941.	1.050	−0.3675	0.4328	−0.8597
1942.	1.105	−0.3150	0.4523	−1.533
1943.	1.104	−0.3139	0.4536	−1.542
1944.	1.131	−0.3281	0.4290	−1.504
1945.	1.234	−0.3721	0.3573	−1.529
1946.	1.317	−0.3936	0.2803	−1.520
1947.	1.324	−0.4666	0.2624	−1.110
1948.	1.321	−0.4272	0.2661	−1.302
1949.	1.232	−0.2045	0.3034	−2.081
1950.	1.188	−0.1522E-01	0.3197	−2.848
1951.	1.193	0.1107	0.3121	−3.468
1952.	1.180	0.1853	0.3138	−3.768
1953.	1.167	0.2293	0.3163	−3.930
1954.	1.132	0.2771	0.3273	−4.017
1955.	1.115	0.3110	0.3317	−4.109
1956.	1.112	0.3250	0.3314	−4.157
1957.	1.102	0.3403	0.3333	−4.188
1958.	1.079	0.3587	0.3405	−4.182
1959.	1.072	0.3669	0.3423	−4.193
1960.	1.070	0.3723	0.3424	−4.207
1961.	1.064	0.3778	0.3444	−4.207
1962.	1.059	0.3824	0.3459	−4.210
1963.	1.057	0.3834	0.3464	−4.210
1964.	1.056	0.3843	0.3470	−4.209
1965.	1.058	0.3824	0.3462	−4.206
1966.	1.059	0.3779	0.3456	−4.189
1967.	1.070	0.3681	0.3412	−4.180
1968.	1.084	0.3575	0.3355	−4.175
1969.	1.080	0.3470	0.3379	−4.110
1970.	1.096	0.3336	0.3329	−4.109
1971.	1.121	0.3218	0.3231	−4.148
1972.	1.134	0.3127	0.3178	−4.151
1973.	1.119	0.3065	0.3250	−4.065
1974.	1.093	0.2968	0.3390	−3.940
1975.	1.102	0.2855	0.3391	−3.934
1976.	1.095	0.2791	0.3439	−3.886
1977.	1.066	0.2761	0.3575	−3.769
1978.	0.9944	0.2779	0.3899	−3.520

Model estimated with OLS adding one observation at a time. For data sources and variable definitions, see Table 1.

TABLE 5
Listing of Recursive Coefficients for Equation 4

Last Year	LNL	LNK	LNM2	YEAR	CONSTANT
1935.	0.7978	−2.125	0.6175	−0.6748E-01	138.9
1936.	1.022	−1.406	0.4744	−0.3457E-01	71.10
1937.	1.141	−0.9158	0.3199	−0.1502E-01	30.94
1938.	1.088	−0.2425	0.2623	0.5877E-02	−12.23
1939.	1.084	−0.2176	0.2716	0.6560E-02	−13.70
1940.	1.086	−0.2205	0.2655	0.6562E-02	−13.68
1941.	1.111	−0.1650	0.2394	0.8356E-02	−17.43
1942.	1.168	−0.3310E-01	0.1449	0.1282E-01	−26.59
1943.	1.153	−0.4299E-01	0.1932	0.1160E-01	−24.34
1944.	1.115	−0.8882E-01	0.2770	0.9009E-02	−19.28
1945.	1.143	−0.6848E-01	0.2346	0.1026E-01	−21.77
1946.	1.206	−0.3933E-01	0.1417	0.1194E-01	−25.06
1947.	1.284	−0.3476	0.2062	0.4518E-02	−9.933
1948.	1.287	−0.3616	0.2097	0.4197E-02	−9.280
1949.	1.214	−0.1904	0.1869	0.7407E-02	−15.81
1950.	1.180	−0.6839E-01	0.1594	0.9798E-02	−20.71
1951.	1.182	−0.6878E-02	0.1334	0.1113E-01	−23.49
1952.	1.173	0.3319E-01	0.1256	0.1168E-01	−24.66
1953.	1.165	0.5997E-01	0.1229	0.1196E-01	−25.26
1954.	1.140	0.8916E-01	0.1254	0.1228E-01	−25.90
1955.	1.127	0.1114	0.1246	0.1253E-01	−26.42
1956.	1.125	0.1169	0.1232	0.1262E-01	−26.59
1957.	1.120	0.1241	0.1225	0.1272E-01	−26.79
1958.	1.104	0.1351	0.1257	0.1284E-01	−27.00
1959.	1.100	0.1385	0.1257	0.1290E-01	−27.11
1960.	1.100	0.1384	0.1257	0.1290E-01	−27.11
1961.	1.098	0.1399	0.1259	0.1293E-01	−27.17
1962.	1.095	0.1418	0.1261	0.1297E-01	−27.23
1963.	1.094	0.1431	0.1268	0.1296E-01	−27.23
1964.	1.090	0.1463	0.1289	0.1293E-01	−27.16
1965.	1.089	0.1472	0.1294	0.1291E-01	−27.13
1966.	1.090	0.1447	0.1281	0.1298E-01	−27.25
1967.	1.095	0.1336	0.1198	0.1336E-01	−27.92
1968.	1.102	0.1208	0.1095	0.1383E-01	−28.76
1969.	1.100	0.1031	0.1002	0.1449E-01	−29.89
1970.	1.111	0.8438E–01	0.8723E-01	0.1509E-01	−30.95
1971.	1.126	0.7046E-01	0.7448E-01	0.1554E-01	−31.78
1972.	1.132	0.5885E-01	0.6483E-01	0.1598E-01	−32.56
1973.	1.123	0.4761E-01	0.6143E-01	0.1646E-01	−33.37
1974.	1.103	0.3095E-01	0.6388E-01	0.1702E-01	−34.26
1975.	1.111	0.1783E-01	0.6071E-01	0.1722E-01	−34.60
1976.	1.105	0.1021E-01	0.6315E-01	0.1734E-01	−34.77
1977.	1.078	0.5790E-02	0.7441E-01	0.1743E-01	−34.84
1978.	0.9992	0.1135E-01	0.1134	0.1721E-01	−34.16

Model estimated with OLS, adding one observation at a time. For data sources and variable definitions, see Table 1.

4. ROLE OF THE FINANCIAL SECTOR ON OUTPUT

Although many writers such as Levhari-Patinkin (1968), Johnson (1969), Friedman (1969), and Bailey (1962, 1971), argued for real balances in the production function, others, such as Pierson (1972), Moroney (1972), and Fisher (1974), raised questions concerning what was being measured by real balances. While Pierson (1972, p. 389) argued that the "appeal of the theory that money belongs in the production function is that it offers a way for monetary growth to affect the real variables in the system," she later noted (p. 391–392) that "credit should also be included. . . ." Her main objection was that a production function model containing real balances neglects "the effects of the credit system or a financial intermediary system and thus claim too much for money." Moroney (1972, p. 342) makes a similar point arguing ". . it may seem justifiable to include real balances as an input of an aggregate production function. Yet by doing so I think that the sources of the productivity of money are not clearly enough exposed. It seems well worthwhile to consider them in more detail than is suggested simply by including real balances as an ordinary input." Fisher (1974, p. 531) while commenting on Sinai-Stokes (1972) work, noted, "The question here is again whether real balances are an adequate index of the resources used in transacting. This is unlikely . . . if there is technical progress in transactions which is not explicitly modelled."

While we concede the validity of the above points, the major objective of our 1972 paper was to "test the hypothesis that real money balances have been mistakenly omitted from the production function." While our results "support the view that real money balances are a producer's good," in no way did we assert that the *only* way to model the effect of a monetary/financial system on real output was to add real balances as an input in the production function. While most researchers allude to the added output available from a more fully developed financial system, there are a number of ways to model the effects of the financial system on production. In our original paper we used m1, m2, and m3 as possible inputs, with the full realization that the aggregation problem increases, the further the monetary measure moves from high powered money. Rather than experimenting with other more aggregate monetary measures, we now propose adding to the model an indirect indication of the degree of optimality of the money supply or functioning of the financial system. We assume that a financial system is more optimal, the less the restriction on monetary or credit creation. The more optimal the financial system, the more the technology of production can shift to utilize the financial system. Conversely, in a credit crunch situation when interest rates increase, the production system is adversely impacted as credit is rationed. The problem now is to develop a measure of the degree of optimality of the financial system.

Friedman (1969, p. 34) developed a measure for the degree of optimality of the money supply. In his view, "our final rule for the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal interest rate equal to zero."[6] As the money supply approaches optimality, producers would presumably hold greater and greater levels of real balances, and the production process would switch technology to use the financial system more intensively.[7] We propose to test this hypothesis with the model

4.1 $$Q = Ae^{\Phi t + \Gamma i}L^{\alpha}K^{\beta}m^{\tau}e,$$

where i = the interest rate and the other variables are defined as before. Equation 4.1 can be estimated as

4.2 $$\ln Q = \text{Ln}\, A + \Phi t + \Gamma i + \alpha \ln L + \beta \ln K + \tau \ln m + u.$$

Equation 4.2 adds the interest rate as an additional shift parameter. We have used the same data and estimation procedure as in our 1972 paper to facilitate comparisons with our prior work. For the interest rate, we have selected the AAA and BBB annual interest rate data from Table C55 of the 1970 *Economic Report of the President.* The results of estimating 4.2 with and without time are given in Tables 6 and 7 below.

Equations 1, 6, 9, and 14 list results reported in Sinai-Stokes (1972, equations 4, 5, 8, and 9), and are repeated to facilitate comparisons. Different forms of equation 4.2 will indicate whether the production function should contain real balances, real balances and a financial sector shift parameter, or just a financial sector shift parameter. All proposed functional forms are estimated containing time or not containing time to test whether the proposed additions are really a proxy for time.

Equations 2 and 3 contain models without time and with two forms of the financial market shift parameter, one with the AAA rate, one with the BBB rate. In these equations, both the financial market shift parameters Γ are significant (t scores of −2.94 and −2.53) and have the expected negative sign. As interest rates fall, there is an increase in output, given the same level of inputs. When time is added to these models, the results, which are reported in equations 10 and 11, show t scores of −1.98 and −1.79, respectively, for Γ. In predictive power, these models are comparable to the equations with only real balances.

Equations 4, 5, 7, and 8 contain both the financial market shift parameters and real

TABLE 6

Estimates of the Parameters of the Cobb-Douglas Production Function With and Without the Interest Rate a Shift Parameter and With and Without Real Money Balances 1929–1967.

$$\ln Q = \text{Ln}\, A + \Gamma i + \alpha \ln L + \beta \ln K + \tau \ln m + u'$$

With	1 m1	2 AAA	3 BBB	4 m1, AAA	5 m1, BBB	6 m2	7 m2, AAA	8 m2, BBB
ln A	−3.022	−2.84	−2.56	−2.82	−2.54	−3.54	−2.94	−2.46
	(−11.44)	(−8.19)	(−5.50)	(−8.53)	(5.80)	(14.13)	(−8.75)	(−5.61)
Γ		−.0333	−.0187	−.0162	−.0102		−.0294	−.0197
		(−2.94)	(−2.53)	(−1.22)	(−1.35)		(−2.65)	(−2.95)
α	.9454	1.052	1.025	.9053	.8206	1.092	.9006	.7722
	(7.72)	(8.27)	(6.88)	(6.79)	(5.57)	(10.79)	(7.08)	(5.35)
β	.5850	.6244	.592	.6395	.6554	.470	.6257	.6379
	(10.12)	(7.85)	(7.57)	(8.44)	(8.76)	(9.55)	(8.20)	(8.56)
τ	.1716			.1308	.1505	.214	.1713	.1991
	(3.84)			(2.30)	(2.77)	(3.52)	(2.77)	(3.22)
$\alpha + \beta + \tau$	1.702	1.676	1.617	1.676	1.627	1.776	1.698	1.609
R^2	.994	.994	.994	.994	.993	.994	.994	.993
R^2 (OLS)	.995	.995	.996	.995	.995	.995	.996	.996
SEE	.03548	.0350	.0366	.0348	.0392	.037	.0356	.0414
SEE (OLS)	.03259	.0315	.0318	.0316	.0310	.034	.0308	.0302
DW	1.43	1.52	1.46	1.38	1.35	1.33	1.47	1.34

t scores have been listed in parentheses. All calculations have been made with B34S (Stokes [1989]) using 2nd-order GLS to be comparable to Sinai-Stokes (1972, 1975, 1981). Data sources and variable definition listed in Sinai-Stokes (1972). AAA is the AAA bond rate. BBB is the BBB bond rate. m1, m2, and m3 are the real values of M1, M2, and M3. R^2 = adjusted coefficient of determination for GLS2 equation. R^2 (OLS) = the adjusted coefficient of determination for OLS equation. SEE = standard error of estimate. SEE (OLS) = SEE for OLS equation. DW = Durbin-Watson for GLS2 equation.

TABLE 7

Estimates of the Parameters of the Cobb-Douglas Production Function With and Without the Interest Rate as a Shift Parameter and With and Without Real Money Balances and With Time 1929–1967.

$$\ln Q = \text{Ln} A + \Phi t + \Gamma i + \alpha \ln L + \beta \ln K + \tau \ln m + u'$$

	9	10	11	12	13	14	15	16
With	m1	AAA	BBB	m1, AAA	m1, BBB	m2	m2, AAA	m2, BBB
ln A	−2.27	−1.92	−1.53	−2.16	−1.91	−2.57	−2.47	−2.19
	(−3.79)	(−3.21)	(−2.36)	(−3.54)	(−2.86)	(−3.73)	(−3.52)	(−2.92)
Φ	.0058	.0073	.0083	.0053	.0055	.0065	.0035	.0025
	(1.47)	(1.86)	(2.03)	(1.32)	(1.27)	(1.58)	(.76)	(.50)
Γ		−.0239	−.0138	−.0133	−.0088		−.0263	−.0182
		(−1.98)	(−1.79)	(−.99)	(−1.12)		(−2.23)	(−2.55)
α	.9662	1.028	.9809	.9238	.8502	1.100	.9118	.8003
	(7.83)	(8.29)	(6.77)	(6.91)	(5.71)	(10.6)	(7.10)	(5.39)
β	.4278	.4247	.3878	.4903	.5010	.3233	.5343	.5687
	(3.68)	(3.23)	(2.93)	(3.64)	(3.61)	(3.32)	(3.80)	(3.90)
τ	.1267			.1005	.1176	.1325	.1405	.1717
	(2.46)			(1.69)	(2.02)	(1.83)	(1.94)	(2.25)
$\alpha + \beta + \tau$	1.521	1.453	1.369	1.515	1.469	1.556	1.587	1.541
R^2	.995	.995	.994	.995	.994	.995	.994	.993
R^2 (OLS)	.995	.995	.996	.995	.996	.995	.995	.996
SEE	.0329	.0333	.0366	.0332	.0369	.0332	.0352	.0399
SEE (OLS)	.0325	.0314	.0307	.0318	.0309	.0331	.0312	.0304
DW	1.45	1.47	1.36	1.42	1.35	1.33	1.49	1.32

t scores have been listed in parentheses. All calculations have been made with B34S (Stokes [1989]) using 2nd-order GLS to be comparable to Sinai-Stokes (1972, 1975, 1981). Data sources and variable definition listed in Sinai-Stokes (1972). AAA is the AAA bond rate. BBB is the BBB bond rate. m1, m2, and m3 are the real values of M1, M2, and M3. t = time trend, 1929 = 0. R^2 = adjusted coefficient of determination for GLS2 equation. R^2 (OLS) = the adjusted coefficient of determination for OLS equation. SEE = standard error of estimate. SEE (OLS) = SEE for OLS equation. DW = Durbin-Watson for GLS2 equation.

balances, while equations 12, 13, 15, and 16 also contain time. The equations containing the financial market shift parameter and m1, both with and without time (equations 4, 5, 12, and 13) show no significance for the financial market shift parameter. The coefficient (τ) for real balances, defined as m1, is significant in all equations without time (see t of 2.30 and 2.77 for m1 in equations 4 and 5, respectively). However, if time is added to these models containing m1, real balances are only significant with the BBB interest rate (see t of 2.02 for m1 in equation 13). However, in all models with real balances measured as m2 and the financial market shift parameter (see equations 7, 8, 15, and 16), *both* the real balances coefficient (see t of 2.77, 3.22, 1.94, and 2.25) and the financial market shift parameter (see t of −2.65, −2.95, −2.23, and −2.55) are significant whether or not time is in the model. These models are superior to the models containing only the real balances variable as measured by the OLS SEE. For example, in models without time, for the OLS form of the model (equation 6) the SEE was .034. If the financial market shift parameter is added, this SEE falls to .0308 and .0302 in equations 7 and 8 respectively. In models containing time, the SEE for OLS (.0331) in equation 14 falls to .0312, and .0304 in equations 15 and 16 respectively, as AAA and BBB are used as the financial market shift variable. From this experiment, we conclude that there is evidence for using a financial market shift parameter to control for changes in production technology *and* real balances in the production function when real balances are defined as m2. The importance of this experiment is that it indicates that there are substantial costs on real output of high interest rates.[8]

Our findings do not support the classical dichotomy that monetary variables have no effect on the level of real output. These results are preliminary in that we have by design assumed away simultaneity problems so as to be comparable to Sinai-Stokes (1972). Future work should address these concerns with more recent data, alternative estimation methods, and different measures for the monetary variable and the financial market shift variables.

5. CONCLUSION

We have surveyed the literature of real balances in the production function since Sinai-Stokes (1972). Additional evidence supporting the basic contention that real balances belong in the production function have been presented by Short (1979), Subrahmanyam (1980) for the original data 1929–1967, Simos (1981) for newer data, Sinai-Stokes (1981a) for Japanese data, Nguyen (1986) for the United States using still newer data, Yoo (1981) for Canadian data and Sinai-Stokes (1981b) for disaggregate United States data. The discussion has now moved beyond the question, "does real balances belong in the production function," to a detailed discussion of how to model the effect of the financial sector on the real economy. In related research, Neuburger-Stokes (1974, 1975) looked at the effect on real output of a discriminatory capital market for Germany and Japan. The present paper builds on this research and on that of Eckstein-Sinai (1986) to argue that the effect of the degree of optimality of the financial sector (as measured by the interest rate) is a significant shift parameter in the production function. While this functional form may be improved on in future work, what seems clear from the research to date is that an aggregate production function that does not take into account the effects of the financial sector is not completely specified. We are currently working on better measures to capture the functioning of the financial market and possibly alternative specifications on how it should be entered into the production function.

NOTES

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1. Sinai-Stokes (1972) footnote 1 contains general theoretical references. Moroney (1972) and Fisher (1974) contain additional references not available in 1972.
2. Sylla (1977) provides a summary of this research.
3. We appreciate obtaining Nguyen's data to extend our work. The questions raised by the Jessen-Kamath Bennett (1987) "Counterexample" are left aside for later separate consideration.
4. Detail on these tests have been removed from the paper to conserve space and are available from the authors.
5. The recursive residual procedure works on OLS, not GLS models. Since serial correlation affects only the standard errors of the coefficients and not the coefficients themselves, it is possible to use the recursive OLS coefficients to detect changes in the structure of the model as new observations are added.
6. In this paper, we do not focus on how the interest rate was lowered.
7. If real balances are in the production function as an input, the assumption is that unless one of the shift parameter variables moves, production will use more real balances relative to capital, using the same technology as real balances increase. Nguyen's model (equation 3.1), which contained both money as an input and the growth of money times time as a shift parameter, allowed for changes in technology if

the growth rate of money changed. In our proposed model (see equation 4.1 below), technology will change as the interest rate changes, or over time in those equations containing time.

8. Eckstein-Sinai (1986 p. 61) present a detailed discussion of the effects of changes in interest rates on the economy. In their discussion, "the 'flow of funds' or 'credit cycle' can be divided into phases of accumulation, developing financial instability or the precrunch period, crunch, and reliquefication. . . . The crunch is characterized by extremely depressed liquidity, and deteriorated balance sheet positions for households, corporations and financial institutions; sharply increased interest rates as all sectors scramble for remaining available funds; . . . and the inability of many borrowers to obtain funds at any cost." In such a situation, it is likely that the production process would shift into other ways to operate. For this reason, we have argued for the interest rate as a shift parameter in the production function. A related question concerns whether to use the real or nominal interest rate to proxie for the financial sector. Friedman (1959) was clear that the nominal interest rates should be used since the "optimum quantity of money" related to money demand which is a function of the nominal interest rate not the real interest rate. If the real interest rate was used a major problem would be calculating the expected price series to form the real interest rate. In view of these problems, only the nominal interest rate was used to proxie for the financial sector in this preliminary paper.

REFERENCES

Bailey, M.J., *National Income and the Price Level,* (New York: McGraw Hill Book Company, 1962, 1971).

Ben-Zion, Uri, and Vernon Ruttan, "Money in the Production Function: An Interpretation of Empirical Results," *Review of Economics and Statistics,* Vol. 61, No. 2, (May 1975), p. 246.

Boyes, William, and David Kavanaugh, "Money and the Production Function: A Test of Specification Errors," *The Review of Economics and Statistics,* Vol. LXI, No. 3 (August 1979), pp. 442–446.

Brown, R., J. Durbin and J. Evans, "Techniques for Testing the Constancy of Regression Relationships Over Time," *Journal of the Royal Statistical Society Series B,* Vol. 37 (1975), pp. 149–192.

Butterfield, David, *Money as a Productive Factor: Study of Business Money Balances,* PhD Thesis, University of California, Berkeley, 1975.

Christensen, L.R., and D.W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929–1967," *Review of Income and Wealth,* series 15 (December 1969), pp. 293–320.

——————, "U.S. Real Product and Real Factor Input, 1929–1967," *Review of Income and Wealth,* series 16 (March 1970), pp. 19–50.

Dufour, Jean-Marie, "Recursive Stability Analysis of Linear Regression Relationships: An Exploratory Methodology," *Journal of Econometrics,* Vol. 19, (1982), pp. 31–76.

Eckstein, Otto, and Allen Sinai, "The Mechanisms of the Business Cycle in the Postwar Era," in Robert J. Gorden, ed. *The American Business Cycle: Continuity and Change,* National Bureau of Economic Research Studies in Business Cycles, Vol. 25 (Chicago: The University of Chicago, Press, 1986), pp. 39–120.

Faddeeva, V.N., *Computational Methods of Linear Algebra,* New York: Diver. 1959.

Fisher, Stanley, "Money in the Production Function," *Economic Inquiry,* Vol. 12 (December 1974), pp. 517–533.

Friedman, Milton, *The Optimum Quantity of Money and Other Essays,* (Chicago: Aldine Publishing Company, 1969).

Friedman, Milton, and Anna J. Schwartz, *Monetary Statistics of the United States.* (New York: Columbia University Press, 1970).

Gerschenkron, Alexander, *Economic Backwardness in Historical Perspective,* (Cambridge: Harvard University Press, 1962).

Jensen, Keith, and Shyam Kamath and Robert Bennett, "Money in the Production Function: An Alternative Test Procedure," *Eastern Economic Journal,* Vol. 13, No. 3 (July–September 1987), pp. 259–269.

Johnson, H.G., "Inside Money, Outside Money, Income, Wealth, and Welfare in Monetary Theory," *Journal of Money Credit and Banking,* Vol. 2, No. 1 (February 1969), pp. 30–45.

Khan, Mohsin, and Pentti Kouri, "Real Money Balances as a Factor of Production: A Note," *Review of Economics and Statistics,* Vol. 61, No. 2, (May 1975), pp. 244–245.

Levhari, David, and Don Patinkin, "The Role of Money in a Simple Growth Model," *American Economic Review,* Vol. 58, No. 4 (September 1968), pp. 713–754.

Moroney, John, "The Current State of Money and Production Theory," *American Economic Review,* Vol. 62, No. 2, (May 1972), pp. 335–343.

Nadiri, M.I., "The Determinants of Real Cash Balances in the U.S. Total Manufacturing Sector," *Quarterly Journal of Economics,* Vol. 83, (May 1969), pp. 173–196.

——————, "Some Approaches to the Theory and Measurement of Total Factor Productivity: A Survey," *Journal of Economic Literature,* Vol. 8, No. 4 (December 1970), pp. 1137–1178.

Neuburger, Hugh, and Houston H. Stokes, "German Banks and German Growth, 1883–1913: An Empirical View," *The Journal of Economic History,* Vol. 34, No. 3 (September 1974), pp. 710–731.

——————, "German Banking and Japanese Banking: A Comparative Analysis," *The Journal of Economic History,* Vol. 35, No. 1 (March 1975), pp. 238–255.

——————, "German Banking and German Growth: Reply to Fremdling and Tilly," *Journal of Economic History,* Vol. 36, No. 2 (June 1976), pp. 425–427.

——————, "German Banks and German Growth: Reply to Komlos," *Journal of Economic History,* Vol. 38, No. 2. (June 1978) pp. 480–482. See also further rejoinder, *Journal of Economic History,* Vol. 38, No. 4, (December 1978) pp. 963–964.

Niccoli, Albert, "Real Money Balances: An Omitted Variable from the Production Function? A Comment," *Review of Economics and Statistics,* Vol. 61, No. 2, (May 1975), pp. 241–242.

Nguyen, Hong, "Money in the Aggregate Production Function: Reexamination and Further Evidence," *Journal of Money, Credit and Banking,* Vol. 28, No. 2 (May 1986), pp. 141–151.

Pierson, Gail, "The Role of Money in Economic-Growth," *Quarterly Journal of Economics,* Vol. 86, (August 1972), pp. 383–395.

Prais, Zmira, "Real Money Balances as a Variable in the Production Function," *Review of Economics and Statistics,* Vol. 61, No. 2, (May 1975), pp. 243–244.

——————, "Real Money Balances as a Variable in the Production Function," *Journal of Money, Credit, and Banking,* Vol. 7 (November 1975), pp. 535–543.

SAS Institute Inc., *SAS/ETS User's Guide,* Version 5 Edition, (Cary, NC: SAS Institute, 1984), 739 pp.

Short, Eugenie, "A New Look at Real Money Balances as a Variable in the Production Function," *Journal of Money, Credit, and Banking,* Vol. 11, No. 3 (August 1979) pp. 326–339.

Simos, Evangelos, "Real Money Balances as a Productive Input: Further Evidence," *Journal of Monetary Economics,* Vol. 7, (1981), pp. 207–225.

Sinai, Allen, and Houston H. Stokes, "Real Money Balances: An Omitted Variable From the Production Function?" *The Review of Economics and Statistics,* Vol. 54, No. 3, (August 1972), pp. 290–296.

——————, "Real Money Balances: An Omitted Variable From the Production Function? Reply to Comments," *The Review of Economics and Statistics,* Vol. 57, No. 2 (May 1975), pp. 247–252.

——————, "Real Money Balances as a Further Variable in the Production Function—A Further Reply," *Journal of Money and Credit,* Vol. IX, No. 2 (May 1977), pp. 372–373.

——————, "Real Money Balances and Production: A Partial Explanation of Japanese Economic Development for 1952–1968," *Hitotsubashi Journal of Economics,* Vol. 21, No. 2 (February 1981a), pp. 69–81.

——————, "Money and the Production Function—A Reply to Boyes and Kavanaugh," *Review of Economics and Statistics,* Vol. LXIII, No. 2, (May 1981b) pp. 313–318.

——————"Reply to Jensen-Kamath-Bennett", *Eastern Economic Journal,* forthcoming 1989.

Stokes, Houston H., "B34S On Line Help Manual," (January 1989) pp. 141.

Subrahmanyam, Ganti, "Real Money Balances as a Factor of Production: Some New Evidence," *Review of Economics and Statistics,* Vol. 62, No. 2 (May 1980), pp. 280–283.

Sylla, Richard, "Financial Intermediaries in Economic History: Quantitative Research on the Seminal Hypotheses of Lance Davis and Alexander Gerschenkron," in *Recent Developments in Business and Economic History: Essays in memory of Herman E. Kross,* R. Gallman, editor (JAI Press, 1971), pp. 55–80.

U.S. Government, *Economic Report of the President,* United States Government Printing office, Washington, D.C., February 1970.

You, Jong, "Money, Technology, and the Production Function: An Empirical Study," *Canadian Journal of Economics,* Vol. 14. No. 3, (August 1981), pp. 515–524.