COMPETITIVE MARKETS AND AGGREGATE INFORMATION

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A frequently asked question is, how informationally efficient are market prices? In answering this question, economists commonly model the price formation process as one of tâtonnement where agents revise their beliefs until a market-clearing price is established. The Walrasian market thus aggregates all relevant information about an asset and yields a strong-form efficient price. A second approach, consistent with the structure of some modern-day securities markets, is to utilize a competitive dealer framework where traders submit orders to market makers who compete for trades. Using the demand and supply orders from agents, market makers compile information by first aggregating demand and, second, by using it as a conditioning variable to determine future payoffs and price.

This paper constrasts the Walrasian and competitive market structures. The motivation for this analysis is threefold: First, it is common for empiricists to assume rational expectations markets in forming their hypotheses, even for markets such as the NASDAQ, which are clearly not Walrasian. It is important to know that this assumption does not adversely affect the validity of empirical results. Second, economists sometimes employ the notion of the rational expectations equilibrium. By contrasting the two market structures, it is possible to infer that prices formulated in markets are indeed rational, and empirical studies that use market data have the underlying assumption of rationality. Third, the analysis shows that aggregating information is a double-edged sword because on one hand it leads to informative prices, but simultaneously, markets can aggregate information so well that there is no incentive to obtain information in the first place (consistent with Grossman [1976]), a result that implies market failure from an informational perspective.

The results demonstrate that competing market makers aggregate information so that market participants have *ex post* homogeneous expectations and equilibrium price is strong-form efficient. Specifically, it is shown that a competitive dealer system mimics the behavior of a Walrasian rational expectations market when the idiosyncratic error components of private signals are identically and independently distributed, risk-averse agents trade to maximize expected utility of terminal wealth, and aggregate demand is equated to supply.

These results are obtained from the simultaneous development of two equilibrium pricing models. For both economies, the market is defined as one with a single type of agent: informed traders receive private signals regarding the future value of a risky asset where the signal is the true value of the asset plus an idiosyncratic

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error. Although each agent trades to maximize expected utility of end-of-period wealth given his information set, equilibrium price is established through different processes. In a competitive dealer environment, market makers set price as a conditional expectation of aggregate demand formed from market orders. Moreover, under perfect competition between market makers, the expected profit on any trade is zero.

The Walrasian equilibrium is rational expectations in the sense that agents conjecture an equilibrium price that contains an average of private signals as in Grossman [1976]. If competitive dealers can observe aggregate demand, they are observing the full set of market information and the resulting price is efficient. Like Grossman's equilibrium price, competitive dealers form a price that summarizes all information so that it is strong-form efficient and a sufficient statistic for future payoff value.

Given the difference between the information sets but a common level of efficiency, it can be argued that the economic role of competitive dealers is to rationally aggregate market information. Specifically, in a competitive market it is not necessary for agents to conjecture an equilibrium price (that is, to guess what each other agent believes *and* the functional form of price) because market makers summarize the market's information for them. Critical to this result is market makers' ability to "learn" by observing trades. Despite the informational efficiency of price the competitive market structure has a perverse outcome: market makers become "better informed" than any individual with private information. While this is "good news" for supporters of the efficient market hypothesis, it is a disincentive for private individuals to obtain costly information.

The finding that a competitive market maker structure produces an informational efficient pricing system bodes well for a market structure such as the NASDAQ. In that market, traders submit demands for posted bid and ask prices. Market makers subsequently compete for trades by lowering the "ask" price and raising the "bid" price (or narrowing the spread). The ensuing competition promotes trade, and since each trade reveals information, market makers learn about the private set of information until they know almost all that there is to know about future payoffs. That is, they know the vector of private signals and can average across private signals to obtain a noisy estimate of the future payoff value. Even though informed agents know that they reveal their private information when they trade, they submit orders because their information is short-lived. Therefore, whatever private information exists must be exploited in the current period, or will expire and be worthless at the end of the first trading period. (The notion of trading strategically across periods for the purpose of camouflaging private information is discussed by Kyle [1985].)

Consider the concept of informational efficiency in the context of recent market events. During the week of 10-14 April 2000 the NASDAQ suffered a 35 percent correction. Given this event, some relevant questions are: "If the pricing system is so efficient, why did such a large correction occur? Moreover, when did price-earning multiples of, say, 31 become appropriate when historically they have been about 15? Surely, market prices are artificially high and thus inefficient." The logic stemming from this analysis is that the problem is not necessarily with the market structure. Instead, the problem is with the precision of information, or even bias within the information. The existence of an equilibrium price does not preclude the possibility of price under- or overreacting to information. In this economy, agents learn from each other–they do not ask whether there is bias attached to the information, nor do they have the means to detect it. When agents receive high quality signals, and they are not consistent with current price levels, prices will adjust as they did that week. It is not a matter of an informationally inefficient market structure–it is a matter of garbage in, garbage out.

In addition to showing that competitive market makers efficiently aggregate individuals' private information, the following list describes characteristics of that equilibrium:

1. Equilibrium price contains a weighted-average of prior and posterior beliefs about the asset's future value where the posterior is an average of informed agents' private signals. Moreover, the price revision process is partially Bayesian, because supply shocks shift the market-clearing price.
2. The revision parameter in a competitive dealer equilibrium price is related to the proportion of "new" information impounded in price.
3. In a small economy, price can be driven away from its intrinsic value by an average error as in DeLong [1990]. In a large economy price is equated to its intrinsic value since the Strong Law of Large Numbers produces an average error of zero and infinite precision of signals, (that is, agents become perfectly informed).
4. And most importantly, a competitive dealer market structure yields a price that summarizes all relevant information, so that it is a sufficient statistic.

The literature has begun to investigate the quality of information used to value securities and the behavior of market participants. For example, Daniel, Hirshleifer and Subrahmanyam [1998] and Barberis, Shleifer and Vishny [1998] present theoretical models of investor behavior that show how deviations from intrinsic values can occur. As well, the bull market and (less frequently) bear market phenomena have precipitated research on how noisy or even biased information spreads across traders [Baigent and Acar, 2000]. The good news is that, in the long run, prices tend to move toward their intrinsic values [DeBondt and Thaler, 1985 and Fama, 1991]–the bad news is that it might take some time to arrive at that value.

ECONOMIC STRUCTURE

Economic Structure

Grossman [1976] considers an economy with two tradable assets: a risk-free asset with known rate of return and a risky asset with unknown future value. Informed agents, indexed by, $i = 1, 2, \ldots, n,$, allocate their initial wealth, W_{0i}, between the safe and risky asset. The budget constraint is given by $W_{0i} + X_{Fi} + P_0 X_0$, where X_{Fi} is the amount invested in the safe assets, and X_i is the number of shares of the risky asset purchased or sold at current price, P_0. The safe asset yields one plus a risk-free rate of return r (normalized to zero for convenience), and the risky asset returns an uncertain end-of-period payoff, P_i, that is a realization of a random variable, $\tilde{P}_1 \sim N(\bar{P}_1, \sigma^2)$. Random end-of-period wealth, $\tilde{W}_{1i}$, is given by

$$(1) \qquad \tilde{W}_{1i} = X_{Fi} + \tilde{P}_1 X_i$$
$$= X_{oi} + (\tilde{P}_1 - P_0)X_i.$$

At time 0 all agents receive a private information set, I_i, regarding the future value of the risky asset. A portion of the information set is a private signal, y_i, which takes the form of the true payoff value, P_1, plus an idiosyncratic error, ε_i, that is,

$$(2) \qquad y_i = P_1 + \varepsilon_i,$$

where ε_i, ~ $N(0,1)$ and $E(\varepsilon_i \varepsilon_j) = E(\varepsilon_i)E(\varepsilon_j) = 0, \forall i \neq j$.

In equation (2), the presence of ε_i, reflects errors of judgment that prevent agents from knowing perfectly the future value of the risky asset.[1]

The equilibrium prices developed subsequently require that the information sets differ between the two markets. In a competitive dealer market, informed agents have the information set $I_i \equiv y_i$ which produces equilibrium price $P_1^c(y)$. In contrast, agents who participate in a Walrasian market have an information set $I_i \equiv [y_i, P_0^w(y)]$ where $P_0^w(y)$ is the conjectured equilibrium price, and $y \equiv (y_1 ... y_n)$ is the complete set of private signals (or market information). The difference between the two information sets reflects the absence (presence) of a market maker. It is shown that in a competitive dealer market, y is aggregated by market makers so that individuals require only their private signals. Conversely, forming a sufficient statistic price in a Walrasian market requires the addition of conjectured equilibrium price. This result is discussed in greater detail in a later section.

Given their information, I_i, informed traders maximize expected utility of end-of-period wealth. The standard assumption that utility of end-of-period wealth is a negative exponential function, $U(\tilde{W}_{1i}) = -\exp(-\alpha \tilde{W}_{1i})$, and normally distributed end-of-period wealth, $\tilde{W}_{1i}$, is employed.

Assuming conditional normality for terminal wealth, maximizing terminal wealth with respect to demand for the risky asset, X_i, yields a familiar demand equation for the risky asset

$$(3) \qquad X_i = [E(\tilde{P}_1 \mid I_i) - P_0]/[(aVar(\tilde{P}_1 \mid I_i)].$$

Hereafter, equilibrium price carries a superscript "w" or "c" representing Walrasian or competitive market structures.

Equilibrium Prices

In this section it is shown that both a Walrasian rational expectations market structure and a competitive dealer market structure yield informationally identically efficient prices. In this analysis the sufficient statistic characteristic of price, initially shown by Grossman [1976], is a non-trivial result. If informed agents can extract the average signal from equilibrium price, they can learn all there is to know about the future payoff value of a security. In this context, individual signals become extraneous information since agents can form expectations about future value using the average signal, with the same precision as using both private signals and equilibrium price. Price, therefore, is strong-form efficient. Derivation of two equilibrium prices is to follow.

Walrasian Equilibrium

Grossman [1976] presents the following equilibrium model. To form a rational expectations equilibrium it is assumed that agents have an information set. This equilibrium requires that agents conjecture a price

(4a) $$P_0^w = \alpha_0 + \alpha_1 \bar{y} \quad \text{where,}$$

(4b) $$\bar{y} = \sum_{i=1}^{n} y_i \Big/ n \,,$$

(4c) $$\alpha_0 = \left(\bar{P}_1 - n^{-1}\sigma^2 S\right) / \left(n\sigma^2 + 1\right), \text{ and}$$

(4d) $$\alpha_1 = n\sigma^2/(n\sigma^2 + 1).$$

The remainder of the proof requires that price be a sufficient statistic. That is, if informed agents know *all* market parameters, they can extract the average signal from price and form expectations of the future value of the risky asset with the same precision as using both private signals and price. In (4c)-(4d), $a_i = 1\ \forall i = 1, n.\ j.$. Moreover, it is easy to show that substituting for $I_i = \bar{y}$ (the sufficient statistic condition) and (4a)-(4d) in (3), and aggregating demand yields the equilibrium condition $\sum_{i=1}^{n} X_i = S$ where S is the fixed supply of the risky asset and is assumed to be an exogenous shock. The resulting equilibrium price is then

(5) $$P_0^w(y) = \alpha_0 + \alpha_1 \bar{y} = \frac{\bar{P}_1}{n\sigma^2 + 1} + \frac{n\sigma^2 \bar{y}}{n\sigma^2 + 1} - \frac{n^{-1}\sigma^2 S}{n\sigma^2 + 1}$$

Equation (5) shows that equilibrium price is formed through a Bayesian updating process. The first two terms in (5) are a weighted-average of prior $\left(\bar{P}_1\right)$ and posterior $\left(\bar{y}\right)$ information. Further, a_i reflects the proportion of "new" information, $\bar{y}$, embedded in price.

In this equilibrium, price is determined when informed agents make a conjecture about price. The result of this conjecture is *ex post* homogeneous expectations because, after equilibrium price is determined, all agents can use the same quantity, average signal, $\bar{y}$, to form expectations about the future payoff value of the risky asset. Moreover, equilibrium price is fully revealing of informed agents' private signals. That is, agents observe $\bar{y} = P_1 + \bar{\varepsilon}$. Because is unobservable, the future payoff value, P_1, is also unobservable.

Competitive Dealer Equilibrium

Here, equilibrium price is formed in a competitive dealer market. To compute this result, return to the demand equation (3) and let $I_i = y_i$ so that individual *i*'s demand for the risky asset is (6) and where each agent is a price taker

$$X_i = [E(\tilde{P}_1 \mid y_i) - P_0^c]/[(Var(\tilde{P}_1 \mid y_i)]. \tag{6}$$

Excess demand (or order flow) is $\sum_{i=1}^{n} X_i - S$ where supply is assumed to be an exogenous and fixed shock. Market makers use a linear pricing schedule (7) as in Kyle [1985] and Admati and Pfeiderer [1988]. Equation (7) is of the form $P_0^c = E(\tilde{P}_1 \mid n\bar{X}) - n\lambda S' = \bar{P}_1 + n\lambda[\bar{X} - E(\bar{X})] - n\lambda S'$. This construct has two important properties: it is efficient and it is regret-free. The regret-free property implies a zero expected profit on any given trade.

$$\begin{aligned} P_0^c(y) &= \bar{P}_1 + \lambda\left[\sum_{i=1}^{n} X_i - S\right] \\ &= \bar{P}_1 + \lambda\left[\sum_{i=1}^{n} X_i\right] - \lambda S = \bar{P}_1 + n\lambda\bar{X} - n\lambda S' \end{aligned} \tag{7}$$

where, $\lambda = Cov(\tilde{P}_1, \bar{X})/Var(\tilde{P}_1, \bar{X}) = \sigma^2/(\sigma^2 + n^{-1})$ and $S' = n^{-1}S$.

Aggregating demand from (6) and using the terminal condition (7) results in equilibrium price (8a, b)

$$P_0^c(y) = \bar{P}_1 + n\lambda\left(\bar{y} - \bar{P}_1\right) - n\lambda S' \text{ (See appendix for proof.)} \tag{8a}$$

Substituting for $\lambda = \sigma^2/(\sigma^2 + n^{-1})$ and simplifying yields an equilibrium price (8b).

$$P_0^c(y) = \bar{P}_1/\left(n\sigma^2 + 1\right) + n\sigma^2\bar{y}/\left(n\sigma^2 + 1\right) - \sigma^2 S/\left(n\sigma^2 + 1\right) \tag{8b}$$

The Market Makers' Dilemma

The problem confronting each dealer is that they do not have private information about the future value of the risky asset, but must trade with individuals who have private information. Because they are at an informational disadvantage, they must (1) learn from those with private information and (2) provide compensation for their potential losses. Each of these issues is dealt with in turn.

First, consider the process by which dealers learn from the trades of informed individuals. It is suggested that the conditioning variable should be the average demand for the risky asset, $\bar{X}$, because it is more informative than individual trades.

Each individual has signal y_i with variance $\sigma^2(\sigma^2 + 1)^{-1}$. While it is possible to extract individual signals, market makers should prefer to average across demand and extract the average signal, $\bar{y}$, with variance $\sigma^2(\sigma^2 + n^{-1})^{-1}$. The intuition for using average demand is clear—the average signal has greater "precision" than individual signals where precision is defined as the reciprocal of the variance. (Further, it is easy to show that conditioning on $\bar{X}$ is identical to extracting the average signal and using it as a conditioning variable.)

Access to aggregate demand implies that market makers learn as much as possible from informed traders to know about the future payoff value for the risky asset. The perverse result, however, is that market makers become "better informed" than those with private information after performing the necessary calculus. Therein lies the dilemma noted by Grossman [1976]. In a Walrasian market private information is revealed in equilibrium price. Alternatively, in the competitive market structure, market makers learn all that there is to know about payoffs. Consequently, there is no incentive to obtain private information, particularly if it is costly (as in Grossman and Stiglitz [1980].

The second issue raised above is now addressed; market makers must provide compensation for potential losses to informed traders. Note that in equation (7), the revision parameter, λ, has two complementary roles. First, it reflects the amount of new information embedded in price through excess demand. Second, it represents the market makers' compensation for bearing the risk of trading with individuals who have private information (that is, the adverse selection cost). If excess demand is ± 1 unit, then $P_0 = \bar{P}_1 \pm \lambda$. The higher price represents the "ask" and the lower price represents the "bid." Also, if we consider the general form of $\lambda = \sigma^2(\sigma^2 + n^{-1}\sigma_\varepsilon^2)^{-1}$ we can see two interesting effects. First, as the precision of informed traders' information increases (σ_ε^2 decreases), the bid-ask spread increases since $\delta\lambda / \delta\sigma_\varepsilon^2 < 0$. Second, as the number of traders increases, the bid-ask spread increases because the potential losses are larger ($\delta\lambda / \delta n > 0$).

DISCUSSION OF EQUILIBRIUM PROCESSES

Since equations (5) and (8b) are both sufficient statistics for the vector of private signals, the main point of this paper has been proven–there are two ways to form a sufficient statistic price. One is through rational expectations and a second is through a competitive dealer framework. In the latter case, competing market makers perform the task of fully aggregating the market's information set. This task is accomplished by observing demand. Since demand is based on private signals, observing each agent's demand and averaging across individual demands allows for the formation of a noisy estimate of the asset's intrinsic value. That is, through observing demand, market makers learn from those with private information and their adverse selection problem is mitigated. (The intuition of this statement is discussed at length in Glosten [1989] who suggests that when the adverse selection cost is high, transactions costs are correspondingly high to compensate market makers for potential losses on informed trades.)

WALRASIAN PRICE VERSUS COMPETITIVE DEALER PRICE

A comparison of the two equilibrium prices reveals several interesting properties. First, the Walrasian auctioneer has one objective–to clear the market. In contrast, competitive dealers establish an informative price while competing for trades. The motivation for trades occurs because market makers have an informational advantage over any given individual trader. Both market structures provide a sufficient statistic price, however, there is a slight difference between the two prices. Consider the difference in prices shown in equation (9) below:

$$\delta = P_0^c - P_0^w = -[\sigma^2 S/(n\sigma^2 + 1)][(n - 1)/n] < 0. \quad (9)$$

Equation (9) reveals that as long as supply is positive, competitive dealers under-price the Walrasian auctioneer. The intuition of this result is that dealers are competitive and lower prices to attract trades, and is a reflection of their differing roles–the Walrasian auctioneer has the purpose of setting a market clearing price in contrast to dealers who compete for trades. Moreover, the difference in price implies that the bid-ask spread will be lower and competition among dealers has a social benefit (lower transactions costs). In a large economy, the two prices converge so that both are fully revealing of the future payoff value. That is, $\lim_{n\to\infty} \delta \approx 0$, and $P_0^c - P_0^w = P_0$.

The second observation is that both prices reflect the same amount of new information in price so that the efficiency of each price is identical. Kyle [1985] defines price efficiency as the reciprocal of the variance of future payoff conditioned on equilibrium price. That is, efficiency is $\Gamma = [Var(\tilde{P}_1 \mid P_0)]^{-1}$. Then,

$$\Gamma^w = \Gamma^c;$$

$$\{Var(\tilde{P}_1) - [Cov(\tilde{P}_1, P_0^w)^2]/[Var(P_0^w)]\}^{-1} = \{Var(\tilde{P}_1) - [Cov(\tilde{P}_1, P_0^c)^2]/[Var(P_0^c)]\};$$

$$[\sigma^2/(n\sigma^2 + 1)]^{-1} = [\sigma^2/(n\sigma^2 + 1)]^{-1}; \quad (10)$$

$$(n\sigma^2 + 1)/\sigma^2 = (n\sigma^2 + 1)/\sigma^2.$$

Finally, note that for each price the conditional variance is less than the unconditional variance so that price is informationally efficient, that is, $\sigma^2/(n\sigma^2 + 1)21 < \sigma^2$.

FURTHER RESEARCH ISSUES

Other issues of concern are the quality of information, regulatory problems, and the effect of Electronic Communications Networks.[3] Each of these issues is discussed in below.

Quality of Information

The information structure is assumed to be comprised of a private signal plus an idiosyncratic error for each individual market participant (equation 2). Moreover, the idiosyncratic portion of signals (the error terms) are assumed to be identically and independently distributed. This suggests that all individuals have the same "skill" at processing the information they are given, or that all traders have the same precision. In a pragmatic sense, this cannot be true, and would be manifested in demand for the risky asset (equation 3). The greater the precision of the signal, the higher (lower) the demand associated with "good" ("bad") news. Thus, with greater precision, individuals will trade more aggressively. At the limit, when precision is infinite (perfect information), price is equal to its intrinsic value and there is no trade (as in Milgrom and Stokey [1982]).

It could also be assumed that signals are not formed independently. Allowing for a positive correlation of idiosyncratic errors is tantamount to suggesting that individuals systematically bias "good" news and "bad" news. It does not prohibit the formation of either a Walrasian or competitive dealer equilibrium, but it does prevent price from fully converging to its intrinsic value, even under large economy assumptions. A further change that would affect price formation is to allow supply to be random, instead of fixed and known as currently modeled. Changing this assumption does not allow a Walrasian equilibrium to be found, but a competitive dealer market equilibrium still exists.

Regulatory Issues

The Securities and Exchange Commission has established practices that promote "full disclosure;" however, it is clear that as corporations become more complex, accurately reporting the state of the firm is more arduous and subject to interpretation. Certainly Enron Energy Corporation and Global Crossing are examples of how information can be misleading or even withheld from investors with devastating results, but these are not the first of such cases (for example, Sunbeam, Waste Management, Boston Chicken, BreX). Former Federal Reserve Chairman Paul Volcker has suggested that there is a conflict of interest between counseling and auditing a firm, and to remedy the conflict, counseling and auditing should not be undertaken by the same consulting firm. How the suggestion of severing activities will be perceived, especially after the long fight to repeal the Glass-Steagall Act, is currently unknown. What is known is that, when full information is eventually disclosed, markets establish an appropriate price. Unfortunately, the time elapsed before that terminal price is revealed may be substantial, and there could be a significant transfer of wealth.

In addition to auditing and counseling, new firms pose a greater problem because they have no historical information to report. Financial statements (balance sheets, income statements and statements of cash flows) cannot exist for a new firm so that the only source of information is analyst forecasts. Currently, the literature is pointing to biases within that information structure [Dreman and Berry, 1995]. As noted earlier, the efficient market hypothesis only posits that information be embedded in prices quickly and fully. It does not speak to the issue of what information is included, nor to the issue of how well participants regulate themselves in the production of information.

Electronic Communications Networks

Mr. Joel Steinmetz, Senior Vice President of Instinet made the following comments to the Sub-Committee on Commerce, Trade, and Consumer Protection:

> As an important part of its services to clients, Instinet acts as an electronic communications network, or "ECN." ECNs are electronic marketplaces that allow institutional, retail and professional market participants to trade securities directly with one another, as well as with other securities firms. ECNs are operated by companies that are registered with the Securities and Exchange Commission as "broker-dealers" and that are members of the National Association of Securities Dealers. ECNs provide electronic agency brokerage services, meaning that they match customer orders as agents, not principals. In other words, an order is executed if a matching order is immediately available from another customer on the ECN. If no matching order is available, the order is displayed in the electronic order "book" and becomes eligible for execution by orders entered by other subscribers. ECNs typically are compensated by commissions paid by the seller and buyer in each transaction, generally on a per share basis. Another consequence of being an agency broker is that, unlike Nasdaq market makers or exchange specialists, ECNs do not trade for their own accounts. In recent years, the SEC has established special additional regulatory requirements applicable to ECNs, as well as to other "alternative trading systems." [19 December 2001]

ECNs clearly have the ability to bypass market makers and trade with each other. As such, they can participate as Walrasian auctioneers, and by their "shopping," can find the intrinsic value at which the market will clear. Moreover, because their orders may not be directly submitted to a market maker (or group of market makers), their private information is less likely to be revealed. This provides a motivation for obtaining costly private information, and is a solution to the Grossman and Stiglitz [1980] problem where individuals have limited motivation to obtain private information.

CONCLUDING REMARKS

This paper shows that competitive dealers can use average demand to form an equilibrium price identical to that established in a rational expectations (Walrasian) market. The resulting equilibrium price is strong-form efficient and the revision process is partially Bayesian. The role of competitive dealers is to aggregate information *fully and rationally* so that, *ex post*, agents have homogeneous expectations.

The evolution of the capital markets literature has been to first assume that markets are efficient and provide countless studies corroborating that school of thought. When anomalies began to surface, however, a stream of studies (theoretical and empirical) that considered the quality of information and behavior of agents originated. This research mostly assumed rational expectations Walrasian markets and, as a result, abstracted itself from the reality of different market structures. The results of this analysis have implications for the study of markets because they show that the Walrasian market assumption of prior studies is not absurd since an equally efficient price can be derived in a system such as the NASDAQ. The inefficiencies in the market therefore, are not necessarily a direct result of the market structure–rather, they can be a result of the quality of information being used to form prices, or the behavior of market participants.

APPENDIX

<u>Expectation of payoff value by conditioning on a sufficient statistic:</u>

Using the sufficient statistic $\bar{y}$, agents can form an expectation of future price as

$$\begin{aligned} E(\tilde{P}_1 \mid \bar{y}) &= E(\tilde{P}_1) + \{[Cov(\tilde{P}_1, y)]/[Var(\bar{y})]\}[\bar{y} - E(\bar{y})] \\ &= \bar{P}_1 + \left\{\sigma^2 / \left(\sigma^2 + n^{-1}\right)\right\}\left[\bar{y} - \bar{P}_1\right] \\ &= \bar{P}_1 / \left(n\sigma^2 + 1\right) + n\sigma^2 \bar{y} / \left(n\sigma^2 + 1\right) \end{aligned}$$

as shown in (5).

<u>Preliminary results and definition of terms:</u>

$$E(\tilde{P}_1 \mid y_i) = (\bar{P}_1 + \sigma^2 y_i)/(\sigma^2 + 1)$$

$$Var(\tilde{P}_1 \mid y_i) = \sigma^2/(\sigma^2 + 1) \text{ where } \tilde{P}_1 \sim N(\bar{P}_1, \sigma^2)$$

For notational simplicity define the following

(i) $h^{-1} = \sigma^2/(\sigma^2 + 1)$, (ii) $h_\sigma = 1/\sigma^2$,

$$\therefore E(\tilde{P}_1 \mid y_i) = h_\sigma h^{-1} \bar{P}_1 + h^{-1} y_i.$$

Also note that

(i) $(h_\sigma + 1)\, h^{-1}$ (ii) $\sigma_{\bar{y}}^2 = \sigma^2 + n^{-1}$ and (iii) $n^{-1} \sum_{i=1}^{n} y_i = \bar{y}$ so that $n\bar{y} = \sum_{i=1}^{n} y_i$

End of preliminary results.

Formal Proof of Equilibrium Price (8a, b):

From (6) and substituting the preliminary results from above, summing over all yields $\sum_{i=1}^{n} X_i = \beta\left[nh_\sigma h^{-1}\bar{P}_1 + h^{-1}\sum_{i=1}^{n} y_i - nP_0^c\right]$.

Using $n\bar{y} = \sum_{i=1}^{n} y_i$ aggregate demand for informed traders is

$$\sum_{i=1}^{n} X_i = n\beta\left[h_\sigma h^{-1}\bar{P}_1 + h^{-1}\bar{y} - P_0^c\right].$$

Dividing by n traders yields average demand for the risky asset

$$\bar{X} = \beta\left[h_\sigma h^{-1}\bar{P}_1 + h^{-1}\bar{y} - P_0^c\right].$$

Further,

(i) $$E\left(\bar{X}\right) = E\left\{\beta\left[h_\sigma h^{-1}\bar{P}_1 + h^{-1}\bar{y} - P_0^c\right]\right\} = \beta\left[\bar{P}_1 - P_0^c\right],$$

(ii) $$Cov(\tilde{P}_1, \bar{X}) = Cov(\tilde{P}_1, \beta(h_\sigma h^{-1}\bar{P}_1 + h^{-1}\bar{y} - P_0^c)] = \beta h^{-1}\sigma^2,$$

(iii) $$Var\left(\bar{X}\right) = \left(\beta h^{-1}\right)^2 \sigma_{\bar{y}}^2, \text{ and}$$

(iv) $$\sigma_{\bar{y}}^2 = \sigma^2 + n^{-1}.$$

Therefore, $$\lambda = Cov(\tilde{P}_1, \bar{X})/Var(\bar{X}) = \sigma^2/(\sigma^2 + n^{-1}).$$

Equation (7) states that $P_0^c = \bar{P}_1 + n\lambda\left[\bar{X} - E\left(\bar{X}\right)\right] - n\lambda S'$.

Substituting for $E\left(\bar{X}\right) = \beta\left[\bar{P}_1 - P_0^c\right]$, the following is obtained

$$P_0^c = \bar{P}_1 + n\lambda\left[\bar{X} - \beta\left(\bar{P}_1 - P_0^c\right)\right] - n\lambda S',$$
$$P_0^c\left(1 - n\lambda\beta\right) = \bar{P}_1\left(1 - n\lambda\beta\right) + n\lambda\bar{X} - n\lambda S', \text{ and}$$
$$P_0^c = \bar{P}_1 + \gamma\bar{X} - \gamma S',$$

where, $\gamma = n\lambda/(1 - n\lambda\beta)$.

Substituting for $P_0^c = \bar{P}_1 + \gamma\left[\bar{X} - S'\right]$ in average demand where informed traders are price takers, it follows that average equilibrium demand for the risky asset is

$$\bar{X} = \beta\left[h_\sigma h^{-1}\bar{P}_1 + h^{-1}\bar{y} - \left(\bar{P}_1 + \gamma\bar{X} - \gamma S'\right)\right]$$
$$= \left(1 + \gamma\beta\right)^{-1}\left\{\beta h^{-1}\left(\bar{y} - \bar{P}_1\right) + \gamma\beta S'\right\}.$$

Next, substitute for $\bar{X}$ and γ in P_0^c which gives price in terms of signals as in the Walrasian market. That is, (8a) obtains

$$\begin{aligned} P_0^c &= \bar{P}_1 + \left[n\lambda/(1-n\lambda\beta)\right]\left[(1+\gamma\beta)^{-1}\left(\bar{y}-\bar{P}_1\right)+\gamma\beta(1+\gamma\beta)^{-1}S'\right]-\gamma S' \\ &= \bar{P}_1 + \left[n\lambda/(1-n\lambda\beta)\right]\left[(1-n\lambda\beta)\beta h^{-1}\left(\bar{y}-\bar{P}_1\right)+(1-n\lambda\beta)\gamma\beta S'\right]-\gamma S' \\ &= \bar{P}_1 + n\lambda\left(\bar{y}-\bar{P}_1\right)+n\lambda\gamma\beta S'-\gamma S' \\ &= \bar{P}_1 + n\lambda\left(\bar{y}-\bar{P}_1\right)-n\lambda S' \end{aligned}$$

The remainder of the proof requires inputting and simplifying to (8b).

NOTES

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1. Baigent [1997] shows that the assumption of agents' idiosyncratic error terms as i.i.d. is not necessary for price to be a sufficient statistic. Under the condition that the correlation between agents' errors is $\rho > 0$, price is still a sufficient statistic; however, convergence to a fully revealing price is not possible.
2. The following information was downloaded from http://www.nasdaq.com. In this excerpt, the competitive process of NASDAQ market makers and the formation of prices is described. See below.
   The Nasdaq Difference - Market Makers
   Essential to Nasdaq's market structure, Market Makers are independent dealers who actively compete for investor orders by displaying quotations representing their buy and sell interest—plus customer limit orders—in Nasdaq-listed stocks. Each Market Maker has equal access to Nasdaq's trading system, which broadcasts their quotations simultaneously to all market participants. By standing ready to buy and sell shares of a company's stock, Market Makers provide to Nasdaq-listed companies a unique service. The result of their combined sponsorship helps meet investor demand and creates an environment of immediate and continuous trading. Currently, more than 500 market making firms provide capital support for Nasdaq-listed stocks. All are required to:
   · Disclose their buy and sell interest by displaying two-sided quotes in all stocks in which they choose to make a market.
   · Display both quotes and orders in Nasdaq, in compliance with the Securities and Exchange Commission's (SEC) Order Handling Rules.
   · Honor their quoted prices and report trading in a timely manner. Failure to do so can lead to disciplinary action.
3. Thanks is given to an anonymous referee for suggesting the inclusion of this discussion.

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