Private Resolution of Production Externalities

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Abstract

Using the techniques of game theory and integer maximization, this paper attempts to provide a truly private characterization of externality conflict resolution. With the explicit introduction of processing and control options in reciprocal cases, we avoid the assignment of liability. Depending on the degree of cooperation between the two involved parties, we display various payoff results. The latter range from the noncooperative Nash solution to the fully cooperative Pareto solution. The intermediate stage of pre-game strategy restriction appears to have relevance in cases of partial cooperation.

A comparison is made with the traditional Pigouvian and Coasian literatures with the apparent result that our technique avoids some of the conceptual problems and employs strategies which are readily available in the "real world."

The multi-faceted nature of the economic externalities problem has been carefully analyzed by E. J. Mishan (1971) and J. M. Buchanan (1973). Together they emphasize that there are conceptual differences between the study of externalities as sources of disruption of the competitive market-Pareto duality through the use of "mechanical" models and the institutional and behavioral aspects which prevent resolution of externality-related conflicts in some efficient manner. The Pigouvian debate makes some of these aspects clear.

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The traditional Pigouvian approach is basically a general equilibrium apparatus. Profit-maximizing behavior is taken as given and well-defined. Firms merely incorporate taxes or subsidies into their pricing structures, and there is no need to look further at the behavior of the individuals.

R. H. Coase (1960) introduced some flexibility (in what appeared to him to be a one-sided, producer-oriented approach) with the notions of merger, reciprocity and bargaining. His work was an attempt to synthesize the allocative and institutional aspects of externality situations. To effect this, the analysis was restricted to two-party cases. Buchanan and W. C. Stubblebine (1962) and O. A. Davis and A. B. Whinston (1962) continued the use of two-party examples. Buchanan and Stubblebine emphasized that a unilateral tax (or subsidy) may cause a movement away from a Pareto equilibrium. Davis and Whinston questioned the generality of the Pigouvian approach by postulating that it could be applied smoothly only in the presence of separability.

The controversy has had a recent revival. W. J. Baumol (1972) decried the use of two-party situations under the belief that most "real-world" externality problems involve more than two parties. Through the manipulation of a simple competitive-Pareto model, he reinforces Pigou's proposal for taxes and subsidies to achieve the social output levels. The analysis leaves out important institutional and behavioral considerations that are usually available in the two-party models.

A major result of the two-party approach has been the Coase Theorem that with costless transactions, liability has no effect on resource allocation. The result was dependent upon the explicit recognition by firms that foregone bribes are costs. J. R. Marchand and K. P. Russell (1973) and A. Gifford and C. C. Stone (1973) formally specified this behavior and reinforced the Coase Theorem.

It is the intent of this paper to examine some of the issues involved in these papers and to propose an alternative way of looking at the problem. To achieve a restatement of the problem, we use some basic tools of game theory and integer maximization.

I. The Formal Structure

In order to establish a frame of reference, from the outset we shall restrict our attention to negative technological externalities between firms. Ignoring the important problems involved with externality-induced nonconvexities, we shall begin with the strictly convex and differentiable unconstrained profit model initiated by Davis and Whinston (1962) and utilized by Marchand and Russell and Gifford and Stone for perfectly competitive firms.

A. The Logical Evolution

Let us briefly reconstruct the precise nature of the externality debate since Pigou. Consider at this point a unilateral externality situation where the profits of the two firms are given as

$$\pi_1 = p_1 q_1 - C_1(q_1) \quad (1)$$

$$\pi_2 = p_2 q_2 - C_2(q_1, q_2). \quad (2)$$

The p_i, q_i and C_i are respectively prices, outputs and cost functions. The externality is represented by the presence of q_1 in C_2. Denote the private solutions to separate maximizations of problems (1) and (2) as $\hat{q}_1$ and $\hat{q}_2$. With these outputs, Firm 1 earns its highest profit $\pi_1(\hat{q}_1)$, and Firm 2 earns its lowest conditional profit $\pi_2(\hat{q}_1, \hat{q}_2)$. Firm 2 would be best off in the absence of Firm 1 producing q_2^0 for a profit of $\pi_2(q_2^0)$.

The traditional Pigouvian approach would propose that Firm 1 be taxed in an amount necessary to insure that the Pareto solution q_1^* (and hence q_2^*) is produced, maximizing the sum of (1) and (2). According to Coase, it is as economically feasible for the recipient to bear the burden. From the Coase Theorem it follows that (q_1^*, q_2^*) will be reached regardless of where liability is laid. As mentioned, the device by which Coase obtains symmetry with the case of damage payments for the externality-producer is the foregone bribe. If Firm 1 produces at $\hat{q}_1$, it loses what Firm 2 would have paid it to reduce output. Marchand and Russell modify the Coase Theorem to read that in a world of zero transactions costs, if either damages or a foregone bribe is explicitly introduced as a cost in the producer's profit function, then only under separability can we say that (q_1^*, q_2^*) will result regardless of liability.

Separability came to the externalities literature from mathematical programming. Multivariate functions that could be written as chains of functions of each of the variables separately could be treated as a simpler set of independent subproblems. Davis and Whinston used this concept to show that the traditional Pigouvian tax/subsidy schemes were more mathematically tractable when the marginal decisions of a recipient do not depend on the other's output. Specifically,[1] this means that $C_2(q_1, q_2)$ may be written $C_2(q_1) + d_2(q_2)$.

In theoretical models, the choice between separable and nonseparable cost functions is somewhat arbitrary. In what we may consider real-world externality situations, separability becomes an empirical question based on the engineering characteristics of the firms' processes and the physical and biochemical nature of the pollutants. We cannot make general

[1] See R. Dusansky and P. J. Kalman (1972) for an extension of the Davis-Whinston usage.

statements about precisely which situations exhibit separable behavior.[2]

Gifford and Stone generalized the Marchand-Russell result in the following manner. Let q_1, q_2 represent any outputs of Firms 1 and 2 such that $0 < q_1 < \hat{q}_1$ and $\hat{q}_2 < q_2 < q_2^0$. Suppose Firm 1 is not liable. Then $\pi_1(\hat{q}_1) - \pi_1(q_1)$ is the least Firm 1 would accept to produce any q_1. The most Firm 2 would pay is $\pi_2(q_1, q_2) - \pi_2(\hat{q}_1, \hat{q}_2)$. With liability on Firm 1, Firm 2 would accept at least $\pi_2(0, q_2^0) - \pi_2(q_1, q_2)$. Firm 1 would pay at most $\pi_1(q_1)$. The resulting bargaining payment must be a linear combination of the limits in each case. Adding these payments to and subtracting them from the profit functions of the payee and payor respectively, Gifford and Stone show that maximizing these adjusted expressions yields the Pareto optimum regardless of liability.

The implication of the Gifford-Stone result was that Marchand and Russell's work was incomplete. The latter had used increased costs and not decreased revenue as damages in the recipient's adjusted profit function. The difference, however, only occurs in the case of nonseparability. With separability, externalities do not alter output, and revenue remains constant.

The strictly noncooperative situation under permissive law[3] yields profits $\pi_1(\hat{q}_1) + \pi_2(\hat{q}_1, \hat{q}_2)$. The strictly noncooperative situation under prohibitive law has Firm 1 shutting down. Total profits are $\pi_1(0) + \pi_2(0, q_2^0)$. The social optimum yields the highest sum $\pi_1(q_1^*) + \pi_2(q_1^*, q_2^*)$.

In the sequel we will do the following: (1) introduce reciprocal models for the separable and nonseparable cases in which the legal structure is not (but can be) a principal feature, and where explicit compensation for damages and enforcement are irrelevant; (2) introduce control and processing alternatives and a bargaining apparatus which functions without initial conditions set by law; (3) give examples; and (4) compare our results with those mentioned above.

B. The Separable Case

In an effort to broaden the problem discussed above, we shall deal with the more general case of reciprocal (two-directional) externalities. This, of course, includes the unilateral situation as a special case. In this subsection, we shall assume that externalities cause separable effects. The cost functions for both firms can be written

$$C_i(q_i, q_j) = c_i(q_i) + d_i(q_j),\ i,j = 1,2; i \neq j. \quad (3)$$

We shall designate $d_i(q_j)$ as full damages imposed by Firm j on i. Avoidance of these damages can occur either from processing by i of the medium containing the externality received from j, or from control of the discharges by j. There are six orderings of the costs associated with full damages, control and processing. Except for very low levels of production, "source-control" costs are assumed to be less than "sink-processing" costs, and the latter are assumed to be less than full damages to the recipient. The fixed costs associated with the former two cause them to exceed full damages for small output levels. We shall see later that if full damages are less than one or both of the other costs, then processing and/or control become irrelevant. If control costs exceed processing costs, our model below has Pareto optimal private behavior at the outset. In either case, the techniques below need not be employed. Many cases where the recipient bears the full burden of the externality exhibit the cost behavior we have assumed here.[4] Let the control costs i be given by

[2] A comment on this shortcoming by an anonymous referee is appreciated. Our defense is that although we cannot express the conditions under which to expect separability, we know that all situations are either separable or nonseparable. We examine the implications of both.

[3] A term used by Mishan (1967) to depict a vacuous legal code toward externalities.

[4] Two possible phenomena that may lead to this ordering are diffusion of contaminants and/or the formation of chemical compounds of greater toxicity or corrosiveness than original discharges.

$$CC_i = f_i(q_i),\ i = 1,2, \tag{4}$$

and the processing costs by[5]

$$PC_i = g_i(q_j),\ i,j = 1,2;\ i \neq j. \tag{5}$$

We assume that f_i and g_i are also convex.

The expanded problem of Firm i ($i,j = 1,2;$ $i \neq j$) may be written

$$\text{maximize } \pi_i = p_i q_i - c_i(q_i) - \alpha_i[d_i(q_j)] - \beta_i[f_i(q_i)] - \gamma_i[g_i(q_j)] \quad q_i, \beta_i, \gamma_i \tag{6}$$

subject to

$$\alpha_i, \beta_i, \gamma_i = 0 \text{ or } 1 \tag{7}$$

and

$$\alpha_i + \gamma_i = 1. \tag{8}$$

The composite constraint (7) refers respectively to Firm i bearing full damages or not, and processing Firm j's externality it receives or not. (8) states that full damages and processing are mutually exclusive. We assume that $c_i'(q_i) = \partial c_i(q_i)/\partial q_i$, $f_i'(q_i) = \partial f_i(q_i)/\partial q_i$ and $\partial g_i(q_j)/\partial q_j$ are all non-negative.

We can now define a strategy for Firm i as the two-tuple (β_i, γ_i) referring to the choices of whether or not to control its own discharge and whether or not to process the other firm's discharges. The set of possible strategies is $S_i = \{(1,1),(1,0),(0,1),(0,0)\}$, $i = 1,2$. The necessary and sufficient conditions for private profit maximization for the use of (1,1) or (1,0) by Firm i are

$$P_i - c_i'(q_i) - f_i'(q_i) = 0 \tag{9}$$

and for (0,1) or (0,0) is

$$P_i - c_i'(q_i) = 0. \tag{10}$$

Suppose that Firms 1 and 2 face each other in a two-player game with strategy sets S_1, S_2. We may consider each strategy (β_i, γ_i) as an offensive-defensive strategy. The offense is control while the defense is processing. The normal form of a subgame of i's offense against j's defense is given by Table 1. FPC_j represents fixed processing costs implicit in (5). d_j reflects full damages to j as in (3). Costs appear instead of profits as the payoffs in Table 1 because the other defensive and offensive parts of each player's strategies are needed to calculate the latter. A purely noncooperative solution to the subgame (and its counterpart with the roles of i and j interchanged) following the example of John Nash (1964), occurs where each firm chooses a strategy which maximizes its payoff, given what the other player has chosen. This occurs at $\beta_i = 0$, $\gamma_j = 1$ yielding $(0, -PC_j)$ and $\beta_j = 0$, $\gamma_i = 1$ yielding $(0, -PC_j)$. The intuition of the Nash strategies (0,1) is that each firm on the offense will avoid controlling its own wastes, but as a defense will process the other's discharges to avoid full damages. $\beta_i = 0$ is a "dominant" strategy for the offense, hence processing becomes the rational choice of the defense. The sum-of-profits social optimum occurs with the use of (1,0) by each firm. There is no a priori reason for profit-maximizing firms to control in order that the other firm need not process. Herein lies the social problem.

TABLE 1.
Offensive-Defensive Subgame

Strategies	$\gamma_j = 1$	$\gamma_j = 0$
$\beta_i = 1$	$(-CC_i, -FPC_j)$	$(-CC_i, 0)$
$\beta_i = 0$	$(0, -PC_j)$	$(0, -d_j)$

[5] It may strike the reader that the externality, say effluent z_j produced by j, should be counted in processing instead of q_j. If $z_j = h_j(q_j)$, g_i is merely written as a composite function.

C. The Nonseparable Case

As noted in Section I. A, the difference in the results of Marchand and Russell and Gifford and Stone lies mainly in the difference between separable and nonseparable external effects. The latter imply that in their approach no distinction need be made. By restricting attention to the unilateral case, they avoid mutual interdependency of the private decisions. The social decisions made after considering bargaining, according to their equations [6], [7], and [12], [13] and our notation in (1) and (2) are

$$p_1 = C_1'(q_1) - C_{21}(q_1, q_2) \tag{11}$$

$$p_2 = C_2'(q_1, q_2) \tag{12}$$

where

$$C_{21}(q_1, q_2) = \partial C_2(q_1, q_2)/\partial q_1 > 0.$$

(11) and (12) are mutually interdependent differential equations, and the Gifford-Stone result depends, as in our case, upon whether or not a solution exists.

When externalities cause nonseparable effects, (3) does not hold. (4) and (5) would remain the same, except that the latter will be functionally related to each firm's own output. Full damages would now include the change in revenue experienced by the recipients. The question recurs as to what should be the reference point to estimate the decreased revenue. Following Marchand and Russell and Gifford and Stone, we choose the isolation solution.

Since (3) does not hold, we must deal with function $C_i(q_i, q_j)$ for each i. Consider, however, that if we are able to avoid external effects we also avoid nonseparability. Hence, let us define $D_i(q_i)$ to be the strictly convex cost function in the absence of the external effect. If $\bar{q}_i$ is i's optimum output in this case, we can now write its profit problem in the more extended version for $i,j = 1,2; i \neq j$;

$$\underset{q_i, \beta_i, \gamma_i, \delta_i}{\text{maximize}}\ \pi_i = p_i q_i - \delta_i[D_i(q_i)] - \alpha_i[p_i(\bar{q}_i - q_i) + C_i(q_i, q_j) - D_i(\bar{q}_i)] - \beta_i[f_i(q_i)] - \gamma_i[g_i(q_i)] \tag{13}$$

subject to

$$\alpha_i, \beta_i, \gamma_i, \delta_i = 0 \text{ or } 1 \tag{14}$$

$$\alpha_i + \gamma_i = 1 \tag{15}$$

$$\alpha_i + \delta_i = 1 \tag{16}$$

The constraint (14) is similar to (7) except that we now have to choose between the full damage nonseparable term and the avoidance cost function. Hence, we include δ_i. (15) and (16) together imply that processing precludes full damages and insures the presence of the avoidance cost function. As a consequence of nonseparability, each q_j is related to each q_i. As we see, for example, in a statement of the first-order conditions for i's four strategies against j's (0,0),

$$p_i - D_i'(q_i) - f_i'(q_i) = 0 \tag{17}$$

$$2p_i - C_i'(q_i, q_j) - f_i'(q_i) = 0 \tag{18}$$

$$p_i - D_i'(q_i) = 0 \tag{19}$$

$$2p_i - C_i'(q_i, q_j) = 0 \tag{20}$$

$$i,j = 1,2;\ i \neq j,$$

the functional relationships of the firms outputs vary with the type of offensive-defensive strategy each employs. (18) and (20) depend explicitly on the other's output. (17) and (19) depend implicitly on the latter as they are solved simultaneously with equations corresponding to the opponent's strategy.

Due to the functional interrelatedness, we must make the results of this section more conditional than with separability. I.e., in cases where solutions exist for the interdependent simultaneous differential equations given above, we can make comments about the Nash and Pareto solutions to a two-firm game corresponding to the separable case.

II. Private Conflict Resolution

The traditional predicament in the literature has been to reconcile the extremely private positions of the two parties in the externality conflict-of-interest. The bargaining process as depicted by Gifford and Stone in the case of liability is a continuum of alternative solutions among the noncooperative ones. The proposals in this section are made without consideration of liability. Once the mechanism of cooperation is defined, we may examine the alternative cases of no partial and full cooperation in the context of the processing and control decisions described above. We shall call the bargaining apparatus pre-game strategy restriction (PGSR), and it will have special meaning in the case of partial cooperation.[6]

Let us suppose, then, that Firms 1 and 2 above enter pre-game negotiations. The tools for such negotiations are called proposals. A *proposal* by a firm is a specific proper subset of its total strategy set. If two proposals are made and approved, the players form a contract. A *contract* is a binding agreement listing the proposed strategy sets and some net payment accruing to either of the firms for making an "over-compensating" strategy restriction.[7] These contracts obviously must be binding to be at all meaningful.

The criterion by which a player evaluates proposals by the opponent is favorability. A proposal is considered *favorable* to a player if the worst it can do given this proposal including any expected net payment is at least as good as that in the original game. Only favorable proposals are considered in contracts.

PGSR can be represented very simply in the above externality games. Each firm can choose not to process and/or not to control. Let us define $T_i^1 = \{(1,1),(1,0)\}$ and $T_i^2 = \{(0,1),(0,0)\}$, $i = 1,2$, accordingly.[8] Each strategy set pair (T_i^r, T_j^s) $r,s = 1$ or 2, $r \neq s$, represents the normal form of a subgame. Let a payoff function M map these subgames into solutions for each i where

$$M^i(T_i^r, T_j^s) = m^i_{rs}. \tag{21}$$

We also define a_{ij} to be the net payment (taken only out of current profits) from Firm i to Firm j. A contract may be written $(T_i^r, T_j^s; a_{ij})$. The conditions for a contract are such that the payment made to the "sacrificing" player must exceed its loss and be less than the gainer's benefit from agreeing to play a restricted subgame.[9]

A. Conflict or Cooperation

(1) *No Cooperation.* When the players will not cooperate by choice, by law or because of prohibitive costs of negotiation (unlikely in the two-firm case), no contracts are made. The overall Nash solution gotten from the offensive-defensive games given by Table 1 for both firms is the result. We shall denote this solution as (N^1, N^2). As noted above, each player expects that the other will not control, and it will process to avoid full damages. In general, this solution will not be the socially optimal solution.

(2) *Partial Cooperation.* Given that the Nash solution (N^1, N^2) is the worst that a firm can expect to do, we give the conditions under which proposals may become contracts. These are listed in Table 2.

By making contracts, the players mutually agree to restrict their strategies and decrease the uncertainty of each other's actions. The resulting subgame is then played. The players will settle in general on the Nash solution to this restricted game. If the contract includes (T_1^2, T_2^2), we will essentially revert to (N^1, N^2). In any movement from this, Firm i would vie for the subgame (T_i^2, T_j^1), reaping the benefit of j's control, $i,j = 1,2, i \neq j$. If the contract includes (T_i^1, T_j^1) the resulting selfish Nash solution will end up being the Pareto optimal solution. Essentially, this is the goal of all approaches to the control of externalities. The drawback of partial cooperation is that although we can achieve the subgame which by rational behavior guarantees us the social solution, we cannot be sure that this subgame will be chosen. Any result in this case must be conditional upon the probability of the right subgame being played. The strictly noncooperative solution is not subject to this uncertainty along with the solution in the third case.

(3) *Full Cooperation.* When the firms have no impediments to complete agreement, proposals will reduce to the Pareto optimal strategies (1,0). If the payoffs from these strategies are denoted by (P^1, P^2), we know that $P^1 + P^2$ under our assumptions is a strict maximum. Since the noncooperative alternative (N^1, N^2) is the basis for bargaining, $P^1 + P^2 > N^1 + N^2$ implies both can be made better off. This means that by contracting to control, each firm can avoid processing as well as full damages. Both can be directly better off because of the move, or any loser may be adequately compensated to make the move. The actual payments depend on the profit functions, specifically on the costs of controlling one's own wastes relative to processing the other's wastes.

TABLE 2.
Conditions for Contracts

Proposals	*Conditions for Contract (i, j = 1,2; i ≠ j)*		
(T_1^1, T_2^1)	$(N^j - m^j_{11}) < a_{ij} <$	$m^i_{11} - N^i$	if $N^i \geq 0$
		m^i_{11}	if $N^i < 0$
(T_1^1, T_2^2)	$(N^j - m^j_{12}) < a_{ij} <$	$m^i_{12} - N^i$	if $N^i \geq 0$
		m^i_{12}	if $N^i < 0$
(T_1^2, T_2^1)	$(N^j - m^j_{21}) < a_{ij} <$	$m^i_{21} - N^i$	if $N^i \geq 0$
		m^i_{21}	if $N^i < 0$
(T_1^2, T_2^2)	None		

B. A Separable Example

Assume that the cost functions in (3) are of the form

$$C_i(q_i, q_j) = A_i q_i^2 + B_i + E_i q_j \tag{22}$$
$$i,j = 1,2; \quad i \neq j.$$

The terms reflect variable, fixed and linear externally-imposed costs. Let control and processing costs be

$$CC_i = a_i + b_i q_i \tag{23}$$
$$PC_i = d_i + e_i q_j \tag{24}$$
$$0 < b_i, e_i < 1; \quad i,j, = 1,2; \quad i \neq j.$$

(7) may now be written as

$$\max_{q_i, \beta_i, \gamma_i} \pi_i = p_i q_i - A_i q_i^2 - B_i - \alpha_i [E_i q_j] - \beta_i [a_i + b_i q_i] - \gamma_i [d_i + e_i q_j] \tag{25}$$

subject to constraints (7) and (8). (9) and (10) become

$$p_i - 2A_i q_i - b_i = 0 \tag{26}$$
$$p_i - 2A_i q_i = 0 \tag{27}$$
$$i,j = 1,2; \quad i \neq j.$$

Due to separability, (26) and (27) may be solved independently.

Consider the following set of data: $p_1 = 10$, $A_1 = .25$, $B_1 = 4$, $E_1 = 5$, $a_1 = 8$, $b_1 = .50$, $d_1 = 16$, $e_1 = .90$; $p_2 = 8$, $A_2 = .50$, $B_2 = 3$, $E_2 = 4$, $a_2 = 6$, $b_2 = .50$, $d_2 = 20$, $e_2 = .67$. These num-

[6] PGSR is motivated in part by each of three related notions. In Davis and Whinston (1962) it was felt that an important factor in cases of reciprocal externalities is the uncertainty involved when the profits (separable case) and the actual outputs (nonseparable case) of each firm depends on the activities of the other. Robert Rosenthal (1970), in developing his theory of games in effectiveness-function form, ciritczed the inherent lack of conditionality in the traditional Von-Neumann-Morgenstern concept of strategies and solutions. He proposed that we consider the notion of "actions" which is more general than that of strategies. To us, one such action is PGSR. Finally, we read in R. D. Luce and H. Raiffa that, "We cannot help feeling that the realistic cases actually lie in the hiatus between strict non-cooperation and full cooperation." (1957, p. 105)

[7] This refers to the fact that some of the compensation is being made "in-kind" or by a corresponding strategy restriction. The magnitude of the payoffs, as in all bargaining situations, depends on the relative power of the players.

[8] If the strategy subsets are singletons, then the game is simply determined.

[9] As pointed out by David Starrett and Starrett and Richard Zeckhauser in a discussion of nonconvexity, firms may be forced to shut down when externalities are excessive. This could occur when negative profits are greater than fixed costs. We do not explicitly introduce the shutdown alternative since the large negative payoffs result only if the recipient continues to exist. If the latter decides to shut down, our game evaporates under the condition that firms not operating at positive outputs cannot claim an identity for bargaining purposes.

bers were chosen according to the assumption above about full damages, control and processing costs.[10] The profit maximizing payoffs are given by Table 3, rounded to the nearest integer. Implicit in the calculations is the fact that $\alpha_i = 0$ and $e_i q_j = 0$ when j controls.

The overall Nash solution yields (75,–4). The best that Firms 1 and 2 can do is 96 and 29 respectively. The worst each firm can do is 36 and –61. The Pareto optimum yields (72,19). According to the above, this occurs if as a result of PGSR the firms play the game (T_1^1, T_2^1) or from full cooperation.

TABLE 3.
The Separable Case Payoff Matrix

Strategies	*(1,1)*	*(1,0)*	*(0,1)*	*(0,0)*
(1,1)	(66,–1)	(66,19)	(58.9)	(58,29)
(1,0)	(72,–1)	(72,19)	(36,9)	(36,29)
(0,1)	(80,–14)	(80,–61)	(74,–4)	(74,–51)
(0,0)	(96,–14)	(96,–61)	(56,–4)	(56,–51)

C. A Nonseparable Example

Consider a slightly more complicated problem where the respective cost functions are

$$C_1(q_1, q_2) = A_1 q_1^2 q_2 + B_1 \qquad (28)$$

$$C_2(q_1, q_2) = A_2 q_2^2 + B_2 + E_2 q_1 q_2. \qquad (29)$$

Let the avoidance cost function discussed in Section I.C. be

$$D_i(q_i) = A_i q_i^2 + B_i \qquad (30)$$

for each i. For Firm 1, (13) may be written

$$\underset{q_1, \beta_1, \gamma_1, \delta_1}{\text{maximize}}\ \pi_1 = p_1 q_1 - \delta_1 [A_1 q_1^2 + B_1] - \alpha_1 [p_1(\bar{q}_1 - q_1) + A_1 q_1^2 q_2 - A_1 \bar{q}_1^2] - \beta [a_1 + b_1 q_1] - \gamma_1 [d_1 + e_1 q_2] \qquad (31)$$

subject to (14)–(16). For Firm 2, the analogous problem is

$$\underset{q_2, \beta_2, \gamma_2, \delta_2}{\text{maximize}}\ \pi_2 = p_2 q_2 - \delta_2 [A_2 q_2^2 + B_2] - \alpha_2 [p_2(\bar{q}_2 - q_2) + A_2 q_2^2 + E_2 q_1 q_2 - A_2 \bar{q}_2^2] - \beta_2 [a_2 + b_2 q_2] - \gamma_2 [d_2 + e_2 q_1] \qquad (32)$$

subject to (14)–(16).

The first-order conditions derived from (31) and (32) depend on the actual strategies used. Recall that if one firm processes while the other controls, only fixed processing costs are experienced. Sample equations are given for

(0,1) (1,0)

$$p_1 - 2A_1 q_1 = 0 \qquad (33)$$

$$2p_2 - 2A_2 q_2 - E_2 q_1 - b_2 = 0 \qquad (34)$$

and

(1,0) (0,1)

$$2p_1 - 2A_1 q_1 q_2 - b_1 = 0 \qquad (35)$$

$$p_2 - 2A_2 q_2 = 0. \qquad (36)$$

The interdependencies, different for different strategies, can be seen in (33), (34) and (35), (36). In cases where Firm i processes or Firm j controls, i's output decision is independent of j's.

In a manner analogous to the separable example, we supply the following example data: $p_1 = 10$, $A_1 = .25$, $B_1 = 14$, $a_1 = 6$, $b_1 = .20$, $d_1 = 20$, $e_1 = .25$; $p_2 = 8$, $A_2 = .25$, $B_2 = 12$, $E_2 = .50$, $a_2 = 10$, $b_2 = .25$, $d_2 = 18$, $e_2 = .20$. The full damage terms are of greater magnitude than in the separable case. Hence, we are assured of their being greater than processing and control costs. The payoff matrix is given by Table 4.[11]

[10] The assumption in Section II. *B* calls for $E_i q_j > d_i + e_i q_j > a_j + b_j q_j$ for all but very low levels of q_j, Firm i being the recipient. Aside from helping to guarantee that the above inequality holds, the restriction of the b_i and e_i, $i = 1,2$, to be between 0 and 1 reflects the relative importance of fixed costs in the control and processing decisions. Once these two restrictions are met, the particular values are irrelevant if they are all (except for the traditionally fractional A_i) of the same order of magnitude. Given specific values, equations (26) and (27) are used to generate profit maximizing outputs. The latter are then translated via (25) into the payoffs in Table 3. Although the relative profit positions of the firms make a difference in terms of the actual monetary payoffs, the progress of the game to the social solution would not be prevented if the roles of Firm 1 and Firm 2 were reversed. Shutdown cases were ruled out above.

TABLE 4.
The Nonseparable Case Payoff Matrix

	(1,1)	*(1,0)*	*(0,1)*	*(0,0)*
(1,1)	(46,21)	(46,39)	(42,34)	(42,52)
(1,0)	(74,21)	(74,39)	(–82,34)	(–82,52)
(0,1)	(80,17)	(80,–51)	(62,30)	(62,–28)
(0,0)	(100,17)	(100,–51)	(–71,30)	(–53,–46)

The Nash solution yields (62,30). The best payoffs are 100 and 52 respectively when one firm does nothing as its opponent controls. The worst payoffs are –82 and –51. The latter occur when a $\beta_i = 0$ meets a $\gamma_j = 0$, and Firm j bears full nonseparable damages. The sum-of-profits optimum yields (74,39). Again, it would result from rational individual behavior if as a result of PGSR, the game (T_1^1, T_2^1) is played.

III. A Comparison

In the presence of market failure, it has been traditional to invoke nonmarket forces to guide the economic system to the allocation representing the maximum sum-of-profits. Implicit in the initial proposal by Pigou is liability. Liability connotes blame, and Pigou felt that in the transition from the state of nature, the prevalence of permissive law should cause blame to be placed on externality-producers. This was reemphasized by Baumol.

In their reciprocity arguments, Coase, Marchand and Russell and Gifford and Stone broaden the problem to include the imposition of liability as a variable. This would logically imply that some broader welfare index be proposed to rank allocations inclusive of a legal structure. Being aware of the problems involved with the latter, however, they reverted to the sum-of-profits index. Each situation, then, calls for each firm to take into account things which are exogenous to it. It seems as though the attempt was made to dualize Pigou-type liability, but then to neutralize liability by saying that it did not matter for resource allocation.

The technique proposed in this paper explicitly introduces the options of processing and control. It appears that this extension is necessary to reconcile discussion of the Coase Theorem and actual environmental problems. Also, the previous literature generally assumes some system of rights. The system is defined up to a point, and then the conflict resolution is left to private behavior. It seems more plausible to allow private resolution from the beginning with means that are readily available in the world.

As we have seen, the bargaining limits (full damages) in Section I.A depend on liability. Since they are a function of the actual outputs produced, Gifford and Stone append them to the private profit functions to get social behavior. We find that by introducing less severe bargaining limits in the absence of liability, we do not obtain the same behavior.

Consider the case where both firms are at (0,0). Rather than receive full damages Firm i, say, will process Firm j's externality. Hence, the most Firm i would pay Firm j to stop producing the externality is $PC_i(q_j)$. Firm j by controlling can avoid damaging Firm i. Hence, the least it would accept to do this is $CC_j(q_j)$. Any payment, a_{ij}, from Firm i and Firm j will be such that

$$CC_j(q_j) < a_{ij} < PC_i(q_j), i,j = 1,2; \quad i \neq j. \qquad (37)$$

[11] The choice of the data for this case generally follows the restrictions of footnote 10. The numbers have been changed to preserve the inequality therein due to the new severity of nonseparable full damages. The data used for Table 3 would cause shutdowns upon solving (31) and (32), thereby dissipating the example. Finding the optimal outputs is more difficult as they depend on the choice of *both* players' strategies. The profit pair for (0,0), (0,0) is gotten by taking the midpoint of profits from the equally likely simultaneous solutions, (q_1, q_2), of (30.7,1.3) and (1.4,28.6).

As the indices indicate, this holds for both firms. Following Gifford and Stone, if we attempted to append a linear combination of the bargaining limits to the profit functions, we would find that for each Firm *i* they are invariant with respect to the actual level of its production. I.e., they depend on the level of q_j. With processing and control, we do not need to make the private marginal conditions equal to the social ones. If we take the sum of profits when each firm chooses the Pareto optimal strategy (1,0) we find that the private and social and marginal conditions are identical. This, coupled with the avoidance of liability, appears to take much of the wind out of the traditional Pigouvian approach as well as the Coase Theorem in cases where our cost ordering holds.

One corollary of this is that we avoid the Baumol debate over whether or not Pigouvian taxes should be paid to the externality recipients. Also, we have no need to create a market for the externalities themselves or for "legal externality rights."

The Pareto solution to externality situations where the processing and control options are explicitly considered under our cost assumptions is different from the traditional one. The sum of profits maximization for each technique respectively in the reciprocal case yields

$$p_i - \frac{\partial D_i(q_i)}{\partial q_i} - \frac{\partial f_i(q_i)}{\partial q_i} = 0 \qquad (38)$$

$$p_i - \frac{\partial C_i(q_i, q_j)}{\partial q_i} - \frac{\partial C_j(q_i, q_j)}{\partial q_i} = 0 \qquad (39)$$

for $i,j = 1,2; i \neq j$. Denote the solution to (38) and (39) as (q_1^c, q_2^c) and (q_1^*, q_2^*) respectively, with c denoting control. We can see that in general they will be different. A comparison of the levels of output for each depends on the relative costs of damages received, damages caused and control of one's own wastes. The actual profit expressions are different. We can say that (q_1^*, q_2^*) maximizes the sum of profits in the traditional sense. We may also say that (q_1^c, q_2^c) maximizes the sum of profits in a process-control sense. Since they are different functions, we cannot make general comparisons of the two without knowledge of the precise functional forms and the parameters. Nonetheless, a simple comparison in the separable example given in Section II. B shows that the traditional Pareto optimum yields a sum of profits of 34 while from Table 3 our approach yields 91.

As a final comparison, if the Marchand and Russell and Gifford and Stone premises are at all uncertain, we must deal with problems of enforcement. If liability exists without enforcement (or the threat thereof), we cannot expect private behavior to be modified to social behavior. If enforcement exists, the costs involved may again invalidate the Coase Theorem. Information problems are also significant as noted in Davis and Whinston (1966).

IV. Concluding Remarks

As a method of capsulating the analysis of the previous sections we put forth our proposition in summary form: If the cost configuration in an externality situation is such that full damages are greater than processing costs which are greater than control costs, then we have the results listed in Table 5.

TABLE 5.
Summary of Results

Degree of Cooperation	*Type of Result*
None	The Nash Strategies (0,1) will yield (N^1, N^2) with no side payment.
Partial	No clear result in terms of strategies, but, some payoffs between (N^1, N^2) and (P^1, P^2) with some probable side payments. There is no guarantee of (P^1, P^2) although it may occur through PGSR.
Full	The Pareto strategies (1,0) and (P^1, P^2) with some side payments.

We believe that the explicit introduction of processing and control alternatives coupled with no reliance on liabilities adds credibility to the private resolution of externality conflicts. If liability is introduced, this automatically restricts the firms to a subgame of the overall game. If externality-producers were liable to control (not pay full damages), only the northwest four cells of Tables 3 and 4 would be relevant. Another alternative would be that the firm with the generally more favorable profit position should be forced to control. In this case, the northeast four cells of Tables 3 and 4 would seem relevant. Here, however, the selfish choice of the social optimum is unlikely. The laws would be arbitrary, of course, unless a more general welfare index provided us with a ranking of such states.

The next step from liability, then, is for the government to dictate a solution and no game need occur. A similar criticism was directed at Pigouvian tax scheme in that if the central authority had enough information to calculate the Pareto optimal taxes, then it could simply dictate the Pareto output levels. In our scheme, the central authority is a passive observer on the externality issue.[12]

The most obvious generalization of the above model is to an $n > 2$-firm situation. Firm i would have a strategy $(\beta_i; \gamma_1, \ldots, \gamma_{i-1}, \gamma_{i+1}, \ldots, \gamma_n)$ if it were possible to detect whose externalities it is processing. Firms wishing to make mutual strategy restrictions would need the assurance that they could control their externality "directed" at some other firm. This presents us with two sides of a public goods problem complicated by the possibility of coalition formation. We are further stifled by the possibility of two-firm or two-coalition bargains being mutually inconsistent.

Perhaps another type of generalization for cases where $n \geqslant 2$ would be to replace the 0,1 variables with probability distributions over the processing and control decisions. This, along with resolution of the large numbers problems, presents an ample target for future research in the attempt to deal with more than two-firm externality dilemmas.

[12] For more on the role of government see the tax versus regulation argument of Buchanan and G. Tullock (1975).

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