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The Rational Expectations Hypothesis and Economic Analysis

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The Rational Expectations Hypothesis (henceforth REH) has generated so much controversy in the economic literature that some economists have referred to its impact as a "revolution." Whether or not one agrees that the hypothesis is correct, it has greatly influenced economic analysis and policy making. Unfortunately many economists have confused the conclusions obtained from rational expectations models with the hypothesis itself, and this has lead to improper criticisms of the REH.

In this paper I hope to eliminate some of the confusion surrounding the REH by separating the hypothesis of rational expectations from the models in which it has been employed. The first section of this paper presents a broad definition of rational expectations to clarify our understanding of the hypothesis. Section two utilizes Cagan's model of money demand to illustrate some of the important issues that have been raised in the rational expectations debate. It is argued that many of the criticisms of the REH are actually criticisms of the structure of the models in which the REH has been employed, or of the specific assumptions made when the REH is applied. Cagan's model of money demand provides a convenient framework for examining some of the problems arising from model structure. The problems arising from modelling information available to agents in the economy are explored in section three and section four surveys some of the applications of the REH in microeconomics and macroeconomics. Conclusions follow in section five.

What Are Rational Expectations?

Economic theory has long recognized the importance of expectations in the economy but it has been difficult for economists to model them. Keynes (1936) argued that expectations cannot be modelled because agents have too little information to form 'strict mathematical expectations' of relevant events. Thus he accorded 'sentiment or chance' an important role in determining public anticipations. The REH takes the opposite view: expectations formed by the public are strict mathematical expectations of the relevant variables.

The REH asserts that information is scarce and is generally not wasted. Furthermore, the way expectations are formed depends "...specifically on the structure of the relevant system describing the economy." (Muth, 1961, p. 316) The REH implies that economic agents do not make systematic errors in forming expectations about economic variables, given the information available. This does not mean that expectations are formed without error, but rather that agents do not consistently over or under predict the value of the economic variable in question. As McCallum has stated," ... the basic idea of the hypothesis is simply that agents behave purposefully in

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collecting and using information, just as they do in other activities." (1980, p. 717) Agents will make forecasting errors, but within the framework of the REH, these errors are random.

If information useful for forecasting becomes available to agents, they will acquire and use that information until the acquisition costs, at the margin, are equal to the marginal benefits. Some problems can arise in practice because it is often difficult to know the value of information before it is acquired. One important point should be noted about the REH as it has been defined above. If economic information is useful in forecasting, then agents who do not utilize it in their forecasts will make systematic forecasting errors. Since the hypothesis assumes that agents do not make systematic errors, those that do will sustain losses and will eventually leave the industry. Alternatively, economists could sell their forecasts to the industry. In either case, the REH implies that the expectations of economic agents are, on average, equal to the mathematical expectation of the relevant economic variable, conditioned upon the information available at the time.

Two aspects of the REH make it attractive to economists as a model of expectations. First, economists are not inclined to argue that agents in a market make systematic forecasting errors a priori, at least not without good reason. Second, if market conditions change, few would argue that expectations would remain the same. The REH assumes that relevant new information will be incorporated into market forecasts while other models of expectations do not. It should be intuitively clear that rational expectations are intimately connected with the relevant economic model in theory. Whether agents actually know the economic model or simply use complicated rules of thumb to eliminate systematic errors is irrelevant. The key issue for economists, especially in policy making, concerns the attractiveness of employing models that assume systematic errors will be made. The REH implies such a situation cannot persist; other models of expectations do not make this assumption.[1]

An Example of the REH Using Cagan's Model of Money Demand

The Cagan model of money demand during hyperinflation is often used to illustrate the REH. Many of the issues raised in this section arise in other models that employ the REH in both microeconomic and macroeconomic applications. Cagan argued that the demand for real balances is a negative function of the expected inflation rate.

$$(1) \qquad m_t - p_t = \gamma - \alpha \pi^e_t + \epsilon_t$$

where m_t and p_t are the logarithms of the money supply, and the price level at time t respectively, π^e_t is the rate of inflation between time t and t+1 expected by the public, and ϵ_t is an error term. The REH requires

$$(2) \qquad \pi^e_t = E_t(p_{t+1} - p_t) \equiv E_t(X_{t+1})$$

where X_{t+1} is the inflation rate between t and t+1 and E_t is the mathematical expectation operator conditioned on information available at time t. The information set is assumed to include all current and past observations on p_t, m_t and the structure of the model.

Differencing equation (1), substituting (2) and solving for the rate of inflation will yield:

$$(3) \qquad X_t = \frac{1}{1+\alpha}(\mu_t + \eta_t) + \frac{1}{1+\alpha}(E_t X_{t+1})$$

where $\mu_t = m_t - m_{t-1}$, $\eta_t = \epsilon_t - \epsilon_{t-1}$, and $E_t X_t = X_t$ by assumption. Equation (3) states that the inflation rate this period depends upon the current growth rate in the money supply minus the difference of the shock to money demand plus the inflation rate expected next period. As Blanchard (1979) has shown, an infinity of solutions exists for the current inflation rate. For example, we could solve (3) **backward** in time and express $E_t(X_{t+1})$ as a function of **past** growth rates of money and **past** errors in the money demand function as follows:

$$(4) \qquad E_t X_{t+1} = \left(-\frac{1}{\alpha}\right) \sum_{i=0}^{\infty} \left(\frac{1}{1+\alpha}\right)^i E_t(\mu_{t-1} - \eta_{t-1}) + B^t$$

where B^t is an arbitrary constant whose value depends on time. Alternatively, we could solve (3) **forward** in time arguing that the future expected rate of inflation depends upon future anticipated growth rates in the money supply and errors in the money demand function.

$$(5) \qquad E_t X_{t+1} = \left(\frac{1}{\alpha}\right) \sum_{i=1}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^i E_t(\mu_{t+i} - \eta_{t+i}) + A_t$$

where, again, A^t is an arbitrary constant. In general, both (4) and (5) are correct solutions for $E_t X_{t+1}$. This indeterminacy arises because the current inflation rate depends upon the inflation rate expected next period. As Blanchard notes, "... there is one more price (or expected price) than markets to clear." (Blanchard, 1979, p. 115)

Several important points concerning the rational expectations debate can be illustrated in the solutions (4) and (5) given for the expected future inflation rate. First, note both solutions for the expected inflation rate depend upon the structure of the model. Many critics have argued that economic agents do not know the correct economic model or that several plausible models exist. How can a rational expectation be formed under these circumstances? As stated before, in theory we must model expectations **as if** the model were known to ensure that theoretically there are no systematic errors made by agents in forecasting. This does not imply that agents form their expectations in this way, nor does it imply that the model is correct in practice. The correctness of any logically consistent model is ultimately an empirical question.

Second, note that the solutions (4) and (5) indicate that the error terms in the money demand equation affect expectations. Under the REH it is not possible to create and solve a model and simply add error terms at the end in an **ad hoc** fashion. The error terms and their properties must be specified throughout the model as their presence directly affects the

rational expectations solution. This result has caused a serious re-thinking of economic theory and econometrics and ultimately requires a unification of both areas. Furthermore, the REH requires economists to recognize the existence and importance of uncertainty in the economy explicitly. Mistaken expectations affect the economy in many REH models (such as Lucas's (1973) model of the Phillips curve). Rather than ignore the role played by uncertainty in the economy, the REH has focused attention on its importance.

Third, the fact that two solutions exist raises serious questions about uniqueness. How do we know which path ((4) or (5)) the economy will choose? Solving this problem requires additional information concerning the dynamic operation of the economy.

The issue of stability is closely intertwined with the question of uniqueness. Depending upon the size of the parameter α, the nature of the money supply process, and the error terms, one or both of the solutions for the expected inflation rate will not converge. If we impose the requirement that the inflation rate should not "explode" if the money supply process does not, and, for simplicity ignore the error terms, then the infinite sum will only converge if the forward solution (5) is used. This occurs because $(1+\alpha)/\alpha > 1$; therefore, the backward solution will "explode" even if the money supply process does not.[2] Most of the literature has simply focused on the convergent path without addressing the problem of how the economy gets to such a path.[3]

Furthermore, the convergence of the solutions (4) and (5) will also depend upon the size of the arbitrary constants A^t and B^t. Generally, if these constants are non-zero, then these terms grow without bound and then, even if the infinite sums in (4) and (5) converge, the expected inflation rate will not. Again, many of the supporters of the REH have implicitly (or explicitly) assumed these terms are zero. The argument provided is that we don't observe market prices or inflation rates growing without bound, and so the market must (somehow) choose a stable path with A=B=0.

Unfortunately, this argument is not entirely convincing and does not rule out the possibility that an economy or market could choose an unstable path for a period of time and then return to the convergent path. These problems are not unique to rational expectations models. As with any new field of research, these issues deserve continued research and development. Much promising work has been done in this area. For example, McCallum (1983) has shown that the uniqueness problem in rational expectations models occurs in **any** dynamic model with expectations. The REH itself does not create uniqueness problems. Furthermore McCallum contends that a wide class of linear models exists where convergence is obtained.[4] More recently, Scarth (1985) has extended McCallum's procedure showing that dynamic restrictions can lead to unique solutions in rational expectations models.

Another closely related issue concerns individual expectations. Given the above model, there is one rationally expected inflation rate for all economic agents if we choose the convergent solution (5), assume a finite money supply process and assume A = 0. Many critics have asserted that such a restriction is unwarranted. However, this criticism is not necessarily justified. As McCallum (1980) has noted, the error term in the money demand equation (1) could reflect a random distribution of agents' expectations about the "rational" expectation. As long as this distribution is uncorrelated with information available to agents and does not lead to systematic errors, then the criticism is not serious. Furthermore, models by Lucas (1973) and Cukierman and Wachtel (1979) have employed the REH allowing expectations to vary across markets due to information differentials. Thus the criticism that the REH requires everyone to have the same expectations about the future is a characteristic of specific models that have employed the REH, it is not a feature of the hypothesis itself.

Another issue concerns the role of speculation in the expectations process. Consider again the solution (5), and substitute that solution into (3). This yields

$$(6) \qquad X_t = \left(\frac{\alpha}{1+\alpha}\right)A^t + \left[\left(\frac{1}{1+\alpha}\right)(\mu_t - \eta_t) + \left(\frac{\alpha}{1+\alpha}\right)\left(\frac{1}{\alpha}\right)\sum_{i=1}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^i E_t(\mu_{t+i} - \eta_{t+i})\right]$$

Flood and Garber (1980) have referred to the terms in brackets above as the **market fundamentals**. These fundamentals are the economic determinants of price (or the inflation rate here) in the market. A **"price bubble"** arises when the actual market price depends positively on arbitrary self-fulfilling expectations of future price changes that are independent of the market fundamentals. Thus a bubble occurs when $A^t \neq 0$, and prices will continue to change over time even if market fundamentals are unchanged. The term A^t thus reflects arbitrary and self-fulfilling elements in expectations.

The interesting point to observe about equation (6) and the definition of a price bubble is that it seems analogous to Keynes' well-known beauty contest problem. Keynes argued that expectations were arbitrary and self-fulfilling to the point that they could not be modelled. He argued further that the current stock price depends upon what future price is expected. Since that future expected price depends upon what other agents think it will be, he argued that no determinate price can exist that is not arbitrary. Essentially, then, Keynes' argument could be interpreted as an assertion that the market fundamentals are dominated by speculative elements (modelled here by the arbitrary constant when it is non-zero).

More recently Ackley (1983) has raised this issue in an interesting comparison of macoreconomics and microeconomics. Essentially Ackley argues that in many markets the quantities supplied or consumed do not change rapidly enough to drive prices to their new equilibrium levels when a change in market fundamentals occurs. In cases such as gold markets, market fundamentals do not dominate price movements. Rather, prices are determined largely by speculative impulses. As a result, price movements can be self-fulfilling and thus consistent with rational expectations even

though these price movements are independent of market fundamentals. Ackley feels that even though such microeconomic price behavior can be rational for specific markets, it yields perverse macroeconomic results.

The Information Problem

Similar conclusions to those discussed by Ackley were reached by De Canio (1979) and Frydman (1982). Their approach was to question how a market can reach a rational expectations equilibrium if agents do not know the complete structure of the model. A crucial issue in the rational expectations literature concerns the information available to agents. In most applications researchers have assumed that agents know the structure of the model including the values of the relevant parameters, the past history of the relevant economic variables and (in macroeconomic models) the policy rules pursued by the monetary authorities. How can agents acquire all of this information?

Here the issue does not necessarily concern the REH itself, but rather, the amount of information researchers assume that agents have optimally acquired. Most of the rational expectations literature has not been concerned with the problem of how agents acquire information. DeCanio (1979) showed that the assumption of rational expectations does not **guarantee** a unique rational expectations solution when information is incomplete. However, convergence to a rational expectations equilibrium is possible if agents follow certain optimal rules in acquiring information.

In a similar context Frydman (1982) argues that agents cannot form optimal forecasts if they are only allowed to observe market prices and the information available in their local market on their cost functions. Frydman further contends that rational "learning" cannot occur because agents "... cannot ascertain the average of forecasts (of that period's price) formed by other agents ('the average opinion')." (1982, p. 654) This is the beauty contest problem discussed by Keynes, and in the model presented by Frydman, it precludes convergence to a rational expectations equilibrium. Sufficient information is unavailable on market fundamentals so the speculative element dominates, and price bubbles are possible.

What these studies imply is that important issues relating to convergence and uniqueness need to be addressed. Although the initial research on these problems has not been overly encouraging, it has not rendered the hypothesis completely inapplicable. Despite the problems on convergence and information, agents do have a great deal of information (cultural, sociological) as well as economy-wide information that leads me to believe that the convergence problems are not as bad as one might think. (McCallum, 1983; Cooley, 1985; Scarth 1985) Undoubtedly, current thinking about these problems will change. What is important is that the REH has gone a long way toward changing the modelling of expectations from an **ad hoc** basis to one more formally grounded within the context of optimizing behavior.

Applications

The REH has been applied in many areas, some with more success than others. In the stock markets, the REH has been known as the "efficient markets hypothesis." This hypothesis has been widely accepted as a model of financial market behavior. Ironically this application of the REH has been most successful in the one market Keynes felt was the most difficult to model. The efficient markets hypothesis asserts that profit opportunities are exploited by the market so that the current price of a stock incorporates all currently available information, and is the best forecast of the future stock price. Although most tests of market efficiency have supported the REH (such as Mishkin (1981)), some tests concerning the variance of stock prices have rejected the REH (Shiller (1981)). At this point it is safe to say that the mean market forecast seems to be efficient, but tests of the variance of stock prices have rejected the REH (see Shiller (1981)). Further evidence along these lines will be required, but many forms of the REH have been supported.[5]

Other research of market efficiency has focused upon the forward market for foreign exchange. If markets are efficient, then the price of a forward contract for foreign exchange should be an unbiased estimate of the future spot price. Again evidence from these studies has generally supported the REH (see Frenkel (1977)). Other tests applied to this market have rejected stronger versions of the REH (Hansen & Hodrick (1980)).

Most of the debate over the REH has arisen in its macroeconomic applications. By the early 1970s most macroeconomists accepted the "natural rate hypothesis" (NRH) argument that the long run Phillips curve for an economy is vertical. In the short run, however, the curve represented the well-known tradeoff between inflation and unemployment. The tradeoff existed between inflation and unemployment in the short run because agents were fooled into believing their real wages are higher than they actually are when prices increase. As a result, unemployment falls when unanticipated inflation occurs. Many macroeconomists accepted the NRH because short run stabilization policy is possible within this framework, and the debate centered on the issue of the length of the "short run."

When the REH was first applied to this problem, the hypothesis asserted that, on average, unanticipated inflation ($p_t - E_{t-1}p_t$) would be a white-noise error term (i.e., have zero mean with no serial correlation). Furthermore, the expectational error could not be influenced by systematic government policy. This became known as the "neutrality proposition" first advanced by Lucas (1973) and by Sargent and Wallace (1975). Together the REH and NRH implied that active stabilization policy would not be effective in reducing macroeconomic fluctuations.

Since many economists who favored active stabilization policy agreed with the NRH, the initial reaction was a severe criticism of the REH itself. Many simply argued it was not an accurate hypothesis concerning expectations formation. However, I believe most of the controversy reflected a disagreement with the conclusions of the models that employed the REH (primarily the neutrality proposition) rather than a rejection of the REH itself. In fact, many economists who once rejected the REH have come to employ it in recent research (see Robert Gordon (1982)). It is possible to create models that use the REH but that do not imply policy neutrality (see Fischer (1977), Shiller (1978) and Gordon (1982)). As this point has become clarified, disputes over the REH have become less important, and issues concerning model structure have come to the

foreground. The REH does not imply that active stabilization policies are neutral, rather the structure of the models leads to this result. However the REH does impose testable restrictions on the data which can be rejected if the REH restrictions are inappropriate. For example, Mishkin (1982) rejected the policy neutrality conclusion while not rejecting the REH. On the other hand, Litterman and Weiss (1982) were unable to reject the neutrality proposition or the REH using vector autoregressions in their work.[6] Naturally, further careful research is crucial.

Conclusions

This paper has argued that the chief controversy about the REH concerns the specific assumptions about the information agents have and the structural characteristics of the models in which the REH has been employed. Given the impact the REH has had on theoretical and empirical research, it can indeed be called a revolution, having greatly changed the way many economists think about markets and the economy. The hypothesis has also lead to advances in econometric modelling and testing that are important for progress in economic theory and practice. One conclusion that the REH has forcefully made clear is that policy evaluation using econometric models cannot be conducted under the assumption that the estimated coefficients of the economy will not change. As the solutions derived earlier have shown, expectations depend upon the structure of the model which includes the anticipated effects of government policy. If those policies are changed, the expectations of agents in the economy will change. This creates serious problems for macroeconomic simulations.

Given the wide support the hypothesis has received in the applications to financial markets and in several tests of macroeconomic models, the future of the REH is promising. This optimistic view is reinforced by the failure of several studies to reject the restrictions implied by the REH. Hopefully, when the fundamental issues concerning the REH are better understood, this research will lead to more widespread applications of the hypothesis in economics.

Footnotes

[1]The adaptive expectations rule is a good illustration. Adaptive expectations of inflation depend only on past values of the inflation rate. Therefore other variables that might be useful in forecasting inflation are ignored by this model. Furthermore, adaptive expectations are backward-looking which can lead to biased forecasts. See Sargent and Wallace (1975).

[2]This is not entirely accurate since restrictions on the money supply process could, in certain cases, lead to convergence. See Blanchard.

[3]Burmeister (1980) has questioned whether it is reasonable to assume that a single path exists that converges to a rational expectations equilibrium. In a situation with multiple paths to an equilibrium, no unique rational expectations solution will exist.

[4]This is important because it rules out speculative bubbles discussed below.

[5]Ron Michener (1982) has recently investigated this problem. He finds that in a simple, one-asset economy it is possible for the price of the asset to violate the theoretical variance - bounds implied by the present value relation. As a result these variance tests do not necessarily imply that markets are inefficient. Marjorie Flavin (1983) discovered that in many cases, these variance bounds tests are biased in small samples toward rejecting the efficient markets hypothesis. Her results suggest that the evidence presented thus far that rejects efficiency in the stock market is unreliable.

[6]As equations (6) and (7) demonstrate, it is difficult to test the REH separately from the model itself since the two are so closely connected. Therefore, most tests of the REH are **joint** tests of rational expectations **and** the model itself. This has caused some problems in testing the REH in general and illustrates the care required in estimating and testing expectations models.

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