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CHARGES, PERMITS AND POLLUTANT INTERACTIONS

ALFRED ENDRES*

I. Introduction

There has been considerable discussion in the literature as to how predetermined environmental quality standards can be attained. Effluent charges and transferable discharge permits (TDPs) are the policy instruments considered most often.

In most of the literature the problem is treated as if different pollutants could be regulated independently.[1] However, even though a useful assumption to facilitate theoretical analysis in the first place, this approach assumes away important aspects of practical pollution problems.

Generally, the environment does not provide special subcapacities for the assimilation of each pollutant. Several pollutants rather draw upon the same capacity of the environment, simultaneously. Moreover, they often react chemically. Their mixtures then generate an environmental impact different from the sum of the impacts that each individual pollutant would have in the absence of the others.

In all these cases, an acceptable level of a pollutant can only be defined for given levels of other pollutants.

In the following analysis it is assumend that a single indicator "I" exists which relates the quantities of n pollutants $(X_1, \ldots, X_n)$ to "load units" of this medium. (The higher the index value the lower is the quality of the environmental medium). This indicator is assumed to take care of the problems of simultaneous environmental capacity use and chemical reactions.

Here, the target of environmental policy can be defined in terms of a predetermined level $\bar{I}$ of this index. It should be noted that this type of a target definition, contrary to the pollutant specific definition traditionally used, is compatible with indefinitely many combinations of n pollutant quantities.

Of course, it cannot be said in general terms what properties the environmental constraint defined for the economic process by setting the target $\bar{I}$ might have. If the indicator would take the linear additive form of $I = a_1X_1 + \cdots + a_nX_n$ where the a_i are constant "load-parameters", the pollutants could be substituted against each other at a constant rate for each given level $\bar{I}$. This very simple type is called "linear interaction", below. Of course,

*Technical University of Berlin, West Berlin, West Germany.
[1]Notable exceptions are [2] and [3].

the marginal rate of substitution among pollutants $-(dX_j/dX_i)_{dI=0}$ may decrease or increase or take non-monotonous forms. All of these types are labeled nonlinear interaction, below.[2] It is even possible that pollutants are complements rather than substitutes. Here, the detrimental effects of different pollutants compensate each other. This case, however, will not be discussed, below.

Henceforth, the existence of a regulatory agency is supposed, aiming at a decrease of the environmental load in its control region from the unregulated level I^* to a predetermined target level $\bar{I}$. There are n pollutants supposed to be generated by n regional industries, one pollutant by each. The unregulated equilibrium pollution levels are denoted $X_1^*, \ldots, X_n^*$. The emissions reduced from their unregulated levels $X_1^*, \ldots, X_n^*$ to any level $\hat{X}_1, \ldots, \hat{X}_n$ are denoted $\hat{x}_1, \ldots, \hat{x}_n$.

Since $\bar{I}$ can be met with many combinations of pollutant quantities it is to be decided which combination the agency is to aim at. It is supposed that the agency is trying to find the pollutant allocation $(X_1^{**}, \ldots, X_n^{**})$ which meets the environmental constraint $\bar{I}$ at minimum cost. Simultaneously with the problem of finding $(X_1^{**}, \ldots, X_n^{**})$ the agency of course has to solve the problem of assigning each industry pollutant quantity X_i^{**} to the individual generators of pollutant i. Since the latter problem is extensively treated the literature using independent targets for the pollutants [1; 5; 9], it is ignored henceforth.

As alternative means to achieve the predetermined standard $\bar{I}$, effluent charges and marketable pollution permits are discussed, below.

II. Properties of the Optimum

Before analyzing the ability of alternative policies to meet $\bar{I}$ at minimum cost the nature of the optimum allocation is to be elaborated. The problem of the environmental agency is

$$C = \sum_{i=1}^{n} C^{(i)}(x_i) = \min! \tag{1}$$

where C is the aggregated abatement cost of all polluters, $C^{(i)}$ is the abatement cost of the industry generating pollutant i and x_i is the abatement quantity of this pollutant.[3] $\partial C^{(i)} \partial x_i > 0$, $\partial^2 C^{(i)} \partial x_i{}^2 > 0$ is supposed to hold.

The first constraint for the cost minimization is the environmental standard to be met, i.e., $\bar{I} - I(x_1, \ldots, x_n) \geq 0$, where $\partial I/\partial x < 0$, for all $i \in \{1, \ldots, n\}$. Moreover, you cannot clean up more mess than generated. Thus, for each abatement activity an "upper boundary condition" $X_i^* - x \geq 0$, for all $i \in \{1, \ldots, n\}$ holds, where X_i^* is the unregulated "status quo ante" equilibrium quantity of pollutant i.

Finally, the levels of pollution abatement are non-negative, i.e., $x_i \geq 0$, for all $i \in \{1, \ldots, n\}$ holds.

[2] Taking the toxicity to fish as an indicator of the environmental load caused by a combination of pollutant concentrations, J. B. Sprague found high evidence for linear pollutant interaction [8]. From the four air quality indices surveyed by A. E. S. Green et al, three take the linear interactive form [7]. In the Soviet Union, interactive ambient air quality standards are used. They all take the form of linear interaction [4, 66].

Of course, the use of linear indicators in biology and other sciences is no proof of the underlying environmental structure being a linear one. Indicators are only proxies, after all. Their quality cannot be assessed by the author, a simple economist only.

[3] The index (i) in $C^{(i)}$ is dropped, below, where no confusion seems to be possible.

As mentioned above, the problem of assigning the maximum allowable emission level for each industry to the members of this industry is not analyzed in this paper. It is, therefore, assumed that within each industry, the pollution allowances are distributed in a manner minimizing intra-industry abatement cost.

The Lagrangean function for this constraint minimization problem is

$$Z = C(x_1, \ldots, x_n) + \mu(I(x_1, \ldots, x_n) - \bar{I}) + \sum_{i=1}^{n} \tau_i (x_i - X^*). \tag{2}$$

The Kuhn-Tucker conditions are

$$\partial C/\partial x_i + \mu \partial I/\partial x_i + \tau_i \geq 0 \quad x_i \geq 0 \quad x_i(\partial C/\partial x_i + \mu \partial I/\partial x_i + \tau_i) = 0 \quad \text{for all} \quad i \in \{1, \ldots, n\} \tag{3}$$

$$I(x_1, \ldots, x_n) - \bar{I} \leq 0 \quad \mu \geq 0 \quad \mu(I(x_1, \ldots, x_n) - \bar{I}) = 0 \tag{4}$$

$$(x_i - X_i^*) \leq 0 \quad \tau_i \geq 0 \quad \tau_i(x_i - X_i^*) = 0, \quad \text{for all} \quad i \in \{1, \ldots, n\}. \tag{5}$$

According to the Arrow-Enthoven Theorem, these conditions are necessary and sufficient for a global solution of our cost minimum problem, given the constraint qualification is met, the objective function $C(x_1, \ldots, x_n)$ is differentiable and quasiconvex and the constraint function $I(x_1, \ldots, x_n)$ is differentiable and quasiconcave. (The condition that there exists an $i \in \{1, \ldots, n\}$ such that $\partial C/\partial x_i > 0$ at the solution, is met anyway, in the problem analyzed here.)

The Kuhn-Tucker conditions allow for interior and corner solutions. In the case of an interior solution ($0 < x_i < X_i^*$, for all i) for the reduction of any pair of pollutants i, $j \in \{1, \ldots, n\}$ it follows that

$$\begin{aligned} \partial C/\partial x_i &= -\mu \partial I/\partial x_i \\ \partial C/\partial x_j &= -\mu \partial I/\partial x_j \\ \rightarrow -(dx_j/dx_i)_{dC=0} &= -(dx_j/dx_i)_{dI=0} \end{aligned} \tag{6}$$

Condition (6) indicates that in the solution the marginal rate at which the two pollutants can be substituted against each other at the predetermined index level $\bar{I}$ (their marginal rate of substitution) equals the marginal rate at which the two pollutants can be substituted against each other at a given level of aggregate abatement cost (their marginal rate of transformation).

Corner solutions may turn up in two kinds of forms. First, in the non-negativity condition(s) of one (several) variable(s) the strict equality sign may hold, second in the upper boundary condition(s) of one (several) variable(s) the strict equality sign may hold (or both). Because of space limitations these situations are not analyzed here, but in another paper [6].

The properties of the solution of the cost minimization problem under environmental restrictions have been established. How about the chances to arrive at this optimum by applying alternative environmental policies? Discussing this question, the cases of linear and nonlinear interaction in the environmental target constraint are separately dealt with, below.

III. Linear Interaction

Effluent Charges

Suppose, the regulatory agency is to use effluent charges as a means to achieve the predetermined interactive quality standard $\bar{I}$. To decide which of the indefinitely many combinations of the n pollutants compatible with $\bar{I}$ it is to aim at, the regulatory agency has to make a guess on the marginal abatement costs of the n polluting industries. On the basis of this estimate and the agency's knowledge of $I(X_1,\dots,X_n)$, the n pollutant target levels $(\bar{X}_1,\dots,\bar{X}_n)$, are defined. These levels are the ones the agency takes to meet the environmental restriction $\bar{I}$ at minimum cost.

The estimate of the abatement cost functions is also needed for another purpose: It provides the basis for a guess on how the n polluting industries will adjust their discharges to alternative levels of tax rates $t_{X_1},\dots,t_{X_n}$, just as in the case of no pollutant interactions: Consider any polluter A in any i-polluting industry wanting to minimize his burden B_A of environmental policy $B_A = t_{X_i}X_i^{(A)} + C_A^{(i)}(x_i^{(A)})$. The first term indicates the firm's emission tax bill and the second term its abatement cost.

Under the above mentioned condition of increasing marginal abatement cost, the burden is minimized for

$$t_{X_i} = \partial C_A^{(i)}/\partial x_i^{(A)}. \tag{7}$$

Thus, under an effluent charge law each polluter will reduce emissions until the marginal abatement cost equals the tax rate, as is well known in the literature. Using this information and having assessed the marginal abatement cost, the regulatory agency sets tax rates $t_{X_1},\dots,t_{X_n}$ equal to (its guess of) the marginal abatement cost of the polluting industries in the target situation $(\bar{X}_1,\dots,\bar{X}_n)$. If the target situation is not attained after the industries' adjustment to the taxes, the tax rates have to be revised. It is hoped that a solution will be attained after an iterative process of trial and error [1]. Figure 1 illustrates this "pricing and standards" technique for the case of linear interaction, the existence of an interior solution and two pollutants X, Y.

The first quadrant of Figure 1 depicts the situation of the X-industry. The unregulated equilibrium emission quantity of that industry is X^*. $C(x)$ is the total, $\partial C/\partial x(x)$ the marginal abatement cost curve. Of course, it is highly unlikely that the regulatory agency's estimate of the abatement cost of the X-industry (and any other industry) would be correct.

Therefore, in the first quadrant of Figure 1, the agency's estimate $C^e(x)$ and $\partial C^e/\partial x(x)$ are distinguished from the true curves $C(x)$, $\partial C/\partial x(x)$. The second quadrant depicts the situation of the Y-industry, analogously.

The solution of the constraint cost minimization (equation (6), above) is graphically illustrated in the third quadrant. $P^{**}(X^{**},Y^{**})$, where an iso-cost curve C^{**} derived from the true abatement cost curves is tangent to the target line $\bar{I}$ represents the genuine optimum. $\bar{P}(\bar{X},\bar{Y})$, where an iso-cost curve $\bar{C}^e$ derived from the agency's estimate of the abatement costs is tangent to the target line represents the optimum as assessed by the agency.

Under these circumstances the regulator will set tax rates $t_X{}^{(1)} = \partial C^e/\partial x(\bar{x})$, $t_Y{}^{(1)} = \partial C^e/\partial y(\bar{y})$, which he expects to induce abatement activities $\bar{x}$, $\bar{y}$ to bring emissions down from X^*, Y^* to the targent levels $\bar{X}$, $\bar{Y}$.

The firms adjust to these taxes by reducing emissions up to the point where their marginal abatement costs equal the tax rates. However, the costs calculated by the firms in their adjustment are the genuine abatement costs, rather than the marginal abatement costs as estimated by the agency.

Thus, the post tax emission equilibrium is X_1, Y_1 with

$$t_X{}^{(1)} = \partial C/\partial x(x_1), \qquad t_Y{}^{(1)} = \partial C/\partial y(y_1).$$

This equilibrium is illustrated as P_1 in Figure 1, missing the target $\bar{P}(\bar{X},\bar{Y})$. Therefore, the tax rates have to be revised.

In the process of restructuring tax rates the regulatory agency can rely upon the following informations, given interaction is linear: The agency knows from (6) that in the cost minimum situation (X^{**},Y^{**})

$$-(dy/dx)_{dC=0}(X^{**},Y^{**}) = -(dy/dx)_{dI=0}(X^{**},Y^{**}) \text{ holds.} \tag{6a}$$

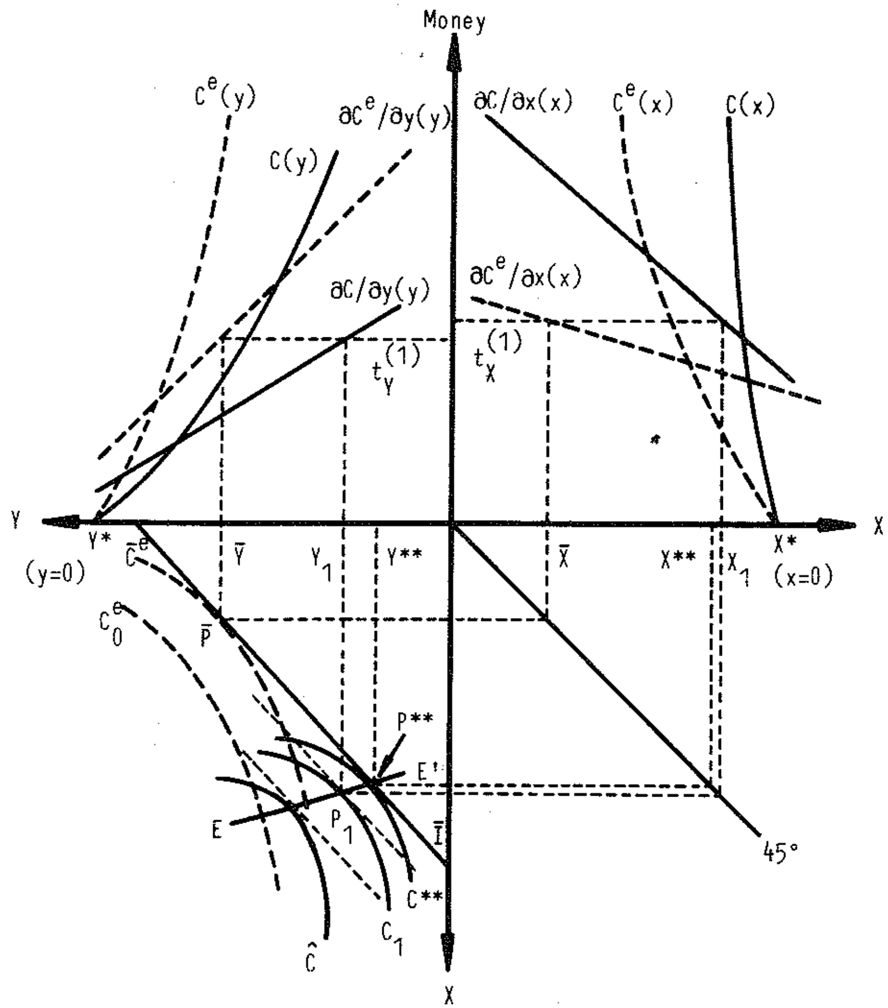

Figure 1 Effluent Charges with Linear Interaction

From (7) it is known that

$$t_X/t_Y = -(dy/dx)_{dC=0}(X^{**},Y^{**}) \text{ holds.} \tag{7a}$$

The agency concludes that

$$t_X/t_Y = -(dy/dx)_{dI=0}(X^{**},Y^{**}) \text{ holds.} \tag{8}$$

Thus, the regulatory agency can take it for granted that in the optimum it is struggling for, realized as emission tax equilibrium, the relative tax rate for the two pollutants equals the marginal rate of pollutant substitution, evaluated in the solution.

Since $I = I(X,Y)$ is known to the agency and the marginal rate of pollutant substitution does not depend on the levels of pollutants in the case of linear interaction, the agency knows the term $-(dy/dx)_{dI=0}$ in (8) without knowing where the solution lies. Therefore, the agency is aware of the relative optimum tax rates without having complete information on the polluting industries' marginal abatement costs. In the abatement equilibrium attained after the agency's first tax rate estimation (P_1 in Figure 1) the condition $t_X{}^{(1)}/t_Y{}^{(1)} = -(dy/dx)_{dI=0}$ holds. Thus, all the agency has to do after realizing that the environmental restriction is not met, in P_1, is to raise the absolute value of the tax rates leaving relative taxes as they are. In Figure 1, this would correspond to a move of the equilibrium allocation along the line EE' from P_1 towards P^{**}.

Even though it is well known that restructuring tax rates may be difficult in practice (with or without pollutant interaction) it is interesting to note that in the case of linear interaction the agency's strive for optimality is not more complicated than in the case of regulating a single pollutant.

The simple decision rule is:

If the load on the environment after adjusting to the tax rates set in the first place is above the target level $\bar{I}$, all tax rates have to be raised in the same proportion, until $\bar{I}$ is met. If the load falls short of the level aimed at (contrary to what is shown in Figure 1) all tax rates may be reduced by the same percentage amount.

Transferable Discharge Permits

Suppose the regulatory agency is using TDPs as a means of environmental policy. Then, a quantity of permits is issued by the agency guaranteeing that the environmental target level $\bar{I}$ is met. The agency has the option of auctioning off the permits, or giving them away free of charge to the polluters. In both schemes permits can be resold. Moreover, the permits may be designed as seperate emission rights, one type of a right for each type of a pollutant, or they may be written in the form of "load-permits" (L-permits), allowing the generation of load-units in terms of the indicator I. Sha..ng L-permits, it would be convenient to use one of the pollutants (say, j) as a numeraire. Then, the permits would be written out in units of pollutant j. One L-permit would certify the right of discharging one unit of pollutant j or, alternatively, the quantity of pollutant i, equivalent to a unit j in terms of the index I.

Assume permits are distributed free of charge to the polluting industries according to $\bar{X}$, $\bar{Y}$, the cost minimal pollutant quantities as supposed by the agency. Then, in Figure 2, a $\bar{X}(\bar{Y})$ shows the initial permit endowment of the $X(Y)$-industry. $\partial C/\partial y$ shows the marginal abatement cost curve of the Y-industry and $dC(x)/dy$ shows the marginal cost of abating Y-equivalents in the X-industry.

It should be noted that the abscissa in the 1st quadrant of Figure 2, showing the X-industry, has been rescaled to Y-equivalents, since Y is the numeraire pollutant for permit transactions.

In the starting situation, the marginal abatement costs of the Y-industry are lower than the marginal abatement costs of the X-industry (in terms of Y-equivalents). Therefore, in the permit market, $\partial C/\partial y$ is the Y-industry's permit supply curve and $\partial C(x)/dy$ is the X-industry's permit demand curve. To read quantities supplied and quantities demanded along the same axis, $\partial C/\partial y$ has been shifted from the 2nd quadrant to $\partial C/\partial y^T$ in the 1st quadrant of Figure 2.

It is illustrated in Figure 2 (and can also be demonstrated analytically) that the permit market is in equilibrium at X^{**}, Y^{**}, the least cost situation.

Thus, it turns out that TDPs are an efficient and target proof policy instrument in the case of linear interaction, as they are in the case of no interaction, traditionally analyzed in the literature.

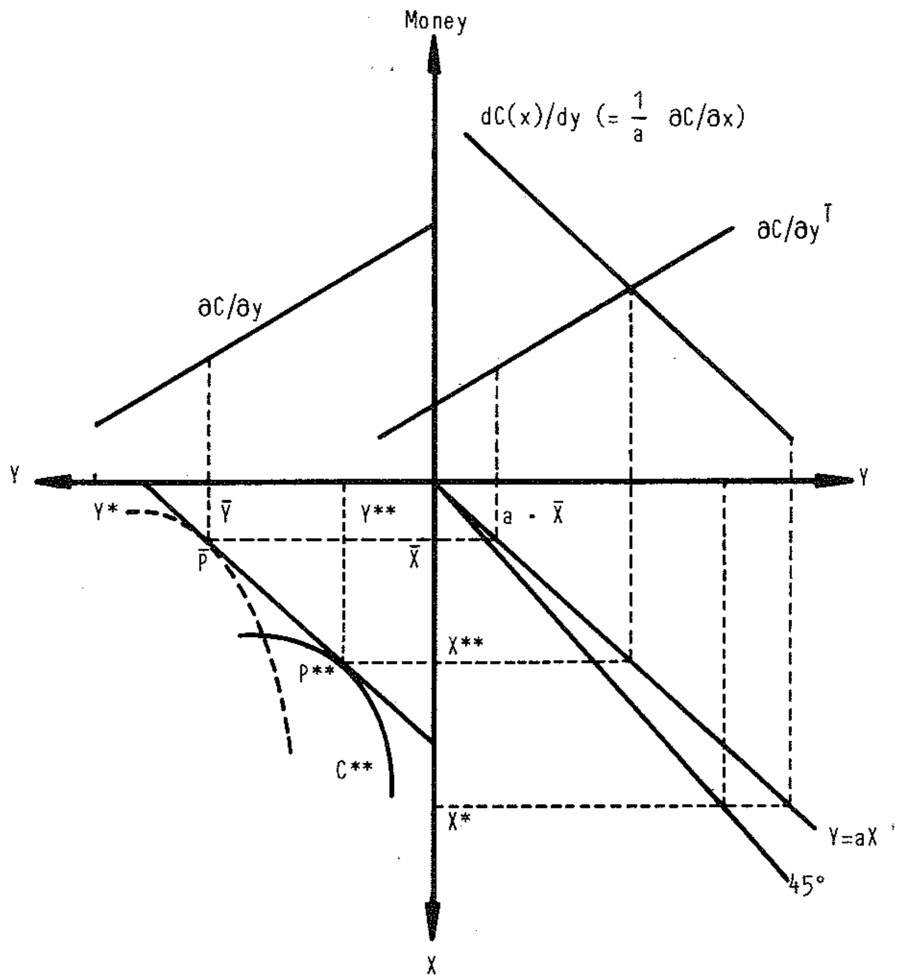

Figure 2 Transferable Discharge Permits with Linear Interaction

IV. Nonlinear Interaction

As an example for nonlinear interaction, below, a case of a target constraint exhibiting a concave and a convex section is considered. (See $\bar{I}$ in Figures 3, 4).

It should be noted that the Kuhn-Tucker conditions, given in section II may not represent the solution of the cost minimum situation in a case like that since the condition of quasi-convexity of the constraint function (with respect to the variables X, Y, i.e., quasi-concavity with respect to the variables x, y) is not met. Apart from that, a convex section in the constraint $\bar{I}$ may intersect the axis in a "cusp", possibly violating the constraint qualification.

Moreover, the problem of multiple optima may arise: In the example of Figure 3, a local optimum occurs at $\hat{P}(\hat{X}, \hat{Y})$ where an iso-abatement cost curve is tangent to the target curve $\bar{I}$. The global optimum, however, is in the corner P^{**} with $X = X^{**}$, $Y = 0$.

To avoid lengthy considerations, below, the discussion is confined to the points different

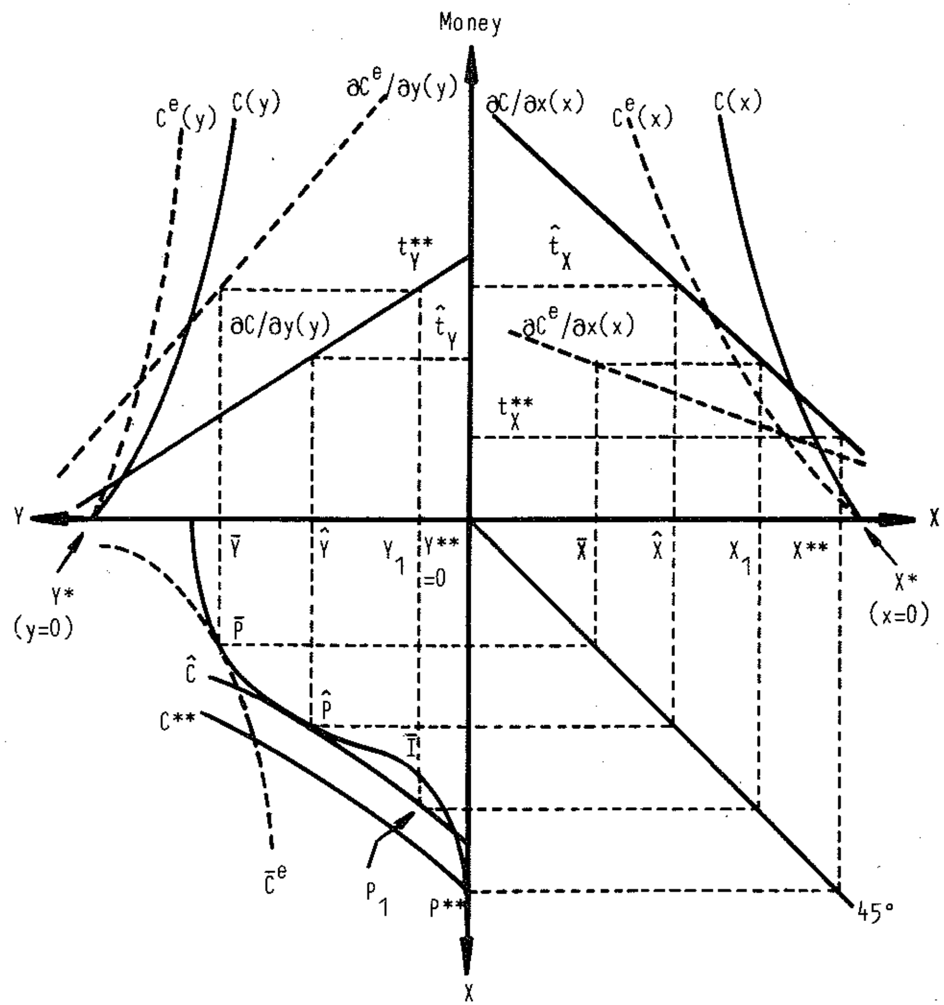

Figure 3 Effluent Charges in a Case of Nonlinear Interaction

from the case of linear interaction.

Effluent Charges

The corner solution $(X^{**}, Y = 0)$ may be achieved by means of effluent charges. In the example of Figure 3, tax rates t_X^{**} and $t_Y \geq t_Y^{**}$ are appropriate. However, opposed to the case of linear interaction, there is no simple decision rule to correct the tax rates if they turn out to have been misspecified in the first place. Moreover, the agency might end up in the local optimum $(\hat{X}, \hat{Y})$ using tax rates $\hat{t}_X$, $\hat{t}_Y$, missing the globally optimal corner solution.

Transferable Discharge Permits

A possible complication for the TDP policy in the nonlinear interactive case arises from the fact that the marginal cost of the X-industry to abate Y-equivalents may not be an

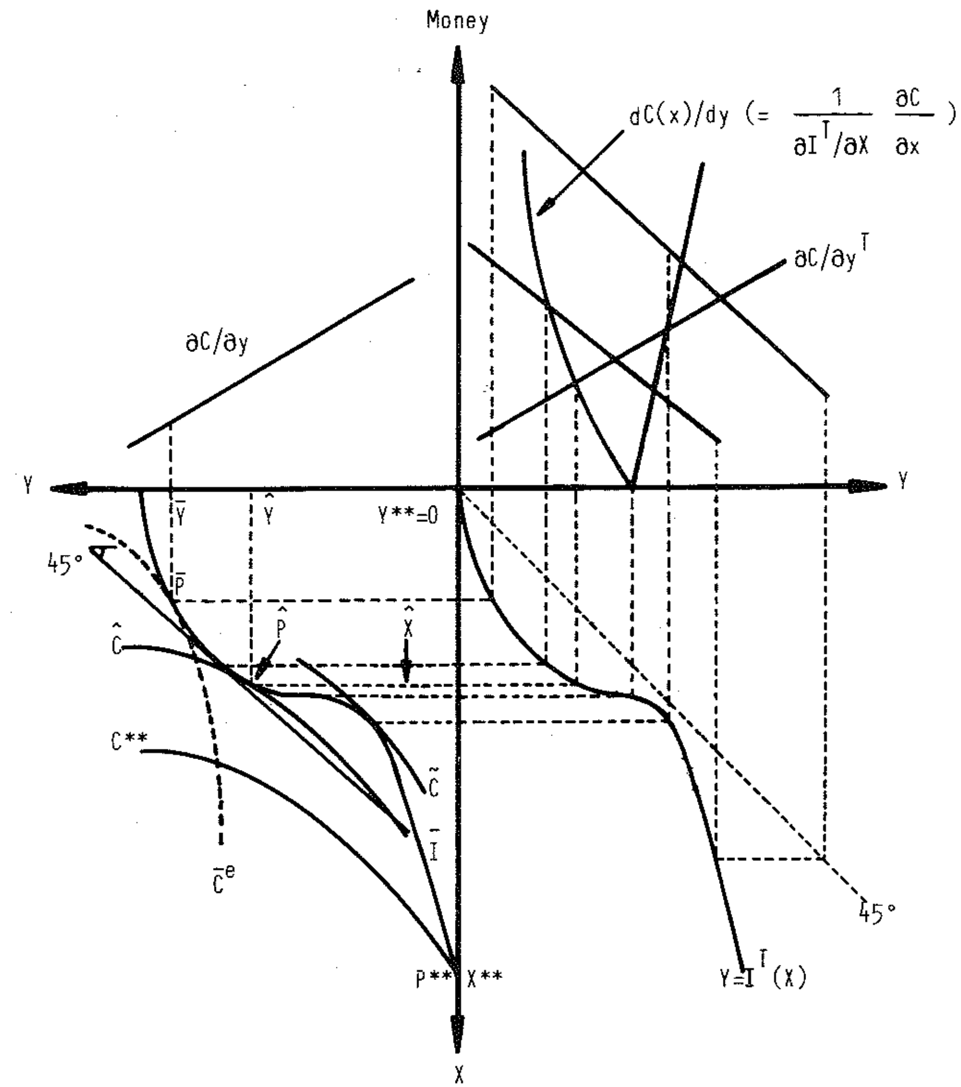

Figure 4 Transferable Discharge Permits in a Case of Nonlinear Interaction

increasing function of the number of Y-equivalents abated:

Still, the marginal abatement costs of the X-industry are supposed to be an increasing function of the quantity of pollutant X abated. This effect, however, may be overcompensated by the fact that to abate an additional Y-equivalent, less and less units of X have to be abated, in the convex part of the constraint $\bar{I}$. If overcompensation occurs, the marginal abatement cost curve of the X-industry in terms of Y-equivalents is U-shaped. In Figure 4, starting from an initial permit endowment of allowing emissions $\hat{X}, \hat{Y}$, just as supposed for the analysis of the linear interactive case, above, $\partial C/\partial y$ is the permit supply curve of the Y-industry, as it was in the linear interactive case. However, $dC(x)/dy$ is the permit demand curve of the X-industry only in its (towards zero emissions) upward sloping part.

Thus, if the globally optimal solution lies in the range of the downward sloping part of the curve, as it does in the example of Figure 4, it cannot be attained as a permit market equilibrium.

In the permit market, supply and demand meet in the local optimum $\hat{X},\hat{Y}$.

V. Conclusion

The possibilities to meet an interactive pollutant constraint at minimum cost have been considered. Effluent charges and transferable discharge permits have been used to represent the environmental policy options.

It turned out that in the case of linear interaction

—effluent charges are efficient, but have a problem of immediate ecological accuracy, well known from the literature on noninteractive pollutants,

—TDPs are efficient and accurate.

In the case of nonlinear interaction generating multiple optima,

—the global optimum is always attainable by effluent charges, in principle. In practice, however, the trial and error process may be very complicated and the agency might end up in a situation only locally optimal, if optimal at all.

—TDPs may loose their efficiency property, as the global optimum may be unattainable as a permit market equilibrium.

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