THE LONG-RUN IMPACT ON POPULATION AND INCOME OF OPEN ACCESS TO LAND IN A MODEL WITH PARENTAL ALTRUISM

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INTRODUCTION

The issue of a steady-state population is one that has been partly assumed away by modern growth theory, starting with Solow [1956], in which a constant returns to scale production function is assumed to exist in labor and capital so that population growth can continue in an unbounded manner and, if technological improvement occurs continuously, income per capita can also rise forever. Of course, these types of models do not appeal to those who see relatively near-term "limits to growth" or problems of sustainability, such as the authors in Costanza [1991] or biologists such as Wilson [1998]. The empirical issue of whether there are near-term limits to growth is not one this paper is intended to address directly. However, it is plausible to most people that the human population level has some finite upper bound, as exemplified by the many estimates of such upper bounds described in Cohen [1995].[1] Whether any ultimate limits are near or far, one can learn something from examining the question of the "optimum" population as it relates to the steady-state equilibrium in models that include both parental altruism and a cost of child-bearing.[2] This type of model was developed in some detail in Becker and Barro [1988]. A key feature of the Becker-Barro model is that parents care about both the number and dynastic utility of their children, which leads to a dynastic utility function which is a function of the number and utility of all future descendants.[3]

Harford [1997] notes that certain versions of this type of model lend themselves to consider population externalities.[4] Specifically, a steady-state population produced when all externalities are properly priced can be interpreted as the optimum population for the representative individual of any generation. This is so, in that such a population causes the representative parent to desire the number of children that will exactly replace her.[5] In the following section, it is assumed that both land and capital are applied to labor in a constant returns to scale production function. A comparison is made between the steady-state population in a world in which all share equally in land (an open access regime) and one in which land is privatized. An additional assumption is that the return to capital investment becomes only partly excludable when land is open-access. In other words, the private, but not the social, return to capital is diminished as a side-effect of the open-access nature of land. This assumption is partly inspired by North's [1981] discussion of how the development of

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more secure property rights was an important factor in encouraging investment and inventions. North specifically states,

> The First Economic Revolution [the transition from hunting and gathering to settled agriculture] was a fundamental change because it made possible the increase in the effective resource base and raised the private rate of return to improving that resource base through the incentives provided by property rights. [1981,67]

As North indicates, the incentive to engage in the sowing of seeds, weeding of crops, and the care of animals to be used for food and production of more animals requires secure private rights in the land upon which these are grown. Without secure property rights in land, building structures for sheltering people, animals, and stored crops becomes problematic. In short, all sorts of other property rights depend upon the notion of clear rights in land. Others have expressed similar views. Deacon quotes Thirgood [1981,58] on this point, "In the Mediterranean environment there is a clear relationship between the security that accompanies stable government and good husbandry of the land. Disruption of settled government has almost inevitably led to an increase in pastoralism" [1999, 344].

Even Julian Simon, noted for his optimistic views regarding progress and the role increased population plays in it, concedes that insecure property rights can destroy the incentives that lead to progress. Referring to Mexico in the 1970s, he states, "There are, however, places where, for negative reasons—usually wars, or fights about land tenure—good land that was formerly cultivated is no longer farmed. Frustrated by the slow pace of agrarian reform, Mexican peasants began seizing land. In fear of more seizures, the big estates then cut their investments" [1996,132].

Another fundamental idea developed within the model is the tendency for, and interpretation of, excessive child-bearing when there is an open-access resource. Dasgupta [1993] and Cohen [1995] are two important authors who have independently argued that open access to natural resource causes excessive child-bearing.[6] From a positive viewpoint, differing degrees of privatization of resources across societies and over time, by affecting the net private cost of bearing a child, can be seen as an explanatory variable for differing rates of child-bearing. An anecdote which indicates this motivation for the model is given by *The Economist*:

> Theodore Panayotou, an economist at the Harvard Institute for International Development, recalls visiting an area of northern Thailand where two hill tribes, the Karen and the Akha, had villages some 10-15 km apart. The Karen tribe had an effective system of communal land management so that each family had access to an agreed amount of land. The average family had three children. The entrance to the Akha village was dominated by a statue of a copulating couple; its men married only when their girlfriends were pregnant; its land management was chaotic, with households farming as much land as they could grab. The average Akha family had eight or nine children—a rational response, because each extra child increased the amount of land and other natural resources that a family could capture. [1994,25]

A dramatic event that may be a relevant example of the interplay between open access resources, population, and negative outcomes is that of the 1994 Rwandan civil war between the Tutsis and Hutus. In that strife a significant fraction of the population was killed or became refugees. Brander and Taylor [1998,134] suggest that the root of the conflict was rising population leading to resource degradation and a filling up of vacant land. The arguments made by Dasgupta [1993] and Cohen [1995], and incorporated into the model developed here, indicate that the presence of open access land was itself a factor in promoting the population growth that pushed Rwanda toward violent civil conflict. These anecdotes provide a vivid illustration that the degree of privatization ought to be seriously considered as a potential part of the explanation of historical changes in birth rates along with (the not entirely independent) factors such as wage and education levels considered by such authors as Becker, et al. [1999] and Galor and Weil [1999].

The following model demonstrates that an open-access policy toward land tends to generate overpopulation when the private productivity of capital is held constant. However, the reduction in capital investment, caused by the tendency of the return to capital to be partly captured by others, can lead to a situation where the population under the open access regime is lower than the steady-state population under the pure private ownership regime. An example is worked to provide some further insight into the relationship among utility and production parameters and steady-state values. A final section offers summary thoughts and conjectures.

AN OPEN-ACCESS PROBLEM IN A MODEL WITH LAND AND CAPITAL

Each individual is assumed to live one generation and have children who live and consume in the following generation. Ignoring individual subscripts, the total utility of an individual depends upon the utility derived from her current consumption $U[c]$, plus the weighted total utility of her children, $B[n]V[k',a'\bar{k}']$. (Brackets are used to enclose arguments of functions.) U is increasing and concave in c. V is the value of total utility for each child and B is an increasing concave function of the number of children n. B represents the total weight on the per child total utility and B/n can be interpreted as the generational discount factor. It is assumed that $B[0] = 0$ and $B[1] < 1$. The symbol k' stands for the capital inherited by each child, a' stands for the land inherited by each child, and $\bar{k}'$ stands for the level of per capita capital in the economy, taken as given by the individual. In recursive formulation, the value function of total utility satisfies Bellman's equation:

(1a) $$V[k,a,\bar{k}] = max\ \{U[c] + B[n]V[k',a',\bar{k}']\}$$

by choice of $[c,n,k']$ subject to

(1b) $$c + (p + k')n \leqslant f[(qk + (1-q)\bar{k}), a] + k$$

where k is capital owned by an individual in the current (parental) generation, a is land owned by an individual in the current generation, $\bar{k}$ is the economy-wide per capita capital in the current generation, p is the basic cost of raising each child, and f is the production function scaled to the individual level.[7]

The formulation is meant to encompass both the case of full and complete property rights in land and capital, and the case of open access to land. In the case of complete property rights, $a' = (1/n)a$, holds. In the open access regime, a' is taken as given but with a known law of motion. In this case, it is assumed that each parent correctly believes that each child will have an equal share in the total land available in the economy. Also, the parameter $0 < q \leqslant 1$ enters the production function as an indicator of the economy's regime. For closed access, $q = 1$ is assumed. Under an open-access regime, however, it is assumed that $q < 1$ obtains. This assumption is pursuant to our discussion indicating that investment in capital complementary to land is discouraged by open access in land. In essence, the fraction $(1 - q)$ of the return to any capital owned by an individual is captured by the rest of society. In turn, $(1 - q)$ of the rest of the economy's capital contributes to the production of each individual indicating the presence of an investment externality. This formulation is "optimistic" in the sense that all capital is assumed to be productive to society, which would not be the case if some capital were in the form of weaponry designed to take goods from others.[8]

The per capita production function is $f = F/N$, where F is a constant returns to scale production function in capital, land, and labor with convex isoquants. N is the relevant population. Each individual is assumed to be endowed with one unit of labor and thus N also represents the amount of labor. Thus, $f = F[kN, aN, N]/N = F[k, a, 1]$ with f a concave function of k and a.

With this formulation, the interior solution to maximization of equation (1a) subject to constraint (1b) can be written (ignoring individual subscripts and letting $s' = [k', a/n, \bar{k}']$):

(2a) $$U'[c] = \lambda,$$

(2b) $$B'[n]V[s'] + B[n]V_a[s'](-a/n^2) = \lambda(k' + p),$$

(2c) $$B[n]V_k[s'] = \lambda n,$$

with envelope conditions

(3a) $$V_k[s] = U'[c](1 + qf_k[qk + (1-q)\bar{k}, a])$$

(3b) $$V_a[s] = (B[n]/n)V_a[s'] + \lambda f_a[qk + (1-q)\bar{k}, a]$$

Condition (2a) indicates that the shadow price of a unit of capital bequest is the marginal utility of consumption by the parent. Condition (2b) indicates that the rate of gain from child-bearing, $B'[n]V[s']$, is reduced by its negative effect on land per child in the closed-access regime, $B[n]V_a[s'](-a/n^2)$, which is equated to direct per-child reduction in current consumption utility from the child-bearing cost and bequest, $\lambda(k' + p)$. In the open-access regime, the term $B[n]V_a[s'](-a/n^2)$ would not appear (be set to zero) as an approximation of the effect that having one more child would have on the amount of land available to other children of the same parent when there are numerous parents in a given generation. Equation (2c) indicates that the rate of gain in utility of the parent from enhancing the total utility of her descendants by increasing the capital bequest per descendent, $B[n]V_k[s']$, should equal the rate at which utility is reduced by increasing each bequest times the number of such bequests, λn.

We will analyze long-run steady-state equilibria in which total population is positive and constant, implying $n = 1$. This is reasonable. Given the assumed lack of any technical progress, it can be shown that a population that grows at a geometrically positive rate forever is simply infeasible given any production function with elasticities of substitution between capital and land, and labor and land that are one or less. This is because the per capita output f would eventually fall below the level $p + k$, the amount necessary for a parent to raise one child with enough capital to maintain positive consumption if it left the same bequest per child. This is due to the fact that per capita capital would have to keep increasing at a geometric rate (at least) in order to simply maintain per capita output by offsetting the geometric decline in land per person. On the other hand, it is necessary for the existence of a positive steady-state population that the utility of consumption available after paying for one child be positive. That is, $U[f^* - p] > 0$ must hold, where f^* is any proposed steady-state per capita output. Under the present formulation, there is no gain to having descendants all of whom will have negative utility given that one's "children" can obtain total utility of zero by not being born.

Given the restriction that we wish to characterize steady states with $0 < a^* < \infty$, one has $n = 1$ and $B[1] \equiv b < 1$. Substitution into the optimizing conditions of (2) and (3) yields the long-run equilibrium conditions:

(4a) $$f_k[k^*, a^*] = (1 - b)/(bq)$$

(4b) $$c^* + p = f[k^*, a^*]$$

(4c) $$1 = (U'[c]/U[c])\{[(1 - b)/b](k^* + p) + f_a[k^*, a^*]a^*\}.$$

For an open-access equilibrium, $q < 1$ and the term $f_a[k^*, a^*]a$ disappears from condition (4c). If $f_{ka} > 0$ along with $f_{kk} < 0$, it follows from (4a) that k^* is a strictly increasing function of a^*. Furthermore, given that f is a positive function of its arguments, assuming $f_{ka} > 0$ also insures that c^* is a strictly increasing function of a^*. Therefore, the question of the uniqueness of the equilibrium is reducible to the question of the uniqueness of a^*. Taking $k^* = k[a^*]$ from condition (4a) and $c^* = c[a^*]$ from conditions (4a) and (4b), if the right-hand side of condition (4c) were monotonic in a, then any solution would be unique. However, the minimal conditions needed to insure this are not clear.

To gain further insight into the possibilities, the special assumptions that $U = c^{\sigma}$, and $f = \mu a^{\alpha} k^{\theta}$ are adopted. This implies that the elasticity of current utility with respect to consumption is $E_{U_c} = \sigma$. The total land in the economy is defined to be A. After significant algebra, one finds that the steady-state population under the private property and open-access regimes are, respectively[9]

(5a) $$N^* = A\Psi\{[1 - \sigma(\alpha + \theta)]/[b + (1 - b)\sigma)p]\}^{(1-\theta)/\alpha}$$

(5b) $$N_O = A\Psi q^{(\theta/\alpha)}\{[1 - \sigma\theta q]/[b + (1 - b)\sigma)p]\}^{(1-\theta)/\alpha}$$

where

(5c) $$\Psi = (\mu b)^{(1/\alpha)}[\theta/(1 - b)]^{\theta/\alpha}$$

The uniqueness of steady-state population implies the uniqueness of $a^* = A/N^*$, and that of k^* and c^*, by the previous argument relating the three endogenous variables. It can be shown that both N^* and N_0 are strictly increasing in the intergenerational discount factor b over the relevant range. Population under either regime is proportional to available land (A), and negatively related to the basic cost per child (p), the elasticity of utility with respect to consumption (σ), and the elasticity of output with respect to land (α).[10] While less clear algebraically, calculations indicate that population is also a generally negative function of the elasticity of output with respect to capital (θ).

The ratio of steady-state population under open access to that of private efficient regime is

(6) $$(N_0/N^*) = q^{(\theta/\alpha)}\{[1 - \sigma\theta q]/[1 - \sigma(\alpha + \theta)]\}^{(1-\theta)/\alpha}$$

If $q = 1$, equation (7) is clearly greater than one, indicating that if the private productivity of capital is unaffected by open access to all land, then the steady-state population is larger under the open-access regime. However, it is clear that for q arbitrarily close to zero, $(N_0/N^*) < 1$ will hold, indicating that the negative effect on investment is sufficient to create a lower, as well as poorer, population under an open-access regime.

The significance of this is in the subtlety it introduces to the notion of overpopulation. An open-access regime may be overpopulated in the sense that a reduction in population would improve the dynastic utility of the representative individual, given the regime where all land is shared equally among however many individuals exist. This is because each individual in the open-access regime ignores the reduced availability of land to other people's descendants caused by an increase in her own descendants. But the steady-state population under the open-access regime can still be below the (efficient) population that would exist under the private ownership regime due to the discouraging effect of the lower incomes brought about by the lower levels

of capital that exist in the open-access regime.[11] In other words, it might simultaneously be true that the open-access regime leads to a population that is too large, and that the steady-state population is below the level that would exist in a regime of complete private property rights.

Turning now to the issue of capital, calculations indicate that the steady-state level of capital in the optimal and open-access cases are respectively:

(7a) $$k^* = \theta p[b + (1 - b)\sigma]/[1 - b][1 - \sigma(\alpha + \theta)]$$

(7b) $$k_o = q\theta p[b + (1 - b)\sigma]/[(1 - b)(1 - \sigma\theta q)]$$

Clearly, k_o is increasing in q over the relevant range. However, $k_o < k^*$ holds even if $q = 1$. Open access implies a higher population for any given capital level than does the closed-access allocation. Because the marginal product of capital is a positive function of land per capita and land per capita decreases in population, open access implies lower per capita capital even if the private return equals the social return to capital. Of course, this is not inconsistent with the possibility of greater total capital in the open-access economy.

Equation (7a) indicates that the optimal steady-state capital per person is proportional to p (the cost per child), an increasing function of σ (the elasticity of current utility with respect to consumption), of α (the elasticity of output with respect to land), and of θ (the elasticity of output with respect to capital) . Furthermore, k^* is an increasing function of the generational discount factor b.

Since steady-state population is a positive function of b, per capita land is a negative function of b. With a higher generational discount factor lowering land per person but raising capital per person, the effect of a higher b on steady-state per capita income is not obvious. However, using steady-state solutions, it can be shown that

$$\partial f^*/\partial b = -(psb^{-2})/[(1 - s(a + q)] = \partial c^*/\partial b < 0.$$

Thus, greater weight on the total utility of children leads to a lower steady-state level of per capita consumption.

Some Calculations with Specific Parameter Values

Further insight can be gained by examining the numerical examples that allow quantitative comparisons of steady-state solutions under differing parameter values. The numerical results are displayed in Table 1. Case I can be considered the base case. Cases II and III vary b above and below the base case value of .5. These latter cases show that the steady-state population is higher for higher values of the generational discount factor, given the other parameter values. Case IV lowers the value of α, which results in a dramatic increase in the population with relatively small declines in per capita levels of capital, production and consumption compared with Case I. Case V increases θ, which leads to a larger population, but also higher levels of per capita capital, production and consumption. Cases VI and VII vary the value of σ

TABLE 1
EXAMPLES OF OPTIMUM STEADY STATE POPULATION AND CAPITAL

CASES:	I	II	III	IV	V	VI	VII	VIII
PARAMETERS (A = 100, μ = 10 all cases; see text for definitions)								
b	0.5	0.7	0.3	0.5	0.5	0.5	0.5	0.5
α	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2
θ	0.2	0.2	0.2	0.2	0.3	0.2	0.2	0.2
σ	0.5	0.5	0.5	0.5	0.5	0.7	0.3	0.5
p	2	2	2	2	2	2	2	1
OPTIMUM STEADY STATE VALUES								
N (thousands)	10.1	54.9	1.0	1661	12.8	4.0	26.2	161.8
k	.75	1.42	.46	.71	1.2	.94	.59	.375
m	3.75	3.04	5.42	3.53	4	4.72	2.95	1.87
c	1.75	1.04	3.42	1.53	2	2.72	1.95	.87
OPEN ACCESS STEADY STATE (q = .5 in all cases)								
N (thousands)	10.1	54.6	.99	1011	9.5	5.6	19.4	160.9
k	.316	.596	.195	.316	.486	.366	.268	.158
m	3.16	2.56	4.56	3.16	3.24	3.66	2.68	1.58
c	1.16	.556	2.56	1.58	1.24	1.66	.680	.579

above and below the Case I value of .5. The results show that population is decreasing in the elasticity of consumption utility, while per capita capital, production and consumption are increasing. Finally, Case VIII halves p, which causes the population to increase by a factor of 16, while per capita capital and production are halved.

For comparison, values for steady-state population, per capita capital, production and consumption are included at the bottom of the table for a value of q = .5. Per capita capital, output and consumption are all lower in the steady state. However, because the private productivity of capital is only half its social value, population is below the level associated with complete private property rights in all but Case VI. Of course, for q = 1 steady-state population would be greater under open access. Thus, the private productivity of capital does not need to deteriorate drastically under open access in order to have a lower steady-state population.

CONCLUSION

Apart from issues of income distribution, the central issue affecting whether an economy can improve social well-being through collective action is the presence of externalities and related phenomena such as increasing returns and public goods. Defining private property rights more effectively and completely in order to reduce a "tragedy of the commons" addresses two problems at once. It discourages overuse of a resource by an existing population, and it discourages excessive child-bearing by imposing on the parent the recognition of how an added child reduces the available land (and other natural resources) per child. The gains from such privatization are even

greater if investments, which will improve the productivity of the resource, are thereby encouraged.[12]

With the insights of North in mind, one might say that if the world is less overpopulated than it was in the time of Malthus, it is due to technological progress; and the incentives for technological progress have been greater in modern times partly because of the more complete specification of property rights. Thus, complete property rights (and time) help to increase incomes, which raises the opportunity cost of each child, tending to reduce the desired number of children. This is a pattern that reflects the insights of both North [1981] and Becker and Lewis [1973] and others. However, it has been shown that a more complete set of property rights, independent of its effect on incomes, can tend to reduce the desired number of children. Thus, a move toward more complete property rights can affect child-bearing choices both indirectly and directly.

NOTES

The author wishes to acknowledge very helpful and extensive comments from the editor of this *Journal* and the referee who went well beyond the usual effort. The final product owes much to them. The author accepts the responsibility for any remaining errors or shortcomings.

1. Stokey [1998] uses various models of growth with pollution to theoretically explore the question of whether there are limits to growth. An important conclusion was that limits to growth exist whenever efficient environmental regulations reduce the rate of return on capital below the individual rate of time preference for some sufficiently large output. An important difference between the models used by Stokey and those presented here is that child-bearing was not endogenous in her models. In the present models, the individual rate of time preference is effectively a function of the number of children, while in Stokey's models it is a parameter. However, why her conclusion would not apply to the models of this paper is not obvious if one defines the rate of time preference as the "generational discount factor" when population and output are constant.
2. Even models that admit unbounded growth will tend to have a nature such that the optimal rate of growth will be affected in a similar way by many of the same factors that affect the optimal level of population in the models to be discussed.
3. The Becker-Barro model was adapted by Harford [1997; 1998] to explore how the traditional Pigouvian taxation of externalities such as pollution did not, by itself, eliminate child-bearing externalities. In these models a tax per child on the parent equal to the discounted present value of net external damages caused by that child and all its descendants was required to induce Pareto efficiency from the viewpoint of the representative individual in the generation making the decision.
4. Razin and Yuen [1995] use a Becker-Barro style model where human capital growth leads to a steady-state growth rate of population and income. They do not include any externality in their model, but focus on the difference in population and income growth rates implied by the Benthamite and Millian version of the dynastic utility function.
5. The question of the optimum population has a long history. Razin and Sadka [1995, 38] attribute to Bentham the idea that the proper social welfare function is one which maximizes the number of individuals times the utility per capita of society by the choice of population. They also indicate that Mill preferred the social objective of maximizing the per capita utility of the average or representative individual. Razin and Sadka [1995], Razin and Yuen [1995], Nerlove et al. [1985], and Palivos and Yip [1993] have demonstrated under various circumstances that the Benthamite objective function leads to a larger population than the Millian objective function. The philosopher Parfit [1984] has labeled the tendency for the Benthamite criterion to lead to a large population with a low standard of living the "repugnant conclusion."

Votey [1969] explored a model that parameterized a range of objective functions that encompassed each of these as special cases. Others, such as Michel and Pestieau [1993] and Samuelson [1975], have explored the problem of the optimum population in overlapping generation models that often assume an absence of parental altruism or a lack of child-bearing cost, in contrast to the type of model to be explored in the present paper. Michel and Pestieau [1993] examine a variant of the model used by Samuelson [1975] and commented upon by Deardorff [1976] in which there is no parental altruism, no child-bearing cost, and no endogenous choice of children. This model has been shown to produce optimality (under the preferences assumed) only by "serendipity." Schweizer [1996] uses several versions of the overlapping generations model to explore how the steady-state conditions relate to the Henry George Theorem (related to the efficiency and financing of club good choices). While some parental concern for children is introduced into some versions of his models, the type and degree of altruism is not the same as assumed in this paper, and no externality issues are addressed.

6. Dasgupta states "...local common property resources in poor countries have in recent studies been found to be a good deal less a source for free-riding than they have traditionally been taken to be; nevertheless, the static mis-allocation, however small, can cumulatively have a large effect on population" [1993,350-351]. Cohen refers to Kenya when he states, "Replacing communal cultivation of lands held by a clan with privately owned lands shifted some costs of large families from the community to the private landowner, giving incentives to limit family size" [1995,66]. Such statements are based upon the same view of incentives as that of Razin and Sadka when they state that, "For example, if poverty, in the sense of low family income per capita, tends to be associated with large family size, a system of child allowances and tax exemptions designed to alleviate poverty and reduce inequality may actually worsen the situation" [1995,136]. In all the statements, the fact that a new child can lay claim to resources that would otherwise go to unrelated individuals is taken to increase the net incentive to have children.
7. In the context of a similar model, Alvarez [1999] points out that the term $k'n$ in the constraint (1b) introduces a non-convexity which can potentially jeopardize the existence of a solution. Also, the fact that B is a function of a choice variable makes the dynamic programming problem non-standard. Alvarez is able to transform his problem to eliminate these technical issues. However, the author decided that the benefits of attempting a similar transformation of the current problem would exceed the expected costs.

   In earlier versions of this paper the author worked with the equivalent objective function written

$$V_t = U[c_t] + \sum_{T=t}^{\infty} \beta_{tT} U_{T+1}$$

where t is time and

$$\beta_{tT} = B_t ... B_T$$

is the discount factor on the utility of the representative descendant's utility $(T-t)$ generations in the future. However, the referee's suggestions led to the less cumbersome presentation seen in the text.

8. Not all possible negative effects of an open-access regime are included here. For example, the standard model of a fishery [Hartwick and Olewiler, 1998, Chapter 11] generally implies that open access will lead to a decline in the stock of fish, which makes each unit of fishing effort less productive. In the present model, this could be crudely captured by assuming that the effective amount of land declined. However, the effects on income of a reduction in the incentive to invest in capital and the incentive to overuse the resource which can both be caused by open access are very much the same.
9. The steady-state value of population for a model in which land and labor are the only productive inputs is easily seen as one where $\theta = 0$.
10. The negative relationship between α and N^* might be less obvious because of the various exponents in which α appears, but it can shown to be true for $N^* > 1$, which will hold in any sensible steady state.
11. If the present model made p a negative function of income, an empirically plausible relationship, the ratio of open-access population to optimal population would tend to be higher.
12. Issues of the tax treatment of children under the Social Security and income tax systems are being ignored. While the income tax in the United States provides tax reductions that are a positive function of the number of children, the effect of the Social Security system, as Becker and Barro [1988] demonstrate, is to reduce the net gain from children since each child is required to help pay for the retirement of all qualified elderly people.

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