AN EXPECTED UTILITY-USER'S GUIDE TO NONEXPECTED UTILITY EXPERIMENTS

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INTRODUCTION

The expected utility model has long been the standard for analyzing choice among risky alternatives. Almost since its inception, however, questions have been raised concerning its predictive validity. Experimental studies have shown that violations of expected utility theory are systematic and predictable. As more and more experiments have been performed, though, the patterns of behavior have become increasingly more complicated. The purpose of this survey is to help clarify some of the patterns emerging from recent experimental work and give examples of problems in which these patterns are relevant. If the patterns violate the assumptions of the expected utility model and expected utility is used anyway, some errors can occur; these errors are also discussed in the paper.

There are already several useful surveys of the nonexpected utility literature, but with different foci. For example, Machina [1987] and Fishburn [1988] survey the reasons why the expected utility hypothesis fails and possible alterations, concentrating mainly on its underlying assumptions. Camerer [1992] discusses some recent experiments to both clarify experimental technique and test alternative choice models. The distinguishing purpose of this survey is to take the experimental evidence as a primitive, find patterns in the evidence, and then illustrate some of the kinds of modeling errors that occur if these patterns hold but the expected utility model is used.

In essence, then, this paper is designed as a guide for those who apply expected utility theory and want to know what bearing the experimental evidence has on the appropriateness of their choice. Expected utility has proven to be extremely useful for analyzing behavior in risky situations, so it would be unfortunate to abandon it in favor of a more complex model. Furthermore, even though many alternatives to expected utility have been proposed, there is no single model which accommodates all of the evidence. As of yet there is no "successor" to the expected utility model. It is argued below that even if there were a successor, the expected utility model still would be useful, and, more importantly, it would still be appropriate for analyzing some types of decisions. Other types of decision problems, however, would be analyzed best using nonexpected utility models. Consequently, expected utility should not be abandoned, but rather applied with a little more caution.

Expected utility theory states that individuals choose among risky alternatives to maximize the mathematical expectation of the utility of the possible outcomes. Specifically, suppose that the individual must choose among lotteries with at most n

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(monetary) outcomes, $x_1,...,x_n$. Then the alternatives among which the individual must choose can be represented as probability vectors of the form $\mathbf{p} = (p_1,...,p_n)$, and the expected utility hypothesis states that there exists a utility function $u(\mathbf{x})$ such that the individual ranks probability vectors (lotteries) according to the preference function

$$EU(p) \equiv \sum_{i=1}^{n} p_i u(x_i) \tag{1}$$

This representation also can be constructed if the *independence axiom* and the *reduction of compound lotteries axiom* are added to the usual axioms of completeness, transitivity and continuity.[1] The preference ordering $\succcurlyeq$ satisfies independence if, for any two lotteries p and q, $p \succcurlyeq q$ if and only if for any third lottery r and any $0 < \alpha \leq 1$, $\alpha p + (1 - \alpha)r \succcurlyeq \alpha q + (1 - \alpha)r$. Informally, independence states that when an individual decides between a probability mixture of p and another lottery r, and the same mixture of q and r, the only thing that affects his choice is his preference between p and q.[2]

The other axiom, reduction of compound lotteries, concerns the way in which individuals treat lotteries whose payoffs are other lotteries. A *single-stage* lottery involves a probability distribution which offers (monetary) payoffs as outcomes, whereas a *compound* lottery involves a probability distribution which offers other lotteries as outcomes. Every compound lottery has an equivalent single-stage representation; that is, there is a single-stage lottery with the same distribution of monetary payoffs as the compound lottery. The axiom states that an individual prefers one compound lottery to a second if and only if he prefers the equivalent single-stage version of the first lottery to the equivalent single-stage version of the second. In other words, the only thing the individual cares about is the distribution of final payoffs, not the way the problem is represented. Both the independence and reduction of compound lotteries axioms are normatively appealing, and, in addition, the resulting model is useful.

The experimental evidence to be discussed in this paper often violates this model, and, because of the evidence, alternative decision models have been proposed. This survey restricts attention to the evidence, even though in many cases the evidence indicates a specific alternative to expected utility. An effort is made to discern patterns of violations, and conclusions are drawn from the patterns instead of the specific models. The evidence is organized into several distinct categories: risk attitudes and the failure of asset integration, evidence involving three-outcome lotteries (probability triangles), problem representation effects, and preference reversals. The paper concludes with a discussion of what this evidence implies for existing research based on expected utility theory.

ASSET INTEGRATION AND RISK ATTITUDES

The first category of evidence concerns the domain and shape of the utility function u. In expected utility theory the carrier of value is the final wealth position, but a body of experimental evidence suggests that the carrier of value is the change in wealth. In the latter case, the argument of the utility function can be thought of as a gain or a loss from some initial wealth position, allowing gains and losses to be treated differently. Specifically, risk attitudes can differ depending on whether the outcome represents a gain or a loss. If, on the other hand, individuals integrate assets before making decisions, so that utility depends on final wealth positions, then risk attitudes should be uniform over gains and losses.

The early consensus was that people are risk averse, since they tend to purchase insurance and avoid fair bets. If asset integration holds, then people should be risk averse over both gains and losses. Experimental evidence, however, shows that choice patterns tend to reverse when gain outcomes are changed to losses of equal magnitude, thereby rejecting the asset integration hypothesis [Kahneman and Tversky, 1979, hypothetical; Camerer, 1989, real; Battalio, Kagel and Komain, 1990, real; Kagel, MacDonald and Battalio, 1990, real].[3] The same pattern holds for laboratory rats [Battalio, Kagel and MacDonald, 1985, real; Kagel, MacDonald and Battalio, 1990, real].[4]

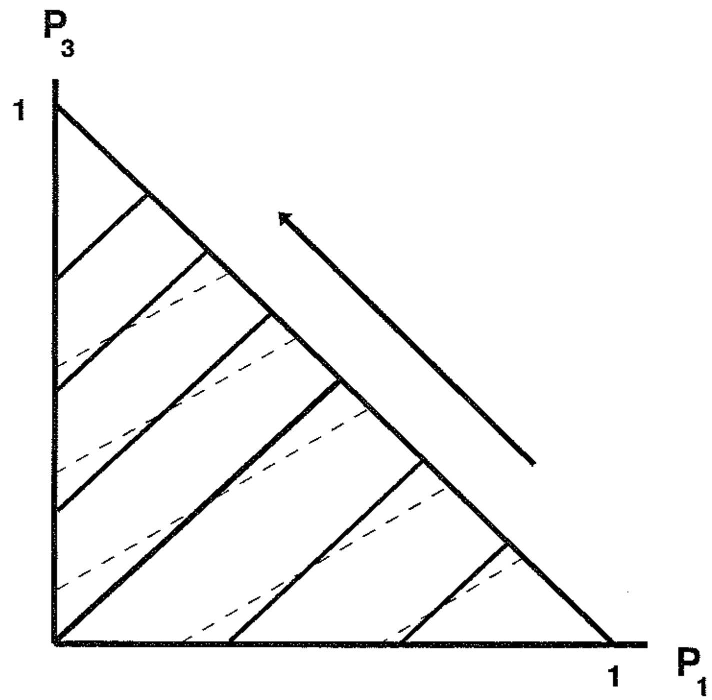
For example, Kahneman and Tversky [1979] find that 92 percent of their subjects prefer a hypothetical 0.8 chance of losing $4000 (with a payoff of $0 otherwise) to losing $3000 for sure. If the payoffs are changed to gains instead of losses, then 80 percent of the subjects prefer the sure gain of $3000 to the 0.8 chance of $4000. Battalio, Kagel and Komain [1990] confirm this pattern with smaller, real payoffs. They find that 65 percent of the subjects prefer a 0.6 chance of losing $20 to a sure loss of $12, while 81 percent prefer a sure gain of $12 to a 0.6 chance of gaining $20.

The above evidence suggests that individuals are risk loving over losses and risk averse over gains [see also Camerer, 1989, real]. Casual observation tells us, however, that people purchase actuarially unfair insurance and also buy lottery tickets. Kahneman and Tversky [1979] and Battalio, Kagel and Komain [1990] find that subjects are more likely to be risk averse when losses are unlikely and more likely to be risk loving when gains are unlikely.[5] The pattern that emerges, then, is that risk attitudes are different depending on the likelihood of an extreme event (either loss or gain) and that attitudes are reversed for gains vs. losses.

PREFERENCES OVER THREE-OUTCOME LOTTERIES

Three-outcome lotteries are especially important because many experimental studies, including the Allais paradox, use lotteries with at most three outcomes, and because these lotteries can be depicted in a simple diagram designed to illustrate indifference curves over lotteries. Let the three outcomes be $x_1 < x_2 < x_3$, so that a lottery p is characterized by its probabilities on each of the three outcomes. Since $p_1 + p_2 + p_3 = 1$, each lottery is completely characterized by just two of the three component probabilities, and, according to convention, we use p_1 and p_3. The space of probability distributions with outcomes x_1, x_2, x_3 can then be represented in (p_1,p_3) space by the triangle with vertices at (0,0), (0,1) and (1,0), that is, the set $P = \{(p_1,p_3) \mid p_1 + p_3 \leq 1\}$, as in Figure 1.

FIGURE 1



To derive the indifference map for an expected utility maximizer, set $p_1u(x_1) + p_2u(x_2) + p_3u(x_3) = v$, use the transformation $p_2 = 1 - p_1 - p_3$, and solve for p_3 to get

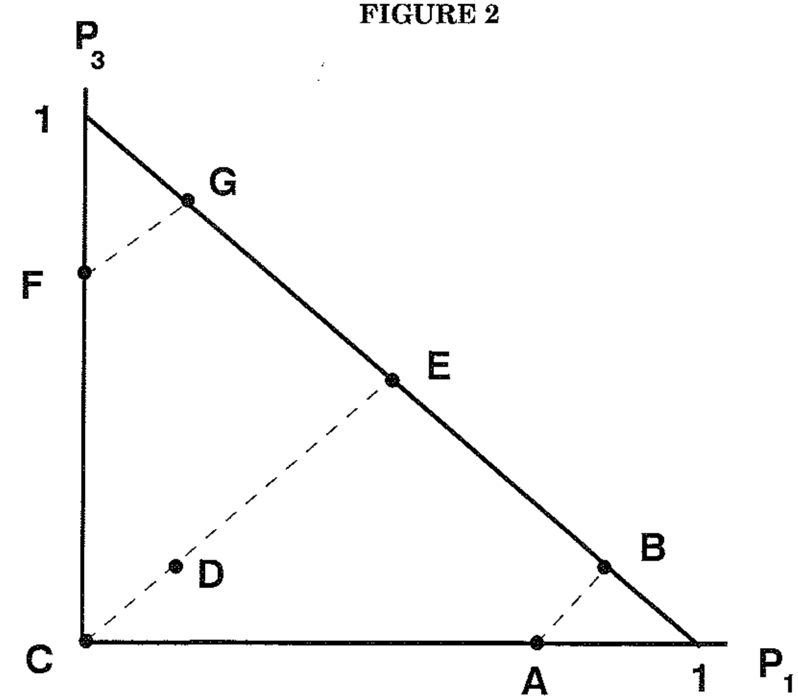
(2)
$$p_3 = \frac{u(x_2) - u(x_1)}{u(x_3) - u(x_2)}p_1 + \frac{v - u(x_2)}{u(x_3) - u(x_2)}$$

From equation (2) it follows that indifference curves are parallel straight lines. If u is an increasing function, movements to the northwest in the triangle correspond to higher expected utility values. The slope of the indifference lines reflect the individual's degree of risk aversion. The individual is risk averse if mean preserving increases in risk make the individual worse off. The set of points in (p_1,p_3) space with expected value μ is given by the equation

(3)
$$p_3 = \frac{x_2 - x_1}{x_3 - x_2}p_1 + \frac{\mu - x_2}{x_3 - x_2}$$

In Figure 1, the dashed lines represent iso-expected value loci and the solid lines represent indifference curves. Movements to the northeast along the dashed lines represent mean preserving increases in risk. For the individual to be risk averse, the solid lines must be steeper than the dashed lines, that is,

FIGURE 2



(4)
$$\frac{u(x_2) - u(x_1)}{u(x_3) - u(x_2)} \geq \frac{x_2 - x_1}{x_3 - x_2}$$

In fact, a straightforward extension of the above analysis shows that an expected utility maximizer with utility function u^* is more risk averse than one with utility function u if and only if u^* generates steeper indifference lines in every triangle than u, so that the slope of the indifference lines really is a measure of the degree of risk aversion.

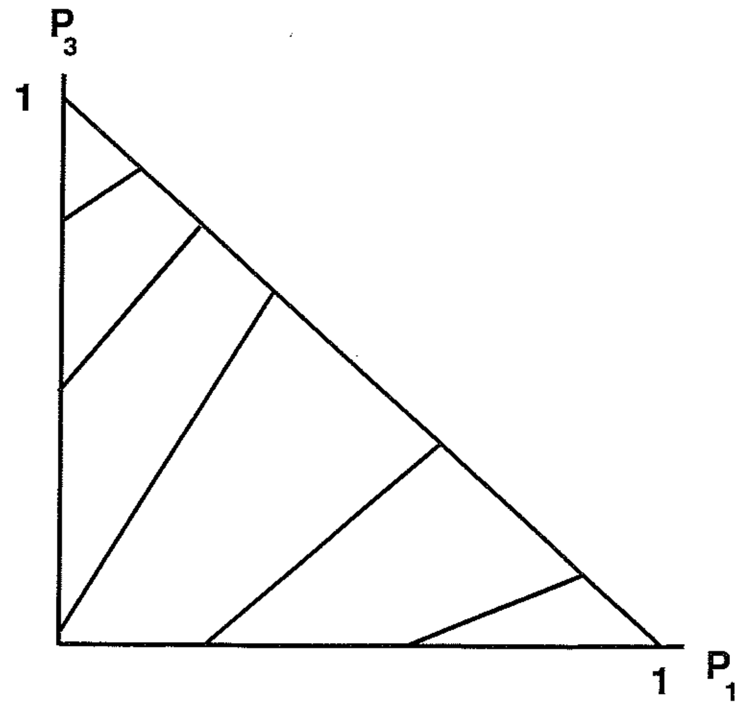
For probability triangles, expected utility theory has two testable implications: (1) indifference curves are straight lines, and (2) they are parallel. These hypotheses can be tested by determining preferences over lottery pairs in different regions of the triangle. For example, consider Figure 2. The individual is asked to state preferences for the choice pairs (A,B); (C,D); (C,E); (D,E); and (F,G). All of the lines connecting the points in the choice pairs are parallel, so an expected utility maximizer must choose either the first member of every pair or the second member of every pair. Any other choice pattern violates expected utility.

The first such experiment was conducted by Maurice Allais, using the choice pairs (A,B) and (C,D). Subjects were given a choice between C, which was $1 million for sure, and D, which had a 0.1 probability of $5 million, a 0.89 probability of $1 million, and a 0.01 probability of $0. Subjects were then given a choice between A, which had a 0.11 probability of $1 million and a 0.89 probability of $0, and B, which had a 0.1 probability of $5 million and a 0.9 probability of $0. Allais and other researchers have found that the modal choice pattern is C over D but B over A, which violates expected utility. Many subjects also choose one of the two patterns consistent with expected utility, but very few subjects choose the pattern of D over C but A over B. The fact that expected utility violations are systematic, rather than random, is what initiated the development of nonexpected utility models.

Experiments of this type have also been performed by Chew and Waller [1986, hypothetical], Camerer [1989, real], Conlisk [1989, hypothetical], Battalio, Kagel and Komain [1990, real], Kagel, MacDonald and Battalio [1990, real], Prelec [1990, hypothetical], MacDonald, Kagel and Battalio [1991, real], and Gigliotti and Sopher [1992, hypothetical]. Although many subjects have expected utility payoffs, the violations tend to be systematic, and when all three outcomes are gains, violations tend to fit the following pattern: B is preferred to A, C is preferred to D, and C is preferred to E. A large amount of evidence also suggests that G is preferred to F. If one draws indifference lines consistent with these choices, then the indifference line through C must be steeper than the indifference line through A and the indifference line through F, rejecting the hypothesis that indifference lines are parallel. Instead, indifference curves seem to start out flat in the southeast region of the triangle, get steeper with movements to the northwest, and then get flatter again.[6] In other terminology, the indifference curves seem to *fan out* (from the origin) for the less-preferred region of the triangle, and *fan in* for the more-preferred region (see Figure 3). If outcomes are losses, then this pattern reverses [Camerer, 1989; Battalio, Kagel and Komain, 1990; Kagel, MacDonald and Battalio, 1990], so that there is fanning out for the more-preferred region and fanning in for the less-preferred region.[7]

As for the curvature of the indifference curves, the property that indifference curves are straight lines is known as *betweenness*.[8] The alternative hypotheses are *quasiconcavity* and *quasiconvexity*. Quasiconcavity is a preference for a probability mixture of two indifferent lotteries to both of the two component lotteries, and can be thought of as a preference for randomization. Quasiconvexity is the opposite and can be thought of as an aversion to randomization. The evidence concerning betweenness is not as clear cut as the evidence for fanning. For example, Coombs and Huang [1976, real] find that 55 percent of subjects satisfy betweenness, Chew and Waller [1986, hypothetical] find 73 percent, Prelec [1990, hypothetical] finds 24 percent, and Gigliotti and Sopher [1992, hypothetical] find 50 percent. The first three studies find that violations are in the direction of quasiconcavity for gains and quasiconvexity for losses; that is, most of the subjects who violate betweenness would prefer D to C and D to E when outcomes are gains in Figure 2. MacDonald, Kagel and Battalio [1991, real] find that rats also exhibit quasiconcavity over gains. Gigliotti and Sopher, on the other hand, find that 42 percent of all subjects have quasiconvex preferences over gains. Conlisk [1987, hypothetical] also finds strong support for quasiconvexity over gains, with 53 percent of subjects in one experiment and 41 percent of subjects in a second experiment exhibiting that property. Camerer [1989, real] supports quasiconvexity over gains and quasiconcavity over losses overall, which has some intuitive appeal because quasiconvexity can be thought of as a dislike for randomization, so that risk aversion and randomization aversion occur in the same domain. However, Camerer also finds different patterns for

FIGURE 3



different regions of the triangle, with quasiconcavity near the axes and quasiconvexity near the hypotenuse when outcomes are gains, and the opposite pattern for losses.[9] The inconclusiveness of the evidence suggests two courses of actions. First, since a large number of subjects do violate betweenness, theorists should explore the implications of the alternative hypotheses, but since neither alternative hypothesis dominates, the implications of both must be explored. Second, because a large number of subjects obey betweenness, and because there is conflicting evidence about the direction of violations when they do occur, theorists may want to retain the betweenness hypothesis when exploring the implications of other nonexpected utility patterns.

Somewhat surprisingly, fanning and curvature violations of expected utility disappear when all of the lotteries in Figure 2 are moved off of the boundary and into the interior of the triangle [Camerer, 1992, real; Conlisk, 1989, hypothetical; Harless and Camerer, 1991, both; Gigliotti and Sopher, 1992, hypothetical; Harless, 1992b, real]. In fact, expected utility theory seems to work extremely well when all lotteries have the same number of outcomes, that is, when points on the boundary are disregarded.[10] This fact has been termed a *boundary effect* [Conlisk, 1989]. For example, when lotteries involve gains, 42 percent of Gigliotti and Sopher's evidence is consistent with expected utility when lotteries are off the boundaries, but only 22 percent of their evidence is consistent when some lotteries are on the boundaries. Similarly, Conlisk finds that 68 percent of subjects behave consistently with expected utility when all lotteries are off the boundaries, and this number falls to 50 percent, when all lotteries are on the boundaries. Harless only tests off-boundary choices, and 50-75 percent of choices are consistent with expected utility theory.

PROBLEM REPRESENTATION EFFECTS

Most choice problems can be stated in several ways without changing the probability distributions underlying the different alternatives. For example, lotteries involving gains can be rephrased by giving the individual a nonstochastic wealth increase before the choice must be made and stating the alternatives in terms of losses. It has been found that this type of change, known as a *framing* effect, often causes individuals to change their preference ranking [Tversky and Kahneman, 1981; 1986, both]. Two other problem representation effects are *juxtaposition* effects, in which it matters how the two lotteries in the choice pair are correlated [Loomes and Sugden, 1987a, real; Starmer and Sugden, 1989, real; Harless, 1992a, real], and *reduction* effects, in which individuals fail to reduce compound lotteries to the equivalent single-stage lottery [Conlisk, 1989, hypothetical].

A typical framing effect is reported by Tversky and Kahneman [1986, both]. Problem 1 asks the subject to assume he is $300 richer than he is now and decide between a sure gain of $100 and a 50:50 chance of $200 or nothing. Problem 2 asks the subject to assume he is $500 richer than he is now and decide between a sure loss of $100 and a 50:50 chance of losing $200 or nothing. In Problem 1, 72 percent of the subjects chose the sure thing whereas in Problem 2 only 36 percent of the subjects chose the sure thing. Given the failure of asset integration, and the fact that individuals tend to be risk averse over gains and risk loving over losses, one can predict the direction of framing effects. If the lotteries are stated in terms of gains, then the person will behave in a risk averse manner, but if the person is given an addition to wealth and the lotteries are stated in terms of losses, we would expect the person to behave in a risk loving manner.[11] The problem is determining why the individual adjusts initial wealth immediately, instead of incorporating it into the probability distribution of possible outcomes relative to the original wealth.

Juxtaposition effects refer to how payoffs from one lottery are correlated with payoffs from a second lottery. Consider two lotteries: A offers payoff a with probability p_a (and zero otherwise), and B offers payoff b with probability p_b, with $a > b > 0$ and $p_a < p_b$. There are four possible states of the world: in state S_1 both lotteries pay, in S_2 only A pays, in S_3 only B pays, and in S_4 neither pays. Letting p_i denote the probability of state S_i, we have $p_1 + p_2 = p_a$, and $p_1 + p_3 = p_b$, which means that $p_2 < p_3$. This can be conveniently shown in a table such as the one below.

	S_1	S_2	S_3	S_4
	p_1	p_2	p_3	p_4
A	a	a	0	0
B	b	0	b	0

Experiments on juxtaposition effects show that changing the probabilities of the states without changing the probabilities of the payoffs may cause individuals to change their preferences over A and B. A notable example can be found in Starmer and Sugden [1989, real], where a = £11, b = £7, p_a = .14, and p_b = .20. Consider the two problems represented below.

Problem 1	0.14	0.06	0.80
A	11	0	0
B	7	7	0

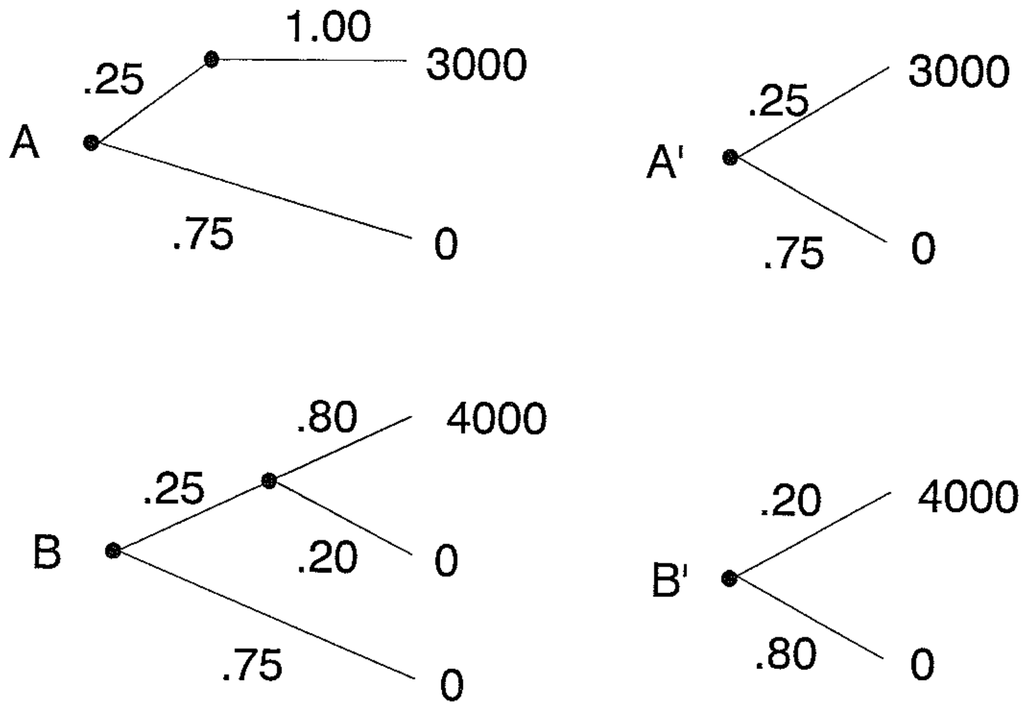
Problem 2	0.14	0.20	0.66
A	11	0	0
B	0	7	0

In Problem 1 there is maximum overlap of payoffs, that is, whenever A has a positive outcome B also has a positive outcome. In Problem 2 there is no overlap of payoffs, that is, A and B have positive outcomes in different states of the world. 61 percent of the individuals chose the same lottery in both problems with 44 percent choosing A both times. Of the 39 percent of the subjects who switched their choices, however, 76 percent switched from B to A. They chose the less risky lottery (B) when payoffs are overlapped and the riskier lottery when payoffs are not overlapped.

Harless [1992a, real] finds that results on juxtaposition effects are very sensitive to the manner in which the problems are represented for the subjects. For example, juxtaposition effects matter when the choices are presented in matrix form, as above, but do not matter when juxtaposition is achieved through ticket numbers; that is, payoffs are assigned to ticket numbers and then a ticket is drawn randomly. In Problem 1 above, there would be 100 tickets, and lottery A would pay £11 for tickets 1-14 and zero otherwise, while lottery B would pay £7 for tickets 1-20. In Problem 2 lottery A would pay £11 for tickets 1-14 and lottery B would pay £7 for tickets 15-34. Harless finds no systematic differences between choices with the ticket representation of the problems. Since matrix representations rarely appear outside of the laboratory, this questions the importance of juxtaposition effects.

The third problem representation effect has to do with reduction of compound lotteries. A *single-stage* lottery involves a probability distribution which offers (monetary) payoffs as outcomes, whereas a *compound* lottery involves a probability distribution which offers other lotteries as outcomes. If individuals convert compound lotteries into equivalent single-stage lotteries before making their decisions, then they are said to *reduce* compound lotteries. An example of a failure to reduce compound lotteries can be found in Kahneman and Tversky [1979, hypothetical],

FIGURE 4



where the subjects are faced with a choice between lottery A and lottery B in Figure 4. Each lottery involves two stages, and the first stage is simply a 0.75 probability of making it to the second stage. For choice A the subject receives a certain $3000 if the second stage is reached, while for choice B the subject faces a 0.8 probability of $4000 and a 0.2 probability of nothing. 78 percent of the subjects chose A. The equivalent single-stage lotteries are shown as A′ and B′, and when faced with A′ and B′ directly, 65 percent chose B′. Both lotteries have the same underlying payoff distribution, and the reduction of compound lotteries axiom predicts that subjects choose either A and A′ or B and B′. The experimental result, which has been substantiated by Conlisk [1989, hypothetical], Starmer and Sugden [1991, real] and Carlin [1992, hypothetical], is interpreted as a failure to reduce compound lotteries.

PREFERENCE REVERSALS

The final category of experimental evidence defines what has become known as the preference reversal phenomenon.[12] Individuals express a preference for one lottery over a second lottery, but assign a higher monetary value to the second lottery. A standard example, due to Grether and Plott [1979, real], involves a P-bet which has a 35/36 chance of winning $4 and losing $1 otherwise, and a $-bet which has an 11/36 chance of winning $16 and losing $1.50 otherwise. The P-bet offers a higher probability of winning, and the $-bet offers a larger prize. Typically individuals prefer the P-bet to the $-bet but assign a higher monetary value to the $-bet, and this behavior has been verified by a large number of experimenters for many different payoff values, although raising the stakes does seem to reduce the frequency of reversals [Pommerehne, Schneider and Zweifel, 1982, real; Reilly, 1982, real].

A number of different explanations of the preference reversal effect have been proposed. The obvious candidate is that preferences are intransitive, since the evidence states that there is some dollar amount X such that $P \succ \$ \succ X \succ P$, where $\succ$ denotes the strict preference relation. However, Holt [1986] and Karni and Safra [1987] show that these effects may also be caused by particular features of the experimental design if the independence axiom fails but transitivity does not. Holt shows how preference reversals can be attributed to how subjects are actually paid during the experiment,[13] and Karni and Safra show how they can be attributed to the manner in which certainty equivalents of lotteries are elicited.[14] In fact, Safra, Segal and Spivak [1990] show that, given the manner in which certainty equivalents are elicited, preference reversals are consistent with preferences satisfying the properties of betweenness, fanning out, and risk aversion. Following a different direction, Segal [1988] shows that preference reversals may be caused by failure to reduce compound lotteries instead of intransitivity.

Cox and Epstein [1989, real] and Tversky, Slovic and Kahneman [1990, real] designed experiments to see if preference reversals are caused by either failure of independence or failure to reduce compound lotteries. They find that reversals persist even when the experimental design eliminates situations in which independence or reduction of compound lotteries may govern choice. Tversky, Slovic and Kahneman further refine their experiment to determine if reversals are caused by intransitivity or by a tendency to either overvalue the $-bet or undervalue the P-bet. They found that only about 10 percent of reversals are caused by intransitivity and about 65 percent are caused by overvaluing the $-bet. This last result suggests that decisions, to some extent, are task dependent because the task of comparing two lotteries is different from the task of assigning certainty equivalents. Given the persistence of preference reversals, the question arises as to whether they can be eliminated in market settings. Chu and Chu [1990, real] argue that preference reversals are eliminated by arbitrage procedures, but Berg, Dickhaut and O'Brien [1985, real] find that arbitrage reduces the dollar amount of reversals but not their frequency. Cox and Grether [1992, real] find that although preference reversals do not disappear in a repeated market setting, they become less frequent, and only about half of the reversals are in the predicted direction.

IMPLICATIONS FOR MODELING BEHAVIOR TOWARD RISK

In this paper several preference patterns which are contrary to traditional expected utility theory have been identified. Choices depend on changes in wealth instead of final wealth positions, indifference curves exhibit fanning properties, and individuals may exhibit quasiconvexity over gains and quasiconcavity over losses. Framing matters, juxtaposition effects may matter, and people do not reduce compound lotteries. Finally, preference reversals are a persistent problem. How important all these violations are to someone who uses expected utility depends on whether or not the choice patterns in question have any relevance in the types of decision problems economists generally analyze.

First look at the problems caused by fanning patterns. When fanning out is exhibited, the indifference curves become steeper with movements to the northwest in the probability triangle. Machina [1982] characterizes this as individuals becoming more risk averse at each payoff level when there is a stochastically dominating shift in the payoff distribution. Stochastically dominating shifts are movements to the northwest in probability triangles, and the steeper the indifference curve the more risk averse is the individual. Similarly, fanning in means that the individual is made less risk averse by stochastically dominating shifts. These definitions can be used to discuss the implications of fanning patterns in a simple portfolio choice problem discussed in Neilson [1992a].

An individual must decide how to divide his wealth between a riskless asset and a risky asset with higher expected returns. If event E occurs, which happens with probability p, the individual's wealth is determined by the selected portfolio. If the event $\sim E$ occurs, then the firm through which he invests declares bankruptcy, and the investor recovers the deterministic amount R, which is less than initial wealth. If F_a is the distribution function of the payoffs from the portfolio in which a fraction a of wealth is invested in the risky asset, the individual chooses the most preferred distribution of the form $pF_a + (1-p)\delta_R$, where δ_R is the degenerate distribution assigning probability one to outcome R. The independence axiom now can be applied directly to this choice problem. If F_{a*} is preferred to F_a, then the independence axiom states that $pF_{a*} + (1-p)\delta_R$ is preferred to $pF_a + (1-p)\delta_R$. Consequently, the optimal portfolio is independent of either the likelihood or the severity of bankruptcy, and thus the effect of a change in either p or R is zero. Neilson [1992a] shows that this result no longer holds in the presence of fanning patterns. If fanning out holds, increases in p or R represent stochastically dominating shifts in the payoff distribution, which make the individual less risk averse, which in turn increases his demand for the risky asset.

It is by no means clear how we should expect decision makers to behave in this situation. Intuition based on years of working with the expected utility model would say that the probability or severity of bankruptcy should have no impact, but this intuition leads us astray when fanning patterns exist. Another case where intuition based on using expected utility fails is in second price auctions [Chew, 1989; Karni and Safra, 1989; Neilson, 1993]. In a second price auction, bidders simultaneously submit sealed bids, the highest bidder wins the prize, and the winner pays the second highest bid. The optimal strategy for an expected utility bidder is to bid his value of the prize, and this strategy is a dominant strategy, so that it does not depend, for example, on how many other bidders there are.[15] When expected utility fails and the prize itself has a random value, however, the optimal strategy is much more complicated.

When the prize is random, the value that an expected utility maximizer bids is the certainty equivalent of the prize, and the certainty equivalent is determined by how risk averse the individual is. Suppose that fanning out holds, for example, and that one more bidder enters. The existence of the new bidder both reduces the individual's probability of winning and increases the expected payment he must make if he does win. These effects push the individual into the region in which he is less risk averse (they both make him worse off), which makes his certainty equivalent of the prize increase, and therefore he bids more. In fact, there is no dominant strategy for bidders when bidders exhibit fanning out, and their optimal bids depend on the behavior of other bidders [Neilson, 1993]. If one of the other bidders increases his bid, for example, this pushes the first individual into the region where he is less risk averse, and he bids more, too. This means that the appropriate concept for analyzing these auctions is Nash equilibrium, instead of the dominant strategy equilibrium concept which can be used when bidders are expected utility maximizers.

These examples show two circumstances in which fanning patterns affect not only the solutions to decision problems, but also the way in which the problems must be thought about. The same thing holds true with violations of betweenness. First, if betweenness is violated, second order conditions for maximization may not hold, and therefore corner solutions may be more common. For example, if an individual is an expected utility maximizer, risk aversion implies that the individual will be a diversifier [Tobin, 1958], which is the proper second order condition for maximization in a portfolio choice problem.[16] Dekel [1989] shows that if betweenness fails, risk aversion may not be enough, and that risk aversion *and* quasiconcavity are needed for the individual to be a diversifier. To see how this works, refer back to Figure 2. There are two assets with payoff distributions corresponding to points C and E in the probability triangle, and assume the individual is indifferent between C and E. Now consider point D, which is a linear combination of C and E. If the individual is an expected utility maximizer, betweenness implies that he is also indifferent between C, D and E, which fits the requirements for being a diversifier. If, however, his preferences are quasiconvex, he strictly prefers C and E to D, so he is not a diversifier.

Crawford [1990] shows that Nash equilibrium may not exist if preferences are quasiconvex, and so another of the usual tools of economic analysis is called into question.[17] The problem arises when trying to find mixed strategies. If one of the players is to play a mixed strategy, he must be indifferent between the actions over which he is mixing. If his preferences satisfy betweenness, he is also indifferent between all of the mixed strategies over those actions. If his preferences are quasiconvex, though, he prefers the two pure actions to all of the mixed strategies, so he will prefer to play a pure strategy. This means that if players' preferences are quasiconvex, Nash equilibrium exists only if there is a pure strategy equilibrium. If preferences are quasiconcave, there is no problem, because then the individual prefers a mixed strategy to any of the pure strategies between which he is indifferent.

Unlike accommodating fanning patterns or violations of betweenness, accommodating failures of asset integration requires only a minor break from expected utility, since the utility function in equation (1) could be easily changed to $u(x,w)$, where w is reference wealth and x is the change in wealth.[18] Reference wealth must be included to accommodate properties such as decreasing risk aversion. As yet, however, it has not been determined how much or when reference wealth changes in response to choice situations. For example, Tversky and Kahneman [1981] present framing problems in which individuals immediately update initial wealth, as above, but it has also been noted that betting on longshots at horse races significantly increases during the course of the racing day. This latter phenomenon has been explained as individuals who have lost large amounts during the day attempting to recoup their losses through longshot bets, which fits well with the joint hypothesis that reference wealth is held constant throughout the day and individuals are risk loving over losses. Together these suggest that reference wealth is updated at the beginning of a decision process, but not during the course of a sequential decision process. If the individual is risk loving over losses and risk averse over gains relative to reference wealth, the exact position of reference wealth is important for analyzing choices.

Preference reversals pose a more fundamental problem. The prevailing wisdom on preference reversals is that people use one choice function to choose between pairs of alternatives and a second choice function to assign monetary values to lotteries. Irwin et al. [1993] investigate the implications for environmental improvements. Subjects were asked which they would prefer, an improvement in a consumer product, such as a VCR, or an improvement in environmental quality, and then they were asked to state how much they were willing to pay for each type of improvement. Most subjects preferred the environmental improvement but were willing to pay more for the consumer product improvement, which is a classic preference reversal. This type of behavior is probably widespread and significant, and its implications must be studied more carefully.

IMPLICATIONS FOR USERS OF THE EXPECTED UTILITY MODEL

Fanning patterns and betweenness violations, at least, significantly impact both the way that choice problems are analyzed and the resulting solutions. The other effects should have equal impact, although they have not received as much attention in papers which attempt to apply nonexpected utility models. In light of all this, what is an economist who uses expected utility to do?

The first step is to determine how one treats the evidence. One interpretation is that the evidence reflects true decision-making behavior, and therefore modelers of decision making must be careful not to make assumptions which contradict the relevant evidence. Another interpretation is that some of the biases are artifacts of faulty experimental motivation and control, and that these biases could be consistent with expected utility if we allow for decision errors which arise as a response to the faulty experimental method. Harless and Camerer [1991] explore this approach, and find that under this hypothesis the calculated decision errors are large, but they are also large for all of the nonexpected utility models which have been designed to accommodate the evidence.

This leaves several options open, including

(1) Treat expected utility violations as decision errors and retain the expected utility model intact.
(2) Continue to use expected utility, remaining conscious of when biases occur.
(3) Modify expected utility by dropping asset integration.
(4) Use a model that is appropriate for the problem being analyzed.

If option (1) is chosen, more work must be done to establish whether the violations of expected utility really can be attributed to decision errors, and this is a problem which must be addressed by experiments. Option (2) makes the solution of problems simpler, but it is unsatisfying to solve problems knowing that the solution is biased. Furthermore, above, we demonstrated that these biases can have a significant impact not only on what behavior is predicted but also on the way in which economists think about the problems. Option (3) is reasonable, but it requires a precise theory of reference wealth, which would also go a long way toward solving framing problems. It will not, however, completely accommodate the evidence. Option (4) requires the use of many different models depending on the circumstances. Nonexpected utility models which accommodate fanning patterns and betweenness violations are surveyed by Machina [1987] and Fishburn [1988], for example. Loomes and Sugden [1982, 1987b] propose a model which accommodates juxtaposition effects, Segal [1990] proposes a model which does not require reduction of compound lotteries, and Neilson (1992b) presents a model incorporating boundary effects.

Nevertheless, option (4) is a good one. The experimental evidence presented in this paper suggests that expected utility works well in a large number of cases (for example, when alternatives have the same large number of outcomes and there is a high degree of overlap across states), so that option (4) prescribes the use of expected utility in many cases. For other cases one should use a model which incorporates an appropriate modification of the expected utility model to find the effects of expected utility violations.

NOTES

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1. Completeness states that any two lotteries can be compared, transitivity states that if $p \succcurlyeq q$ and $q \succcurlyeq r$, then $p \succcurlyeq r$, and continuity states that if $p \succ q \succ r$ then there exists $\alpha \in (0,1)$ such that $q \sim \alpha p + (1-\alpha)r$. Together these axioms imply that there exists a function, defined over lotteries, which represents preferences. Without some additional axioms, they do not imply that there is an expected utility representation.

2. The normative appeal of this axiom arises from thinking about the probability mixtures as flipping a coin and playing lottery r if the coin lands tails (which occurs with probability $1-\alpha$) and playing either p or q if the coin lands heads. Independence states that the individual should choose the mixture which is preferred conditional on the coin landing heads.
3. The listing of citations for experimental papers specifies whether the subjects chose among lotteries with real payoffs, hypothetical payoffs, or a mixture of both.
4. Experiments involving laboratory rats are important for several reasons. First, the experimenter can affect the rat's body weight severely, so that payoffs are both real and large for the rat, unlike with human subjects (where payoffs are typically either small or imaginary). Second, rats do not react to the experimental environment in ways that may be adverse to the experiment, such as by trying to outguess the experimenter. Third, if rats and humans exhibit the same behavioral patterns, then the patterns are considered more robust than if the two species behave differently.
5. By conducting an experiment in the People's Republic of China, where they were able to offer rewards as high as three times the average monthly salary of the subjects, Kachelmeier and Shehata [1992, real] show that risk loving behavior toward large, unlikely gains persists even when the gains are quite large.
6. Prelec [1990, hypothetical] finds the opposite pattern for the southeast corner of the triangle.
7. Historically, the first evidence violating expected utility was consistent with fanning out, and during the 1980s elaborate models were constructed to accommodate this evidence. Consequently, all of the early nonexpected utility models allow fanning out, but many of these theories are violated by the new evidence.
8. Formally, the betweenness axiom states that for any two lotteries p and q, $p \succ q$ implies that for all $0 < \alpha < 1$, $p \succ \alpha p + (1-\alpha q) \succ q$, and $p \sim q$ implies that for all $0 < \alpha < 1$, $p \sim \alpha p + (1-\alpha q) \sim q$. Intuitively, this means that if the individual is indifferent between p and q, he is indifferent between p, q and any probability mixture of the two.
9. Hey and Strazzera [1989, hypothetical] take an alternative approach to finding the shapes and slopes of indifference curves in probability triangles. Instead of evaluating pairwise choices, they asked nine subjects questions to find indifferent points within probability triangles involving gains. Their results shed some light on why contradictory patterns emerge across studies: some individuals exhibit one pattern and other individuals exhibit a different pattern.
10. Harless and Camerer [1991] analyze 23 experimental data sets involving several thousand choices, and their strongest conclusion is that the expected utility model should never be used when alternative lotteries have different numbers of possible outcomes.
11. Framing effects clearly violate any choice model which assumes asset integration (such as the traditional expected utility model), because then the individual must make identical choices over identical wealth distributions.
12. This was first identified by Lindeman [1971] and Lichtenstein and Slovic [1971]. The first economists to examine the problem were Grether and Plott [1979].
13. Typically subjects make choices in a large number of pairs, and then one of the pairs is chosen randomly, and the subjects' payoffs are determined by that pair. Holt [1986] argues that this procedure makes the subjects treat choices as a portfolio instead of as a set of independent selections, but Starmer and Sugden [1991, real] run an experiment that shows that subjects ignore any portfolio effects.
14. Certainty equivalents are elicited using the incentive compatible Becker-DeGroot-Marschak mechanism [1964]. The subject is given the right to play a lottery p, and then is asked to declare the lowest price c for which he is willing to sell the right to play p. A random offer price is then drawn from the lottery q, and if the drawn offer price r is above c the individual exchanges p for the amount r. If, instead, the drawn offer price is below c, the individual plays the lottery p.
15. If bidder i bids less than v_i, his value of the item, there is positive probability that he is beaten by someone who bids less than v_i. If, instead, he had bid v_i, he would have won the prize and paid less than his value, which is a situation he prefers to losing the auction and receiving zero. If he bids more than v_i, there is positive probability that he wins and must pay more than v_i, which makes him worse off than losing the auction.
16. An individual is a diversifier if he weakly prefers a linear combination of two indifferent probability distributions.
17. He does, however, provide an alternative solution concept which does not rely on quasiconcavity of preferences.
18. For an early example of this, see Markowitz [1952], and for a later example see Kahneman and Tversky [1979].

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