IN DEFENSE OF IGNORANCE:

ON THE SIGNIFICANCE OF A NEGLECTED FORM OF INCOMPLETE INFORMATION

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INTRODUCTION

The purpose of this paper is to compare and contrast two substantially different kinds of incomplete information. Both kinds of incomplete information imply that the world can potentially be better understood as additional information becomes available. Both allow the possibility that individuals may invest in new information and that behavior may be changed in light of the information acquired. Both imply that individuals can make mistakes. However, the two have significant differences. Incomplete information of the sort most widely studied by economists affects the *precision* of estimates over well-understood possibilities. Here, incomplete information is a consequence of unrestricted, but *finite samples of the entire range of possibilities*, as in Stigler's [1961] original model of the economics of search. Incomplete information of the sort discussed in this paper, termed *ignorance*, is a consequence of samples that are *restricted to a subset* of the potentially available data.

Individual efforts to collect and process data, for many reasons, may be restricted to a subset of the potentially available data. For example, it is generally far less costly to sample or experiment locally than globally. As a consequence, even experienced individuals tend to know more about their own national cuisine than all others, and to have a broader knowledge of their "native" tongue than of other potentially more useful or richer languages. Moreover, and partly for similar reasons, it is often the case that individuals are simply unaware of potential data that might be acquired or the cost of acquiring it. We are all born into the world in an extremely ignorant state, that is to say without a clear sense of the broad range of possibilities that we might potentially exploit.

In a statistical or mathematical sense, the data restrictions that generate ignorance involve restrictions on the domain of the information collected or analyzed. Such data limitations affect the precision of estimates in a manner analogous to that generated by finite but unrestricted samples, but also affect the assignment of probabilities to possibilities in a manner distinct from sample size.

The latter is an important distinction between the ignorance and finite sampling conceptions of incomplete information. Ignorance of relevant possibilities affects the *domain* over which an individual's subjective probability function *can be* defined. In

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economic terms, this aspect of informational boundedness determines whether individuals are aware that some opportunities even potentially exist. Ignorance, consequently, may have effects on individual decision making that go beyond those associated with more or less accurate probability estimates defined over already well-understood possibilities.

The economic significance of this aspect of ignorance has been largely neglected in most neoclassical economic analysis. The overwhelming majority of work in neoclassical economics is grounded on models of rational decision making within very well-informed circumstances. Examples of this abound: the classroom presentation of perfect competition is generally the first model of markets that students of economics confront. Here, all consumers and firms are assumed to have perfect information about their own preferences, the best production technologies, and all prices. Implications of this textbook model are widely used to analyze antitrust, environmental, and other regulatory policies.

In more sophisticated intertemporal models and decision models that allow for uncertainty, decision makers are routinely assumed to have rational expectations based on perfect knowledge of the model of interest and complete knowledge of all relevant probability functions. Bayesian learning and statistical representations of expectations have been applied to many microeconomic and game theory settings in order to analyze individual decisions in ever more complex, but still relatively well-informed, circumstances. If any informational problems exist, players are assumed to be able to calculate all the probability distributions required to characterize expected opponent strategies and payoffs. For example, the usual characterization of a mixed strategy equilibrium assumes that each player calculates and adopts the correct probability distribution over the entire strategy set, even in cases where they have no clear incentive to do so.

In some of these models, individual knowledge may be said to be incomplete insofar as learning remains possible as new data become available or are acquired. In such models, individuals may also make informational decisions regarding how large a data set to acquire, which is usually modeled as a fairly direct extension of the usual resource allocation problem. Learning takes place in such models, but what is learned is general, rather than anything *truly unanticipated or new*; each individual's estimated probability function, defined over well-understood strategy and outcome domains, become more precisely characterized through Bayesian updating. There are no surprises in economic games; the basic decision problem is not really changed by new information.[1]

Nothing about game theory precludes a variety of informational assumptions, Binmore [1992]. In the models most widely used by economists, however, game players are assumed to know the incentives and strategy sets of other players. And much has been learned about rational and rationally adaptive behavior through the use of Bayesian and other rational models of learning. This paper is not an effort to replace that research agenda with another. However, it should be acknowledged that statistical models of imperfect information and learning can not fully represent all the knowledge problems confronted by economic agents.

This paper demonstrates that the implications of ignorance are often distinct from those associated with the usual statistical representations of imperfect information.

The next two sections note several different implications of the finite sample and restricted sample conceptions of incomplete information. These sections demonstrate that ignorance of relevant facts or of procedures for analyzing data can lead individuals to have opinions and take actions that are systematically biased. It turns out that the implications of ignorance parallel and complement those that follow from imperfect or bounded rationality as in Rubinstein [1998], Conlinsk [1996], Fox and Tversky [1995], or Simon [1955]. However, ignorance does not imply that individuals are necessarily irrational or computationally limited in any way, but merely informationally constrained in a manner that many models seem to neglect.

Informationally-bounded decision making allows the possibility that individuals would have made "better decisions" had they known all relevant facts or properly understood all available methods for analyzing the facts at their disposal. This is not to say that individuals are perfectly rational, but that a good deal of what appears to be non-rational behavior may be the effects of ignorance. A good deal of what have come to be regarded as instances of bounded rationality may actually be instances of informational boundedness that prevent individuals from making accurate calculations.

The second half of the paper explores the different implications of ignorance and search conceptions of incomplete information for economic development, welfare, education, and long-term growth. For example, I demonstrate that many market phenomena can be more readily explained as consequences of ignorance and changes in the level of ignorance than as consequences of statistical learning. Although the analysis developed below is based on the usual neoclassical behavioral assumptions, its conclusions generally affirm the Austrian argument about the economic significance of limited knowledge [Hayek, 1945; Shackle, 1969; Kirzner, 1973; O'Driscoll and Rizzo, 1996].

Ignorance of the sort discussed by the Austrians has been so neglected in the training of so many bright neoclassical economists that "ignorance" has apparently become at most a temporary phenomenon that *can not be relevant* for economic activity.[2] In my experience, this opinion is especially widespread among economists trained after the rational expectations revolution of the mid 1980s, because rational expectations models implicitly rule out the possibility of data sets that may give rise to systematically biased expectations. Ignorance can not be a significant factor in such models. The earth could never have been "flat." The analysis of the second half of the paper demonstrates that ignorance has had very substantial effects on public policy and long-term economic development.

The final section concludes the analysis and summarizes the paper's effort to increase our knowledge about ignorance.

TWO KINDS OF IMPERFECT INFORMATION

When individuals search for data in the widely used sense pioneered by Stigler [1961], it is generally assumed that the relevant probability distribution is already known by the persons doing the searching, or at least can be readily determined from a small enough number of observations that such knowledge is economically feasible. Increases in sample size (or experience) take place without changing the number of characteristics that are tabulated and without significantly changing the domain of sampling. Becoming better informed in this context has a clear meaning derived from sampling theory. The more one knows (the larger is the sample), the more accurate one's estimates of the underlying real phenomena tend to become. In this finite sample sense, imperfect information may be said to exist if the precision of one's perceptions of the world can be increased by further sampling.

Knowledge based on a restricted subset of the potentially available data implies a substantially different form of incomplete information. The domain of possible events is only partially known by persons who are constrained by ignorance. The usual probabilistic characterization of generalized search models implicitly rules out this possibility. Knowledge of a probability distribution implies that searchers are aware of *every* possible event (price, product, temporal sequence, invention, context) that *potentially can occur* or be found. Ignorance implies that individuals can not fully know the domain of the distribution that is relevant for the choice at hand, or the dimensionality of possibilities within the perceived domain. In other words, the existence of ignorance generally implies *a complete lack of knowledge* about a variety of real or imagined possibilities.

To eliminate or reduce this form of incomplete information requires changing the range of possibilities considered or the number of characteristics observed. Interpreted in sampling terms, reductions in ignorance can be accomplished by tabulating *new features* of the sample at hand or by reducing constraints that previously limited the sampling of features already tabulated. Note that both these data set problems imply that *ignorance does not necessarily diminish as sample size,* per se, *increases.*

The distinction between these two types of incomplete information would be of little interest for economists or social scientists if their implications for rational decision making were identical, or if the same activities always induced similar changes in both types of incomplete knowledge. But neither of these conclusions holds universally. In many cases, ignorance has implications that are significantly different from those associated with the finite sample representations of incomplete information.

To see this, consider a simple price search model. Suppose that someone is attempting to purchase some widely sold commodity, say a coffee maker. Suppose also that there are two kinds of shopping places: "malls" and "discount stores." Now imagine that a person is familiar with malls but is not aware of discount stores (for example, my daughter). Imagine that she is shopping for coffee makers in malls, but believes that only malls exist, and consequently that the price distribution of malls is the entire price distribution for coffee makers. Being a careful (rational) shopper, she obtains a sufficient sample of prices to form a cost-effective estimate of the distribution of prices at malls, $f(P \mid M)$, and purchases the lowest-cost coffee maker that she manages to find. However, because of her ignorance, she does not realize that the distribution of prices in discount stores, while perhaps similarly shaped, $f(P \mid D)$, lies generally to the left ("is below") that of the stores in the malls (for example, the conditional minimum expected price at the mall P^e_M is above that of the discount stores P^e_S. *She is unaware that she has learned a conditional probability distribution rather than an unconditioned distribution.*

Because she has an unbalanced sample of the distribution of prices among all stores, she not only is *ex ante* uncertain about the best price she will find, but also

makes a biased estimate of the price distribution, and is consequently likely to make mistakes.

One likely consequence of ignorance is biased expectations and systematic *mistakes*. Within the finite sampling representation of incomplete information, rationality implies that only nonsystematic mistakes can arise, because an unrestricted domain of sampling allows individuals to acquire data about the entire probability distribution of interest. Ignorance implies that the underlying model (conditional probability distribution) that informs one's sampling is missing relevant dimensions, or that the data set itself is constrained in a manner not fully understood. As a consequence, the individual mistakes a conditional distribution for the whole distribution of interest.

This kind of mistake is impossible in rational expectations based micro- and macroeconomics analyses because individuals are assumed to know essentially as much as can be known about the model at the time they make their decisions. Such "fully rational" individuals are assumed (i) to know the general features of the entire distribution relevant for a given decision, (ii) to use all the information possessed to make unbiased estimates of relevant stochastic phenomena, and (iii) to make perfect (rational) use of those estimates.[3] Consequently, on average, their decisions are always correct.

Individuals who are less than fully rational might make mistakes insofar as they violate the last two assumptions, but a fully rational individual who does not know the entire probability distribution of interest can not avoid systematic mistakes except through blind good fortune. Fully rational individuals who are imperfectly informed in this sense *can only form biased estimates* because the conditional distribution that they know differs from the unconditional distribution that they should know. In contrast, individuals who use finite but complete samples would make systematic errors only if they were not fully rational. Even a single complete observation is an unbiased estimate of a distribution's mean and mode. Only ignorance leads rational individuals to draw systematically mistaken conclusions about the world.

It bears noting that the ignorance in the illustration is not equivalent to wrongly assuming that discount stores are part of the same price distribution encountered at the mall. In that case, sampling at both malls and discount stores together with Bayesian updating would eventually disprove the initial hypothesis, and the perceived price distribution would converge to the actual. In the case of interest, the person in question remains fundamentally *ignorant* of the *existence* of discount stores, so those stores do not exist as far as that person is concerned, and *will never be sampled.*

Although the above reasoning may seem unfamiliar, even wrong, insofar as it implies the possibility that individuals may repeatedly make mistakes, it bears noting that similar reasoning is often used to criticize research carried out by economists who engage in empirical work. Econometricians are routinely criticized for using only a subset of individuals, markets, time periods, or governments to reach general conclusions. Empiricists in other sciences are also routinely chided for not getting ever more complete data sets.

Evidently, the problems feared by critics are not problems associated with sample size, since only a few dozen observations will generally suffice for statistical inference, but rather implicitly concern the sampling procedure or data limitations. Improving data sets to address these problems requires collecting data about previously neglected variables or from neglected subdomains of the general distribution of interest (other time periods or countries). Critics often suspect that inclusion of such neglected data will *change* the conclusions drawn—not simply reduce estimation error.[4]

TWO TYPES OF LEARNING

In addition to differences in the expectations or estimates that can be developed from restricted and unrestricted sampling, there are also differences in the methods by which new knowledge is acquired. The accumulation of personal knowledge through time often combines repetition (increased sampling) with the incorporation of totally new ideas or phenomena into one's world view (reductions in ignorance). The first sort of learning can be modeled using the conventional statistical (Bayesian) models of learning. However, reductions in ignorance can not be so readily modeled, which is one reason why ignorance tends to be neglected in economic analysis. Eliminating ignorance involves a quite different process of learning than does increased sampling.

The conceptual limitations of the usual statistical representations of learning are most apparent in cases where statistical search or signaling models fail to operate. For example, learning is ruled out within a Bayesian framework whenever some dimensions of the prior probability distribution are missing because the person in question is ignorant of their existence. Conventional Bayesian updating in such cases does not occur, because *regardless of the next event observed* the posterior on the missing dimensions remains zero. In such cases, knowledge can not be accumulated by experience of the sort analyzed in statistical models of learning.

To see this, recall that the posterior probability of event s given that m has occurred is the probability of s times the probability of observing m given that s is true divided by the probability of event m, [Hirshleifer and Riley, 1992].

$$P(s \mid m) = [P(s)\ F(m \mid s)/\ F(m)\ ] \tag{1}$$

Obviously if the probability of s is initially, implicitly or explicitly, assigned a value of zero (for example, the prior $P(s) = 0$) the posterior probability will always be zero whatever the actual probabilities of m and m given s may be. This holds regardless whether $P(s)$ is assumed to be zero, or whether one is totally ignorant of the existence of *s and so no probability is assigned to s*. That is to say, Bayesian updating allows refinements of theories (which can generally be represented as conditional probability functions) over events that *are known* to be possible, but *not* over events completely ignored or completely ruled out *a priori*.

Learning these "missing dimensions" involves a reduction in ignorance, which is fundamentally different from Bayesian updating and similar statistical representations of learning. Priors are not updated when ignorance is reduced, but, rather, *new priors are created* for previously unrecognized possibilities.[5]

It also bears noting that reducing ignorance does not always increase one's sense of certainty. This contrasts with most statistical notions about information, which are based upon Shannon and Weaver's [1949] pioneering work on information theory. Within the context of most statistical characterizations of learning, additional information tends to reduce the variance of the estimates of relevant model parameters, as in ordinary sampling theory. In the price search illustration, as is often the case for discrete reductions of ignorance, the discovery of new possibilities simultaneously provides new information *and increases uncertainty*. The combined mall-discount store price distribution has a larger variance than that of the mall alone. The larger world is often more complex and uncertain than previously appreciated.

SOME IMPLICATIONS: IGNORANCE, MARKETS AND MARKETS INSTITUTIONS

Hayek [1945] has argued that the problem of knowledge—the converse of ignorance in the sense used here—is fundamental to economic life and economic prosperity. Economic prosperity clearly requires tapping the talent and energy of the many and varied individuals in a society. Yet prosperity clearly requires more than talent and energy as these are, and always have been, more or less uniformly distributed about the world. Institutions have to encourage those talents and energies to be employed in productive activities, in a setting where "productive activities" are themselves subject to both types of incomplete knowledge problems—both at the level of individuals and groups.

Clearly, individual differences in knowledge (ignorance and experience) and in abilities to use and process the knowledge at their disposal affect both production and consumption opportunities. Modern production relies heavily on specialization and contracts. A good deal of specialization involves the accumulation of complementary bits of knowledge (education) and job experience that increases the precision with which an individual understands a few narrow relationships largely unknown to those pursuing other careers. Similarly, as noted in a growing literature, contracts are, to a substantial degree efforts to overcome problems of imperfect information. Contracts attempt to make particular future behaviors more predictable by agreeing to incentive structures (contingent prices) for services and other inputs. That is to say, contracts increase certainty by solving coordination and public good problems within the firm and among firms. In this sense, information in the Shannon sense is one of the outputs of the contracting process. It bears noting that what can and should be done in the future is often partly learned by the parties as negotiation takes place and areas of mutual ignorance are reduced, especially in contracts between experts in rapidly changing markets. Prior to the contract, what can and will happen are only imperfectly understood. Insofar as new production possibilities are learned, the informational aspects of negotiation can be as important as the formalized commitments agreed on. Both will affect future patterns of exchange, particularly in contracts among agents with diverse expertise.[6]

Consumer choices are also directly affected by informational problems, and market practices adopted in part to reduce them. Store fronts, regular business hours, and posted prices reduce knowledge requirements by making it easier for individuals to determine the bounds and dimensionality of their budget set, while also allowing the price distribution to be more easily estimated. Storefronts and other visual product displays make it easier to discover previously unknown products, to estimate quality without a specific program of research, and to engage in price/quality search over products already known. The modern graphical images, detailed descriptions and prices listings of modern web-merchants serve similar functions. Without posted prices and some direct observations about what is to be sold, not only do prices have to be learned one by one, but haggling takes time away from learning more about the range of goods and services that might be acquired.[7]

Many of the market institutions that we observe around the world have evolved to address both ignorance and search problems associated with trade and production. Profiting from specialization requires an organizational structure that can coordinate and use disparate specialists without knowing precisely what the specialists are doing. Modern markets allow people who remain completely *ignorant* of one another's existence—let alone of their subjective payoffs and incentives—to trade indirectly, because a long series of middlemen and other experts bridge the ignorance gaps that we all must acknowledge. Potential reputation effects and warrantees reduce, but do not eliminate, the risks associated with dealings between strangers.

Most of us do not know, estimate, or imagine the personal lifestyle and histories of the particular individuals who produce the goods and services that we consume at home. When we travel abroad, we are necessarily even more ignorant of the persons with whom we deal. Those who choose to be taxicab drivers or shopkeepers in Istanbul, Prague, or Moscow remain largely unknown to us, which in many cases is probably just as well. Nonetheless, travelers can purchase goods and services in most countries around the world, even when they can not speak to one another. Many international travelers do not know, and have never known, the language of the shopkeepers with whom they must deal. They can not read the labels of the products they wish to purchase nor directly bargain with the seller over price, quantity, or quality. (Only the partly informed can even make an educated guess about a word or phrase in a foreign language. Those completely ignorant of a language can not even guess.)

The market conventions of display and location that emerge from competition allow us to hire a car and driver, to travel, shop, and sleep in places where one is largely ignorant of travel times, prices, reputations, and specific locations. Storefronts and formal product arrangements signal willingness to sell. Arabic numbers are widely used to post money prices for both familiar and unfamiliar products. Pointing to specific products or menu items serves as a primitive, but nearly universal, form of communication *within a marketplace* because ordinary methods of display allow this language to function. These market conventions allow strangers to engage in the *types* of transactions they are familiar with (and therefore which can be expected in a statistical sense to be carried out far from home) in spite of complete ignorance over what would seem to be relevant details.

These and many other supporting cultural, legal, and political institutions have evolved to facilitate exchange while economizing on knowledge—not just in the statistical sense that high-variance estimates will function nearly as well as low-variance estimates, but in the sense that transactions between complete strangers (persons about whom we are totally ignorant) over previously unknown products may take place routinely and smoothly.

IGNORANCE, WELFARE AND GROWTH

That imperfect information can affect individual welfare has long been acknowledged by educators and economists, although differences between the ignorance and search notions of incomplete information have not been fully appreciated. Nearly everyone appears to believe that imprecise and biased estimates can reduce personal welfare in most areas of life by reducing the returns from personal resources. Increased knowledge allows individuals to choose among alternative careers and productions that were previously unknown, and increased precision allows individuals to more effectively pursue those activities chosen. Consequently, increases in both kinds of information tend to increase individual welfare in conventional neoclassical terms by enlarging and sharpening perceived opportunity sets. However, there are some differences between the welfare implications of incomplete search and ignorance.

For example, knowledge that expands the range of new products, quality distinctions, occupations, or technologies can not make one worse off because knowledge of this sort provides additional possibilities. This conclusion does not imply that reduced ignorance necessarily makes each person feel better off. Correcting biased expectations may make one subjectively worse off insofar as the future may appear substantially less rosy than it had previously. Perceived opportunities may be more complex and difficult to decide among than one had imagined. However, it may be argued that reductions in ignorance can not make one objectively worse off, insofar as the future really was, in any case, bound to be less rosy or more complex than previously recognized. (Reductions in the ignorance *of others* can make one objectively worse off by reducing one's own relative informational advantage.)

Note that the widening possibilities generated by reduced ignorance implies that individuals generally benefit from such new knowledge *regardless of their preferences for risk*. This is not the case with the conventional statistical representation of incomplete information. Here, the use of unbiased estimation procedures assures that *ex ante* estimates are unbiased and that expected utility is maximized. Additional information in this context reduces the variance of estimators but does not, by assumption, diminish bias or alter expected values. Consequently, a risk-neutral person's welfare can not be improved by an increase in sample size. A larger sample does not change the expected values of the parameters of interest, only the variance, and only the risk-averse care about the variance of the estimates used.[8]

The welfare implications of these two different notions of incomplete information also differ after a decision is made. Both statistical notions of incomplete information and abject ignorance allow the possibility of *ex post* regrets. Bayesian updating may imply that a past decision was in error because one's estimated assessments of the relative merits of alternatives were a bit off. One may have "bad luck" in the stock market. Similarly, learning new possibilities may undermine the initial rationale for an activity by bringing new costs or benefits to the fore. In either case, "error correction" implies that one may learn by doing.

However, it is clear that the potential for *ex post* regret is larger for the case of ignorance than for the case of finite samples. In the Bayesian context, posteriors adjust gradually as events are observed. The Bayesian calculus gives one an unbiased expectation at the time of choice, even if the estimates are not as precise as they eventually will be. Revised probabilities, consequently, may suggest that mistakes at the margin were made *ex ante*, but generally not that avoidable errors have been made. By contrast, ignorance can cause one to be completely and systematically wrong. Reductions in ignorance can add entirely new possibilities and, consequently, lead to very significant revisions in plans. Consequently, heartfelt regrets, or genuine mistakes, are potentially larger when they are the result of ignorance than when they are the result of modest errors in the estimated probabilities characterizing the perceived stochastic process generating payoffs. (This is, of course, one rationalization for public education.)

If we extend these ideas from the level of individuals to the level of society, it is clear that unbiased expectations imply that on average the "market" gets it right, and that individuals will generally not have *ex post* regrets that affect behavior. In such cases, the market outcomes and new information that emerges from such decisions will not cause individuals to significantly revise their plans. On the other hand, reductions in ignorance that change the perceived range of market possibilities can clearly cause individuals to radically change their consumption, investment, and career plans in ways that they had not imagined. (College and international travel often have such effects.)

Perhaps the clearest evidence of the significance that ignorance has on market outcomes and individual welfare is the difference between the pattern of ordinary life in 2000 and 1900. There is essentially universal agreement that the average person's opportunity set in 2000 is strictly preferred to that available in 1900. Again, both types of incomplete information can be said to have played a role in the increase in average personal welfare. That is to say, part of this improvement is the result of the refinement of products available in 1900 (and the accumulation of capital): farm equipment and railroads are better than they used to be, and literacy rates are generally higher. Another part of the improvement is the result of increased knowledge that allowed new, *previously unrecognized*, possibilities to be pursued.

Reductions in ignorance were extremely important. Many existing production technologies and products were refined, but many more were created of whole cloth. The computer on which this paper is being composed is not a refinement of the typewriter of 1900. Jet aircraft are not refinements of the horse and carriage, or chariot, although they are alternative modes of transportation. Modern medical practices are not generally refinements of previously existing herbal remedies and surgical methods. In these cases, economic progress has been largely the result of innovation (that is, unexpected insights and discoveries that reduce the unknown in productive ways) [Schumpeter, 1942; Solow, 1956; Dudley, 1991]. Reductions in ignorance that took place in the past century substantially improved material well-being in most areas where new knowledge was applied.[9]

Had all the possibilities that we understand today been recognized a century ago, surely the world would have looked a lot different in 1900. That is to say, the pattern of consumption and production in 1900 *failed to realize* all the potential gains from trade and production that truly existed. Many of these possibilities were blocked by constraints on available knowledge—that is to say, by ignorance. With each economically relevant reduction in ignorance, a new pattern of exchange emerged. And, to the extent that the "old" remains objectively possible, the "new" pattern may be said to have increased welfare.

The industrial expansion of the past two centuries, thus, reveals the economic significance of the ignorance that previously constrained economically relevant decisions. Even today, as suggested by Kirzner's [1988] perspective, many unrealized potential gains to trade are bound to remain unknown, and therefore unrealizable because of ignorance that is beyond the scope of conventional search and Bayesian models of learning. Ignorance allows the possibility of genuine progress. *We don't know what we are missing.*

RATIONAL IGNORANCE, NATURAL IGNORANCE, AND EDUCATION

The boundary between known and unknown can be can be analyzed to some extent using the rational choice calculus of modern economics, although truly "rational ignorance" is possible only within fairly narrow limits, Congleton [2001]. Under rational ignorance, an individual makes a conscious decision to remain completely ignorant of the details of an area of knowledge that is known to exist, but within which he or she is otherwise completely uninformed. For many readers, this may include such areas as: the cha cha, gourmet cooking, corporatism, the calculus of variations, chaos theory, celestial mechanics, and Mandarin Chinese.[10]

The costs and benefits of becoming informed in such areas may be analyzed using subjective probability functions defined over dimensions that are initially known: for example, benefits and costs. However, it clear that none of the relevant benefit or cost functions that map new knowledge into benefits can be determined very accurately. Complete ignorance of all the specific facts that might be learned implies that the domain of these functions and their values can not be directly assessed nor estimated in much detail. Individuals may be said to "choose" continued ignorance in such areas, but it must be acknowledged that estimates used for such decisions are necessarily of poor quality.

In areas of ignorance well away from the margin, ignorance is so great as to preclude even a cursory analysis of unimagined possibilities. (Consider the cave man's "decision" not to learn about stainless steel or helicopters.) The bulk of ignorance is natural in the sense that man is born into the world in a largely ignorant state. Initially, we do not possess even such fundamentally useful information about which foods are best for health or entertainment, or indeed what things are food. (The health sections of modern book stores attest to the difficulty of discovering the former even at quite advanced stages of life.)

That ignorance is largely natural rather than rational is of interest because it implies that one can not simply expect self-interest to overcome all relevant knowledge problems. Thus, parents do not simply leave their children in rooms with piles of open books, but spend considerable time and energy reading to them and teaching them what to read. This pattern of investment in the moral and practical training of the young is evidently ancient. If all knowledge problems could be overcome by increased sampling, such directed programs of education would provide little improvement over undirected self-education. Clearly direct experience would not take "students" very far in reducing their ignorance, especially early in life when personal ignorance is greatest.

What to eat, how to dress, how to interact with people, the rules of algebra and spelling, the ideas of chemistry and geography, and the importance of playing by the rules are all matters transmitted from one generation to the next rather than directly embedded in the genetic code of humanity. Parents consequently invest significant resources in educating their children, through organized schooling and other means.[11] Left to their own devices, most parents evidently believe that children would remain more ignorant than would best serve their children's own interests.

Consequently, reducing ignorance appears to be the principal aim of both private and public schools. Surprisingly little of higher education involves increasing the sample size used to estimate phenomena that are well-recognized by the student. Even most rote memorization is generally an effort to cause students to remember new facts, words, or ideas rather than an effort to increase sample size.

The education industry devotes most of its information-oriented resources to inducing students to learn "new" facts and subtle complex relationships previously discovered or invented by others. This procedure spares new economic students from the daunting task of reinventing the observations of Smith, Jevons, Marshall, Samuelson, Friedman, Arrow, Lucas, North, and Buchanan from their own direct experience. The normal methods of education economize on resources by allowing previously identified facts and theories to be learned in a manner that is far more extensive, and far less costly, than possible through personal rediscovery.

This is not to say that statistical learning plays no role in education, but rather that the education industry is better understood as a systematic effort to reduce ignorance than as an effort to increase sample sizes over phenomena already understood by students. Once new theories and possibilities are learned, individual assessments of them may be refined by experience as characterized by statistical models of learning. Understanding which theories best apply to given circumstances often appears to be the result of statistical learning.[12] However, by and large, the productivity of the education industry comes from systematic reductions in ignorance. Interpreted in statistical terms, this aspect of education increases each student's understanding of the world by adding *new* conditioning variables, and creating new priors on possible relationships among the new variables acknowledged.[13]

It also bears noting that *formal education often reduces the peak of a student's subjective likelihood function*, with the result that well-educated students may become increasingly modest about that which they actually claim to "know." For example in economics, once introduced to the possibility of negative income effects, students are no longer as confident that all demand curves slope downward. This is, of course, one explanation of the conundrum "the more we know the less we know." As rational ignorance replaces natural ignorance, one becomes more aware of the limits of one's own knowledge. Such a conclusion would be unlikely within a finite sampling perspective, where larger samples tend to increase certainty rather than reduce it.

As society's knowledge base has increased in the past two centuries, the systematic reduction of ignorance through organized education has led to the development of a very large industry, largely paid for and mandated by public policy. The latter

would be totally unnecessary, as would curriculum design, if ignorance was not generally a binding constraint on personal decisions to accumulate human capital.

SOME BROADER IMPLICATIONS OF IGNORANCE REDUCTION

More generally, scientific progress can be understood as a combination of increased sampling in known domains and expansions of the domains in which samples may be knowingly acquired. What Kuhn [1995] calls ordinary scientific progress is generally not a matter of the elimination of ignorance in the sense used here, but rather of gradual increases in precision. A good deal, perhaps most, of scientific progress is the result of gradually refining theories over event spaces that have been fully appreciated for a long time. Everyone knew there were stars long before the geocentric interpretation of stellar motion was replaced with heliocentric ones. The basic ideas of agriculture have been appreciated for millennia. Many manufactured products such as pottery, clothing, or books are the result of successful efforts to refine technologies and possibilities long acknowledged to exist. Gradual learning is also clearly evident in the slow refinement of most methods for constructing bridges, buildings, gardens, jewelry, and pastries. In all such areas of progress, rational search and the Bayesian representation of learning are very powerful and useful models of the incremental improvement in our understanding of familiar phenomena.

On the other hand, it must be acknowledged that technological progress can also be the result of genuine innovation and discovery. The iron age evidently replaced the stone age because new possibilities for using particular kinds of rocks were discovered rather than old ones refined. Moreover, in many cases, reductions in one kind of ignorance lead to unanticipated increases in knowledge in other areas. Technological progress often reduces ignorance indirectly by allowing new, *previously unimagined* phenomena to be considered. The compass, the telescope, the microscope, probability theory, satellites, submersibles, and other recent information gathering innovations have allowed previously unobserved—indeed unobservable—phenomena to be seen and analyzed for the first time.

New intellectual developments or theories, what Kuhn [1995] calls paradigm shifts, may similarly provide such radical reinterpretations of familiar data that entirely new issues and possibilities are brought to the fore—as an example, modern chemistry allowed previously unimagined possibilities to be evaluated. Such instances of intellectual and technological advancement both reduce ignorance and provide new processes by which ignorance—fundamental ignorance—may be reduced in the future. These processes are not directly amenable to Bayesian analysis insofar as *new* phenomena or hypotheses are created rather than old ones reassessed.

It is possible to acknowledge the existence of ignorance within a statistical learning model by using a probability distribution defined over a known range of possibilities plus some residual event, say "exceptions", "the unknown" or "supernatural." As possibilities other than those previously recognized occur, the posterior probability for "the unknown" would gradually increase. In this manner, one may come to recognize that a good deal of experience is unexplained—beyond the reach of the current domains of accepted theories: "unreal," or supernatural. (Religion may thus be given a Bayesian foundation!) However, until new possibilities within the unknown are recognized and itemized in some detail, no new theories would emerge from this Bayesian methodology. The "residuals" would remain anomalies, flukes, miracles.

Until the dimensionality of the probability function describing the world is expanded, no new patterns of conditional probability (causal theories) can emerge. A subset of the unknown has to become known or at least hypothesized for this to occur. In this manner, ignorance clearly limits the range of hypotheses that may be contemplated and tested regarding both very general and narrow features of the world. Popper [1957] and Shackle [1969] argue that one can prepare for and perhaps even expect to be surprised (that is to say, expect to learn something new—not simply update priors over possibilities already known) but one can never know what surprising result or observation will be stumbled upon.

The empirical relevance of surprise discoveries is emphasized by Burke [1995] who catalogues many historically significant examples of serendipity: where a series of very unlikely "connections" inadvertently led to major innovations during the early stages of the industrial revolution. More recently, the modern age of polymer plastics was launched by the unexpected discovery of Nylon at Dupont, and the civilian Internet emerged out of security concerns of the department of defense which lead to the creation of the DARPAnet.

CONCLUSION

The present essay has attempted to persuade the reader that ignorance has economically significant impacts, and that our understanding of many economic phenomena can be deepened by taking account of this neglected kind of incomplete information. The analysis has noted several cases in which the implications of ignorance differ significantly from those associated with the finite sampling representation of incomplete information. For example, ignorance may lead to mistakes that are *not self-corrected* by experience. Consequently, efforts to reduce ignorance often come from "outside" the individual as with training programs and advertising by firms, and with the unsolicited advice of colleagues, parents, and friends. The existence of libraries and the education industry are also most easily explained as efforts to reduce some of the worst consequences of ignorance.

For the most part, the aim of the present analysis has not been to reduce the reader's ignorance of basic facts, but rather to suggest a series of behaviors and institutions that can be readily explained when the concept of ignorance is distinguished from other notions of imperfect information.

Although all information in some sense can be said to be subjective, insofar as every person's collection of information is unique, there is another sense in which information can be said to be objective. That is to say, it is often possible for one person to recognize another's lack of knowledge, and to contribute directly or indirectly to increasing the knowledge of other persons. This objective aspect of incomplete information has played a role in the present analysis insofar as the reader's own experience has been implicitly used to provide empirical support for the different implications of the search and ignorance notions of incomplete information. The analysis has shown how a series of ideas that were largely familiar to most readers can be linked together by analyzing some of the implications of ignorance: the accumulation of knowledge, innovation, economic progress, the role of education, and rational regrets. (Most honest readers will acknowledge that many of these connections were formerly unrecognized, even if they now seem obvious.)

The purpose of this essay is not to chide economists for neglecting the significance of ignorance in economic institutions and activities. Obviously, much useful work has been facilitated by assuming that decision makers are completely informed. Work based on complete information and Bayesian models of learning can, and perhaps paradoxically, have clearly reduced our *ignorance* about economic phenomena by bringing new possibilities and conclusions to our attention.

The purpose of this paper is best summarized as an effort to reduce our ignorance about ignorance. This paper attempts to sharpen our understanding of what it means to be less than perfectly informed, and to indicate a few areas where a neglected form of incomplete knowledge has clear economic significance. Additional research on the implications of ignorance may be expected to lead both to new insights and to a deeper understanding of economic phenomena—implications that tend to be missed under methodological conventions that rule out the possibility or importance of ignorance, *a priori*. Ignorance is not bliss, but neither is it irrelevant.

NOTES

The argument presented in this paper has benefited from conversations with many colleagues over the years including, but not limited to: Gordon Tullock, I. J. Goode, Israel Kirzner, James Buchanan, Ronald Wintrobe, David Levy, Bryan Caplan and Tyler Cowen. Helpful comments were also received from the editor and an anonymous réferee. Of course, the argument and conclusions remain my own.

1. It may be argued that rational expectations models assume greater information on the part of decision makers than the core neoclassical models, insofar as agents were not previously assumed to know anything globally about economics or markets. Moreover, the pre-rational expectations models implicitly assume that economists know a bit more than market participants insofar as they are able to recognize the existence of the invisible hand, and explain this to non-economists.
2. See, for example my colleague's, [Caplan, 1999], defense of neoclassical representations of imperfect information in his critique of the Austrian school.
3. Assumptions (ii) and (iii) characterize two different aspects of the economic definition of rationality. (ii) implies that individuals will not make systematic information processing errors (i.e., will form "rational expectations"). (iii) implies that individuals have well-understood and consistent objectives (that is, will maximize utility).

   Fox and Tversky [1995] suggest that ignorance may lead individuals to behave irrationally. Experiment indicate that many individuals are more inclined to bet on what they know than on what remains unknown or uncertain. They term this form of risk-averse behavior "ambiguity aversion." Of course, notions of rationality extend well beyond economics. For a discussion of various meanings of rationality in the social sciences, see Simon [1978]. Good [1976] provides a nice overview of the Bayesian interpretation of rationality which differs in significant respects from that of the modern economic one.
4. I only consider honest mistakes in the data collection procedure here. Feigenbaum and Levy [1996] discuss how preferences over estimates may affect scientific work.
5. Binmore [1992, 488] suggests that a complete list of the possible outcomes is not even conceptually possible, or at least is beyond the capacity of real decision makers.

   Cyert and DeGroot [1974] provide one of the earliest analyses of the relationship between Bayesian learning functions and Rational Expectations. Although Cyert and DeGroot find many instances where Bayesian learning converges toward Rational Expectations and market clearing prices, they report several failures (inconsistencies) in cases where the original model used by individuals (firms) is incorrect. Frydman [1982] argues that agents can not generally compute optimal forecasts in competitive markets because they can not estimate the forecasts of all other agents.

6. The role that external institutions play in affecting transactions costs has been previously emphasized, for example, in North [1990]. The significance of transactions costs in the organization of production has its origin in Coase [1937]. Modern contributions are well summarized in Williamson and Winter [1993]. These transactions cost assessments are still not included in mainstream text books, but even these related analyses do not directly address the significance of ignorance in the development of market institutions.

   Recent progress in the areas of incomplete and incentive compatible contracts also suggests that our understanding of market relationships is incomplete without consideration of the different degrees to which asymmetric information affects the real and imagined possibilities among which that individuals choose [Mas-Colell et al., 1995, Ch. 13]. Differences in knowledge creates opportunities for arbitrage and innovation, for fraud and shirking, and as noted above for institutional evolution.

7. Of course, not all markets use posted prices. Many of these are widely and correctly regarded as "old-fashioned" markets. Tourists often find haggling in such markets to be entertaining, as for example, in many tourist areas along the Mexican boarder with the United States. Markets without posted prices also potentially allow proprietors to exploit the ignorance of potential customers in such circumstances by charging the ignorant higher prices than the well-informed, as in used car markets. Less obvious but fundamentally similar price discrimination takes place in "modern" market settings where prices appear to be posted. Recently, it has become commonplace for stores to "match" prices by offering price "guarantees." Such programs clearly benefit particular consumers who know of lower prices available at other stores. Others pay a premium for their ignorance.

8. Of course, sampling may do more than improve an individual's estimation of parameters of interest. For example, a consumer that engages in price search is generally more interested in his or her *sample minimum* than in the quality of the estimated average price. Insofar as the expected sample minimum tends to fall as sample size increases, additional price information can still be worthwhile for a risk neutral shopper. However, in cases where the only purpose of sampling is improved estimation, it is clear that risk neutral persons will prefer very small samples, in the limit samples of one.

9. Of course, new knowledge can also magnify one's ability to do harm, as is clearly the case with the last century's many innovations in weapons of mass destruction. Even with this point acknowledged, however, it remains the case today that the average person's opportunity set is much wider and more desirable than was the case in 1900.

10. Once possibilities are acknowledged to exist, individuals can choose how large a sample to acquire, and how much effort to devote to processing the data acquired. It is possible that individuals may choose to invest in a *sample of zero*, or to collect or process only a constrained sub-sample. In such cases, natural ignorance replaces rational ignorance.

    See Fremling and Lott [1996] or Congleton [2001] for formal treatments of how rational ignorance may affect expectations.

11. This educational impulse may have some genetic basis in that parents who educate their children are most likely to find their genes present in future generations insofar as the transmission of such knowledge increases the fitness of their children. For example, it is not obvious that human children would long survive without being taught what items in the local environment may usefully ingested (what food is). This point seems to be neglected in most discussion of genetic foundations of human behavior. Economists are not the only ones who have been neglecting the implications of ignorance.

12. Once a domain of possibilities is expanded, experience can improve a student's understanding of the likelihood of the "new possibilities" as experience increases. For example, we may note that most economists have gradually come to discount the possibility of upward-sloping demand curves—a "possibility" once emphasized by an intermediate micro professor or two in their distant past.

13. Of course, educators do occasionally pass on incorrect inferences about the world, and may induce a bit of confusion by inculcating the "wrong" priors, or "mistaken" facts. However, the ability of educators to "get it wrong" is further evidence of the non-statistical nature of most formal educational technologies. Formal education allows students to form "priors" that are not connected with their own direct experience and observation.

    Once outside the classroom, subsequent experience does allow students to more perfectly understand what their teachers have taught them (by revising their priors) at least in areas where students make their own direct observations.

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