Economic Model of Solid Waste Disposal and Recycling*

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Introduction

The purposes of this paper are to develop a joint production model of solid waste recovery and extend the model to include some rules of efficient resource allocation with regard to waste products. Part II is a brief discussion of the taxonomy of solid waste management. Part III develops a model of joint production, and Part IV extends the joint production model adding externalities (social costs) resulting in a recycling production model. Aspects of the model examined include assumptions of the model, joint production of finished goods, intermediate goods and solid waste, and, decision rules (marginal conditions for optimization).

Taxonomy of Solid Waste Management

Solid waste includes both household and commercial wastes e.g., garbage (wet) or rubbish (dry). Waste may be either combustible, e.g., paper (largest component), wood, rubber and plastic or non-combustible, e.g., metallic and nonmetallic (construction rubble predominant) glass, porcelain, and plastic.[1] Handling systems are integral to solid waste management. Included in sewer systems are the storage function, collection and transport function (which composes 80% of handling cost), processing function—compacting, grinding and shredding, and, treatment function (disposal).

Typical treatment processes are varied and are essential to recycling.[2] The best known of these include landfill or dumping (most prevalent),[3] and, incineration with residue making up about 40% of original weight of waste.[4] This consists of metals, glass, ash and slag (for roadbeds, cinder blocks, or cement).[5] Less well

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[1] R. J. Black, "The Solid Waste Problem in Metropolitan Areas," *California Vector Views* 11:51 (September 1964).

[2] This is a result of two factors. First, landfill is cheaper than any other system. Second, without recycling, as much as 50% of all waste generated must be buried or dumped in the ocean, even with the use of incineration or composting. Indiscriminate deposition of waste in the ocean has been shown widely to be deleterious in many ways. (See Golueke and McGauhey, 1:5, also A. V. Hirschheydt, "Problems of Refuse Disposal," and G. Sudhoff, "Methods of Planning and Preparing Plants for Processing or Disposal of Communal Refuse; both in International Research Group on Refuse Disposal, pp. 71–72, 359, September 1968.)

[3] The fill settles and decomposes over time, emitting methane gas. This means that a waiting period, which varies in length with the climate, is necessary before the fill can be used. Methane emission, which may last more than ten years, makes building dangerous; therefore, park reforestation are more appropriate. (Methane and carbon dioxide may contaminate ground water and lateral soil; so the choice of location is dependent on hydrological and other geological parameters.) See M. W. First, F. J. Viles, S. Levin, "Control of Toxic and Explosive Hazards in Buildings Erected on Landfills," *Public Health Reports* 81:419 (May 1966); Golueke and McGauhey, 1:87–88; F. Heigl, "Organization of Refuse Disposal," in International Research Group on Refuse Disposal, pp. 30, 33, December 1969; C. Schneider, "Sanitary Fill Reused Safely," *The American City* 68:83 (October 1953).

[4] F. A. Govan, "High Rise Disposal Problems," *Refuse Removal Journal* 10:6 (March 1967).

[5] Municipal refuse has just under half the heat content of an equivalent weight of soft coal. This will

known treatment processes include: pyrolysis—thermo-chemical waste reduction without oxidation—where waste is reduced to basics (e.g., charcoal, gas, tar, oil) and recycled for chemical firms,[6] biochemical treatment which yields solid conditioners or low quality fertilizer,[7] lagooning—degradation of waste in ponds[8] and recycling—salvage returned to productive sector after additional process of treatment.[9]

Urban growth and waste generation—aggravated by lack of municipal planning resulting from fragmented jurisdiction (e.g., 6.5 million people in Northeast Illinois are governed by 249 municipalities and 1060 local governments) is one major factor in waste problems.[10] Economic stability based on rising consumption, hence rising waste—is a second factor aggravating waste problems.[11] Rising expectations for environmental quality have brought pressures to expand and improve waste handling systems. A final factor is the continual resource depletion which has fostered a need for resource recovery using waste as a substitute input for production or as a marketable output.[12]

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generate about 1.5 pounds of steam per pound of refuse. If used to generate electricity, each ton of refuse will produce 200 kw-hrs. of electricity. The steam, however, cannot be transported far. "Combustible Rubbish Contents Favor Using Heat Recovery," *Refuse Removal Journal* 10:32 (November 1967); "Navy to Incinerate Rubbish for Power," *Refuse Removal Journal* 10:19 (April 1967); E. T. Hayes, "Man-Made Ores" in *Proceedings of the Symposium: Mineral Waste Utilization*, p. 5; E. R. Shequine, "Steam Generation from Incineration," *Public Works* 95:92 (August 1964).

[6] The gas can be used for a fuel, a reagent in reduction or ores or other reduction processes. As a fuel, the gasses have about 400–500 Btu/ft. This would generate 400,000 kw-hrs./100 tons of waste. All of these gases burn cleanly. The charcoal can be used for briquettes, a soil additive, or an industrial purifying filter. The oils can be used for pesticides, wood preservation, and industrial chemicals. Among these chemicals are benzene, styrene, butadiene, methane, ethane, methanol, and acetic acid. "Discarded Tires Will Become a Basic Chemical Feedstock Instead of a Junkyard Eyesore," *Chemical Engineering* (2 December 1968), p. 53; "Pyrolitic Decomposition of Solid Wastes," *Public Works* 99(8):82 (August 1968); Gentile, p. 188; Golueke and McGauhey, 1:96.

[7] From 1962 to 1964, nine American composting plants closed down, leaving only fifteen in operation today. J. S. Wiley, F. E. Gartnell, H. G. Smith, "Concept and Design of the Joint U.S. Public Health Service-Tennessee Valley Authority Composting Project, Johnson City, Tenn." in *International Research Group on Refuse Disposal*, September 1968, p. 300.

Composting produces a bulky product high in potassium and phosphates, but low in nitrogen. Composting with sewage sludge improves the nitrogen content. Depending on the length of processing, its use yields different results.

[8] Use for irrigation can stabilize a falling level of ground water, but this type of operation presumes a soil structure which will retain nutrients and filter the water not used by crops. These nutrients are the same as those which support choking algae to the detriment of fish and fowl when released directly into rivers and lakes. Potential pollution of ground and surface water requires special construction and/or preparation of digesting lagoons. J. R. Sheaffer, "Reviving the Great Lakes," *Saturday Review* (Reprint) (November 7, 1970): McGauhey, pp. 153, 156.

[9] Recycling is used more extensively in Europe than here. Salvage operations have actually decreased here as manual sorting became too expensive, synthetics reduced the need for used paper and rags, ink build-up in used paper became too high for reuse, and alloys became so sophisticated that used contaminated and bonded metals could not be used profitably in the new processes. Nonexistence of effective markets and sporadic supply are also problems. Even so, Collins estimates that as much as 50% of our wastes could be salvaged. C. Collins, "Regional Systems Approach and Market Development Seen as Key to Waste Recycling," *Waste Age* 1(1):20 (April 1970); Golueke and McGauhey, 1:105–6, 116; C. A. Rogus, "Refuse Collection and Disposal in Western Europe—Part III; Salvaging, Composting, and Landfilling," *Public Works* 93:139 (June 1962); J. E. Ullman, "Waste Disposal and Social Costs" in J. E. Ullman (ed.), *Waste Disposal Problems in Selected Industries* (Hempstead, N. Y.: Hofstra University Yearbook of Business, 1969), p. 283.

[10] J. R. Sheaffer, "Metropolitan Problems in Refuse Disposal" in *Proceedings of the National Conference on Solid Waste Research* (December 1963), p. 203.

[11] J. K. Galbraith, "How Much Should a Country Consume?" in H. Jarrett (ed.), *Perspectives on Conservation* (Baltimore: Johns Hopkins University Press, 1958), p. 97; Golueke and McGauhey, 1:3; Kapp, p. 64; Packard, p. 160.

[12] E.g., H. J. Bennett and C. Morse, Scarcity and Growth: The *Economics of Natural Resource Availability* (Baltimore: Johns Hopkins University Press, 1963). "Mountains of Solid Wastes," Bionomics Briefs 1(5):5 (September 1967); V. Packard, pp. 178–179.

Pursuant to clarification of the economic basis of such problems we examine a model of solid waste recycling.

Joint Production As An Economic Model of Solid Waste Recycling

A. Assumptions and Qualifications

The model of solid waste disposal requires, for purposes of analysis, a framework of assumptions and qualifications:

1. We accept maximization of behavior on the part of both firms and individuals as analogous to real experience. Firms maximize profit and individuals maximize utility.
2. We assume the existence of definable individual preferences which are free of bandwagon, snob, or Veblen effects. We expect, however, that individual preferences about freedom can be changed if loss of freedom is compensated adequately.
3. We accept the distribution of income as it stands and will attempt not to change it *via* any given waste handling method.
4. We assume that society's goal is to maximize the joint utility of its members. To the extent that this means future members, we note the delimitation established by the problem of "option demand." We expect to account for every cost generated by an action.
5. We assume that an approach of partial equilibrium can achieve maximized joint utility; i.e., the rest of the system will be assumed to be in equilibrium conditions in order to make waste handling efficient.
6. We assume that continuous substitution is possible among factors and among goods.
7. We assume that institutions are not fixed, but can be changed at a price; *i.e.*, institutional change will be expressed as a cost. This cost includes the direct costs of reorganizing (*e.g.*, relocation, wages) and the costs of compensation for changes in individual utilities.
8. We assume that technology is changing but is fixed at any given point in time. This means that at any given point in time, all existing technology may be utilized, and improvements may be anticipated. These improvements carry a cost, however; following Galbraith, the input-output relationship for investment in innovation is probably as stable as any others in the long run, so that technological improvement has a definable cost.[13]
9. We assume that marginal cost is a negative marginal revenue. Therefore, everything may be expressed as a marginal revenue of some sort.

A breakdown of costs is also required by the economics of solid waste disposal, for purposes of analysis:

1. Internal costs to the firm are those of land, labor, capital, material inputs to manufacturing, and transportation. Transportation costs may contribute to the costs of labor, materials, or finished goods. Land and capital are assumed to be fixed in location.
2. Household costs involve land, labor, materials, time, and transportation. Capital is amortized and included with materials as goods. Labor *per se* means labor brought into the household. Cost of labor within the household is measured by the time cost.
3. External costs are more complex. Current externalities are divisible: human health, property damage, and aesthetic/recreation value. Health costs are incurred by the public or by sanitation workers, and are reflected both in medical costs and in the wages of sanitation workers. Property damage is both direct and indirect. Recreational values are affected by changed inclinations on the part of individuals. Aesthetic externalities assess cost in terms of reduced individual utilities.

All of these factors can be projected into the future, because of the accumulation of solid wastes. In addition, there are two explicit costs: (1) option demand, which can only be mentioned in passing, and (2) pecuniary externality, or "backshift" cost, when the cost is

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[13] Galbraith (1958), pp. 90-91.

shifted forward in time because profitable markets do not exist in the present. However, for brevity, we will not develop either of these concepts in this paper.

Joint production is a process of production involving more than one output for a given number of inputs. It relates to manufacturing production.

Solid waste, viewed as a "man-made ore" or a "product for which no use has yet been found" is essentially one product of a manufacturing firm's output. If a company produces one major product, and its production technology entails waste residues, then the firm in fact produces two joint products. The assumption is that each firm uses only one product process, but several inputs may be employed in the production of both goods outputs and waste products.

The following relation can be used to represent this production process:[14]

$$\phi(x_1,\ldots,x_n)=\psi(x_1,\ldots,x_n) \qquad (1)$$

where x_i represent commodities and services, ϕ is the input function and ψ, the output function. $x_1,\ldots,x_n$ appears on both sides of the equation. This is useful because in many cases, output residues such as iron scrap or glass cullet are recycled and used as inputs later. By not classifying the x's into several groups, we are judging all inputs and outputs equally. Three basic categories of outputs exist: (1) marketable finished goods, (2) intermediary goods, (3) nonmarketable residues. It is not necessary to differentiate, but only to describe goods of the same genre but different quality, such as anthracite and bituminous coal.

It is useful to combine two functions of the equation:

$$f(x_1,\ldots,x_n)=\psi(x_1,\ldots,x_n) - \phi(x_1,\ldots,x_n) \qquad (2)$$

Then, if x_1 occurs both as an input and an output, its magnitudes in each role may be compared, and may be referred to as net input or net output, or net product function, e.g. as in equation 2, (Waste) = (Output) - (Input). In essence, we have created an input-output relation in which final output is analogous to Leontief's final demand.

What conditions prevail if the firm maximizes profit? Profit may be expressed as:

$$\pi=\sum_{i=1}^{n} p_i(x_i)x_i$$

where $p_i(x_i)$ is the unit price of x_i as a function of x_i, the quantity of x_i. This is possible because input quantities are negative. Maximization is facilitated through a Langrangian function encorporating implicit costs which is less straightforward than a general function but generates shadow prices for waste:

$$\Pi=\sum_{i=1} P_i(X_i)X_i - \lambda f(X_1,\ldots,X_n) \text{ such that}$$

First order conditions are

$$o=\frac{\partial\Pi}{\partial X_i}=\frac{\partial P_i}{\partial X_i}X_i+P_i-\lambda\frac{\partial f}{\partial X_i}(i=1,\ldots,n) \qquad (3)$$

and,

$$o=\frac{\partial\Pi'}{\partial\lambda}=f(X_1,\ldots,X_n)$$

For any X_i and X_j where $(\partial P_i/\partial X_i)=P_i'$ and $(\partial f/\partial X_i)=f_i'$

$$P_i'X_i+P_i=\lambda f_i' \text{ and } P_j'X_j+P_j=\lambda f_j' \qquad (3a)$$

or

$$\frac{P_i'X_i+P_i}{P'X_j+P_j}=\frac{f_i'}{f_j'} \text{; we get the joint product.} \qquad (3b)$$

Taking total differential of (2), the net product function,

$$\begin{aligned} df=0&=f_1dX_k+\cdots+f_idX_i\\ &=f_jdX_j+\cdots+f_ndX_n \end{aligned}$$

and dividing by dX_i and holding all other dX's except dX_j equal to zero, we get:

$$f_i'+f_j'\frac{\partial X_j}{\partial X_i}=0, \text{ or, } \frac{f_j'}{f_i'}=-\frac{\partial X_j}{\partial X_i} \qquad (4)$$

Hence:

$$\frac{P_i'X_i+P_i}{P_j'X_j+P_j}=-\frac{\partial X_j}{\partial X_i} \text{, the elasticity of output.}$$

If X_i and X_j are both outputs, then $P_i'X_i+P_i=MR_i$ (the marginal revenue product) and likewise for X_j, $P_j'X_j+P_j=MR_j$ for a single firm. The ratio of MR's must equal the marginal rate of product substitution. If X_i and X_j are inputs, then $P_i'X_i+P_i=MC_i$ and the ratio of MC's must equal the marginal rate of factor substitution. If X_i is an input, X_j an output, equation (4) becomes $MC_i=\frac{\partial X_j}{\partial X_i}MR_j$ ie with inputs negative, MC of an input must equal its marginal revenue product. This completes first order conditions.

Second order conditions follow from the Bordered Hessian determinants, D_k, alternate in sign, starting with $D_2>0$ up to

$$(-1)^n\begin{vmatrix} \lambda f_{11}\cdots\lambda f_{1n}f_1\\ \cdot \qquad\qquad \cdot\\ \cdot \qquad\qquad \cdot\\ \lambda f_{n1}\cdots\lambda f_{nn}f_n \end{vmatrix}>0 \qquad (5)$$

where f_{ij} is second partial of f to X_i and X_j. λf_{ij} has an interesting interpretation.

Restating (3a) to interpret λ

$$\lambda=\frac{P_i'X_i+P_i}{f_i'}=\frac{\partial\Pi/\partial X_i}{\partial f/\partial X_i}=\frac{\partial\Pi}{\partial f} \qquad (6a)$$

and

$$f_{ij}=\frac{\partial\Pi}{\partial f}\cdot\frac{\partial^2 f}{\partial X_i\partial X_j}. \qquad (6b)$$

We have the dominant of a closed connected, non negative matrix with a unique λ. The positive valued λ indicates the second order conditions for normal production are present. A negative or zero valued λ would indicate output is an input.

(6b) indicates the change in X_j's marginal contribution to profit for a change in X_i.

X_i and X_j are (1) complements, (2) substitutes or (3) independent, respectively, if f_{ij} is greater, (2) less, or (3) equal to zero given $\lambda>0$. In case (1), their MC's are lower when used together than used separately (if either is recycled waste, it is productive). In case (2), MC's are higher when used together than apart. In case (3), they have no effect on each other. If X_i and X_j are outputs, we have similar effects. Case (1) means their joint marginal revenue product is higher than their separate ones; case (2) means the opposite. Case (3) means they are unrelated. Hence λf_{ij} determines internal economies in joint production. For example, let X_1 and X_2 respectively be inputs of molten sodium silicate and cullet. $\lambda f_{1,2}$ is positive because production efficiency is enhanced by their joint use. $\lambda f_{1,2}$ is higher if X_1 and X_2 are combined to make glass because adding cullet to new material speeds glass production. λf_{ij} is negative for detinned steel scrap and pig iron because new steel is more efficiently produced if the inputs are used separately. All this assumes normal production conditions i.e. $\lambda>0$, without the existence of second order conditions i.e. $\lambda\leqslant 0$, effects of X_i and X_j are difficult to interpret.

These are the basic conditions for efficiency in joint production. They are no different from those involved in the production of one good, which is just one special case of joint production. Joint production does, however, present extra problems. If X_i is an input and X_j is an output, how do we decide how much X_i is being used specifically to produce X_j? This difficulty can be illustrated from the production function in standard input-output form:

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[14] Cf. S. V. Ciriacy-Wantrup, "Economics of Joint Costs in Agriculture," *Journal of Farm Economics* 23(4):778n (November 1941).

$AX_i = d_i$ or

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} - \begin{pmatrix} d_1 \\ \cdot \\ \cdot \\ d_4 \end{pmatrix}$$

Input-output provides a connected, closed, nonnegative matrix which furnishes the signs of the partial cross elasticities of output computed in equation 4. Following (2), this becomes an input-output matrix with net input x_i's negative and net output x_j's positive: $X = (I - A)^{-1} d$

$$\text{OR} \begin{pmatrix} 1 - a_{11} & - a_{12} & - a_{13} & - a_{14} \\ - a_{21} & 1 - a_{22} & - a_{23} & - a_{24} \\ - a_{31} & - a_{32} & 1 - a_{33} & - a_{34} \\ - a_{41} & - a_{42} & - a_{43} & 1 - a_{44} \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \begin{pmatrix} d_1 \\ \cdot \\ \cdot \\ d_4 \end{pmatrix} \qquad (7)$$

The similarity to a Leontief model implies four separate production processes—one for each $x_i \cdot a_{ij}$'s take on a specific meaning which is not immediately evident in joint production. If coal and iron ore are being used to produce pig iron and slag, what is the meaning of ∂ (tons of slag)/∂ (tons of coal)?

In a fixed technology, where all inputs and outputs occur in constant proportions, $\frac{\partial x_j}{\partial x_i}$ cannot be calculated. But in this case, the vector of outputs can be treated as a single output. Total revenue, for example, is a price-weighted average of the outputs. It could just as well be revenue from a single output, since output proportions do not change and the joint production problem does not arise. If the output mix can be varied, however, by variation of the input mix, then certain inputs must have a differential effect on the output mix. We can establish this effect using (7) where $\frac{\partial x_j}{\partial x_i}$ is simply a_{ji}, a constant. Unfortunately, (7) implies constant returns both to factors and to scale, and neither of these prevails in industry. This is especially true for the waste handling industry which, if it uses recycling, has a wide range of sharply increasing returns to scale.[15] However, we can use a system similar to (7) for estimating $\frac{\partial x_j}{\partial x_i}$.

Given a level of output, an output mix, and the associated input level and mix, we should in practice be able to determine incrementally $\frac{\partial x_j}{\partial x_i}$ and the point at which total marginal revenue equals total marginal costs. Pareto-optimality requires that the sum of marginal prices of jointly produced goods must equal the marginal cost of producing them.[16]

There are practical limits in developing from neo-classical welfare theory an estimate of external costs which may be feasibly used in an operational model of solid waste. This leads us to consider a more limiting but operational measure, an effluent charge representing the marginal external cost of joint production which we will express as $-MR_E$, a negative marginal revenue of externalities. This measure will prove most useful in the extended model which follows.

Recycling Production Model

A. Prices and Production Levels

The recycling model is an extension of the joint production model. For the effects of joint production in terms of waste products, the following conditions are to be noted:

$$MR_1 + MR_2 + \cdots + MR_n = 0, \qquad (1)$$

and

$$MR_i = -\frac{\partial X_j}{\partial X_i} MR_j \qquad (2)$$

where MR_i is the marginal revenue of the ith good or service. Externalities are considered services or disservices, so that they may be included in these conditions.

Four goods are to be assumed: X_P, X_W, X_1, and X_E representing a finished good, a waste good, an input, and an externality associated with production of X_W. In respect to the status of MR_W, if positive, X_W is an output; if negative, it is an input. That is, a positive quantity (output) with a negative price has the same effect on revenue as a negative quantity (input) with a positive price, so that they are essentially the same.

In using this convenience, the cost incurred by X_W is an imputed cost. That is, some "real" input must be used to dispose of this waste material, and the negative price is really the cost of this input. Nothing is lost by assuming that X_1 can be used to this end.

When a firm recycles waste within its properties, it is not accounted for in (1) since waste output becomes an input. Waste appears as one or the other only in net status.

How is the price of waste determined for a firm? Price must be the same as market value. If its internal price or cost of disposal were higher, the firm would not use it; if lower, a demand would arise on the outside and the price would rise. Therefore, it does not need to appear in the production function.

B. Decision Rules

What, then, is the effect of X_W's presence in the production process? Suppose it is not marketable. Buchanan and Stubblebine's condition[17] requires that

$$-MR_W - MR_1 - MR_P = MR_E.$$

We can restate this as

$$MR_P = -MR_W - MR_1 - MR_E. \qquad (3)$$

Then total marginal revenue equals total marginal cost, and MR_E becomes an internal cost. This is simply restating the joint production propositions following from equation (4), P. 64 in terms of MR's, i.e. $(P'_P X_P + P_P) = -(P' X_W + P_W) - (P'_1 X_1 + P_1) - (P'_E X_E + P_E)$, where $P'_i = \frac{\partial P_i}{\partial X_i}$.

It will be useful to define more precisely what MR_E is. Stating MR_E as an input implies that the elements affected by the externality are also inputs. This becomes clearer when we consider that without the externality, the resources affected can be used for some other purpose. This corresponds well to Rothenberg's concept of a "quantity of assimilating medium."[18] This

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[15] W. W. Walters, "Production and Cost Functions," *Econometrica*, January–April, 1963, pp. 1–33.

[16] E. J. Mishan, "The Relationship Between Joint Products, Collective Goods, and External Effects," *Journal of Political Economy* 77(3):331 (May–June 1969).

[17] Buchanan and Stubblebine did for Coase what Meade has done for Pigou. They derived the concept of a Pareto-relevant externality, defining it as a condition which exists between two individuals A and B.

Using Pigou's system of taxes and subsidies, they assume a diseconomy and internalize it as an externality, reducing the effects of A to zero. When B maximizes his utility internally, Pareto-efficiency is achieved. If however, the tax is not returned, and B maximizes a disequilibrium occurs. Pigou's tax is not Pareto-efficient. Two-way compensation is necessary.

Further, the marginal conditions and use of marginal rate of substitution indicate the importance of opportunity cost and the level of reduction. Reduction of the externality must take place only to the point where the system becomes an equality. This implies that an externality will continue to exist even in Pareto-optimality.

[18] Rothenberg, p. 116.

medium is being used as an input; as its supply decreases, its price climbs, and justification for its use depends on how much is added to revenue. For efficiency, $MR_E = -\frac{\partial X_P}{\partial X_E} MR_P$, so that the extent of an externality depends on the value of the good produced.

Another interesting conclusion can be drawn. Efficiency requires that $MR_W = -\frac{\partial X_I}{\partial X_W} MR_1$. But from above, $\frac{\partial X_I}{\partial X_W} = 1$, so that $MR_W = MR_I$. In (3), MR_I refers only to the production of X_P. We find also that $MR = -\frac{\partial X_P}{\partial X_I} MR_P$ and $MR_E = \frac{\partial X_P}{\partial X_E} MR_P$. This states the conclusion that the whole cost of an externality falls on the consumer, and none of it is absorbed by the factors.

X_I is affected indirectly because waste and externalities increase the marginal revenue which must be gained from X_P. Assuming that the whole industry is subject to the same technology and externalities, the firm faces a decreasing demand curve and production must be cut back. This reduces the need for X_I without compensation. Hence, waste generation and externalities affect both the consumers and the factors.

Now suppose the waste generated is marketable. Assume five goods: X_P, X_W, X_K, X_L, and X_E (a finished product, a waste product [marketable], two inputs, and an externality). The condition for efficiency is now

$$MR_P + MR_W = -MR_K - MR_L - MR_E. \quad (4)$$

That is, just because MR_W is marketable does not mean that it will not produce externalities. These externalities, however, will be the same as those produced by any finished product and therefore we will assert that $X_E = 0$.

Remember that if the output mix can be changed by a change in the input mix, then $\frac{\partial X_P}{\partial X_L}$ and $\frac{\partial X_P}{\partial X_K}$ are meaningful.[19] Of course, they may vary with the level of production. Our product and factor mix, then, will be controlled by the conditions

$$MR_L = -\frac{\partial X_P}{\partial X_L} MR_P \qquad MR_L = -\frac{\partial X_W}{\partial X_L} MR_W$$

$$MR_K = -\frac{\partial X_P}{\partial X_K} MR_P \qquad MR_K = -\frac{\partial X_W}{\partial X_K} MR_W$$

Is this consistent with our previous conditions? Yes.

Substitute for MR_L and MR_K. Then

$$\begin{aligned} MR_P + MR_W &= -\frac{\partial X_P}{\partial X_L} MR_P - \frac{\partial X_W}{\partial X_L} MR_W \\ &\quad - \frac{\partial X_P}{\partial X_K} MR_P - \frac{\partial X_W}{\partial X_K} MR_W \\ &= -\left(\frac{\partial X_P}{\partial X_L} + \frac{\partial X_P}{\partial X_K}\right) MR_P \\ &\quad - \left(\frac{\partial X_W}{\partial X_L} + \frac{\partial X_W}{\partial X_K}\right) MR_W \\ &= MR_P + MR_W \end{aligned}$$

The price of factors, then, is determined by the revenue from our products and their relative contribution to each. And our waste product has become a normal good.

An intermediate good is easy to deal with because it is very similar to the case of waste produced in conjunction with production of a finished good. Input (the intermediate good) has, however, a propensity for creating waste, and is directly responsible for added costs (*e.g.*, then added costs will be associated with use of the intermediate good). It may, therefore, be more efficient to use an input (*e.g.*, coal with a

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[19] Euler's theorem would normally require $\left(\frac{\partial x_P}{\partial x_L}\right) + \left(\frac{\partial x_P}{\partial x_K}\right) = 1$. However, since our input quantities are negative, this expression equals -1.

low sulfur content) with a higher initial cost, but one which imposes fewer added costs. The use of two inputs will be adjusted until the total costs associated with each are equal. The external costs of waste still act as input costs to the final product, and the consumer of the finished product (*via* the intermediate good) bears the cost. Inputs bear the external costs in an indirect manner.

While clearer quantification of *MR* amplifies the usefulness of the recycling model, all we can suggest is use of accurate data on factor and output prices and simulation of as nearly perfect conditions of partial equilibrium as possible to increase the accuracy of the partial cross elasticity of output measures.

Conclusions

There is a growing interest in waste as a valuable resource and it is drawing large firms and modern technology into the field. In this paper, we have approached the economic model of recycling with a joint production model where marginal conditions are identified. Internal costs and a practical measure of external costs have been incorporated into the model.

The applicability of the model relates to the question of what combination of methods deals most efficiently with heterogeneous waste. With regard to a special process, separation can be considered in the form of a joint-producing firm. The value of each of its products can be positive or negative which is shown by the model. The problem of pricing can be handled in one of two ways, depending on whether waste is separated before or after collection. Assuming that waste is separated after collection, the material-input has a potential for causing waste handling and external costs. The components show varying potentials, but it is impractical to determine how much of each component is contributed by a given waste generator. The costs of negative-valued outputs, therefore, are charged against the revenue of positive-valued outputs as shown by the model.

If, on the other hand, waste is separated before collection, the inputs in the model can be given different prices. These prices will influence the household's valuation of a good which has a propensity for causing waste handling and external costs.

Industry is seeking new revenue sources and the quantity of solid waste could be reduced substantially if the value of new waste materials were realized. Such realization is feasible by making the joint production model operational. Waste will, in the future, be treated increasingly as a normal good and will be considered in the market organization of industries. A less obvious result is an increase in the efficient use of resources for solid waste management. Establishment of waste handling as a normal industry will effect regional structure. This will affect industrial location, local prices and local tastes, as well as introduction of new products in the region as a result of recycling, in consonance with growing interest in regional importance. However, new opportunities must have public support to bring about appropriate accounting of external effects. Needed is a realization of the potential of solid waste management, and that proper economic concepts and tools can provide a rational approach to the escalating problem of solid waste management.

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[20] In the separation process, the picture was in the past one of intensive labor; that is, components were either collected separately from households (intra-household labor-inputs and collection labor-inputs) or separated by hand on a conveyor belt in treatment plant after collection. The growing expense of time as labor-inputs discouraged these modes, and the health danger of the conveyor belt was another negative factor. Capital intensive methods were the next development, for the purpose of specific separation. Capital intensive methods generally require grinding or shredding before separation, and utilize magnetism, specific gravity, and/or grating for separation. The initial investment is large, and a large waste load is required to bring marginal cost above average cost. The problem is one of rising production.

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