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NIPE WP 12/ 2010

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS
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A Growth Model for the Quadruple Helix Innovation Theory

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Abstract: We propose a theoretical growth model with which to frame analytically the Quadruple Helix Innovation Theory (QHIT). The aim is to emphasise the investment in innovation transmission mechanisms in terms of economic growth and productivity gains, in one-high-technology sector, by stressing the role played by the helices of the Quadruple Helix Innovation Model: Academia and Technological Infrastructures, Firms of Innovation, Government and Civil Society. In the existing literature, the relationship between the helices and respective impacts on economic growth does not appear clear. Results are fragile due to data weakness and the inexistence of a theoretical framework to specify the relationship between the helices. Hence our motivation for providing the QHIT with a theoretical growth model. Our intent is to model the importance of emerging, dynamically adaptive, and transdisciplinary knowledge and innovation ecosystems to economic growth. We find that higher economic growth rate is obtained as a result of an increase in synergies and complementarities between different productive units, or an increase in productive government expenditure.

Keywords: Economic Growth; Quadruple Helix Innovation Model; Innovation Ecosystems.

JEL Classifications: O10, O18, O31

1 Introduction

According to the Quadruple Helix Innovation Theory (QHIT), a country's economic structure lies on four pillars/helices: Academia; Firms, Government and Civil Society, and economic growth is generated by the clustering and concentration of talented and productive people. Creative cities and knowledge regions are thus considered the true engines of economic growth. Academia and Firms, together with Technological Infrastructures of Innovation, provide the integrated innovation ecosystem where all forms of creativity can rise. In turn, Governments provide the financial support and the regulation system for the definition and implementation of innovation activities. Civil Society demands for ever innovating goods and services.

In the existing literature, the relationship and the interaction between the different helices and economic growth is not obvious, since the majority of studies lead to fragile conclusions due to data weakness and the inexistence of a theoretical model that highlights the relations between the four different helices.

We hence wish to provide the QHIT with a theoretical framework with which to investigate analytically the economic assumptions, results and predictions of this young but thriving theory. Our intent is to highlight and model the role of emerging, dynamically adaptive, and transdisciplinary knowledge and innovation ecosystems in economic growth.

In modelling the role of innovation ecosystems in economic growth, we propose a theoretical development on the macroeconomic transmission mechanisms of investment in innovation, in terms of economic growth and productivity gains in one-high-technology-sector model.

We also wish to emphasise the importance of public investments in Technological Infrastructures of Innovation and the role of Firms in increasing productivity growth in certain regions, in strict coordination with local governmental policies for innovation and entrepreneurship and with the national plans for science and technology. Local economic development is, in fact, nowadays promoted through various initiatives that link universities to industrial innovation based on university research, such as the creation of science parks, business incubators and other bridge-institutions.

Gibbons et al. (1994) argue that scientific research is a powerful, but not the only organized human activity that produces knowledge relevant to the economy, economic policy and other society needs. Indeed, according to Carayannis and

Campbell (2006), innovation ecosystems are systems in which a variety of organizations and institutions in public and private sectors – namely governments, universities, research institutions, business communities, and funding/ financing organizations –, collaborate and compete between each other, thus creating an environment that fosters innovation through free interaction of information, human resources, financial capital and institutions. Economic growth arises as a result of the formation of specialised differentiated productive units that interact with one other and complement each other. Carayannis and Campbell (2009) add that the participating elements in the Quadruple Helix Innovation Model (QHIM) are government, research and development (R&D) facilities, industrial R&D facilities, university laboratories and civil-society based sources of innovation and knowledge.

Despite improvements in the scientific-knowledge-base and in organizational know-how, innovation processes are not easy to delineate or manage. Generally, Academia consensually play an important role as a source of fundamental knowledge and, occasionally, industrially relevant technology. Nevertheless, the relationship between entities can be difficult to illustrate, in particular, the university-industry relationships can be difficult for firms to manage. New fields of knowledge with high rates of technological advance have been developed, like Nano-Bio-TIC, offering rich opportunities for commercial exploitation, but they still pose a problem of coordination and translation among the different entities.

According to the latest version of Oslo Manual (OECD, 2005), a basic reference for the measurement of scientific and technological activities including guidelines for innovation, the strict definition of innovation is difficult to attain due to the complexity of innovation processes and the different ways in which they can occur according to types of firms and industries. In this sense, Schumpeter (1934, p. 66) provides a useful definition of innovation as the “carrying out of new combinations to result in dynamic or discontinuous development”. West and Farr (1989, p. 16) define innovation as the “. . . intentional introduction and application within a role, group or organization of ideas, processes, products or procedures, new to the relevant unit of adoption, designed to significantly benefit role performance, the group, the organization or the wider society”. For Johnson (1992), innovation is a continuous cumulative process involving not only radical and incremental innovation but also the diffusion, absorption and use of innovation. Accordingly, innovation is an interactive learning process taking place in connection with ongoing activities in procurement, production

and sales.

Before the 2000's, we had the national system of innovation - a set of distinct institutions which jointly or individually contribute to the development and diffusion of new technologies and which provides the setting within which Government implements policies to influence the innovation process. In the 2000's, a series of change in views regarding innovation systems have taken place - global networking in value added and innovation, centres and innovation ecosystems, customers and users, systemic thinking and sustainable innovation.

Moreover, a new setting for innovation has emerged, consisting in global networks of knowledge hubs, where sustainable innovation is based on ethically, socially, economically, and environmentally sustainable principles. In addition, value chains are becoming specialised and distributed around the world, while the collaboration is facilitated by cross-regional technical communities based on "brain circulation". Powell and Grodal (2005) argue that networks contribute significantly to the innovative skills of firms by exposing them to new sources of ideas, speeding up access to resources, and facilitating the transfer of knowledge. In this sense, the nature of knowledge, conceptualised in terms of tacitness or explicitness, is a crucial factor in determining whether members of a network can effectively share information and skills. The Technological Infrastructures of Innovation are also crucial in the codification of the tacit knowledge in the form of finished inputs.

Aiming to capture such wide scope of definitions for innovation and innovation activities, in our developed model we assume that the whole economic society takes part in the innovation process, that is, we assume the one-high-technology-sector structure in that innovation is undertaken with the same technology as that of the final good and inputs.

The QHIT is a development of the Triple Helix Innovation Theory. According to the National Institute for Triple Helix Innovation, the triple helix innovation process is based on three pillars, Academia, Government, and Industry, which play integrated and sometimes overlapping roles. It consists in the establishment of creative links between the three above helices in order to develop or discover new knowledge, technology or products and services that are conveyed to final users in fulfilment of society needs. Final users consume the knowledge, technology, or products and services or use them to produce new goods and services that are ultimately sold or consumed.

Etzkowitz and Leydesdorff (2000), among others, have defended the applica-

bility of the triple helix model. Economies where triple helix applies have high levels of skilled labour, knowledge-based and innovation-driven industry and service sectors, technology-intensive universities, governments and industries. Their model focuses on innovative firms and the support they may obtain from state authorities, in particular, from universities and research institutions. Educational institutions of higher learning represent Academia. Government may be represented by any of the three levels of government and their owned corporations as national, regional and local. There are no restrictions on the types of industry involvement in triple helix innovation processes, i.e., industry may be represented by private corporations, partnerships or sole ownerships (e.g., Etzkowitz and Klofsten, 2005).

Arguing that the triple helix is not a sufficient condition for long-term innovative growth, the QHIT adds a fourth helix, Civil Society, which takes part in the knowledge creation process (e.g., Lijemark, 2004). Khan and Al-Ansari (2005), too, consider the interaction between Firms, Academia, Government and Civil Society as a requirement for sustainable growth. The “quadruple helix” emphasises the importance of integrating the perspective of the media-based and culture-based public, the Civil Society.

Within the QHIT, Delman and Madsen (2007) also consider one kind of organizations which lead to quadruple helix structures. They are independent, non-profit, member-based organizations which combine funding from government and private sector. They have the important task of translation and coordination, in the emerging fields of knowledge, between the four helices. They implement shared-cost R&D programs; build R&D Infrastructures; and supply technical products and services. They create networks and build partnerships and associations to undertake R&D. They contribute to a national cross-sectoral vision of R&D excellence, and develop, attract and retain highly qualified people. These hybrid entities constitute the Technological Infrastructures of Innovation. Etzkowitz and Leydesdorff (2000), too, propose different possible interactions between the helices: In a centralised model, government controls academia and industry, whereas in a decentralised model, each of the helices develops independently; with hybrid organizations – the Technological Infrastructures of Innovation – playing the important interface role. Interaction and cooperation between the different four helices fosters the co-evolution of Government, Academia, Industry and Civil Society. The quadruple helix model offers, then, a useful framework of orientation for policy and policymaking.

The model that we propose, a Quadruple Helix Innovation Model (QHIM),

captures the quadruple helix innovation process by specifying, on the production side, a one-high-technology-sector structure in which Academia, Government, and Firms produce, in an integrated and overlapping manner, innovation, new knowledge, technology and products and services, all together forming a final good - aggregate output -, which can be consumed or invested in the production of more innovation, new knowledge, technology and products and services. The cluster effects between the different productive entities of the QHIM are captured by the assumption of complementarities between all the entities which contribute in an intermediate level to the final good production, which we name the Intermediate Productive Units (IPUs). Additionally, we assume that there are internal costs to investment in both manufacture and R&D, which captures the relevant economic nature of costly investment in research. The role of Civil Society is specified on the consumption side of our economy, where households demand for and consume the innovation, knowledge, technology, products and services, in the form of the final good, the aggregate output of our economy.

The remainder of the paper is organised as follows. Section 2 presents the model and its results. Here, the production and the consumption side are described and the general equilibrium is derived. Section 3 ends the paper with some final remarks and references about the public policies coordination.

2 Specification and Results of the Model

2.1 Production Side - Technology Curve

2.1.1 Government Expenditure

The government's role in this economy (our ecosystem of innovation) consists in providing a pure public good, in the form of government expenditure on education, health, technological infrastructures of innovation and innovation services, which increases the productivity of all inputs in the same way. We follow Barro (1990) and assume that productive government expenditure is a flow variable. For all t , the current flow of productive government expenditure, $G(t)$, is a constant fraction of current output, $Y(t)$, that is:

$$G(t) = \tau Y(t) , \quad 0 < \tau < 1. \tag{1}$$

The government's budget is balanced in all periods. Assuming zero-public-debt, and zero-consumption-taxes, for simplicity, the government's budget con-

straint is:

$$G(t) = T(t) = \tau Y(t). \quad (2)$$

2.1.2 Final Good

There is one final good, $Y(t)$, whose production requires labour, $L(t)$, public expenditure, $G(t)$, and the inputs, $x_i(t)$, of a number $A(t)$ of intermediate productive units i ($i = 0 \dots A$). Each intermediate productive unit is associated with one innovation i ($i = 0 \dots A$).

2.1.3 Intermediate Productive Units (IPUs)

We assume that Academy, Firms and Technological Infrastructures of Innovation have an identical productive role in this economy. They constitute the intermediate productive units i ($i = 0 \dots A$), and contribute to output production, $Y(t)$, by producing inputs $x_i(t)$.

With the goal of capturing the synergy effects that are nurtured by and between the existing intermediate productive units (IPUs), we assume that there are complementarities between the IPUs. Matsuyama (1995), for instance, regards complementarities as an essential feature in explaining economic growth, business cycles and economic development. Indeed they constitute a very relevant feature of industrialised economies. Building on Evans et al. (1998), we specify that the inputs of the IPUs enter complementarily in the production function. Hence the production function is:

$$Y(t) = L(t)^{1-\alpha-\beta} G(t)^\beta \left(\int_0^{A(t)} x_i(t)^\gamma di \right)^\phi,$$

which, substituting $G(t)$ by its equivalent according to equation (1), becomes:

$$Y(t) = \tau^{\frac{\beta}{1-\beta}} L(t)^{\frac{1-\alpha-\beta}{1-\beta}} \left(\int_0^{A(t)} x_i(t)^\gamma di \right)^{\frac{\phi}{1-\beta}}, \quad \gamma\phi = \alpha, \quad \frac{\phi}{1-\beta} > 1. \quad (3)$$

The parameter restriction $\gamma\phi = \alpha$ is imposed in order to preserve homogeneity of degree one. And the assumption $\frac{\phi}{1-\beta} > 1$ is made so that the inputs x_i are complementary to one another, that is, so that an increase in the quantity of one input increases the marginal productivity of the other inputs.

Assuming that it takes one unit of physical capital $K(t)$ to produce one physical unit of any type of intermediate productive units' input, physical capital

is related to inputs $x_i(t)$ by the rule:

$$K(t) = \int_0^{A(t)} x_i(t) di. \quad (4)$$

2.1.4 Innovation

Regarding innovation, we wish to frame the idea that the whole society is involved in the innovation process, because we wish to accommodate Florida's (2002) idea that creativity comes from people and people are the critical resources of the new age economies. Indeed as Barroso (2010) points out, countries' new agendas require a coordinated response with social partners and civil society in order to achieve sustainable and equitable growth. Following Rivera-Batiz and Romer (1991), we assume the one-high-technology-sector structure in that innovation is undertaken with the same technology as that of the final good and inputs. We further assume that innovation i requires $P_A i^\xi$ units of foregone output, where P_A is the fixed cost of one new innovation-design in units of foregone output, and i^ξ represents an additional cost of innovation i in terms of foregone output, meaning that there is a higher cost for higher indexed innovations. Like in Evans et al. (1998), this extra cost is introduced in order to avoid explosive growth.

Accommodating Anagnostopoulou (2008)'s argument, innovation expenses are thus specified as part of total capital investment expenses. With zero depreciation, for simplicity, total investment in each period, $\dot{W}(t)$, is equal to physical capital accumulation, $\dot{K}(t)$, plus innovation expenditure, $P_A(t)\dot{A}(t)^\xi$:

$$\dot{W}(t) = \dot{K}(t) + P_A(t)\dot{A}(t)^\xi. \quad (5)$$

It follows that total capital $W(t)$ is equal to physical capital plus innovation capital:

$$W(t) = K(t) + P_A \frac{A(t)^{\xi+1}}{\xi + 1}. \quad (6)$$

It will be later shown that Y and W growth at the same rate, which means that we can write aggregate output as a function of total capital, in the following form:

$$Y(t) = BW(t), \quad (7)$$

where B , the marginal productivity of total capital, is constant.

2.1.5 Internal Costly Investment

Agreeing with Benavie et al. (1996) and Romer (1996), our model contemplates costs to capital accumulation. Following Thompson (2008), we consider that investment in total capital $W(t)$ involves an internal cost. With zero capital depreciation, installing $I(t) = \dot{W}(t)$ new units of total capital requires spending an amount given by:

$$J(t) = I(t) + \frac{1}{2}\theta \frac{I(t)^2}{W(t)}, \quad (8)$$

where $C(I(t), W(t)) = \frac{1}{2}\theta \frac{I(t)^2}{W(t)}$ represents the Hayashi's (1982) installation cost.

The equilibrium investment rate is the one that maximises the present discounted value of cash flows. The *current-value Hamiltonian* is:

$$H(t) = BW(t) - I(t) - \frac{1}{2}\theta \frac{I(t)^2}{W(t)} + q(t)I(t), \quad (9)$$

where $q(t)$ is the market value of capital and the transversality condition of this optimization problem is $\lim_{t \rightarrow \infty} e^{-rt}q(t)W(t) = 0$.

We solve the model for a particular solution, the Balanced Growth Path, for which growth rates are constant. We will suppress the time argument, from now onwards, whenever that causes no confusion. Having in mind that the growth rate of output is $g_Y = g_W = g = \frac{I}{W}$, the first-order condition, $\frac{\partial H}{\partial I} = 0$ is equivalent to:

$$q = 1 + \theta g, \quad (10)$$

which says that in a balanced growth path solution, q is constant.

The co-state equation, $\frac{\partial H}{\partial W} = rq - \dot{q}$, is equivalent to:

$$\dot{q} = rq - \left(B + \frac{1}{2}\theta g^2 \right),$$

which, in a balanced growth path solution, becomes:

$$q = \frac{B + \frac{1}{2}\theta g^2}{r}, \quad (11)$$

which also implies that r is constant in a balanced growth path solution.

Closing up the model, we have the economy's budget constraint:

$$I(t) + \frac{1}{2}\theta \frac{I(t)^2}{W(t)} = Y(t) - G(t) - C(t). \quad (12)$$

Continuing with solving the model, final good producers are price takers in the market for inputs. In equilibrium they equate the rental rate on each input with its marginal productivity, so the demand curve faced by each IPU is:

$$\frac{\partial Y(t)}{\partial x_j(t)} = R_j(t) = \frac{\alpha}{1-\beta} \tau^{\frac{\beta}{1-\beta}} L(t)^{\frac{1-\alpha-\beta}{1-\beta}} x_j(t)^{\gamma-1} \left(\int_0^{A(t)} x_i(t)^\gamma di \right)^{\frac{\phi}{1-\beta}-1}. \quad (13)$$

Turning now to the IPU's production decisions. Once invented, the physical production of each unit of the specialised input requires one unit of capital. So, in each period, the monopolistic IPU maximises its profits, taking as given the demand curve for its good:

$$\max_{x_j(t)} \pi_j(t) = R_j(t)x_j(t) - rqx_j(t),$$

which leads to the markup rule:

$$R_j = \frac{rq}{\gamma}. \quad (14)$$

At time t , in order to enter the market and produce the A th input, an IPU must spend up-front an amount given by $P_A A(t)^\xi$, where, as mentioned earlier, P_A is the fixed cost of one new innovation-design, in units of foregone output, and i^ξ represents an additional cost of patent i in terms of foregone output. Hence, the dynamic IPU's zero-profit condition is:

$$P_A A(t)^\xi = \int_t^\infty e^{-r(\tau-t)} \pi_j(\tau) d\tau,$$

which, assuming no bubbles, is equivalent to:

$$\xi g_A = r - \frac{\pi_j}{P_A A^\xi}. \quad (15)$$

The symmetry of the model implies that $R_j(t) = R(t)$, $x_j(t) = x(t)$ and $\pi_j(t) = \pi(t)$. Hence we can rewrite $R(t)$ as:

$$R = \Omega_R A^{\frac{\phi-1+\beta}{1-\beta}} x^{\frac{\alpha-1+\beta}{1-\beta}}, \quad (16)$$

where $\Omega_R = \frac{\alpha}{1-\beta} \tau^{\frac{\beta}{1-\beta}} L^{\frac{1-\alpha-\beta}{1-\beta}}$ is a constant, $\pi(t) = (1-\gamma)R(t)x(t)$ is equivalent to:

$$\pi = \Omega_\pi A^{\frac{\phi-1+\beta}{1-\beta}} x^{\frac{\alpha}{1-\beta}}, \quad (17)$$

where $\Omega_\pi = (1 - \gamma)\Omega_R$. and x is given by:

$$x = A^\xi \left(\frac{\Omega_R}{R} \right)^{\frac{1-\beta}{(1-\beta)-\alpha}}, \quad (18)$$

where we impose the parameter restriction $\xi = \frac{\phi - (1-\beta)}{(1-\beta) - \alpha}$.

In a balanced growth path, the interest rate and the shadow-value of capital are constant and hence so is R . It follows from expression (16), that we then must have:

$$\left(\frac{\phi - 1 + \beta}{1 - \beta} \right) g_A = - \left(\frac{\alpha - 1 + \beta}{1 - \beta} \right) g_x,$$

that is:

$$g_x = \xi g_A, \quad \xi = \frac{\phi - (1 - \beta)}{(1 - \beta) - \alpha}.$$

Symmetry also implies that equation (4) simplifies to $K = Ax$, which means that $g_K = (1 + \xi)g_A$. Likewise, the production function (3) is rewritten as:

$$Y = \tau^{\frac{\beta}{1-\beta}} L^{\frac{1-\alpha-\beta}{1-\beta}} A^{\frac{\phi}{1-\beta}} x^{\frac{\alpha}{1-\beta}}, \quad (19)$$

whose time-differentiation gives $g_Y = \left(\frac{\phi + \alpha\xi}{1-\beta} \right) g_A = (1 + \xi)g_A$. As a result, equation (15) becomes:

$$g_Y = \frac{1 + \xi}{\xi} \left(r - \frac{\Omega_Y}{R^{\frac{\alpha}{(1-\beta)-\alpha}}} \right), \quad \Omega_Y = \frac{(1 - \gamma)}{P_A} \Omega_R^{\frac{1-\beta}{(1-\beta)-\alpha}} \quad (20)$$

Equation (20) unites the equilibrium balanced growth path pairs (r, g) on the production side of this economy. We call it Technology curve, after Rivera-Batiz and Romer (1991).

2.2 Consumption Side - The Euler Equation

Infinitely lived homogeneous consumers/householders constitute another pillar of the Quadruple Helix Innovation Theory. They want to consume a distinct and high quality good, that is, they demand to consume a final product, Y , whose production requires innovation. Analytically, we can adopt the standard specification for intertemporal consumption. Consumers maximise, subject to a budget constraint, the discounted value of their representative utility, subject

to a budget constraint:

$$\max_{C(t)} \int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\sigma}}{1-\sigma} dt \quad (21)$$

$$s.t. \quad \dot{E}(t) = rE(t) + w(t) - C(t), \quad (22)$$

where variable $C(t)$ is consumption of $Y(t)$ in period t , ρ is the rate of time preference and $\frac{1}{\sigma}$ is the elasticity of substitution between consumption at two periods in time. Variable $E(t)$ stands for total assets, r is the interest rate, $w(t)$ is the wage rate, and it is assumed that households provide one unit of labour per unit of time.

The transversality condition of this optimisation problem is $\lim_{t \rightarrow \infty} \mu(t)E(t) = 0$, where $\mu(t)$ is the shadow price of assets, and consumption decisions are given by the familiar Euler equation:

$$g_C = \frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho). \quad (23)$$

2.3 General Equilibrium

Time-differentiation of the investment equation (5) tells us that W grows at the same rate as Y :

$$\frac{\dot{W}}{W} = \frac{\dot{K}}{K} \frac{K}{W} + \frac{\dot{A}}{A} \frac{A^{1+\xi}}{W} P_A,$$

which is equivalent to:

$$g_W = (1 + \xi)g_A$$

Then the economy's budget constraint (12) tells us that a constant growth rate of W implies that consumption grows at the same rate as output:

$$\dot{W} = Y - G - C - \frac{1}{2}\theta \frac{I(t)^2}{W(t)},$$

which is equivalent to:

$$g_W = \frac{Y}{W} - \frac{G}{W} - \frac{C}{W} - \frac{1}{2}\theta g^2. \quad (24)$$

According to (24) a constant g_W requires that:

$$\left(\frac{\dot{Y}}{W}\right) = \left(\frac{\dot{G}}{W}\right) + \left(\frac{\dot{C}}{W}\right),$$

which, as G and W grow at the same rate as Y , implies that C , too, grows at the same rate as Y . As labour is constant, the per-capita economic growth rate is then given by:

$$g_C = g_Y = g_K = g_W = g = (1 + \xi)g_A.$$

The general equilibrium solution is obtained by solving the system of the two equations, (20) and (23), in two unknowns, r and g . Recalling equation (10), the system to be solved is:

$$\begin{cases} g = \frac{1}{\sigma}(r - \rho) \\ g = \frac{1+\xi}{\xi} \left[r - \frac{\Omega}{(r+r\theta g)^{\frac{\alpha}{(1-\beta)-\alpha}}} \right] \end{cases}, \quad r > g > 0, \quad (25)$$

where $\Omega = \gamma^{\frac{\alpha}{(1-\beta)-\alpha}} \frac{(1-\gamma)}{P_A} \Omega_R^{\frac{1-\beta}{(1-\beta)-\alpha}}$ and $\Omega_R = \frac{\alpha}{1-\beta} \tau^{\frac{\beta}{1-\beta}} L^{\frac{1-\alpha-\beta}{1-\beta}}$.

The restriction $r > g > 0$ is imposed so that present values will be finite, and also so that our solution(s) have positive values for the interest rate and the growth rate.

While the Euler equation is linear and positively sloped in the space (r, g) , the Technology curve is nonlinear, as shown in the Appendix. Although the Technology curve is nonlinear, the model has a unique solution.

Proposition 1 *The Quadruple Helix Innovation Model has a unique solution for $\sigma > 1$ and $\Omega^{\frac{1-\alpha-\beta}{1-\beta}} > \rho$.*

Proof. Defining two new variables and rewriting our system, we can show that the proposed model has a unique solution. Our new variables are:

$$Y = \theta g; \quad Z = r(1 + \theta g),$$

which allows us to rewrite the system as:

$$\begin{cases} Z = \frac{\sigma}{\theta} (Y + 1) (Y + \eta) \\ Z^\omega = \frac{\lambda}{Y + \mu} \end{cases}, \quad (26)$$

where $\omega = \frac{\alpha}{(1-\beta)-\alpha}$, $\lambda = \frac{\theta\Omega}{\sigma-\frac{\xi}{1+\xi}}$, $\mu = \frac{\rho\theta}{\sigma-\frac{\xi}{1+\xi}}$ and $\eta = \frac{\rho\theta}{\sigma}$.

Our restrictions become:

$$Y > 0 ; Z > \frac{1}{\theta}Y(Y + 1).$$

To ensure that $r > g$, we impose $\sigma > 1$ so that the Euler equation (23) lies above the 45° line. This implies that λ, μ and η are all positive. Hence the first equation of the rewritten system defines a strictly decreasing curve $Y \mapsto Z(Y)$ from $Z(0) = \left(\frac{\Omega}{\rho}\right)^{\frac{1}{\omega}}$ to $Z(\infty) = 0$, while the second equation defines a strictly increasing curve $Y \mapsto Z(Y)$ from $Z(0) = \rho$ to $Z(\infty) = \infty$. Hence the system has a unique solution in the region $Y > 0$ iff $\Omega > \rho^{\omega+1}$ (which is equivalent to $\Omega^{\frac{1-\alpha-\beta}{1-\beta}} > \rho$). The second restriction is also met because $Z = \frac{\sigma}{\theta}(Y + 1)(Y + \eta) > \frac{1}{\theta}Y(Y + 1)$. ■

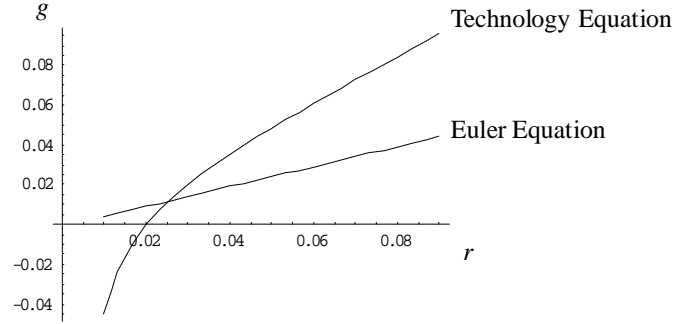
Since the nonlinearity of the Technology curve does not allow for the analytical derivation of the equilibrium solution(s), we resort to solving the system through a numerical example. The chosen values for our parameters are:

$$\begin{aligned} \sigma &= 2 ; \rho = 0.02 ; \alpha = 0.4 ; \beta = 0.3 ; \gamma = 0.1 ; \phi = 4 ; \\ \xi &= 11 ; L = 1 ; \theta = 3 ; P_A = 15 ; \tau = 0.15, \end{aligned}$$

where the values for α, γ and consequently $\phi = \frac{\alpha}{\gamma}$ are the same as those used by Evans et al. (1998) in their numerical example. In turn $\xi = \frac{\phi-(1-\beta)}{(1-\beta)-\alpha} = 11$. The values for the preference parameters σ and ρ are in agreement with those found in empirical studies such as Barro and Sala-i-Martin (2004). Population is often chosen to have unity value. The value for parameter τ is in agreement with Irmen and Kuehnel (2009). And the values for θ and P_A are chosen so as to give us realistic values for the equilibrium growth rate and interest rate. With the chosen parameter values, system (25) becomes:

$$\left\{ \begin{array}{l} g = 0.5r - 0.01 \\ g = \left(\frac{12}{11}\right) \left[r - \frac{(0.1)^{\frac{4}{3}}(0.009)\left(\frac{4}{7}\right)^{\frac{7}{3}}}{(r+3rg)^{\frac{4}{3}}} \right] \end{array} \right. , \quad r > g > 0,$$

Figure 1, with r on the horizontal axis and g on the vertical axis, helps us visualise this economy's balanced growth path general equilibrium solution,



which, for the adopted parameter values, is:

$$r = 0.025253 ; g = 0.0116265$$

Proposition 2 *Everything else constant, an increase in the public investment parameter, τ , leads to an increase in the equilibrium growth rate.*

Proof. Looking at the rewritten model (26):

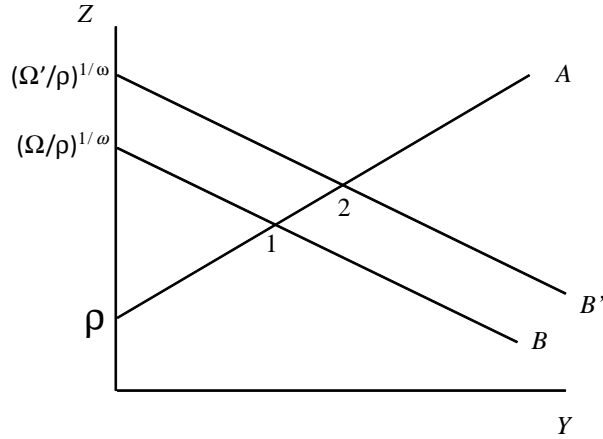
$$\begin{cases} Z = \frac{\sigma}{\theta} (Y + 1) (Y + \eta) & (A) \\ Z^\omega = \frac{\lambda}{Y + \mu} & (B) \end{cases},$$

naming our curves (A) and (B), curve (A) is positively sloped and curve (B) is negatively sloped in the space (Z, Y) . An increase in τ implies an increase in Ω , meaning that curve (B) shifts to the right. The new equilibrium has higher values for Z and Y , as illustrated in Figure 2. Given that $g = \frac{Y}{\theta}$, this implies a higher value for the growth rate ($dr = 0$).

Proposition 3 *Everything else constant, an increase in the complementarities parameter, $\frac{\phi}{1-\beta}$, leads to an increase in the equilibrium growth rate.*

Proof. As in Proposition 2, an increase in $\frac{\phi}{1-\beta}$ implies an increase in α , hence an increase in Ω , meaning that curve (B) shifts to the right. The new equilibrium has higher values for Z and Y , as illustrated in Figure 2. Given that $g = \frac{Y}{\theta}$, this implies a higher value for the growth rate ($dr = 0$). ■

■



3 Final Remarks

We have provided the QHIT with a first theoretical framework, where we have integrated the questions concerning public investment relevance, the importance of complementarities between the different economic helices, the importance of the costly nature of investment, and of policies to achieve a new equilibrium in economic growth.

According with Carayannis and Campbell (2009), quadruple helix refers to structures and processes of the gloCal Knowledge Economy and Society – gloCal according to Carayannis and Von Zedwitz (2005) underscores the potential benefits of a mutual and parallel interconnection between different productive levels. Innovation ecosystems reveal the importance of pluralism and diversity of agents, and organisations arranged along the matrix of innovation networks and knowledge clusters, and this all may result in a democracy of knowledge, driven by pluralism of innovation and knowledge. Yawson (2009), for example, considers that advances in biotechnology, ICT and nanotechnology have stimulated innovation and convergence, but at the same time, have brought to light the importance of adequate regulations, and have introduced a need for society awareness about their risks and benefits, which has called for the Civil Society as an essential fourth helix of the national ecosystem of innovation. In fact, Knowledge creation is now transdisciplinary, non-linear and hybrid, hence inclusion of the fourth helix becomes critical since scientific knowledge is increasingly evaluated by its social robustness, equitableness and inclusiveness.

Our developed model proposed to capture this complex dynamics by assuming a one-high-technology-sector structure.

There has been a plethora of open innovation policies and emerging public-people-partnerships, sometimes referred to as quadruple-helix models that are currently being adopted in an attempt to implement the Lisbon Strategy. According to Chesbrough (2003), innovation no longer flows top-down and from the core of the company outward, but rather bottom-up and from the outside towards that core. In this sense, we can also cite Von Hippel (2005), who defends the democratisation of innovation, in which the role of users is primordial when it comes to creating new products and concepts. With the introduced model, we have confirmed analytically that increased coordination and complementarities between different actors are necessary and do increase the growth rate.

According to Yawson (2009), the ecosystem of innovation answers the question of how economic policies should be implemented. The ecosystems of innovation start with a national innovation goal, that is interpreted through the four helices' perspectives, Academia, Government, Firms and Civil Society, and recognizes that innovation by creative citizens underpin the success of a country's innovation goal and strategy. With our proposed Quadruple Helix Innovation Model, we have found analytically that an increase in productive public expenditure does increase the economic growth rate of quadruple helix's economies.

Existing literature on innovation and innovation policies indicate a growing interest in knowledge economies and knowledge societies that can enhance citizens' social/cultural/economic development and well being. Future research will be dedicated to testing our model based on empirical findings.

Appendix

In order to analyse the shape of the Technology curve (20), and as it is impossible to isolate r on one side of the equation, we rewrite it as $F(r, g) = 0$ and apply the implicit function theorem, so as to obtain, in the neighbourhood of an interior point of the function, the derivative $\frac{dr}{dg}$:

$$F(r, g) = \xi g - (1 + \xi) r + (1 + \xi) \Omega_Y r^{\frac{-\alpha}{1-\beta-\alpha}} (1 + \theta g)^{\frac{-\alpha}{1-\beta-\alpha}} = 0,$$

which leads to:

$$\frac{dr}{dg} = -\frac{\frac{dF(r,g)}{dg}}{\frac{dF(r,g)}{dr}} = \frac{\xi - \left(\frac{\alpha}{1-\beta-\alpha}\right) \theta (1 + \xi) \Omega_Y r^{\frac{-\alpha}{1-\beta-\alpha}} (1 + \theta g)^{\frac{\beta-1}{1-\beta-\alpha}}}{(1 + \xi) + \left(\frac{\alpha}{1-\beta-\alpha}\right) (1 + \xi) \Omega_Y r^{\frac{\beta-1}{1-\alpha}} (1 + \theta g)^{\frac{-\alpha}{1-\beta-\alpha}}}.$$

Hence, our nonlinear Technology curve is positively sloped when:

$$r^{\frac{-\alpha}{1-\beta-\alpha}} (1 + \theta g)^{\frac{\beta-1}{1-\beta-\alpha}} < \frac{\xi}{\left(\frac{\alpha}{1-\beta-\alpha}\right) \theta (1 + \xi) \Omega_Y},$$

and negatively sloped otherwise.

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