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**TO NEGOTIATE OR TO GAME THEORIZE:**

**Negotiation vs. Game Theory Outcomes for Water Allocation Problems in the Kat Basin,  
South Africa**

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Abstract. Common property resource disputes are increasing, due to increase in competition and deterioration of quality. Cooperative arrangements have long been in the center of public interest regarding the mechanisms used by communities that share them. Two main approaches have been applied separately, namely negotiation approaches and cooperative game theory. Although the two approaches depart from different directions and are based on different assumptions, they complement each other as they are based on similar principles of fairness and efficiency and can both be seen as leading to cooperative outcomes. In this paper we apply the Role-Playing Game that is a component of the Companion Modeling approach--a negotiation procedure, and Cooperative Game Theory (Shapley value and the nucleolus) to a water allocation problem in the Kat watershed in South Africa. We use simplifying assumptions to allow a comparable solution. The negotiation and the cooperative game theory provide similar trends vis a vis the various players and their outcomes. Our conclusion is that Cooperative Game Theory and Negotiation approaches may be complementary to each other.

**JEL references:** C61, C71, C78, Q25, Q56, R14

**Key Words:** Negotiation; Role-playing game; Core; Nucleolus; Shapley value; Water allocation; Economic efficiency, Planning models

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## **TO NEGOTIATE OR TO GAME THEORIZE:**

### **Negotiation vs. Game Theory Outcomes for Water Allocation Problems in the Kat Basin, South Africa**

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#### ***1. Introduction***

Common property resources have long been in the center of public interest regarding the mechanisms used by communities that share them (e.g., Ostrom et al., 1994 and many citations used there). The framework suggested in Ostrom et al. (1994) is one of interaction among the users of the resource, assuming a common interest and thus, an arrangement that will be acceptable to all. Two main approaches can be identified, that help in analytically understanding the outcome of an allocation problem of such character—negotiation approaches (NA) and cooperative game theory (CGT). Although the two approaches depart from different directions and are based on different assumptions, they complement each other based on the principle that was suggested by Carraro et al (2005b:39) “The perception of fairness plays a crucial role in determining how a surplus is divided, and the potential allocation rules must be perceived as ‘equitable’ and ‘envy-free’ by all parties”.

NA focus on the resolution of conflicts originating from stakeholders’ different perceptions and uses of natural resources. Recent literature on environmental management and catchment management in particular, place strong emphasis on achieving negotiated settlements to such conflicts (Becu et al., 2003). In the water sector, there have been many applications of both NA and CGT at various levels, starting from sectoral through international (Carraro et al., 2005a; Parrachino et al., 2006). So far, the literature has applied separately NA and CGT to solving allocation issues and has not attempted at comparing the NA and CGT solutions. Other authors argue that participation in knowledge-sharing for water is a fundamental requisite because of the cumulative effects of individual actions on the patterns of water use at the irrigation system and river basin scale (Lankford and Watson, 2006). Although it is expected that under certain conditions of interaction among the parties, NA and the CGT will lead to similar allocation solutions, it is still not clear if empirically this is the case.

In this paper we apply a negotiation procedure and a CGT approach to a water allocation problem in the Kat watershed in South Africa. We use simplifying assumptions to allow a comparable solution. We apply a Role-Playing Game that is a component of the Companion Modeling approach (Barreteau et al. 2004), which has already demonstrated its capacity for promoting discussion among stakeholders with contrasted and eventually conflicting viewpoints (Dray et al. 2006). We formulate the same allocation problem also as a CGT, evaluating allocations (Shapley value and the nucleolus), which in our case are contained in the core, thus showing a relevant stability property. We compare the allocation solutions and explain sources for differences. The next section provides a short description of the geographical, historical, political and institutional aspects of the Kat watershed. Then, the water allocation issues in the watershed are discussed. The fourth section formulates the water allocation problem as a negotiation game and the fifth section presents the negotiated solution to the allocation problem, Section 6 compares and explains differences, and section 7 provides possible extensions. The paper is then concluded.

## ***2. The Kat watershed in South Africa<sup>5</sup>***

The Kat River valley (Figure 1), a tributary of the Great Fish River, is situated in the Eastern Cape province of South Africa. Although the watershed has a relatively high rainfall, much of the climate of the 1,700 km<sup>2</sup> catchment is sub-humid to semi-arid. The fertile valley land can be utilized only through irrigation, using water from the Kat River. Prior to 1969, irrigators relied on the natural flow in the river, but since 1969 water from the Kat Dam (a 24 million cubic meters (Mm<sup>3</sup>) storage capacity) became available. While irrigation takes up by far the majority of water in the catchment, domestic water users (about 49,500 inhabitants in 2001) represent an important component in the demand for water in the catchment.

Four groups of irrigators can be identified in the Kat Valley: small scale black farmers, often forming cooperatives, large scale ‘emerging’ black farmers<sup>6</sup>, white commercial farmers with scheduled water rights, white commercial farmers without scheduled water rights. The main water related stakeholders in the Kat Valley are therefore (1) the four groups of irrigators, (2) domestic water users and (3) the Municipality of Nkonkombe. The South Africa Department of

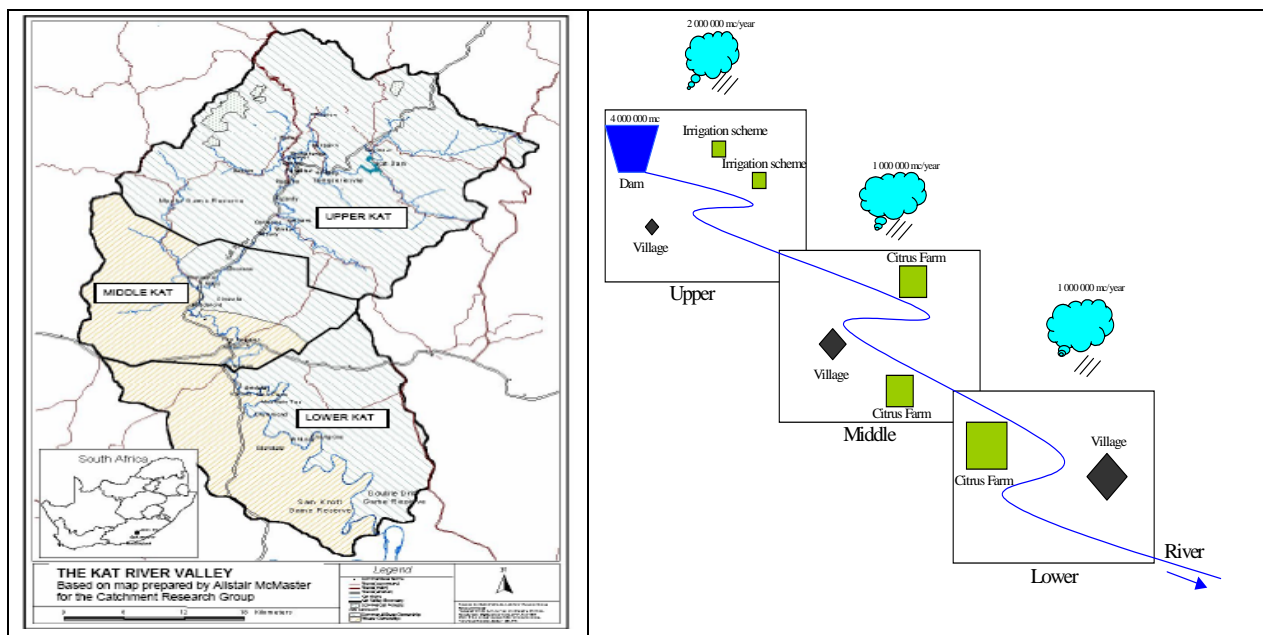
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<sup>5</sup> Modified from Farolfi and Rowntree (2005).

<sup>6</sup> “Emerging” citrus farmers are black farmers that after 1994 had the right to exploit (and not own) citrus farms located during Apartheid period in the Ciskei Bantustan and previously owned and managed by the public Bantustan’s administration.

Water Affairs (DWA), currently operating the Kat Dam, is considered the 4th important stakeholder in the system. The complex and contentious political history of the valley has given rise to a power dynamic that historically has favored the white commercial farmers producing citrus at the expense of the black population. These white farmers controlled water use through the Kat River Irrigation Board (IB).<sup>7</sup>

Figure 1: Map of the Kat Catchment and the stylized sub-catchments in the RPG (for the conversion factors used see Table 1)



### 3. Water allocation issues in the Kat watershed

Water sources in the Kat River valley are currently almost exclusively from surface water. Some groundwater developments are foreseen in the near future and this could increase water availability in the catchment by nearly 10 percent (DWA, 2001). As mentioned above, decisions about water allocation strategies are taken by the recently established Water User Association (WUA), which represents the various groups of stakeholders in the catchment.

<sup>7</sup> The IB was substituted lately by a more participatory Water User Association (WUA) including representatives of all main groups of stakeholders in the Kat. This organization is now in charge of defining the water “business plan” indicating water allocation strategies and resource management for the catchment. While the difference between the IB and WUA may not be essential for the CGT, it is very important for the negotiation process, as will be discussed later.

Irrigated agriculture is the largest water user in the catchment, accounting for 60% of the water requirements, including the ecological reserve<sup>8</sup> (12%). Domestic uses (13%), afforestation (10%) and losses due to alien vegetation (5%) are the other requirements in the catchment. The recent history of South Africa led to the co-existence of different groups of irrigators in the catchment. These farmers are located in specific portions of the catchment<sup>9</sup> (Figure 1), namely smallholders and emerging farmers in the Upper Sub catchment (U), emerging and large-scale farmers in the Middle Sub catchment (M) and large-scale farmers in the Lower Sub catchment (L) (Farolfi and Rowntree, 2005).

Domestic water consumption per capita is low by existing standards, mainly due to striking rural poverty and non-existence of services (Farolfi and Abrams, 2005). A crucial issue for the National Water Resource Strategy of South Africa is the protection of environmental and ecological needs, which is translated into the definition and the respect of the ecological reserve in each catchment.

The Kat River Dam is certainly the main tool for water supply management in the catchment. The dam is currently operated by DWAF based on a mechanism of water licenses and scheduled-non scheduled users. It is expected that soon the WUA will be responsible for the management of the dam. Therefore, in this paper we assume that it is the WUA and not DWAF that decides on Dam water allocation. Future demands were identified (Farolfi-Abrams, 2005) to include increase in citrus surface; increase in small-scale irrigation schemes; increase in domestic uses, particularly in rural areas; more tourism, game farms and, possibly, golf courses. These will be addressed in future analyses.

#### ***4. Formulating water allocation decision as a negotiation process***

To facilitate discussions about water allocation strategies within the Kat River WUA, stakeholders' representatives accepted to take part in a process of Companion Modeling (Barreteau et al., 2003) consisting of an iterative and participatory development of a simulation model—KAT AWARE (Farolfi and Bonté, 2005) that illustrates alternative scenarios of water allocation in the catchment (Farolfi and Rowntree, 2005).

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<sup>8</sup> Defined in the National Water Act as “the portion of the resource kept unavailable for allocation and dedicated to basic ecological purposes”.

<sup>9</sup> These portions of the catchment correspond to the three voting areas identified to nominate the Kat River Valley Water Users Association representatives (Figure 1).

An important component of Companion Modeling is the use of Role-Playing Games (RPG) to facilitate stakeholders' comprehension of the developing model and to allow modelers better understanding of stakeholders' strategies and behavior. For this purpose, a RPG was constructed based on the model KAT AWARE, which is being developed with the WUA following the Companion Modeling approach.

In the RPG, the Kat catchment (Figure 1) is divided into three sub-catchments corresponding to the three mentioned voting areas: Upper, Middle and Lower. Two smallholding irrigation schemes (20 ha each) are located in the Upper Sub-catchment (U), two citrus farmers (30 ha each) are located in the Middle Sub-catchment (M) and a citrus farmer (40 ha<sup>10</sup>) is in the Lower Sub-catchment (L). Farmers have irrigated land on which they produce Cabbages if they are smallholders or Citrus if they are large-scale farmers. Domestic water users live in three villages<sup>11</sup>: one in U (3 000 hab), one in M (5 000 hab) and one in L (15 000 hab). An average amount of rainfall equivalent to 2 million m<sup>3</sup>/year falls on U, whilst annual rainfall equivalent to 1 million m<sup>3</sup>/year falls on the M and on L. A dam with a storage capacity of 4 million m<sup>3</sup> is located in U. A Water Users Association (WUA) exists in the catchment and is responsible for water management and allocation according to the principles of Social Equity, Environmental Sustainability and Economic Efficiency as indicated in the Water Legislation of South Africa. All players are members of the WUA.

To simplify the game while reflecting its reality, data representing the actual catchment are transformed into values that are proportional to the real ones, and only 8 players are left to interact (table 1)<sup>12</sup>. The primary goal of the game is for the catchment as a whole to manage in a sustainable way the available water resource, taking into consideration the above-mentioned principles. At the same time, the goal for each player is to maximize their individual economic gain and if they are a village manager to maximize also villagers' satisfaction (see additional discussion in the CGT section), within the context of the group goal. The game is set to allow sessions of 7 simulated years. This choice is a compromise between playability and a time span

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<sup>10</sup> Ha=hectare. 1 ha=2.5 acre.

<sup>11</sup> The figure for the inhabitants in the villages and for the dam storage are those used in the RPG and will be used also for the CGT approach.

<sup>12</sup> The conversion factors for the different variables range between 2.1 (Inhabitants in catchment) and 13 (Surface citrus). The arbitrary choice of the conversion factors was due to the trade-off between having an RPG that represents the reality and an RPG that is "playable". It may be noticed that players validated the RPG representation of the Kat during the first session. The only criticism was for the disproportionately high importance of domestic water consumption in the RPG compared to the real one. This issue was addressed in a second version of the RPG by changing the conversion factor of "Inhabitants in catchment" from 2.1 to 6.

sufficient to provide elements of discussion on mid-term consequences of water allocation. The presented RPG session spans over 6 years.

Table 1: Actual and transformed values for use in the RPG of main variables in the Kat Basin

Variable	Actual values	Values in the RPG
Dam Capacity (Mm <sup>3</sup> )	24,000,000	4,000,000
Natural runoff (Mm <sup>3</sup> )	13,500,000	3,300,000
Domestic consumption (Mm <sup>3</sup> )	1,500,000	580,000
Irrigation consumption (Mm <sup>3</sup> )	11,000,000	1,064,000
Cabbage area (ha)	180	40
Citrus area (ha)	1,300	100
Inhabitants in catchment	49,000	23,000
Annual outflow	1,600,000	550,000

At the beginning of the game farmers receive a number of hectares corresponding to their farm (or irrigation scheme) and for each ha a symbol corresponding to their production (cabbage or citrus). Farmers also receive an amount of money corresponding to their previous year's profit and a number of workers indicating how many permanent or seasonal employees they hired the previous season<sup>13</sup>. Every year farmers may decide to increase or reduce their irrigated surface. They can also decide to change their production (cabbage to citrus or vice versa). If they decide to plant new citrus, they can choose an innovative irrigation technology (drip), which would cost more but will save water. Cabbage producers can decide to have 1 cycle, 2 cycles or 3 cycles of cabbage production per year on their fields. Budgets and water consumption data for Citrus and Cabbage are provided to farmers at the beginning of the game. Village managers receive, and pay for, bulk water from the WUA that manages the entire water in the catchment and provide water services (including water distribution) to the households of each village. They start with a given ratio of water sources for the households of their village. These water sources are: river water; collective tap; indwelling tap. Each water source has a different cost (investment + operating cost) that has to be added to the cost of the bulk water the managers “buy” from the WUA. Village managers can charge their habitants with a per capita tariff for the water services they provide, and this corresponds to their annual income. The households derive a certain level

<sup>13</sup> For production and profit functions used in the KarAWARE model that supports the RPG, see the characteristic functions used also for the CGT model.

of satisfaction (utility) from their income that can be spent on consumption goods. Because households have different levels of effort associated with the various water sources they are provided, they also obtain different levels of utility from each of the three sources of water.

The village manager objective is twofold. They want to maximize their profit resulting from the difference between the tariff collection and the water provision cost + bulk water cost and at the same time they want to maximize the sum of households' utility. Elements of budgets and utility values for households are provided to local village managers at the beginning of the game.

A number of factors varying annually, such as rainfall, market prices and population dynamics can influence players' strategies.

#### ***4.1 Negotiation results***

This section illustrates some outcomes of a RPG session held in the Kat catchment in November 2005. The set-up of the game and the players participating in the session are presented in the right-hand side rectangle of figure 1. The initial values characterizing each player are presented in the left column of table 2, which also includes final values (end of year 6 of the negotiation game) and averages for the economic values, providing an indication of the strategies and trends followed by players during the RPG session. In the tables that follow, year 1 is the initial state and was set by the game facilitators; years 2 to 6 were actually played.

Table 2 shows the initial and final values of the exogenous factors controlled by the game operators. Average values are also provided in the right column of table 1. The game facilitators introduced a general trend of increasing water scarcity. This stress was produced by a combination of lower rainfall and increasing population in the catchment. Some marginal changes (mainly reductions) affected crop prices. A relatively low level of uncertainty was introduced in the session, corresponding to a small difference between expected (forecasted) and actual exogenous factors to which stakeholders were confronted.

Table 2 – Exogenous factors in the RPG session: initial, final and average values

Variable	Initial	Final	Difference	Average
			%	
Rainfall Upper (m3)	2,000,000	1,400,000	-30	1,483,333
Rainfall Middle (m3)	1,000,000	600,000	-40	650,000
Rainfall Lower (m3)	1,000,000	600,000	-40	666,667



Population Upper (inhab.)	3.000	3.500	17	3.250
Population Middle (inhab.)	5.000	5.500	10	5.250
Population Lower (inhab.)	15.000	16.000	7	15.500
Market Price Citrus (R/ton)	2.000	2.000	0	1.950
Market Price Cabbage	6.00	5.00	-17	5.67

Note: R=Rand, the South African currency. 1 USD=6 Rand

Clear differences in behavior and strategies among players were observed for different sectors and in the three sub-catchments (table 3 and 4). In the U the two irrigation schemes opted first for an intensification of their cabbage productions (from 2 to 3 cycles per year). Only at the end of the RPG session the second irrigation scheme decided to reduce the cultivated surface by 50%. In the M, the two citrus farmers adopted two very different strategies, the one oriented first towards diversification (cabbage in addition to citrus) and then abandoning citrus, whilst the other one kept constant the citrus surface but also planted an equivalent surface at cabbage. In L, the large citrus farm adopted a quite “conservative” strategy consisting in reducing only by 25% the planted surface at citrus and not moving to cabbage. All new citrus plants in the three farms were equipped with innovative irrigation technologies, consisting in drip systems, more costly in terms of investment, but water saving.

Table 3: Strategies and outcomes for the 5 farms during the RPG session: initial and final values

	<b>Initial</b>	<b>Final</b>	<b>Differenc</b>	<b>Annual</b>
			%	
<b>Irrigation Scheme 1 (U)</b>				
Ha citrus old technology	0	0	0.0	
Ha citrus new technology	0	0	0.0	
Ha cabbage	20	20	0.0	
Cycles cabbage	2	3	50.0	
Total Ha	20	20	0.0	
Employment (n)	51	76.6	50.1	84
Cumulated Profit (R)	64,208	250,000	289.4	43,640*
<b>Irrigation Scheme 2 (U)</b>				
Ha citrus old technology	0	0	0.0	
Ha citrus new technology	0	0	0.0	
Ha cabbage	20	10	-50.0	
Cycles cabbage	2	2	0.0	
Total Ha	20	10	-50.0	
Employment (n)	51	25	-51.0	53.2
Cumulated Profit (R)	64,208	250,000	289.4	42,406

Citrus Farm 1 (M)				
Ha citrus old technology	30	0	100.0	
Ha citrus new technology	0	5		
Ha cabbage	0	30		
Cycles cabbage	0	1		
Total Ha	30	35	16.7	
Employment (n)	46	46	0.0	55.4
Cumulated Profit (R)	829,300	3,290,00	296.7	554,299
Citrus Farm 2 (M)				
Ha citrus old technology	30	0	100.0	
Ha citrus new technology	0	30		
Ha cabbage	0	30		
Cycles cabbage	0	1		
Total Ha	30	60	100.0	
Employment (n)	46	84	82.6	73.8
Cumulated Profit (R)	829,300	740,000	-10.8	126,052
Citrus Farm 3 (L)				
Ha citrus old technology	40	0	100.0	
Ha citrus new technology	0	30		
Ha cabbage	0	0	0.0	
Cycles cabbage	0	0	0.0	
Total Ha	40	30	-25.0	
Employment (n)	62	44	-29.0	71.2
Cumulated Profit (R)	1,105,700	2,710,00	145.1	455,562

\* Annual profit is the net financial result of a specific year. It is not cumulated.

Table 4 shows the dynamics in the village managers' decisions regarding water services and tariffs for their households. As a general trend, better water provision was introduced in all villages, and this was accompanied by an increase in water tariffs required from the households. In some cases the increase in domestic water tariffs was perceived too high by local residents (village L), affecting negatively their utility. On the other hand, this water tariff increase in village L triggered a huge improvement in the village manager's profit.

Table 4: Strategies and outcomes for the three villages during the RPG session: initial and final values

	Initial	Final	Difference %	Annual average
<b>Village 1 (U)</b>				
Population (Inhab.)	3,000	3,500	16.7	
Share of river source	0.8	0.0	-80.0	

Share of collective tap	0.2	0.2	0.0	
Share of indwelling tap	0.0	0.8	80.0	
Water tariff (R/m3)	1	2	100.0	1.7
Satisfaction index	40.6	41.7	2.8	41.3
Manager's cum. Profit (R)	20,500	420,000	1,948.8	70,900*
<b>Village 2 (M)</b>				
Population (Inhab.)	5,000	5,500	10.0	
Share of river source	0.8	0	-80.0	
Share of collective tap	0.2	0.2	0.0	
Share of indwelling tap	0	0.8	80.0	
Water tariff (R/m3)	1	1.7	70.0	1.5
Satisfaction index	40.6	42.89	5.7	42.2
Manager's cum. Profit (R)	34,180	300,000	777.7	54,600
<b>Village 3 (L)</b>				
Population (Inhab.)	15,000	16,000	6.7	
Share of river source	0.1	0	-10.0	
Share of collective tap	0.4	0	-40.0	
Share of indwelling tap	0.5	1	50.0	
Water tariff (R/m3)	1.5	2	33.3	1.8
Satisfaction index	42.7	41.9	-1.8	42.4
Manager's cum. Profit (R)	128,130	2,110,000	1,546.8	351,400

\* Annual profit is the net financial result of a specific year. It is not cumulated.

It was clear that the WUA gave priority to the domestic uses of water, not hampering any initiative of water provision enhancement by the local managers. The respect of an ecological reserve set at 500,000 m<sup>3</sup>/year in years of drought and 750,000 in normal years was another WUA priority. Agricultural uses were more controlled and the release of new water licenses to farmers was less automatic, especially when the dam reserve became scarce (last three years of the RPG session).

The water allocation policy by the WUA allowed positive results in terms of economic outputs for four farms out of five (cumulated profit). Nevertheless, the average annual profit referred to the played period shows a lower performance with respect to the initial state for all farms, particularly those cultivated at citrus. Cabbage was more profitable due to a relatively steady trend in market price (excluding the final two years) and, more important, because no investment is required for new plantations. Farm 4 in the M subcatchment registered the worst performance paying the cost of heavy investment in new hectares planted at citrus combined with lower market prices in years 3 and 4. In addition, the session was too short to allow the

farmer recovering the investment through new citrus plants production (in the RPG, citrus takes 2 years after plantation to become productive).

Job creation was generally positive for all farms (average higher than initial value). The water shortage provoked by the WUA decision to stop releasing water the last year had very negative impacts on job creation, particularly in the M and the L sub-catchments, where citrus is cultivated.

Table 5 shows also the dynamics of water consumption in the three sub catchments. At year 1 L is the most water consuming (large village and large citrus farm) followed by M and U. The latter increases consistently water consumption during years 2 to 4 due mainly to the intensification of cabbage production. The slight increase in water consumption in the remaining sub catchments is due to domestic better provision and demographic positive trends. At year 5, water consumption in U contracted due to a change of strategy in one of the two irrigation schemes. In year 6 the WUA decided to stop releasing water from the dam in order to allow refilling.

The increasing water demand in the three sub-catchments is partially compensated by water releases from the Dam decided by the WUA (table 5). During the first 4 years of the game the WUA opted for a use of the Dam water to satisfy users' water demand and to provide a water flow in the river able to preserve ecological equilibriums (the ecological reserve).

At the end of year 5, when the Dam level reached 1.3 million cubic meters, the WUA decided to stop suddenly and completely water flushes. This decision determined an improvement of the dam water quantity, but had an immediate and dramatic consequence on the socio-economic and environmental indicators in the catchment.

Table 5: Profit (R), job creation (n. of employees), water consumption (m<sup>3</sup>) in the three sub catchments; Dam level and Ecological reserve (m<sup>3</sup>) for the whole basin

<i>YEAR</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>AVERAGE</i>
<b>Profit U</b>	148,924	191,235	258,121	328,450	142,437	-127,311	<b>156,976</b>
<b>Profit M</b>	1,692,874	1,089,725	332,920	1,047,169	1,196,990	-949,883	<b>734,966</b>
<b>Profit L</b>	1,233,926	688,012	337,207	1,330,596	1,585,825	-333,844	<b>806,954</b>
<b>Profit catchment</b>	3,075,724	1,968,972	928,248	2,706,215	2,925,252	-	<b>1,698,895</b>

<b>Agric. profit U</b>	128,416	168,774	184,898	228,657	40,474	-234,938	<b>86,047</b>
<b>Agric. profit M</b>	1,658,694	1,081,258	256,006	991,829	1,132,383	-	<b>680,351</b>
<b>Agric. profit L</b>	1,105,796	525,873	143,245	833,148	1,031,818	-906,511	<b>455,562</b>
<b>Employment U</b>	102.1	140.39	178.67	191.43	108.48	102.09	<b>137.19</b>
<b>Employment M</b>	92.4	119.26	132.72	141.68	156.44	129.82	<b>128.72</b>
<b>Employment L</b>	61.59	75.71	76.99	84.05	84.69	44.27	<b>71.22</b>
<b>Annual water cons. U</b>	376,893	557,852	754,369	886,709	568,552	280,028	<b>570,734</b>
<b>Annual water cons. M</b>	464,526	612,535	657,732	800,720	843,208	564,019	<b>657,123</b>
<b>Annual water cons. L</b>	784,335	994,810	998,942	1,148,847	1,193,328	658,260	<b>963,087</b>
<b>Dam level</b>	4,000,000	3,674,800	2,774,144	1,814,240	1,368,704	2,361,648	<b>2,665,589</b>
<b>Ecological reserve</b>	1,700,000	1,500,000	750,000	850,000	950,000	350,000	<b>1,016,667</b>

Table 5 allows formulating some considerations on the general socio-economic and environmental trends by sub-catchment. Job creation is linked to the surfaces cropped and to the intensity of production (cycles of cabbages on the same surface); it therefore follows closely the dynamics of water consumption. As a general trend (average on the played period), job creation is positive in all catchments, particularly in sub-catchments U and in M. Profit is more sensitive to water availability and during the first years of game is (negatively) influenced by high investments in the citrus farms.<sup>14</sup> This is also why the performances of M and L are worst than the one of U (average annual profit). The annual figures show clearly the dramatic impact of the WUA decision at year 5 on profit generation for the three sub catchments. Again, M and L where citrus farms are located suffer particularly for the water shortage.

Finally, it is worthwhile noticing that the decision to stop completely water flushes from the dam had a negative impact also on the ecological reserve, well below the limit of 500,000 cubic meters at year 6. As an average, the ecological reserve was kept at about 1 million m<sup>3</sup>/year, corresponding to 40% less than its level at the beginning of the negotiation game session, but 33% more than the limit set for wet years and 100% more than the limit set for dry years.

<sup>14</sup> In this game, profit=total income-total costs. If a farmer invests in citrus plantations, therefore, his annual income during the first years of new orchards is constant (no production) whilst the costs increase. It was noticed by citrus farmers during the game debriefing that this is not really how they see things because an investment is calculated as a positive asset in their budget, whereas here is a negative (cost) one. They suggested calling "cash-flow" what we call "profit" in the game outcomes.

### ***5. Formulating water allocation decision as a cooperative game***

The Cooperative Game Theory (CGT) model will introduce several assumptions. We assume that the players are rational, price takers and profit maximizers. They will engage in cooperative arrangements only if it can improve their situation compared with the status quo.

The watershed includes three players (each with several water activities), Upper sub-watershed, Middle sub-watershed, and Lower sub-watershed—U, M, L, as is described in the left rectangle of Figure 1 (we will use  $i=1$  for U,  $i=2$  for M, and  $i=3$  for L). There is a water storage (dam) in the U sub-watershed, and an outlet of the river beyond the L sub-watershed (we will refer from hereafter only to watershed). There is a deterministic (at this point) rainfall quantity that falls on the area of each watershed and ends up in the river. There are no losses of water and all the rainfall can be used as a source for the water-activities (this assumption can also be modified by having a fraction of the rainfall available for use, assuming losses and evaporation). So, each player refers to the amount of rainfall on its area as water in the river available for use. There are also ground water sources, but for simplicity, they are not included. Players can also use water from the reservoir, that is released (flushed) by the WUA upon a request from the player (if supplies last). The WUA can refuse supplying water from the reservoir if the amount in the reservoir is below a given level.

The WUA is actually the authority that oversees and regulates the players' behavior. From the point of view of CGT, the WUA could decide on an allocation of water that respects principles on which it has been agreed upon (e.g., Social Equity, Environmental Sustainability and Economic Efficiency). It is assumed that the players obey the WUA rules of behavior. The objective of each player is to maximize annual profits subject to water availability, prices and costs, and the WUA rules of behavior.

Rules of behavior that are respected by the players include: (1) No player extracts water that runs in the river that doesn't belong to that player. Such water include the water that the WUA dedicates to run in the river to keep water flow in the river in the segments belonging to U, M, and L, (2) water that has to be released to the Fish River below the L watershed, (3) water that was released from the reservoir for use by a given player and has to run in the river through the 'territory' of another player.

Constraints for each player include (1) fixed Citrus land, (2) limited land for cabbage, (3) fixed number of inhabitants consuming water, (4) given amount of rainfall that can be utilized via the flow of the river.

Constraints or rules for the WUA (or at the Kat watershed) include: (1) a given amount of water in the storage at the beginning of the year, (2) a given amount of water required in the storage at the end of the year, (3) a given amount to be released from the Kat to the Fish river, (4) a given amount to be left in the river for local benefits, (5) a minimum annual amount of water per inhabitant to fulfill human needs. (6) a given amount of available laborers to work in Citrus or cabbage operations. It is assumed that these laborers can move freely between the three watersheds. Additional assumptions represent the linkage between the water use and water flow in the river. These linkages are expressed in the equations of the model in the Annex.

The paper investigates the likelihood of cooperation among the three players. CGT introduces the concept of the characteristic function for each set of coalitional arrangements among the players in the watershed. A characteristic function is the best outcome of a coalition. Further assumptions are needed. In our CGT analysis we shall consider a one year game period. The essential aspect of this choice is that we shall not consider investment decisions<sup>15</sup>. This is a clear limitation, but we consider that in the NA players are not in the condition of making optimal investment decisions (especially due to the short time span allowed). Hence, the one year timespan that we use in the CGT approach should offer a more appropriate benchmark as far as comparison with the NA is sought for.

Let us start with the status quo, individual coalitions. The status quo is represented by each player (watershed) working on their own to maximize their utility/profits from the available water they have, subject to individual constraints of each player and rule constraints imposed by the WUA. There are no special treatment of the water activities and the land constraints. However, in the maximization problem for each player in the case of the status quo, we need to add several assumptions to deal with the watershed constrains: Labor, minimum water flow in the river, and the environmental flow to be released for the Fish River. We will assume a total amount of available labor in the watershed and a total amount of environmental flow to be released to the Fish River. The total available labor to each player will be based on the relative

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<sup>15</sup> We should stress that the formulation of our CGT problem allows considering longer time horizons by creating for each player and coalitions optimization problems that span over T years but what the player considers is the expected annualized value.

total available land of that player (indicating the potential employment ability of that player). The allocation of the environmental flow amount will be made based on the basis of the total amount of water used by that player. The minimum flow in the Kat river will be simply deducted from the available amount of water for each player.

In the next step we move to the calculation of the characteristic functions of the partial coalitions. We assume that all permutations are possible, even that between player 1 (U) and player 3 (L). However, for these players we shall simply assume that what they can achieve in cooperating with each other is just the sum of what they can get separately. The difference between the calculation of the value of the other partial coalitions and the individual coalitions will be that the decision on water allocation and the total amounts of the minimum flow and the environmental flow can be made jointly rather than individually. Additionally, the allocation of the rule values for labor, minimum flow and environmental flow will follow the pattern suggested in the case of the individual coalition calculations.

And finally we move to calculate the characteristic function of the grand coalition. In this case the labor constraint is at the watershed level, the minimum flow constraint is also at the watershed level, and the environmental flow constraint can be met at the outlet of the Kat when it leaves the area of Player 3 (L).

Let us now introduce the variables.

$F_{i\alpha}$  is the (natural) available flow (with probability greater or equal than  $\alpha$ ) in watershed  $i$ .

$S_i$  is the stream entering part  $i$  ( $S_U = 0$ ).

$E_i$  is ecological reserve constraint for part  $i$  (flow leaving part  $i$ ). Since the ecological reserve level that is of interest is the quantity that leaves the watershed, each watershed  $i$  is expected to release to the next one, and L is expected to release to the environment that same quantity, it will be denoted by E. This is another simplifying assumption, as the level of E in each watershed  $i$  could quite be a decision variable.

$C_i$ : water for civil use in part  $i$ .

$W(D)$  is the additional water available from the dam.



A player, or a coalition, will use its available water, to maximize its revenue. Actually, we assume that each player solves an optimization problem to maximize the use of the available water via allocation among all possible water uses. The plan that maximizes the returns, is called the characteristic function of the coalition/player. A very general exposition of the optimization problem (generalized to each coalition) is as follows:<sup>16</sup>

$$\begin{array}{l} \text{Max} \\ \text{water from dam, water fee,} \\ \text{water from river} \end{array} \left\{ \begin{array}{l} (\text{land under citrus}); (\text{land, water intensity,} \\ \text{cycles of cabbage}); (\text{water consumption in vilages by source}); \\ (\text{environmental release}) \end{array} \right\}$$

*Subject to :*

- (1) *environmental reserve outflow*
- (2) *rain availability*
- (3) *water in river*
- (4) *land*
- (5) *labor force*
- (6) *other institutional and technical constrains*

Such an optimization problem is solved to yield solutions to the following coalitions (individuals, partial, and grand coalition) {U}, {M}, {L}, {U, M}, {M, L}, and {U, M, L}. The solution for each coalition, the characteristic functions are denoted by:  $v(\{U\})$ ,  $v(\{M\})$ ,  $v(\{L\})$ ,  $v(\{U, M\})$ ,  $v(\{M, L\})$ , and  $v(\{U, M, L\})$ . Note that according to our simplifying assumption  $v(\{U, L\})$  will be replaced by  $v(\{U\})+v(\{L\})$ .

Using various game theory solution concepts, allocations of payoffs are made among the three players. We use for demonstration the Shapley Value solution concept (Shapley, 1971):

$$\phi_i = \frac{1}{n} \sum_{s=1}^n \frac{1}{c(s)} \sum_{\substack{i \in S \\ |S|=s}} [v(S) - v(S \setminus \{i\})], \quad i = U, M, L$$

where  $c(s)$  is the number of coalitions of size  $s$  containing player  $i$  :

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<sup>16</sup> A multi-year problem will be developed in our future work.

$$c(s) = \binom{n-1}{s-1} \equiv \frac{(n-1)!}{(n-s)!(s-1)!}$$

Another CGT solution concept that is used in the paper is the nucleolus (we don't provide the definition of the nucleolus: the interested reader can find it in standard references, as Owen, 1995). Remember that this allocation always lies in the core, provided that it is non empty. In our results, the values of the nucleolus differ very little from those of the Shapley value.

### 5.1 CGT results

Based on simplified calculations (Annex), the following are the characteristic values of the Kat CGT (in Rands):

$$v(\{U\}) = 336,060$$

$$v(\{M\}) = 1,758,946$$

$$v(\{L\}) = 1,185,693$$

$$v(\{U, M\}) = 2,341,140$$

$$v(\{U, L\}) = 1,521,753$$

$$v(\{M, L\}) = 2,944,639$$

$$v(\{U, M, L\}) = 3,552,913$$

The resulting Shapley allocation is:

$$\phi_U = 467,820.33$$

$$\phi_M = 1,890,706.33$$

$$\phi_L = 1,194,386.33$$

with

$$\phi_U + \phi_M + \phi_L = v(\{U, M, L\}), \text{ since the Shapley value provides an efficient allocation,}$$

$$\phi_i \geq v(\{i\}), \quad i = U, M, L, \text{ which suggest individual rationality, and}$$

$$\phi_U + \phi_M \geq v(\{U, M\}), \quad \phi_U + \phi_L \geq v(\{U, L\}), \quad \text{and} \quad \phi_M + \phi_L \geq v(\{M, L\}) \text{ which suggests group rationality (otherwise stated, the Shapley value lies in the core).}$$

The payoff is distributed among the three players such that U, M, and L get 13, 53 and 34 percent of the total cooperative profits, respectively. U is clearly the main beneficiary from the CGT allocation, increasing its share in the cooperative payoff by 39 percent compared with the

non cooperation payoff, while both M and L gained 7 and 1 percent, respectively. The CGT assumes utility transfer in the form of payments (or compensations).

For the nucleolus, the allocation is: 465,647 for *U*, 1,888,533 for *M*, 1,198,733 for *L*. Since the core is non empty (the Shapley value, as seen, lies in it), the nucleolus is also in the core. Clearly, the fact of having a non-empty core, and of having found a couple of (close) allocations belonging to the core can be seen as “good news”, proving that there exist incentives for the players to cooperate. Since the allocation provided by the nucleolus are so close to those for the Shapley value (the differences are by far smaller than variations due to the approximations used or to the assumptions done), the same comments as for the Shapley value apply. Let’s stress that the nucleolus and the Shapley value incorporate different views about fairness. So, the fact that they are quite close each other means that the answers provided by these two views practically coincide, in this case, providing strong reasons in favor of such allocation(s).

#### ***6. Comparison of negotiation and cooperative game theory allocations***

Any comparison between the NA/RPG and CGT outcomes has to be subject to several caveats. This is especially true at this early stage of our work. First, the RPG has a dynamic nature that we cannot capture with the CGT model as simple as it is now. Second, the main differences in the calculations of the profits of the players lead to possible discrepancies between the individual profits. While the calculation of profits to the *U*, *M*, and *L* players in the case of the CGT are a result of an optimization process that takes into account very strict set of variables, the RPG process incorporates ‘real’ players that take into consideration many more factors than the algorithm used in the CGT. Just these two caveats may explain possible differences in catchment outcomes. As for the behavior of individual players, observations from the RPG session show that both irrigators and village managers aim at improving their respective indicators of performance (profit for irrigators; profit+residents’ satisfaction for village managers) without necessarily maximizing them. This might be due to a lack of information on the possible alternative strategies they could adopt during the RPG session and refers to a behavior called “satisficing”, where satisficing is an alternative to optimization for cases where there are multiple and competitive objectives in which one gives up the idea of obtaining a "best" solution (Simon, 1992). Players therefore adopt year after year strategies of incremental

improvement of their indicators. These strategies take into account external factors and must be discussed within (and cleared by) the WUA before they can be put in practice. In addition, willingness to reach an improved state does not correspond necessarily to an improved state of the players all along the RPG session: lack of play skills or external factors' dynamics worst than forecasted can be the causes of performances less positive than expected.

As can be seen from a first comparison, the catchment profit (outcome), based on the RPG outcomes is 1.699 million Rand on average for the 6 years (varies between a low of 0.928 in year 3 to a high of 3.075 million Rands in year 1. We should mention that year 6 was a loss due to a decision by the WUA not to release water from the dam). The calculated profit was 3.552 million Rand using the first year's water availability values in the CGT application.<sup>17</sup>

While the total payoff at the catchment level may be different in the RPG and CGT procedures, due to use of different assumptions, we would have a more useful insight into the distribution of the payoff among the players. In our case, the three sub-catchments U, M, and L shares in the catchment total profit was 9, 44 and 47 percent respectively in the RPG average year solution (Or 5, 55, 40 percent in the first year), and 13, 53 and 34 percent respectively in the CGT solution.

## ***7. Conclusions and extensions***

This paper developed a framework for comparison of CGT and RPG outcomes to a problem of water allocation among competing uses. Such a framework is useful for several reasons. First it allows the analysts to assess the nature of the assumptions made during the calculations or negotiation session. Second it creates feedback loops between the CGT and the RPG to consider in further development of the tools. And third, it may suggest complementary roles for each approach under different conditions that the parties in the allocation problem face.

Acknowledging the overly simplified optimizations procedure in the case of the CGT, it is suggested that the baseline scenario in CGT will be modified in order to address new constraints and scenarios that have been considered in the RPG session. This will include the dynamic nature of the allocation problem, and various structures to consider the environmental flow needs. Future sessions of the RPG will take CGT results into consideration, such as allowing for

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<sup>17</sup> Consider the 1<sup>st</sup> year of RPG (Table 4), results are very close to CGT. The main difference is that during the game, investments for citrus are made in the RPG leading to no production for the young trees and changes in market prices reduce the profit, whereas these aspects have not been introduced in the CGT.)

negotiation among sub-catchments, and integrating the WUA to be part of the sub-catchments to eliminate independent decisions such as the one to stop release of water from the dam.

Additional extensions to be considered in future developments include:

1. Developing a multi-year optimization model for each possible coalition in the catchment;
2. Inclusion of additional CGT solution concepts and a stability index;
3. Having the environmental flows level an endogenous part the optimization planning model;
4. Having the village water supply and the urban residents utility a decision variable rather than a constraint; and
5. Incorporating the stochastic nature of the rainfall and the water in the dam at the beginning of the period.

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## ***Annex: The formulation of the characteristic function***

### *The physical model*

Denote by  $D^0$  the water stock behind the dam at the beginning of the planning period.  $R_i$  ( $i = 1,2,3$ ) is the amount of rainfall on the area of player  $i$ . We interpret that variable also to be the amount of water available for use by player  $i$ , and also the amount of water in the river that player  $i$  can use.  $D_i$  is the request for flashes of water from the dam operator by player  $i$ .  $R_1^D$  is the amount of water stored behind the dam during the planning period. As noticed, only rainfall in the part of the area of player 1 can be dammed.  $D$  will be the amount of water behind the dam that can be used during the planning period.

Several relationship hold so far:

$$[1] \quad R_1^D \leq R_1$$

$$[2] \quad D = D^0 + R_1^D \geq \sum_i D_i$$

We further denote by  $W_i$  the total demand for water, during the planning period, by player  $i$  and by  $E$  the demand (allocation) for environmental needs where the watershed ends and the Kat River becomes a tributary of the Fish River. This occurs after player  $UL$ 's area.

Here we add several more relationships:

$$[3] \quad W_i \leq S_i,$$

Where,  $S_i$  is the supply of water available to player  $i$  as follows:

$$[4] \quad S_1 = D_1 + R_1 - R_1^D$$

$$[5] \quad S_2 = D_2 + R_2 + (S_1 - W_1)$$

$$[6] \quad S_3 = D_3 + R_3 + (S_2 - W_2), \text{ and finally}$$

We need now to include more specific relationship representing the total water demand by each player. Given the water use patterns by each player, there are agricultural uses (Citrus and Cabbage), and domestic uses (in the villages and urban centers). This amount is imposed on the watershed and is subject to a policy decision.



For our purposes, let  $V_i^v$ ,  $V_i^u$ ,  $C_i^C$ , and  $C_i^B$  be the amounts of water used by the village, the citrus, and the cabbage, respectively, for player  $i$ . The total demand for water by player  $i$  is:

$$[7] \quad W_i = V_i^v + V_i^u + C_i^C + C_i^B.$$

*Additional constraints:*

The WUA imposes minimum flow ( $MF_i$ ) to keep the river flowing in each subwatershed. This minimum flow amount is deducted from each player's amount of available water  $S_i$ .

$$[8] \quad S_i^{MF} = S_i - MF_i$$

In addition, the WUA imposes the environmental flow constraint. We will handle that constraint in the following way:

A total of  $EF$  has to leave the Kat to the Fish river. This amount has to fulfill the following relationship:

$$[9] \quad EF \leq \sum_i EF_i$$

*The optimization process in each sub watershed*

*The villages*

Villages. Assume that each village has a given population  $N_i^v$ , that the annual water consumption per person is  $v_i^v$ , the cost of providing each unit of water is  $P^{v_i}$ , and that the utility per inhabitant in the village from being provided a unit of water is  $u^{v_i}$ . We assume that the utility is linear in money. For simplicity assume that the ratio is 1:1. The total 'benefits' from providing water to the village is therefore:

$$U_i^v = V_i^v \cdot (u^{v_i} - P^{v_i}), \text{ where}$$

$$V_i^v = N_i^v \cdot v_i^v.$$

We introduce a constraint on the minimal amount per year that a village inhabitant should be receiving per year.

$$[10] \quad v_i^v \geq \underline{v}.$$

### *The citrus industry:*

Assume that citrus is grown with three factors of production, namely land, water and labor. Since we deal with a perennial crop and we assume no investment in new plantation, we will have the area of citrus in each sub-watershed fixed at  $L_i^C$ . Since in our model the area is *fixed*, the decision growers make is how much water per hectare to apply, including also no irrigation that end up with a minimum yield (Farolfi and Bonte, 2006).

The payoff for citrus production in watershed i is:

$$F_i^C = L_i^C \cdot \left[ Y_i^C(C_i^C) \cdot P^C - C_i^C \cdot P_i^{C_i^C} - B_i^C \cdot P^B \right], \text{ where}$$

$Y_i^C(C_i^C)$  is the citrus production function. It has a positive intercept at  $C_i^C = 0$ ;  $P^C$  is the price per unit of citrus produced, which is a function of the amount of water applied per hectare,  $C_i^C$ ;  $P_i^{C_i^C}$  is the cost of water charged to the citrus operation in subwatershed i. Note that we allow different water charges per crop and subwatershed;  $B_i^C$  is the labor per hectare of citrus; and  $P^B$  is the cost per unit of labor, assuming the same for the entire Kat.

### *The cabbage industry*

The cabbage industry production is very similar to that of the citrus except that, under our set of assumptions, the land for growing cabbage is not fixed (however, for realism we impose a constraint of 60 ha on the extension of land that can be used for cabbage; this constraint turns out to be binding only for player  $U$  and for the subcoalition  $\{U, M\}$ ). In the case of cabbage, the growers do not vary the amount of water per hectare, but decide only the area to be cultivated with cabbage with a given amount of water per ha. The payoff per hectare of cabbage is:

$$F_i^B = L_i^B \cdot \left[ C_i^B \cdot P^B - C_i^B \cdot P_i^{C_i^B} - B_i^B \cdot P^B \right], \text{ where}$$

$P^B$  is the cost per unit of cabbage produced (cost of labor has the same symbol;  $C_i^B$  is the amount of water applied per hectare of cabbage;  $P_i^{C_i^B}$  is the cost of water charged to the cabbage operation in subwatershed i;  $B_i^B$  is the labor per hectare of cabbage. Cabbage growers decide on the area they plant with cabbage  $L_i^B$ .

*The objective function of watershed i:*

Watershed i maximizes payoff from the three activities, subject to physical and institutional constraints:

$$[11] Y_i = \underset{V_i^v + C_i^C + C_i^B}{Max} \{U_i^u + U_i^v + F_i^C + F_i^B\}, i=1, 2, 3$$

*s.t.* the relevant constraints in [1]-[10]

*The characteristic functions*

The characteristic function of the individual coalitions are actually a solution to an LP problems that are based on the coalition at stake. In the case of the individual coalitions we solve [11] subject to relevant constraints in [1]-[10], and imposed rules of allocation of water from the Dam, Allocation of the minimum flow and allocation of the environmental flow among *players 1, 2, 3*.

Then we have the possibility of subcoalitions. Clearly a coalition of {1, 2} and a coalition of {2, 3} can be envisioned. As said, a coalition of {1, 3} is less obvious. We can include such coalition on the premise that the WUA enforces rules and water transfers that are respected by its members.